Bank Runs, Fragility, and Regulation *

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Abstract

We develop a theory of banking regulation in a general equilibrium model in which banks may be vulnerable to self-fulfilling runs. We show that the competitive equilibrium is constrained efficient in the absence of runs, but the presence of runs renders the equilibrium leverage decisions inefficient. Individual banks fail to internalize that reducing leverage raises future asset prices, which improves other banks' liquidity in a run—making them less vulnerable and reducing the probability of costly defaults in equilibrium. Introducing minimum capital requirements, even in the absence of bailouts, improves the market outcome.

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1 Introduction

The run on Silicon Valley Bank in March 2023—and subsequent failures of Signature Bank, First Republic and Credit Suisse—once again exposed the fragility of the banking system. These events have reignited a debate over banking regulation, particularly the role of stricter capital requirements in absorbing losses and deterring runs.[1] While there is broad agreement that higher equity buffers reduce banks' vulnerability, a question at the center of current policy debates is whether banks would choose to hold sufficient capital in the absence of regulation.

In this paper, we develop a theory of banking regulation in a general equilibrium environment where banks are vulnerable to self-fulfilling runs, as in Diamond and Dybvig (1983). We show that the answer to the question above is no: in the presence of run risk, banks choose too little capital relative to what is socially optimal. The inefficiency arises from the following mechanism. When banks face a run, they must sell assets to meet deposit withdrawals; hence, when banks collectively hold more equity, asset prices are higher, which improves liquidity for institutions facing withdrawals and discourages depositors from running. However, individual banks fail to internalize that collectively raising equity buffers boosts asset prices and, through general equilibrium effects, lowers the likelihood of runs on other banks. As a result, they hold too little equity relative to the social optimum.

Environment. Our framework is a macroeconomic model of self-fulfilling bank runs, building on Amador and Bianchi (2024). A bank's vulnerability depends on its individual leverage as well as asset prices, which are endogenously determined in a Walrasian market where banks can trade capital. Our earlier work considered a situation where banks start with a high initial debt level and studied how a policy of credit easing can reduce the incidence of bank runs ex post. This paper shifts to an ex-ante macroprudential perspective: we analyze banks' leverage decisions under the risk of costly default and assess whether and why a social planner would opt for a different leverage level than the competitive equilibrium.

The model has three periods and a continuum of banks, which have limited commitment and may default due to idiosyncratic fundamental shocks or self-fulfilling runs. In the initial period, banks decide on capital investment, equity payouts, and leverage. The bond price at which a bank issues deposits reflects the perceived probability of default in the subsequent period. In the intermediate period, a bank default can be triggered by a fundamental shock or by a self-fulfilling

[1] In July 2023, the Federal Reserve, the Federal Deposit Insurance Corporation (FDIC), and the Office of the Comptroller of the Currency (OCC) made a joint proposal to raise capital requirements. More recently, the Vice Chair for Supervision Michelle Bowman has advocated rolling back increases in capital requirements introduced after Dodd-Frank. See also the July 2023 statement by Governor Chris Waller.

run, as in Cole and Kehoe (2000).[2] When a bank faces a run, it must sell some of its assets to raise the liquidity needed to repay depositors. Even if assets are fully liquid, the bank may remain vulnerable to runs whenever a positive equilibrium spread exists between the return on capital and the cost of deposits. This is because the inability to issue new deposits reduces banks' profits and operating value, making them more prone to default during a run. In the final period, banks collect the proceeds from their capital holdings, choose consumption, and decide whether to repay or default on their deposits. The terminal value affects asset prices in earlier periods and influences the ex-ante leverage decisions.

We characterize the competitive equilibrium of this economy and compare it with a constrained-efficient allocation, in which a social planner chooses borrowing decisions on behalf of banks in the initial period to maximize their ex-ante welfare. In the constrained-efficient allocation, the capital market clears competitively in each period, banks retain the decision of whether to repay or default, and remain vulnerable to runs. The key distinction is that the planner internalizes how initial leverage choices affect the future path of asset prices and, in turn, the continuation values of banks.

Constrained efficiency without runs. Our normative analysis begins by examining a version of the model where defaults occur solely due to fundamentals. We establish a benchmark result: in the absence of runs, competitive equilibria are constrained efficient. Although banks' default decisions depend on market-clearing asset prices and the planner internalizes such effects, it chooses the same leverage as individual banks.

Crucial to this constrained efficiency result is that repaying banks are neither net buyers nor net sellers of capital in equilibrium. To understand the argument, consider an individual bank's payoff as a function of its leverage. Since the planner's leverage choice is feasible for the individual bank, the bank's payoff from choosing the competitive equilibrium leverage must be at least as high as its payoff from choosing the planner's leverage when facing the competitive equilibrium price of capital. Moreover, because repaying banks are neither net buyers nor net sellers of capital, any deviation of the capital price from its market-clearing level would increase the value of repayment in period 1 and improve the bond price in period 0. It follows that choosing the planner's leverage when facing the competitive equilibrium price yields a higher payoff than choosing the same leverage at the constrained-efficient price. Therefore, the bank's payoff in the competitive equilibrium must be at least as high as in the constrained-efficient allocation. But since the planner can also choose the competitive equilibrium leverage, the two payoffs must be

[2] Cole and Kehoe (2000) shares similarities with Diamond and Dybvig (1983), but a key difference is that default is strategic in the former. In the context of our model, this means that banks' shareholders can choose to let the bank fail or take lower dividend payments and keep the bank operating.

equal—confirming that the competitive equilibrium is constrained efficient.

Furthermore, we establish both the existence of the competitive equilibrium (under an additional condition) and uniqueness. In effect, we present versions of the classic welfare theorems extended to an environment with *equilibrium default*. Our constrained efficiency result differs from the ones in Kehoe and Levine (1993) and Alvarez and Jermann (2000) where the possibility of default constrains the set of equilibrium allocations, but the existence of a complete set of state-contingent securities implies that default does not occur along the equilibrium path.

Constrained inefficiency with runs. We then turn to examine the economy with runs, which is our main focus. In our model, defaults are costly, and individual banks have incentives to reduce leverage to make themselves less vulnerable to future defaults. In the absence of runs, the constrained efficiency result tells us that these private incentives perfectly align with the social ones. With runs, this is no longer the case.

In particular, reducing leverage for all banks in the initial period increases the demand for capital in the second period, raising asset prices. Higher asset prices benefit banks facing runs because they are net sellers of capital—seeking liquidity to cover withdrawn deposits. Therefore, a reduction in aggregate leverage improves banks' liquidity in the second period and reduces their vulnerability to runs. Since individual banks fail to account for these positive general equilibrium effects, the competitive equilibrium is *constrained inefficient*. We show that implementing the constrained-efficient allocation requires introducing a positive tax on leverage in the initial period or, equivalently, a minimum capital requirement.

It is worth highlighting three points from our analysis.

First, the presence of equilibrium default is crucial for our constrained inefficiency result in the economy with bank runs. Specifically, we show that in a version of the model without uncertainty (and thus without equilibrium default), the competitive equilibrium remains constrained efficient, even when banks face borrowing constraints linked to asset prices that account for the possibility of runs.

Second, by construction, our model features no capital sales or "fire sales" in equilibrium (Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997). As noted earlier, the fact that banks are neither net buyers nor net sellers in equilibrium is central for the constrained efficiency of the competitive equilibrium in the absence of runs. However, when a run occurs, banks are forced to sell assets, and lower asset prices make repayment more costly, thereby increasing their vulnerability to a run and a costly default in equilibrium. It is the effect of asset prices on the off-equilibrium repayment value during a run, combined with the presence of equilibrium default, that renders the competitive equilibrium constrained inefficient.

Third, our theory yields distinctive implications for the cyclicality of optimal regulation. In existing models, leverage restrictions are warranted when financial constraints are currently slack but may bind in the future due to aggregate shocks. Since the likelihood of binding constraints increases in downturns, these frameworks typically imply that regulation should be procyclical—tightening in bad times (see Schmitt-Grohé and Uribe, 2017). In contrast, our model shows that leverage restrictions can be desirable even in the absence of aggregate shocks. Moreover, the relationship between fundamentals and optimal policy is non-monotonic. When fundamentals are strong, default incentives are limited, and regulation is unnecessary. When fundamentals are weak, defaults are driven by fundamentals alone, leaving little scope for policy intervention. It is at intermediate levels of fundamentals—where coordination failures become relevant—that tighter leverage restrictions are most effective.

Literature. Our paper is related to several strands of the literature. Following Diamond and Dybvig (1983), a substantial body of literature has analyzed bank runs driven by self-fulfilling expectations and evaluated the role of government policies. Much of this literature has focused on policies designed to prevent runs on *individual banks*, including deposit insurance, suspension of convertibility, bailouts, lender-of-last-resort facilities, regulatory forbearance, and capital requirements (e.g., Cooper and Ross, 1998; Ennis and Keister, 2009; Keister, 2016; Dávila and Goldstein, 2023; Schilling, 2023; Diamond and Kashyap, 2016; Kashyap, Tsomocos and Vardoulakis, 2024). In contrast to this literature, we study a general equilibrium model where the interaction between aggregate leverage and asset prices gives rise to the need for regulation.[3]

Our paper is related to the literature that analyzes the welfare properties of competitive equilibrium in economies with imperfect financial markets. In the general equilibrium literature with incomplete markets, Hart (1975) and Stiglitz (1982) provide early examples of constrained inefficiency and Geanakoplos and Polemarchakis (1985) establish the generic constrained inefficiency of competitive equilibria. In contrast, we analyze constrained efficiency in an economy with limited commitment. The studies by Kehoe and Levine (1993) and Alvarez and Jermann (2000) provide versions of welfare theorems in environments with limited commitment.[4] However, in these environments, borrowers can trade complete state-contingent securities, which means that default does not occur in equilibrium. Our contribution is to examine the welfare properties of competitive equilibrium in an environment where default occurs in equilibrium.

In a similar vein, the macroprudential policy literature has examined environments without

[3] Dávila and Goldstein (2023) examines spillovers from bankruptcies, but abstracts from modeling the general equilibrium.

[4] Kehoe and Levine (1993) show that welfare theorems fail with multiple goods (see also, Jeske, 2006; Bloise and Reichlin, 2011; Gottardi and Kubler, 2015; Martins-Da-Rocha, Phan and Vailakis, 2025).

equilibrium default, where financial constraints and aggregate shocks give rise to inefficiencies—i.e., pecuniary externalities—that can be mitigated through ex-ante borrowing restrictions. Dávila and Korinek (2018) classify the underlying channels into two categories: redistributive externalities—where price changes reallocate wealth among agents with different marginal rates of substitution (e.g., Caballero and Krishnamurthy, 2003; Lorenzoni, 2008)—and collateral externalities—where price movements affect financial constraints by altering the market value of collateral assets (e.g., Bianchi, 2011; Stein, 2012).[5] In contrast to these studies—where the need for regulation arises from aggregate shocks, fire sales, or collateral constraints—our paper uncovers an inefficiency resulting from the interaction between equilibrium default and self-fulfilling runs. A distinct implication of our framework is that the externality is stronger in periods of normal fundamentals and gives rise to a non-monotonic relationship between fundamentals and borrowing restrictions.

A different strand of the literature examines banking regulation in the presence of moral hazard arising from the expectation of bailouts. Since at least Bagehot (1873), it has been recognized that rescuing distressed banks creates an incentive problem, prompting the need for regulatory constraints such as leverage restrictions. Building on this insight, a large literature has studied how the anticipation of government guarantees distorts risk-taking incentives—both in partial equilibrium models focused on individual institutions (e.g., Kareken and Wallace, 1978; Dovis and Kirpalani, 2022) and in general equilibrium frameworks (e.g., Farhi and Tirole, 2012; Acharya and Yorulmazer, 2007). In contrast, the theory developed in this paper identifies a role for banking regulation even in the absence of government bailouts, arising from general equilibrium effects and the endogenous fragility generated by self-fulfilling runs.

In a related paper, Gertler, Kiyotaki and Prestipino (2020) study how countercyclical capital requirements can reduce vulnerability to banking panics in a calibrated model. Their analysis builds on the framework developed by Gertler and Kiyotaki (2015), where strategic complementarities operate across banks, giving rise to multiple equilibria—one with high asset prices and all banks repaying, and one with low asset prices and all banks defaulting. In contrast, our model features runs on individual banks, triggered by strategic complementarities among their depositors, with distinct implications for policy.[6] Beyond these differences in the environment, we also

[5] Lanteri and Rampini (2023) study the interaction between these two types of externalities and find that investment subsidies are optimal. Kurlat (2021) and Asriyan et al. (2024) also consider settings where asset prices are inefficiently low. Other examples characterizing constrained-inefficient borrowing include Gromb and Vayanos (2002), He and Kondor (2016), Bianchi and Mendoza (2018), Kilenthong and Townsend (2021), Itskhoki and Moll (2019), Ottonello, Perez and Varraso (2022), and Bocola and Lorenzoni (2023). An alternative strand of the literature study pecuniary externalities in economies with asymmetric information (Farhi, Golosov and Tsyvinski, 2009; Gersbach and Rochet, 2012; Di Tella, 2019).

[6] For example, in their framework, a maturity extension or a liquidity provision does not prevent bankruptcy, since a poorly capitalized bank remains tempted to divert funds and default. In our model, by contrast, these policies can prevent runs.

provide a formal characterization of the constrained-efficient allocation and show that the optimal macroprudential policy takes the form of minimum capital requirements.

Several recent papers have developed models in which banks may face runs despite holding liquid assets (Amador and Bianchi, 2024; Drechsler et al., 2023; Haddad et al., 2023; Jiang et al., 2024, 2025).[7] Our model builds on our earlier work, which examines a setting where banks make leverage decisions in the absence of uncertainty. In contrast, the present paper analyzes a model where leverage decisions are made under uncertainty, introducing the potential for equilibrium defaults. It is this element that is crucial for the inefficiency in ex-ante borrowing choices we uncover in this paper.

There are several strands of literature on corporate, household, and sovereign borrowing where equilibrium default plays a prominent role. Much of this work, similar to ours, follows Eaton and Gersovitz (1981), where default constitutes a strategic decision by the borrower, and lenders charge interest rates that compensate for the associated risk. Key theoretical contributions in this tradition address the existence (Chatterjee et al., 2007; Chatterjee and Eyigungor, 2012), uniqueness (Auclert and Rognlie, 2016; Aguiar and Amador, 2019; DeMarzo, He and Tourre, 2023), and efficiency (Dovis, 2019; Aguiar, Amador, Hopenhayn and Werning, 2019) of equilibrium.[8] Our paper differs by considering a general equilibrium model with multiple borrowers where endogenous fluctuations in asset prices play a central role.

Finally, our paper is related to historical discussions on the origins of banking crises, especially the debate on whether banking crises occur because of poor fundamentals (e.g., Calomiris and Mason, 2003; Baron, Verner and Xiong, 2021; Correia, Luck and Verner, 2024), self-fulfilling runs (e.g., Friedman and Schwartz, 1963; Bernanke, 2018). In our model, worse fundamentals expand the region where self-fulfilling runs can emerge, thereby linking these views of banking crises.[9]

To summarize, our analysis provides a theory of banking regulation in general equilibrium. Our model starting point is motivated by widespread policy concerns about the risks of short-term

[7]Following the bankruptcy of several institutions in March 2023, a recent literature has investigated important aspects of the latest sequence of runs (see e.g., Llambias and Ordonez, 2024; Arfaoui and Uhlig, 2025).

[8]A few papers study the implications of equilibrium default in quantitative macroeconomic models (Arellano, Bai and Kehoe, 2019; Gomes, Jermann and Schmid, 2016; Khan, Senga and Thomas, 2021; Ottonello and Winberry, 2020). There are other important approaches to model equilibrium default. Dubey, Geanakoplos and Shubik (2005) adapt a standard incomplete market economy to allow for default, with investors pricing the risk associated with pooled assets rather than individual bonds. Geanakoplos and Zame (2014) introduce a collateral equilibrium where default results in the transfer of assets from borrowers to lenders, but without deadweight losses, and present a proof of existence of equilibrium (see also Araujo, Páscoa and Torres-Martínez, 2002). Fostel and Geanakoplos (2015) present a Binomial No-Default Theorem that states that any equilibrium is equivalent to another equilibrium in which there is no default. One key difference in our environment is that default is costly.

