Banks, Liquidity Management and Monetary Policy

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September 26, 2017

Abstract

We develop a new tractable model of banks' liquidity management and the credit channel of monetary policy. Banks finance loans by issuing demand deposits. Because loans are illiquid, deposit transfers across banks must be settled with reserves. Deposit withdrawals are random, and banks manage liquidity risk by holding a precautionary buffer of reserves. We show how different shocks affect the banking system by altering the trade-off between profiting from lending and incurring greater liquidity risk. Through various tools, monetary policy affects the real economy by altering that trade-off. In a quantitative application, we study the driving forces behind the decline in lending and liquidity hoarding by banks during the 2008 financial crisis. Our analysis underscores the importance of disruptions in interbank markets followed by a persistent decline in credit demand.

*We would like to thank Harjoat Bhamra, Luigi Bocola, John Cochrane, Dean Corbae, Itamar Dreschler, Huberto Ennis, Xavier Freixas, Anil Kashyap, Nobu Kiyotaki, Arvind Krishnamurthy, Ricardo Lagos, Ellen McGrattan, Thomas Philippon, Chris Phelan, Tomek Piskorski, Erwan Quintin, Ricardo Reis, Chris Sims, Harald Uhlig, and Mike Woodford for helpful discussions. We also wish to thank participants at various seminars and conferences. Jorge Mondragon provided excellent research assistance. We are grateful for financial support by the Fondation Banque de France, the Smith Richardson Foundation. Bianchi acknowledges the support of National Science Foundation under award 1324395. E-mails: javieribianchi@gmail.com and sbigio@econ.ucla.edu.

1 Introduction

Banks' liquidity management is central to the transmission and implementation of monetary policy. By affecting the trade-off between lending and holding liquid assets, central banks affect the supply of loanable funds, and through this channel they affect the real economy. Understanding how banks manage liquidity is, thereby, paramount to understanding the transmission and implementation of monetary policy. However, macroeconomic models, especially those used for monetary policy analysis, largely abstract from how banks manage liquidity.

This paper provides a tractable model of the banking system, featuring a liquidity management problem. In our theory, banks operate in competitive markets for loans and deposits and trade in an over-the-counter (OTC) market for central bank reserves. A spread between loans and deposits leads banks to engage in maturity transformation, but at the same time, this exposes them to idiosyncratic liquidity risk. Through various tools, central banks alter the trade-off between profiting from lending and incurring greater liquidity risk, giving rise to a credit channel of monetary policy.

The first building block of our model is the liquidity management problem of an individual bank. When a bank grants a loan, it simultaneously creates demand deposits—or credit lines. These deposits can be used by the borrower to perform transactions at any time. When deposits are transferred out of a bank, that bank must transfer an asset to the bank that absorbs the liabilities. Because loans cannot be sold immediately, to settle the transaction, the bank needs to transfer liquid assets which in our model correspond to central bank reserves. If the bank receives a large withdrawal of deposits, it may fall short of holdings of reserves. Being short of reserves, that bank must incur expensive borrowing from other banks—or the central bank's discount window. By holding a large precautionary buffer of reserves, the bank can reduce this liquidity risk. The opportunity cost of this buffer is that it reduces the profits from intermediation. This is the classic bank's liquidity management problem.

In the second building block, we embed this liquidity management problem into a dynamic equilibrium model of industry dynamics. Banks are subject to idiosyncratic withdrawals of deposits. Following the realization of these shocks, banks trade in an OTC market for reserves to accommodate their reserve surpluses or deficits. Matching probabilities, as well as the interbank market rate (i.e., the federal funds rate) depend on the abundance or scarcity of reserves. When few banks have a surplus of reserves, it becomes increasingly expensive for a bank to be short of reserves. The banking system also faces a demand schedule for loans and a supply schedule for deposits. The amount of bank equity is key to determining banks' portfolio decisions and the market clearing returns for deposits, loans, the federal funds rate, and the price level.

The third building block of the model is the central bank. In the theory, the central bank has access to various tools. A first set of instruments are reserve requirements, discount window rates, and interest payments on reserves. This first set of instruments affects the demand for reserves by altering the relative return on reserves. A second set of instruments are open-market operations (OMO) that involve exchange between liquid and illiquid assets. This second set of instruments alters the volume of reserves in the system. Both sets of instruments carry real effects by tilting the liquidity management trade-off and affecting the aggregate supply of bank lending.

Overall, the model has several features both theoretically and quantitatively: it introduces an analytically tractable OTC market for assets into a general equilibrium theory of banking; it delivers an endogenous liquidity premium and money multiplier; it allows for a tractable solution of stationary equilibrium and transitional dynamics; it allows for interaction between liquidity and the real economy; and it allows us to analyze how monetary policy affects the banking sector and the real economy through the credit channel.

Despite the richness of bank portfolio decisions, idiosyncratic withdrawal risk, and an OTC interbank market, we are able to reduce the state space into a single aggregate endogenous state: the aggregate value of bank equity. Moreover, the bank's problem satisfies portfolio separation. In turn, this allows us to analyze the liquidity management problem through a portfolio problem with non-linear returns that depend only on aggregate market conditions. Being analytically tractable, this makes the analysis of the model transparent and amenable to various applications both theoretically and quantitatively.

Quantitative Application. As an application of our model, we investigate qualitatively and quantitatively what explained the deep and protracted decline in bank lending during the Great Recession. Through the lens of the model, we evaluate the plausibility of the following hypotheses:

Hypothesis 1 - Low Bank Equity: Lack of lending responds to the equity losses suffered in 2008 and the tightening of capital requirements.

Hypothesis 2 - Increased Precautionary Holdings of Reserves: Banks hold more reserves and less loans because they now face greater liquidity risk.

Hypothesis 3 - Interest on Reserves and Fed Policy: Interest payments on excess reserves have led banks to substitute reserves for loans.

Hypothesis 4 - Weak Loan Demand: Banks face a weaker demand for loans.

To evaluate these hypotheses, we calibrate our model and simulate it with shocks associated with each hypothesis. To weigh on hypotheses 1 and 3, we feed the model with shocks to equity losses and a Fed policy that we observe directly from the data. To weigh on the other two hypotheses, we consider shocks to the dispersion of deposit withdrawals and the efficiency of the interbank market (hypothesis 2) and to the loans demand (hypothesis 4). We extract the path of these shocks that can explain the behavior of bank credit, discount-window loans, and the volume of the interbank market. To see the logic for the indentification of these unobserved shocks, consider first a negative shock to loan demand. Given banks' supply schedule, the decline in loan demand causes a decline in loan returns, which in turn leads banks to substitute reserves for loans. As a result, banks resort less to the discount window. Consider instead a reduction in the efficiency of the interbank market or an increase in volatility of deposit withdrawals. Both of these shocks also lead to a contraction in banks' supply of loans but, at the same time, lead banks to resort more heavily to the discount window. In turn, reductions in interbank market efficiency can be disentangled from increases in withdrawal volatility because lower efficiency depresses interbank market lending volumes whereas increases in volatility produce the opposite effect.

We feed the observed and extracted series of shocks into the model, and turn shocks on and off, to measure the relative importance of each shock. Our analysis favors an early disruption in the interbank market—concentrated during September 2008—followed by a substantial and persistent shock to loan demand as main drivers of the crisis.Although not targeted, the model also explains the persistent increase in the liquidity ratio and the relatively higher liquidity premium around Lehman Brothers' bankruptcy in September 2008.

Related Literature. A tradition in macroeconomics dating back to at least Bagehot (1873) stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework to study policy with a full description of households, firms, and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was abandoned from macroeconomics for many years. Until the Great Recession, the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.

In the aftermath of the global financial crisis, numerous calls have been made for the development of macroeconomic models with an explicit role for banks (see e.g. Woodford, 2010).

Some early steps were taken by Gertler and Karadi (2011) and Curdia and Woodford (2009), who show how shocks that disrupt financial intermediation can have important effects on the real economy. Following these papers, a large literature has studied how various policies affect bank equity and macroeconomic outcomes. Our model also belongs to the banking channel view, but it emphasizes instead how monetary policy affects the trade-off banks face in holding assets of different liquidity. In turn, this approach relates our model to classic models of bank liquidity management and monetary policy.[1] Our contribution to this literature is to bring the classic insights from the liquidity management literature into a modern dynamic general equilibrium model that can be used for policy analysis and the study of banking crises.[2]

Our paper also builds on the search theoretic literature of monetary exchange (see the survey by Williamson and Wright, 2010). Williamson (2012) studies an environment in which assets of different maturity have different properties as mediums of exchange. In Cavalcanti, Erosa, and Temzelides (1999) reserves emerge as a disciplining device to sustain credit creation under moral hazard and to guarantee the circulation on deposits. Atkeson, Eisfeldt, and Weill (2015) present a tractable model to study trading decisions in an OTC market where agents have different credit risk exposures. Afonso and Lagos (2015) develop an OTC model of the federal funds market and use it to study the intraday evolution of the distribution of reserve balances and the dispersion in loan sizes and federal funds rates. Our market for reserves is a simplified version of that model, which we embed in a fully dynamic general equilibrium model.[3]

We share common elements with recent work at the intersection of money and banking. Brunnermeier and Sannikov (2017) introduce inside and outside money into a dynamic macro model and study the real effects of monetary policy through the redistributive effects of inflation.

[1] Classic papers that study static liquidity management—also called reserve management—by individual banks in static settings are Poole (1968) and Frost (1971). Bernanke and Blinder (1988) present a reduced-form model that blends reserve management with an IS-LM model. Many modern textbooks for practitioners deal with liquidity management. For example, Saunders and Cornett (2010) and Duttweiler (2009) provide managerial and operations research perspectives. Many modern banking papers have focused on bank runs. See, for example, Diamond and Dybvig (1983), Allen and Gale (1998), Ennis and Keister (2009), or Holmstrom and Tirole (1998). Gertler and Kiyotaki (2015) is a recent paper that incorporates bank runs into a dynamic macroeconomic model. Del Negro et al. (2017) study a rich dynamic stochastic general equilibrium model with shocks to the resaleability of assets as in Kiyotaki and Moore (2008).

[2] There is large empirical literature that provides underpinning for the monetary policy transmission mechanism that we study here. Bernanke and Blinder (1988) and Kashyap and Stein (2000) are early studies on the bank lending channel of monetary policy. In recent work, Nagel (2016) documents significant time variation in liquidity premium and how it relates to monetary policy. Jiménez, Ongena, Peydró, and Saurina (2012) and Jiménez et al. (2014) exploit both firm heterogeneity in loan demand and variation in bank liquidity ratios to identify the presence of the bank lending channel in Spain. Chodorow-Reich (2014) analyzes the effects of credit contractions on employment outcomes.

[3] Ashcraft and Duffie (2007) and Afonso and Lagos (2014) provide empirical support for search frictions in the federal funds market and the presence of substantial liquidity costs.

In contrast to their work, reserves and deposits are not perfect substitutes in our model, giving rise to a liquidity management problem. Piazzesi and Schneider (2016) studies the link between the payments system and securities markets with a focus on asset pricing. One important consideration in their work that is not present here, is that interbank-market loans require collateral assets.

Outline. The paper is organized as follows. Section 2 presents the model, and Section 3 provides theoretical results. Section 4 reports the calibration exercises. There, we study the steady state and policy functions under that calibration. In Section 6, we analyze the transitional dynamics generated after shocks associated with each hypothesis. In Section 7, we evaluate and discuss the plausibility of each hypothesis. Section 8 presents extensions of our baseline model, and Section 9 concludes. All proofs are in the appendix.

2 The Model

We propose a dynamic equilibrium model of the banking system, in which banks are subject to idiosyncratic withdrawal shocks. The central feature of a bank's portfolio problem is a decision on how to allocate assets between reserves (liquid) and loans (illiquid). Liquidity risk generates a precautionary buffer stock of reserves. There is a single final consumption good and no aggregate uncertainty.

To present the model, we first describe in detail the dynamic portfolio problem of an individual bank, followed by the description of the interbank market. The model is closed by considering the policies of the central bank—which we refer to as the Fed—and introducing a demand schedule for loans and a supply schedule for deposits.

2.1 Banks: Preferences and Budgets

There is a unit-mass continuum of banks indexed by j. Banks operate in competitive markets for loans, deposits, and reserves.

Timing. Time is discrete, indexed by t, and there is an infinite horizon. Each period is divided into two stages: the lending stage, (l), and the balancing stage, (b). At the lending stage, banks make portfolio decisions and solve a liquidity management problem. At the balancing stage, banks are subject to a random idiosyncratic withdrawal of deposits. A deposit withdrawn from one bank is transferred to another bank. That transaction is settled with reserves. Reserves are issued by the central bank and serve as the numeraire. If banks lack the reserves to settle that transaction, they can borrow them from other banks in the interbank market or from the central bank.

Preferences. Banks' preferences over a stochastic stream of dividend payments $\{c_t^j\}$ are given by

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t u(c_t^j), \tag{1}$$

where $\beta < 1$ is the time discount factor, and $u(c) \equiv \frac{c^{1-\gamma}-1}{1-\gamma}$ is the period utility function.[4]

Lending stage. Banks enter the lending stage of period t with a portfolio of assets and liabilities and collect or make associated interest payments. On the asset side of their balance sheet, banks hold loans, b_t, and reserves, m_t. On the liability side, they issue demand deposits, d_t, discount window loans, w_t, and net interbank loans, f_t. If the bank has borrowed funds, f_t is positive, and if it has lent reserves, f_t is negative. Loans and deposits are denominated in nominal terms and pay respectively i_t^b and i_t^d in units of reserves. We denote by P_t the price level (the price of consumption goods in terms of reserves).[5]

During the lending stage, banks choose real dividends, c_t, and a new portfolio of loans, reserves, and deposits. Their portfolio is the triplet $\{b_{t+1}^j, m_{t+1}^j, d_{t+1}^j\}$. We use $\tilde{x}_{t+1}$ to denote a portfolio variable chosen in the lending stage and x_{t+1} to denote the end-of-period portfolio variable chosen in the balancing stage. The bank's nominal budget constraint in the lending stage is

$$P_t c_t^j + \tilde{b}_{t+1}^j + \tilde{m}_{t+1}^j - d_{t+1}^j = (1+i_t^b) b_t^j + (1+i_t^{ior}) m_t^j - (1+i_t^d) d_t^j - \left(1 + \bar{i}_t^f\right) f_t^j - \left(1 + i_t^{dw}\right) w_t^j - P_t T_t^j. \tag{2}$$

The nominal rates i_t^{ior} and i_t^{dw} are, respectively, the interest rates on reserves and discount window loans. These rates are set by the Fed. We assume $i_t^{dw} \geq i_t^{ior}$ so that there is a positive spread between the discount window rate and the interest on reserves, as occurs in practice.

[4] Curvature in the objective of the bank is important in generating dividend smoothing and slow-moving bank equity, as observed empirically. One way to rationalize these objectives is through undiversified bank owners.

[5] Without aggregate shocks, this would be equivalent to a model with only real assets. However, in presence of unanticipated shocks, the denomination of debt matters for the transitional dynamics because of valuation effects.

Outstanding interbank market loans earn a weighted average of many interbank market rates, denoted by $\bar{i}_t^f$. This rate is the average rate among multiple transactions in the interbank market, which will be described in the next section. Finally, T_t^j represents bank-specific taxes.

Banks are subject to a capital requirement constraint,

$$\tilde{d}_{t+1}^j \leq \kappa \left(\tilde{b}_{t+1}^j + \tilde{m}_{t+1}^j - \tilde{d}_{t+1}^j \right), \tag{3}$$

which imposes an upper bound, κ, on the stock of deposits relative to the value of equity at the end of the lending stage.

The problem of the bank in the lending stage is to choose the portfolio $\{b_{t+1}^j, m_{t+1}^j, d_{t+1}^j\}$ and dividend payments c_t, subject to the budget constraint (2) and the capital requirement (3). Notice that for an individual bank, it is budget feasible to increase one unit of loans by issuing one unit of deposits, as long as the capital requirement (3) does not bind. When (3) binds, a bank can only finance a fraction of loans with deposits. The residual fraction needs to be financed by cutting dividends or by a reduction in reserves, which, as we show next, will expose the bank to higher liquidity risk in the balancing stage.

Balancing stage. Banks enter the balancing stage with the portfolio $\{\tilde{b}_{t+1}^j, \tilde{m}_{t+1}^j, \tilde{d}_{t+1}^j\}$ chosen at the lending stage. Then, at the beginning of the balancing stage, banks face a withdrawal shock ω_t^j. The shock induces a random inflow/withdrawal of deposits $\omega_t^j \tilde{d}_{t+1}^j$. Given this shock, the end-of-balancing-stage deposits, d_{t+1}^j, are

$$d_{t+1}^j = \tilde{d}_{t+1}^j (1 + \omega_t^j). \tag{4}$$

When ω_t^j is positive (negative), the bank receives (loses) deposits from (to) other banks. The ω shock has a cumulative distribution $\Phi(\cdot)$ common to all banks.[6] The support of Φ is $[\omega_{\min}, \infty)$ where $\omega_{\min} \geq -1$ and $\int_{\omega_{\min}}^{\infty} \omega_t \Phi_t(\omega) = 0, \forall t$.[7] Because the withdrawal shock is idiosyncratic and has a zero expectation, deposits are only reshuffled across banks and hence preserved within banks.

Withdrawal shocks capture an essential element of the payment system: the circulation of

[6] We could allow Φ to be a function of the bank's liquidity or leverage ratio. This would add complexity to the bank's decisions but would not break aggregation. What is crucial for tractability is that Φ_t is not a function of the bank's size.

[7] Restricting the support by setting $\omega_{\min} > -1$ would capture that not all liabilities can be withdrawn immediately from the bank —for example, because a fraction of the deposits are term deposits. In this case, i^d would correspond to the weighted average of the demand and term deposits.

deposits. Deposit circulation is the fundamental feature that enables banks to facilitate transactions between third parties. When a bank issues a loan, a borrower is credited with deposits at the issuing bank. As the borrower makes payments to third parties, those deposits may end up being transferred to other banks. In turn, the withdrawal of a deposit from one bank is an inflow of that deposit to another bank. Because the receptor bank absorbs the liabilities of the bank that issued the deposit, an asset needs to be transferred to settle the transaction. We assume that loans cannot be sold during the balancing stage—that is, loans are illiquid.[8] As occurs in practice, reserves are the asset that is used to settle positions. Therefore, the bank that faces a withdrawal transfers reserves to the deposit receptor bank. In this environment, the randomness in ω captures the complexity of the payment system. A deposit withdrawal captures a negative payment shock, or alternatively a negative confidence shock, that leads depositors to switch accounts to a different bank.

We adopt the convention that the bank that issues deposits pays for the interest on those deposits. Thus, a transfer of one unit of deposits is settled with $\left(1+i_{t+1}^{d}\right)/\left(1+i_{t+1}^{ior}\right)$ reserves, which guarantees that the bank that receives the deposit is compensated by the interest it will pay the depositor at $t+1$ and does not earn the interest on the reserves used in the settlement.

By the end of the balancing stage, banks must maintain a minimum of reserve balances. Specifically, banks must satisfy

$$m_{t+1}^{j} \geq \rho_t d_{t+1}^{j}, \tag{5}$$

with $\rho_t \in [0,1]$. The case where $\rho = 0$ corresponds to a system without reserve requirements, in which case banks are solely required to finish with a positive balance of reserves; banks cannot issue reserves.[9] An alternative regulatory constraint links a minimum amount of liquid assets (i.e., reserves) to illiquid assets (i.e., loans). As we show in Section 8, this has very similar implications. Whether reserve requirements are present does not affect our analysis: the demand for reserves is not generated by policy. A precautionary demand for reserves emerges as long as there is a non-negativity constraint on reserves.

Because of the withdrawal shock ω, banks will have uncertain reserve balances after the shock. This shock will map into a reserve surplus (or deficit) which is the final reserve balance in excess

[8]The lack of a liquid market for loans during the balancing stage can be explained by several market frictions discussed by the literature. For example, loans may be illiquid assets if banks specialize in particular customers, if they face agency frictions, if there is asymmetric information, or if these transfers take time. Reserves, instead, are special assets issued by the central bank and are fully liquid in the balancing stage.

[9]In fact, monetary policy in many advanced economies operates without a reserve requirement—for example, Canada or the United Kingdom.

of required reserves:

$$s_t^j = s(\tilde{m}_{t+1}^j, \tilde{d}_{t+1}^j, \omega_t^j) \equiv \underbrace{\left(\tilde{m}_{t+1}^j + \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}\right)\omega_t^j \tilde{d}_{t+1}^j\right)}_{\text{reserve balance after } \omega \text{ shock}} - \underbrace{\rho_t \tilde{d}^j{}_{t+1}(1+\omega_t^j)}_{\text{required reserves after } \omega \text{ shock}} . \tag{6}$$

The surplus, s_t^j, depends on the lending-stage choices of $\tilde{m}_{t+1}^j$, $\tilde{d}_{t+1}^j$, and ω_t^j. The first term in (6) is the reserve balance, given by the initial reserve position plus the reserves transferred from/to other banks. The second term is the required reserves, the product of the reserve requirement, ρ_t, and the outstanding amount of deposits after the withdrawal shock, given by (4). From (6), it follows that if a bank faces a large withdrawal, the level of reserves falls below the reserve requirement. In fact, shocks $\omega < \omega^* \equiv \rho_t - (m^j/d^j)\left[(1+i_{t+1}^d)/(1+i_{t+1}^{ior}) - \rho_t\right]$ translate into a reserve deficit. Moreover, if a bank accumulates few reserves in the lending stage or, likewise, issues many deposits, it is more likely to incur a reserve deficit.

Banks with a positive s_t^j will try to lend their excess reserves. Banks with a negative s_t^j must obtain reserves to satisfy the reserve requirements (5). Banks in deficit obtain reserves by borrowing from surplus banks in the interbank market, or by ultimately borrowing from the Fed's discount window. Considering the interbank and discount window loans, reserves evolve from the balancing stage to the next lending stage as follows:

$$m_{t+1}^j = \tilde{m}_{t+1}^j + \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}\right)\omega_t^j \tilde{d}_{t+1}^j \ + f_{t+1}^j \ + w_{t+1}^j. \tag{7}$$

This law of motion states that the end-of-period reserves, m_{t+1}^j, are the reserves left after the withdrawal shock plus reserves borrowed in the interbank market and the discount window.[10] Next, we describe how the interbank market operates.

Interbank market. At the beginning of the balancing stage, the realization of idiosyncratic withdrawal shocks generates a distribution of banks with reserve surpluses and deficits, s_t^j. Banks with a shock $\omega > \omega^*$ have a reserve surplus and therefore want to lend reserves; banks with $\omega < \omega^*$ are in deficit and must borrow reserves. Because there are matching frictions in the interbank market, banks on either side of the market may be unable to lend/borrow all of their surplus/deficit. If a bank in deficit cannot obtain enough funds in the interbank market, it can borrow the difference from the discount window as the last resort. Similarly, if a bank in surplus

[10] Notice that in order to satisfy (5), the sum of funds borrowed in the interbank market loans and at the discount window must satisfy $f_{t+1}^j \ + w_{t+1}^j \geq \rho_t d_{t+1}^j - \left[\tilde{m}_{t+1}^j + \omega_t^j \tilde{d}_{t+1}^j \left(\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}}\right)\right]$.

is unable to lend all of its surplus, it can keep its balance at the central bank and earn the interest on reserves. Because in equilibrium the interbank market rate lies strictly between the interest on reserves and the discount window rates, banks seek to trade in the interbank market before trading with the Fed. Interbank market and discount window loans are repaid in the next lending stage.

The interbank market is an OTC market, as advocated by Ashcraft and Duffie (2007). We follow Afonso and Lagos (2015) but make departures to obtain a closed-form solution for the interbank rate. Because the complete description of the interbank market is intensive in notation, we only provide a brief description here; all details can be found in a companion paper (Bianchi and Bigio, 2017).

The interbank market operates in a sequential way. At the beginning of the balancing stage, each bank instructs a continuum of traders with a trading order. Each trader must close an infinitesimal position, as in Atkeson et al. (2015). There are N trading rounds. Matches are formed at random according to a matching process in each round, and a number of trading positions close accordingly. The probability of a match at a given round is determined by an efficiency parameter λ and a matching function which in turn depends on the aggregate amount of surplus and deficit positions that remain open at that round. When traders meet, they bargain over the rate used and split the dynamic surplus according to Nash bargaining. The bargaining power for borrowers is denoted by η.[11]

This OTC market generates a sequence of volumes of interbank market loans and terms of trade throughout the trading rounds within each balancing stage. In the following proposition, we present a formula for the pair of interbank market and discount window loans $\{f^j, w^j\}$ as a function of market tightness (i.e., the relative magnitudes of banks in deficit and surplus) that we denote by θ. Let the aggregate amounts of surplus and deficit by respectively $S_t^+ \equiv \int_0^1 \max\left\{s_t^j, 0\right\} dj$ and $S_t^- \equiv \int_0^1 \min\left\{s_t^j, 0\right\} dj$. Then, the market tightness is $\theta_t \equiv S_t^- / S_t^+$.

Proposition 1. *Given θ_t, the amount of interbank market loans and discount window loans for a bank of surplus s^j is*

$$(f_t^j, w_t^j) = \begin{cases} -s^j(\Psi^-(\theta_t), 1 - \Psi^-(\theta_t)) \text{ for } s^j \leq 0 \\ -s^j(\Psi^+(\theta_t), 0) \text{ for } s^j > 0, \end{cases} \tag{8}$$

[11] Using the large family assumption of Atkeson et al. (2015) simplifies the analytical solution substantially. The assumption makes matching probabilities linear in the deficit or surplus position, regardless of the size of the surplus or deficit of a bank. This also avoids having match-specific terms in the bargaining problem. Without this assumption, the combinatorial problem of determining the distribution of matches becomes intractable.

and the average interbank market rate is

$$\bar{i}_t^f(\theta_t) = i_t^{ior} + (1 - \phi(\theta_t))(i_t^{dw} - i_t^{ior}), \tag{9}$$

where the formulas for $\{\Psi^+(\theta_t), \Psi^-(\theta_t), \phi(\theta_t)\}$ *are described in Appendix E.*

According to Proposition 1, a bank short of reserves ($s_t^j < 0$) is able to patch the fraction, Ψ_t^-, of its deficit with interbank market loans and the rest, $1 - \Psi_t^-$, from the Fed. Similarly, a bank with surplus lends its fraction Ψ_t^+ in the interbank market and keeps the difference in an account at the Fed. These fractions depend endogenously on the abundance of reserves. If there are many banks in deficit (surplus), the probability that a bank in deficit finds a match is low (high). In addition, the federal funds rate is a weighted average of the i^{ior} and i^{dw} with endogenous weights as a function of the abundance of reserves. As in Afonso and Lagos (2015), the weight $\phi(\theta_t)$ can be interpreted as an effective bargaining power for borrower banks in a one time match. The federal funds rate is closer to i^{dw} if more banks are in deficit because this lowers the bargaining power of the lenders. Conversely, the federal funds rate is closer to i^{ior} if more banks are in surplus. As shown in Appendix E, the functional forms for the time invariant functions $\phi(\theta_t)$ and $\{\Psi_t^-(\theta_t), \Psi_t^+(\theta_t)\}$ depend onthe two deepparameters of the matching market: efficiency of matching λ and the bargaining power η, In particular, a higher efficiency leads to higher fractions of matches $\Psi_t^- \Psi_t^+$ in the interbank market. Likewise, ϕ_t increases with the bargaining power of borrowers η, which makes the federal funds rate closer to the lower band of the corridor. Naturally, the volume of interbank market loans (interbank market) is given by $\Psi_t^+(\theta_t)S_t^+ = -\Psi_t^-(\theta_t)S_t^-$.

To summarize the costs/benefits of being short/long in reserves, we introduce the liquidity yield function. This object is useful once we express the law of motion for equity in the recursive formulation.

Definition 1. The liquidity yield function for a bank with a surplus s is

$$\begin{aligned}
\chi_t(s) &= \begin{cases} \chi_t^+ s & if \ s \geq 0 \\ \chi_t^- s & if \ s < 0 \end{cases}, \\
\chi_t^- &= \Psi_t^- \left(\bar{i}_t^f - i_t^{ior}\right) + \left(1 - \Psi_t^-\right)\left(i_t^{dw} - i_t^{ior}\right), \\
\chi_t^+ &= \Psi_t^+ \left(\bar{i}_t^f - i_t^{ior}\right).
\end{aligned} \tag{10}$$

The term χ_t is precisely the marginal cost/benefit of having a reserve deficit/surplus. An im-

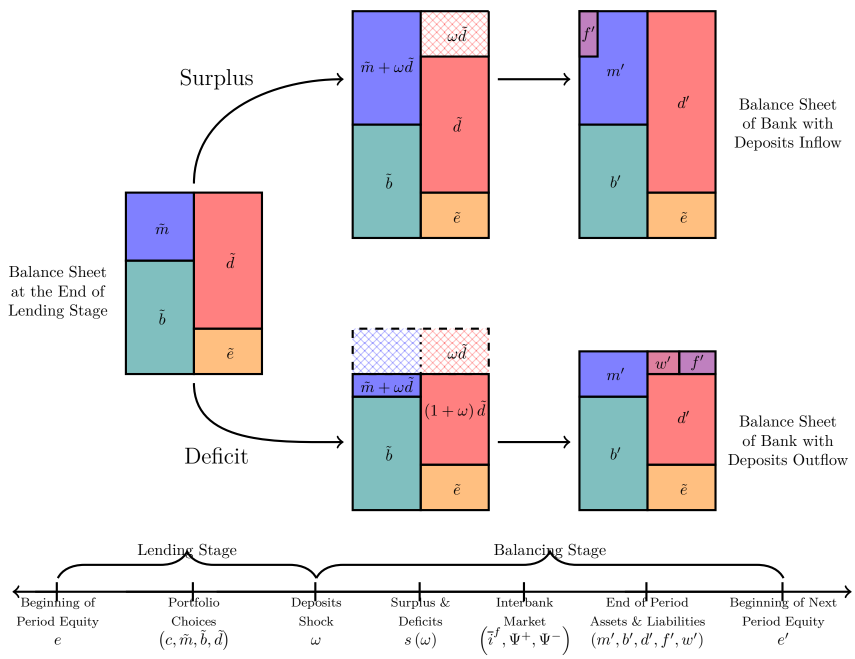

Figure 1: Timeline diagram and banks' balance sheet. For illustration purposes, it is assumed that the portfolio chosen in the lending stage is $\tilde{m} = \rho\tilde{d}$.

portant observation is that interbank market frictions will produce a positive wedge between the marginal cost reserve deficits and the marginal benefit of surplus.

