Bank Runs, Fragility, and Regulation

Manuel Amador   Javier Bianchi

Motivation

What is the optimal banking regulation when there is a risk of runs?

- Should regulators impose minimum capital requirements?

- Standard view: Moral hazard from bailouts or deposit insurance
    - Analysis focuses on a single intermediary
- Today:
    - A theory of banking regulation in general equilibrium
    - Banks **over-leverage**, even absent bailouts

In this Paper

- General equilibrium model of banks runs (Amador-Bianchi 2024)
  - Default is strategic (Cole-Kehoe)
  - Runs can happen despite liquid assets ⇒ require equilibrium profits
  - Endogenous liquidation value/asset prices

    + Risky leverage choice

Ex-ante efficiency?

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Ex-ante efficiency?

In the absence of runs:

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With **runs**:

- Banks are over-leveraged

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- Three periods $t = 0, 1, 2$
- One final consumption good and one factor (capital)
- K units of capital in fixed supply

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- Can default at $t = 1, 2$ (Cole-Kehoe timing)
- Creditors: linear utility and discount rate R

Roadmap

1. Bank problem in partial equilibrium, for given price of capital $\{p_t\}$
   - Characterize "region of vulnerability"
2. General equilibrium: market clearing for capital, determination of $\{p_t\}$
3. Normative analysis: socially optimal ex-ante leverage choices

Banks' Preferences and budget constraints

- Preferences

$$u(c_0) + \beta \mathbb{E} u(c_1) + \beta^2 \mathbb{E} u(c_2), \quad \text{where} \quad u = \log \quad [\text{tractability}]$$

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$$c_0 = (z + p_0)k_0 - Rb_0 + q_0(b_1, k_1)b_1 - p_0 k_1,$$

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last period, no more borrowing or investment

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$$n_t = (z + p_t)k_t - Rb_t$$

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V^D independent of prices and increasing in k

Period 2

Simple static problem

$$V_2(b_2, k_2) = \max_{d_2 \in \{0,1\}} \left\{ (1 - d_2) u(z k_2 - R b_2) + d_2 u(z_2^D k_2) \right\}$$

Default choice:

$$d_2(b_2, k_2) = \begin{cases} 1 & \text{if } \; R b_2 > \phi k_2, \text{ where } \phi \equiv z - z_2^D \\ 0 & \text{otherwise,} \end{cases}$$

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Borrowing limit at $t = 1$ $R b_2 \leq \phi k_2$

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Without a run

$$\text{s.t.} \quad c_1 = n_1 + b_2 - p_1 k_2$$

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Value functions indexed by p_1

Default Thresholds

- Given p_1, two default thresholds

$$\text{Fundamental:} \quad V_1^R(n_1) = V_1^D(k_1, \hat{z}^F)$$

$$\text{Run:} \quad V_1^{Run}(n_1) = V_1^D(k_1, \hat{z}^{Run})$$

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Result: $\hat{z}^F(n_1, k_1) \geq \hat{z}^{Run}(n_1, k_1)$

- If $\frac{z}{p_1} > R$, then $\hat{z}^F(n_1, k_1) > \hat{z}^{Run}(n_1, k_1)$

If $\frac{z}{p_1} \geq R$:

$$V_1^{Run}(n_1) = A + (1+\beta)\log(n_1) + \beta \log\left(\frac{z}{p_1}\right)$$

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excess return from leverage

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cannot borrow

$$\Rightarrow V_1^R(n_1) = V_1^{Run}(n_1)$$

If $\frac{z}{p_1} > R$:

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$\Rightarrow V_1^R(n_1) > V_1^{Run}(n_1) \Rightarrow$ A run lowers the value of the bank because profits vanish

$\Rightarrow$ A run can lead the bank to default

$\Rightarrow$ Bank vulnerable to runs, despite assets being liquid (Amador and Bianchi 2024)

Self-Fulfilling Runs

$\hat{z}^{Run}(k_1, n_1)$

$\hat{z}^{F}(k_1, n_1)$

z_1^D

Self-Fulfilling Runs

Default

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$V_1^D(k_1, z_1^D) > V_1^R(n_1)$

Self-Fulfilling Runs

Safe

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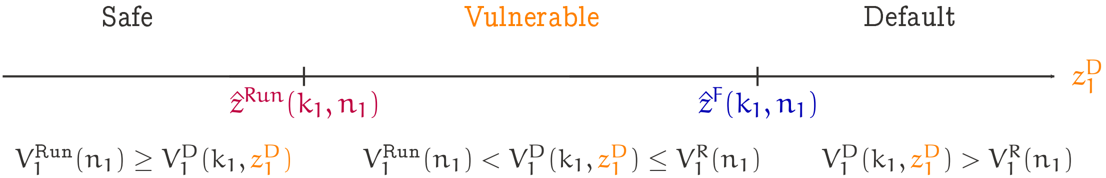
[Run & repay is an off-equilibrium event]

Self-Fulfilling Runs



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- **Sunspot:** a vulnerable bank faces a run with probability λ.