[9]This connection between fundamentals and self-fulfilling crises dates back to second generation models of currency crises, such as Obstfeld (1996), and is present in many other studies.

debt,[10] but emphasizes the efficiency implications of default risk and self-fulfilling runs in general equilibrium. This interaction between general equilibrium, default risk, and self-fulfilling runs distinguishes our approach from previous work that either narrows attention to individual banks or focuses on the inefficiencies that arise due to aggregate shocks, fire sales, collateral constraints, or bailouts.

Outline. Sections 2 and 3 present the model. Section 4 provides the theoretical characterization of the economy without runs and establishes a benchmark efficiency result. Section 5 presents the theory of optimal banking regulation in the presence of runs. Section 6 concludes. The main proofs are included in the body of the paper. All other proofs are collected in an online appendix.

2 Model

The economy has three periods, $t \in \{0, 1, 2\}$, and is populated by a continuum of banks and investors, both of measure one. There is a single consumption good produced using capital with a linear technology. We assume that banks have direct access to the production technology, in line with the most recent strands of macro-finance models. Capital does not depreciate, and it is in fixed aggregate supply, equal to K.

Banks. Banks' preferences over a stream of dividend payments are given by

$$u(c_0) + \beta \mathbb{E} u(c_1) + \beta^2 \mathbb{E} u(c_2),$$

where $\beta > 0$ is the discount factor, and $u = \log$.

In period 0, all banks start with b_0 units of maturing debt, referred to interchangeably as bonds or deposits, and capital holdings $k_0 = K$. Banks use the capital to produce zk_0 units of the final good. We assume that the productivity of a bank remains equal to $z > 0$ as long as the bank does not default. Banks can issue one-period bonds to investors, which promise a payment of $R > 0$ in the subsequent period.[11]

A bank chooses dividend payments, c_0, issues new short-term bonds, b_1, and selects a new level of capital, k_1, which is purchased in competitive markets. The bank's budget constraint is

[10] In his Nobel lecture, Douglas Diamond emphasizes that "Private financial crises are everywhere and always due to the problems of short-term debt."

[11] A one-period debt/deposit contract is akin to a demand-deposit contract in which depositors are owed repayment in the final period but may withdraw early. This contract is often motivated by preference shocks to early versus late consumption, which are private information (Diamond and Dybvig, 1983). Here, we take the deposit contract as given, which is common in the banking literature (e.g., Dávila and Goldstein, 2023).

given by

$$c_0 = (z + p_0)k_0 - Rb_0 + q_0(b_1, k_1)b_1 - p_0 k_1,$$

where p_0 is the price of capital, and q_0 represents the bond price schedule as a function of the portfolio chosen by the bank. We assume that banks cannot default in this initial period.[12]

In period 1, banks may repay or default on their debt. If a bank defaults, it cannot borrow or save in bonds and cannot trade in the market for capital. We also assume that the defaulting bank keeps a fraction of its capital, while the remaining capital is lost. The value of defaulting for a bank in period 1 is

$$V_1^D(k_1, z_1^D) = u(z_1^D k_1) + \beta u(z_2^D k_1), \tag{1}$$

where z_1^D and z_2^D encapsulate the fraction of capital kept after default, or equivalently, the productivity during default. In what follows, we assume that z_1^D is drawn from some distribution with support in $[\underline{z}, \overline{z}] \subset [0, z]$, i.i.d. across banks, while $z_2^D \in (0, z)$ is predetermined and common between banks. We let F denote the c.d.f. of z_1^D and f its density.[13]

If the bank repays its deposits and its budget constraint is given by

$$c_1 = (z + p_1)k_1 - Rb_1 + q_1(b_2, k_2)b_2 - p_1 k_2, \tag{2}$$

where c_1 is the level of dividends in period 1, p_1 is the price of capital, q_1 is the bond price schedule, and b_2 and k_2 are the new choices of short-term bonds and capital. That is, the bank collects the return on its assets, pays its deposits, and then issues new deposits and buys new capital. As in period 0, the bond price depends on the portfolio chosen by the bank.

In period 2, the bank can choose again whether to repay or default. If the bank defaults, its value is given by

$$V_2^D(k_2) = u(z_2^D k_2).$$

If the bank repays, it consumes the output produced minus the deposits repaid with interest:

$$c_2 = zk_2 - Rb_2.$$

Investors. Investors are risk neutral and discount future consumption at a rate $1/R$ across two subsequent periods. They buy the bonds supplied by the banks and do not hold any capital stock.

[12] The absence of default in period 0 can be rationalized with a low enough initial debt b_0.

[13] Generating equilibrium default in the model requires a shock to the bank. We consider a shock that affects the outside option value for the bank as opposed to a shock to the productivity of a repaying bank because it results in a more tractable characterization. Additionally, the assumption that the defaulting bank retains a fraction of its capital is a common one in the literature (e.g., Gertler and Kiyotaki, 2010, 2015).

The Banks' Problem and Bond Prices

We describe the bank's problem starting from the final period.

Bank Problem at $t = 2$

In the final period, the bank's value is given by

$$V_2(b_2, k_2) = \max_{d_2 \in \{0,1\}} \left\{(1 - d_2)u(zk_2 - Rb_2) + d_2 u(z_2^D k_2)\right\},$$

where d_2 represents the default decision, and the value of repayment reflects that in the final period, the bank consumes the output net of the liabilities.

The optimal default decision is given by:

$$d_2(b_2, k_2) = \begin{cases} 1 & \text{if } \; zk_2 - Rb_2 < z_2^D k_2, \\ 0 & \text{otherwise,} \end{cases}$$

where we assumed that the bank repays if indifferent.

Given that there is no uncertainty in the final period, the bond price at $t = 1$ is 1 if $zk_2 - Rb_2 \geq z_2^D k_2$ and 0 otherwise. We can express the bond price schedule as

$$q_1(b_2, k_2) = \begin{cases} 1 & \text{if } \; Rb_2 \leq \phi k_2, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

where $\phi \equiv z - z_2^D$ represents the maximum leverage. Using that $z_2^D \in (0, z)$, it follows that the maximum leverage is such that $\phi \in (0, z)$.

Bank Problem at $t = 1$

In period 1, a bank can be subject to a run. Following Cole and Kehoe (2000) and Amador and Bianchi (2024), we assume that the bank issues bonds before a repayment or default decision has been made. The fact that investors are atomistic introduces the possibility of a coordination failure: if investors suddenly panic and become unwilling to roll over the bonds, the bank faces a liquidity problem and may be pushed to default on its debt, which in turn rationalizes investors' initial panic.

To determine whether a bank is vulnerable to a self-fulfilling run or not, we need to distinguish the value of repayment in the case when the bank faces a run and when it does not.

Consider first the case without runs. Define the net worth of the bank as

$$n_1 \equiv (z + p_1)k_1 - Rb_1.$$

Using (2) and (3), we obtain that the value of repayment for the bank in this case is

$$V_1^R(n_1) = \sup_{c_1 \geq 0, k_2 \geq 0, b_2} \Big\{ u(c_1) + \beta u(zk_2 - Rb_2) \Big\}, \tag{4}$$

subject to:

$$c_1 = n_1 + b_2 - p_1 k_2,$$
$$Rb_2 \leq \phi k_2.$$

The borrowing constraint in the bank's problem reflects that the bank can borrow at a price of 1, provided that promised repayments do not exceed ϕk_2. A bank would not choose to borrow more than this amount, as the bond price would otherwise drop to zero, as shown in (3). If the constraint set is empty, we set $V_1^R(n_1) = -\infty$.

The following lemma presents the solution to the value function of the bank.

Lemma 1. *The value function $V_1^R(n_1)$ is such that*

$$V_1^R(n_1) = \begin{cases} \infty & \text{if } p_1 < \phi/R \text{ or } p_1 = \phi/R, n_1 > 0; \\ A + (1+\beta)\log(n_1) + \beta \log\left(\frac{z-\phi}{p_1 - \phi/R}\right) & \text{if } \frac{\phi}{R} < p_1 \leq \frac{z}{R}, n_1 > 0; \\ A + (1+\beta)\log(n_1) + \beta \log(R) & \text{if } p_1 > \frac{z}{R}, n_1 > 0; \\ -\infty & \text{if } p_1 \geq \phi/R, n_1 \leq 0, \end{cases}$$

where $A \equiv \beta \log \beta - (1+\beta)\log(1+\beta)$.

Proof. In Online Appendix B.1. □

Let us first discuss the second case in the lemma, which will be the relevant one in equilibrium. If $p_1 > \phi/R$ and $n_1 > 0$, given log utility, the optimal dividend policy conditional on repayment is

$$c_1(n_1) = \frac{1}{1+\beta} n_1. \tag{5}$$

while the bank portfolio satisfies

$$\frac{\beta}{1+\beta}n_1 = p_1 k_2(n_1) - b_2, \qquad b_2(n_1) \leq \frac{\phi}{R}k_2(n_1), \qquad k_2(n_1) \geq 0. \tag{6}$$

If $p_1 < \frac{z}{R}$, the return on capital exceeds the one on deposits, pushing the bank up against its leverage constraint. The optimal policies are then given by

$$k_2(n_1) = \frac{\beta}{1+\beta}\left(\frac{n_1}{p_1 - \frac{\phi}{R}}\right), \qquad b_2(n_1) = \frac{\phi}{R}k_2(n_1). \tag{7}$$

If $p_1 = \frac{z}{R}$, the return on capital is equal to the return on deposits, making the bank indifferent across portfolios as long as they are consistent with (6). Replacing these policies in the bank objective in (4) yields the value in the second case in Lemma 1.

The remaining cases in the lemma are as follows. The first case corresponds to the case where the return on capital is so high that the collateral constraint does not restrict borrowing and investment. In this case, the payoff to the bank is infinite and can be achieved by ever increasing levels of investment and borrowing. The third case corresponds to the case where capital has a lower return than deposits. In this case, banks do not invest in capital and save $b_2(n_1) = -\frac{\beta}{1+\beta}n_1$. The last case corresponds to the case where the bank cannot avoid zero dividends in the first period or where the constraint set is empty, resulting in a value of $-\infty$.

Runs. Consider now the case where the bank faces a run and cannot issue deposits. A bank that repays its maturing bonds when facing a run has to pay dividends and purchase capital using only its available net worth. The payoff is given by

$$V_1^{Run}(n_1) = \sup_{c_1 \geq 0, k_2 \geq 0, b_2} \Big\{u(c_1) + \beta u(zk_2 - Rb_2)\Big\}, \tag{8}$$

subject to:

$$c_1 = n_1 + b_2 - p_1 k_2,$$

$$b_2 \leq 0.$$

The continuation payoff reflects that the bank does not default in period 2 (as it carries no liabilities) and consumes the return on its investments. Note that the only difference between the problems (4) and (8) is the final inequality constraint. Namely, the bank cannot borrow when facing a run.

The value function for a bank that repays during a run is then:

$$V_1^{Run}(n_1) = \begin{cases} A + (1+\beta)\log(n_1) + \beta\log\left(\frac{z}{p_1}\right) & \text{if } p_1 \leq \frac{z}{R}, n_1 > 0, \\ A + (1+\beta)\log(n_1) + \beta\log(R) & \text{if } p_1 > \frac{z}{R}, n_1 > 0, \\ -\infty & \text{if } n_1 \leq 0. \end{cases} \tag{9}$$

The first case in (9) reflects that when $p_1 < \frac{z}{R}$, the bank facing a run only invests out of its available net worth. The other two cases mirror the last two cases in Lemma 1, except that $V_1^{Run}(n_1) = -\infty$ when $n_1 < 0$, regardless of p_1.

Default thresholds. We can use the value functions solved above to characterize default thresholds: the levels of productivity (under default) that make a bank indifferent between repaying and defaulting. These thresholds depend on whether the bank is facing a run or not.

Let us start by examining a situation without runs. Consider a bank with net worth n_1 and capital k_1. The bank defaults for values of z_1^D such that $V_1^D(k_1, z_1^D) > V_1^R(n_1)$; and repays otherwise. Hence, there exists a threshold $\hat{z}^F$ such that a bank that draws a default productivity higher than $\hat{z}^F$ defaults, while a bank that draws a default productivity below or equal to $\hat{z}^F$ repays. If $k_1 = 0$, then $V_1^D(k_1, z_1^D) = -\infty$, and we set $\hat{z}^F = \infty$. For $k_1 > 0$, the threshold is given by

$$\hat{z}^F(n_1, k_1) = \begin{cases} \infty & \text{if } p_1 < \frac{\phi}{R} \text{ or } p_1 = \frac{\phi}{R}, n_1 > 0; \\ e^A \left(\frac{n_1}{k_1}\right)^{1+\beta} \left(\frac{R}{Rp_1 - \phi}\right)^{\beta} & \text{if } \frac{\phi}{R} < p_1 \leq \frac{z}{R}, n_1 > 0; \\ e^A \left(\frac{n_1}{k_1}\right)^{1+\beta} \left(\frac{R}{z - \phi}\right)^{\beta} & \text{if } p_1 > \frac{z}{R}, n_1 > 0; \\ 0 & \text{if } p_1 \geq \frac{\phi}{R}, n_1 \leq 0. \end{cases} \tag{10}$$

We call $\hat{z}^F(\cdot)$ the *"fundamental default threshold."*

Next, let us examine the situation where the bank faces a run. A bank with net worth n_1 and capital k_1 defaults for values of z_1^D such that $V_1^D(k_1, z_1^D) > V_1^{Run}(n_1)$; and repays otherwise. If $k_1 = 0$, then $V_1^D(k_1, z_1^D) = -\infty$, and we set $\hat{z}^{Run} = \infty$. For $k_1 > 0$, the threshold in case of a run is given by

$$\hat{z}^{Run}(n_1, k_1) = \begin{cases} e^A \left(\frac{n_1}{k_1}\right)^{1+\beta} \left(\frac{z/p_1}{z - \phi}\right)^{\beta} & \text{if } p_1 \leq \frac{z}{R}, n_1 > 0; \\ e^A \left(\frac{n_1}{k_1}\right)^{1+\beta} \left(\frac{R}{z - \phi}\right)^{\beta} & \text{if } p_1 > \frac{z}{R}, n_1 > 0; \\ 0 & \text{if } n_1 \leq 0. \end{cases} \tag{11}$$

We call $\hat{z}^{Run}(\cdot)$ the *"run threshold."*

It follows from (10) and (11) that $\hat{z}^{Run}(n_1, k_1) \leq \hat{z}^F(n_1, k_1)$; that is, a bank defaults over a larger range of z_1^D shocks. Moreover, if $k_1 > 0$, the inequality is *strict* if and only if

$$\text{(i) } p_1 < \frac{z}{R} \text{ and } n_1 > 0; \text{ or (ii) } p_1 < \frac{\phi}{R}. \tag{12}$$

In both cases, the return on capital, z/p_1, exceeds the borrowing cost, R, implying that the inability to leverage due to a run strictly reduces the value of repayment by preventing the bank from profiting from the spread between returns and borrowing costs. Case (i) corresponds to a situation where the bank has positive net worth and can leverage up to a limit ($\phi > 0$) while (ii) corresponds to a situation where a bank that is not facing a run can leverage without limits, regardless of its net worth. As we will see below, the latter case is not compatible with general equilibrium.

A bank with net worth n_1 and capital k_1 will always default if it draws a default productivity z_1^D above the fundamental threshold $\hat{z}^F(n_1, k_1)$, and will always repay if it draws z_1^D below the run threshold $\hat{z}^{Run}(n_1, k_1)$. However, if z_1^D falls between these two thresholds, the outcome is indeterminate, and we refer to the bank as *vulnerable* to a run. If investors expect the bank to be able to borrow and repay, they are willing to lend, and the bank repays. Conversely, if investors expect the bank to be unable to borrow and default, they are unwilling to lend, and the bank defaults. In both scenarios, the bank's behavior validates investors' expectations, making them self-fulfilling. The outcome in which the bank ends up defaulting reflects a coordination failure among investors, as in Cole and Kehoe (2000) and Diamond and Dybvig (1983).[14]

Figure 1 illustrates the three possible regions a bank may fall into, given its portfolio and the prevailing market price p_1. For low values of z_1^D, the bank repays (safe region); for high values, it defaults unambiguously (default region). For intermediate values, the bank is vulnerable to a self-fulfilling run.