This wedge is endogenous and depends on the relative abundance of reserves and policy. In frictionless environments, that wedge disappears because there is no kink in χ. For example, in a Walrasian interbank market, when $\lambda \to \infty$, $\bar{i}^f$ equals i^{ior} if $S^+ > S^-$ or equals i^{dw} if $S^+ < S^-$. In the other extreme in which there is no interbank market, the wedge becomes such that $\chi^- = i_t^{dw} - i_t^{ior}$ and $\chi^+ = 0$. This kink is critical for the liquidity management problem and the transmission of monetary policy, as we will argue in the policy analysis section. An exact formula for χ as a function of policy rates $\{\lambda, \eta\}$ and θ_t is presented in Appendix E. Figure 1 summarizes the timing of events.[12]

2.2 Central Bank Policies

The Fed issues reserves, sets corridor rates, and also purchases private loans. The Fed's balance sheet is analogous to the balance sheet of a bank, with the difference that the Fed does not

[12]We note that there are potentially other frictions that can create a wedge between lending and borrowing rates in the interbank market related (e.g. default risk). What is key for our liquidity management problem is the existence of such a wedge, and hence our analysis also applies to these other contexts.

issue deposits. Instead, the Fed issues reserves, M_t^{Fed}. On the asset side, the Fed holds discount window loans W_t^{Fed} and private loans B_{t+1}^{Fed}.[13]

The budget constraint of the Fed is[14]

$$(1+i_t^{ior})M_t^{Fed} + B_{t+1}^{Fed} + W_{t+1}^{Fed} = M_{t+1}^{Fed} + (1+i_t^b)B_t^{Fed} + (1+i_t^{dw})W_t^{Fed} + P_t T_t^{Fed}. \tag{11}$$

The Fed generates operating profits/losses from its balance sheet position and from activity in the interbank market: it pays interest on the outstanding stock of reserves, i_t^{ior}, and collects revenues from the interest from discount window loans, i_t^{dw}, and from holdings of private loans, i_t^b. To fund new discount window loans W_{t+1}^{Fed} and purchases of loans B_{t+1}^{Fed}, the Fed issues new reserves M_{t+1}^{Fed}. To balance the budget constraint, we endow the Fed with taxes/transfers, T_t^{Fed}.[15]

Monetary policy can be specified in various ways. We focus on a monetary dominant regime in which T_t^{Fed} is a choice for the Fed. In a stationary equilibrium, the inflation rate will be constant and equal to g, the nominal growth rate of reserves set by the Fed. Notice that at a steady state with $g = 0$, using subscript ss to denote steady-state variables, we have that the tax at steady state is given by

$$T_{ss}^{Fed} = \frac{i_{ss}^{ior} M_{ss}^{Fed} - i_{ss}^b B_{ss}^{Fed} - i_{ss}^{dw} W_{ss}^{Fed}}{P_{ss}}.$$

That is, taxes finance the interest payment of reserves net of the return on the stock of discount window and private loans. Away from a stationary equilibrium, we will consider two classes of policies. In the first class of policies, we keep the nominal balance sheet of the Fed growing at the long-run value. In this economy, movements in the real demand for reserves is accommodated with movements in the price level, following a quantity theory relationship, as we will explain below. Taxes follow from the operating profits/losses of the Fed and the balance sheet policies,

[13] Incorporating Treasury bills (T-bills) and conventional open-market operations into our model is relatively straightforward. If T-bills are illiquid in the balancing stage, T-bills and loans become perfect substitutes from a bank's perspective and the model becomes equivalent to our baseline model—with an additional market clearing condition for T-bills. If T-bills are perfectly liquid but are not counted as part of the reserve balance, we can show that banks that have a deficit in reserves will first sell their holdings of T-bills in exchange for reserves to banks that have surplus and only then go to the federal funds market. Because our focus is more broadly on liquidity management rather on the composition of liquid assets, we prefer to keep only one type of liquid assets in the model.

[14] To derive this condition, we combine the budget constraint of the Fed in the lending and balancing stage: $\tilde{M}_{t+1}^{Fed} + B_t^{Fed} + (1+i_t^{dw})W_t^{Fed} = (1+i_t^{ior})M_t^{Fed} + B_{t+1}^{Fed} + P_t T_t$, $M_{t+1}^{Fed} = \tilde{M}_{t+1}^{Fed} + W_{t+1}^{Fed}$.

[15] We completely abstract in the paper from frictions between the monetary authority and fiscal authority by considering a consolidated budget constraint.

according to (11). In a second class of policies, we consider an inflation targeting regime, in which the Fed alters the nominal supply of reserves to stabilize the inflation at its long-run value. To accomplish this, the Fed exchanges reserves for deposits with banks. Appendix M.3 provides the details on how the Fed accomplishes this in the model and the numerical algorithm used to compute the transitional dynamics with this inflation targeting regime.

2.3 Loan Demand and Deposit Supply

To close the model, we need a loan demand and deposit supply schedule. We assume that there is a downward-sloping loan demand and an upward-sloping deposit supply. In both cases, we consider constant elasticity functions. The loan demand schedule is

$$\frac{B_{t+1}^{d}}{P_t} = \Theta_t^b \left(\frac{1}{1+i_{t+1}^b} \frac{P_{t+1}}{P_t} \right)^{\epsilon}, \ \epsilon > 0, \ \Theta_t^b > 0, \tag{12}$$

where ϵ is the semi-elasticity of credit demand with respect to the real return on loans. The intercept Θ_t^b captures possible credit demand shifts.

The deposit supply schedule is

$$\frac{D_{t+1}^{S}}{P_t} = \Theta_t^d \left(\frac{1}{1+i_{t+1}^d} \frac{P_{t+1}}{P_t} \right)^{-\varsigma}, \ \varsigma > 0, \ \Theta_t^d > 0, \tag{13}$$

where ς is the semi-elasticity of deposit supply with respect to their real return.

Given these schedules for loan demand and deposit supply, and using the bank's optimal portfolios, we can solve for market clearing returns for loans and deposits. In Appendix C, we offer a simple microfoundation for the demand schedule for loans and supply for deposits. The advantage of this microfoundation is that these supply and demand schedules are static and thus do not impose additional dynamic restrictions on the model. Throughout the paper, however, we will work directly with the exogenous schedules.

2.4 Competitive Equilibrium

The initial conditions for an equilibrium are a distribution of $\{d_0^j, b_0^j, f_0^j, w_0^j\}$ over banks and a balance sheet for the Fed, $\{B_0^{Fed}, M_0^{Fed}, W_0^{Fed}\}$. Taking as given returns and Fed policies, banks choose $\left\{\tilde{d}_t^j, \tilde{b}_t^j, \tilde{m}_t^j, c_t^j, f_t^j, w_t^j\right\}$ contingent on their history of idiosyncratic shocks to maximize expected lifetime utility. Macroeconomic aggregates are deterministic since there is no aggregate risk. We adopt the convention of denoting aggregate variables in uppercase letters, and they are defined as

$$B_{t+1} \equiv \int_j b^j_{t+1} dj, \ M_{t+1} \equiv \int_j m^j_{t+1} dj, \ D_{t+1} \equiv \int_j d^j_{t+1} dj, \text{ and } W_t \equiv \int_j w^j_t dj.$$

The competitive equilibrium is formally defined below.

Definition 2. Given $\{d^j_0, d^j_0, b^j_0, f^j_0, w^j_0\}$ and a deterministic sequence of government policies $\left\{\rho_t, B^{Fed}_t, M^{Fed}_t, W^{Fed}_t, T^j_t, \kappa_t, i^{ior}_t, i^{dw}_t\right\}_{t\geq 0}$, a **competitive equilibrium** is a deterministic sequence of interest rates and prices $\left\{i^b_t, i^d_t, \bar{i}^f_t, P_t\right\}_{t\geq 0}$, a deterministic sequence of matching probabilities $\{\Psi^+_t, \Psi^-_t\}$, a deterministic path for aggregates $\{D_{t+1}, B_{t+1}, M_{t+1}, W_t\}$, and a stochastic sequence of bank policy variables $\left\{b^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, c^j_t, f^j_t, w^j_t\right\}_{t\geq 0}$ such that

(i) Banks' policies $\{b^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_t, c^j_t\}$ solve the banks' problems 1 and 2, and $\{f^j_t, w^j_t\}$ are given by the formula in Proposition 1.

(ii) The central bank's budget constraint (11) is satisfied and $\int_j T^j dj = T^{Fed}_t$.

(iii) Aggregate loans are consistent with (12) and aggregate deposits are consistent with (13) .

(iv) Markets clear $\forall t \geq 0$:

$$\int_j b^j_{t+1} dj + B^{Fed}_{t+1} = B^d_{t+1} \quad \text{(loan market clearing)}$$

$$\int_j d^j_t dj = D^S_{t+1} \quad \text{(deposit market clearing)}$$

$$\int_j m^j_{t+1} dj = M^{Fed}_{t+1} \quad \text{(reserve market clearing)}$$

$$\int_j f^j_t dj = 0 \quad \text{(interbank market clearing)}$$

$$\int_j w^j_t dj = W^{Fed}_{t+1} \quad \text{(discount window clearing)}$$

(v) The matching probabilities $\left\{\Psi^+_t, \Psi^-_t\right\}_{t\geq 0}$ and the federal funds rate $\bar{i}^f_t$ are consistent with the surplus and deficit masses S^-_t and S^+_t, as given by Proposition 1.

Definition 3. A **stationary equilibrium** is a competitive equilibrium where $\{D_{t+1}, B_{t+1}, M_{t+1}, W_t\}$ and P_t grow at rate g. A **steady state equilibrium** is a competitive equilibrium with $g = 0$.

Once we characterize equilibria in the next section, it will become clear that equilibrium in the reserve market, condition (reserve market clearing), is in fact a quantity theory equation. The bank's problem delivers a real demand for reserves for banks, which, combined with the nominal quantity of reserves set by the Fed, pins down the price level. A definition of equilibrium that considers the non-financial side of the economy, as in Appendix C, would include (34) and (35) and an additional price, the wage, as part of the equilibrium definition.

3 Theoretical Analysis

In this section, we derive theoretical properties of the model. We start with a description of the features that make the model tractable. In particular, we show an aggregation result where the only state variable is real aggregate equity. We explain how the bank's problem can be separated into a consumption-saving problem and a portfolio problem. The portfolio problem, in turn, can be reduced to a choice of liquidity and leverage ratio for banks. We end the theoretical study of the model by deriving some properties of classic monetary policy exercises.

3.1 Recursive Bank Problems

It is convenient to describe the banks' optimization problem in recursive form. Denote by V^l and V^b the banks' value functions during the lending and balancing stages, respectively. To keep track of aggregate states, which vary deterministically over time, we index policy functions, value functions, and prices by t. To ease notation, we omit superscript j in the Bellman equations.

At the beginning of each lending stage, the individual states are $\{b, m, d, f, w\}$. Choices in the lending stage are consumption, c, and portfolio variables $\{\tilde{b}, \tilde{m}, \tilde{d}\}$. These portfolio variables, together with the idiosyncratic shock, ω, become the initial states in the balancing stage. The continuation value is given by the expected value at the balancing stage V_t^b, under the probability distribution of ω. The bank problem for the lending stage is

Problem 1. *[Lending Stage Bank Problem]*

$$V_t^l(b, m, d, f, w) = \max_{\{c, \tilde{b}, \tilde{d}, \tilde{m}\} \geq 0} u(c) + \mathbb{E}\left[V_t^b(\tilde{b}, \tilde{m}, \tilde{d}, \omega)\right] \tag{14}$$

$$P_t c + \tilde{b} + \tilde{m} - \tilde{d}, \qquad \text{(Budget Constraint)}$$

$$= (1 + i_t^b)b - (1 + i_t^d)d + (1 + i_t^{ior})m - (1 + \bar{\imath}_t^f)f - \left(1 + i_t^{dw}\right)w - P_t T_t,$$

$$\tilde{d} \leq \kappa\left(\tilde{b} + \tilde{m} - \tilde{d}\right). \qquad \text{(Capital Requirement)}$$

Considering the solution to the interbank market found in Proposition 1, the bank problem at the balancing stage is

Problem 2. *[Balancing Stage Bank Problem]*

$$V_t^b(\tilde{b},\tilde{m},\tilde{d},\omega) = \beta V_t^l(b',m',d',f',w') \tag{15}$$

$$b' = \tilde{b} \qquad \text{(Evolution of Loans)}$$

$$d' = \tilde{d} + \omega\tilde{d} \qquad \text{(Evolution of Deposits)}$$

$$m' = \tilde{m} + \left(\frac{1+i_{t+1}^{d}}{1+i_{t+1}^{ior}}\right)\omega\tilde{d} + f' + w' \qquad \text{(Evolution of Reserves)}$$

$$s = \tilde{m} + \left(\frac{1+i_{t+1}^{d}}{1+i_{t+1}^{ior}}\right)\omega\tilde{d} - (1+\omega)\,\rho\tilde{d} \qquad \text{(Reserve Balance)}$$

$$m' \geq \rho d' \qquad \text{(Reserve Requirement)}$$

$$(f',w') = \begin{cases} -s(\Psi_t^-, 1-\Psi_t^-) \text{ for } s<0 \\ -s(\Psi_t^+, 0) \text{ for } s \geq 0. \end{cases} \qquad \text{(Interbank Market Transactions)}$$

Notice that in the balancing stage problem, there is no longer a maximization condition since the optimal choices of interbank loans and discount window loans have already been taken into account. The continuation value in the balancing stage is the value of the bank in the lending stage, which depends on the end-of-period portfolio variables.

3.2 Model Solution

Here we summarize the bank's problem in one Bellman equation with a single individual state variable. The single state is real equity, which we denote by e. The first step is to substitute V^b defined in (15) into (14). The second step is to express the right-hand side of the bank's budget constraint as a function of e. For this, we allow taxes on banks to be proportional to bank equity. Thus, define for

$$e_t \equiv \frac{(1+i_t^{ior})m_t + \left(1+i_t^b\right)b_t - \left(1+i_t^d\right)d_t - \left(1+\bar{i}_t^f\right)f_t - \left(1+i^{dw}\right)w_t}{P_t}(1-\tau_t), \tag{16}$$

where τ is the tax rate on bank equity. For future reference, we denote by E_t the aggregate level of real equity $\int_j e_t^j dj$.

The third step is to construct a law of motion for the individual equity of a bank. For that we update (16) one period forward and use the definition of the liquidity yield function (10) and the laws of motion of deposits and reserves, (4) and (7), to express

$$e_{t+1}=\frac{\left(1+i_{t+1}^{ior}\right)\tilde{m}_{t+1}+\left(1+i_{t+1}^{b}\right)\tilde{b}_{t+1}-\left(1+i_{t+1}^{d}\right)\tilde{d}_{t+1}+\chi_{t+1}\left(s_{t}^{j}\right)}{P_{t+1}}(1-\tau_{t+1}). \tag{17}$$

This law of motion for equity depends only on lending stage choices and the withdrawal shock in the balancing stage. Now, we can define the gross real returns of all assets:

$$R_t^m\equiv\frac{1+i_{t+1}^{ior}}{1+\pi_{t+1}},\ R_t^b\equiv\frac{1+i_{t+1}^{b}}{1+\pi_{t+1}},\ R_t^d\equiv\frac{1+i_{t+1}^{d}}{1+\pi_{t+1}},\ \text{and }\bar{\chi}_t(\tilde{m},\tilde{d},\omega)\equiv\frac{\chi_t(s(\tilde{m},\tilde{d},\omega))}{1+\pi_{t+1}},$$

where $1+\pi_t\equiv P_{t+1}/P_t$ is the gross inflation rate.

Proposition 2 makes use of the law of motion for future equity, the definition of real returns and shows that we can summarize the value functions in (14) and (15) in a single Bellman equation written in real terms.

Proposition 2. *[Single State Representation]*

$$V_t(e)=\max_{\left\{c,\tilde{m},\tilde{b},\tilde{d}\right\}\geq 0} u(c)+\beta\mathbb{E}_{\omega}\left[V_{t+1}(e')\right], \tag{18}$$

$$\frac{\tilde{m}}{P_t}+\frac{\tilde{b}}{P_t}-\frac{\tilde{d}}{P_t}+c=e, \tag{19}$$

$$e'=\left[R_t^m\frac{\tilde{m}}{P_t}+R_t^b\frac{\tilde{b}}{P_t}-R_t^d\frac{\tilde{d}}{P_t}+\bar{\chi}_t\left(\frac{\tilde{m}}{P_t},\frac{\tilde{d}}{P_t},\omega\right)\right](1-\tau_{t+1}), \tag{20}$$

$$\tilde{d}\leq\kappa\left(\tilde{b}+\tilde{m}-\tilde{d}\right). \tag{21}$$

This problem is a portfolio savings problem with a leverage constraint. The bank starts with equity, e, that can be allocated into dividends or investments. In turn, investments can be allocated into loans, $\tilde{b}$, and reserves, $\tilde{m}$, and the bank can leverage its position by issuing deposits $\tilde{d}$. Next period equity e' depends on the return realizations of the portfolios. Leverage is limited by the capital requirement. A non-standard feature is the presence of a non-linear return given by the kink in χ. Next, we show that despite the kink, we can still aggregate banks into a representative bank.

Proposition 3. *[Homogeneity and Portfolio Separation] The bank Bellman equation (18) can be characterized as follows:*

(i) The certainty equivalent of the bank's equity return solves the following problem:

$$\Omega_t \equiv (1-\tau_t) \max_{\{\bar{b},\bar{m},\bar{d}\}\geq 0} \left\{ \mathbb{E}_\omega \left[R_t^b \bar{b} + \ R_t^m \bar{m} - \ R_t^d \bar{d} + \bar{\chi}_t(\bar{m},\bar{d},\omega) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \tag{22}$$

$$\bar{b} + \bar{m} - \bar{d} = 1,$$

$$\bar{d} \leq \kappa \left(\bar{b} + \bar{m} - \bar{d} \right).$$

(ii) The value function $V_t(e)$ is

$$V_t(e) = v_t\,(e)^{1-\gamma} - 1/(1-\beta)(1-\gamma), \tag{23}$$

where v_t is

$$v_t = \frac{1}{1-\gamma} \left[1 + \left(\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1} \right)^{\frac{1}{\gamma}} \right]^{\gamma}. \tag{24}$$

(iii) The optimal bank dividend–equity ratio $\bar{c} \equiv c/e$ is

$$\bar{c}_t \ = \frac{1}{1 + [\beta(1-\gamma) v_{t+1} \Omega_t{}^{1-\gamma}]^{1/\gamma}}. \tag{25}$$

(iv) Policy functions for $\{\tilde{b},\tilde{m},\tilde{d}\}$ from (18) can be recovered from the optimal portfolio weights $\left\{\bar{b},\bar{m},\bar{d}\right\}$ obtained in (22) and consumption decisions $\{\bar{c}\}$ obtained in (25):

$$\begin{aligned}
\tilde{b}_{t+1}(e_t) &= P_t \bar{b}_t (1-\bar{c}_t) e_t, \\
\tilde{m}_{t+1}(e_t) &= P_t \bar{m}_t (1-\bar{c}_t) e_t, \\
\tilde{d}_{t+1}(e_t) &= P_t \bar{d}_t \ (1-\bar{c}_t) e_t.
\end{aligned}$$

Key to this proposition is that the budget constraint is linear in e and the objective is homothetic. Alvarez and Stokey (1998) show that the standard properties of dynamic programming on bounded spaces apply to homogeneous dynamic programming problems such as the ones here. This implies that the solution here is unique and policy functions are linear. Although there is a kink in the liquidity yield function, the bank's problem is homethetic and thus satisfies these properties.

Equation (22) represents the liquidity management problem. This problem consists of the choice of portfolio weights that maximize the risk-adjusted return on equity. The certainty equivalent portfolio value of the bank can be expressed as the sum of the returns of the individual assets plus the liquidity cost that depends on the surplus of reserves. The portfolio of the bank pins down a *real* demand for reserves in addition to a real demand for deposits and a real supply of loans. In the next section, we present properties of this portfolio problem.

An important implication of Proposition 3 item (iv) is that policy functions are linear in equity. As a result, two banks with different levels of equity are scaled versions of a bank with one unit of equity. This implies that the distribution of equity is not a state variable. Rather, the aggregate equity is a sufficient state for the banking side of the model.[16] As we show in Appendix I, aggregate bank equity evolves according to

$$E_{t+1} = (R^b_{t+1}\bar{b}_t - R^d_{t+1}\bar{d}_t)E_t(1-\bar{c}_t) + \frac{\tilde{M}_{t+2} + (B^{Fed}_{t+2} - B^{Fed}_{t+1}(1+i^b_{t+1}))}{P_{t+1}}. \tag{26}$$

This law of motion is obtained after combining the banks and the Fed budget constraints, and using market clearing for reserves and interbank market loans. This law of motion states that the equity of the bank tomorrow is given by the claims against firms (loans) net of the claims owed to households (deposits), and the net claims with the Fed plus transfers, which are determined by next-period balance-sheet policies.

Price-Level Determinacy. The equilibrium price level P_t is determined by a quantity-theory equation that resembles classic money demand but where the demand for money emerges from the real reserve balance of banks:

$$P_t \cdot \underbrace{\bar{m}_t(1-\bar{c}_t)\int e^j_t dj}_{\text{demand for real balances}} = \underbrace{M_t}_{\text{supply of nominal balances}}. \tag{27}$$

In this quantity equation, the left-hand side is the demand for real reserve balances in nominal terms. The real demand for reserves is given by the value of equity after paying dividends, $(1-\bar{c}_t)\int e^j_t dj$, times the portfolio weight on reserves, $\bar{m}_t$, as stated in Proposition 3. The reserve demand, $\bar{m}_t$, follows from the liquidity management portfolio problem Proposition 3 (item i), which encodes future information on returns and policies. Given the nominal supply of reserves

[16] Studying differences between large and small banks is beyond the scope of this paper. See Corbae and D'Erasmo (2014) and Corbae and D'Erasmo (2013) for a model where bank size matters.

set by the Fed, this quantity-theoretic equation determines the price level.[17]

There is, of course, a connection with cash-in-advance and money-search frameworks. Those models feature a demand for real balances that emerges from a transactions demand. Here, instead the reserve demand depends on the use of reserves as a settlement instrument for the interbank market.

3.3 Portfolio Management and Liquidity Premia

We analyze the bank's portfolio problem. As outlined in Proposition 3 (item i), the portfolio management problem consists of choosing portfolio weights on loans $\bar{b}$, reserves $\bar{m}$, and deposits $\bar{d}$ to maximize the risk-adjusted return on equity:

$$R^e \equiv R^b \bar{b} + \ R^m \bar{m} - R^d \bar{d} + \bar{\chi}_t(\bar{m}, \bar{d}, \omega).$$

If we substitute out loans, $\bar{b}$, from the budget constraint and suppress time subscripts, we have that the bank's objective is

$$\max_{\substack{\bar{d} \in [0,\kappa] \\ \bar{m} \in [0, 1+\bar{d}]}} \left(\mathbb{E}_\omega \left[\left(\underbrace{R^b}_{\text{Return on Loans}} - \underbrace{\left(R^b - R^m\right)}_{\text{Liquidity Premium}} \bar{m} + \underbrace{\left(R^b - R^d\right)}_{\text{Loan-deposit spread}} \bar{d} + \underbrace{\bar{\chi}_t(\bar{m}, \bar{d}, \omega)}_{\text{Liquidity Yield}} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.$$

If the bank only invests in loans and $\bar{d} = \bar{m} = 0$, it obtains a return on equity equal to the return on loans, $R^e = R^b$. If the return on loans exceeds the return on deposits, issuing deposits provides an *external finance premium* of $R^b - R^d$. However, it also exposes the bank to greater liquidity risk. Banks can self-insure against the liquidity risk by holding more reserves because the penalty from a deficit in reserves is decreasing in $\bar{m}$ (i.e., $\chi\left(\bar{m}, \bar{d}, \omega\right)$ is increasing in $\bar{m}$).

Leverage Decision. We have the following first-order condition for deposits:

$$R^b - R^d = \mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \cdot \frac{\partial \bar{\chi}_t\left(\bar{m}, \bar{d}, \omega\right)}{\partial \bar{d}} \right] + \mu \frac{\kappa}{1+\kappa}, \tag{28}$$

[17]With binding capital requirements, the price level remains determined at steady state. Bank wealth and liquidity management pin down the aggregate demand for reserves. In fact, in a steady state without withdrawal risk, the price level can be obtained analytically: equity is at steady state only when its return is equal to $1/\beta$. Without risk, banks maintain reserves equal to the reserve requirement and steady state equity and portfolios are determined in closed form. This leads to a static quantity equation. In the absence of capital requirements, and with positive interest on reserves, which eliminates the spread between loans and reserves, there is indeterminacy. See Ennis (2014) for a thorough analysis of these aspects.

where μ denotes the Lagrange multiplier on the capital requirement constraint. The loan-deposit spread (28) involves two terms. The first term is the risk-weighted marginal cost of issuing depositsincluding the expected payments in the interbank market. The second term is the shadow cost of tightening the capital requirement constraint.

Liquidity Decision and Liquidity Premium. When portfolio weights on loans and reserves are strictly positive, we have an expression for the liquidity premium (LP), the spread between loans and reserves:

$$\begin{aligned}\underbrace{R^b - R^m}_{\text{liquidity premium}} &= \frac{\mathbb{E}_\omega\left[(R^e_\omega)^{-\gamma}\cdot\frac{\partial\bar{\chi}_t\left(\bar{m},\bar{d},\omega\right)}{\partial\bar{m}}\right]}{\mathbb{E}_\omega\left(R^e\right)^{-\gamma}}, \\ &= \underbrace{\mathbb{E}_\omega\left[\frac{\partial\bar{\chi}_t\left(\bar{m},\bar{d},\omega\right)}{\partial\bar{m}}\right]}_{\text{interbank market return}} + \underbrace{\frac{\mathbb{COV}_\omega\left[(R^e)^{-\gamma},\frac{\partial\bar{\chi}_t\left(\bar{m},\bar{d},\omega\right)}{\partial\bar{m}}\right]}{\mathbb{E}_\omega\left[(R^e)^{-\gamma}\right]}}_{\text{risk premium}}.\end{aligned} \tag{29}$$

The liquidity premium can be decomposed into two components. The first component, which is the first term on the right-hand side of (29), is the *interbank market return.* This term represents the expected marginal benefit of holding an extra unit of reserves in the interbank market. Using the definitions for the liquidity yield and the reserve surplus, this term can be expressed as

$$\mathbb{E}_\omega\left[\frac{\partial\bar{\chi}_t\left(\bar{m},\bar{d},\omega\right)}{\partial\bar{m}}\right] = \frac{1}{1+\pi_t}\left[\chi^+\left(1-F\left(\omega^*(\bar{m},\bar{d})\right)\right)+\chi^- F\left(\omega^*(\bar{m},\bar{d})\right)\right],$$

where $\omega^*(\bar{m},\bar{d})$ is the threshold at which the reserve balance changes from surplus to deficit for a given portfolio $\{\bar{m},\bar{d}\}$. Given a withdrawal shock ω, an additional unit of reserves allows the bank to save χ^- if the bank is in deficit, which occurs with probability $F\left(\omega^*\right)$, and allows the bank to earn χ^+ if the bank is in surplus, which occurs with probability $\left(1-F\left(\omega^*\right)\right)$.

The second term in the LP is the *liquidity risk premium.* Since $\partial\bar{\chi}_t/\partial\bar{m}$ is decreasing in ω and the return on equity, R^e, is increasing in ω, the risk premium is positive. Intuitively, reserves provide a higher return when the bank suffers adverse withdrawal shocks. This premium disappears if banks are risk neutral or if there is no kink in the liquidity yield function. In the latter case, $\partial\bar{\chi}_t/\partial\bar{m}$ becomes a constant, and hence the covariance term vanishes.

The decomposition of the liquidity premium clarifies the role of the friction in the interbank market. In a Walrasian limit, $\chi_t^+ = \chi_t^-$, so the kink and therefore the risk premium disappear.

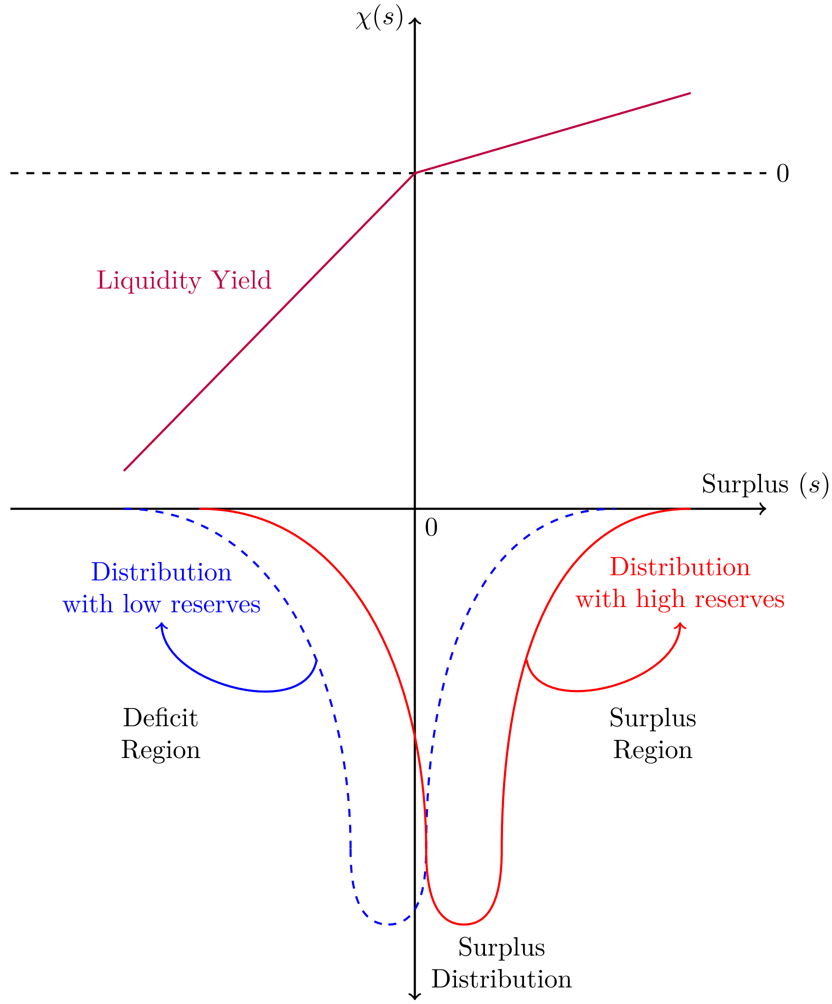

Figure 2: Description of Liquidity Management Problem

Changes in the volatility or other higher moments of ω have no effects on the LP and consequently do not affect bank lending decisions.[18]

Graphical illustration. The key trade-off characterized by the first-order conditions can be understood visually through Figure 2. The x-axis in the figure corresponds to the reserve balance s of a given bank. Above the x-axis we plot $\chi(s)$, the total interest earned or paid in the interbank market as a function of the reserve surplus. The slope of this liquidity yield is given by the market conditions and the Fed's policy. The reserve surplus depends on its portfolio choices in the lending stage and the realization of the withdrawal shock. Below the x-axis we plot the probability distributions for s for two different choices of $\{\bar{m}, \bar{d}\}$. The region to the left of the

[18] See Bianchi and Bigio (2017) for a discussion of the Walrasian limit of the interbank market. In a Walrasian limit, the federal funds rate is always i^{ior}(when there is an aggregate surplus of reserves) or i^{dw} (if there is an aggregate deficit of reserves).

y-axis represents the probability of ending up in deficit. The figure depicts how, by increasing m, the distribution, depicted with a dashed red line, shifts to the right, meaning that surpluses are more likely. When a bank chooses between lending more or holding more reserves, it compares the spread between the return on loans and reserves with the increase in the probability of ending in a surplus. With risk aversion, probabilities are weighted by marginal utilities. This is precisely the trade-off expressed condition (29). In summary, the LP indicates that banks are willing to sacrifice the premium on illiquid assets to insure against the possibility of adverse withdrawal shocks.