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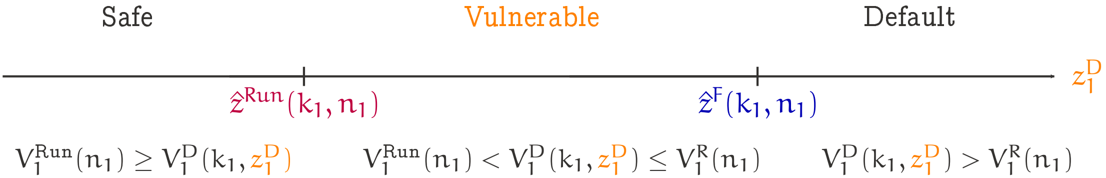

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  Hence the default probability is

$$d_1(n_1, k_1, z_1^D) = \begin{cases} 0 & \text{if } z_1^D \leq \hat{z}^{Run}(n_1, k_1) \\ \lambda & \hat{z}^{Run}(n_1, k_1) < z_1^D \leq \hat{z}^{F}(n_1, k_1) \\ 1 & \text{if } z_1^D > \hat{z}^{F}(n_1, k_1) \end{cases}$$

Period 0: Value and Leverage Choice

$$V_0(n_0) = \max_{c_0 \geq 0, k_1 \geq 0, b_1} u(c_0)$$

$$+ \beta \int_{\underline{z}}^{\overline{z}} \left[d_1(n_1, k_1, \tilde{z}) V_1^D(k_1, \tilde{z}) + (1 - d_1(n_1, k_1, \tilde{z})) V_1^R(n_1) \right] dF(\tilde{z})$$

subject to

$$c_0 = n_0 + q_0(n_1, k_1) b_1 - p_0 k_1,$$

$$n_1 = (z + p_1) k_1 - R b_1.$$

where the bond price schedule is given by

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Deposits allow for higher portfolio returns and c_0, but raises exposure to default

Roadmap

1. Bank problem in partial equilibrium, for given asset prices $\{p_t\}$
    - Characterize “region of vulnerability”
2. General equilibrium: market clearing for capital, determination of $\{p_t\}$
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Definition

Given B_0, and a run probability, λ, a *symmetric competitive equilibrium* consists of $\left\{p_0, p_1, q_0, \hat{z}^F, \hat{z}^{Run}, d_1, d_2, V_1^R, V_1^D, b_1, k_1, b_2, k_2\right\}$ such that:

(a) Banks choose portfolios and repayment optimally

(b) Investors break even

$$q_0(n_1, k_1) = (1 - \lambda)F(\hat{z}^F(n_1, k_1)) + \lambda F(\hat{z}^{Run}(n_1, k_1))$$

(d) The market for capital clears

- Aggregate demand for capital equals K at $t = 0, 1$.

Equilibrium at $t = 1$

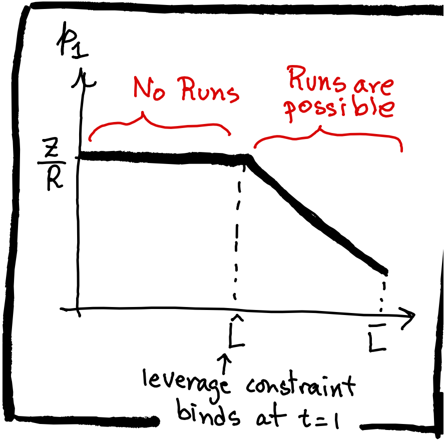
- Characterization in terms of **leverage** $l_1 = b_1/k_1$
  - Redefine thresholds as $\hat{z}^F(l_1|p_1)$, $\hat{z}^{Run}(l_1|p_1)$
  - In the aggregate $L_1 = b_1/K$
- Share of banks defaulting is increasing in L_1:

$$\underbrace{\left[1 - F(\hat{z}^F(L_1|p_1)\right]}_{\text{Fundamentals}} + \underbrace{\lambda\left[F(\hat{z}^F(L_1|p_1) - F(\hat{z}^{Run}(L_1|p_1)\right]}_{\text{Runs}}$$

Equilibrium Asset Price at $t = 1$

$$K_2 = \frac{\beta}{(1+\beta)(p_1 - \phi/R)} N_1.$$

Lower net worth (and higher leverage) leads to lower demand for capital



Equilibrium Asset Price at $t = 1$

$$\mathcal{P}_1(L_1) \equiv \begin{cases} \frac{z}{R} & \text{if } L_1 \leq \hat{L}, \\ \beta z + (1+\beta)\frac{\phi}{R} - \beta R L_1 & \text{if } L_1 \in (\hat{L}, \bar{L}). \end{cases}$$