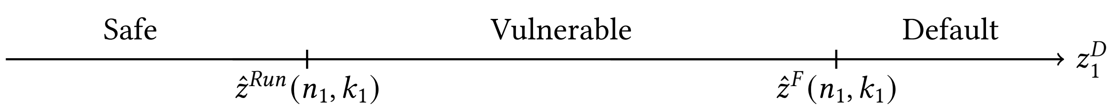

Figure 1: Fundamental and run thresholds for given asset price p_1

[14] As we discussed in Amador and Bianchi (2024), in this model, banks can be subject to runs even when holding only liquid assets, a feature that resonates with the failure of Silicon Valley Bank (see e.g., Metrick, 2024). When the return on capital exceeds the cost of borrowing, banks benefit from leveraging and exploiting the interest rate margin. If a bank faces a run, however, it experiences a loss in franchise value, which can leave it more prone to default and, therefore, vulnerable to a self-fulfilling run. This feature of our model is also present in Drechsler et al. (2023) and Jiang et al. (2024) who explore how higher interest rates may generate simultaneously higher franchise value and lower asset values, thus leaving banks more vulnerable to runs. Abstracting from runs, Di Tella and Kurlat (2021) find that the optimal hedging strategy by banks is consistent with long-duration assets and short-duration liabilities.

As in Cole and Kehoe (2000), we assume that a sunspot determines the outcome when a bank is vulnerable to a run.[15] Specifically, with probability λ, a run occurs, and the vulnerable bank defaults. With complement probability $1-\lambda$, a run does not occur and the vulnerable bank is able to borrow. We assume that this sunspot draw is i.i.d. across banks.

Using the default thresholds, we define $d_1(n_1, k_1, z_1^D)$ as the probability that an individual bank with net worth n_1, capital k_1, and default productivity z_1^D will default in period $t = 1$. Then,

$$d_1(n_1, k_1, z_1^D) = \begin{cases} 0 & \text{if } z_1^D \leq \hat{z}^{Run}(n_1, k_1), \\ \lambda & \text{if } \hat{z}^{Run}(n_1, k_1) < z_1^D \leq \hat{z}^F(n_1, k_1), \\ 1 & \text{if } z_1^D > \hat{z}^F(n_1, k_1). \end{cases} \tag{13}$$

Because investors are assumed to be risk neutral, the bond price in period 0 is given by the expected value of repayment in period 1. Abusing notation, we can express it as

$$q_0(n_1, k_1) = (1-\lambda)F(\hat{z}^F(n_1, k_1)) + \lambda F(\hat{z}^{Run}(n_1, k_1)). \tag{14}$$

A bank that has no leverage clearly does not default, and is not vulnerable to a run. The following lemma formalizes this simple point.

Lemma 2. *A bank without deposits never defaults. That is,*

$$\hat{z}^F(n_1, k_1) \geq \hat{z}^{Run}(n_1, k_1) > z$$

for $n_1 = (z + p_1)k_1 - Rb_1$ *and* $b_1 \leq 0$.

Proof. In Online Appendix B.2. □

Bank Problem at $t = 0$

At $t = 0$, the problem of a bank consists of choosing dividends, deposits, and investment to maximize its expected payoff. *Unlike the problem at $t = 1$, the bank problem in period $t = 0$ faces uncertainty.* Specifically, when choosing its portfolio, the bank must consider how the outside option shock z_1^D and the sunspot at $t = 1$ will affect its decision to repay or default. Moreover, the bond price schedule at which the bank will be able to borrow from investors, $q_0(n_1, k_1)$, will reflect the risk of default.

[15] This is a somewhat arbitrary equilibrium selection device, and alternative assumptions are possible. See, for example, the specification in Bocola and Dovis (2019).

The bank's problem at $t = 0$ can be formulated as follows:

$$V_0(n_0) = \max_{\substack{c_0 \geq 0, \\ k_1 \geq 0, b_1}} \left\{ u(c_0) + \beta \int_{\underline{z}}^{\overline{z}} \left[d_1(n_1, k_1, \tilde{z}) V_1^D(k_1, \tilde{z}) + (1 - d_1(n_1, k_1, \tilde{z})) V_1^R(n_1) \right] dF(\tilde{z}) \right\}, \quad (15)$$

subject to

$$c_0 = n_0 + q_0(n_1, k_1) b_1 - p_0 k_1,$$
$$n_1 = (z + p_1) k_1 - R b_1,$$

where $n_0 = (z + p_0) k_0 - R b_0$.

A reader may wonder why the value function of a bank that repays when facing a run, V_1^{Run}, does not appear in (15). The reason is that V_1^{Run} represents the payoff of an off-equilibrium choice, as a bank that repays in the presence of a run will not face one. Although not appearing directly in (15), V_1^{Run} is nonetheless essential for determining the run threshold, which in turn affects the default probability and the bond price schedule faced by the bank.

Discussion on Bank Defaults

A central ingredient of our model is that banks take leverage decisions subject to default risk. In particular, banks have the choice to repay or default on their obligations. This contrasts with much of the banking literature following Diamond and Dybvig (1983) where if a bank's assets cannot cover the liabilities, it must default.[16] To the extent that shareholders can choose in practice whether to capitalize a distressed bank or let it fail, we see this as a desirable feature of the model. Moreover, a model where default is privately optimal is helpful in tracing the normative implications and determining the scope for policy intervention.

However, our model is obviously stylized and does not capture detailed institutional features on actual bank's default and resolution policies. Our specific assumption regarding the outside option value for a defaulting bank enhances the tractability of the model—specifically, by making default decisions solely a function of leverage.[17] Similarly, the assumption of log utility for banks implies that they must always ensure they can consume strictly positive amounts under both

[16] More generally, in most macro-finance models default does not occur in equilibrium because the existence of a complete set of state-contingent securities rule out default in equilibrium (e.g., Kehoe and Levine, 1993; Lorenzoni, 2008) or simply the possibility of default is not considered.

[17] The assumption that shareholders in our model continue to receive payments after default is not necessarily unrealistic. Lehman Brothers and Bear Stearns paid almost as many dividends in the run-up to the 2008 crisis as in the years preceding the crisis, effectively transferring resources from bondholders to shareholders (Acharya et al., 2017, 2022). More recently, Credit Suisse shareholders in March 2023 perceived positive payoffs, while some of the bondholders did not.

repayment and default. This allows us to obtain closed-form expressions for banks' policies and values.

We highlight that what is central in our normative analysis is the *differential impact of aggregate leverage on the value of repaying banks*, depending on whether they are subject to runs or not. In this vein, Appendix D shows how the analysis can be extended to environments with arbitrary default costs.

3 Competitive Equilibrium

We have examined the problem of individual banks taking as given prices for capital $\{p_0, p_1\}$. We now turn to the determination of these prices in general equilibrium. The market clearing condition for capital requires that banks' demand for capital equals K in both periods $t = 0$ and $t = 1$.

In what follows, we narrow attention to *symmetric pure-strategy equilibria*: banks with the same state variables choose the same policies in all periods, and such policies do not involve mixed strategies. We can define a (symmetric, pure-strategy) competitive equilibrium as follows.

Definition 1 (Competitive equilibrium)**.** Given an initial level of deposits, B_0, and a run probability, λ, a competitive equilibrium consists of prices of capital in the two periods, $\{p_0, p_1\}$, borrowing choices in the two periods for repaying banks, $\{B_1, B_2\}$, capital choices in the two periods for repaying banks, $\{K_1, K_2\}$, initial net worth in the two periods $\{N_0, N_1\}$, default threshold functions $\{\hat{z}^F, \hat{z}^{Run}\}$, a default probability function d_1, a period-1 repayment value function V_1^R, a period-1 default value function V_1^D, and a price schedule q_0 such that the following conditions obtain:

(a) Repaying banks optimize.

For $t = 0$, given $\{p_0, q_0, V_1^R, V_1^D, d_1\}$, there exist optimal policy functions $\{k_1, b_1\}$ that solve the bank's problem, (15). In addition, $K_1 = k_1(N_0)$ and $B_1 = b_1(N_0)$.

For $t = 1$, given p_1, there exist optimal policy functions $\{k_2, b_2\}$ that solve the repaying bank's problem (4) where V_1^R is the resulting value function.

In addition, $K_2 = k_2(N_1)$ and $B_2 = b_2(N_1)$.

(b) Default is optimal. That is, given p_1 and λ, V_1^D is given by (1); the default threshold functions $\{\hat{z}^F, \hat{z}^{Run}\}$ are given by (10) and (11), respectively; and the default probability function d_1 is given by (13).

(c) Investors' pricing condition holds. That is, the bond price schedule q_0 satisfies (14) given λ and the default threshold functions.

(d) The market for capital clears at $t = 0$ and $t = 1$. That is, $K_1 = K$ and

$$\left[(1-\lambda)F(\hat{z}^F(N_1, K_1)) + \lambda F(\hat{z}^{Run}(N_1, K_1))\right](K_2 - K) = 0,$$

and where $N_0 = (z + p_0)K - RB_0$ and $N_1 = (z + p_1)K_1 - RB_1$.

The market clearing conditions in part (d) ensures that the total demand for capital equals the total supply of capital. As long as a positive mass of banks does not default in period $t = 1$, the second condition requires that $K_2 = K$.

A first result is that a default in period $t = 1$ with probability one is not part of an equilibrium: a bank could instead choose not to borrow at all in period $t = 0$ and have a higher payoff.

Lemma 3. *In any competitive equilibrium,* $q_0(N_1, K_1) > 0$.

Proof. In Online Appendix B.3. □

The above implies that in an equilibrium, there will be a strictly positive fraction of banks that do not default in period $t = 1$. With this, we can establish the following result:

Lemma 4. *In any competitive equilibrium,* $p_1 \in (\phi/R, z/R]$.

Proof. In Online Appendix B.4. □

If $p_1 < \phi/R$, there would be an infinite demand for capital. Conversely, if $p_1 > z/R$, there would be zero demand for capital. Market clearing therefore requires $p_1 \in (\phi/R, z/R]$. Combining Lemmas 3 and 4, it follows that $N_1 > 0$ in equilibrium.

3.1 The Initial Portfolio Choice

We now analyze the bank's portfolio decision in period $t = 0$—that is, its choice of borrowing b_1, capital k_1, and consumption c_0. Toward a characterization, it is useful to reformulate the problem in terms of choosing leverage and capital.

Redefining functions in terms of leverage. We know that in period $t = 0$, equilibrium requires that the demand for capital be $K_1 = K > 0$. We thus focus on the case where $k_1 > 0$, and define leverage l_1 as

$$l_1 \equiv \frac{b_1}{k_1}.$$

By Lemmas 3 and 4, equations (10) and (11) imply that the default thresholds that are relevant in equilibrium are just functions of n_1/k_1, or equivalently functions of l_1 alone.[18]

Abusing notation, we let $\hat{z}^F(l_1|p_1)$ and $\hat{z}^{Run}(l_1|p_1)$ refer to the thresholds as functions of leverage, where we make explicit the dependence on the price of capital in period 1. It is then immediate from equation (13) that the default probability and the bond price are also a function of leverage, which we redefine by $d_1(l_1, z_1^D|p_1)$ and $q_0(l_1|p_1)$, respectively. Again, making explicit the dependence on the price of capital in period 1, we let $V_1^R(n_1|p_1)$ denote the repayment value function in period $t = 1$.

Optimality at $t = 0$. We can now analyze the bank's problem in period $t = 0$, defined in (15). We have the following result.

Lemma 5. *In any competitive equilibrium, there is no l_1 such that $q_0(l_1|p_1)l_1 \geq p_0$.*

Proof. In Online Appendix B.5. □

If the inequality in the lemma were violated, banks would demand an unbounded amount of capital, and the market would fail to clear. It follows from this lemma that, in a competitive equilibrium, initial net worth must be non-negative, $n_0 \geq 0$. Otherwise the feasible set in period $t = 0$ is empty, and the bank's problem in period $t = 0$ would not be well defined. We will restrict attention to competitive equilibria with strictly positive net worth, $n_0 > 0$, which guarantees strictly positive consumption in period $t = 0$, and thus a payoff to the banks that is bounded below.

Given a choice of leverage l_1, we can solve for the optimal capital and consumption choice in period $t = 0$:

$$c_0(n_0) = \frac{1}{1 + \beta(1 + \beta)} n_0, \tag{16}$$

$$k_1(n_0, l_1|p_0, p_1) = \frac{\beta(1 + \beta)}{1 + \beta(1 + \beta)} \left(\frac{n_0}{p_0 - q_0(l_1|p_1)l_1} \right). \tag{17}$$

Despite facing default risk, the optimal policies for consumption and capital follow a simple expression, owing to log utility and the fact that the probability of default is only a function of leverage.

[18] To see this, note that $n_1/k_1 = ((z + p_1)k_1 - Rl_1k_1)/k_1 = z + p_1 - Rl_1$.

When choosing leverage, a key consideration is how leverage affects the bond price, $q_0(l_1|p_1)$, given by (14). Because $\hat{z}^F$ and $\hat{z}^{Run}$ are weakly decreasing in l_1, the bank takes into account that higher leverage reduces the price at which it can issue bonds. To inspect these implications, we examine the optimality condition for leverage. The following assumption guarantees the differentiability of the bank's problem.

Assumption 1. *The probability distribution function f is continuous and such that $f(\underline{z}) = f(\overline{z}) = 0$.*

We are now ready to state the following result.

Lemma 6 (Necessary condition for l_1). *Suppose that $n_0 > 0$ and that Assumption 1 holds. Then, a level of leverage l_1 that solves problem (15) must satisfy*

$$\frac{1}{c_0} - \frac{\beta R}{c_1} = -\frac{\overbrace{(1-\lambda) f(\hat{z}^F(l_1|p_1)) \frac{\partial \hat{z}^F(l_1|p_1)}{\partial l_1} + \lambda f(\hat{z}^{Run}(l_1|p_1)) \frac{\partial \hat{z}^{Run}(l_1|p_1)}{\partial l_1}}^{\textit{Change in default probability}}}{(1-\lambda) F(\hat{z}^F(l_1|p_1)) + \lambda F(\hat{z}^{Run}(l_1|p_1))} \frac{l_1}{c_0}$$

$$- \frac{\overbrace{\lambda f(\hat{z}^{Run}(l_1|p_1)) \frac{\partial \hat{z}^{Run}(l_1|p_1)}{\partial l_1}}^{\textit{Change in run probability}}}{(1-\lambda) F(\hat{z}^F(l_1|p_1)) + \lambda F(\hat{z}^{Run}(l_1|p_1))} \frac{\beta}{k_1} \Big[\overbrace{V_1^R(n_1|p_1) - V_1^D(k_1, \hat{z}^{Run}(l_1|p_1))}^{\textit{Additional loss in case of runs}} \Big], \quad (18)$$

where $n_1 = (z + p_1 - Rl_1)k_1$, $c_0 = c_0(n_0)$, $c_1 = c_1(n_1)$, and $k_1 = k_1(n_0, l_1|p_0, p_1)$.

Proof. In Online Appendix B.6. □

The bank's Euler equation (18) reflects that when the bank changes its leverage, it brings more resources to the present period, promises future repayments, and alters the probability of default, which in turn affects the price at which it borrows. If leverage does not affect the default probability at the *margin*, the right-hand side of (18) is zero. Given that bond prices are actuarially fair, we obtain a standard inter-temporal Euler equation with an intertemporal price given by R. Notice that even if leverage does not affect the default probability at the margin (and thus default risk does not appear explicitly in the Euler equation), the level of default risk still influences the optimal choice of leverage, as the costs associated with default incentivize banks to reduce leverage.

If leverage increases the default probability at the margin, the right-hand side of (18) becomes positive. In this case, the marginal utility benefits of borrowing and consuming today exceed the marginal cost of borrowing and reducing consumption to service the debt in repayment states. This gap emerges because when higher leverage increases the default probability at the margin, leverage introduces an additional cost. Specifically, to compensate bondholders for the increased default risk, the bank must issue the infra-marginal unit of deposits at a lower price. While the bank pays, on average, the risk-free rate, it still absorbs the default cost when it defaults.

In the absence of runs, the cost of raising leverage is given solely by the reduction in the bond price at which the bank issues deposits, as reflected in the first line of (18). A marginal increase in leverage in this case does not alter the bank's continuation value, as a bank at the threshold is indifferent between defaulting and repaying when there are no runs. *However, when a bank is subject to runs, a fall in the run threshold introduces an additional cost. At the run threshold, the bank faces a discrete drop in its continuation value in the event of a run.* This occurs because, at the threshold, the bank would prefer to repay if it could roll over its deposits. The additional cost is then given by the product of the marginal change in the run probability and the loss in the event of a run at the threshold (i.e., the reduction in the value function), as shown in the second line of (18).

3.2 Asset Prices and Aggregate Leverage

Next, we show the equilibrium relationship between the price of capital, p_1, and aggregate leverage, $L_1 \equiv B_1/K$—that is, the level of aggregate borrowing in period $t = 0$ per unit of capital.