3.4 Monetary Policy Analysis

This section analyzes the transmission of monetary policy and highlights the central role played by the liquidity premium. We show how Fed policies alter the liquidity premium and carry real effects, even in the long run. In addition, we provide conditions under which certain Fed policies reproduce classic neutrality results.

It is useful to start with conditions under which the liquidity premium vanishes (i.e., if $R_t^b = R_t^m$). When the liquidity premium vanishes, we say that the banking system is satiated with reserves. Satiation is attained under one of the following two conditions.

Proposition 4 (Conditions for satiation)**.** *Banks are satiated with reserves at t if either (i) $i_t^{dw} = i_t^{ior}$ or (ii) $\bar{m}_t \geq \rho\omega_{\min}\bar{d}_t$ holds.*

Under condition (i), the Fed lends reserves at the discount window at the same rate it remunerates reserves, i_t^{ior}. This means that the cost of the reserve deficit is zero, and in that case there is no liquidity risk and hence no premium.[19] Under condition (ii), banks hold sufficient reserves to be in surplus for any withdrawal shock. For condition (ii) to hold, the Fed must carry operations that ensure that this is an equilibrium outcome.[20]

Before considering policies that deliver real effects via changes in the liquidity premium, we establish a classic neutrality result.

[19] Remunerating reserves at the same rate of deposits, rather than loans, is not sufficient to ensure satiation in the presence of capital requirements because a wedge would remain between the return on reserves and loans, as implied by (29) and (28).

[20] We do not model why the Fed might choose to induce a positive liquidity premium. We simply take as given that this is a standard policy instrument to affect credit creation. This can be motivated by a fire sale externality that arises because of a marked-to-market capital requirement constraint (see, e.g., Stein (2012) and Bianchi and Mendoza (2017)). Another natural motivation for a policy that induces a positive premium is aggregate demand management for macroeconomic and price stability.

Proposition 5 (Conditions for Policy Neutrality). *Consider the real asset positions* $\{B_t/P_t, D_t/P_t, M_t/P_t\}$ *in a stationary equilibrium induced by a Fed policy with a balance sheet* $\{M_{t+1}, W_{t+1}, B_{t+1}\}$ *that grows at rate* g *and nominal policy rates* $\{i_t^{dw}, i_t^{ior}\}$*. Then,*

i) An increase in the initial balance sheet by a multiple $k > 0$ *increases the price level by* k *without any effects on the real asset positions in the stationary equilibrium.*

ii) Away from satiation, an increase in g *has no effects on real asset positions if and only if real policy rates* $\{\frac{1+i_t^{ior}}{1+g}, \frac{1+i_t^{dw}}{1+g}\}$ *are constant.*

Part i) establishes long-run money neutrality. A qualification for this neutrality result is that it applies only to the stationary equilibrium. Because loans and deposits are denominated in nominal terms, the change in policy affects the initial price level and, through this, it affects the real equity of the bank. In the long run, however, changing the nominal balance sheet of the Fed by a multiple k leads to the same initial stationary equilibrium. Part ii) is concerned with the issue of super-neutrality or non-super-neutrality. If nominal rates are adjusted by the increase in inflation, then there are no real effects. Instead, an increase in the growth rate of the nominal balance, for given nominal rates i_t^{dw}, i_t^{ior}, affects the real return on reserves and the discount window, and hence delivers real effects.

An important distinction relative to classical results with cash-in-advance constraints as well as the New Keynesian literature is that permanent increases in inflation do affect real returns.[21] The reason is that in those environments, the real interest rate is the rate of time preference in a stationary equilibrium. Given the real rate, there is a one-to-one relation between inflation and nominal rates through the Fisher equation. The endogenous liquidity premium that arises because of the frictions in the interbank market breaks this tight link and allows the Fed to affect real rates in the long-run . Next, we consider how open-market operations that exchange loans for reserves have real effects.

Proposition 6 (Real Effects of Open-Market Operations). *Consider a competitive equilibrium induced by balance sheet policies* $\{M_{t+1}, W_{t+1}, B_{t+1}\}$ *and policy rates* i_t^{dw}, i_t^{ior}*. Consider also a time-zero open-market operation of size* ΔB_0^{Fed} *reversed the following period, that is, a policy sequence* $\{\tilde{M}_{t+1}, \tilde{W}_{t+1}, \tilde{B}_{t+1}\}$*, such that*

1. $\tilde{B}_1^{Fed} = B_1^{Fed} + \Delta B^{Fed}$, $\tilde{M}_1^{Fed} = M_1^{Fed} + \Delta M^{Fed}$*, and* $\Delta M^{Fed} = \Delta B^{Fed} \geq 0$*.*
2. $\{M_{t+1}, W_{t+1}, B_{t+1}\} = \{\tilde{M}_{t+1}, \tilde{W}_{t+1}, \tilde{B}_{t+1}\}$ *for all* $t > 1$*.*

[21] In these environments, changes in the permanent rate of inflation affect the economy because inflation acts like a tax on certain transactions carried out in non-interest-bearing assets or because of price stickiness.

The operation is neutral if and only if banks are satiated with reserves at $t = 0$ in the equilibrium induced by $\{M_{t+1}, W_{t+1}, B_{t+1}\}$.

When banks are satiated with reserves, open-market operations are irrelevant. For every unit of loans the Fed purchases, the banks reduce their holdings of loans by one unit. Away from satiation, however, open market operations alter the liquidity premium and induce a change in the total amount of loans.

Overall, our model articulates a credit channel of monetary policy. The model puts liquidity management by banks at the center stage of the transmission of monetary policy. Shocks to withdrawals of deposits and frictions in the interbank market create a special role for central bank reserves. Importantly, withdrawal risk generates a precautionary motive for holdings for reserves well and above the amounts required by regulation. Having a monopoly over the supply of reserves allows monetary policy to alter the volume of credit by shifting the liquidity premium. Next, we describe the calibration of the model and proceed with a quantitative application.

4 Quantitative analysis

4.1 Calibration

We calibrate the model so that its steady state fits regularities of the pre-crisis US financial system, in particular, the federal funds market. We take 2006-2007, the last two years before the recent US financial crisis, as the reference period.

Model period. We define the time period to be a month. In the US, the Fed funds market operate daily, and reserve requirements are computed by averaging end-of-day balances over a two-week window. On the other hand, bank portfolio decisions and loan sales are likely to take longer than two weeks. Capturing these institutional details would require a more complex structure with multiple balancing stages and more complicated reserve requirements. We view a monthly model as a reasonable middle ground between the daily nature of the Fed funds market and the lower frequency of bank decisions. The choice of a monthly model is also practical once we turn to the application in Section 7: data is available only monthly and having a lower time period would complicate the numerical implementation. As long as interbank-market positions are sufficiently persistent within lending stages, a higher frequency of interbank trades will not lead to quantitatively significant differences in our model.

Distribution of withdrawal shock. For the distribution of the withdrawal shocks to deposits, Φ_t, we assume $1+\omega$ follows a log-normal distribution with standard deviation σ and where the mean is chosen so that given σ, ω has a zero mean. The calibration of σ is explained below. A log-normal distribution is convenient because it delivers a distribution of excess reserves that fits the empirical counterpart.

Parameter values. The values of all parameters are listed in Table 1. In summary, we need to assign values to 16 parameters that we divide in two subsets $\{g, i^{ior}, i^{dw}, B^g, \kappa, \rho, \beta, \gamma, \Theta^b, \Theta^d, \epsilon, \eta\}$ and $\{\zeta, \sigma, \lambda\}$. The parameters in the first subset are chosen independently of model simulations: we set the nominal growth of reserves g to 2 percent to obtain a steady state inflation rate of 2 percent. We set $i^{ior} = 0$ because the Fed did not pay interest on reserves prior to 2008. Accordingly, we also set $B^g = 0$, in line with the close to nil holdings of private securities by the Fed before the crisis. The discount window rate is set to $i^w = 6$ percent in annualized terms, which was the nominal primary credit discount rate during 2006. We set the bargaining parameter to $\eta = 0.5$ as the baseline value. An equal bargaining power to banks in surplus and deficit leaves the federal funds rate in the middle of the corridor when the market tightness is close to one. We set $\rho = 0.10$, which is the reserve requirement that applies to roughly the entire banking system.[22] We set the capital requirement to $\kappa = 0.10$ to have a capital adequacy ratio of 9 percent, in line with Basel regulation.[23]

Two parameters, $\{\beta, \gamma\}$, govern banks' preferences. The discount factor β determines the dividend rate, which at steady state must be equal to the return on equity. Accordingly, we set the annualized discount factor to match an annual return on equity of 8 percent, which is approximately the sum of common and preferred dividends over equity (i.e., $\beta = 1 - (1 + 0.08)^{1/12} = 0.993$). We choose 1 for the value of the intertemporal elasticity of substitution, γ. A unit value has the advantage of simplifying the computations by making dividend payments only a function of the level of equity, as substitution and income effects cancel out.

The calibration of the loan demand and deposit supply schedules requires four parameter values: two scale and two elasticity parameters. The scale of the deposit supply Θ^d is set to obtain an annualized real deposit interest rate of 1 percent, which is in the range of the interest

[22] For banks with net transactions over USD 48.3 million as of 2006, the reserve requirement is 10 percent (see Federal Reserve Bulletin, Table 1.15 https://www.federalreserve.gov/pubs/supplement/2006/02/table1_15.htm).

[23] Basel regulation features various capital requirements that banks simultaneously need to satisfy, some of which feature different risk weights when computing the value of banks' assets. We see 9 percent as appropriate given these different requirements. Notice that implicitly we are applying the same risk weights to loans and reserves, which is sensible in our model because both reserves and loans are risk free. Below, we discuss an extension of the model with risky loans.

rate on deposits, according to balance sheet data (Drechsler, Savov, and Schnabl, 2016). The scale of the loan supply Θ^b is normalized so that the level of equity is 1 at steady state. Neither elasticity plays a role at steady state, but these elasticities do matter in the transitions. Because the capital requirement is binding at steady state, the elasticity of the deposit supply does not have an impact on the banks' portfolios after an unanticipated shock. For simplicity we set it equal to the elasticity of loan demand, to be calibrated below. In fact, the only effect of the elasticity of the deposit supply is on the speed of convergence of bank equity to its steady state level. If equity is below steady state, a relatively lower elasticity leads to a bigger contraction in the interest rate of deposits, which increases banks' profits and speeds up convergence. The elasticity of loan demand also affects the speed of convergence in a similar way, but in addition is more important for the response in the volume of loans after a shock.[24] We calibrate this elasticity below.

The second set of parameters, $\{\lambda, \zeta, \sigma\}$, is chosen to match empirical features of the federal funds market. The three moments we target are: (i) the ratio of discount window loans to reserves; (ii) the distribution of excess reserves at the beginning of a trading session; and (iii) the response of bank credit to an increase in the federal funds rate. While this is a joint calibration exercise, each moment is particularly sensitive to a certain parameter, as we explain below.

The parameter λ, governs how quickly the interbank market trades, and hence is set to match the fraction of discount window loans given by the Fed as a fraction of the total amount of reserves. In 2006, this ratio was equal to 2 percent, which is obtained by setting $\lambda = 2.1$.

Next, we describe the choice of the volatility parameter σ. Afonso and Lagos (2014) describe how the distribution of excess reserve balances evolve throughout a typical federal funds trading session. Because their data are daily, we implicitly assume that the distribution within a business day is the same as the distribution within a month, the model frequency. The volatility of the withdrawal shock is set to minimize the discrepancy between the distribution of excess reserves at the beginning of each trading session in the model vis-à-vis the empirical counterpart. To achieve this, we follow a two-step iterative procedure. First, given σ, we set the value of λ that delivers the targets for discount window loans. Second, we compute the mean squared difference between the distribution of excess reserves in the model and the data, and pick the value of σ that minimizes this discrepancy. The resulting value is $\sigma = 5$ percent. The resulting distribution of the withdrawal shock and the equilibrium excess reserves vis-à-vis the empirical distribution

[24] In simulations in which the capital requirement constraint does not bind, there are effects on banks' portfolios on impact, as variations in the interest rate on deposits affect banks' willingness to leverage up.

Table 1: Calibration

	Value	Source/Target
Capital requirement	$\kappa = 10$	Regulatory parameter
Discount factor	$\beta = 0.993$	Dividend ratio = 8%
Risk aversion	$\gamma = 1$	Constant dividend-equity ratio
Reserve requirement	$\rho = 0.1$	Regulatory parameter
Deposit supply intercept	$\Theta^d = 9.6$	Annual deposit rate = 1%
Loan demand intercept	$\Theta^b = 10.4$	Unit steady state equity
Discount window rate (annual)	$i^{dw} = 6\%$	2006 value
Interest on reserves (annual)	$i^{ior} = 0\%$	2006 value
Bargaining parameter	$\eta = 0.5$	Baseline value
Inflation	$g = 0.085\%$	Long-run inflation target=2%
Matching friction	$\lambda = 2.1$	DW to reserves W/M =2%
Volatility	$\sigma = 0.05$	Reserve-balance distribution
Loan demand deposit supply elasticities	$\zeta = -\epsilon = 25$	Bank credit response to policy rate

is presented in panel (b) of Figure 3.

Finally, the elasticity of loan demand is set to be consistent with vector autoregression evidence on the response of bank credit to a monetary policy shock.[25] In Bernanke and Blinder (1988), a 1 percentage point increase in the nominal policy rates produces a decline in bank credit of 2 percent within a one-year horizon. We replicate this response in our model by settting $\zeta = 25$. Given the monthly frequency, this implies that a 1/12 percentage point increase in the rate of loans, reduces the stock of loan demand by 1/12*25 $\sim$ 2%.

5 Sensitivity

We first show that the model fits the targeted moments in steady state. Panel (a) of Figure 3 shows the calibrated distribution of withdrawal shocks. The standard deviation is about 5 percent. Panel (b) shows the fit to the distribution of excess reserves in the data in Afonso and Lagos (2014)—recall that volatility is calibrated to minimize the distance between the model and the empirical distributions. Panel (b) also shows how increasing or reducing the volatility of the shock by 50 leads to a distribution of excess reserves that departs further from the data. Panel (c) shows the mapping between the severity of the friction in the interbank market, parameterized by λ, and the amount of discount window loans as a fraction of total reserves —our first target.

[25] An alternative to our macro approach to discipline the elasticity of loan demand would be to use loan-level demand elasticities. The challenge is to disentangle supply from demand effects.

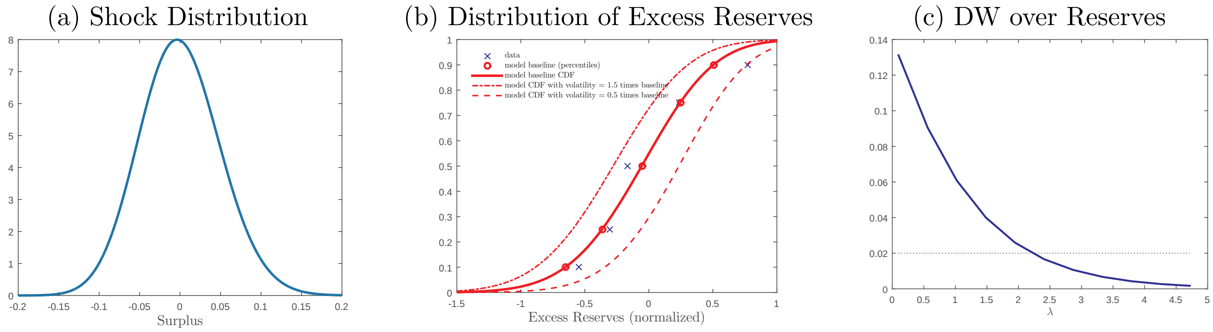

Figure 3: Shock distribution, parameters and moments

Note: For panel (b), the distribution of excess reserves in the model and the data has been normalized by the mean levels. Data source: Afonso and Lagos (2014).

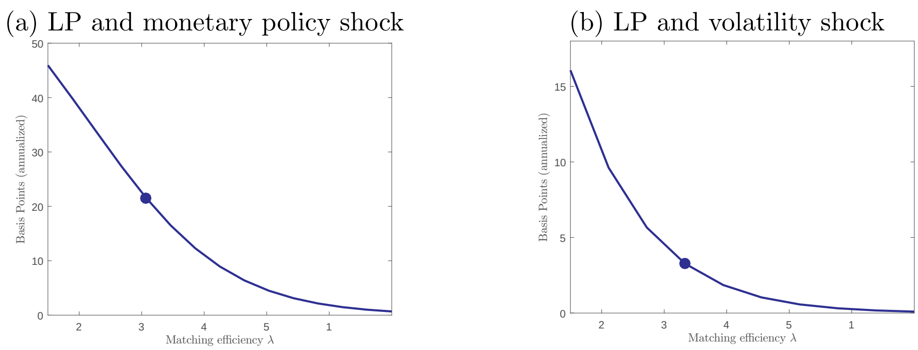

Figure 4: Sensitivity to matching friction λ.

Note: The liquidity premium (LP) is expressed as deviations from the steady state liquidity premium associated with each value of λ. Solid dots are the baseline values of λ in our calibration. For the monetary policy shock, we consider an increase in the discount window rate that, at the baseline calibration, produces an increase of 100 annualized bps in the Fed funds rate —the required increase in the discount window is 300 bps. For the effects of volatility, we consider an increase volatility of 10 percentage points, which is the increase in volatility that we estimate in our quantitative application of Section 7.

As λ rises, the interbank market becomes more efficient and there is less use of the discount window. At $\lambda = 2.1$, the model matches the empirical target of 2%.

The model also generates long-run moments of other (non-targeted) important statistics that are in line with the data (corresponding to US 2007 values). In particular, the model generates a volume of interbank market loans as a fraction of deposits of 1.6% (versus 1.8% in the data). In addition, the liquidity premium, is 2.8% (versus 3.8% in the data). In the data, we measure the liquidity premium as the difference between the return of an illiquid asset and the interest on reserves. Following Nagel (2016), we treat the three-month general collateral repurchase agreements as the data analogue of the return on illiquid assets.

Next, we show how monetary policy shocks and withdrawal volatility shocks affect the economy as a function of the magnitude of the friction of the interbank market, as captured by λ. We compute transitional dynamics away from steady state, in response to a one-period shock, and report the impact effects on the quantity of loans and the liquidity premium.[26] Panels (a) and (b) of Figure 4 show how the effects on liquidity premium vary for a range of λ for the two shocks. This figure shows how the response to these shocks is amplified as the value of λ is reduced (i.e., the friction in the interbank market becomes stronger).

6 Dynamic Responses

This section studies the economy's response to shocks that are associated with the hypotheses described in the introduction. The goal of the exercise is to show how the model works and to provide a basis for the identification that will be examined in more detail when we turn to infer several shocks from the data in Section 7.

General details of the experiments. All the shocks are unanticipated and arrive at $t = 0$. Their paths are deterministic thereafter. In all cases, shocks follow an autorregressive process $\varepsilon_t = \varrho \varepsilon_{t-1}$, $\forall t \geq 1$, where ε_0 is expressed as percentage deviations from the steady state. We set $\varrho = 0.8$ so that the half-life of all shocks is three years. The size of the shocks we consider are in the range of those we infer from the data in Section 7.

To compute the transitional dynamics in response to the shocks, we also need to specify the Fed policy. We assume that the Fed keeps a constant growth rate for reserves, M^{Fed}, equal to steady state inflation. In addition, the Fed adjusts the nominal discount window rate i^{dw} to keep a fixed real discount window rate and sets $i^{ior} = 0$. This is a natural benchmark because it helps us to isolate the endogenous response of lending and deposit creation to different shocks.

In this section, for each experiment, we report transitions for real equity and loans (in percentage deviations from the steady state), discount-window and interbank-market loans, the liquidity premium and liquidity ratio (in levels), and inflation (expressed as a simple deviation from steady state). The liquidity premium and inflation are annualized.[27]

Naturally, transitions depend on the initial level of equity, and we initiate equity so that real equity after the shock equals its steady state value, which we normalize to one. The reason for

[26] In this excercises, we assume that the Fed's policy is to keep inflation at its target value by varying the nominal quantity of reserves. This implies that the real return on reserves stays constant, and that variations in the liquidity premium are accounted exclusively by variations in the real return on loans.

[27] The responses to the full set of variables, and for the alternative policy where the Fed varies the amount of reserves to maintain an inflation target is available from the authors upon request.

this normalization is the following. Because assets are denominated in nominal terms, changes in the initial price level—as a result of shocks—alter the real value of equity. If we initiate the economy at steady state and the price level adjusts upon the arrival of a shock, equity adjusts automatically away from steady state. By setting $E_0 = E^{ss}$, we can distill the direct effect of each shock from its indirect effect via the valuation of equity.[28]

6.1 Transitions after Shocks to Bank Funding

Next, we study the effects of two shocks that affect the bank's ability to borrow: a shock that lowers the bank's equity and a shock to the capital requirement.

Equity Losses/Convergence. We begin with the analysis of a transition to steady state when the initial level of equity is 1 percent below steady state. This shock captures an unexpected rise in non-performing loans, for example. Because equity is the only endogenous state, these transitional dynamics are important in understanding the model's internal dynamics after all other shocks. The responses of some key variables are reported in Figure 5.

How does the economy return to steady state when equity is below steady state? To understand these dynamics, recall that Proposition 3 demonstrated that bank policies are linear in equity. This means that if portfolios weights are kept constant, a 1 percent drop in equity translates into a 1 percent contraction in all, the supply of loans, the demand for deposits, and the demand for reserves. For the loans market to clear after a contraction in the loan supply, an increase in the return for loans is needed. Similarly, the deposit rate must fall in order to clear the deposit market. Finally, for the reserve markets to clear, the initial price level needs to jump above steady state. With this increase in the price level, the real supply of reserves falls and equilibrium is restored.

Over time, bank equity increases: this is because the real lending rate rises as the deposit rate falls.[29] This greater spread makes intermediation more profitable. Over time, we can also see that the price level reverts to its stationary path and deflation keeps the real return on reserves relatively high. The overall effect of the shock to bank equity on the portfolio weights for reserves and loans—and thus the liquidity ratio—depends on the elasticities of the loan

[28] A constant time-zero equity can be produced by the model if (a) all the assets and liabilities are indexed to inflation or (b) a tax/transfer from households is introduced to keep the bank equity constant.

[29] Appendix J provides parameter conditions that guarantee monotone convergence to a unique steady state for a policy where the Fed induces satiation and earns no profits from its portfolio. We expect that for small distortions and shocks, the exercises has the same properties, a behavior that we verified numerically.

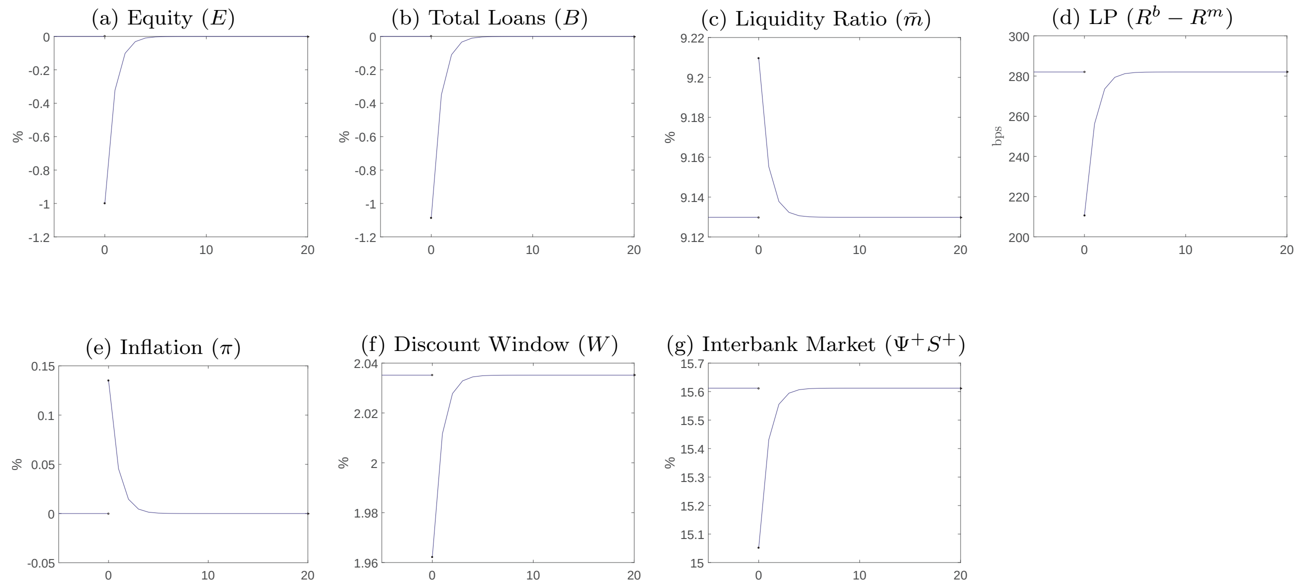

Figure 5: Transition after Equity Loss

Note: equity, loans, discount window loans, and interbank market are in real terms. Equity and loans are expressed in percentage deviations from steady state, while discount window loans, and interbank market are in levels. Inflation is expressed as a deviation from steady state.

demand and deposit supply. For our baseline calibration, the real quantity of reserves falls less than the real quantity of loans. This, in turn, results in a higher liquidity ratio and a lower liquidity premium. The latter result is inconsistent with the observed patterns during the crisis, as discussed in Appendix L.[30]

Capital Requirements. Next, we analyze a shock that increases the required level of equity by considering a reduction of 10 percent in κ. (Figure 13 in Appendix A reports the simulations.) The shock produces an immediate 10 percent decrease in bank leverage, which, like the equity shock, reduces the funds available to the bank. On impact, the general equilibrium effects are similar to those after equity losses and produce the same effects: a reduction in the supply of loans, a loan rate increase, and a deposit rate decline, as well as a contraction in the demand for reserves, which again produces a jump in the price level, an increase in the liquidity ratio and a fall in the liquidity premium. Immediately after the shock, equity grows beyond its steady state

[30]For lower loan demand elasticities, we can produce a decrease in the liquidity ratio. However, the liquidity ratio and the liquidity premium always move in opposite directions, which again is inconsistent with the observed patterns during the Great Recession. For example, consider the extreme case of when loan demand is perfectly inelastic at some quantity $\overline{B}$. Then, the real volume of loans should be unaltered at equilibrium by the decline in equity. Since $\bar{B} = \beta \bar{b}_t E_t / P_t$, the equilibrium can only be restored with a decrease in the liquidity ratio, with an increase in $\bar{b}_t$. This in turn would lead to an increase in the liquidity premium. For higher elasticities, the phenomenon is reversed.

value. This happens because the borrowing-lending spread increases. Eventually, the increase in equity overcomes the tightening of capital requirements and the sign of the effects reverses before the economy returns to steady state. Since the pattern of capital requirement shocks is similar to that of a decline in bank equity, neither hypothesis can be the predominant factor that explains the data.

6.2 Transitions after Shocks to Interbank Markets

We consider two sources of disruptions in the interbank market: a shock to the volatility of withdrawals and a shock to the matching efficiency λ. Figure 6 presents the transitions to these two shocks.

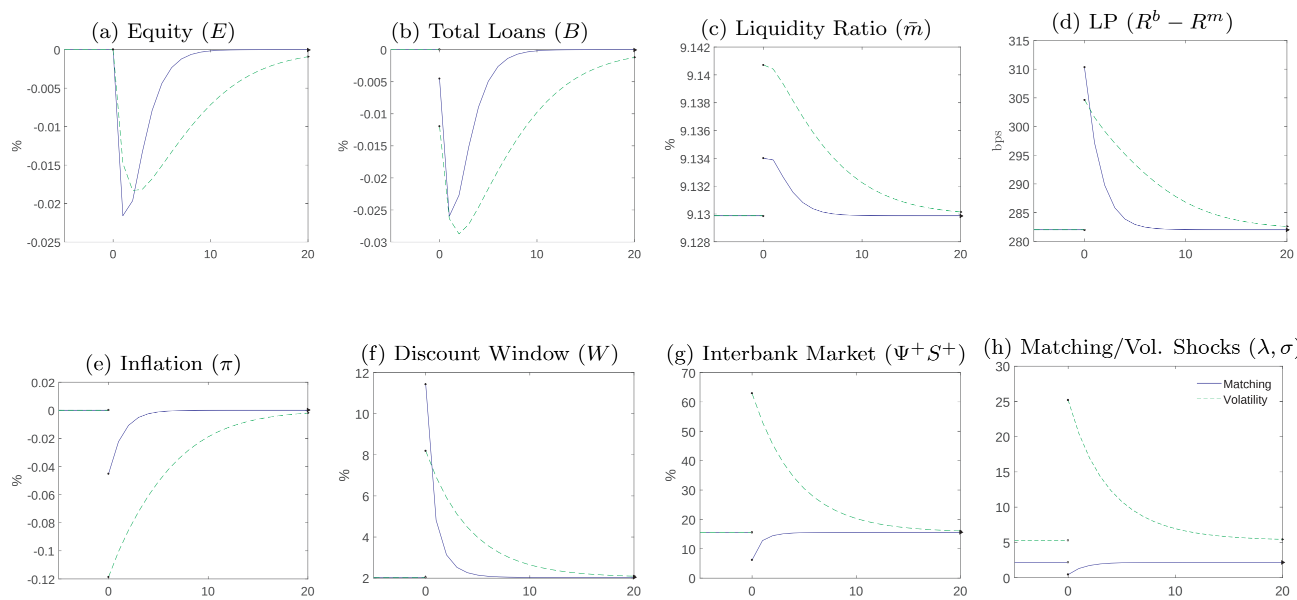

Figure 6: Transition after interbank market shocks

Increased Volatility. For a given portfolio, the increase in volatility generates an increase in liquidity risk. In response, banks increase their buffer of reserves and reduce the supply of lending. Since the Fed keeps the nominal supply growing at a constant rate, the higher demand for reserves is accommodated with a drop in the price level. Likewise, the decline in the loan supply generates an increase in the loan rate. In the aftermath of the shock, equity declines because banks allocate a lower fraction of their portfolio to loans, the high-return asset. We also observe that banks borrow more from each other and from the Fed, as a result of the larger withdrawal shocks that are realized (panels f and g). As the shock dissipates, the dynamics become similar to the case when equity is below steady state.

One important takeaway is that the increase in deposit instability can explain the effect on the liquidity premium and liquidity hoarding, but in contrast to the crisis, it leads to a counterfactual increase in the activity of the federal funds market. According to these predictions of the models, the observed pattern in the crisis appears to call for a shock to the efficiency in the interbank market, λ, which we explore below.

