Appendix to "Overborrowing, Financial Crises and Macroprudential Policy"

by

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This Appendix discusses two alternative formulations of the financial regulator's problem. In the formulation examined in the paper, the regulator values collateral in the credit constraint using the asset pricing function of the unregulated decentralized competitive equilibrium. The problem of this regulator is time-consistent by construction and its allocations efficient, conditional on that pricing function. For this reason, we label this the "conditional efficient" (CE) financial regulator's problem. Its main limitation is that the regulator is prevented from using policy to manipulate the price of collateral in its efforts to address the effects of the pecuniary externality and improve efficiency. Hence, in the two formulations we discuss in this Appendix we relax this assumption by allowing the asset pricing function to be endogenous to the financial regulator's problem. As we noted in the paper and explore more here, the ability of the financial regulator to influence the asset pricing function creates a time inconsistency problem. Thus, in these two formulations we consider a regulator that solves a problem akin to a Ramsey problem, in which the time-inconsistency problem remains, and a regulator that solves an optimal, time-consistent problem. The comparison across the two helps us analyze how the ability to influence the asset pricing function affects policy depending on the ability to commit.

To simplify the analysis, we consider an environment with an exogenous dividend stream and a collateral constraint that depends on the economy's aggregate stock of land, instead of the individual holdings. That is, households in the unregulated decentralized equilibrium solve the following problem:

1 Decentralized Equilibrium

Households solve:

$$
\begin{aligned}
\max_{\{c_t, L_{t+1} b_{t+1}\}_{t\geq 0}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t) && (1)\\
s.t. \quad c_t + q_t L_{t+1} + \frac{b_{t+1}}{R} &= L_t(q_t + z_t) + b_t \\
\frac{b_{t+1}}{R} &\geq -\kappa q_t \bar{L}
\end{aligned}
$$

Land pays a linear return z_t, where z_t follows a Markov process.

The first-order conditions are:

$$
\begin{aligned}
u'(c_t) &= \beta R \mathbb{E}_t \left[u'(c_{t+1}) \right] + \mu_t \\
q_t u'(c_t) &= \beta \mathbb{E}_t \left[u'(c_{t+1}) \left(z_{t+1} + q_{t+1} \right) \right] \\
\mu_t \left(\frac{b_{t+1}}{R} + \kappa q_t \bar{L} \right) &= 0 \\
\mu_t &\geq 0
\end{aligned}
$$

A competitive equilibrium is a set of allocations and prices so that these optimality conditions are satisfied and there is market clearing in the market for assets $L_{t+1} = \bar{L}$.

2 Financial Regulator's Problem

We consider now a social planner who can choose directly the level of debt subject to two restrictions: a) resource constraint, b) competitive market clearing in land market. That is, the planner chooses borrowing decisions and transfers the balance from credit market operations $T_t = b_t - \frac{b_{t+1}}{R}$. In this way, the social planner internalizes the price effects of financing decisions over the borrowing ability of the economy.

In this environment, households solve the following problem:

$$\max_{\{c_t, L_{t+1}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2}$$

$$s.t. \quad c_t + q_t L_{t+1} \quad = \quad L_t(q_t + z_t) + T_t \tag{3}$$

First order conditions are:

$$q_t u'(c_t) = \beta \mathbb{E}_t \left[u'(c_{t+1}) \left(z_{t+1} + q_{t+1} \right) \right]$$

This condition will be the key implementability constraint for the financial regulator as it will link the planner's policies to the market price for land.

2.1 Financial Regulator under Commitment

We consider first a social planner akin to a Ramsey planner. This planner chooses at time 0 future policies and is assumed to be able to commit to such policies. As described above, the market for land remains competitive, i.e., households choose their land holdings according to (2). The planner solves the following problem:

$$\max_{\{c_t, q_t, b_{t+1}\}_{t\geq 0}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

$$\begin{aligned}
s.t. \quad c_t + \frac{b_{t+1}}{R} \quad &= \quad \bar{L}_t z_t + b_t \\
\frac{b_{t+1}}{R} \quad &\geq \quad -\kappa q_t \bar{L} \\
q_t u'(c_t) \quad &= \quad \beta \mathbb{E}_t \left[u'(c_{t+1}) \left(z_{t+1} + q_{t+1} \right) \right]
\end{aligned}$$

Notice the last equation is like an implementability constraint, stating that the planner must choose allocations and prices that are consistent with the optimality condition for the price of land in the competitive market.