In a scenario where the price is such that $z/R > p_1 > \phi/R$, we know from (7), that aggregate investment in capital is given by

$$K_2 = \frac{\beta}{(1+\beta)(p_1 - \phi/R)} N_1.$$

Using $N_1 = (z + p_1 - RL_1)K$ and $K_2 = K$, we have an expression for the market clearing price of capital $p_1 = \beta z + (1+\beta)\phi/R - \beta R L_1$. Given that $z/R > p_1 > \phi/R$, this requires that $L_1 \in (\hat{L}, \bar{L})$, where

$$\hat{L} \equiv ((1+\beta)\phi - z(1-\beta R))/(\beta R^2), \qquad \bar{L} \equiv (z + \phi/R)/R.$$

For $p_1 = z/R$, banks become indifferent across bonds and capital as the portfolio is consistent with its optimal consumption and the borrowing constraint. Using (5), (6), and $K_2 = K$, the borrowing constraint can be written as $(p_1 - \phi/R)K \leq \beta N_1/(1+\beta)$, which leads to $L_1 \leq \hat{L}$.

The lemma below formalizes these results and presents the equilibrium price of capital.

Lemma 7. *In equilibrium, $L_1 < \bar{L}$, and the price of capital is given by*

$$p_1 = \mathcal{P}_1(L_1) \equiv \begin{cases} \frac{z}{R} & \text{if } L_1 \leq \hat{L}, \\ \beta z + (1+\beta)\frac{\phi}{R} - \beta R L_1 & \text{if } L_1 \in (\hat{L}, \bar{L}). \end{cases} \tag{19}$$

Proof. In Online Appendix B.7. □

The price of capital in period 1 is continuous with respect to aggregate leverage L_1, but it features a kink at $\hat{L}$, where the leverage constraint becomes binding. For $L \leq \hat{L}$, the leverage constraint does not bind, and arbitrage requires that the return on capital equals the return on bonds. For $L_1 > \hat{L}$, the return on capital exceeds the return on bonds, and market clearing requires a price of capital that is decreasing in aggregate leverage. The result that the price of capital is decreasing in leverage is a central one that arises in economies with financial frictions (Shleifer and Vishny, 1992; Kiyotaki and Moore, 1997) and will play an important role in our analysis.

The value of aggregate leverage L_1 is sufficient to characterize the rest of a competitive equilibrium. Specifically, given L_1, the price of capital in period 1, p_1, is determined by (19). In turn, given p_1, we can determine the default thresholds, $\hat{z}^F$ and $\hat{z}^{Run}$, by (10) and (11), as well as the default probability function d_1 and the bond price schedule q_0 using (13) and (14). The value of b_2 can then be found using (6) given that $k_2(n_1) = K$. Additionally, V_1^R is specified in Lemma 1 and V_1^D is specified in (1). Finally, p_0 is determined using (17) and $l_1 = L_1$:

$$p_0 = \mathcal{P}_0(L_1) \equiv (1 + \beta(1+\beta))q_0(L_1|\mathcal{P}_1(L_1))L_1 + \beta(1+\beta)\left[z - \frac{RB_0}{K}\right]. \tag{20}$$

Given p_0, p_1, and L_1 that satisfies the above, an equilibrium requires that, in addition, L_1 corresponds to banks' optimal choice of leverage.

4 Equilibrium and Efficiency Without Runs

In this section, we analyze the properties of the competitive equilibrium without runs by setting $\lambda = 0$. We will establish both existence and uniqueness of a competitive equilibrium and show that, in the absence of runs, the economy is constrained efficient. After providing these welfare theorem results, we will return to the economy with runs in Section 5 and show that the competitive equilibrium is constrained inefficient.

4.1 Constrained Efficiency Without Runs

We consider the problem of a planner that chooses leverage in period 0 on behalf of banks, while leaving all other choices unrestricted, and letting all markets clear competitively. Crucially, banks retain the ability to make default decisions, and therefore, the bond price at which the planner issues bonds in period 0 continues to reflect the probability of default in period 1, characterized above.

As we discussed at the end of the previous section, leverage is all that is needed to characterize an equilibrium. Notice in particular that given the choice of leverage by the planner, the consumption and investment decision of banks at $t = 0$ is given by (16) and (17), and the market clearing prices for capital, p_0 and p_1, are then given respectively by (20) and (19).

We assume that the planner maximizes the welfare of banks. Note, however, that because investors have linear utility, they always break even ex ante and therefore remain indifferent among any level of leverage chosen by the planner. The planning problem is

$$\max_{L_1, c_0 \geq 0, n_1, p_1} \left\{ u(c_0) + \beta \int_{\underline{z}}^{\overline{z}} \max\{V_1^R(n_1|p_1), V_1^D(K, \tilde{z})\} dF(\tilde{z}) \right\}, \tag{21}$$

subject to

$$c_0 \leq zK - RB_0 + q_0(L_1|p_1)L_1K,$$

$$L_1 < \overline{L},$$

$$n_1 = (z + p_1)K - RL_1K,$$

$$p_1 = \mathcal{P}_1(L_1).$$

In formulating the continuation value in (21), we have used that the default decision at $t = 1$ is based on the comparison between V_1^R and V_1^D, given that there are no runs. In addition, we have set the budget constraint as an inequality (at a solution, it will always be binding).

The problem is similar to that of an individual bank, with two key differences. First, the planner is subject to a resource constraint in period 0 that reflects that banks must hold K units of capital (note that p_0 cancels out in this constraint). Second, when choosing leverage, the planner internalizes that this affects the price of capital in period 1. This, in turn, influences the default threshold in period 1 and consequently affects the bond price in period 0.

We will impose the following assumption:

Assumption 2. *The parameters are such that* $zK - RB_0 + \overline{m} > 0$*, where*

$$\overline{m} \equiv \max_{L_1 \leq \overline{L}} q_0(L_1|\mathcal{P}_1(L_1))L_1K.$$

This assumption guarantees that a feasible choice (c_0, L_1) exists that strictly dominates any allocation with $c_0 = 0$. We can show that the constraint set is bounded and closed and the objective is continuous and bounded anywhere except at $c_0 = 0$.[19] This guarantees that there is a solution to the planner's problem. We define such solutions as representing constrained-efficient levels of leverage.

Definition 2 (Constrained-efficient leverage)**.** A competitive equilibrium is *constrained efficient* in the absence of runs if the equilibrium leverage level L_1 solves the planning problem in (21).

We can use Assumption 1, which guarantees differentiability of the objective function, to state a necessary condition for a constrained-efficient leverage:

$$\frac{1}{c_0} - \frac{\beta R}{c_1} = -\frac{f(\hat{z}^F(L_1|p_1))}{F(\hat{z}^F(L_1|p_1))} \frac{\partial \hat{z}^F(L_1|p_1)}{\partial L_1} \frac{L_1}{c_0}. \tag{22}$$

This Euler equation is analogous to the competitive equilibrium condition (18), once we set $\lambda = 0$. The reader may have expected to see terms in the planner's optimality condition that capture the combined impact of leverage on the price of capital and the effect of this price on default thresholds (specifically, chain rule terms involving derivatives of the thresholds with respect to the price p_1 and of the price p_1 with respect to L_1). However, these first-order effects are equal to zero at equilibrium.

To see this, consider first the case $L_1 \in (\hat{L}, \overline{L})$. Noting that this implies $p_1 < z/R$ and using (10),

[19] Specifically, we can enlarge the constraint set to be $L_1 \leq \overline{L}$ (now with a weak inequality), as the boundary choice of $L_1 = \overline{L}$ is dominated by the choice of $L_1 = 0$. To see this, we start by by substituting n_1 and p_1 as functions of L_1. It is possible to show that $V_1^R(n_1|p_1)$ and $\hat{z}^F(L_1|p_1)$ are continuous and differentiable functions of L_1 after this substitution. In addition, as $L_1 \nearrow \overline{L}$, $V_1^R(n_1|p_1)$ tends to minus infinity, and thus default occurs with certainty at the limit (that is, $\hat{z}^F(L_1|\mathcal{P}_1(L_1))$ tends to zero). Thus, banks raise no revenue in period $t = 0$ and are certain to default in period $t = 1$ as $L_1 \nearrow \overline{L}$, and thus the choice of $\overline{L}$ is dominated. Similarly, we can also show that there is lower bound on leverage as $L_1 \geq \min\{-(z - RB_0/K), 0\}$ as for $L_1 \leq 0$, $q(L_1|p_1) = 1$. Note that q_0 is bounded, and thus the set of (c_0, L_1) that satisfies the constraints is bounded. Given that all inequalities in the constraint set are weak, the feasible set is closed. Finally, the functions q_0 and $\mathcal{P}_1$ are continuous, and the objective function is continuous in c_0 and L_1 and is bounded everywhere except at $c_0 = 0$ (where it takes a minus infinity value). Note also that a simpler sufficient condition for Assumption 2 is that $zK - RB_0 > 0$.

we have that

$$\frac{\partial z^F(L_1|p_1)}{\partial p_1} = \hat{z}^F(L_1|p_1)\left[\frac{1+\beta}{z+p_1-RL_1} - \frac{\beta R}{Rp_1-\phi}\right], \tag{23}$$

Evaluating this expression at the equilibrium price $p_1 = \mathcal{P}_1(L_1)$, we can see that the change in the threshold is zero.

For the case $L_1 < \hat{L}$, banks are unconstrained in period 1 and p_1 is constant. That is, $\mathcal{P}_1'(L_1) = 0$. Together with (23), it thus follows that for any $L_1 < \bar{L}$, we have

$$\left.\frac{\partial z^F(L_1|p_1)}{\partial p_1}\right|_{p_1=\mathcal{P}_1(L_1)} \mathcal{P}_1'(L_1) = 0.$$

which shows that *there is no first-order effect of leverage on the default threshold operating through the price, p_1.*[20] The reason for this result is that, in an equilibrium, banks are neither net sellers nor net buyers of capital. Recall that defaulting banks are assumed to retain their capital holdings, which implies that in equilibrium, repaying banks must also maintain their capital holdings.

This result that (22) coincides with the competitive equilibrium condition (18), once we set $\lambda = 0$, suggests that the equilibrium level of leverage is constrained efficient. However, given that the individual bank problem and the planner problem are non-convex, the fact that the first-order condition for leverage is the same in both problems is not sufficient to establish that the resulting allocations are the same.[21] We proceed to prove efficiency through an alternative route. To this end, we first present the following lemma.

Lemma 8. *Suppose $\lambda = 0$. Let $p_1 = \mathcal{P}_1(L_1)$ for $L_1 < \bar{L}$. For any $\hat{p}_1 \in (\phi/R, z/R]$, we have:*

(i) $V_1^R((z+p_1)K - RKL_1|p_1) \leq V_1^R((z+\hat{p}_1)K - RKL_1|\hat{p}_1)$, *and*

(ii) $q_0(L_1|p_1) \leq q_0(L_1|\hat{p}_1)$,

with the first inequality is strict if $\hat{p}_1 \neq p_1$.

Proof. First, let us define

$$v(p) \equiv \hat{A} + (1+\beta)\log(z+p-RL_1) + \beta\log\left(\frac{z-\phi}{p-\frac{\phi}{R}}\right),$$

[20] This also explains why the objective function is differentiable even though the function $\mathcal{P}_1(L_1)$ is not differentiable at $L_1 = \hat{L}$.

[21] See Bloise, Martins-da Rocha and Vailakis (2025) for examples where relying on first-order conditions can give rise to misleading conclusions when studying constrained social optima.

for $p > \phi/R$ and $p > RL_1 - z$, and where $\hat{A} = A + (1+\beta)\log K$.

The function $v(p)$ is differentiable. And $v'(\bar{p}) = 0$ at $\bar{p} \equiv \beta z + (1+\beta)\phi/R - \beta RL_1 = \phi/R + \beta R(\bar{L} - L_1) > \phi/R$. Note also that

$$v''(\bar{p}) = \frac{1}{\beta(1+\beta)(R(\bar{L} - L_1))^2} > 0.$$

It follows that the function $v(p)$ achieves a strict local minimum at $\bar{p}$. Given that $v'(p)$ has a unique zero, it follows that the function $v(p)$ is strictly decreasing in $p \in (\phi/R, \bar{p})$ and strictly increasing in $p > \bar{p}$.

Now if $p > \phi/R$, it follows that

$$z + p - RL_1 > z + \phi/R - RL_1 = R[(z + \phi/R)/R - L_1] = R(\bar{L} - L_1),$$

Hence, for $L_1 < \bar{L}$, we have that $p > RL1 - z$.

Using Lemma 1, we have that

$$V_1^R((z+p)K - RKL_1 | p) = v(p),$$

For $L_1 \in (\hat{L}, \bar{L})$, $p_1 = \bar{p} \in (\phi/R, z/R)$, and the result in part (i) is immediate, as $v(p)$ has a strict minimum at $p = \bar{p} = p_1$. That is, for any $\hat{p}_1 \in (\phi/R, z/R]$, $v(\hat{p}_1) \geq v(p_1)$ with strict inequality if $\hat{p}_1 \neq p_1$.

For $L_1 \leq \hat{L}$, we have $p_1 = z/R$ and $\bar{p} \geq z/R$. Then, the result in part (i) is immediate, as $v(p)$ is strictly decreasing in $p \in (\phi/R, z/R]$, and thus for any $\hat{p}_1 \in (\phi/R, z/R]$, $v(\hat{p}_1) \geq v(p_1)$ with strict inequality if $\hat{p}_1 \neq p_1$.

Given that V_1^R encounters a minimum at p_1 and V_1^D is independent of p_1, it follows that the fundamental default threshold, $\hat{z}^F$, also encounters a minimum at p_1, as can be seen from (23). Part (ii) then follows. That is, the bond price q_0 also encounters a minimum at p_1, although it does so weakly, given that F could be constant in the relevant range. □

Part (i) of Lemma 8 establishes that the value function of repayment is minimized at the equilibrium price (within the range of prices between ϕ/R and z/R). Part (ii) shows that the bond price is minimized at the equilibrium price (for prices in the same range).

The underlying economics at work in the lemma are as follows. In equilibrium, repaying banks are neither net buyers nor net sellers of capital in period 1. If the price of capital were to deviate from its equilibrium value, a bank that repays in period 1 is able to choose the same allocation as in equilibrium. However, the bank would strictly benefit by buying capital when its price decreases or selling capital when its price increases, thereby achieving a strictly better outcome if the price deviates from its equilibrium level.

With this result in hand, we can prove a first welfare theorem-like result for our economy.

Proposition 1 (Constrained-efficiency)**.** *Suppose $\lambda = 0$. Any competitive equilibrium is constrained efficient.*

Proof. Consider an equilibrium. Let L^E denote the equilibrium level of leverage, and let $p_1^E = \mathcal{P}_1(L^E)$ denote the equilibrium price of capital in period 1. In addition, let L^P denote a level of leverage that solves the planner's problem (21), and let $p_1^P = \mathcal{P}_1(L^P)$ denote the price of capital in period 1 that corresponds to L^P.

Let $\mathbb{E}V_1(l_1, k_1|p_1)$ represent the expected value for the bank in period 1 as a function of the leverage and capital chosen in period 0. Using $\lambda = 0$,

$$\mathbb{E}V_1(l_1, k_1|p_1) \equiv \int_{\underline{z}}^{\overline{z}} \max\{V_1^R((z + p_1)K - Rl_1K|p_1), V_1^D(K, \tilde{z})\}dF(\tilde{z}),$$

where we have used that $\hat{z}^F$ is the point of indifference between repayment and default.

Part (ii) of Lemma 8 implies that $q_0(L^P|p_1^E) \geq q_0(L^P|p_1^P)$. Using in addition that $q_0(L^P|p_1^P) = q_0(L^P|p_1^E) = 1$ for $L^P \leq 0$ by Lemma 2, it follows that

$$q_0(L^P|p_1^E)L^PK \geq q_0(L^P|p_1^P)L^PK.$$

And hence,

$$zK - RB_0 + q_0(L^P|p_1^E)L^PK \geq zK - RB_0 + q_0(L^P|p_1^P)L^PK. \tag{24}$$

This implies that L^P is feasible when banks face the equilibrium prices p_1^E. Now, in the competitive equilibrium, banks at time $t = 0$ prefer to borrow $B_1 = L^EK$ rather than L^PK when facing equilibrium prices p_1^E. That is, it must be that

$$\begin{aligned} &u(zK - RB_0 + q_0(L^E|p_1^E)L^EK) + \beta\mathbb{E}V_1(L^E, K|p_1^E) \\ &\qquad \geq u(zK - RB_0 + q_0(L^P|p_1^E)L^PK) + \beta\mathbb{E}V_1(L^P, K|p_1^E), \end{aligned} \tag{25}$$

Part (i) of Lemma 8 also implies that $\mathbb{E}V_1(L^P, K|p_1^E) \geq \mathbb{E}V_1(L^P, K|p_1^P)$. Taken together, (24) and (25) yields

$$\begin{aligned} &u(zK - RB_0 + q_0(L^E|p_1^E)L^EK) + \beta\mathbb{E}V_1(L^E, K|p_1^E) \\ &\qquad \geq u(zK - RB_0 + q_0(L^P|p_1^P)L^PK) + \beta\mathbb{E}V_1(L^P, K|p_1^P) \end{aligned} \tag{26}$$

which implies that banks in the competitive equilibrium achieve a utility weakly higher than in the constrained-efficient allocation.