Matching Frictions. Like the increase in volatility, more frictions in the interbank market generate hoarding of reserves. Qualitatively, the responses are therefore similar with an important difference: while an increase in volatility raises trade in the interbank market, the reduction in λ has the opposite effect (panel g). In the following section, we calibrate paths of shocks to σ and λ to match the observed features of the interbank market and show their contribution to the decline in lending.

6.3 Transitions after Shocks to Credit Demand

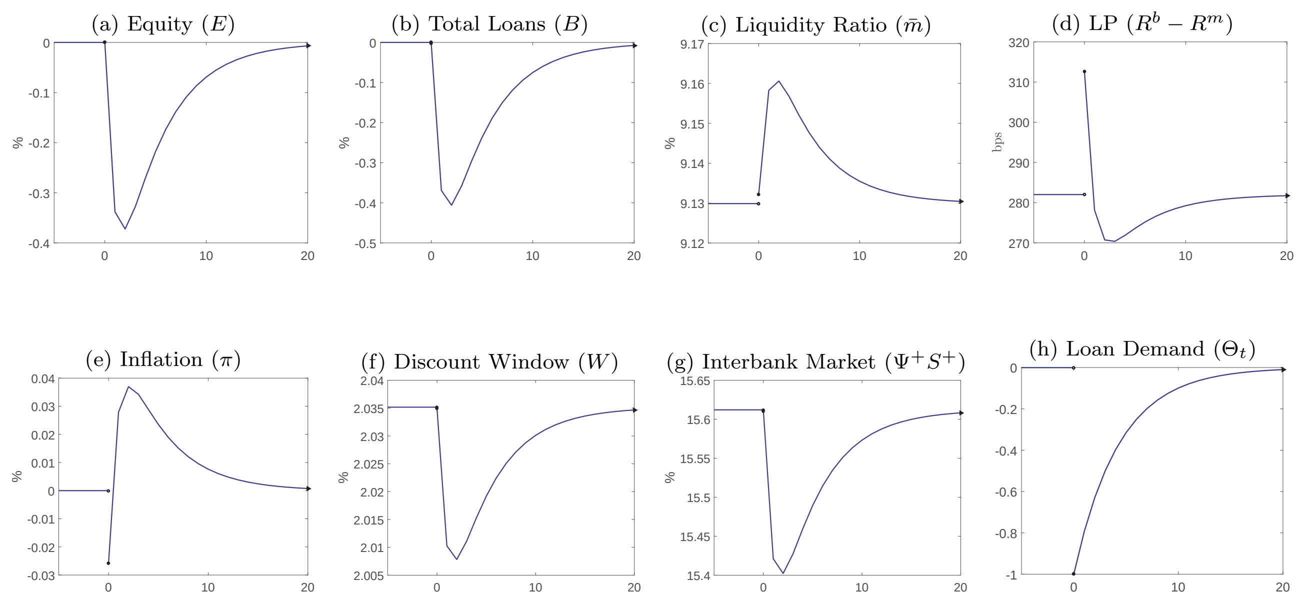

Figure 7: Transition after credit demand shock

The effects of a negative credit demand shock are captured through a decline in Θ_t. Figure 7 illustrates the effects of a negative temporary shock to loan demand. The effects contrast sharply with the effect after the shocks considered above because, there, the supply rather than the demand for loans contracts. As a result, a key difference is that the demand shocks produce a decline in the return on loans. Since loans become less attractive, this leads to a higher liquidity ratio, and lower discount window and interbank market loans.

6.4 Transitions after Policy Shocks

Interest on Reserves. The next experiment presents a policy increase in the interest on reserves, i_t^{ior}, from 0 to 100 basis points in annualized terms. Given that the nominal return on reserves increases, banks respond by allocating a larger fraction of their portfolio to reserves. Since the supply of reserves is fixed, the price level declines. Simultaneously, because of this desired substitution, the supply of loans contracts.[31] To restore market clearing in the loans market, the return on loans increases. As the liquidity ratio increases, there are fewer interbank market loans and discount window loans. We can also observe a decline in the liquidity premium. Figure 14 in Appendix A illustrates the transition.

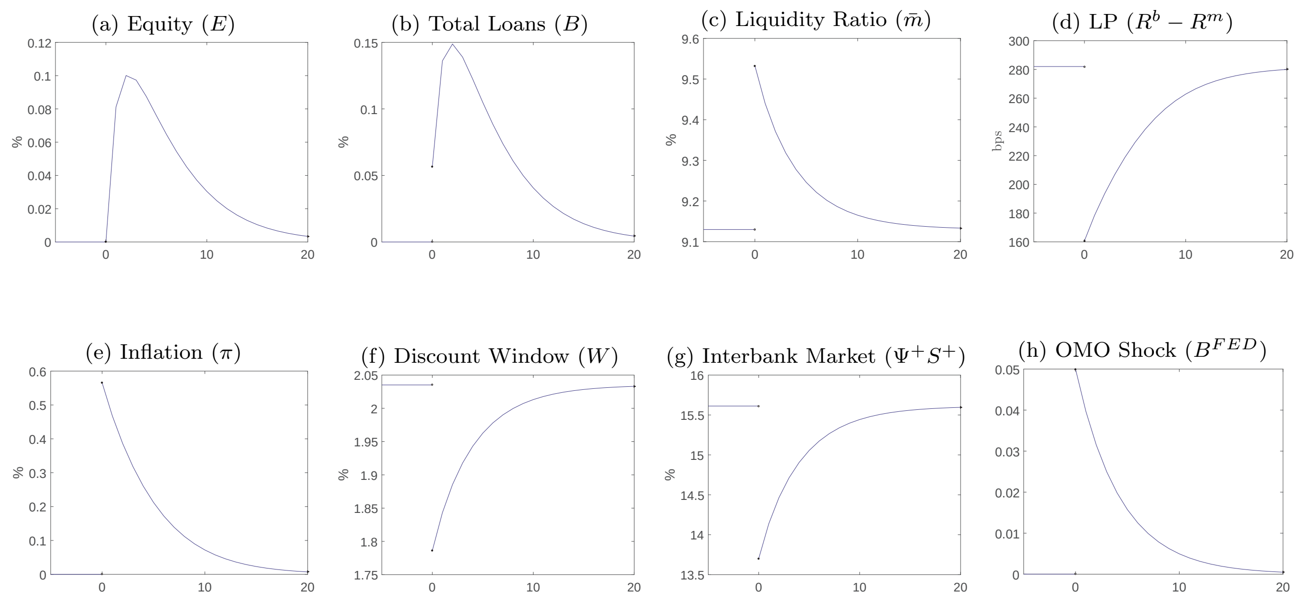

Figure 8: Transition after Open-Market Operations

Open-Market Operations. The final policy experiment is an open-market operation where the Fed exchanges reserves for loans. The Fed's real purchases of loans are shown in Panel (h) of Figure 8. The Fed's portfolio profits are rebated back to banks every period. In essence, this operation exchanges illiquid assets for liquid assets. The overall effect is to increase the total outstanding loans in the economy, as shown in Figure 8. There is a crowding-out effect of private holding of loans, though: as the Fed's purchases put downward pressure on the return on loans, banks reduce the amount of loans they provide. Bank equity increases, reflecting the larger overall investment in loans and the fact that the Fed rebates profits to banks. Finally, there are

[31] If the capital requirement constraint was not binding, banks could end up taking more deposits and increasing lending.

Table 2: Summary of Response to Shocks

	Loans	Reserves	Interbank Loans	DW Loans	LP
Data	↓	↑	↓	↑	↑
Equity loss	↓	↓	↓	↓	↓
Capital requirement	↓	↓	↓	↓	↓
Volatility	↓	↑	↑	↑	↓
Matching efficiency	↓	↑	↓	↑	↑
Credit demand	↓	↑	↓	↓	↑
Interest on reserves	↓	↑	↓	↓	↓

Note: The table lists the effects of shocks on different variables on impact.

inflationary effects from this policy resulting from the increase in the supply of reserves.

6.5 Summary and Discussion of Transitional Dynamics

Table 2 summarizes the response of the economy to different shocks considered. For reference, we also include a row with the movements observed in the data during the crisis. As the table shows, several of these shocks are candidates for explaining the decline in lending during the crisis. However, these shocks have different implications for other key variables. in the In the next section, we exploit the lessons from these transitional dynamics, to examine the shocks that led to the collapse in lending during the crisis.

7 Inspecting the Decline in Lending

We now turn to the quantitative application. We draw on the lessons from the transitional dynamics to quantitatively examine the causes of the joint contraction in lending and increased liquid holdings by banks during the 2008 crisis in the United States. We consider four hypotheses: (i) low bank equity, (ii) precautionary holdings of reserves, (iii) Fed policies, and (iv) weak credit demand. We discuss other alternatives in Section 8. Details on the motivating facts are discussed in Appendix L. The data sources are described in Appendix K.

7.1 Procedure

To weigh each hypothesis, we feed an associated sequence of shocks to parameters into the model. We should note that while these are transitional dynamics, i.e., not shocks in the technical sense, we still refer to "shocks" for the ease of exposition. Hypotheses (i) and (iii) are relatively straightforward to evaluate by feeding an observable path for equity losses (hypothesis i), and Fed policies (hypothesis iii) into the model.[32] Evaluating hypotheses (ii) and (iv) is more challenging because these shocks are unobservable. To evaluate hypothesis (ii), we associate it with shocks to the withdrawal volatility and the matching efficiency, whereas for hypothesis (iv), we associate it with a shock to loan demand. To discipline these unobservable shocks, we infer the sequences of shocks that allow the model to reproduce the time series for the volumes of discount window loans, interbank market loans, and bank credit.

We consider the period February 2008 through February 2010, which centers around the Lehman Brothers' bankruptcy. We thus have 25 months and three shocks so overall there are $25x3$=75 shock values to back out from a numerical procedure. Because we have as many shocks as observables, the model can, in principle, perfectly reverse engineer the sequence of values of that replicate the path of the series in the data.[33] Below we explain how the model indeed renders identification. Throughout the experiment, we assume that the Fed keeps inflation at the steady state value by varying the nominal quantity of reserves. This assumption is numerically convenient because, with it, we do not need to solve for the initial price level as we search for shocks. This policy is also consistent with the stable inflation path we observe in the data.

7.2 Identification

We have three shocks to match three series. To explain how we can identify the matching friction, volatility and loan demand shocks, we take two shocks at a time and plot the combinations of the value of the shocks that deliver the same data target. When we produce plots, we fix the third shock at its steady state value. Our goal is to show how the two shocks uniquely pin down two moment targets.

[32] The increase in capital requirements is part of hypothesis 1. However, capital requirements turn out to not play an important role for two reasons: First, the effective increase in capital requirements takes place starting in 2013 based on Basel, and even the anticipation of these policies has negligible effects. Moreover, if there had been a tightening of market-based constraints, this would have been offset in our model because the other shocks we estimate make these capital requirements slack for many periods.

[33] This is not guaranteed, however, because there is a limited support of the realization of the endogenous variables that the model can match. For example, as λ becomes large, the interbank market behaves similarly to a Walrasian market, in which case volatility shocks do not affect credit.

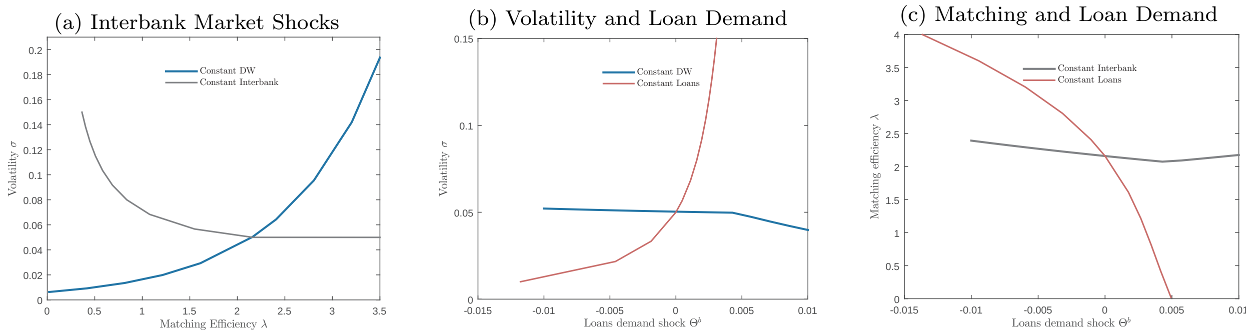

Figure 9: Identification for Quantitative Application

Let us use Panel (a) in Figure 9 as a first example. Panel (a) shows combinations of matching friction λ and volatility σ that deliver the same volumes of trade in the discount window (the Constant DW curve) and interbank market (the Constant Interbank curve).[34] As we can see, increasing the matching efficiency requires an increase in volatility to keep the discount window loans constant, which reflects an upward-sloping Constant DW curve. The opposite relation holds for the Constant Interbank curve: since an increase in matching efficiency increases the interbank market volume, volatility needs to decrease to keep the interbank market constant. As a result, there is only one crossing between the two curves. A single shock combination produces those two moments.

Panels (b) and (c) show that, similarly, one can separate the loan demand and volatility, and the loan demand and matching efficiency. As loan demand decreases, pushing down the equilibrium level of loans, one requires a reduction in volatility and an increase in matching efficiency to keep the level of credit constant. This is reflected in an upward-sloping curve for the Constant Loans curve in Panel (b) and a downward-sloping one in Panel (c). At the same time, discount window and interbank market loans are relatively less sensitive to changes in loan demand. Hence, the Constant DW and Constant Interbank curves are relatively flat in Panels (b) and (c).[35] Based on this, we conclude that discount window loans and interbank loans are more informative about λ and σ than about Θ^b.

[34] Volumes are expressed in terms relative to deposits, that is, we target $W/D = 0.2$ percent and $\min\{S^-, S^+\}/D = 2$ percent.

[35] Qualitatively, a decrease in loan demand also increases the level of reserves, and through this effect, it reduces discount window loans: hence, the negative slope in the constant DW curve in Panel (b). The effect of a decrease in loan demand over the interbank market depends on whether the market has a deficit or an excess of reserves. For the relevant case during the crisis period, in which banks had excess reserves, one obtains a positive relationship between the loans demand and the matching efficiency.

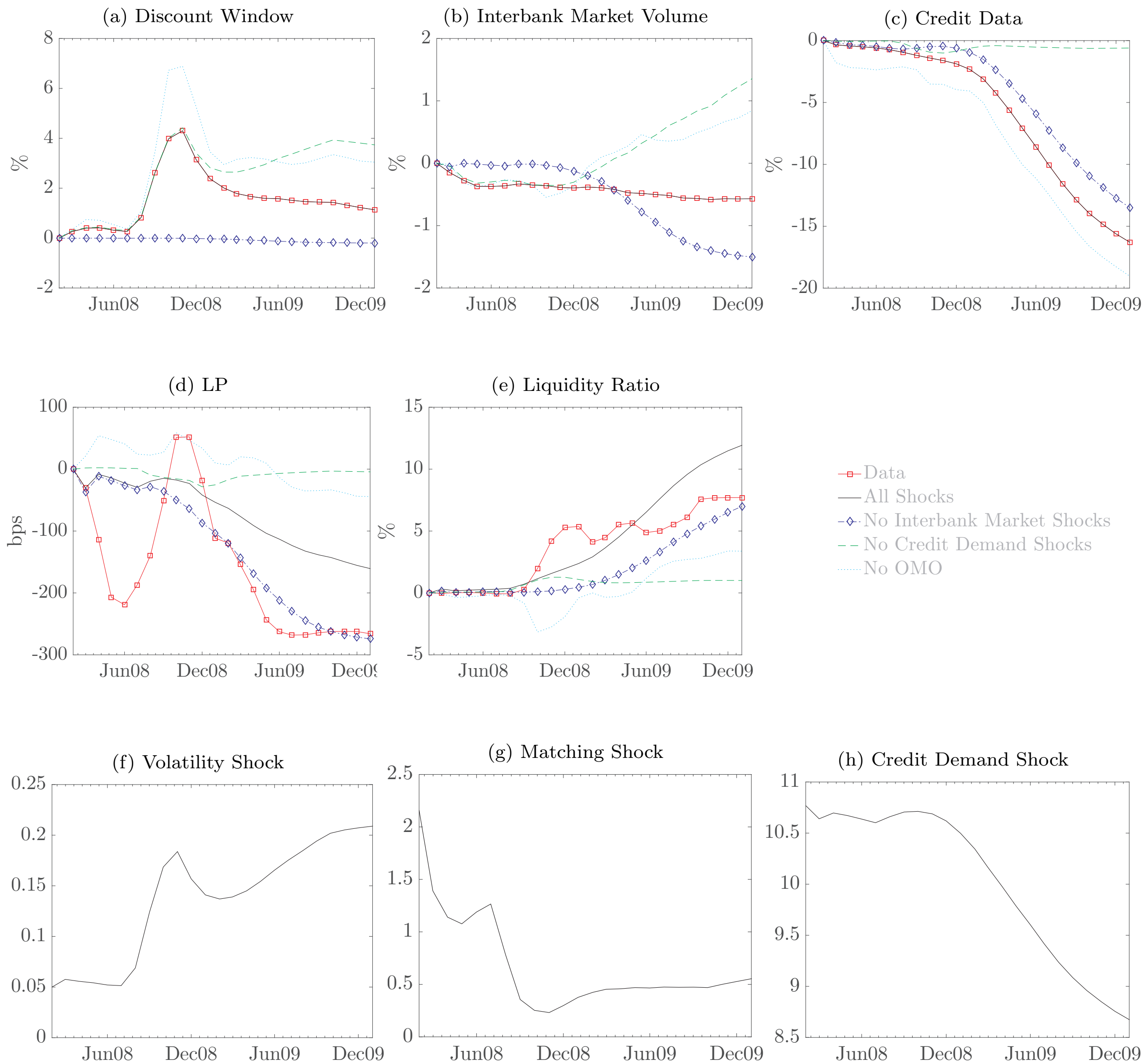

Figure 10: All Experiments

Note: discount window loans and interbank market loans are expressed as simple deviations from steady state, while loans are expressed as a fraction of the steady state level of loans. Interbank market loans are expressed in percent. Variables in the data have been smoothed using a 3-month rolling window.

7.3 Results

Figure 10 reports the results of our main experiment. In each panel of the upper row, we consider a different variable. Within each panel, we report the data series and the model analogue under three different scenarios. For comparison, model variables are expressed as deviations from the steady state, whereas the data series is reported in deviations relative to the value in January 2008 (see Figure 15 in Appendix L for details on the data series). In the lower row, we present the series for $\{\lambda, \sigma, \Theta^b\}$ that the model needs to fit the data. To evaluate each hypothesis, we evaluate the importance of each shock by turning off one shock at a time. The figure reports the simulations when we turn off interbank market shocks (dash-dotted line), demand shocks (dash line), and open market operations (dotted line). Although we do not report their corresponding experiments, it is important to report that neither equity losses nor the increase in interest on reserves played a quantitatively important role. The reasons is that, as summarized in Table 2, these hypothesis produce counterfactual predictions and a modest quantitative impact on loan provision as seen in Section 6.

By construction, the benchmark case that includes all shocks (the matching friction, withdrawal volatility and loan demand) matches the three data targets perfectly (in the upper panel). In the lower panels, we report the shocks that the model needs to fit the data: the model needs a gradual decline in matching efficiency, an increase in withdrawal volatility that spikes around September 2008 and partially reverts after, and a decline in the loans demand that escalates starting in 2009. As an external validation, we also plot the model's fit to the liquidity premium in the data (Panel d) and the liquidity ratio (Panel e). Quantitatively, the model tracks the liquidity ratio well. The model also produces a relatively higher liquidity premium around the Lehman bankruptcy, although the model's premium is not nearly as volatile as in the data.

The counterfactual simulations show that both the interbank market and demand shocks had a prevalent role and their timing is instructive. Before the Lehman bankruptcy in September 2008, neither shock produces a substantial decline in credit. However, to match interbank market features, the model needs a significant matching shock to produce the early pronounced decline in the volume of interbank credit. In the run-up to the crisis, we also observe a substantial increase in deposit volatility, consistent with the spike in discount-window loans. Absent these interbank market shocks, the access to the discount-window and the interbank market would have remained essentially flat throughout the period.

After the Lehman crisis, both shocks contributed to the decline in lending, as Panel (c) shows. If we rank the shocks, the credit demand shock produces the lion's share of the impact, except for the period around September 2008 in which the interbank market is more important role. As a policy counterfactual, we can observe that open-market operations were important to mitigate the collapse in total credit. To get a sense of their quantitative effect, the model suggests that the negative effect of the interbank market shock was about the same magnitude as the positive effect of the open-market operations.

Discussion of the Results. A natural interpretation of the the shocks inferred is that the 2008 crisis was characterized by an episode of increased counterparty risk in the interbank market and by increased deposit instability. These disruptions in the interbank market would manifest in our model through an increase in the volatility of deposits and a lower matching probability in the interbank market. The timing of the shocks is suggestive of a deeper economic phenomenon in which an initial contraction in the supply of loans, produced by disruptions in the interbank market, eventually lead to a collapse in credit demand.

8 Extensions of the Baseline Model

8.1 Liquidity Coverage Ratio

The new regulatory framework following the recent financial crisis requires banks to hold a minimum fraction of total assets in reserves, the so-called liquidity coverage ratio (LCR). In contrast with the reserve requirement in the baseline model, the required reserve holdings are tied to the amount of assets rather than liabilities. We show next how this feature is easily accommodated into our model, and how the introduction of this new regulatory tool affects the banking system.

In addition to the reserve requirement (5), banks are subject to

$$m_{t+1}^{j} \geq \rho^{lcr} b_{t+1}^{j}. \tag{30}$$

Here, ρ^{lcr} is a parameter that works analogously to reserve requirements but is applied to loans instead of deposits.[36] Since banks need to satisfy both the reserve requirement (RR) and the LCR, we can redefine the surplus function as

[36] The parameter ρ^{lcr} could be linked to the volatility of deposit withdrawals, so as to capture constraints related to the Net-Stable-Funding Ratio (NSFR). This constraint could also be imposed in the lending stage instead. If the constraint binds at the lending stage, shocks need to be large enough to have effects on the reserve-to-loans ratio.

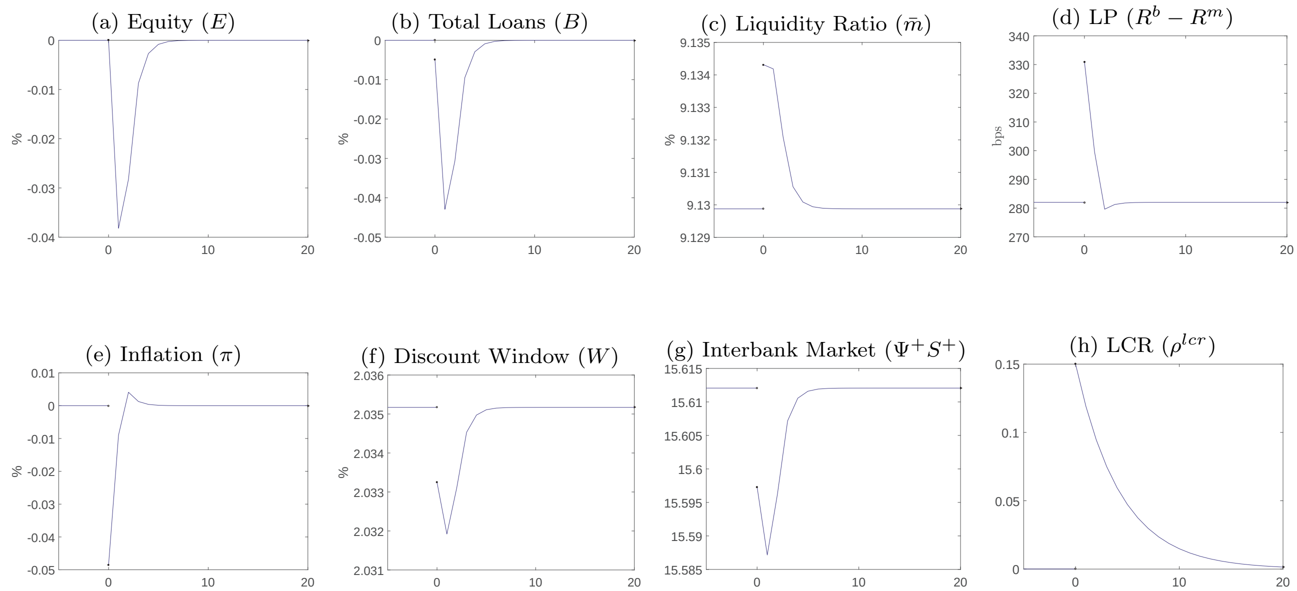

Figure 11: Liquidity Coverage Ratio

$$s(\omega) \equiv \min\left\{s^{rr}(\omega), s^{lcr}(\omega)\right\}, \tag{31}$$

where $s^{lcr}(\omega_t^j) = \rho^{lcr}\tilde{b}^j_{t+1} - (\tilde{m}^j_{t+1} + \frac{\omega_t^j \tilde{d}^j_{t+1}(1+i^b_{t+1})}{1+i^{ior}_{t+1}})$, and $s^{rr}(\omega)$, as defined in (6). The characterization of the individual bank problem and the determination of equilibrium remain essentially unchanged except that the surplus function in Proposition 3 has to be redefined following (31).[37] The key difference in the portfolio problem is that reserves carry an additional premium over loans because an increase in loans tightens the LCR constraint in states where the surplus is given by $s(\omega) = s^{lcr}(\omega)$.

As Figure 11 shows, the introduction of an LCR increases the demand for reserves and reduces bank lending. While the LCR was not active throughout the crisis, and hence we do not include it as one of the hypotheses for the contraction in bank lending, this regulatory tool will be playing an important role going forward.

[37] Because s^{RR} and s^{LCR} are linear in ω, the surplus function is characterized simply as

$$s(\omega) = \begin{cases} s^{LCR}(\omega) & if\ \omega < \bar{\omega} \\ s^{RR}(\omega) & if\ \omega \geq \bar{\omega} \end{cases}$$

where $\bar{\omega}$ is the value of ω such that $s^{LCR}(\omega) = s^{RR}(\omega)$.

8.2 Credit Risk and Risk Aversion

We introduce two other sources of time-varying risk: idiosyncratic shock to loan returns and variations in risk aversion. On the former, we assume that each bank's portfolio of loans is subject to an idiosyncratic "repayment shock" at the beginning of each period. As is the case for the withdrawal shock, the shock to loan returns is assumed to have zero mean. Besides the modification to the portfolio problem of the bank, which now incorporates an additional source of uncertainty, the model remains essentially the same.[38] In particular, because the shock has mean zero, the law of motion for aggregate equity continues to be determined by (26).Figure 12 (straight line) shows the effects of an increase in the variance of this shock: it goes from zero, as in our benchmark, to 20 bps. Because of banks' risk aversion, the increase in the risk of loan returns leads to a decline in total lending and to a concomitant increase in the liquidity ratio. The liquidity premium also rises on impact, reflecting that banks require a higher premium on loans to be willing to absorb the increase in risk. As banks have higher reserves, discount window loans and interbank market loans are reduced on impact. Finally, as equity falls because of the lower investment in loans, the impact of credit risk on the liquidity premium is eventually reversed.

To analyze the case of an increase in risk aversion, we consider Epstein-Zin preferences, keeping the intertemporal elasticity of substitution equal to unity and consider a level of risk aversion equal to 100.[39] Figure 12 (dashed line) shows that the dynamics after an increase in risk aversion are similar to the dynamics resulting from an increase in credit risk.

In terms of the hypothesis for the contraction in bank lending analyzed in Section 6, the response of the banking system to an increase in credit risk or risk aversion delivers outcomes that are qualitatively fairly consistent with various banking variables in the crisis. To the extent that the financial crisis was characterized by increased uncertainty about loan repayments by borrowers at the cross section of banks and an increase in risk aversion, this is a shock that could have played an active role in the crisis.

[38] Let z be the idiosyncratic default rate on bank loans, with zero mean and variance σ_z. The portfolio problem now becomes

$$\Omega_t \equiv (1-\tau_t) \max_{\{\bar{b},\bar{m},\bar{d}\}\geq 0} \left\{ \mathbb{E}_{\omega,z}\left[R_t^b(1+z)\bar{b} + R_t^m\bar{m} - R_t^d\bar{d} + \bar{\chi}_t(\bar{m},\bar{d},\omega)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}, \tag{32}$$

$$\bar{b}+\bar{m}-\bar{d}=1,$$

$$\bar{d}\leq\kappa\left(\bar{b}+\bar{m}-\bar{d}\right).$$

[39] A version of Proposition 3 applies with Epstein-Zin preferences, and hence tractability is not lost in this case.

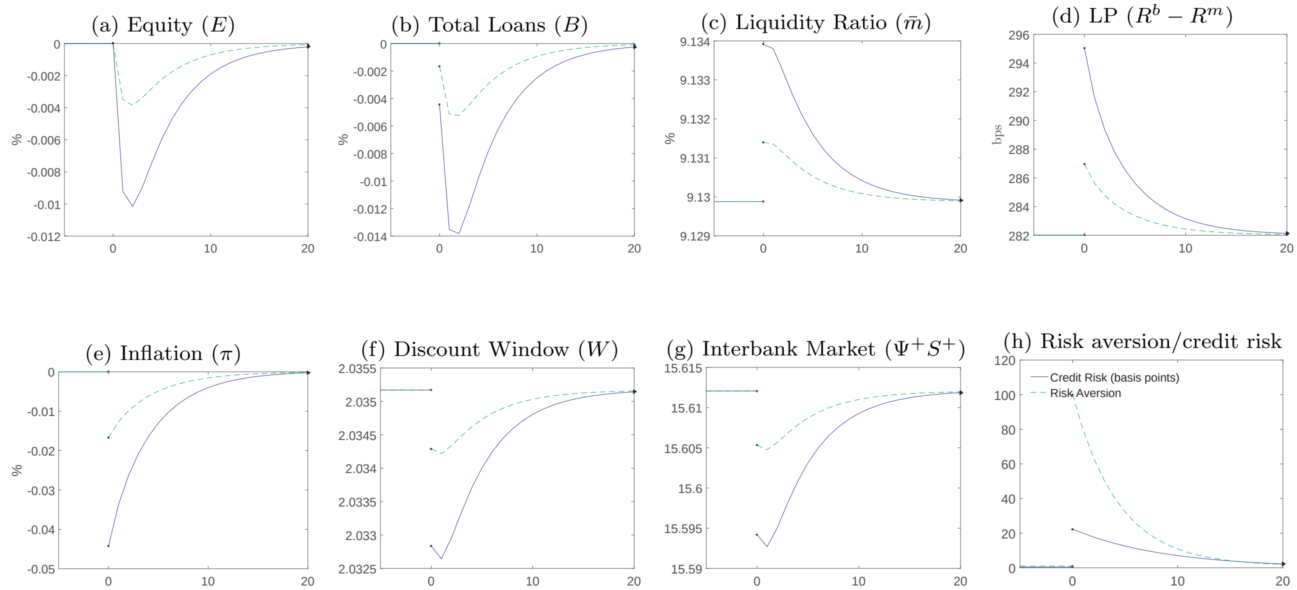

Figure 12: Credit Risk and Risk Aversion

9 Conclusion

Historically, the topics of money and banking had been studied and taught together. Despite this historical connection, modern monetary models developed independently from banking. The financial crises of the last decades in the United States, Europe, and Japan, however, have revealed the need for a unified framework.