The first-order conditions of this problem (using ξ_t for the multiplier on the asset pricing condition and μ_t for the multiplier on the collateral constraint) are:

$$q_t : \quad \xi_t = \xi_{t-1} + \frac{\mu_t \kappa}{u'(c_t)} \quad \forall t > 0 \tag{5}$$

$$b_{t+1} :: \lambda_t = \beta R_t E_t \lambda_{t+1} + \mu_t \tag{6}$$

$$c_t : u'(c_t) - \xi_t q_t u''(c_t) + u''(c_t)\xi_{t-1}(q_t + z_t) = \lambda_t \quad \forall t > 0 \tag{7}$$

According to 7, the planner internalizes how a change in consumption at time t helps relax the borrowing constraint at time t and makes it tighter at $t-1$. The time inconsistency problem is evident from the presence of the lagged multipliers in these optimality conditions. Hence, the planner would like to promise lower consumption in the future to relax the current collateral constraint. However, the planner has incentive to re-optimize in the future.

Notice that ξ_t is a non-decreasing sequence, and strictly increasing whenever the collateral constraint binds. In particular,

$$\xi_t = \frac{\mu_0 \kappa}{u'(c_0)} + \frac{\mu_1 \kappa}{u'(c_1)} + \frac{\mu_2 \kappa}{u'(c_2)} .... \geq 0$$

This means that the shadow value from relaxing the asset pricing condition equals the sum of all future shadow values of relaxing the credit constraint.

Rearranging the first order conditions, we obtain:

$$u'(c_t) - \xi_t q_t u''(c_t) + u''(c_t)\xi_{t-1}(q_t + z_t) = \beta \mathbb{E}_t R \left(u'(c_{t+1}) - \xi_{t+1} q_{t+1} u''(c_{t+1}) + u''(c_{t+1})\xi_t (q_{t+1} + z_{t+1}) \right) + \mu_t \tag{8}$$

2.1.1 Decentralization

A state contingent tax on debt can decentralize this financial regulator's problem. With a tax on debt, the first order condition with respect to bonds become:

$$u'(c_t) = \beta R(1 + \tau_t) \mathbb{E}_t u'(c_{t+1}) + \mu_t \tag{9}$$

By inspecting (8) and (9), we can derive the tax that implements the financial regulator's problem:

$$\tau_t = \frac{\xi_t q_t u''(c_t) + u''(c_t)\xi_{t-1}(q_t + z_t) + R\left(u'(c_{t+1}) - \xi_{t+1}q_{t+1}u''(c_{t+1}) + u''(c_{t+1})\xi_t(q_{t+1} + z_{t+1})\right) + \mu_t}{\beta \mathbb{E}_t u'(c_{t+1})} \tag{10}$$

2.2 Financial Regulator under Discretion

We characterize the optimal plans of a time-consistent financial regulator in the form of a Markov perfect equilibrium in which government cannot commit to future borrowing decisions. All decisions depend on the payoff relevant state variables (b, z). For a given $B' = \mathcal{B}(b, z)$ describing the optimal plans of future governments, the current government and household will take their current decisions. We solve for a fixed point. Given $B' = \mathcal{B}(b, z)$, we solve for current choices of the government. The choice for bonds need to coincide with $\mathcal{B}(b, z)$ in a Markov perfect equilibrium.