By definition, constrained efficiency requires that

$$u(zK - RB_0 + q_0(L^P|p_1^P)L^P K) + \beta \mathbb{E} V_1(L^P, K|p_1^P) \geq u(zK - RB_0 + q_0(L^E, p_1^E)L^E K) + \beta \mathbb{E} V_1(L^E, K|p_1^E), \tag{27}$$

since the planner could always have chosen the equilibrium level of leverage.

We then have that (26) and (27) imply that the competitive equilibrium choice of leverage also solves the planning problem. □

4.2 Uniqueness and Existence of Competitive Equilibria Without Runs

In the previous section, we showed that the competitive equilibrium, if it exists, is constrained efficient. That is, the equilibrium leverage choice must solve the planning problem (21). Given that the leverage choice uniquely determines the rest of the equilibrium allocation, this immediately implies that if the planner problem has a unique solution, then the competitive equilibrium, if it exists, is also unique. This already implies that the competitive equilibrium is generically unique. In the following proposition, we formalize and further strengthen this result.

Proposition 2 (Uniqueness)**.** *Suppose $\lambda = 0$ and that either*

(i) there is a unique solution to the planner problem (21)*, or*

(ii) there exists a competitive equilibrium with leverage $L_1 = B_1/K > \hat{L}$.

Then, there is at most one (symmetric pure-strategy) competitive equilibrium.

Proof. In Online Appendix B.8. □

Let us now turn to existence of the competitive equilibrium. As we argued above, there exists a solution to the planner's problem as long as Assumption 2 is satisfied. So let us consider a planner's solution with leverage L_1. Given the associated capital prices, for a competitive equilibrium to exist, we require that the optimal bank's choice of leverage at time $t = 0$ coincide with L_1. The payoff function of the bank problem equals that of the planner at L_1. And in addition, both the planner's and the bank's payoffs have a zero derivative at L_1 (a result that we will show below). However, this is not enough to guarantee existence. A second welfare theorem-like result requires additional restrictions to ensure that the bank's problem has a *global maximum at* L_1. The following condition on the density of z_1^D is sufficient:

Assumption 3 (Density)**.** *The probability distribution function f is such that $\frac{1-F(z)}{1+\beta} + \frac{f(z)}{F(z)}z$ is decreasing in z for any $z \in [\underline{z}, \overline{z}]$.*

We can now show existence of equilibrium.

Proposition 3 (Existence of competitive equilibrium)**.** *Suppose $\lambda = 0$ and that Assumptions 1, 2, and 3 hold. If, in addition,*

$$\mathcal{P}_0(L_1) > \sup_{l_1 < (z+p_1)/R} \{q_0(l_1|\mathcal{P}_1(L_1))l_1\},$$

where L_1 solves the planning problem, then there exists a competitive equilibrium.

Proof. In Online Appendix B.9. □

In the proof, we show that Assumption 2, together with the condition stated in the proposition, implies that the derivative of the bank's objective in (15) with respect to leverage can switch signs only once (from a positive value to a negative one). This derivative is zero at the planner's solution L_1, and thus $l_1 = L_1$ is a global optimum for the bank's problem. Note that the condition stated in the proposition is also necessary for an equilibrium to exist, as specified in Lemma 5.

Taking stock. To summarize, this section studied the efficiency, existence, and uniqueness of competitive equilibrium in the absence of runs. In particular, we provided a benchmark result where the competitive equilibrium is constrained efficient, indicating that banks' privately optimal leverage choices align with those of a benevolent planner. As we will show next, this result does not hold in the presence of runs: even if banks are neither net buyers nor net sellers in equilibrium, the interplay between runs and equilibrium default renders the competitive equilibrium constrained inefficient.

5 Optimal Regulation with Bank Runs

We now examine the constrained-efficient leverage in the environment with runs, $\lambda > 0$. We will show that the competitive equilibrium may no longer be constrained efficient and trace out the implications for banking regulation.

5.1 The Constrained-Efficient Allocation with Runs

As in Section 4.1, we consider the problem of a planner who chooses banks' leverage at date 0, while all other decisions are made by banks and markets clear competitively. Banks retain the ability to default, but a key distinction in this environment is that the default decision at $t = 1$ may now be triggered by a run.

With $\lambda > 0$, the planner's problem is now formulated as follows:

$$\max_{\substack{L_1, c_0 \geq 0, \\ n_1, p_1}} \left\{ u(c_0) + \beta \int_{\underline{z}}^{\overline{z}} \left[d_1(L_1, \tilde{z}|p_1) V_1^D(K, \tilde{z}) + (1 - d_1(L_1, \tilde{z}|p_1)) V_1^R(n_1|p_1) \right] dF(\tilde{z}) \right\}, \tag{28}$$

subject to

$$c_0 \leq zK - RB_0 + q_0(L_1|p_1)L_1K,$$
$$L_1 < \overline{L},$$
$$n_1 = (z + p_1)K - RL_1K,$$
$$p_1 = \mathcal{P}_1(L_1),$$

and where d_1 follows (13). As in the individual bank problem (15), the choice of the planner's leverage at $t = 0$ affects the default probability through the effects on the fundamental and run thresholds. While the individual bank internalizes that lower leverage reduces the default probability for given asset prices, the planner also takes into account how leverage affects asset prices, which in turn shift the fundamental and run thresholds. We have established that in the absence of runs, the fact that aggregate leverage affects the default probability has no welfare implications. That is, without runs, the planner would choose the same leverage as the competitive equilibrium. As we will show below, however, this is no longer the case when banks are subject to runs.[22]

5.2 Constrained Inefficiency with Runs

Consider an equilibrium where $p_1 < z/R$, or equivalently, $L_1 > \hat{L}$. Recall from condition (12) that if the price of capital in a competitive equilibrium with runs is such that $p_1 = z/R$, the presence of runs does not impact the equilibrium, as $\hat{z}^F = \hat{z}^{Run}$, and thus the competitive equilibrium remains

[22]Note that the planner's utility would be highest if λ were 0. That is, the planner would prefer that banks operate in an environment without runs. To see this, note that p_1 is independent of λ given L_1. For the same L_1, the planner attains a higher revenue per bond issuance in period $t = 0$, $q_0(L_1|p_1)L_1$, in the model without runs ($\lambda = 0$). Given that $V_1^R(n_1|p_1) \geq V_1^D(K, \tilde{z})$ for all $\tilde{z}$ where $d_1(L_1, \tilde{z}|p_1) > 0$, the planner prefers that banks operate in an environment without runs.

constrained efficient.

Assumption 1 guarantees the differentiability of the objective in the constrained-efficient problem. The first-order optimality condition yields:

$$
\begin{aligned}
\frac{1}{c_0}-\frac{\beta R}{c_1} &= -\frac{(1-\lambda)f(\hat{z}^F(L_1|p_1))\frac{\partial \hat{z}^F(L_1|p_1)}{\partial L_1}+\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{L_1}{c_0} \qquad (29)\\
&-\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{\beta}{K}\Big[V_1^R(n_1|p_1)-V_1^D(K,\hat{z}^{Run}(L_1|p_1))\Big]\\
&\underbrace{-\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))}{q_0(L_1|p_1)}\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial p_1}\mathcal{P}_1'(L_1)}_{\text{General eqm. effect on run probability}}\underbrace{\left[\frac{L_1}{c_0}+\frac{\beta}{K}\Big[V_1^R(n_1|p_1)-V_1^D(K,\hat{z}^{Run}(L_1|p_1))\Big]\right]}_{\text{Loss from runs}},
\end{aligned}
$$

for $L_1 \in (\hat{L},\overline{L})$ and where $c_0 = zK - RB_0 - q_0(L_1|p1)L_1K$, $c_1 = \frac{1}{1+\beta}(z+p_1-RL_1)K$, $n_1 = (z+p_1)K - RL_1K$, and $p_1 = \mathcal{P}_1(L_1)$. See Appendix C.2 for the derivation.

The first two lines in (29) mirror the optimality condition for individual banks (18). Recall that banks face an additional marginal cost from leverage when increasing borrowing raises the default probability and leads to a higher probability of a run. Just like individual banks, the planner faces this additional cost.

The third line in (29) encodes the general equilibrium impact of leverage that the planner internalizes. Unlike an individual bank, the planner recognizes that changing leverage affects the price of capital in period 1, which in turn shifts the run threshold and expected losses from runs. This general equilibrium effect can be decomposed into two terms.

The first term reflects the effect of leverage on the probability of a run. The sign of this effect depends on how leverage affects the price of capital, and how the price of capital, in turn, influences the run threshold. We now characterize these two relationships in turn.

We first differentiate (11) and obtain

$$
\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial p_1} = \hat{z}^{Run}(L_1|p_1)\frac{(1+\beta)\phi}{R(z+p_1-RL_1)p_1} > 0. \qquad (30)
$$

That is, an increase in the price of capital raises the run threshold and makes banks less vulnerable to runs. When a bank faces a run, it needs to sell capital to be able to repay the deposits being withdrawn. In this context, a higher price of capital benefits the bank facing a run, reducing the incentive for investors to withdraw their funds.[23]

We then differentiate (19), and obtain

$$\mathcal{P}_1'(L_1) = -\beta R < 0, \tag{31}$$

for $L_1 \in (\hat{L}, \bar{L})$. That is, the price of capital is decreasing in leverage. To the extent that banks are borrowing constrained in period 1, an increase in leverage requires lower asset prices in period 1 to clear the market for capital.

Combining (30) *and* (31) *shows that higher aggregate leverage increases banks' exposure to runs.*

The second term reflects the deadweight loss incurred when banks default under a run. This includes both the loss to lenders, which imply a lower bond price at time 0, and the reduction in banks' continuation value. Recall that when banks default due to fundamentals, they are ex post strictly better off than if they had repaid. In contrast, when default is triggered by a run, the bank would prefer to repay—provided that investors were willing to continue lending.

Taken together, these two terms establish that the final line in (29) is strictly positive, implying that the competitive equilibrium induces excessive leverage. The planner perceives a higher marginal cost of leverage because it internalizes that higher leverage—through general equilibrium effects—lowers asset prices and raises the probability of runs. While banks account for their own default risk, they do not internalize the effects of their leverage choices on asset prices and the vulnerability of other banks. This externality renders the competitive equilibrium constrained inefficient.

We summarize these results in the following proposition.

Proposition 4 (Constrained inefficiency in the presence of runs)**.** *Consider the case where $\lambda > 0$. Any competitive equilibrium with leverage L_1 such that $z/\mathcal{P}_1(L_1) > R$ and where $f(\hat{z}^{Run}(L_1|\mathcal{P}_1(L_1)) > 0$ is constrained inefficient.*

Proof. This follows directly from noticing that the necessary conditions for the equilibrium choice of leverage and the constrained-efficient level of leverage differ. □

Numerical illustration. Figure 2 illustrates the differences between the competitive equilibrium and constrained-efficient solutions. Panel (a) compares the levels of leverage under both allocations for $\lambda \in [0, 1]$. We observe that at $\lambda = 0$, the leverage levels in the competitive equilibrium and the constrained-efficient allocation coincide. As the probability of a run increases, leverage decreases

[23] In our analysis of credit easing in Amador and Bianchi (2024), we also highlighted the differential impact of the price of capital on banks facing a run because of their need for funds.

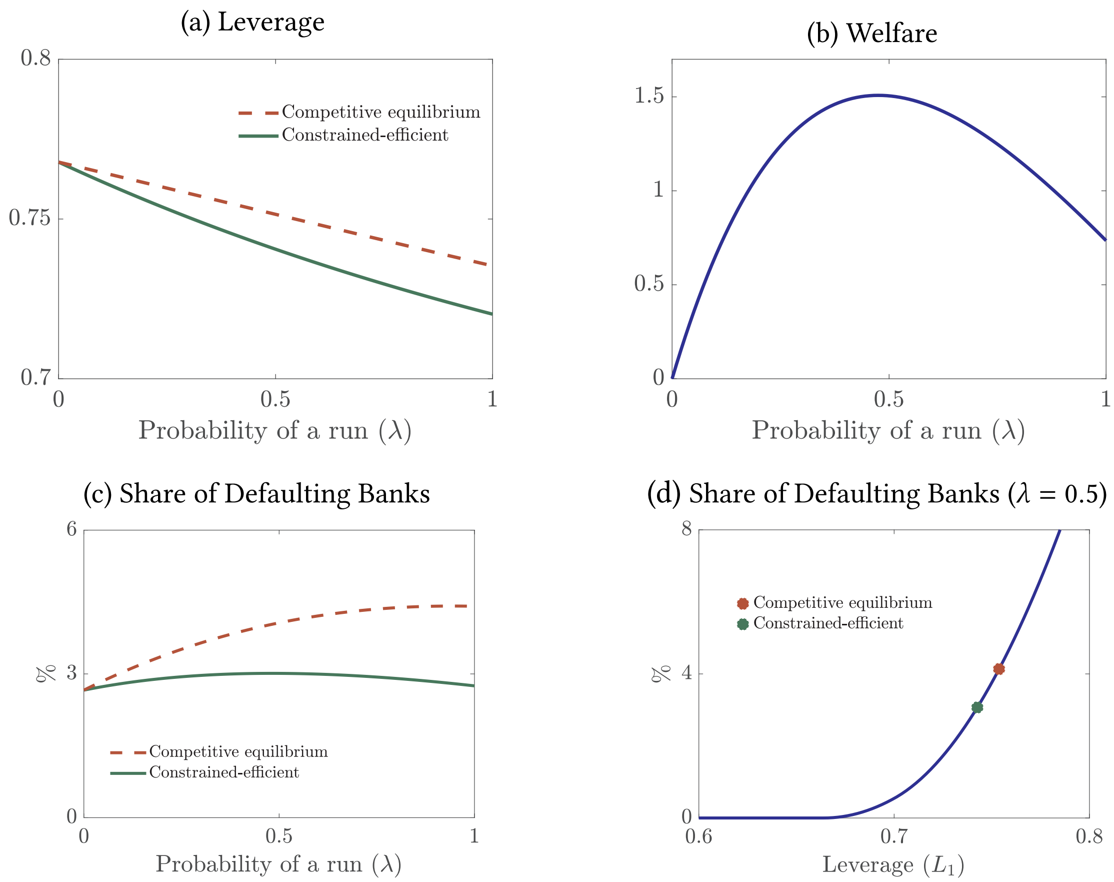

Figure 2: Competitive equilibrium vs. Constrained-efficient allocations

Notes: Panels (a) and (c) present, respectively, leverage and the share of defaulting banks for the competitive equilibrium and constrained-efficient solution as a function of λ. Panel (b) presents the gain from optimal regulation as a function of λ. Panel (d) sets $\lambda = 0.5$ and presents the share of defaulting banks for a range of leverage values; The solid dots represent the competitive equilibrium and constrained-efficient leverage for $\lambda = 0.5$. The simulation is generated using $\beta = 0.90, R = 1.04, z = 1, \phi = 0.2$, and $B_0 = 1.5$. The density for z_1^D is a triangular distribution with support [0,0.6].

in both scenarios, but, in line with the theory, the planner reduces leverage by more. That is, a higher probability of a run makes borrowing more costly for banks, but the planner perceives an even higher marginal cost.

Panel (b) shows the welfare gains from optimal policy, measured in terms of consumption-equivalent units. The figure reveals that welfare gains are non-monotonic in the probability of runs. That is, for low values of λ, an increase in the probability of a run results in larger welfare gains from policy. However, beyond a certain threshold, further increases in the probability of a run lead to smaller welfare gains from policy. This non-monotonicity arises because, as the probability of runs becomes very high, banks respond by endogenously reducing leverage due to a stronger precautionary motive, leaving less scope for welfare-improving policy interventions.