This paper presents a tractable quantitative model of banks' liquidity management and the credit channel of monetary policy. In the model, banks engage in maturity transformation, which exposes them to liquidity risk. To insure against unexpected deposit withdrawals, banks hold reserves as a precautionary buffer. Banks that face large withdrawals deplete their reserves and must resort to costly interbank market and discount window borrowing. Monetary policy has the power to alter the liquidity premium and, in that way, to affect real economic activity. As an application, we study the driving forces behind the decline in bank lending and liquidity hoarding by banks during the 2008 financial crisis. We argue that this pattern was the result of an early disruption in the interbank market, followed by a persistent decline in credit demand.

The application we carried out in the paper is one of many possible ones.[40] Our model could be used to shed light on classic historical debates. For example, it could be used to evaluate the hypothesis in Friedman and Schwartz (2008) that an increase in the deposit-to-currency ratio was responsible for the colossal credit crunch during the Great Depression. Also,

[40] We thank an anonymous referee for providing a rich list with possible extensions and applications.

Friedman and Schwartz (2008) argued that the Fed's increase in reserve requirements in 1937 was a serious policy mistake, but Tobin (1965) opposed that view, arguing that banks held considerable excess reserves—such as during the episode we studied here. One could also use the model to study how different monetary policy and regulatory regimes affect the stability of bank credit. For example, monetary policy regimes around the world have evolved from a gold standard system to one that targets monetary aggregates and, more recently, to one that targets interest rates. One could also use the model to evaluate the desirability and implementation of liquidity regulations for financial stability, such as LCR. Along similar lines, Safonova (2017) studies how the network of the interbank affects the transmission of monetary policy. Chen, Ren, and Zha (2017) use a similar model to study whether tight monetary policy in China induced shadow banking activities. Exploring these kinds of applications is a future task.

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Appendix to Banks, Liquidity Management and Monetary Policy

Not for Publication

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UCLA and NBER

Revised September 2017

Contents

A Additional Plots

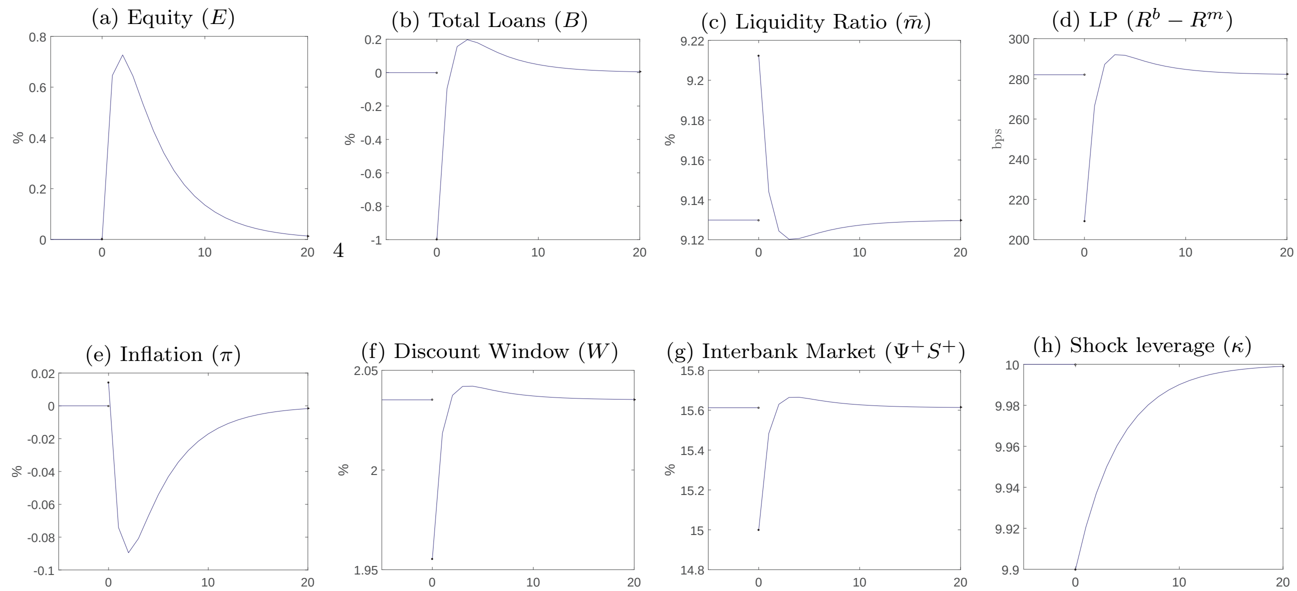

Figure 13: Transition after tightening of Capital Requirement

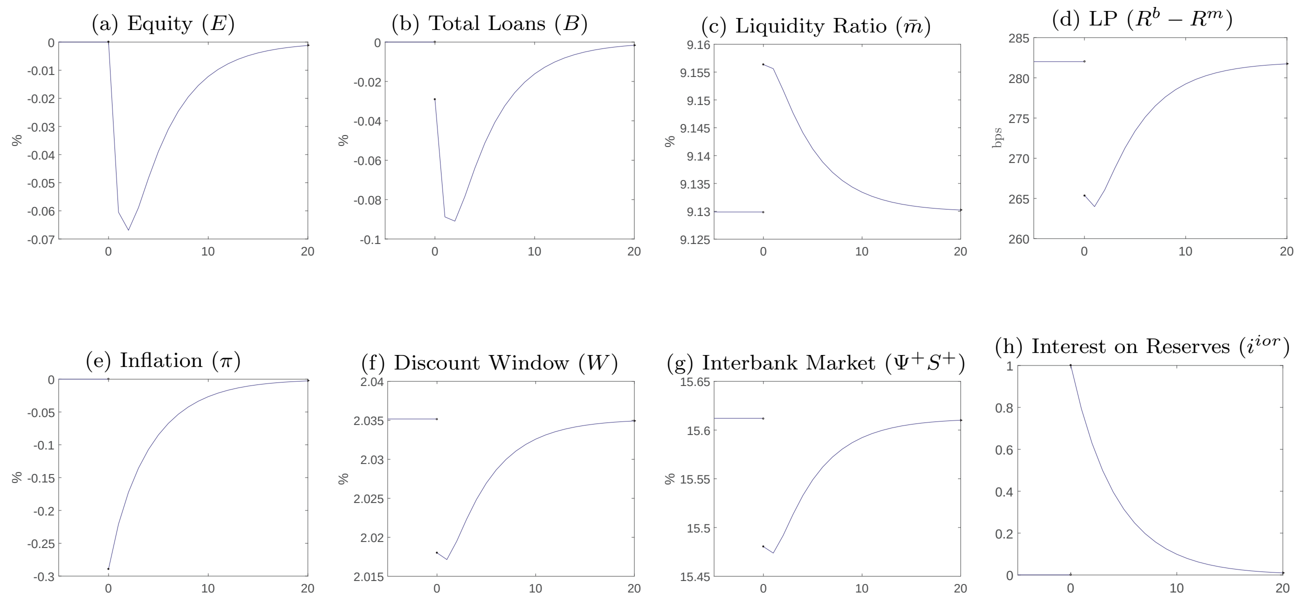

Figure 14: Transition after Increase on Interest on Reserves

B List of Variables

Table 3: List of variables

Interest rates	
$\bar{i}^f$	average interbank market rate
i^b	nominal interest rate on loans
i^d	nominal interest rate on deposits
Individual bank variables	
Portfolios	
$\tilde{b}$	bank loans in lending stage
$\tilde{m}$	reserves held by banks at end of lending stage
$\tilde{d}$	bank deposits at end of lending stage
b	loans at beginning of lending stage
m	reserves held at beginning of lending stage
d	deposits owed at beginning of lending stage
$\bar{b}$	portfolio share in loans
$\bar{m}$	portfolio share in reserves
$\bar{d}$	portfolio share in deposits
Others	
c	consumption
Ω	risk-adjusted value of bank equity
R^e	return on equity
e	real equity
μ	Lagrange multiplier on capital requirement constraint
V^l .V^b	value of the bank at lending/balancing stage
V	value of the bank as a function of bank equity
Interbank market	
ω	withdrawal shock
s	surplus at beginning of balancing stage after shock ω
θ	market tightness in interbank market
f	interbank market loans
w	discount window loans
Ψ^+	probability that a bank with surplus finds a match
Ψ^-	probability that a bank with deficit finds a match
Aggregates	
E	bank equity
Θ_t^b	intercept loan demand
ϵ	elasticity loan demand
Θ_t^d	intercept deposit supply
ς	elasticity deposit supply
B,B^d	loan supply/demand
D^s, D	deposits supply/demand
P	price level
π	inflation
Fed policies	
i^{ior}	nominal interest rate on reserves
i^{dw}	nominal interest rate on discount window loans
M^{Fed}	supply of reserves
B^{Fed}	Fed holdings of private loans
W^{Fed}	discount window loans

C Microfoundations: Loan Demand/Deposit Supply

In this section, we provide a microfoundation for the loan demand and deposit supply schedule for the interested reader. In this extension, in addition to bankers, the economy is populated by overlapping generations of workers and entrepreneurs. Each group has a unit measure, and agents live for two periods.

Workers. Workers belong to a single family and maximize

$$\max_{\left\{c_t^w, c_{t+1}^w h_t^w, d_{t+1}\right\}\geq 0} c_t^{w,t} - \frac{(h_t^w)^{1+\nu}}{1+\nu} + \beta \frac{\left(c_{t+1}^{w,t}\right)^{1-1/(\varsigma+1)}}{1-1/\left(\varsigma+1\right)} \tag{33}$$

subject to

$$d_{t+1} + P_t c_t^{w,t} = z_t h_t \text{ and } P_{t+1} c_{t+1}^{w,t} \leq \left(1+i_{t+1}^d\right) d_{t+1}.$$

The first constraint is the time t budget constraint: given a nominal wage z_t and a supply of hours, h_t, the worker distributes labor income between time t consumption, $c_t^{w,t}$, at a price P_t and deposits d_{t+1}. At $t+1$, deposits earn a nominal return $\left(1+i_{t+1}^d\right)$. Those savings are used to buy $t+1$ consumption at price P_{t+1}. Here, ς governs the intertemporal elasticity of substitution, which in turn maps into the elasticity of the deposit supply presented in (13). In turn, ν is the inverse labor supply elasticity, which also affects the loan demand.

Entrepreneurs. The entrepreneur born at t has linear utility in consumption when old and does not consume when young. The entrepreneur has access to a production technology that uses h_t^d units of labor that are transformed into $t+1$ output via a production function $y_{t+1} = A_{t+1} h_t^\alpha$. Production is scaled by A_{t+1}, a productivity shock that works as a loan demand shifter. The term A_{t+1} is known at t. Entrepreneurs use bank loans to pay workers in the first period. The entrepreneur born at t solves

$$P_{t+1} c_{t+1}^{e,t} = \max_{\{B_{t+1}^d, h_t\}\geq 0} P_{t+1} y_{t+1} - \left(1+i_{t+1}^b\right) B_{t+1}^d + \left(1+i_{t+1}^d\right) D_{t+1}$$

subject to

$$z_t h_t \leq B_{t+1}^d \text{ and } D_{t+1} = B_{t+1}^d - z_t h_t.$$

The objective is to maximize $t+1$ profits to maximize consumption when old. Profits are the sum of sales, minus financial expenses, plus earnings on deposits. The entrepreneur borrows B_{t+1}^d at an interest rate cost $\left(1+i_{t+1}^b\right)$ and uses these funds to finance payroll, $z_t h_t$, or save in a deposit account. To finance this payroll, entrepreneurs borrow from banks. In particular, they obtain a loan from the banks in the form of a number of deposits that can be used to pay workers. The loan is a promise to repay $\left(1+i_{t+1}^b\right)$ by $t+1$.

When we consider the non-financial side of the model, there are now two additional market clearing conditions: labor market clearing and goods market clearing. The goods market clearing condition is

$$A_{t+1} h_t^\alpha = c_t^{e,t-1} + c_t^{w,t} + c_t^{w,t-1} + c_t. \tag{34}$$

This equation states that output is used either as intermediate inputs or for consumption of old entrepreneurs, young and old workers, and bankers. The market clearing condition in the labor market is given by

$$h_t = h_t^w. \tag{35}$$

Equilibrium in the labor market and the optimal policy functions of entrepreneurs and workers yield an autonomous system of demand and supply equations for loans and deposits.

Proposition 7. *The equilibrium loan demand and deposit supply take the form of (12) and (13), and the reduced form-parameters are given by*

$$\Theta_t^b = (\alpha A_{t+1})^{\epsilon}\,,\ \epsilon = \left(\frac{\alpha}{(\nu+1)} - 1\right)^{-1} \text{ and } \Theta_t^d = \beta^{1+\varsigma}.$$

Output and labor are decreasing functions of $(1+i_t^b)/(1+\pi_t)$.

In the paper we do not make reference to this microfoundation and work with the exogenous demand schedule for loans (12) and exogenous supply schedule for deposits (13). However, the definition of equilibria is consistent with the general equilibrium version of the model where we consider this non-financial sector. Note that Proposition 7 uses a labor market clearing condition. Then, clearing in the loans and deposit markets, by Walras's law, imply clearing in the goods market. Once we compute equilibria taking the schedules as exogenous in the bank's problem, it is possible to obtain output and household consumption from the equilibrium rates. One important observation is that an equilibrium real rate for loans, R_t^b, immediately maps into an equilibrium output. In particular, the higher the rate, the lower the output. When monetary policy induces a higher rate, it has the power to reduce output in this environment. This is a notion of the credit channel.

D Equilibrium Conditions

D.1 Transitions

We characterize the set of equilibrium conditions in the main paper. Here, we present a summary of the conditions in one place. Given a sequence of government policy $\left\{\rho_t, \kappa, i_t^{ior}, i_t^{dw}, W_t, B_t^{Fed}, M_t^{Fed}, T_t\right\}$ that satisfies the Fed's budget constraint, the system that characterizes equilibrium yields a solution for individual bank variables,$\left\{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\right\}$, aggregate variables, $\{B_t, M_t, D_t, E_t\}$, and a system of prices and real returns $\{P_t, R_t^b, R_t^m, R_t^d, \bar{\chi}_t\}$. The system features 15 unknowns to be determined for all t. However, there is only one endogenous state variable. The Fed's budget constraint adds one restriction to the set of policy sequences. Other variables that follow from definitions are described in Appendix B.

Individual Bank Variables

The portfolio solution to $\left\{\bar{b}_t, \bar{m}_t, \bar{d}_t\right\}$ and the value of Ω_t are the solutions and value of the following problem:

$$\begin{aligned}
&\Omega_t \equiv \max_{\left\{\bar{b}, \bar{m}, \bar{d}\right\} \geq 0} \left\{\mathbb{E}_\omega \left[R_t^b \bar{b} + \ R_t^m \bar{m} - R_t^d \bar{d} + \bar{\chi}_t(\bar{m}, \bar{d}, \omega)\right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}, \\
&\bar{b} + \bar{m} - \bar{d} = 1, \\
&\bar{d} \leq \kappa_t \left(\bar{b} + \bar{m} - \bar{d}\right).
\end{aligned} \tag{D.1.1}$$

The value of the bank's problem is

$$v_t = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1}\right)^{\frac{1}{\gamma}}\right]^{\gamma}. \tag{D.1.2}$$

Dividends depend on $\{\Omega_t, v_t\}$ via

$$\bar{c}_t \ = \frac{1}{1 + \left[\beta(1-\gamma) v_{t+1} {\Omega_{t+1}}^{1-\gamma}\right]^{1/\gamma}}. \tag{D.1.3}$$

This block of equations yields the equations needed to obtain $\{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$ for a given path for real rates $\left\{R_t^b, R_t^m, R_t^d, \bar{\chi}_t\right\}$.

Aggregate Banking Variables

Next, homogeneity in policy functions gives us the aggregate bank portfolio:

$$B_{t+1} = P_t \bar{b}_t (1 - \bar{c}_t) E_t \tag{D.1.4}$$

$$M_{t+1} = P_t \bar{m}_t (1 - \bar{c}_t) E_t \tag{D.1.5}$$

$$D_{t+1} = P_t \bar{d}_t \ (1 - \bar{c}_t) E_t. \tag{D.1.6}$$

Real aggregate equity evolves according to

$$E_{t+1} = \frac{P_t \left((1 + i_{t+1}^b)\bar{b}_t + (1 + i_{t+1}^{ior})\bar{m}_t - (1 + i_{t+1}^d)\bar{d}_t\right)(1 - \bar{c}_t) E_t - (1 + i_{t+1}^{dw}) W_{t+1} - P_t T_t}{P_{t+1}}. \tag{D.1.7}$$

This block of equations determines $\{B_t, M_t, D_t, E_t\}$ given a path for inflation and nominal rates—which together determine real rates—and transfers.

Market Clearing Conditions

The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

$$\frac{B_{t+1}+B_t^{FED}}{P_t} = \Theta_t^b \left(R_t^b\right)^{\epsilon}, \tag{D.1.8}$$

$$\frac{D_{t+1}}{P_t} = \Theta_t^d \left(R_t^d\right)^{\zeta}, \tag{D.1.9}$$

$$M_t^{Fed} = M_t, \tag{D.1.10}$$

$$R_t^m = \frac{1+i_t^{ior}}{P_{t+1}/P_t}. \tag{D.1.11}$$

The last term is the definition of R_t^m. This block determines $\{P_t, R_t^b, R_t^m, R_t^d\}$ given aggregate bank variables. Notice that $M_t^{Fed} = M_t$ pins down the price level using $P_t \bar{m}_t (1-\bar{c}_t) E_t$. To close the system, we need the equations that determine χ_t.

Interbank Market Block

We need to determine $\bar{\chi}_t$. This follows from the conditions obtained from Proposition 1:

$$S_t^- = \int_1^{\frac{\bar{m}/\bar{d}-\rho_t}{(1-\rho_t)}} s(\omega) d\Phi \text{ and } S_t^+ = \int_{\frac{\bar{m}/\bar{d}-\rho_t}{(1-\rho_t)}}^{\infty} s(\omega) d\Phi.$$

The market tightness is defined as

$$\theta_t = S_t^- / S_t^+.$$

From here, discount window loans are

$$W_t = (1-\Psi^-(\theta_t)) S_t^-, \tag{D.1.12}$$

and the average interbank market rate, $\bar{i}_t^f$, is

$$\bar{i}_t^f = \phi(\theta_t) i_t^{ior} + (1-\phi(\theta_t)) i_t^{dw}.$$

This system of equations gives us

$$\chi_t^- = \Psi_t^- \left(\bar{i}_t^f - i_t^{ior}\right) + \left(1-\Psi_t^-\right)\left(i_t^{dw} - i_t^{ior}\right) \text{ and } \chi_t^+ = \Psi_t^+ \left(\bar{i}_t^f - i_t^{ior}\right). \tag{D.1.13}$$

Note that here we take the probabilities Ψ_t^- and Ψ_t^+ as given functions of market tightness, as in the main text. This block determines $\bar{\chi}_t$ and the amount of discount window loans, W_t. Note that so far, we have provided enough equations to solve for $\{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$, $\{B_t, M_t, D_t, E_t\}$, and $\{P_t, R_t^b, R_t^m, R_t^d, \bar{\chi}_t\}$. The value of W_t enters in the Fed's budget constraint.

Fed Budget Constraint

The government's budget policy sequence $\left\{\rho_t, \kappa_t, i_t^{ior}, i_t^{dw}, W_t, B_t^{Fed}, M_t^{Fed}, T_t\right\}$ satisfies the following constraint:

$$\begin{aligned} & M_t(1+i_t^{ior}) + \ B_{t+1}^{Fed} + W_{t+1}... \\ = \quad & M_{t+1} + D_t^{Fed}(1+i_t^d) + B_t^{Fed}(1+i_t^b) + W_t(1+i_t^{dw}) + P_t T_t. \end{aligned}$$

Law of Motion for Aggregate Equity

A useful expression is obtained combining the individual laws of motion with the Fed's budget constraint:

$$E_{t+1} = (R_{t+1}^b \bar{b}_t + R_{t+1}^m \bar{m}_t - R_{t+1}^d \bar{d}_t) E_t (1-\bar{c}_t) - \frac{B_{t+2}^{Fed} - \tilde{M}_{t+2} - (B_{t+1}^{Fed}(1+i_{t+1}^b) - \tilde{M}_{t+1}^{Fed}(1+i_{t+1}^{ior}))}{P_{t+1}} \tag{D.1.14}$$

Equation (D.1.14) shows that portfolio choices, market returns, and next-period Fed policies and price level determine next-period aggregate real equity.

D.2 Stationary Equilibrium

Consider now the equilibrium conditions for a stationary equilibrium. These are summarized by replacing time subscripts for steady state subscripts ss.

Individual Bank Variables

For the individual bank variables, we have

$$c_{ss} = 1 - \beta^{\frac{1}{\gamma}} \Omega_{ss}^{1/\gamma - 1} \tag{D.2.1}$$

$$v^{ss} = \frac{1}{1-\gamma} \left(\frac{1}{1 - \left(\beta \Omega_{ss}^{1-\gamma}\right)^{\frac{1}{\gamma}}} \right)^{\gamma} \tag{D.2.2}$$

$$\Omega_{ss} \equiv \max_{\bar{b}, \bar{m} \geq 0, \bar{d}} \left\{ \mathbb{E}_\omega \left[\frac{\left(\ (1+i_t^b)\bar{b} + \ (1+i_t^{ior})\bar{m} - \ (1+i_t^d)\bar{d} + \chi_t(\bar{m}, \bar{d}) \right) (1-\tau^{ss})}{1+\pi_t} \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \tag{D.2.3}$$

$$\bar{b} + \bar{m} - \bar{d} = 1,$$

$$\bar{d} \leq \kappa \left(\bar{b} + \bar{m} - \bar{d} \right),$$

where $\{\bar{b}_{ss}, \bar{m}_{ss}, \bar{d}_{ss}\}$ are the optimal choices of $\{\bar{b}, \bar{m}, \bar{d}\}$ in the problem above.

Aggregate Bank Variables and Market Clearing Conditions

The nominal rates and price sequences are given by

$$(1 - \ c_{ss}) \bar{b}_{ss} E_{ss} = \Theta^b \left(\frac{1+i_t^b}{1+\pi_t} \right)^{\epsilon} - B_{t+1}^{Fed}/P_t \tag{D.2.4}$$

$$(1 - \ c_{ss}) \bar{d}_{ss} E_{ss} = \Theta^d \left(\frac{1+i_t^d}{1+\pi_t} \right)^{\zeta} \tag{D.2.5}$$

$$(1 - \ c_{ss}) \bar{m}_{ss} E_{ss} = M_t^{Fed}/P_t. \tag{D.2.6}$$

Interbank Market Block

The interbank market tightness is

$$\theta_t \equiv S_t^- / S_t^+, \tag{D.2.7}$$

and the interbank market conditions are

$$W_t^{Fed} = \Psi_t^- S_t^- \tag{D.2.8}$$

$$\Psi_t^- = \Psi^-(\theta_t) \text{ and } \Psi_t^+ = \Psi^+(\theta_t) \tag{D.2.9}$$

$$S_t^- = \int_1^{\frac{\bar{m_{ss}}/\bar{d_{ss}} - \rho}{(1-\rho)}} s(\omega) d\Phi$$

$$S_t^+ = \int_{\frac{\bar{m_{ss}}/\bar{d_{ss}} - \rho}{(1-\rho)}}^{\infty} s(\omega) d\Phi$$

$$\bar{i}_t^f = \phi(\theta_t) i_{ss}^{ior} + (1 - \phi(\theta_t)) i_t^{dw}, \tag{D.2.10}$$

$$\chi_t^- = \Psi_t^- \left(\bar{i}_t^f - i_t^{ior} \right) + \left(1 - \Psi_t^- \right) \left(i_t^{dw} - i_t^{ior} \right) \text{ and } \chi_t^+ = \Psi_t^+ \left(\bar{i}_t^f - i_t^{ior} \right). \tag{D.2.11}$$

Government Budget Constraint and Aggregate Equity

The government budget constraint and the law of motion for equity are given by

$$B_t^{Fed} \left(\frac{1 + i_t^b}{1 + \pi_t} \right) + W_t^{Fed} \left(\frac{1 + i_t^{dw}}{1 + \pi_t} \right) = M_t^{Fed} \left(\frac{1 + i_t^{ior}}{1 + \pi_t} \right) + P_t E_{ss} \frac{\tau_t}{1 - \tau_t}, \forall t \tag{D.2.12}$$

$$E_{ss} = \frac{\left((1 + i_t^b) \bar{b}_{ss} + (1 + i_t^{ior}) \bar{m}_{ss} - (1 + i_t^d) \bar{d}_{ss} \right) (1 - \bar{c}_{ss}) E_{ss}}{1 + \pi_t} - \frac{1 + i_t^{dw}}{1 + \pi_t} W_t^{Fed} / P_t - E_{ss} \frac{\tau_t}{1 - \tau_t} \tag{D.2.13}$$

$$1 + \pi_t = P_{t+1} / P_t. \tag{D.2.14}$$

E Expressions for $\{\Psi^+, \Psi^-, \phi, \bar{i}^f, \chi^+, \chi^-\}$ in Proposition 1

Here we reproduce formulas derived from Proposition 1 in the companion paper, Bianchi and Bigio (2017). The companion paper includes the market structure that delivers these functional forms. This proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market. The formulas are the following.

Given θ, the market tightness after the federal funds trading session is

$$\bar{\theta} = \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ \left(1 + (\theta^{-1} - 1)\exp(\lambda)\right)^{-1} & \text{if } \theta < 1 \end{cases}.$$

Trading probabilities are given by

$$\Psi^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1 - e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \Psi^- = \begin{cases} \left(1 - e^{-\lambda}\right)\theta^{-1} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}.$$

The reduced-form bargaining parameter is

$$\phi = \begin{cases} \frac{\theta}{\theta - 1}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta > 1 \\ \eta & \text{if } \theta = 1 \\ \frac{\theta(1-\bar{\theta}) - \bar{\theta}}{\bar{\theta}(1-\theta)}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta < 1 \end{cases},$$

and $\bar{i}^f = (1 - \Phi)i^{dw} + \Phi i^{ior}$. The slopes of the liquidity yield function are given by

$$\chi^+ = \left(i^{dw} - i^{ior}\right)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \chi^- = \left(i^{dw} - i^{ior}\right)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right).$$

F Proof of Proposition 7

Worker's Problem. We substitute $t+1$ consumption of worker t's problem into his objective to obtain that his objective equals

$$c_t^{w,t} - \frac{h_t^{1+\nu}}{1+\nu} + \beta\mathbb{U}\left(\frac{\left(1 + i_{t+1}^d\right)d_{t+1}}{P_{t+1}}\right).$$

Then, taking first-order conditions with respect to $c_t^{w,t}$, we obtain

$$1 = P_t\zeta_t^w, \tag{F.1}$$

where ζ_t^w is a Lagrange multiplier on the worker's time t budget constraint. Then, the first-order condition with respect to labor supply yields a labor supply that only depends on the real wage:

$$h_t^{\nu} = z_t\zeta_t^w = z_t/P_t. \tag{F.2}$$

Next, we take the first-order condition with respect to deposits:

$$\zeta_t^w = \beta \mathbb{U}' \left(\frac{\left(1+i_{t+1}^d\right)}{P_{t+1}} d_{t+1} \right) \frac{\left(1+i_{t+1}^d\right)}{P_{t+1}}.$$

We rewrite this condition as

$$P_t \zeta_t^w = \beta \mathbb{U}' \left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t} \frac{d_{t+1}}{P_t} \right) \frac{\left(1+i_{t+1}^d\right)}{P_{t+1}/P_t}.$$

Thus, noticing that $P_t \zeta_t^w = 1$, from (F.1) this expression becomes

$$\left(\frac{d_{t+1}}{P_t}\right)^{1/(\varsigma+1)} = \beta \left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t} \right)^{1-1/(\varsigma+1)}.$$

Clearing d_{t+1} we obtain

$$\frac{d_{t+1}}{P_t} = \beta^{\varsigma+1} \left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t} \right)^{\varsigma}. \tag{F.3}$$

Thus, setting $\beta^{\varsigma+1} = \Theta_t^d$, we obtain the functional form in Proposition 7.