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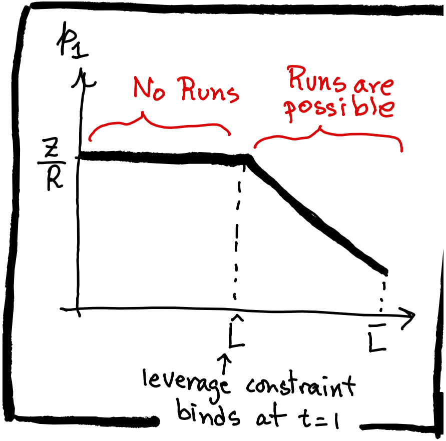

Price for capital p_1 decreasing in L_1 when banks are constrained

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$$\max_{c_0, L_1} \left\{ u(c_0) + \beta \int_{\underline{z}}^{\overline{z}} \left[d_1(L_1, \tilde{z}|p_1) V_1^D(K, \tilde{z}) + (1 - d_1(L_1, \tilde{z}|p_1)) V_1^R(n_1|p_1) \right] dF(\tilde{z}) \right\},$$

subject to:

$$c_0 = zK - RB_0 + q_0(L_1|p_1) L_1 K,$$

and where:

GE piece

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Creditors remain indifferent

Analysis without Runs

Proposition (Constrained-efficiency)

Suppose $\lambda = 0$. *Any competitive equilibrium is constrained efficient.*

Preliminary Lemma

Lemma: Consider any aggregate leverage L_1 and its associated price $p_1 = \mathcal{P}_1(L_1)$:

- If a bank takes $l_1 = L_1$, the value goes up if the price deviates from the eqm.

(i) $V_1^R((z + p_1)K - RKL_1 | p_1) \leq V_1^R((z + \hat{p}_1)K - RKL_1 | \hat{p}_1)$;

strict inequality if $p_1 \neq \hat{p}_1$

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Why?

- In equilibrium, banks are neither net buyers nor net sellers
  - If price deviates from eqm., bank will buy/sell and strictly raise value

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(ii) $q_0(L_1 | p_1) \leq q_0(L_1 | \hat{p}_1)$,

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Proof of Constrained-Efficiency with $\lambda = 0$

Let L^E and L^P be the compet. eqm. and planner's leverage

Associated prices: $p_1^E = \mathcal{P}(L^E)$ and $p_1^P = \mathcal{P}(L^P)$

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- In the competitive eqm., banks prefer (L^E, K) rather than (L^P, K) when facing p_1^E:

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But planner can also choose L^E.

$\Rightarrow$ L^E must solve the planner's problem

Uniqueness and Existence

Proposition (Uniqueness)

Suppose that: (i) there is a unique solution to the planner problem, or (ii) there exists a competitive equilibrium with leverage $L_1 = B_1/K > \hat{L}$.

Then, there is at most one (symmetric pure-strategy) competitive equilibrium.

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Proposition (Existence)

Suppose that Assumption 2 holds and

i) f *is continuous and such that* $f(\underline{z}) = f(\overline{z}) = 0$.

ii) $\left[\frac{1-F(z)}{1+\beta} + \frac{f(z)}{F(z)} z\right]$ *is decreasing in* z *for any* $z \in [\underline{z}, \overline{z}]$.

Then, there $\exists$ a competitive equilibrium.

Taking stock so far

- In the absence of runs, competitive equilibrium is constrained efficient
- No need for leverage restrictions

Economy with runs $\lambda > 0$

Thresholds as a Function of Aggregate Leverage

Consider a reduction in L_1 $\Rightarrow \hat{p}_1 = \mathcal{P}(L_1 - \Delta) > \mathcal{P}(L_1) = p_1$

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Safe

Vulnerable

Default

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$\hat{z}^{F}(l_1|p_1)$

z_1^D

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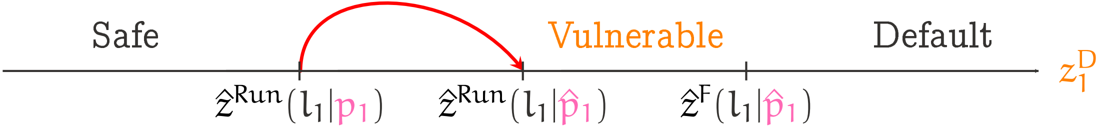
- Zero *first-order* effects on $\hat{z}^F$
- $\frac{\partial \hat{z}^{Run}}{\partial p_1} = \hat{z}^{Run} \frac{(1+\beta)\phi}{R(z+p_1-RL_1)p_1} > 0$ $\Leftarrow$ banks are net sellers in a run