Consistent with the result that banks choose excessive leverage in the competitive equilibrium, panel (c) shows that the share of defaulting banks is higher in the competitive equilibrium than under the planner's allocation. Notably, a higher probability of a run does not necessarily translate into a higher share of defaulting banks. As panel (d) illustrates, holding leverage fixed, the share of defaulting banks increases with λ. However, the increase in λ leads to a lower desired leverage as a precautionary response to the higher likelihood of runs. The interaction of these two forces implies that, under the constrained-efficient allocation, the share of defaults is inverted-U shaped in λ: initially increasing, then declining as the precautionary response dominates.

Decentralization. We now turn to the decentralization of the constrained-efficient allocation. We focus on two policies: a tax on leverage and a capital requirement.

Consider a linear tax on banks' borrowing at $t = 0$, rebated lump-sum. In this case, the banks' budget constraint in period $t = 0$ becomes:

$$c_0 = (z + p_0)k_0 - Rb_0 + \frac{q_0(b_1, k_1)}{1 + \tau_0} b_1 - p_0 k_1 + T_0,$$

where τ_0 denotes the tax rate on borrowing and T_0 the lump-sum transfer. In equilibrium, the government budget constraint implies $T_0 = \frac{\tau_0}{1+\tau_0} q_0(L_1) L_1 K$.

Implementing the constrained-efficient leverage requires setting the tax such that

$$\tau_0^* = -\frac{\partial \hat{z}^{Run}(L_1^* | p_1^*)}{\partial p_1} \mathcal{P}_1'(L_1^*) \left[\frac{\lambda f(\hat{z}^{Run}(L_1^*|p_1^*))}{(1-\lambda)F(\hat{z}^F(L_1^*|p_1^*)) + \lambda F(\hat{z}^{Run}(L_1^*|p_1^*))} \right] \frac{c_1^*}{\beta R} \times \left[\frac{L_1^*}{c_0^*} + \frac{\beta}{K} \Big(V_1^R(n_1^*|p_1^*) - V_1^D(K, \hat{z}^{Run}(L_1^*|p_1^*)) \Big) \right], \quad (32)$$

where $\left\{L_1^*, c_1^*, n_1^*, p_1^*\right\}$ corresponds to the solution of the planner's problem. This expression corresponds to the wedge between the Euler equation for the planner (29) and for individual banks (18), normalized by period 1 marginal utility. If $\lambda > 0$ and $f(\hat{z}^{Run}) > 0$, then $\tau_0^* > 0$, indicating that implementing the constrained-efficient leverage requires *a strictly positive tax on borrowing*.

Looking at the tax formula in (32), we note that three elements determine the magnitude of the tax: how the run threshold varies with the asset price, how the price varies with leverage, and the losses incurred from a run. In particular, the tax is higher when the price of capital is more responsive to leverage, when the run threshold is more responsive to asset prices, and when there is a larger loss from runs.

An alternative policy that can implement the constrained-efficient leverage is a minimum capital requirement constraint. In particular, suppose that at $t = 0$, banks are required to maintain a net worth-to-assets ratio greater than κ:

$$\frac{(z + p_1)k_1 - Rb_1}{p_0 k_1} \geq \kappa.$$

By setting $\kappa = (z + p_1^* - RL_1^*)/p_0^*$, the central bank can implement the desired leverage. The idea is that when facing asset prices as in the constrained-efficient solution, banks would find it individually optimal to borrow more than the planner. Imposing a minimum capital requirement constraint thus prevents banks from leveraging excessively.

5.3 Discussion

Before concluding, let us discuss key features of our analysis and the implications for policy.

Short-term debt. The reliance on short-term borrowing is central to the emergence of endogenous runs in our model, as in Diamond and Dybvig (1983) and Cole and Kehoe (2000), and is therefore crucial for the inefficiency results we derive. If long-term deposits were available at the same borrowing costs as short-term deposits, banks would have incentives to borrow long and eliminate runs. In practice, however, short-term debt is often cheaper for banks due to several reasons, including liquidity benefits of demand deposits (Stein, 2012) and the alignment of incentives under asymmetric information (Diamond and Rajan, 2000; Calomiris and Kahn, 1991).

Our framework could be extended to incorporate these considerations. In particular, consider the case where banks have a choice in period 0 between short-term or long-term deposits, and suppose that investors value short-term deposits (linearly) in their utility.[24] Under this setup, there is a constant liquidity premium between short- and long-term deposits. If the liquidity premium is sufficiently large, banks will be at a corner where they issue only short-term deposits, and our equilibrium analysis remains unchanged.[25]

Default risk. The presence of risk gives rise to costly defaults in equilibrium, and is crucial for our inefficiency results in the presence of runs. In Appendix A, we show that in the absence of uncertainty (and thus equilibrium default), the competitive equilibrium is constrained efficient. In the absence of uncertainty, banks still face a borrowing constraint in period 0 given their inability to commit to repay in period 1. The possibility of runs tightens this borrowing constraint, which

[24]Notice that the fact that investors obtain a liquidity value from deposits does not alter the form of the planner's problem (28). This is because investors are assumed to be linear, and thus the presence of a liquidity premium just reduces the deposit rate in equilibrium to keep them indifferent.

[25]For a relatively smaller liquidity premium where the bank's portfolio is interior, our normative results on constrained inefficiency should remain as long as banks remain exposed to runs in equilibrium.

is itself affected by the endogenous price of capital. But without uncertainty, banks borrow up to the constraint in period 0, and their leverage choices remain constrained efficient.[26]

Policy implications. Our analysis provides theoretical foundations for the macroprudential approach to banking regulation. Notably, Basel III—introduced after the global financial crisis—requires banks to build capital buffers when there is a rise in the risk of financial instability.[27] The underlying principle is that curbing banks' risk-taking helps mitigate the social costs associated with financial distress (see e.g., Hanson, Kashyap and Stein, 2011). The framework developed in this paper supports this view by highlighting how individual banks fail to internalize that reducing their own leverage lowers the run risk faced by others. When banks collectively maintain higher equity buffers, asset prices rise, improving banks' liquidity during runs—since banks must sell assets to meet deposit withdrawals—which in turn reduces the incentive for depositors to run.

The framework also yields distinctive implications for the cyclicality of regulation. Unlike existing models that call for tighter leverage constraints in response to negative aggregate shocks, our model predicts a non-monotonic relationship between fundamentals and optimal policy: regulation is unwarranted when fundamentals are either strong or very weak, but most effective at intermediate levels where banks are most vulnerable to failures triggered by self-fulfilling runs.

6 Conclusions

We develop a theory of banking regulation in a general equilibrium model where banks may be vulnerable to self-fulfilling runs. We show that the competitive equilibrium is constrained efficient in the absence of runs, but the presence of runs renders the equilibrium leverage decisions inefficient. Central to our theory is the general equilibrium interaction between leverage, asset prices, and banks' default incentives: higher leverage reduces the demand for capital and lowers

[26] Dávila and Korinek (2018) introduce the terms "distributive externalities" and "collateral externalities" when discussing inefficiencies that arise in general equilibrium models with financial frictions. They define distributive externalities as the inefficiencies that arise when changes in prices redistribute wealth across agents with different marginal rates of substitution, whereas collateral externalities are those that arise when changes in prices affect financial constraints, such as the market value of assets used as collateral. The inefficiency in our model is not generated by a distributive externality because changes in prices do not lead to a redistribution of wealth—recall that banks are neither net buyers nor net sellers in equilibrium. It is also not generated by a collateral externality: while prices in our model do affect banks' ability to borrow (despite the absence of collateral), this feature is not the reason for the inefficiency. It is the existence of equilibrium default—and the interaction with bank runs—that renders the market outcome inefficient. See Appendix A for an analysis showing that, absent equilibrium default, the competitive equilibrium remains constrained efficient.

[27] Taxes on leverage have received relatively less attention in policy discussions, with some exceptions (e.g., the Minneapolis Plan to End Too Big To Fail). An alternative is a reserve requirement, under which banks must hold a fraction of their deposits as unremunerated reserves at the central bank.

asset prices, which in turn makes banks more vulnerable to runs as they are forced to sell capital to meet deposit withdrawals. A social planner internalizes that maintaining lower leverage raises future asset prices, thereby improving banks' liquidity during runs and making them less vulnerable.

Our findings contribute to the renewed policy debate on banking regulation in the wake of the March 2023 banking turmoil. Going forward, one important direction is to enrich our model to allow for a quantitative assessment of its policy implications. It would also be valuable to incorporate banks' maturity choices, in order to study how the composition of liabilities affects vulnerability to runs and the design of optimal regulation. Another promising avenue is to use our model to examine how capital regulation interacts with other policy tools, such as liquidity requirements, deposit insurance, or government guarantees.

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Appendix

A Case without Risk

We consider here a version of the model where there is no uncertainty. In this case, the model collapses to one with borrowing constraints; and financial constraints will still be relevant in an equilibrium. We will show that the equilibrium is *constrained efficient*.

Toward this, let us assume that z_1^D is a constant. We will also assume that $\lambda = 1$ so that if a bank is vulnerable, a run occurs with probability 1.[28] So, in the absence of risk, the bond price schedule is given by

$$q_0(b_1, k_1) = \begin{cases} 1 & \text{if } V_1^{Run}(b_1, k_1) \geq V_1^D(k_1), \\ 0 & \text{otherwise.} \end{cases}$$

That is, investors lend at a price of 1 if they expect the bank to repay and at a price of 0 if they expect the bank to default, resulting effectively in a borrowing constraint for the bank. Because in our baseline framework all uncertainty is realized in period 1, we can use the expressions for $V_1^D(k_1)$ and $V_1^{Run}(b_1, k_1)$ characterized respectively in equations (1) and (9) to derive the endogenous borrowing limit in period 0. Using $n_1 = (z + p_1)k_1 - Rb_1$ and $l_1 = b_1/k_1$, we have that banks repay

[28] Recall that we have shown that for $\lambda = 0$, that is, only fundamental defaults, the competitive equilibrium is constrained efficient.)

in period 1 if and only if

$$\Phi(l_1, p_1) \equiv A + (1+\beta)\log(z + p_1 - Rl_1) + \beta \log\left(\frac{z}{p_1}\right) - \left[\log(z_1^D) + \beta \log(z_2^D)\right] \geq 0,$$

for any relevant price $p_1 \in (\phi/R, z/R]$. Note that $d\Phi(l_1, p_1)/dl_1 < 0$, and hence, given p_1, the constraint $\Phi(l_1, p_1) \geq 0$ imposes an upper bound on leverage.

The problem of a bank at time $t = 0$ can be written as follows:

$$V_0(n_0) = \max_{c_0 \geq 0, k_1 \geq 0, l_1} \left\{u(c_0) + \beta V_1^R(n_1|p_1)\right\} \tag{A.1}$$

subject to

$$c_0 = n_0 + (l_1 - p_0)k_1,$$
$$n_1 = (z + p_1 - Rl_1)k_1,$$
$$\Phi(l_1, p_1) \geq 0,$$

where $n_0 = (z+p_0)k_0 - Rb_0$ and $V_1^R(n_1|p_1)$ is given by Lemma 1 and where we make the dependence on p_1 explicit.

As before, the equilibrium price of capital remains a function of the leverage chosen by banks in period 0. That is, $p_1 = \mathcal{P}_1(L_1)$ as in (19), and

$$p_0 = \mathcal{P}_0(L_1) = (1 + \beta(1+\beta))L_1 + \beta(1+\beta)\left[z - \frac{RB_0}{K}\right],$$

which takes into account that the bond price in period 0 is equal to 1 in equilibrium.

A competitive equilibrium can then be characterized by prices $\{p_0, p_1\}$ and leverage L_1, such that the policy $(l_1 = L_1, k_1 = K)$ solves (A.1) with $p_0 = \mathcal{P}_0(L_1)$ and $p_1 = \mathcal{P}_1(L_1)$.

The planning problem (28) in the absence of risk reduces to:

$$\max_{c_0 \geq 0, L_1 \leq \bar{L}, p_1} \left\{u(c_0) + \beta V_1^R(N_1|p_1)\right\},$$

subject to

$$c_0 = zK - RB_0 + L_1K,$$
$$N_1 = (z + p_1 - RL_1)K,$$
$$\Phi(L_1, p_1) \geq 0,$$
$$p_1 = \mathcal{P}_1(L_1).$$

The main difference with the competitive equilibrium is that the planner takes into account the role of p_1 in the borrowing constraint.

Arguments similar to the one in the main text show that if the market for capital clears, a higher price of capital leads to a relaxation of the borrowing constraint. In our case:

$$\left.\frac{d\Phi(L_1, p_1)}{dp_1}\right|_{p_1=\mathcal{P}_1(L_1)} = -\frac{\beta}{\mathcal{P}_1(L_1)} + \frac{1+\beta}{z + \mathcal{P}_1(L_1) - RL_1} > 0, \text{ for } L_1 \in [\hat{L}, \bar{L}], \tag{A.2}$$

where the restricted range represents the values for L_1 for which the equilibrium price is sensitive to leverage. Even though the borrowing constraint is affected by the equilibrium price, the only tool the planner has to increase the price in period 1 is *to borrow less*, which defeats the benefits of relaxing the constraint. This implies that the planner solution can be decentralized as a competitive equilibrium, a result we show next.

Let c_0^*, L_1^* and p_1^* be a solution to the planner's problem. Consider the effect of a change in L_1^* in utility (ignoring the borrowing constraint). The marginal effect for $L_1^* \leq \bar{L}$ and $L_1^* \neq \hat{L}$ (the range of values where the $\mathcal{P}_1$ is differentiable) is:

$$u'(c_0^*)K - \beta R V_1^{R\prime}(N_1^*|p_1^*)K + \beta \frac{dV_1^R((z + p_1^* - RL_1^*)K|p_1^*)}{dp_1}\frac{d\mathcal{P}_1(L_1^*)}{dL_1}.$$

For $L_1^* < \hat{L}$, $\frac{d\mathcal{P}_1(L_1^*)}{dL_1} = 0$. For $L_1^* \in (\hat{L}, \bar{L}]$, using that $p_1 = \mathcal{P}_1(L_1)$, we have that:

$$\frac{dV_1^R((z + p_1^* - RL_1^*)K|p_1^*)}{dp_1} = -\frac{\beta R}{R\mathcal{P}_1(L_1^*) - \phi} + \frac{1+\beta}{z + \mathcal{P}_1(L_1^*) - RL_1^*} = 0.$$

Taken together, it follows that the objective is differentiable for all values of $L_1 \leq \bar{L}$, and the marginal effect of a change in L_1 is

$$u'(c_0^*)K - \beta R V_1^{R\prime}(N_1^*|p_1^*)K.$$

This should not come as a surprise. As in the analysis in text, given that repaying banks are neither seller nor buyers of capital in period 1, the general equilibrium effect of p_1 in the payoff function is zero.

Now, suppose that the planner's solution satisfies $\Phi(L_1^*, p_1^*) > 0$. Then, the planner could increase or decrease L_1 at the margin. The effect in the objective of each of these changes must be non-positive, and as a result, it is necessary that

$$u'(c_0^*) - \beta R V_1^{R\prime}(N_1^*|p_1^*) = 0. \tag{A.3}$$

If instead $\Phi(L_1^*, p_1^*) = 0$, then it is feasible for the planner to decrease L_1^*. To see this, note that either $L_1^* \leq \hat{L}$, and the decrease in L_1^* has no effect on p_1^*. Or $L_1^* \in (\hat{L}, \bar{L}]$, in which case the increase in the price strictly relaxes the constraint by (A.2). We must have that such a feasible perturbation must not increase utility, and

$$u'(c_0^*) - \beta R V_1^{R\prime}(N_1^* | p_1^*) \geq 0. \tag{A.4}$$

Now, we can now show that the planner's solution is a competitive equilibrium. Recall that the price $p_0 = \mathcal{P}_0(L_1)$ is such that the market clearing condition for capital holds in period 0. Hence, we only need to check that the leverage decision L_1^* is optimal for the banks given prices. Given that the bank's problem is convex, it suffices to check that the first-order condition for bank's leverage holds at L_1^*. But this condition coincides with the planner condition above (that is, equations (A.3) and (A.4), depending on whether the constraint binds), guaranteeing that the L_1^* choice is also optimal for banks.

To summarize, in the absence of risk, our model collapses to a model with a financial constraint in period 0 that depends on the equilibrium price of capital in period 1. In this case, the solution to the planning problem can be decentralized as a competitive equilibrium, even when the financial constraint arises because of the potential for self-fulfilling runs in period 1. This shows that the presence of default risk, and equilibrium default, is the key element in the constrained-inefficiency results we have obtained.