Notice that by choosing b', the government sets the current asset price given future government policy functions by affecting the stochastic discount factor that determines the current price. That is:

$$q_t = \frac{\beta \mathbb{E}_t \left[u'\left(b_{t+1} + \bar{L}_t z_{t+1} - \frac{\mathcal{B}(b_{t+1}, z_{t+1})}{R}\right)(z_{t+1} + q_{t+1})\right]}{u'\left(b_t + \bar{L}_t z_t - \frac{b_{t+1}}{R}\right)} \tag{11}$$

Given a policy function implemented by future governments $\mathcal{B}(\cdot)$ and the above pricing equation, the optimization problem of the current government is:

$$\begin{aligned} V(b,z) &= \max_{c,b'} u(c) + \beta \mathbb{E}_{z'|z} V(b', z') \qquad (12) \\ c + \frac{b'}{R} &= bR + \bar{L} z_t \\ \frac{b'}{R} &\geq -\kappa \bar{L} q \end{aligned}$$

$$u'(c)q = \beta \mathbb{E}_{z'|z} u'(b' + \bar{L}z' - \mathcal{B}(b', z'))(\mathcal{Q}(b', z') + z')$$

First order conditions are:

$$\frac{\lambda_t}{R} = \beta \mathbb{E} V_b(t+1) + \tfrac{\mu_t}{R} + \beta \xi_t E_t \left\{ u'(c_{t+1}) \mathcal{B}_b(\mathcal{Q}(b_{t+1}, z_{t+1}) + z_{t+1}) + \mathcal{Q}_b(b_{t+1}, z_{t+1}) u'(c_{t+1}) \right\} \tag{13}$$

$$u'(c_t) - \xi_t u''(c_t) q_t = \lambda_t \tag{14}$$

$$\kappa \mu_t = \xi_t u'(c) \tag{15}$$

Comparing (14) to the anlogous condition under commitment (7) one can see that the planner does not internalize how current consumption affect previous period asset prices and previous tightness of the constraint. It is clear from this equation that the planner values consumption more than in the decentralized equilibrium when the constraint binds, since higher consumption pushes up asset prices and relax the constraint. As we show in the numerical analysis, the differences in valuation ex post does not lead to different consumption allocations because the constraint is already binding and the increase in the asset price is not enough to support higher consumption. However, there are significant differences in the borrowing decisions ex ante.

Putting these two conditions together with the Envelope condition $V_b(b, z) = \lambda$ we arrive at an Euler Equation or Generalized Euler Equation (GEE)

$$\frac{u'(c_t) - \xi_t u''(c_t) q_t}{R} = \beta \mathbb{E} \left(u'(c_t) - \xi_t u''(c_t) q_t \right) + \frac{\mu_t}{R} + \beta \xi_t E_t \left\{ u' \left(\mathcal{C}(b_{t+1}, z_{t+1}) \right) \mathcal{B}_b(b_{t+1}, z_{t+1}) \right. \tag{16}$$

$$\left. \left(\mathcal{Q}(b_{t+1}, z_{t+1}) + z_{t+1} \right) + \mathcal{Q}_b(b_{t+1}, z_{t+1}) u' \left(\mathcal{C}(b_{t+1}, z_{t+1}) \right) \right\} \tag{17}$$

2.2.1 Decentralization

A state contingent tax on debt can decentralize this financial regulator's problem. By inspecting (9) and (17), we can derive the tax that implements the financial regulator's problem:

$$\tau_t = \frac{\beta R \mathbb{E}\left(u'(c_t) - \xi_t u''(c_t) q_t\right) + \mu_t + \xi_t u''(c_t) q_t + \beta R \xi_t E_t \left\{ u'(\mathcal{C}(b_{t+1}, z_{t+1})) \mathcal{B}_b(\mathcal{Q}(b_{t+1}, z_{t+1}) + z_{t+1}) + \right.}{\beta \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))}$$

$$\frac{\left. \mathcal{Q}_b(b_{t+1}, z_{t+1}) u'(\mathcal{C}(b_{t+1}, z_{t+1})) \right\}}{\beta \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))} \tag{18}$$

If the constraint does not currently bind, the optimal tax is strictly positive. Notice that $\mathcal{B}_b > 0, \mathcal{Q}_b > 0$ a (in our numerical analysis).

3 Numerical Results

Our results at this point suggest that the solution of the time-consistent planner is almost undistinguishable from the CE financial regulator defined in the text. The analysis under commitment is work in progress.