Next, we move to derive the demand for loans. From the entrepreneur's problem, we have that

$$\max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1} A_{t+1} h_t^{\alpha} - \left(1+i_t^b\right) B_{t+1}^d + \left(1+i_t^d\right) \left(B_{t+1}^d - z_t h_t\right)$$

subject to

$$z_t h_t \leq B_{t+1}^d.$$

First, observe that

$$\begin{aligned} P_{t+1} A_{t+1} h_t^{\alpha} - \left(1+i_t^b\right) B_{t+1}^d + \left(1+i_t^d\right) \left(B_{t+1}^d - z_t h_t\right) & \\ = P_{t+1} A_{t+1} h_t^{\alpha} - z_t h_t - \left(i_t^b - i_t^d\right) \left(B_{t+1}^d + z_t h_t\right). & \end{aligned}$$

Since $i_t^b \geq i_t^d$, without loss of generality, $z_t h_t = B_{t+1}^d$. Thus, the objective is

$$P_{t+1} A_{t+1} h_t^{\alpha} - \left(1+i_t^b\right) B_{t+1}^d$$

with $z_t h_t = B_{t+1}^d$. Suppose h_t was already chosen by the entrepreneur. Thus, back in the objective function, we have that

$$P_{t+1} A_{t+1} h_t^{\alpha} - \left(1+i_t^b\right) z_t h_t.$$

The first-order condition in h_t yields

$$P_{t+1} \alpha A_{t+1} h_t^{\alpha} = \left(1+i_t^b\right) z_t h_t.$$

Dividing both sides by P_t, we obtain

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^{\alpha} = \left(1+i_t^b\right) \frac{z_t}{P_t} h_t.$$

Now, employing the labor supply function (F.2), we have

$$\begin{aligned} \frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha} &= \left(1+i_t^b\right)h_t^{\nu+1} \rightarrow \\ \frac{1+i_t^b}{P_{t+1}/P_t} &= \frac{\alpha A_{t+1}h_t^{\alpha}}{h_t^{\nu+1}}. \end{aligned}$$

Now, we finally deduce that

$$\begin{aligned} \frac{B_{t+1}^d}{P_t} &= \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow \\ h_t &= \left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{1}{\nu+1}}. \end{aligned}$$

Combining, we obtain

$$\begin{aligned} \frac{1+i_t^b}{P_{t+1}/P_t} &= \alpha A_{t+1}\left(\frac{B_{t+1}^d}{P_t}\right)^{-1}\left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{\alpha}{\nu+1}} \rightarrow \\ \frac{B_{t+1}^d}{P_t} &= \Theta_t\left(\frac{1+i_t^b}{P_{t+1}/P_t}\right)^{\epsilon}. \end{aligned}$$

Thus, the microfoundation yields

$$\begin{aligned} \Theta_t^b &= \left(\alpha A_{t+1}\right)^{\epsilon} \text{ and} \\ \epsilon &= \left(\frac{\alpha}{\nu+1}-1\right)^{-1}. \end{aligned}$$

G Proofs of Propositions 2 and 3 and Liquidity Premium

The proofs of Proposition 2 and Proposition 3 make use of the following two lemmata. First, we establish the homogeneity in χ:

Lemma 1. *The function $\bar{\chi}_t$ is homogeneous of degree 1 in (m, d).*

Proof. We need to show $\bar{\chi}_t\left(am, ad, \omega\right) = a\bar{\chi}_t\left(m, d, \omega\right)$ for any $a > 0$. By definition:

$$\begin{aligned} \bar{\chi}_t(am, ad, \omega) &= \begin{cases} \chi_t^+ s & if\ s \geq 0 \\ \chi_t^- s & if\ s < 0 \end{cases}, \\ s &= am + a\omega d\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}} - \rho ad\left(1+\omega\right) \end{aligned} \tag{G.1}$$

where χ_t^- and χ_t^+ are functions of $\left\{\Psi_t^-, \Psi_t^+, \bar{\iota}_t^f, \theta_t\right\}$ and independent of m and d. We can factor the constant a from the right-hand side of (G.1) and obtain

$$s = a\left(m + \omega d\frac{1+i_{t+1}^d}{1+i_{t+1}^{ior}} - \rho d\left(1+\omega\right)\right).$$

Define the position without the scaling factor a as $\tilde{s}$ given by

$$\tilde{s} = \left(m + \omega d \frac{1 + i_{t+1}^{d}}{1 + i_{t+1}^{ior}} - \rho d\left(1 + \omega\right) \right).$$

Observe that $(s > 0) \longleftrightarrow (\tilde{s} > 0)$, $(s < 0) \longleftrightarrow (\tilde{s} < 0)$ and $(s = 0) \longleftrightarrow (\tilde{s} = 0)$. Thus,

$$\begin{aligned} \bar{\chi}_t(am, ad, \omega) &= \begin{cases} \chi_t^{+} s & if\ s \geq 0 \\ \chi_t^{-} s & if\ s < 0 \end{cases} = \begin{cases} \chi_t^{+} a\tilde{s} & if\ s \geq 0 \\ \chi_t^{-} a\tilde{s} & if\ s < 0 \end{cases} \\ &= \begin{cases} \chi_t^{+} a\tilde{s} & if\ \tilde{s} \geq 0 \\ \chi_t^{-} a\tilde{s} & if\ \tilde{s} < 0 \end{cases} = a \begin{cases} \chi_t^{+} \tilde{s} & if\ \tilde{s} \geq 0 \\ \chi_t^{-} \tilde{s} & if\ \tilde{s} < 0 \end{cases} \\ &= a\bar{\chi}_t(m, d, \omega). \end{aligned}$$

The last line verifies that χ is homogeneous of first degree. **QED.**
The next lemma establishes that an increase in the (gross) nominal policy rates by a constant scales χ_t by that constant. We use this lemma in the policy analysis results when we discuss the neutrality of inflation.

Lemma 2. *Let χ_t be given by two policy rates, $\left\{i_t^{ior}, i_t^{dw}\right\}$, given θ_t. Consider alternative rates $\left\{i_{a,t}^{ior}, i_{a,t}^{dw}\right\}$ such that they satisfy $\left(1 + i_{a,t}^{ior}\right) \equiv k\left(1 + i_t^{ior}\right)$ and $\left(1 + i_{a,t}^{dw}\right) \equiv k\left(1 + i_t^{dw}\right)$ for some k. Then, the $\bar{\chi}_{a,t}$ associated with $\left\{i_{a,t}^{ior}, i_{a,t}^{dw}\right\}$ for the same θ_t satisfy $\bar{\chi}_{a,t} = k\bar{\chi}_t$.*

Proof. Observe that χ_t in Definition 1 (which follows from Proposition 1) is a function scaled by the width of the corridor system $(i_t^{dw} - i_t^{ior})$. Then,

$$i_{a,t}^{dw} - i_{a,t}^{ior} = (1 + i_{a,t}^{dw}) - (1 + i_{a,t}^{ior}) = k((1 + i_t^{dw}) - (1 + i_t^{ior})) = k(i_t^{dw} - i_t^{ior}).$$

Then the result follows immediately from the functional form of χ_t in Proposition 1. **QED.**

G.1 Proof of Proposition 2

We have to show that the recursive problems of banks during the lending and balancing stages can be summarized as a single Bellman equation $V_t(e)$ where e is a single state variable and V_t the value at the lending stage. To show this, define the after-tax real value of equity at the start of a lending stage:

$$e_t \equiv \frac{(1 + i_t^{b})b_t + (1 + i_t^{ior})m_t - (1 + i_t^{d})d_t - \left(1 + i_t^{f}\right) f_t - \left(1 + i_t^{dw}\right) w_t - P_t T_t}{P_t}.$$

This term is the right-hand side of equation (14) in Problem 1 over the price level. If we use this definition, the budget constraint of a given bank satisfies

$$c_t + \frac{\tilde{b}_t + \tilde{m}_t - \tilde{d}_t}{P_t} = e_t. \tag{G.1}$$

The capital requirement constraint only depends on $\left\{\tilde{b}_t, \tilde{m}_t, \tilde{d}_t\right\}$, and the budget constraint is independent of the composition of real equity; the value $V_t^{l}(b, m, d, f, w)$ depends only on e, not its composition. Therefore, this implies the relation $V_t(e) \equiv V_t^{l}(b, m, d, f, w)$. This shows that V_t is the value at the lending stage. Next, we try to find a recursive expression for V_t.

The next step shows that, indeed, $V_t(e)$ can be written recursively. The value of real equity at $t + 1$ can be written in terms of variables determined at the lending stage of period t and the shock

ω_t:

$$e_{t+1} = \frac{(1+i^b_{t+1})b_{t+1} + (1+i^{ior}_{t+1})m_{t+1} - (1+i^d_{t+1})d_{t+1} - \left(1+i^f_{t+1}\right) f_{t+1} - \left(1+i^{dw}_{t+1}\right) w_{t+1} - P_{t+1}T_{t+1}}{P_{t+1}}.$$

By assumption, the tax T_t is proportional to real equity. Thus, we can write $T_{t+1} = \frac{\tau_{t+1}}{1-\tau_{t+1}} e_{t+1}$ for some convenient choice of τ_{t+1}. Multiplying both sides by $1-\tau_{t+1}$ and rearranging, we obtain

$$e_{t+1} = \frac{(1+i^b_{t+1})b_{t+1} + (1+i^{ior}_{t+1})m_{t+1} - (1+i^d_{t+1})d_{t+1} - \left(1+i^f_{t+1}\right) f_{t+1} - \left(1+i^{dw}_{t+1}\right) w_{t+1}}{P_{t+1}} \left(1-\tau_{t+1}\right). \tag{G.2}$$

Now, observe that by definition of $\tilde{m}_{t+1}$ and $\tilde{d}_{t+1}$,

$$m_{t+1} = \tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \left(\frac{1+i^d_{t+1}}{1+i^{ior}_{t+1}}\right) + f_{t+1} + w_{t+1},$$

and

$$d_{t+1} = \tilde{d}_{t+1} + \omega_t \tilde{d}_{t+1}.$$

Substituting these last two expressions in the evolution of equity (G.2), we obtain the after-tax value of equity $\left(1+\tau_{t+1}/\left(1-\tau_{t+1}\right)\right) P_{t+1} e_{t+1}$:

$$\begin{aligned}
= \;& (1+i^b_{t+1})\tilde{b}_{t+1} + (1+i^{ior}_{t+1}) \left(\tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \frac{1+i^d_{t+1}}{1+i^{ior}_{t+1}} + f_{t+1} + w_{t+1}\right) \\
& -(1+i^d_{t+1})(1+\omega_t)\tilde{d}_{t+1} - \left(1+\bar{\imath}^f_{t+1}\right) f_{t+1} - \left(1+i^{dw}_{t+1}\right) w_{t+1} \\
= \;& (1+i^b_{t+1})\tilde{b}_{t+1} + (1+i^{ior}_{t+1}) \left(\tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \frac{1+i^d_{t+1}}{1+i^{ior}_{t+1}}\right) - (1+i^d_{t+1})(1+\omega_t)\tilde{d}_{t+1} \\
& - \left(\bar{\imath}^f_{t+1} - i^{ior}_{t+1}\right) f_{t+1} - \left(i^{dw}_{t+1} - i^{ior}_{t+1}\right) w_{t+1} \\
= \;& (1+i^b_{t+1})\tilde{b}_{t+1} + (1+i^{ior}_{t+1})\tilde{m}_{t+1} - (1+i^d_{t+1})\tilde{d}_{t+1} - \left(\bar{\imath}^f_{t+1} - i^{ior}_{t+1}\right) f_{t+1} - \left(i^{dw}_{t+1} - i^{ior}_{t+1}\right) w_{t+1}.
\end{aligned}$$

By Proposition 1 and by the definition of $\chi\left(s\right)$, (10), the law of motion for e_{t+1} satisfies

$$e_{t+1} = \frac{\tilde{b}_{t+1}(1+i^b_{t+1}) + \tilde{m}_{t+1}(1+i^{ior}_{t+1}) - \tilde{d}_{t+1}(1+i^d_{t+1}) + \bar{\chi}_t\left(\tilde{m}_{t+1}, \tilde{d}_{t+1}, \omega\right)}{P_{t+1}}(1-\tau_{t+1}). \tag{G.3}$$

Now, since we already showed that there exists a function $V_t\left(e\right) = V^l_t(b, m, d, f, w)$, the value function at the balancing stage can be written in terms of the single state—future equity—as

$$V^b_t(\tilde{b}, \tilde{m}, \tilde{d}, \omega) = \beta V_{t+1}(e') \tag{G.4}$$

$$e' = \frac{\tilde{b}(1+i^b_{t+1}) + \tilde{m}(1+i^{ior}_{t+1}) - \tilde{d}(1+i^d_{t+1}) + \bar{\chi}_t\left(\tilde{m}, \tilde{d}, \omega\right)}{P_{t+1}}(1-\tau_{t+1}). \tag{G.5}$$

From here, we substitute this value function at the balancing stage into the value at the lending stage and obtain

$$\mathbb{E}_t\left[V^b_{t+1}(\tilde{b},\tilde{m},\tilde{d},\omega)\right]=\mathbb{E}_t\left[V_{t+1}(e')\right],$$

and thus, we have that

$$\begin{aligned}
V_t(e) = & \max_{c,\tilde{m},\tilde{b},\tilde{d}} u(c)+\beta\mathbb{E}_t\left[V_{t+1}(e')\right],\\
& e = \frac{\tilde{b}+\tilde{m}-\tilde{d}}{P_t}+c,\\
& \tilde{d}\leq\kappa\left(\tilde{b}+\tilde{m}-\tilde{d}\right)\\
& e' = \frac{\tilde{b}(1+i^b_{t+1})+\tilde{m}(1+i^{ior}_{t+1})-\tilde{d}(1+i^d_{t+1})+\bar{\chi}_t\left(\tilde{m},\tilde{d},\omega\right)}{P_{t+1}}(1-\tau_{t+1}).
\end{aligned}$$

Using the definitions of real returns in the main text is enough to establishes the claim in the proposition. **QED**.

G.2 Proof of Items (i)-(iv) in Proposition 3

This section presents a proof of Proposition 3. Here we show that the single state representation satisfies homogeneity. We follow the guess-and-verify approach, common to all dynamic programming models. Our guess is that the value function satisfies $V_t\left(e\right)=v_t e^{1-\gamma}-1/((1-\beta)\left(1-\gamma\right))$, where v_t is a time-varying scaling factor in the value function, common to all banks. From Proposition 1, the bank's problem is summarized by

$$\begin{aligned}
V_t(e) = & \max_{c,\tilde{m},\tilde{b},\tilde{d}} u(c)+\beta\mathbb{E}_t\left[V_{t+1}(e')\right],\\
& \text{subject to}\\
& c+\frac{\tilde{b}+\tilde{m}-\tilde{d}}{P_t}=e,\\
& \tilde{d}\leq\kappa\left(\tilde{b}+\tilde{m}-\tilde{d}\right)\\
& e' = \left((1+i^b_{t+1})\tilde{b}+(1+i^{ior}_{t+1})\tilde{m}-(1+i^d_{t+1})\tilde{d}+\bar{\chi}_t\left(\tilde{m},\tilde{d},\omega\right)\right)\frac{(1-\tau_{t+1})}{P_{t+1}}.
\end{aligned}$$

Note that multiplying and dividing by P_t, we have that e' can also be written as

$$e'=\frac{\left(\tilde{b}(1+i^b_{t+1})+\tilde{m}(1+i^{ior}_{t+1})-\tilde{d}(1+i^d_{t+1})+\bar{\chi}_t\left(\tilde{m},\tilde{d},\omega\right)\right)}{P_t}\frac{(1-\tau_{t+1})}{(1+\pi_{t+1})}, \tag{G.1}$$

where $(1+\pi_{t+1})=P_{t+1}/P_t$.

If the conjecture for the value function is correct, then this condition satisfies

$$v_t e^{1-\gamma} - \frac{1}{(1-\beta)(1-\gamma)} = \max_{c,\tilde{m},\tilde{b},\tilde{d}} \frac{c^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} (e')^{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \right],$$

$$\begin{aligned}
&\text{subject to} \\
&c + \frac{\tilde{b} + \tilde{m} - \tilde{d}}{P_t} = e, \\
&\tilde{d} \leq \kappa \left(\tilde{b} + \tilde{m} - \tilde{d} \right) \\
&e' = \frac{\left(\tilde{b}(1+i^b_{t+1}) + \tilde{m}(1+i^{ior}_{t+1}) - \tilde{d}(1+i^d_{t+1}) + \bar{\chi}_t \left(\tilde{m}, \tilde{d}, \omega \right) \right)}{P_t} \frac{(1-\tau_{t+1})}{(1+\pi_{t+1})}.
\end{aligned}$$

Observe that we can factor out constants from the objective:

$$\begin{aligned}
& \frac{c^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} (e')^{1-\gamma} - \frac{1}{(1-\gamma)(1-\beta)} \right] \ldots \\
= \quad & \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \left[v_{t+1} (e')^{1-\gamma} \right] - \frac{1}{(1-\beta)(1-\gamma)}.
\end{aligned}$$

Then, if we substitute the evolution of e' in (G.1), we obtain

$$v_t e^{1-\gamma} = \max_{c,\tilde{m},\tilde{b},\tilde{d}} \frac{c^{1-\gamma}}{1-\gamma} + \tag{G.2}$$

$$\beta \mathbb{E}_\omega \left[v_{t+1} \left(\frac{\left(\tilde{b}(1+i^b_{t+1}) + \tilde{m}(1+i^{ior}_{t+1}) - \tilde{d}(1+i^d_{t+1}) + \bar{\chi}_t \left(\tilde{m}, \tilde{d}, \omega \right) \right)}{P_t} \frac{(1-\tau_{t+1})}{(1+\pi_{t+1})} \right)^{1-\gamma} \right]$$

subject to

$$e = \frac{\tilde{b} + \tilde{m} - \tilde{d}}{P_t} + c \tag{Budget Constraint}$$

$$\tilde{d} \leq \kappa \left(\tilde{b} + \tilde{m} - \tilde{d} \right) \tag{Capital Requirement}$$

Let us define variables in terms of equity, $\bar{c} = c/e$. Also, define $\bar{b} = \tilde{b}/\left((1-\bar{c})eP_t\right)$, $\bar{m} = \tilde{m}/\left((1-\bar{c})eP_t\right)$, and $\bar{d} = \tilde{d}/\left((1-\bar{c})eP_t\right)$, as in the statement of Proposition 3. By Lemma 1, we can factor constants $(1-\bar{c})eP_t$ from $\bar{\chi}_t$ and express it as

$$(1-\bar{c})eP_t \bar{\chi}_t \left(\frac{\tilde{m}}{P_t(1-\bar{c})e}, \frac{\tilde{d}}{P_t(1-\bar{c})e}, \omega \right) = P_t(1-\bar{c})e\bar{\chi}_t \left(\bar{m}, \bar{d}, \omega \right).$$

Using this observation, we can replace $\bar{c}$ in the value function to obtain

$$v_t e^{1-\gamma} = \max_{c,\tilde{m},\tilde{b},\tilde{d}} e^{1-\gamma} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} \left((1-\bar{c}) e \right)^{1-\gamma} \mathbb{E}_{\omega} \ldots \tag{G.3}$$

$$\left(\left[\frac{\tilde{b}(1+i^b_{t+1})/P_t}{(1-\bar{c}) e} + \frac{\tilde{m}(1+i^{ior}_{t+1})/P_t}{(1-\bar{c}) e} - \frac{\tilde{d}(1+i^d_{t+1})/P_t}{(1-\bar{c}) e} + \chi_t \left(\bar{m}, \bar{d}, \omega\right) \right] \frac{(1-\tau_{t+1})}{(1+\pi_t)} \right)^{1-\gamma}$$

subject to:

$$\frac{\tilde{b}+\tilde{m}-\tilde{d}}{(1-\bar{c}) e P_t} = 1 \tag{Budget Constraint}$$

$$\frac{\tilde{d}/P_t}{(1-\bar{c}) e} \leq \kappa \left(\frac{\tilde{b}/P_t}{(1-\bar{c}) e} + \frac{\tilde{m}/P_t}{(1-\bar{c}) e} - \frac{\tilde{d}/P_t}{(1-\bar{c}) e} \right) \tag{Capital Requirement}$$

From this expression, we can cancel out $e^{1-\gamma}$ from both sides of (G.3), which verifies that the objective is scaled by $e^{1-\gamma}$. Thus, we verify the guess that $V_t(e) = v_t e^{1-\gamma} - ((1-\beta)(1-\gamma))^{-1}$.

Next, we derive the policies that attain $V_t(e)$ and the value of v_t. If the conjecture is correct, using the definition of $\bar{b}$, $\bar{m}$, and $\bar{d}$, we obtain

$$v_t = \max_{\left\{\bar{c},\bar{b},\bar{m},\bar{d}\right\} \geq 0} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} \ldots \tag{G.4}$$

$$\mathbb{E}_{\omega} \left(\left[(1+i^b_{t+1})\bar{b} + (1+i^{ior}_{t+1})\bar{m} - (1+i^d_{t+1})\bar{d} + \chi_t \left(\bar{m}, \bar{d}, \omega\right) \right] \frac{(1-\tau_{t+1})}{(1+\pi_t)} \right)^{1-\gamma}$$

subject to (G.5)

$$\bar{b}+\bar{m}-\bar{d} = 1 \tag{Budget Constraint}$$

$$\bar{d} \leq \kappa \tag{G.6}$$

Thus, any solution to $V_t(e)$ must be consistent with the solution of v_t if the conjecture is correct. Define real return on equity as follows:

$$R^E_t(\bar{b}, \bar{m}, \bar{d}, \omega) \equiv R^b_t \bar{b} + R^m_t \bar{m} - R^d_t \bar{d} + \bar{\chi}_t(\bar{d}, \bar{m}, \omega),$$

where $R^b_t = \left(1+i^b_{t+1}\right) \frac{(1-\tau_{t+1})}{(1+\pi_t)}$, $R^m_t = \left(1+i^{ior}_{t+1}\right) \frac{(1-\tau_{t+1})}{(1+\pi_t)}$, $R^d_t = \left(1+i^d_{t+1}\right) \frac{(1-\tau_{t+1})}{(1+\pi_t)}$, and

$$\bar{\chi}_t(\bar{d}, \bar{m}, \omega) = \chi_t \left(\bar{m}, \bar{d}, \omega\right) \frac{(1-\tau_{t+1})}{(1+\pi_t)}.$$

Then, the value function can be written as

$$v_t = \max_{\left\{\bar{c},\bar{b},\bar{m},\bar{d}\right\}} \frac{\bar{c}^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} \mathbb{E}_{\omega} \left[R^E_t(\bar{b}, \bar{m}, \bar{d}, \omega)^{1-\gamma} \right].$$

We now use the principle of optimality. Let Ω_t be the certainty equivalent of the bank's optimal portfolio problem, that is,

$$\Omega_t \equiv \max_{\left\{\bar{b},\bar{m},\bar{d}\right\}} \left[\mathbb{E}_{\omega} \left[R^E_t(\bar{b}, \bar{m}, \bar{d}, \omega)^{1-\gamma} \right] \right]^{\frac{1}{1-\gamma}}$$

subject to $\bar{b}+\bar{m}-\bar{d}=1$ and $\bar{d}\leq\kappa$. Assume $\bar{c}$ is optimal. If $\gamma<1$, the solution that attains v_t must maximize $\mathbb{E}_\omega\left[R_t^E(\bar{b},\bar{m},\bar{d},\omega)^{1-\gamma}\right]$ if v_{t+1} is *positive*. If $\gamma>1$, the solution that attains v_t must minimize $\mathbb{E}_\omega\left[R_t^E(\bar{b},\bar{m},\bar{d},\omega)^{1-\gamma}\right]$ if v_{t+1} is *negative*. We guess and verify that when $\gamma<1$, the term v_{t+1} is positive and v_{t+1} is negative when $\gamma>1$. Under this assumption, if $\gamma<1$, we have that $v_{t+1}>0$, so $1-\gamma>0$. Thus, by maximizing Ω_t, we are effectively maximizing the right-hand side of v_t. Instead, when $\gamma>1$, we have that $v_{t+1}<0$, so $1-\gamma<0$. Thus, by maximizing Ω_t, we are minimizing $\Omega_t^{1-\gamma}$, which multiplied by a negative number—v_{t+1}—maximizes the right-hand side of v_t.

Hence, the Bellman equation becomes

$$v_t=\max_{\left\{\bar{c},\bar{b},\bar{m},\bar{d}\right\}\geq 0}\frac{\bar{c}^{1-\gamma}}{(1-\gamma)}+\beta v_{t+1}\left(1-\bar{c}\right)^{1-\gamma}\Omega_t^{1-\gamma}.$$

This yields the statements in items (i) and (ii), provided that v_t inherits the sign of $(1-\gamma)$.

To prove item (iii), we take the first-order conditions with respect to $\bar{c}$, and raising both sides to the $-\frac{1}{\gamma}$ power, we obtain

$$\bar{c}=\left(\beta v_{t+1}\right)^{-1/\gamma}\Omega_t^{-(1-\gamma)/\gamma}\left(1-\bar{c}\right)\left(1-\gamma\right)^{-\frac{1}{\gamma}}.$$

We can rearrange terms to obtain

$$\bar{c}=\frac{1}{1+\left[\beta v_{t+1}(1-\gamma)\Omega_t^{1-\gamma}\right]^{1/\gamma}}.$$

Define $\xi_t=(1-\gamma)\beta v_{t+1}\Omega_t^{1-\gamma}$. Under the conjectured sign of v_t, the term ξ_t is always positive. Substituting this expression for dividends, we obtain a functional equation for the value function

$$v_t=\frac{\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)}}{(1-\gamma)}+\beta v_{t+1}\Omega_t^{1-\gamma}\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)}=\frac{\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)}}{(1-\gamma)}+\frac{\xi_t}{(1-\gamma)}\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)}$$

and finally,

$$=\frac{1}{(1-\gamma)}\left[\left(1+\xi_t^{1/\gamma}\right)^{-(1-\gamma)}+\xi_t\left[\frac{\xi_t^{1/\gamma}}{1+\xi_t^{1/\gamma}}\right]^{(1-\gamma)}\right].$$

Thus, we obtain

$$\begin{aligned}v_t&=\frac{1}{(1-\gamma)}\left[\frac{1}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}}+\frac{\xi_t^{1/\gamma}}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}}\right]\\&=\frac{1}{(1-\gamma)}\frac{1+\xi_t^{1/\gamma}}{\left(1+\xi_t^{1/\gamma}\right)^{(1-\gamma)}}=\frac{1}{(1-\gamma)}\left(1+\xi_t^{1/\gamma}\right)^{\gamma}.\end{aligned}$$

This verifies that v_t inherits the sign of $(1-\gamma)$. Thus, we can use Ω^* directly in the value function.

Furthermore, v_t satisfies the following difference equation:

$$v_t = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1}\right)^{\frac{1}{\gamma}}\right]^{\gamma}. \tag{G.7}$$

We can treat the right-hand side of this functional equation, solved independently of consumption. If we solve for this equation independently of the banker's consumption, we can obtain a solution to the banker's consumption policy via

$$\bar{c} = \frac{1}{1 + \left[\beta v_{t+1}(1-\gamma)\Omega_t^{1-\gamma}\right]^{1/\gamma}}.$$

This concludes the proof of items (i)-(iv), for all cases except $\gamma \to 1$. We work out that case next.

Log-Case. Observe that as $\gamma \to 1$, then v_t in (G.7) explodes. However, we can guess and verify that

$$\lim_{\gamma \to 1} v_t(1-\gamma) = \frac{1}{1-\beta}.$$

This assumption can be verified in equation (G.7). In this case,

$$\lim_{\gamma \to 1} (1-\gamma)\, v_t = \lim_{\gamma \to 1}\left[1 + \left(\beta(1-\gamma)\Omega_t^{*1-\gamma} v_t\right)^{\frac{1}{\gamma}}\right]^{\gamma} = 1 + \beta/\left(1-\beta\right) = 1/\left(1-\beta\right).$$

Thus, as $\gamma \to 1$, we have that $c = (1-\beta)$. Thus,

$$\Omega_t \equiv \max_{\{\bar{b},\bar{m},\bar{d}\}} \exp\left(\mathbb{E}_\omega\left[\log\left(R_t^E(\bar{b},\bar{m},\bar{d},\omega)\right)\right]\right).$$

QED.

G.3 Derivation of the Liquidity Premium and Its First-Order Term

The derivation of the liquidity premium is immediate from the first-order condition in the problem that solves Ω_t. The first-order term can be expressed as

$$\begin{aligned}
\mathbb{E}_\omega\left[\frac{\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{m}}\right] &= \frac{1}{1+\pi_t}\mathbb{E}_\omega\left[\frac{\partial \chi_t(m,d,\omega)}{\partial \bar{m}}\right] \\
&= \frac{1}{1+\pi_t}\mathbb{E}_\omega\left[\frac{\partial\left(\chi_t^+ s\mathbb{I}\left[s>0\right] + \chi_t^- s\mathbb{I}\left[s<0\right]\right)}{\partial \bar{m}}\right] \\
&= \frac{1}{1+\pi_t}\mathbb{E}_\omega\left[\frac{\partial\left(\chi_t^+ s\mathbb{I}\left[s>0\right] + \chi_t^- s\mathbb{I}\left[s<0\right]\right)}{\partial \bar{m}}\right] \\
&= \frac{1}{1+\pi_t}\mathbb{E}_\omega\left[\underbrace{\frac{\partial s}{\partial \bar{m}}}_{=1}\left(\chi_t^+ \mathbb{I}\left[s>0\right] + \chi_t^- \mathbb{I}\left[s<0\right]\right)\right] \\
&= \frac{1}{1+\pi_t}\left[\chi_t^+\left(1-F\left(\omega^*\right)\right) + \chi_t^- F\left(\omega^*\right)\right].
\end{aligned}$$

This expression appears in the main text immediately after the derivation of the liquidity premium. In fact, the entire liquidity premium—considering all non-linear terms—can be written in the same way:

$$\mathbb{E}_{\omega}\left[\frac{\left(R^e_{\omega}\right)^{-\gamma}}{\mathbb{E}[(R^e_{\omega})^{-\gamma}]}\frac{\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{m}}\right]=\frac{1}{1+\pi_t}\left[\chi^+_t\left(1-\tilde{F}\left(\omega^*\right)\right)+\chi^-_t\tilde{F}\left(\omega^*\right)\right],$$

where $\tilde{F}$ is the risk-adjusted distribution of withdrawal shocks:

$$\tilde{F}\left(\omega\right)=\int_{-\infty}^{\omega}\frac{\left(R^e_{\omega}\right)^{-\gamma}f(\omega)}{\mathbb{E}[(R^e_{\omega})^{-\gamma}]}d\omega.$$

A corresponding condition can be found for the liquidity premium in terms of deposits. This condition is given by

$$\mathbb{E}_{\omega}\left[\frac{\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{d}}\right]=\frac{1}{1+\pi_t}\mathbb{E}_{\omega}\left[\left(\chi^+_t\mathbb{I}\left[s>0\right]+\chi^-_t\mathbb{I}\left[s<0\right]\right)\left(\frac{R^d_t-\rho R^m_t}{R^m_t}\omega-\rho\right)\right].$$

Similarly, the entire deposit premium can be expressed in terms of a risk-neutral measure.

H Proofs for the Monetary Policy Analysis of Section 3.4

To present formal proofs, we define two important concepts: reserve satiation and neutrality.

Definition 4 (Satiation)**.** Banks are **satiated** with reserves at period t if the liquidity premium is zero, that is, if $R^b_t = R^m_t$.

To discuss policy effects, we compare an original policy sequence—with subindex o—with an alternative (shocked) policy—subindex s in all of the exercises. We mean that a policy is neutral relative to the other in the following sense.

Definition 5 (Neutrality)**.** Consider original and alternative policy sequences:

$$\left\{\rho_{o,t},B^{Fed}_{o,t},M_{o,t},W_{o,t},T_{o,t},\kappa_{o,t},i^{ior}_{o,t},i^{dw}_{o,t}\right\}\text{ and }\left\{\rho_{s,t},B^{Fed}_{s,t},M_{s,t},W_{s,t},T_{s,t},\kappa_{s,t},i^{ior}_{s,t},i^{dw}_{s,t}\right\}.$$

Policy s is **neutral**—relative to o—if the induced equilibria satisfy

$$\left\{e_{s,t},c_{s,t},\bar{b}_{s,t},\bar{d}_{s,t},\bar{m}_{s,t}\right\}=\left\{e_{o,t},c_{o,t},\bar{b}_{o,t},\bar{d}_{o,t},\bar{m}_{o,t}\right\}\text{ for all }t\geq 0.$$

When the condition holds, real aggregate loans and deposits are also determined. The rest of this appendix shows the proofs for the classic exercises in monetary policy analysis that we studied in the main text.

H.1 Proof of Proposition 4 (Conditions for Satiation)

By definition of satiation, the right-hand side of (29) equals zero under satiation:

$$R^b_t-R^m_t=0=\frac{\mathbb{E}_{\omega}\left[\left(R^e_{\omega}\right)^{-\gamma}\cdot\frac{\partial R^{\chi}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{m}}\right]}{\mathbb{E}_{\omega}\left(R^e\right)^{-\gamma}}.$$

Since $(R^e_\omega)^{-\gamma}$ is a strictly positive function for any ω and $\frac{\partial R^{\bar{\chi}}(\bar{m},\bar{d},\omega)}{\partial \bar{m}}$ is weakly positive, we must show that $\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)/\partial \bar{m}=0$. By definition,

$$\frac{\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{m}} = \frac{1}{1+\pi_t}[(\chi^{-}\mathbb{I}\left[\omega<\omega^*\right]+\chi^{-}\mathbb{I}\left[\omega>\omega^*\right])], \text{ for any } \omega\neq\omega^*.$$

Since, $\omega\neq\omega^*$ is a zero-probability event,

$$\begin{aligned}\mathbb{E}_\omega\left[(R^e_\omega)^{-\gamma}\cdot\frac{\partial R^{\bar{\chi}}\left(\bar{m},\bar{d},\omega\right)}{\partial \bar{m}}\right] &= \frac{1}{1+\pi_t}[\chi^{-}\mathbb{E}_\omega\left[(R^e_\omega)^{-\gamma}|\omega<\omega^*\right]\Pr\left[\omega<\omega^*\right]+\\ &\qquad \chi^{+}\mathbb{E}_\omega\left[(R^e_\omega)^{-\gamma}|\omega>\omega^*\right]\Pr\left[\omega>\omega^*\right]].\end{aligned}$$

This expression equals zero in two cases:

Case 1. If $i^{dw}_t=i^{ior}_t$, then the condition holds immediately since $\chi^-=\chi^+=0$. This case is condition (i) in the proposition.