Online Appendix to "Bank Runs, Fragility, and Regulation

Manuel Amador and Javier Bianchi

B Omitted Proofs

B.1 Proof of Lemma 1

Consider first the case where $p_1 < \phi/R$. Let k_2 be any finite number. Then, the bank can choose $b_2 = \phi k_2/R$ and $c_1 = n_1 + b_2 - p_1 k_2 = n_1 + (\phi/R - p_1)k_2$. This implies that $V_1^R(n_1) \geq u(n_1 + (\phi/R - p_1)k_2) + \beta u((z-\phi)k_2)$. Given that $\phi/R > p_1$ and $z > \phi$ (since $z > z_2^D$) it follows that $V_1^R(n_1) \to \infty$ as $k_2 \to \infty$ for any n_1.

The same argument applies when $p_1 = \phi/R$ and $n_1 > 0$. In that case $V_1^R(n_1) \geq u(n_1) + \beta u((z-\phi)k_2)$. Note that in both cases, the value would be less than infinity for finite values of k_2; thus, $k_2 \to \infty$ to achieve the optimum.

Consider next the case $p_1 \geq \phi/R$ and $n_1 \leq 0$. Then, from the constraint set, we have that

$$c_1 = n_1 + b_2 - p_1 k_2 \leq n_1 + (\phi/R - p_1)k_2 \leq n_1 \leq 0,$$

where we used that $k_2 \geq 0$. Hence, the constraint set is either empty or, at best, $c_1 = 0$. In both cases, $V_1^R(n_1) = -\infty$.

In the case with $p_1 = z/R$ and $n_1 > 0$, the bank is indifferent between savings or holding capital. In the case where $p_1 > z/R$, the bank strictly prefers to hold bonds, and thus $k_2 = 0$. In both cases, the shape of the value function follows directly from optimization and log utility.

In the case where $\phi/R < p_1 < z/R$ and $n_1 > 0$, the bank's borrowing constraint binds; that is, $Rb_2 = \phi k_2$. The value function can then be solved for using optimization and log utility. □

B.2 Proof of Lemma 2

We have already argued that $\hat{z}^F(n_1, k_1) \geq \hat{z}^{Run}(n_1, k_1)$. So we just need to show that $z^{Run}(n_1, k_1) > z$. Note that if $k_1 = 0$, the result is immediate, as $\hat{z}^{Run}(n_1, 0) = \infty$. So consider $k_1 > 0$. Given $b_1 \leq 0$, it follows that $n_1 > 0$.

First, note that by (11), $\hat{z}^{Run}(n_1, k_1) \geq \hat{z}^{Run}((z + p_1)k_1, k_1)$. Thus, it is enough to show that $\hat{z}^{Run}((z + p_1)k_1, k_1) > z$.

When $p_1 < z/R$, $\hat{z}^{Run}((z+p_1)k_1, k_1) > z$ is equivalent to

$$e^A (z+p_1)^{1+\beta} \left(\frac{z/p_1}{z-\phi}\right)^\beta \frac{1}{z} > 1. \tag{B.1}$$

Using that $\phi > 0$, we have that

$$e^A (z+p_1)^{1+\beta} \left(\frac{z/p_1}{z-\phi}\right)^\beta \frac{1}{z} > e^A (z+p_1)^{1+\beta} \left(\frac{1}{p_1}\right)^\beta \frac{1}{z} \equiv H(p_1).$$

Note that

$$H'(p_1) = e^A \frac{1}{z} \frac{(z+p_1)^\beta}{p_1^{1+\beta}} (p_1 - \beta z).$$

And thus, $H'(p_1) < 0$ for $0 < p_1 < \beta z$, $H'(p_1) > 0$ for $p_1 > \beta z$, and $H'(p_1) = 0$ for $p_1 = \beta z$. Hence, H achieves a minimum for $p_1 \geq 0$ at $p_1 = \beta z$. Thus, $H(p_1) \geq H(\beta z) = 1$, where the last equality follows from substitution and manipulation of H. Thus, (B.1) holds.

When $p_1 \geq z/R$, $\hat{z}^{Run}((z+p_1)k_1, k_1) > z$ becomes

$$e^A (z+p_1)^{1+\beta} \left(\frac{R}{z-\phi}\right)^\beta \frac{1}{z} > 1.$$

Similarly, then

$$e^A (z+p_1)^{1+\beta} \left(\frac{R}{z-\phi}\right)^\beta \frac{1}{z} > e^A (z+p_1)^{1+\beta} \left(\frac{R}{z}\right)^\beta \frac{1}{z} \geq e^A (z+p_1)^{1+\beta} \left(\frac{1}{p_1}\right)^\beta \frac{1}{z} = H(p_1),$$

where the first inequality uses $\phi > 0$, and the second uses that $R/z \geq p_1$. Then, $H(p_1) \geq 1$ delivers the result. □

B.3 Proof of Lemma 3

Toward a contradiction, suppose that N_1, K_1 is such that $q_0(N_1, K_1) = 0$. This requires that $B_1 > 0$, as otherwise $q_0(N_1, K_1) = 1$. For the bank problem in period $t = 0$ to have a solution and for $K_1 = K > 0$, we need that $N_0 > 0$. The bank's equilibrium payoff at $t = 0$ is:

$$V_0 = \log(N_0 - K_1) + \beta \int_{\underline{z}}^{\bar{z}} V_1^D(K_1, \tilde{z}) dF(\tilde{z}).$$

Consider then the alternative where the bank chooses $B_1 = 0$ and $B_2 = 0$ and chooses the same K_1 and $K_2 = K_1$. This strategy is feasible, and the $t = 0$ payoff to the bank is:

$$\hat{v}_0 = \log(N_0 - K_1) + \beta \left[\log(zK_1) + \beta^2 \log(zK_1)\right].$$

But we have that $z > \overline{z}$, and thus $\hat{v}_0 > V_0$, a contradiction of banks' optimality. □

B.4 Proof of Lemma 4

If $p_1 < \phi/R$, then repaying banks in period $t = 1$ (of which there is a positive mass of them) attain an infinite payoff while demanding an infinite amount of capital, a violation of the market clearing condition $K_2 = K < \infty$.

If $p_1 > z/R$, then repaying banks in period $t = 1$ demand zero capital, a violation of the market clearing condition $K_2 = K > 0$.

Suppose now that $p_1 = \phi/R$. Then by Lemma 1, it follows that $N_1 > 0$, or else $V_1^R = -\infty$ and all banks will default in period $t = 1$ (as $K_1 > 0$ by market clearing). But if $p_1 = \phi/R$ and $N_1 > 0$, the banks attain an infinite value in period $t = 0$ while demanding infinite capital, a violation of market clearing. □

B.5 Proof of Lemma 5

Suppose that a bank in period $t = 0$ chooses $l_1 = l_1^\star$ and $k_1 = K$. If $q_0(l_1^\star|p_1)l_1^\star \geq p_0$, then the bank can increase its demand for capital without changing c_0. This increases its continuation value without bounds as the default probability at $t = 1$ is less than one (as $q_0(l_1^\star|p_1) > 0$); thus, $k_1 = K$ is not optimal. Similarly, if the bank chooses a $l_1^\star$ such that $q_0(l_1^\star|p_1)l_1^\star < p_0$, then the bank can instead choose l_1 such that $q_0(l_1|p_1)l_1 \geq p_0$ without changing its consumption in period $t = 0$. As was the case above, the bank can then increase k_1 without limits and thus increase its continuation value without bounds. Taken together, the demand for capital in period $t = 0$ is not bounded if $q_0(l_1|p_1)l_1 \geq p_0$ for some l_1, and thus cannot be part of a competitive equilibrium. □

B.6 Proof of Lemma 6

The objective function of the bank's problem in period $t = 0$ is (ignoring the dependence on p_1 for notational simplicity) given by:

$$\log(n_0 + q_0(l_1)l_1k_1 - p_0k_1) + \beta F(\hat{z}^{Run}(l_1))V_1^R(n_1) \\ + \beta \int_{\hat{z}^{Run}(l_1)}^{\hat{z}^F(l_1)} [(1-\lambda)V_1^R(n_1) + \lambda V_1^D(k_1,\tilde{z})]f(\tilde{z})d\tilde{z} + \beta \int_{\hat{z}^F(l_1)}^{\infty} V_1^D(k_1,\tilde{z})f(\tilde{z})d\tilde{z}.$$

The thresholds $\hat{z}^F$ and $\hat{z}^{Run}$ are differentiable in l_1, and so is the bond-price function $q_0(l_1)$ given that $f(\underline{z}) = f(\overline{z}) = 0$. It follows that the objective function is differentiable in l_1.

The choice of leverage l_1 must be strictly less than $(z + p_1)/R$ for the bank to have a positive net worth in period 1. If this were not the case, the bank would default for sure in period 1, and we know from the argument in the proof of Lemma 3 that this cannot be optimal. That is, any optimal choice of leverage l_1 is such that $\hat{z}^F(l_1) > \underline{z}$.

Any choice of leverage also must lead to strictly positive consumption in period 0, as otherwise the bank's payoff is $-\infty$ and dominated by $l_1 = 0$ for some $k_1 > 0$, given $n_0 > 0$. It follows that the choice of l_1 must be interior, and the first-order condition with respect to l_1 leads to the condition in the Lemma. □

B.7 Proof of Lemma 7

Consider first the case where $z/R > p_1 > \phi/R$. From (7), market clearing for capital requires that

$$K_2 = \frac{\beta}{(1+\beta)(p_1 - \phi/R)} N_1 = K.$$

Using that $N_1 = (z+p_1-RL_1)K$, this implies $p_1 = \beta z + (1+\beta)\phi/R - \beta RL_1$. Given that $z/R > p_1 > \phi/R$, this requires that $L_1 \in (\hat{L}, \overline{L})$ where the expressions are provided in the text.

Consider next the case where $z/R = p_1$. In this case, a bank is indifferent between investing in capital and bonds, as long as it is consistent with its optimal consumption and the borrowing constraint. Using (5) and (6), and $K_2 = K$, we can see that it follows that $(p_1 - \phi/R)K \leq \beta N_1/(1+\beta)$, which implies $L_1 \leq \hat{L}$.

B.8 Proof of Proposition 2

For part (i). This follows immediately from the fact that a competitive equilibrium is constrained efficient (Proposition 1) and that there is a unique leverage choice that solves the planning problem.

For part (ii). Suppose we have two distinct equilibria with elements indexed by A and B. Let $L^A < \bar{L}$ and $L^B < \bar{L}$ be the associated levels of leverage. Given that equilibria are characterized by the level of leverage, L^A and L^B must be different. Without loss of generality, we let $L^B < L^A$.

Given that $L^A > \hat{L}$, equation (19) implies that $p_1^A \neq p_1^B$. Facing prices for capital (p_0^A, p_1^A) banks must be willing to borrow $B_1^A = L^A K$ and invest K in period $t = 0$. This must be preferred to borrowing $B_1^B = L^B K$ and investing K in period $t = 0$ given prices (p_0^A, p_1^A). That is,

$$u(zK - RB_0 + q_0(L^A|p_1^A)L^A K) + \beta \mathbb{E} V_1(L^A, K|p_1^A) \\ \geq u(zK - RB_0 + q_0(L^B|p_1^A)L^B K) + \beta \mathbb{E} V_1(L^B, K|p_1^A),$$

where we have used the definition of $\mathbb{E}V_1$ in the proof of Proposition 1. Lemma 8 implies that $q_0(L^B|p_1^A) \geq q_0(L^A|p_1^B)$. Part (i) of the lemma then implies that $\mathbb{E}V_1(L^B, K|p_1^A) > \mathbb{E}V_1(L^B, K|p_1^B)$, given that $p_1^B \neq p_1^A$ and that default does not occur with probability one in any equilibrium (Lemma 3). Thus, it follows that

$$u(zK - RB_0 + q_0(L^B|p_1^A)L^B K) + \beta \mathbb{E} V_1(L^B, K|p_1^A) \\ > u(zK - RB_0 + q_0(L^B, p_1^B)L^B K) + \beta \mathbb{E} V_1(L^B, K|p_1^B).$$

Taken together, the above implies that the time 0 utility to banks in the competitive equilibrium A is strictly higher than the one in the competitive equilibrium B. A contradiction of Proposition 1 as we found a competitive equilibrium that delivers strictly lower welfare than the constrained-efficient solution. □

B.9 Proof of Proposition 3

Assumption 2 guarantees a solution to the planning problem with $n_0 > 0$. Let L_1 be a planner's solution and $p_0 = \mathcal{P}_0(L_0)$ and $p_1 = \mathcal{P}_1(L_1)$ be the associated capital prices. We are now going to show that $l_1 = L_1$ is a solution to the bank's problem given the prices.

We already know that $c_0 = c_0(n_0)$ and that $k_1 = k_1(n_0, l_1|p_0, p_1)$, as given by (16) and (17). The only remaining choice is l_1. We have already argued that a choice of leverage that leads to default for certain is not optimal (see proof of Lemma 3). This implies that $n_1 > 0$, given (10). Thus we can restrict the choice of l_1 to $l_1 < (z + p_1)/R$ or, more strictly, to values such that $F(\hat{z}^F(l_1|p_1)) > 0$.

We can then rewrite the objective function of the bank's problem as:

$$W(l_1|p_1) = \log(c_0(n_0)) + \beta F(\hat{z}^F(l_1|p_1))V_1^R\Big((z + p_1 - Rl_1)k_1\Big|p_1\Big) + \beta\int_{\hat{z}^F(l_1|p_1)}^{\infty} V_1^D(k_1, \tilde{z})dF(\tilde{z}),$$

with $k_1 = k_1(n_0, l_1|p_0, p_1)$.

Using the functional forms, and that $p_1 \le z/R$, we can rewrite the above as

$$W(l_1|p_1) = \log(c_0(n_0)) + \beta(1+\beta)\log k_1 + \beta F(\hat{z}^F(l_1|p_1))V_1^R\Big((z + p_1 - Rl_1)\Big|p_1\Big) + \beta\int_{\hat{z}^F(l_1|p_1)}^{\infty} V_1^D(1, \tilde{z})dF(\tilde{z}).$$

Using the definition of $\hat{z}^F$ in (10), and that $p_1 \le z/R$, we have that

$$\log(z^F(l_1|p_1)) = (1+\beta)\log(z + p_1 - Rl_1) + \text{constant}(p_1).$$

where the constant term depends on the prices. Taking the derivative of $\hat{z}^F$ with respect to l_1, we have that

$$\frac{\partial \hat{z}^F(l_1|p_1)}{\partial l_1} = -\hat{z}^F(l_1|p_1)\frac{(1+\beta)R}{z_1 + p_1 - Rl_1},$$

We can use this to write (eliminating the dependence on p_1 of the functions for simplicity):

$$W'(l_1|p_1) = \frac{\beta(1+\beta)^2 F(\hat{z}^F(l_1))}{(z + p_1 - Rl_1)(p_0 - F(\hat{z}^F(l_1))l_1)} \times\left[\frac{z + p_1 - Rp_0}{1+\beta} - Rl_1\left(\frac{f(\hat{z}^F(l_1))\hat{z}^F(l_1)}{F(\hat{z}^F(l_1))} + \frac{1 - F(\hat{z}^F(l_1))}{1+\beta}\right)\right]$$

where we used that $q(l_1) = F(\hat{z}^F(l_1))$ and that $p_0 - F(\hat{z}^F(l_1))l_1 > 0$ for all $l_1 < (z + p_1)/R$ by the assumption in the proposition. We also used that $F(\hat{z}^F(l_1)) > 0$, as it is not optimal for the bank to borrow into full default. Assumption 1 guarantees that W is indeed differentiable at the choice of l_1 so that $\hat{z}^F(l_1) = \overline{z}$.