Safe Vulnerable Default

z_1^D

$\hat{z}^{Run}(l_1|p_1)$ $\hat{z}^{Run}(l_1|\hat{p}_1)$ $\hat{z}^F(l_1|\hat{p}_1)$

Over-leverage with $\lambda > 0$

$$\frac{1}{c_0} - \frac{\beta R}{c_1} = -\frac{(1-\lambda) f(\hat{z}^F)\frac{\partial \hat{z}^F}{\partial L_1} + \lambda f(\hat{z}^{Run})\frac{\partial \hat{z}^{Run}}{\partial L_1}}{q_0}\frac{L_1}{c_0}$$

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Higher L_1 raises run prob.
(privately efficient)

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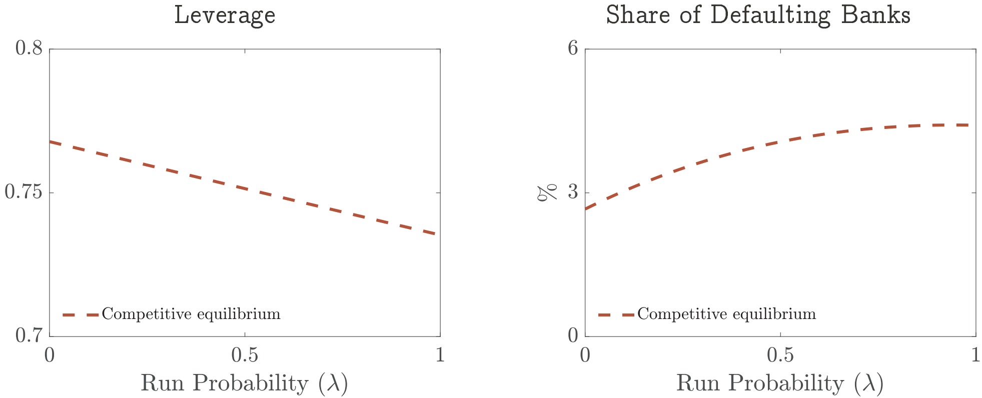
Loss from runs

Planner internalizes that ↓ L_1 leads to ↑ p_1 and fewer runs

Competitive Eqm. vs. Constrained Efficient



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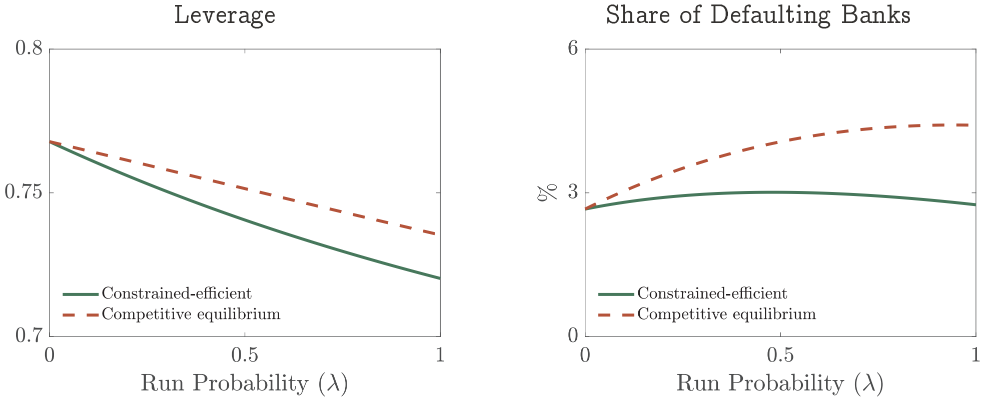

Leverage

Share of Defaulting Banks

Some Connections

- **Bank runs:** Diamond and Dybvig (1983); Ennis and Keister 2009; Diamond and Rajan 2012; Allen and Gale 2000; Gertler and Kiyotaki 2012; Kashyap et al. 2024, etc.

  Here: self-fulfilling runs on individual banks in **general equilibrium**

  - Scope for macroprudential policy from GE effects on asset prices

- **Macroprudential policies:** Caballero and Krishnamurthy 2003; Lorenzoni 2007; Bianchi 2011; Stein 2012; Davila and Korinek 2018; He and Kondor 2016; etc.

  Here: pecuniary externality *without* fire-sales, redistributive effects, or collateral constraints

  - Bank runs + equilibrium default $\Rightarrow$ Excessive leverage

Conclusions

- A macroprudential theory of banking regulation under self-fulfilling runs
- Capital requirements are desirable, even absent bailouts