Case 2. If $i^{dw}_t>i^{ior}_t$, then it must be that no bank can have a reserve deficit with positive probability. Recall that $\{\chi^-,\chi^+\}$ satisfy

$$\begin{aligned}\chi^-_t &= \Psi^-_t\left(\bar{i}^f_t-i^{ior}_t\right)+\left(1-\Psi^-_t\right)\left(i^{dw}_t-i^{ior}_t\right)\\ \chi^+_t &= \Psi^+_t\left(\bar{i}^f_t-i^{ior}\right).\end{aligned}$$

Suppose that $\omega<\omega^*$ in an event with non-zero mass. Then, it must be the case that $\chi^-_t=0$. A necessary condition is that $\Psi^-_t=1$ because $\left(i^{dw}_t-i^{ior}_t\right)>0$. This is not possible since $\Psi^-_t<1$ when market tightness $\theta_0>0$ and $\bar{\lambda}$ is finite. Hence, $[\omega>\omega^*]$ should occur with probability 1. Thus, we need to argue that the aggregate conditions must guarantee that there are enough reserves so that all banks can have a positive balance of reserves, for any ω. Clearly, in that case, ω must be bounded below. Let that lower bound be ω_{min}, as described in the main text. Under condition (ii) of the proposition, no bank is in deficit even for the worst shock. **QED.**

H.2 Proof of Proposition 5 Item (i)

Consider a policy sequence $\{o\}$ and an alternative policy $\{s\}$ such that

1. $X_{s,t}=kX_{o,t}$ for some $k>0$ for the balance sheet variables $X\in\left\{B^{Fed},M,W\right\}$,
2. policies are identical for non-balance-sheet variables $\left\{\rho_{o,t},\kappa_{o,t},i^{ior}_{o,t},i^{dw}_{o,t}\right\}=\left\{\rho_{s,t},\kappa_{s,t},i^{ior}_{s,t},i^{dw}_{s,t}\right\}$.

The proposition states that the stationary equilibrium induced by either policy features identical real asset positions and price levels that satisfy $P_{s,t}=kP_{o,t}$.

The proof is by construction and requires us to verify that the equilibrium conditions that determine $\{\bar{b}_{ss},\bar{m}_{ss},\bar{d}_{ss},\bar{c}_{ss},E_{ss}\}$ in Section D.2 are satisfied by any pair of policy sequences $\left\{M_{o,t},B^{Fed}_{o,t},W_{o,t}\right\}_{t\geq0}$ and $\left\{M_{s,t},B^{Fed}_{s,t},W_{s,t}\right\}_{t\geq0}$ that satisfies the relationship above. We proceed to check that $\left\{\bar{b}_{ss},\bar{m}_{ss},\bar{d}_{ss},\bar{c}_{ss},E_{ss}\right\}$ solves the set of equilibrium equations in Section D.2 in both cases.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both satisfy

$$M_{a,t}=M_{a,t-1}(1+\pi_{ss}),\ B^{Fed}_{a,t}=B^{Fed}_{a,t-1}(1+\pi_{ss}) \text{ and } W_{a,t}=W_{a,t-1}(1+\pi_{ss}) \text{ for some } \pi_{ss} \text{ and } a\in\{o,s\}.$$

By hypothesis, inflation and nominal rates are equal under both policies. Thus, the real interest rate on reserves is equal under both policies. We check the equilibrium conditions in the order in which they appear in Section D.1.

First, we guess and verify that the real returns on loans and deposits are also equal under both policies. If both policies yield the same real rates, the solution for bank portfolios (the solution for Ω_t) must also be equal in both equilibria:

$$\left\{\bar{b}_{o,ss}, \bar{m}_{o,ss}, \bar{d}_{o,ss}, \bar{c}_{o,ss}\right\} = \left\{\bar{b}_{s,ss}, \bar{m}_{s,ss}, \bar{d}_{s,ss}, \bar{c}_{s,ss}\right\}.$$

Consider now the aggregate supply of loans and reserves under either policy:

$$(1 - c_{ss})\bar{b}_{ss}E_{ss} = \Theta^b \left(R^b_{ss}\right)^{\epsilon} - B^{Fed}_{t+1}/P_t.$$

That equation can be satisfied under both policies because $B^{Fed}_{o,t+1}/P_{o,t} = (1+g)B^{Fed}_{o,t+1}/(1+g)P_{o,t} = B^{Fed}_{s,t+1}/P_{s,t}$. This verifies that the real rate on loans is equal under both policies.

The same steps verify that R^d_{ss} is the same under both policies. Similarly, the demand for reserves can be satisfied in both equations because

$$(1 - c_{ss})\bar{m}_{ss}E_{ss} = M_{o,t}/P_{o,t} = M_{a,t}/P_{a,t}.$$

This verifies market clearing for reserves. Thus, asset market clearing is satisfied under both policies.

Now, the ratio of surpluses to deficits is also equal under both policies:

$$\theta_{ss} \equiv S^-_{a,t}/S^+_{a,t} \text{ for } a \in \{o, s\}.$$

Because θ and policy rates are equal, the liquidity cost function χ is also equal under both policies. Observe that χ is a function of θ only. With equal inflation under both policies, the liquidity return R^χ must also be equal. This verifies that all the real rates in both equilibria are the same under both policies.

We now need to verify that the steady state condition for bank equity is also satisfied and that both policies are feasible. Observe that the interbank market loans under both policies satisfy $W^{Fed}_{o,t}/P_{o,t} = W^{Fed}_{s,t}/P_{s,t}$. Therefore, the law of motion for steady state equity is given by

$$E_{ss} = \frac{\left((1+i^b_t)\bar{b}_{ss} + (1+i^{ior}_t)\bar{m}_{ss} - (1+i^d_t)\bar{d}_{ss}\right)\left(1-\bar{c}_{ss}\right)E_{ss}}{1+\pi_{ss}} - \frac{1+i^{dw}_t}{1+\pi_{ss}}W^{Fed}_t/P_t - E_{ss}\frac{\tau_{ss}}{1-\tau_{ss}}.$$

Since the nominal rates, portfolios, inflation and real discount loans, W^{Fed}_t/P_t, are the same under both policies, the equation must yield the same solution E_{ss} under both policy o and policy s. Finally, we verify that the government budget constraint is satisfied under both policies. In any steady state,

$$B^{Fed}_t\left(\frac{1+i^b_{ss}}{1+\pi_{ss}}\right) + W^{Fed}_t\left(\frac{1+i^{dw}_t}{1+\pi_{ss}}\right) = M^{Fed}_t\left(\frac{1+i^{ior}_t}{1+\pi_{ss}}\right) + P_tE_{ss}\frac{\tau_{ss}}{1-\tau_{ss}}.$$

Once we divide both sides of the equation by P_t, we verify that the budget constraint is satisfied by the second sequence if it is satisfied by the first sequence. **QED.**

H.3 Proof of Proposition 5 Item (ii)

This statement of the proposition regards superneutrality and non-superneutrality. The proof closely follows the proof of Proposition 5, item (i). The main difference is that we prove neutrality along an equilibrium sequence, not only in stationary equilibrium. The proof is again by construction and only requires that we verify that the equilibrium conditions that determine $\{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, E_t\}$ in Section D.1 lead to the same values under both policies. Let $\left\{M_{o,t}, B^{Fed}_{o,t}, W_{o,t}\right\}_{t\geq 0}$ and $\left\{M_{a,t}, B^{Fed}_{a,t}, W_{a,t}\right\}_{t\geq 0}$ be two policy sequences. Again, to ease notation, we follow the order of the equations in Section D.1.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both equilibria satisfy

$$X^{Fed}_{a,t} = M^{Fed}_{a,t-1}(1+g_a), \; B^{Fed}_{a,t} = B^{Fed}_{a,t-1}(1+g_a), \text{ and } W^{Fed}_{a,t} = W^{Fed}_{a,t-1}(1+g_a) \text{ for some } g_a \text{ and for } a \in \{o, s\}.$$

Also, let both economies feature identical time-zero conditions: $X_{s,0} = X_{o,0}$ for $X \in \left\{M^{Fed}, B^{Fed}, W^{Fed}\right\}$.

Then, the condition for the Fed's balance sheet implies that

$$\begin{aligned} X_{s,t+1} &= (1+g_s)^t X_{s,0} = (1+g_s)^t X_{o,0}, \\ X_{o,t+1} &= (1+g_o)^t X_{o,0}, \end{aligned}$$

for $X \in \left\{M^{Fed}, B^{Fed}, W^{Fed}\right\}$. Thus, we can relate both balancesheet variables via

$$X_{s,t+1} = \left(1 + \frac{g_s - g_o}{1+g_o}\right)^t X_{o,t+1}$$

Through the proof, we guess and verify the following:

A.1 $\left\{R^b_{o,t}, R^d_{o,t}, R^m_{o,t}, R^{\bar{\chi}}_{o,t}\right\} = \left\{R^b_{s,t}, R^d_{s,t}, R^m_{s,t}, R^{\bar{\chi}}_{s,t}\right\}$.

A.2 $P_{o,0} = P_{s,0} = P_0$.

A.3 $(1+\pi_{s,t}) = (1+\pi_{o,t})\left(1 + \frac{g_s - g_o}{1+g_o}\right)$.

First, we verify (A.1). Under the conjecture that real returns are the same along a sequence, we have that

$$\left\{\bar{b}_{o,t}, \bar{m}_{o,t}, \bar{d}_{o,t}, \bar{c}_{o,t}\right\} = \left\{\bar{b}_{s,t}, \bar{m}_{s,t}, \bar{d}_{s,t}, \bar{c}_{s,t}\right\},$$

so the optimality conditions are satisfied in both cases.

Next, consider the aggregate supply of loans and reserve demand. Equilibrium in the loans market requires

$$(1 - \; c_t)\bar{b}_t E_t \; = \Theta^b \left(R^b_t\right)^{\epsilon} - B^{Fed}_{t+1}/P_t.$$

If the equation is satisfied under both policies, then we must verify that $B^{Fed}_{o,t+1}/P_{o,t} = B^{Fed}_{s,t+1}/P_{s,t}$. To see that this condition holds, recall that

$$B^{Fed}_{s,t+1} = \left(1 + \frac{g_s - g_o}{1+g_o}\right)^t B^{Fed}_{s,0}.$$

Now, if $\pi_{s,t} - \pi_{o,t} = (g_s - g_o)/(1+g_o)$, by (A.2) we have that

$$P_{a,t} = \prod_{\tau=0}^{t} (1+\pi_{a,\tau}) P_0 \text{ for } a \in \{o, s\}.$$

Combined with the guess (A.3) above, we obtain

$$P_{s,t}=\prod_{\tau=0}^{t}(1+\pi_{o,t})\left(1+\frac{g_s-g_o}{1+g_o}\right)P_0=P_{o,t}\left(1+\frac{g_s-g_o}{1+g_o}\right)^t.$$

Therefore,

$$B^{Fed}_{s,t+1}/P_{s,t}=\left(1+\frac{g_s-g_o}{1+g_o}\right)^t B^{Fed}_{s,0}/P_{s,t}=B^{Fed}_{0,t+1}/P_{o,t},$$

which shows that the real holdings of loans under both policies are equal. This is the condition we needed to verify that under our guess, R^b_t is the same under both policies. That R^d_t is the same under both policies follows immediately using the same steps. Next, by assumption, note that R^m_t is the same under both policies because

$$R^m_{o,t}=\left(1+i^{ior}_{o,t+1}\right)/\left(1+\pi_{o,t+1}\right)=\left(1+i^{ior}_{s,t+1}\right)\left(1+\frac{g_s-g_o}{1+g_o}\right)/\left(1+\pi_{o,t+1}\right),$$

and by assumption (A.3), the condition is also equal:

$$\left(1+i^{ior}_{s,t+1}\right)\frac{(1+\pi_{o,t+1})}{(1+\pi_{s,t+1})}/\left(1+\pi_{o,t+1}\right)=R^m_{s,t}.$$

Note that this condition is true because it is one of the assumptions of Proposition 5 that we are proving.

Next, consider the condition for an equilibrium in the reserve market that gives rise to the quantity. In any equilibrium, it must satisfy

$$(1-\ c_t)\bar{m}_t E_t\ =M_{o,t}/P_{o,t}=M_{s,t}/P_{s,t}.$$

This condition is used to verify our guess (A.3). The condition above requires

$$\frac{P_{s,t+1}}{P_{o,t+1}}=\frac{M_{s,t+1}}{M_{o,t+1}}=\left(1+\frac{g_s-g_o}{1+g_o}\right)^t\frac{M_{o,t+1}}{M_{o,t+1}}=\left(1+\frac{g_s-g_o}{1+g_o}\right)^t.$$

Then, since by Assumption (A.2), initial prices are the same, we have that

$$\frac{P_{s,t+1}}{P_{o,t+1}}=\frac{\prod\limits_{\tau=0}^{t}(1+\pi_{s,t})P_0}{\prod\limits_{\tau=0}^{t}(1+\pi_{o,t})P_0}=\left(1+\frac{g_s-g_o}{1+g_o}\right)^t\Rightarrow\prod_{\tau=0}^{t}(1+\pi_{s,t})=\prod_{\tau=0}^{t}(1+\pi_{o,t})\left(\frac{1+g_s}{1+g_o}\right).$$

Since the condition holds for all t, then A.3 is deduced from the quantity equation of reserves.

The next step is to verify that $R^{\bar{\chi}}_t$ is constant under both policies. For that, observe that the interbank market tightness is the same under both economies. To see that, simply note that the ratio of reserves to deposits is the same under both policies, and that this is enough to guarantee that θ_t is equal under both policies. By Lemma 2 and the condition for policy rates in the proposition—$\left(1+i_{o,t}^{x}\right)=\left(1+i_{s,t}^{v}\right)\left(\frac{1+g_s}{1+g_o}\right)$ for $x\in\{dw,ior\}$—in states away from satiation,

$$\chi\left(.;i_{s,t}^{dw},i_{s,t}^{ior}\right)=\left(\frac{1+g_s}{1+g_o}\right)\chi\left(.;i_{o,t}^{dw},i_{o,t}^{ior}\right).$$

Therefore, we have that

$$R_{o,t}^{\chi}=\frac{\chi\left(.;i_{o,t}^{dw},i_{o,t}^{ior}\right)}{1+\pi_{o,t}}=\frac{\left(\frac{1+g_s}{1+g_o}\right)\chi\left(.;i_{o,t}^{dw},i_{o,t}^{ior}\right)}{\left(\frac{1+g_s}{1+g_o}\right)\left(1+\pi_{o,t}\right)}=\frac{\chi\left(.;i_{s,t}^{dw},i_{s,t}^{ior}\right)}{\left(1+\pi_{s,t}\right)}=R_{s,t}^{\chi}.$$

This step verifies that $R_{o,t}^{\bar{\chi}}=R_{s,t}^{\bar{\chi}}$. So far, we have checked the consistency of assumptions (A.1) and (A.3), and that the policy rules for $\left\{\bar{b}_t,\bar{m}_t,\bar{d}_t,\bar{c}_t\right\}$ and the real rates are the same under both equilibria. We still need to show that the sequences for E_t are the same under both policies, that the initial price level is the same, and that the Fed's budget constraint is satisfied under both policies. We verify these conditions jointly. Consider the law of motion for aggregate real equity,

$$E_{t+1}=\left(R_t^b\bar{b}_t+R_t^m\bar{m}_t-(1+R_t^d)\bar{d}_t\right)\left(1-\bar{c}_t\right)E_t-\frac{1+i_t^{dw}}{1+\pi_t}\frac{W_t^{Fed}}{P_t}-E_t\frac{\tau_t}{1-\tau_t},$$

and the Fed's budget constraint in real terms

$$\tfrac{B_t^{Fed}}{P_t}R_t^b+\tfrac{W_t^{Fed}}{P_t}\left(\tfrac{1+i_t^{dw}}{1+\pi_t}\right)=\tfrac{M_t^{Fed}}{P_t}R_t^m+E_t\tfrac{\tau_t}{1-\tau_t}.$$

We have already verified that $B_{s,t+1}^{Fed}/P_{s,t}=B_{s,0}^{Fed}/P_{o,t}$. Following the same steps, we can show that real reserves M_t^{Fed}/P_t and discount loans W_t^{Fed}/P_t are identical in both equilibria. Away from satiation, $R_{o,t}^{\chi}=R_{s,t}^{\chi}$, so that means that real income from the discount window, $\frac{W_t^{Fed}}{P_t}\left(\frac{1+i_t^{dw}}{1+\pi_t}\right)$, is constant under both policies. Provided that τ_t is constant under both sequences, the Fed's budget constraint is satisfied under both policies. Similarly, the law of motion for real aggregate equity is the same, provided that E_0 is the same under both policies. Consider now $\left(B_0,D_0,M_0,W_0\right)$, the initial condition under both policies. If P_0 is same initial price under both policies, $E_{o,0}=E_{s,0}$. This is precisely the pair of initial conditions that we need to confirm our guess $E_{o,0}=E_{s,0}$ and $P_{o,0}=P_{s,0}$. This finalizes the proof that equlibria are the same along both policies. **QED.**

H.4 Proof of Proposition 6

Formally, we prove the following result. Consider two policies, o and s, and let the alternative policy feature an open-market operation performed at $t=0$ and reverted at $t=1$ in the sense that

1. $B_{s,0}^{Fed}=B_{o,0}^{Fed}+\Delta B^{Fed}$, $M_{s,0}^{Fed}=M_{o,0}^{Fed}+\Delta M^{Fed}$, for some loan purchase where $\Delta M^{Fed}=\Delta B^{Fed}\geq 0$.

2. $\left\{\rho_{o,t},\kappa_{o,t},i_{o,t}^{ior},i_{o,t}^{dw}\right\}=\left\{\rho_{s,t},\kappa_{s,t},i_{s,t}^{ior},i_{s,t}^{dw}\right\}$ for all $t\geq 0$.

3. $\left\{\rho_{o,t},B_{o,t}^{Fed},M_{o,t}^{Fed},...\right\}=\left\{\rho_{s,t},B_{s,t}^{Fed},M_{s,t}^{Fed},...\right\}$ for all $t>1$.

The statement of the proposition says that the operation is neutral only if banks are satiated with reserves at time zero. Away from satiation, the policy has real effects.

The proof is in two steps. First, we show that if the policies induce identical allocations, the equilibrium prices $P_{o,0}=P_{s,0}$ must also be equal. Then, we show by contradiction that if the price is constant, the open-market operation must have real effects away from satiation. If banks are satiated, the policy has no effect.

Next, we prove the auxiliary lemma corresponding to the first step of the proof.

Lemma 3. *Consider two arbitrary policy sequences o and s as described above. If real loans, deposits, dividends, reserves, and bank equity are the same in all periods in both equilibria, then* $P_{o,0} = P_{s,0}$.

Proof. We proceed by contradiction. Let us avoid the use of the time subscript and remember that the policy change is at time zero. Without loss of generality, normalize the price in the original equilibrium to $P_o = 1$. If both policies have no effects on real loans and real deposits, then, given the demand and supply functions, $R_o^b = R_s^b$ and $R_o^d = R_s^d$. These rates are consistent with a real quantity of loans B and deposits D.

Consider now the representative bank. By hypothesis, real dividends are equal in both equilibria $c_o = c_s$ and $e_o = e_s$. Let $\left\{b_o^{Fed}, b_o\right\}$ and $\left\{b_s^{Fed}, b_s\right\}$ be the holding of real loans by the Fed under the original policy and the alternative policy. Also, let $\{m_o, m_s\}$ be the real balances under both policies. Then, we know that by market clearing in the loans market,

$$\begin{aligned} B &= b_o^{Fed} + b_o \\ &= b_s^{Fed} + b_s. \end{aligned} \tag{H.1}$$

Since equity, dividends, and real deposits are constant, from the bank's budget constraints we obtain

$$b_o - b_s = m_s - m_o. \tag{H.2}$$

From the quantity equation we obtain

$$\begin{aligned} M_o + \Delta M &= P_s m_s \\ &= P_s \left(b_o + m_o - b_s\right) \\ &= P_s \left(b_o + m_o - \left(B - b_s^{Fed}\right)\right). \end{aligned} \tag{H.3}$$

The second equality follows from (H.2) and the third from (H.1). Now consider the definition of real loans held by the Fed in the alternative policy:

$$b_s^{Fed} = \frac{b_o^{Fed} + \Delta B}{P_s} = \frac{b_o^{Fed} + \Delta M}{P_s} = \frac{b_o^{Fed}}{P_s} + \frac{\Delta M}{P_s}.$$

Substituting the last term into (H.3), we obtain

$$\begin{aligned} M_o + \Delta M &= P_s \left(b_o + m_o - B + \frac{b_o^{Fed}}{P_s} + \frac{\Delta M}{P_s}\right) \\ &= P_s \left(b_o + m_o - B + \frac{b_o^{Fed}}{P_s}\right) + \Delta M. \end{aligned}$$

Thus, we have that the price under the alternative policy satisfies

$$m_o = P_s \left(b_o + m_o - \bar{B} + \frac{b_o^{Fed}}{P_s}\right).$$

Because this equation is independent of ΔM, it must hold for any open-market operation, in particular, for $\Delta M = 0$. Therefore, it must be that $P_o = P_s = 1$. **QED.**

Next, we establish that if the policy is neutral, we reach a contradiction. To reach that contradiction, assume that the policy is neutral. First, observe that if policy s is neutral with respect to policy o, real assets and bank equity must be equal in both equilibria. For that to hold, portfolios must be the same in both equilibria. Thus, consider the first-order condition for loans and reserves. Then, under the original policy,

$$R^b - R^m = \frac{\mathbb{E}_\omega \left[(R^e_\omega)^{-\gamma} \cdot \frac{\partial \bar{\chi}_t\left(\bar{m}, \bar{D}, \omega\right)}{\partial \bar{m}} |_{m_o} \right]}{\mathbb{E}_\omega \left(R^e\right)^{-\gamma}} \equiv \Lambda\left(m_o, \bar{D}\right).$$

Assume the economy is away from satiation, and assume the false hypothesis that the two policy sequences lead to the same real loans, deposits, and dividend sequences. Now, since banks are away from satiation but hold different portfolios, there are differences in the discount window loans. Since the policy is identical from $t = 1$ onward, any difference in Fed income from discount window loans must be offset with $t = 1$ transfers to banks under the alternative policy. That means that if the hypothesis is right, the policy leads to the same equity growth sequence. Since the increase in discount loans is rebated to banks, the return on bank equity is the same if the bank's portfolio is the same. By Lemma 3, the price sequence is the same under both policies. This implies that R^m is the same in both equilibria. Since the aggregate amount of loans is constant, the liquidity premia must be constant. This is where the contradiction appears: away from satiation $\frac{\partial \Lambda\left(m, \bar{D}\right)}{\partial m} \leq 0$, and $m_s > m_o$, the first-order condition cannot hold under both policies. This contradiction proves that away from satiation, the policy must have real effects.

Next, we verify that under satiation, the policy change has no effects. We verify that under satiation, the quantity and real rate of deposits are invariant under both policies. Under satiation, $\Lambda\left(m_o, \bar{D}\right) - \frac{\partial \Lambda\left(m_o, \bar{D}\right)}{\partial m_o} = 0$ and $R^b = R^m = (1 + i^{ior})$. Since R^b depends on real quantities of loans, we have that

$$R^b = \Theta^b \left(b_o + b_o^{fed}\right)^\epsilon.$$

Now, consider the alternative policy. Under the hypothesis that the policy has no effects, the price is constant and equal to 1. Thus, $\Delta \bar{b}^{fed} = \Delta \bar{m}$. Then, the balance sheet changes to

$$b_o^{fed} - m_o = b_o^{fed} + \Delta b^{fed} - (m_o + \Delta \bar{m}).$$

Since banks are indifferent between holding loans and reserves, as long as $R^b = R^m$, we must verify that

$$R^b = \Theta^b \left(b_o + b_o^{fed}\right)^\epsilon = \Theta^b \left(b_s + b_o^{fed} + \Delta b^{fed}\right)^\epsilon.$$

From the budget constraint of the bank,

$$b_s = b_o - \Delta m = b_o - \Delta b^{fed}.$$

Thus, $\Theta^b \left(b_s + b_o^{fed} + \Delta b^{fed}\right)^\epsilon = \Theta^b \left(b_o - \Delta b^{fed} + b_o^{fed} + \Delta b^{fed}\right)^\epsilon$, which is precisely what we needed to show to verify that loans remain constant. Since under both policies real asset returns are the same, real deposit rates are also the same. The same is true about dividends. Finally, since the Fed earns zero profits from the discount window under both policies and the Fed buys assets with equal returns, the operation leads to the same transfers. This verifies that the policy has no effects under satiation. **QED.**

I Law of Motion for Aggregate Equity

To derive the law of motion for aggregate equity, we combine the Fed's budget constraint with the bank's budget constraint and use the market clearing conditions. The budget constraint for the Fed during the balancing and lending stages is

$$
\begin{aligned}
M_{t+1}^{FED} &= \tilde{M}_{t+1}^{FED} + W_{t+1}^{FED} && \text{(I.1)}\\
\tilde{M}_t^{FED}(1+i_t^{ior}) + B_{t+1}^{Fed} &= \tilde{M}_{t+1}^{FED} + W_t^{FED}(1+i_t^{dw}) + B_t^{Fed}(1+i_t^b) + P_t T_t. && \text{(I.2)}
\end{aligned}
$$

Combining these two constraints, we obtain

$$
(1+i_t^{ior})M_t^{Fed} + B_{t+1}^{Fed} + W_{t+1}^{Fed} = M_{t+1}^{Fed} + (1+i_t^b)B_t^{Fed} + (1+i_t^{dw})W_t^{Fed} + P_t T_t^{Fed}. \tag{I.3}
$$

Iterating forward for one period and using (I.2), we obtain

$$
T_{t+1} = \frac{-\tilde{M}_{t+1}^{Fed}(1+i_{t+1}^{ior}) - W_{t+1}^{Fed}(i_{t+1}^{dw} - i_{t+1}^{ior}) - B_{t+1}^{Fed}(1+i_{t+1}^b) + \tilde{M}_{t+2} - B_{t+2}^{Fed}}{P_{t+1}}. \tag{I.4}
$$

On the bank's side, recall that individual equity is defined as

$$
e_t^j = \left(\frac{\tilde{m}_t^j(1+i_t^{ior}) + \tilde{b}_t^j(1+i_t^b) - \tilde{d}_t^j(1+i_t^d) + w_t^j(1+i_t^{dw}) + f_t^j(\bar{i}_t^f - i_t^{ior}) - P_t T_t^j}{P_t} \right) \tag{I.5}
$$

Iterating one period forward, integrating across banks, and the market clearing for reserves, discount window, and interbank market loans, we obtain:

$$
E_{t+1} = \frac{\tilde{M}_{t+1}(1+i_{t+1}^{ior}) + \tilde{B}_{t+1}(1+i_{t+1}^b) - \tilde{D}_{t+1}(1+i_{t+1}^d) + W_{t+1}(i_{t+1}^{dw} - i_{t+1}^{ior}) - P_{t+1}T_{t+1}^j}{P_{t+1}}. \tag{I.6}
$$

Multiplying and dividing by P_t in the denominator, and using definition of real returns,

$$
E_{t+1} = (R_{t+1}^b \bar{b}_t + R_{t+1}^m \bar{m}_t - R_{t+1}^d \bar{d}_t)E_t(1-\bar{c}_t) - \frac{W_{t+1}^{Fed}(i_{t+1}^{dw} - i_{t+1}^{ior})}{P_{t+1}} - T_{t+1}. \tag{I.7}
$$

If we replace this condition by (I.4), we obtain

$$
E_{t+1} = (R_{t+1}^b \bar{b}_t - R_{t+1}^d \bar{d}_t)E_t(1-\bar{c}_t) + \frac{\tilde{M}_{t+2} + (B_{t+2}^{Fed} - B_{t+1}^{Fed}(1+i_{t+1}^b))}{P_{t+1}}. \tag{I.8}
$$

which is equation (26).

Law of Motion under Satiation and $\tilde{M}_t = B_t^{Fed}$. Following Proposition 4, consider a Fed policy of $i_{t+1}^{dw} = i_{t+1}^{ior}$ to generate satiation, and in addition, assume $\tilde{M}_t = B_t^{Fed}$ (for any t). From (I.4), we must then have that $T_t = 0$. Thus, by (I.7) we have that

$$
E_{t+1} = (R_{t+1}^b \bar{b}_t + R_{t+1}^m \bar{m}_t - R_{t+1}^d \bar{d}_t)E_t(1-\bar{c}_t).
$$

Thus, under satiation we have that $R_t^b = R_t^m$, and hence we obtain

$$
E_{t+1} = (R_{t+1}^b(\bar{b}_t + \bar{m}_t) - R_{t+1}^d \bar{d}_t)E_t(1-\bar{c}_t),
$$

and because $\bar{b_t} + m_t = \bar{1} + \bar{d}$, we have that

$$E_{t+1} = (R^b_{t+1} + \bar{d}(R^b_{t+1} - R^d_{t+1}))E_t(1 - \bar{c}_t).$$

However, we know that under satiation, either $R^b_{t+1} = R^d_{t+1}$ or capital requirements bind and $\bar{d} = \kappa$. Thus, this law of motion is written as

$$E_{t+1} = (R^b_{t+1} + \kappa(R^b_{t+1} - R^d_{t+1}))E_t(1 - \bar{c}_t). \tag{I.9}$$

J Dynamical Properties

In this section, we study the dynamical properties of the model. We fully characterize these dynamics when banks have log utility and the Fed carries out a policy of no distortions in the interbank market. Both assumptions simplify the analysis. Although the results are not general, for small deviations around that policy, the dynamic properties should be similar.

Stationary Equilibrium and Policy Effects with Satiation. We begin describing the transitional dynamics of the model when the Fed carries out a policy that satiates the market with reserves and sets $M_t = B_t^{Fed}$. By satiating the market for reserves and maintaining an equal amount of reserves as Fed loans, the Fed induces no distortions in the credit market. A spread between loans and deposits only results from capital requirements. This characterization is useful because it describes the dynamics of the model in absence of any distortions. As long as the Fed does not deviate too much from this policy, the properties should go through.