Given that $\hat{z}^F(l_1)$ is weakly decreasing in l_1, Assumption 3 guarantees that

$$\frac{f(\hat{z}^F(l_1))\hat{z}^F(l_1)}{F(\hat{z}^F(l_1))} + \frac{(1 - F(\hat{z}^F(l_1)))}{1+\beta},$$

is weakly increasing in leverage. Note also that this term is zero for any $l_1 \leq 0$ (as the bank is defaulting with probability 0 for $l_1 \leq 0$, Lemma 2). It follows that

$$Rl_1 \left[\frac{f(\hat{z}^F(l_1))\hat{z}^F(l_1)}{F(\hat{z}^F(l_1))} + \frac{(1 - F(\hat{z}^F(l_1)))}{1 + \beta} \right].$$

The planner solution L_1 is such that $W'(L_1) = 0$, a result that follows from (22). So, $W'(l_1) \leq 0$ for $l_1 \leq L_1$ and $W'(l_1) \geq 0$ for $l_1 \geq L_1$, and thus the planner's solution L_1 is a global maximum of the bank's problem, confirming that last condition for the existence of an equilibrium. □

C Additional Derivations

C.1 Derivation of Equation (18)

Let us define

$$\hat{V}_1^R(l_1) = A + (1+\beta)\log(z_1 + p_1 - Rl_1) + \beta\log\left(\frac{z-\phi}{p_1 - \frac{\phi}{R_2}}\right),$$
$$\hat{V}_1^D(z_1^D) = \log(z_1^D) + \beta\log(z_2^D),$$

where we have removed the dependence on p_1 for simplicity (as it is taken as given in the banks' problem).

Using these two definitions and (1) and (4), we can first express (15) as

$$V_0(K,B_0) = \max_{k_1,l_1}\Bigg\{\log((z+p_0)K + q_0(l_1)k_1l_1 - p_0k_1) + \beta(1+\beta)\log(k_1) + \beta F(\hat{z}^{Run}(l_1))\hat{V}_1^R(l_1)$$
$$+\beta\int_{\hat{z}^F(l_1)}^{\infty}\hat{V}_1^D(z_1^D)dF(z_1^D) + \beta\int_{\hat{z}^{Run}(l_1)}^{\hat{z}^F(l_1)}\left[\lambda\hat{V}_1^D(z_1^D) + (1-\lambda)\hat{V}_1^R(l_1)\right]dF(z_1^D)\Bigg\}.$$

Taking first-order condition with respect to l_1 yields

$$k_1\frac{q_0^{'}(l_1)l_1 + q_0(l_1)}{c_0} + \beta f(\hat{z}^{Run}(l_1))\hat{V}_1^R(l_1)\frac{\partial\hat{z}^{Run}(l_1)}{\partial l_1} + \beta F(\hat{z}^{Run}(l_1))\frac{\partial\hat{V}_1^R(l_1)}{\partial l_1}$$
$$-\beta f(\hat{z}^F(l_1))\hat{V}_1^D(\hat{z}^F(l_1))\frac{\partial\hat{z}^F(l_1)}{\partial l_1} + \beta f(\hat{z}^F(l_1))[\lambda\hat{V}_1^D(\hat{z}^F(l_1)) + (1-\lambda)\hat{V}_1^R(l_1)]\frac{\partial\hat{z}^F(l_1)}{\partial l_1}$$
$$-\beta f(\hat{z}^{Run}(l_1))\left[\lambda\hat{V}_1^D(\hat{z}^{Run}(l_1)) + (1-\lambda)\hat{V}_1^R(l_1)\right]\frac{\partial\hat{z}^{Run}(l_1)}{\partial l_1}$$
$$+\beta(1-\lambda)\int_{\hat{z}^{Run}(l_1)}^{\hat{z}^F(l_1)}\frac{\partial\hat{V}_1^R(l_1)}{\partial l_1}dF(z_1^D) = 0.$$

Using that $\hat{V}_1^R(l_1) = \hat{V}_1^D(\hat{z}^F(l_1))$, the expression above can be written as

$$k_1\frac{q_0^{'}(l_1)l_1 + q_0(l_1)}{c_0} + \beta f(\hat{z}^{Run}(l_1))\lambda\left[\hat{V}_1^D(\hat{z}^{Run}(l_1)) - \hat{V}_1^R(l_1)\right]\frac{\partial\hat{z}^{Run}(l_1)}{\partial l_1}$$
$$+\beta\left[(1-\lambda)F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1))\right]\frac{\partial\hat{V}_1^R(l_1)}{\partial l_1} = 0. \quad \text{(C.1)}$$

In addition, using the definition of $\hat{V}^R$ and equation (14), we obtain

$$\frac{\partial \hat{V}_1^R(l_1)}{\partial l_1} = -\frac{(1+\beta)R}{n_1}k_1,$$
$$q_0(l_1) = (1-\lambda)f(\hat{z}^F(l_1))\frac{\partial \hat{z}^F(l_1)}{\partial l_1} + \lambda f(\hat{z}^{Run}(l_1))\frac{\partial \hat{z}^{Run}(l_1)}{\partial l_1}.$$

Replacing these two conditions in (C.1), we obtain

$$\frac{k_1}{c_0}\left[\frac{\left[(1-\lambda)f(\hat{z}^F(l_1))\frac{\partial \hat{z}^F(l_1)}{\partial l_1} + \lambda f(\hat{z}^{Run}(l_1))\frac{\partial \hat{z}^{Run}(l_1)}{\partial l_1}\right]l_1}{(1-\lambda)F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1))} + 1\right]$$
$$-\beta\frac{\lambda f(\hat{z}^{Run}(l_1))\frac{\partial \hat{z}^{Run}(l_1)}{\partial l_1}}{(1-\lambda)F(\hat{z}^F(l_1)) + \lambda F(\hat{z}^{Run}(l_1))}\left[\hat{V}_1^D(\hat{z}^{Run}(l_1)) - \hat{V}_1^R(l_1)\right] = \beta\frac{(1+\beta)R}{n_1}k_1.$$

Finally, replacing c_1 from (5) and rearranging, we arrive at (18).

C.2 Derivation of Equation (29)

Let us define

$$\tilde{V}_1^R(L_1, p_1) = A + (1+\beta)\log(K) + (1+\beta)\log\left((1+\beta)\left(z_1 - RL_1 + \frac{\phi}{R}\right)K\right)$$
$$\tilde{V}_1^D(z_1^D) = (1+\beta)\log(K) + \log(z_1^D) + \beta\log(z_2^D),$$

where we remove the dependence from K in the value functions as K is given for the planner.

We can express the planning problem (28) as

$$\max_{L_1}\Bigg\{\log(zK - RB_0 + q_0(L_1|\mathcal{P}_1(L_1))KL_1) + \beta F(\hat{z}^{Run}(L_1|\mathcal{P}(L_1)))\tilde{V}_1^R(L_1, \mathcal{P}_1(L_1))$$
$$+\beta\Big[F(\hat{z}^F(L_1|\mathcal{P}(L_1))) - F(\hat{z}^{Run}(L_1|\mathcal{P}(L_1)))\Big](1-\lambda)\tilde{V}_1^R(L_1, \mathcal{P}_1(L_1))$$
$$+\beta\lambda\int_{\hat{z}^{Run}(L_1|\mathcal{P}(L_1))}^{\hat{z}^F(L_1|\mathcal{P}(L_1))}\tilde{V}_1^D(z_1^D)dF(z_1^D) + \beta\int_{\hat{z}^F(L_1|\mathcal{P}(L_1))}^{\infty}\tilde{V}_1^D(z_1^D)dF(z_1^D)\Bigg\}.$$

The first-order condition with respect to L_1 yields

$$\frac{1}{c_0}\left[q_0(L_1|\mathcal{P}(L_1))+\left(\frac{\partial q_0(L_1|\mathcal{P}(L_1))}{\partial L_1}+\frac{\partial q_0(L_1|\mathcal{P}(L_1))}{\partial p_1}\mathcal{P}_1'(L_1)\right)L_1\right]K$$
$$+\beta f(\hat{z}^{Run})\lambda(\tilde{V}_1^R(L_1,p_1)-\tilde{V}_1^D(\hat{z}^{Run}))\left[\frac{\partial\hat{z}^{Run}}{\partial L_1}+\frac{\partial\hat{z}^{Run}}{\partial p_1}\mathcal{P}_1'(L_1)\right]$$
$$+\beta[\lambda F(\hat{z}^{Run})+(1-\lambda)F(\hat{z}^F)]\frac{\partial\tilde{V}_1^R}{\partial L_1}=0, \quad \text{(C.2)}$$

where we have used that

$$\frac{\partial\tilde{V}_1^R(L_1,\mathcal{P}_1(L_1))}{\partial p_1}=0$$

and

$$\frac{\partial\hat{z}^F(L_1|p_1)}{\partial p_1}=0.$$

In addition, we have

$$\frac{\partial\tilde{V}_1^R(L_1,\mathcal{P}_1(L_1))}{\partial L_1}=-u'(c_1)R.$$

$$\frac{\partial q_0(L_1,p_1)}{\partial p_1}=\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial\hat{z}^{Run}(L_1|p_1)}{\partial L_1}$$

Replacing these expressions together with (14) in (C.2), and rearranging we arrive at

$$\frac{1}{c_0}-\frac{\beta R}{c_1}=-\frac{(1-\lambda)f(\hat{z}^F(L_1|p_1))\frac{\partial\hat{z}^F(L_1|p_1)}{\partial L_1}+\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial\hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{L_1}{c_0}$$
$$-\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial\hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{\beta}{K}\Big[V_1^R(n_1|p_1)-V_1^D(K,\hat{z}^{Run}(L_1|p_1)))\Big]$$
$$-\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))}{q_0(L_1|p_1)}\frac{\partial\hat{z}^{Run}(L_1|p_1)}{\partial p_1}\mathcal{P}_1'(L_1)\frac{1}{K}\left[\frac{L_1K}{c_0}+\beta\Big[V_1^R(n_1|p_1)-V_1^D(K,\hat{z}^{Run}(L_1|p_1))\Big]\right],$$

which is equation (18).

D Constrained efficiency with general default value

A central feature of the model is the endogeneity of a bank's default decision. Specifically, a bank compares the value of repaying its deposits with the value of its outside option, which is the value of default. Our model makes several simplifying assumptions that allow us to obtain closed-forms solutions for the main equilibrium objects. In particular, we assume that upon default, a bank retains its capital and continues operating it at a lower productivity while excluded from raising new deposits.

In this section, we study how our normative results can be extended and generalized when banks face arbitrary value functions of repayment and default $V_1^R(n_1|p_1)$ and $V_1^D(K, p_1)$ where we only assume, for simplicity, that these functions give rise to default thresholds that depend solely on leverage. That is, we have that a bank not facing a run repays if $z_1^D < \hat{z}^F(l_1|p_1)$ and a bank facing a run repays if $z_1^D < \hat{z}^{Run}(l_1|p_1)$. The goal is to show how the main results can be extended to setups where these functions have a more general form, in particular, without imposing specific assumptions about the penalties after a bank defaults.

Consider problem (28). Assuming only that default thresholds are decreasing in leverage, we can write the problem at period 0 for arbitrary value functions in period 1 as:

$$\max_{L_1} \Bigg\{ \log(zK - RB_0 + q_0(L_1|\mathcal{P}_1(L_1))KL_1) + \beta F(\hat{z}^{Run}(L_1|\mathcal{P}(L_1)))\tilde{V}_1^R(L_1, \mathcal{P}_1(L_1))$$

$$+ \beta\Big[F(\hat{z}^F(L_1|\mathcal{P}(L_1))) - F(\hat{z}^{Run}(L_1|\mathcal{P}(L_1)))\Big](1-\lambda)\tilde{V}_1^R(L_1, \mathcal{P}_1(L_1))$$

$$+ \beta \int_{\hat{z}^{Run}(L_1|\mathcal{P}(L_1))}^{\hat{z}^F(L_1|\mathcal{P}(L_1))} \lambda \tilde{V}_1^D(z_1^D, \mathcal{P}(L_1))dF(z_1^D) + \beta \int_{\hat{z}^F(L_1|\mathcal{P}(L_1))}^{\infty} \tilde{V}_1^D(z_1^D, \mathcal{P}(L_1))dF(z_1^D) \Bigg\}$$

where as above $\tilde{V}_1^D, \tilde{V}_1^R$ are the period 1 value functions when individual capital equals K.

The first-order condition with respect to L_1 yields:

$$0 = \frac{1}{c_0}\Big[q_0(L_1|\mathcal{P}(L_1)) + \Big(\frac{\partial q_0(L_1|\mathcal{P}(L_1))}{\partial L_1} + \frac{\partial q_0(L_1|\mathcal{P}(L_1))}{\partial p_1}\mathcal{P}'(L_1)\Big)L_1\Big]K$$

$$+ \beta\lambda[\tilde{V}_1^D(\hat{z}^{Run}, p_1) - V_1^R(L_1, p_1)]\, f(\hat{z}^{Run})\Big[\frac{\partial \hat{z}^{Run}}{\partial L_1} + \frac{\partial \hat{z}^{Run}}{\partial p_1}\mathcal{P}'(L_1)\Big]$$

$$+ \beta[\lambda F(\hat{z}^{Run}) + (1-\lambda)F(\hat{z}^F)]\Big[\frac{\partial V_1^R(L_1, \mathcal{P}(L_1))}{\partial L_1}\Big]$$

$$+ \beta\mathcal{P}'(L_1)\Big[\frac{\partial V_1^R(L_1, \mathcal{P}(L_1))}{\partial p_1} q_0(L_1|\mathcal{P}(L_1))$$

$$+ \lambda\int_{\hat{z}^{Run}}^{\hat{z}^F} \frac{\partial \tilde{V}_1^D(z_1^D, \mathcal{P}(L_1))}{\partial p_1} dF(z_1^D) + \int_{\hat{z}^F}^{\infty} \frac{\partial \tilde{V}_1^D(z_1^D, \mathcal{P}(L_1))}{\partial p_1} dF(z_1^D)\Big]$$

In addition, using (14), we have

$$\frac{\partial q_0(L_1, p_1)}{\partial p_1} = (1-\lambda)f(\hat{z}^F(L_1|p_1))\frac{\partial \hat{z}^F(L_1|p_1)}{\partial p_1} + \lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial L_1}$$

where now the first term is potentially different from zero.

Replacing this last equation in the first-order condition for L_1 above and rearranging we obtain

$$\frac{1}{c_0} - \beta\frac{\partial V_1^R(L_1, \mathcal{P}(L_1))}{\partial L_1} = -\frac{(1-\lambda)f(\hat{z}^F(L_1|p_1))\frac{\partial \hat{z}^F(L_1|p_1)}{\partial L_1} + \lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{L_1}{c_0},$$

$$-\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial L_1}}{q_0(L_1|p_1)}\frac{\beta}{K}\Big[V_1^R(n_1|p_1) - V_1^D(K, \hat{z}^{Run}(L_1|p_1)))\Big]$$

$$-\left[\frac{\lambda f(\hat{z}^{Run}(L_1|p_1))}{q_0(L_1|p_1)}\frac{\partial \hat{z}^{Run}(L_1|p_1)}{\partial p_1}\right]\mathcal{P}_1'(L_1)\left[\frac{L_1}{c_0} + \frac{\beta}{K}\Big[V_1^R(n_1|p_1) - V_1^D(K, \hat{z}^{Run}(L_1|p_1))\Big]\right]$$

$$-\frac{(1-\lambda)f(\hat{z}^F(L_1|p_1))}{q_0(L_1|p_1)}\frac{\partial \hat{z}^F(L_1|p_1)}{\partial p_1}\mathcal{P}'(L_1)\left(\frac{L_1}{c_0}\right)$$

$$+ \beta\mathcal{P}'(L_1)\Big[\frac{\partial V_1^R(L_1, \mathcal{P}(L_1))}{\partial p_1}\Big((1-\lambda)F(\hat{z}^F(n_1, k_1)) + \lambda F(\hat{z}^{Run}(n_1, k_1))\Big)$$

$$+ \lambda\int_{\hat{z}^{Run}}^{\hat{z}^F} \frac{\partial V_1^D(z_1^D, \mathcal{P}(L_1))}{\partial p_1} dF(z_1^D) + \int_{\hat{z}^F}^{\infty} \frac{\partial V_1^D(z_1^D, \mathcal{P}(L_1))}{\partial p_1} dF(z_1^D)\Big]$$

The first three lines are identical to those in (29). In particular, the first and second line capture the net private marginal benefit from borrowing, while the third line captures the general equilibrium effect of changes in aggregate leverage on the run threshold—an effect that individual banks do not internalize.

In addition, now changes in the asset price affect the fundamental threshold giving rise to two additional effects. First, the change in aggregate leverage alters the bond price in period 0 by affecting the probability of fundamental defaults, as captured by the fourth line. Depending on whether asset prices lower or raise the fundamental threshold, this may lead banks to over or under-borrow. Second, the fact that asset prices may affect banks that repay and default in equilibrium differently, imply that changes in aggregate leverage give rise to redistributive effects between repaying and defaulting banks. To the extent that these banks have different marginal utilities, the planner will internalize these general equilibrium effects, as captured by the last two lines.