For this section, it is useful to define the inverse demand elasticity of loans and supply elasticity of deposits, $\bar{\epsilon} \equiv \epsilon^{-1}$ and $\bar{\varsigma} \equiv \varsigma^{-1}$, respectively, as well as the intercept of the inverse demand for loans and supply of deposits, $\bar{\Theta}^b \equiv \left(\Theta^b\right)^{1/\epsilon}$ and $\bar{\Theta}^d \equiv \left(\Theta^d\right)^{1/\varsigma}$. We obtain the following characterization for a transition:

Proposition 8. *[Transitions under Satiation] Consider a policy sequence where the Fed induces satiation at all t and satisfies $M_t = B_t^{Fed}$. Then:*

1. **Dynamics.** Real aggregate bank equity follows:

$$E_{t+1} = \left(R_t^b + \kappa \min\left\{\left(R_t^b - R_t^d\right), 0\right\}\right) \beta E_t, \text{ with } E_0 > 0 \text{ given.}$$

2. **Existence and uniqueness.** There $\exists!$ steady state level of $E_{ss} > 0$. The steady state features binding capital requirements if and only if

$$\left(\bar{\Theta}^b\right)^{\frac{\bar{\varsigma}}{\bar{\varsigma}+\bar{\epsilon}}} \left(\bar{\Theta}^b\right)^{\frac{\bar{\epsilon}}{\bar{\varsigma}+\bar{\epsilon}}} \kappa^{\frac{\bar{\epsilon}\bar{\varsigma}}{\bar{\varsigma}+\bar{\epsilon}}} \left(1+\kappa\right)^{(-\bar{\epsilon})\left(\frac{\bar{\varsigma}-\bar{\epsilon}}{\bar{\varsigma}+\bar{\epsilon}}\right)} < \beta. \tag{J.1}$$

3. **Sufficient condition for monotone convergence**. If $\left(1-\bar{\epsilon}\right)/\left(1+\bar{\varsigma}\right) > \kappa/\left(\kappa+1\right),$ then E_t converges to E_{ss} monotonically.

In the paper, we satisfy the parameter restrictions of items 2 and 3 in Proposition 8.

J.1 Proof of Proposition 8

The proof of the proposition is presented in three steps. First, we derive a threshold equity level where capital requirements are binding. Second, we prove that there can be at most one steady state. Third, we provide conditions such that the equilibrium features binding reserve requirements. Finally, we derive the sufficient condition for monotone convergence. We then establish the result for the rate of inflation and the determination of the price level.

Part 1 - Law of Motion of Bank Equity. As shown in the Proof of Proposition 3, under log utility $\bar{c}_t = (1-\beta)$. Then, the law of motion in (I.9) becomes

$$E_{t+1} = \left(R_t^b + \kappa \min\left\{\left(R_t^b - R_t^d\right), 0\right\}\right) \beta E_t.$$

This shows that the law of motion of bank equity satisfies the differential equation in the proposition. Thus, we have obtained a law of motion for bank equity in real terms. We use this to establish convergence. Consider now the condition such that capital requirements are binding for a given $E_t = E$. For that we need that $R_t^b > R_t^d$. Using the inverse of the loan demand function, R_t^b can be written in terms of the supply of loans using the market clearing condition:

$$R_t^b = \bar{\Theta}^b \left(\bar{b} \beta E_t + \frac{B_t^{Fed}}{P_t} \right)^{-\bar{\epsilon}},$$

but since $B_t^{Fed} = \tilde{M}_t$,

$$R_t^b = \bar{\Theta}^b \left(\beta E_t \left(1 + \kappa \right) \right)^{-\bar{\epsilon}}.$$

Using the result that capital requirements are binding, $R_t^b > R_t^d$, we obtain

$$\bar{\Theta}^b \left(\beta E_t \left(1 + \kappa \right) \right)^{-\bar{\epsilon}} > \bar{\Theta}^d \left(\beta E_t \kappa \right)^{\bar{\varsigma}}.$$

Clearing E at equality delivers a threshold,

$$E_\kappa = \frac{1}{\beta} \left[\frac{\bar{\Theta}^b / \bar{\Theta}^d}{\left(1+\kappa\right)^{\bar{\epsilon}} \kappa^{\bar{\varsigma}}} \right]^{\frac{1}{\bar{\varsigma}+\bar{\epsilon}}},$$

such that for any $E < E_\kappa$, capital requirements are binding. Thus, the law of motion of capital is broken into a law of motion for the binding and non-binding capital requirements regions.

We obtain

$$E_{t+1} = \bar{\Theta}^b \left(\beta E_t \left(1 + \kappa \right) \right)^{1-\bar{\epsilon}} - \bar{\Theta}^d \left(\beta E_t \kappa \right)^{1+\bar{\varsigma}} \text{ for } E_t \leq E_\kappa$$

and

$$E_{t+1} = \bar{\Theta}^b \left((1+d_t) \beta E_t \right)^{-\bar{\epsilon}} \beta E_t \text{ for } E_t > E_\kappa.$$

Here, we substituted $\bar{d} = \kappa$ in (I.9) for the law of motion in the constrained region and $\bar{d}_t \left(R_t^b - R_t^d \right) = 0$ in the second region.

Part 2 - Uniqueness of Steady State. Here we show that there cannot be more than one steady state level of real bank equity. We prove this in a couple of steps. First, we ask whether there can be more than one steady state in each region—in the binding and non-binding regions. We show that there can be only one steady state in each region. Then, we ask if two steady states can co-exists, given that they must lie in separate regions. The answer is no.

To see this, define

$$\Gamma \left(E \right) \equiv \bar{\Theta}^b \left(\beta \left(1 + \kappa \right) \right)^{1-\bar{\epsilon}} E^{-\bar{\epsilon}} - \bar{\Theta}^d \left(\beta \left(1 + \kappa \right) \right)^{1+\bar{\varsigma}} E^{\bar{\varsigma}}.$$

If a steady state exists in the binding region, it must satisfy the following condition:

$$1 = \Gamma \left(E_{ss} \right) \text{ and } E_{ss} \leq E_\kappa.$$

It is straightforward to verify that

$$\Gamma' \left(E \right) < 0, \lim_{E \to 0} \Gamma \left(E \right) \to \infty, \text{ and } \lim_{E \to \infty} \Gamma \left(E \right) \to -\infty.$$

Since the function is decreasing and starts at infinity, and the function ends at minus infinity, there can be at most one steady state—with positive E—in the constrained region, $E_{ss} < E_\kappa$.

In the unconstrained region, $E_{ss} \geq E_\kappa$ a steady state is occurs only when

$$1 = R_t^b \beta.$$

We need to find the level of equity that satisfies that condition. Also, we know that $R^d = R^b$ in the unconstrained region. Thus, the supply of loans in the unconstrained region is given by

$$\beta E_t + \left(R^d\right)^{-}\left(R^b\right),$$

the sum of real bank equity plus real deposits. Thus, we can define the equilibrium rate on loans through the implicit map, $\tilde{R}^b(E)$, that solves

$$\tilde{R}^b(E) \equiv \left\{\tilde{R}|\tilde{R} = \bar{\Theta}^b\left(\beta E + \left(R^d\right)^{-}\left(\tilde{R}\right)\right)^{-\bar{\epsilon}}\right\}.$$

If we can show that $\tilde{R}^b(E)$ is a function and $\tilde{R}^b(E) = \beta^{-1}$ for only one E, then we know that there can be at most one steady state in the unconstrained region. To show that $\tilde{R}^b(E)$ is a function, we must show that there is a unique value of $\tilde{R}^b$ for any E. Note that $\tilde{R}^b(E) = \tilde{R}$ for $\tilde{R}$ that solves

$$\left(\tilde{R}\right)^{-\frac{1}{\epsilon}} - \left(R^d\right)^{-}\left(\tilde{R}\right) = \beta E.$$

Thus, since $(R)^{-\frac{1}{\epsilon}}$ is decreasing and $-\left(R^d\right)^{-}(R)$ is decreasing, $\tilde{R}^b(E)$ is a function. Observe that

$$\lim_{\tilde{R}\to 0}\left(\tilde{R}\right)^{-\frac{1}{\epsilon}} - \left(R^d\right)^{-}\left(\tilde{R}\right) = \infty, \text{ and } \lim_{\tilde{R}\to\infty}\left(\tilde{R}\right)^{-\frac{1}{\epsilon}} - \left(R^d\right)^{-}\left(\tilde{R}\right) = -\infty,$$

so $\tilde{R}^b(E)$ exists for any positive E. Since $\tilde{R}^b$ is decreasing in E and defined everywhere, there exists at most one value for E such that $\tilde{R}^b(E) = (\beta)^{-1}$. This shows that there exists at most one steady state in the unconstrained region.

Next, we need to show that if there exists a steady state where $E_{ss} \leq E_\kappa$, there cannot exist another steady state where $E_{ss} \geq E_\kappa$. To see this, suppose that there $\exists$ a steady state in the unconstrained region. Thus, there exists some value $E_u > E_\kappa$ such that

$$\tilde{R}^b(E_u) = 1/\beta.$$

Since $\tilde{R}^b$ is decreasing and $E_u > E_\kappa$, by assumption we obtain that

$$1/\beta < \tilde{R}^b(E_\kappa) = R^b(\beta E_\kappa(1+\kappa)), \tag{J.1}$$

where the equality follows from the definition of E_κ.

As a false hypothesis, suppose that there is another steady state where $E_c < E_\kappa$. Then, using the law of motion for equity in the constrained region,

$$\begin{aligned} R^b(\beta E_c(1+\kappa)) &= 1/\beta - \kappa\left(R^b(\beta E_c(1+\kappa)) - R^d(\beta E_c\kappa)\right) \\ R^b(\beta E_c(1+\kappa)) &< 1/\beta, \end{aligned} \tag{J.2}$$

where the second line follows from $R^b > R^d$ for any $E_c < E_\kappa$. Thus,

$$R^b(\beta E_\kappa(1+\kappa)) < R^b(\beta E_c(1+\kappa)) < \beta^{-1}$$

because R^b is decreasing. However, (J.2) and (J.1) cannot hold at the same time. Thus, there $\exists!$ steady state with positive real equity.

Part 3 - Conditions for Capital Requirements Binding at steady state. Since $\tilde{R}^b$ is decreasing, it suffices to show that if

$$R_t^b\left(E_\kappa\right) < \beta,$$

there exists no steady state with $E_\kappa > \bar{E}$. This condition is guaranteed if

$$\begin{aligned}
\bar{\Theta}^b\left(\left[\frac{\bar{\Theta}^b/\bar{\Theta}^d}{\left(1+\kappa\right)^{\bar{\epsilon}}\kappa^{\bar{\varsigma}}}\right]^{\frac{1}{\bar{\varsigma}+\bar{\epsilon}}}\left(1+\kappa\right)\right)^{-\bar{\epsilon}} &< \beta \rightarrow \\
\left(\bar{\Theta}^b\right)^{\frac{\bar{\varsigma}}{\bar{\varsigma}+\bar{\epsilon}}}\left(\bar{\Theta}^b\right)^{\frac{\bar{\epsilon}}{\bar{\varsigma}+\bar{\epsilon}}}\kappa^{\frac{\bar{\epsilon}\bar{\varsigma}}{\bar{\varsigma}+\bar{\epsilon}}}\left(1+\kappa\right)^{(-\bar{\epsilon})\left(\frac{\bar{\varsigma}-\bar{\epsilon}}{\bar{\varsigma}+\bar{\epsilon}}\right)} &< \beta,
\end{aligned}$$

which is the condition in the statement of the proposition.

Part 4 - Conditions for monotone convergence. Assume that parameters satisfy the conditions for a steady state with binding capital requirements. Observe that if $E_t > E_\kappa$, then $E_{t+1} < E_t$ since $R_t^b < (\beta)^{-1}$ for all $E > E_\kappa$. Thus, any sequence that starts from $E_0 > \bar{E}$ eventually abandons the region. Thus, without loss of generality, we only need to establish monotone convergence within the $E < E_\kappa$ region.

Now consider $E_t < E_{ss}$. We must show that E_{t+1} also satisfies $E_{t+1} < E_{ss}$ if that is the case. Employing the law of motion of equity in the constrained region, notice that

$$E_{t+1} - E_{ss} = \bar{\Theta}^b\left(\beta E_t\left(1+\kappa\right)\right)^{1-\bar{\epsilon}} - \bar{\Theta}^d\left(\beta E_t \kappa\right)^{1+\bar{\varsigma}} - E_{ss}.$$

Define $g\left(E\right) \equiv \Gamma\left(E\right)E$. Thus,

$$\begin{aligned}
E_{t+1} - E_{ss} &= \Gamma\left(E_t\right)E_t - E_{ss} \\
&= -\int_{E_t}^{E_{ss}} g'\left(e\right)de.
\end{aligned}$$

It is enough to show that $g'\left(e\right) > 0$ for any e. We verify that under the parameter assumptions, this is indeed the case. Note that

$$\begin{aligned}
g'\left(e\right) &= \left(1-\bar{\epsilon}\right)\bar{\Theta}^b\left(\beta\left(1+\kappa\right)\right)^{1-\bar{\epsilon}}e^{-\bar{\epsilon}} - \left(1+\bar{\varsigma}\right)\bar{\Theta}^d\left(\beta\kappa\right)^{1+\bar{\varsigma}}e^{\bar{\varsigma}} \\
&= \left(1-\bar{\epsilon}\right)R^b\left(\beta\left(1+\kappa\right)e\right)\beta\left(1+\kappa\right) > \left(1+\bar{\varsigma}\right)R^d\left(\beta\kappa e\right)\beta\kappa,
\end{aligned}$$

where the second line follows from the definition of R^b and R^d and the result that capital requirements are binding in $E < E_{ss}$. Furthermore, since in this region, $R^b > R^d$ for all $E < E_\kappa$, then a sufficient condition for $g'\left(E\right) > 0$ is simply that,

$$\left(1-\bar{\epsilon}\right)\beta\left(1+\kappa\right) \geq \left(1+\bar{\varsigma}\right)\beta\kappa.$$

Thus, a sufficient condition for monotone convergence is

$$\frac{\left(1-1/\epsilon\right)}{\left(1+1/\varsigma\right)} \geq \frac{\kappa}{\left(1+\kappa\right)}.$$

K Data Sources

Except for the liquidity premium and the equity losses, all the macroeconomic variables we use are obtained from the Federal Reserve Bank of St. Louis Economic Research Database (FRED ©) and are available at the FRED ©website. The original data sources for each series are collected by the Board of Governors of the Federal Reserve System (US). We use the following series corresponding to:

- the volume of interbank market loans:
  - Board of Governors of the Federal Reserve System (US), Interbank Loans, All Commercial Banks [IBLACBW027NBOG], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/IBLACBW027NBOG

- the volume of discount window loans:
  - Discount Window Borrowings of Depository Institutions from the Federal Reserve [DISCBORR], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/DISCBORR

- the interest on discount window loans:
  - Board of Governors of the Federal Reserve System (US), Primary Credit Rate [DPCREDIT], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/DPCREDIT

- the interest on reserves:
  - Board of Governors of the Federal Reserve System (US), Interest Rate on Required Reserves [IORR], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/IORR

- bank deposits:
  - Board of Governors of the Federal Reserve System (US), Deposits, All Commercial Banks [DPSACBM027NBOG], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/DPSACBM027NBOG

- the T-Bill rate is:
  - Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/TB3MS

- bank loans:
  - Board of Governors of the Federal Reserve System (US), Commercial and Industrial Loans, All Commercial Banks [BUSLOANS], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/BUSLOANS

The series that corresponds to open-market operations is the ratio of a measure of Fed's assets, normalized by total bank credit. The references for these series are:

- total bank credit
  - Board of Governors of the Federal Reserve System (US), Bank Credit of All Commercial Banks [TOTBKCR], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/TOTBKCR

- Fed's assets are the sum of (WSRLL) securities, unamortized premiums and discounts, repo, and loans held by the fed minus treasury securities (WSHOTS)
  - Board of Governors of the Federal Reserve System (US), Assets: Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans [WSRLL], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/WSRLL

  - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: U.S. Treasury Securities [WSHOTS], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/WSHOTS

The return on the illiquid bond is obtained from Nagel (2016), which uses the return of a the three-month general collateral repurchase agreements. The liquidity premium corresponds to the difference between the return of this asset and the interest on reserves.

- liquidity ratio
  - Board of Governors of the Federal Reserve System (US), (Cash Assets, All Commercial Banks/Total Assets, All Commercial Banks)*100, retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/graph/?g=IW4

L Background Facts

To frame the discussion in the quantitative application, we collect some features of the data recorded during the crisis. The data for the period of June 2007 to February 2010. is presented in Figure 15

Fact 1: Depressed Lending. Panel (a) presents the normalized series for commercial and industrial (C&I). We observe a decline in lending that begins around in late 2007. The decline continues through mid 2008 until there's a partial recovery during the last quarter of 2008, possibly accounted for the drawdown of bank credit lines. Nevertheless, the decline in lending accelerates dramatically from January 2009 until the end of our sample in mid 2009.

Fact 2: Deposit Expansion. Panel (b) presents the normalized series for deposits. We can observe that throughout the period, banks continued to issue deposits, suggesting that there was not a systemic problem of bank funding.

Fact 3: Increased Discount Window Loans. Panel (c) plots the monthly series the ratio of Fed discount window loans relative to total deposits. The figure shows an increase in discount loans by the Fed that begun in early 2008. The the volume of discount window loans jumps rapidly around the Lehman Brothers crisis in September 2008 and then reverts to pre-crisis levels.

Fact 4: Depressed Fed Funds Market Borrowing. Panel (d) plots the monthly series for total Fed Fund market loans relative to deposits. The figure shows a continuous decline that begins in early 2007. The decline persists throughout the entire sample, as emphasized in Afonso and Lagos (2015) and Afonso and Lagos (2014). Together with Fact 3, this figure shows a substitution away from interbank lending to discount window lending. The figure also shows that the decline in interbank lending precedes the expansion of the Fed's balance sheet observed in Panel (f). This and the series in Panel (c) are used to reproduce a series for the withdrawal volatility of deposits in the application of Section 7.

Fact 5: Increased interest on reserves. Panel (e) plots the monthly series for the Fed's interest payment on reserves. The figure shows how the Fed began paying interest on reserves since October 2008. Following an initial increment, the rate on reserves dropped to a floor of 25 bps.

Fact 6: Fed Balance Sheet Expansion. Panel (f) shows the large scale open-market operations of the Fed. We can see how since the beginning of our sample, the Fed carried out a sequence of expansions of its balance sheet. The series begins with a balance sheet which amounts to 0.5 per cent of total bank loans but by the end of the sample, the size of the Fed's balance sheet is 12 percent of private loans.

Fact 7: Increase Liquidity Ratio. Panel (g) shows the substantial increase in the the liquidity ratio, which is the counterpart of the decline in bank lending.

Fact 8: Increased Liquidity Premium. Panel (h) presents the liquidity premium. We compute the liquidity premium as the difference in return between the three-month general collateral repurchase agreements (GC repo) , and reserves at the Federal Reserve. The GC repo is effectively an interbank loan collateralized with Treasury securities. This definition of liquidity premium follows from Nagel (2016) except that we use the interest on reserves as opposed to the return on T-bills. We observe that following an initial decline in the first quarter of 2008, the premium spikes during the third quarter of that year and begins a decline in 2009. For more details, see Nagel (2016) or Del Negro et al. (2017).

Fact 9: Bank Equity Losses. Begenau et al. (2017) report that during the crisis, banks losses reached a peak of about 7% relative to bank book equity. That paper also shows that the entirety of those losses were offset by equity issuances —both preferred and common equity. That paper shows that market-value losses where much greater. In our experiments, we consider a 2 per cent loss in bank equity as middle grounds between losses before and after recapitalizations.

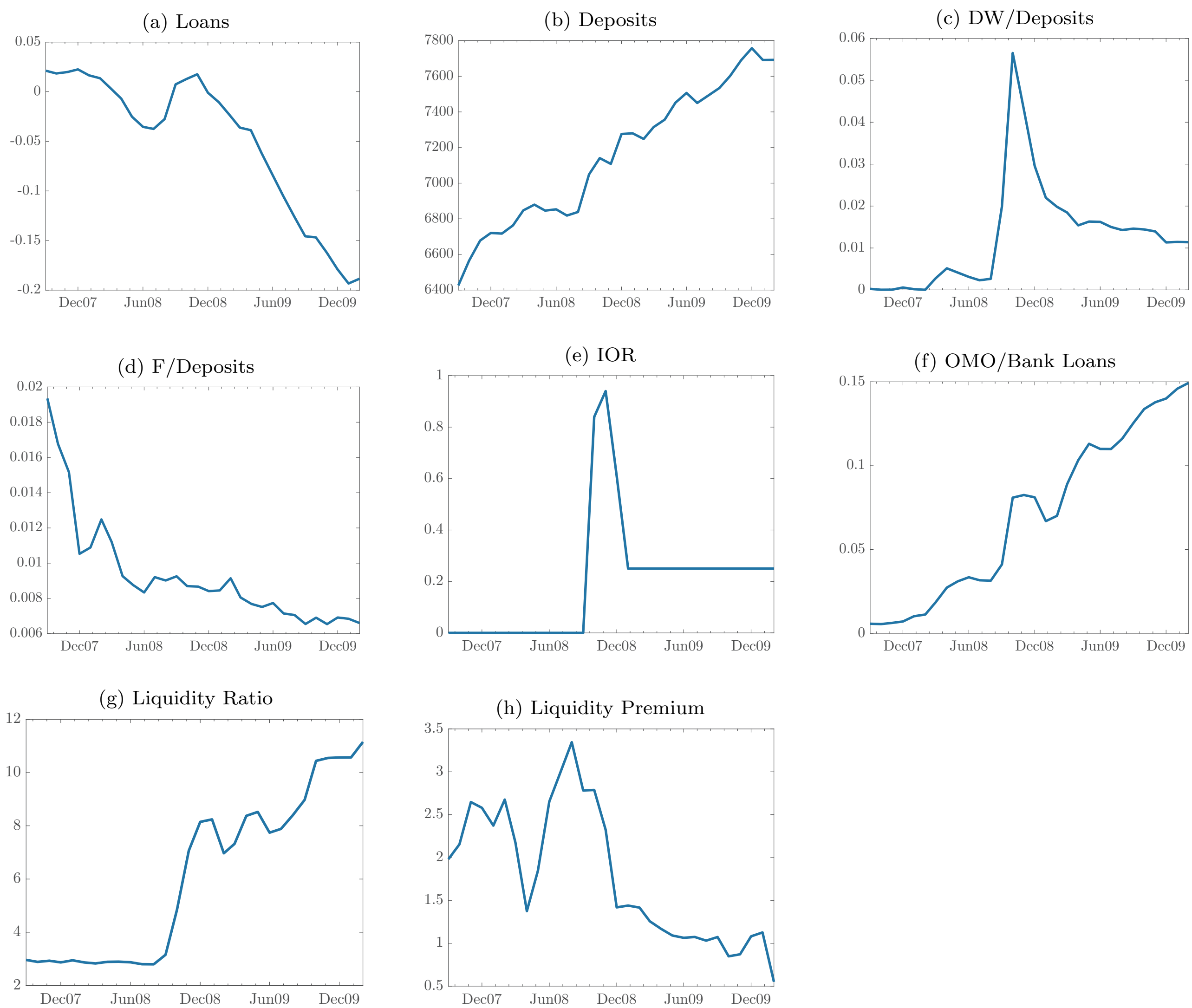

Figure 15: Data

M Algorithms

This appendix presents the numerical algorithms that we use to solve the model. We first present the algorithm to solve the stationary equilibrium in which the Fed's nominal balance sheet grows at rate g_m. We then present the algorithm to solve for transitional dynamics.

M.1 Computation: Stationary Equilibrium

We describe the algorithm to solve for the stationary equilibrium. In a stationary equilibrium, all nominal variables grow at rate g_m and real variables are constant. We also set a value for $R^d$and then obtain the value for Θ^d that is consistent with that value.

1. Set the growth rate of the Fed's nominal balance sheet to g_m and fix a level for B^{Fed}, M^{Fed}. Assume $B^{Fed} = 0$.

2. Guess a stationary value for the return on loans R^b_{ss} and market tightness θ_{ss}.

3. Given market tightness, policy rates, and R^d, compute the liquidity yield function using (10).

4. Solve banks' optimization problem:

   (a) Compute portfolio weights $\{\bar{b}, \bar{d}, \bar{m}\}$ and certainty equivalent value Ω with (22) with conjectured real return on loans and liquidity yield function computed in step 3.

   (b) Compute value of the bank v and consumption using (D.2.2) and (D.2.1), respectively.

5. Check whether banks' policies are consistent with steady state:

   (a) Compute aggregate gross equity growth as

   $$(1-\tilde{c})\,\mathbb{E}_{\omega}(R^b_{ss}\bar{b} + \bar{m} - R^d\bar{d}).$$

   (b) Compute implied market tightness:

   $$S^- = \int_1^{\frac{\bar{m}/\bar{d}-\rho}{(1-\rho)}} s(\omega)d\Phi \text{ and } S^+ = \int_{\frac{\bar{m}/\bar{d}-\rho}{(1-\rho)}}^{\infty} s(\omega)d\Phi.$$

   Market tightness is defined as

   $$\tilde{\theta} = S^-/S^+.$$

6. If the equity growth rate is zero and $\tilde{\theta} = \theta_{ss}$, move to step 7. Otherwise, adjust the guess for R^b_{ss} and θ_{ss} and go to step 3.

7. Compute the nominal amount of reserves and the intercepts of the loan demand and deposit supply functions using that real equity and the initial price level are normalized to one and

$$\begin{aligned}
\tilde{M}^{Fed} &= E(1-\tilde{c})\bar{m}EP, \\
\Theta^b\left(\frac{1}{R^b}\right)^{\epsilon} &= \left(E\bar{b}(1-\tilde{c})\right) - \frac{B^{Fed}}{P}, \\
\Theta^d\left(\frac{1}{R^d}\right)^{-\varsigma} &= E\bar{d}(1-\tilde{c}).
\end{aligned}$$

8. Compute nominal returns using definitions of real returns and T from the Fed budget constraint:

$$W^{Fed}(i^{dw}-\pi)\ +B^{Fed}(i^{b}-\pi)+PT=M^{Fed}(i^{ior}-\pi),$$

where

$$\begin{aligned} M^{Fed} &= \tilde{M}^{Fed}+W, \\ W= &= (1-\Psi^{-}(\theta))S^{-}. \end{aligned}$$

To compute expectations, we use a Newton-Cotes quadrature method. Specifically, we apply the trapezoid rule with a grid of 2,000 equidistant points. To specify the lower and upper boundary of the grid, we take the shock values that guarantee 10^{-5} mass in the tails of the distribution.

M.2 Computation: Transitional Dynamics under Baseline Policy

The algorithm to solve for transitional dynamics starts by conjecturing an initial price level, and then solves for all sequences of prices and quantities using market-clearing conditions and bank problems. After that, we check that the initial price leads the economy to converge to the stationary equilibrium after many periods. The balance sheet of the Fed grows at rate g and sets $1+i^{dw}_{t+1}=(1+r^{dw}_{t+1})(1+\pi_{t+1})$. We assume log utility.

1. Establish a finite period $T\in\mathbb{N}$ for steady state convergence and convergence criterion ε.

2. Set initial deviation from real steady state equity $\delta\in(0,1)$ such that $E_1=(1-\delta)E_{ss}$.

3. Guess an initial price level P_0.

4. Set $t=1$.

5. Given E_t, P_t, $\tilde{M}^{Fed}_{t+1}$, $\forall t=1,...T$ define the real fixed supply of reserves share:

$$\tilde{\mathbf{m}}\equiv\frac{\tilde{M}^{Fed}_{t+1}}{\beta P_t E_t}.$$

6. Given $\tilde{\mathbf{m}}$ compute $(\bar{b}_t,\bar{m}_t,\bar{d}_t,r^b_{t+1},r^{ior}_{t+1},r^d_{t+1})$, which solve

$$\begin{aligned} \bar{m}_t &= \tilde{\mathbf{m}}, \\ \beta E_t\bar{d}_t &= \Theta^d_t\left(1+r^d_{t+1}\right)^{\zeta}, \\ \beta E_t\bar{b}_t &= \Theta^b_t\left(\frac{1}{1+r^b_{t+1}}\right)^{\epsilon}+\frac{B^{Fed}_{t+1}}{P_t}, \\ \bar{b}_t+\bar{m}_t-\bar{d}_t &= 1, \end{aligned}$$

where

$$\begin{gathered}\left(\bar{b}_t,\bar{m}_t,\bar{d}_t\right)=\arg\max_{\bar{b},\bar{m},\bar{d}}\left\{\mathbb{E}_{\omega}\left[\ln\left(R^b_{t+1}\bar{b}+R^b_{t+1}\bar{m}-R^d_{t+1}\bar{d}+\chi(s_t)\right)\right]\right\} \\ \text{s.t. } \bar{b}+\bar{m}-\bar{d}=1 \\ \bar{d}\le\kappa,\end{gathered}$$

This is a system of six equations and six unknowns. Notice that if the capital requirement constraint binds, the system can be reduced to one unknown R^m_{t+1} and one equation,

$$\arg\max_{\bar{m}} \mathbb{E}_\omega \left[\ln \left(\left(\frac{\beta E_t}{\Theta^b_t} - \frac{B^{Fed}_{t+1}}{P_t} \right)^{-\frac{1}{e}} (1+\kappa-\bar{m})^{\frac{e-1}{e}} + R^m_{t+1}\bar{m} - \left(\frac{\beta E_t}{\Theta^d_t} \right)^{\frac{1}{\zeta}} \kappa^{\frac{\zeta+1}{\zeta}} + \chi(s_t) \right) \right] = \tilde{\mathbf{m}}$$

If the capital requirement does not bind, the system can be reduced to two unknowns $\{R^b_{t+1}, R^m_{t+1}\}$ and two equations,

$$\arg\max_{\bar{m},\bar{d}} \left\{ \mathbb{E}_\omega \left[\ln \left(R^b_{t+1} \left(1+\bar{d}-\bar{m} \right) + R^m_{t+1}\bar{m} - \left(\frac{\beta E_t}{\Theta^d_t} \right)^{\frac{1}{\zeta}} \bar{d}^{\frac{\zeta+1}{\zeta}} + \chi(s_t) = \right. \right. \right.$$

$$\left(\tilde{\mathbf{m}}, \frac{\Theta^b_t \left(R^b_{t+1} \right)^{-e} + \frac{B^{Fed}_{t+1}}{P_t}}{\beta E_t} + \tilde{\mathbf{m}} - 1 \right)$$

7. Given R^m_{t+1} and the nominal interest on reserves set by the Fed (i^{ior}_{t+1}), compute inflation as

$$\pi_{t+1} = \left(\frac{1+i^{ior}_{t+1}}{R^m_{t+1}} \right) - 1.$$

8. Given π_{t+1} and P_t, compute next period price $P_{t+1} = (1+\pi_{t+1})\, P_t$.

9. Compute next period equity

$$E_{t+1} = \beta \left(R^b_{t+1}\bar{b}_t + \bar{m}_t - R^d_{t+1}\bar{d}_t \right) E_t - \left(\frac{B^{FED}_{t+2}}{P_{t+1}} \right) - \left(\frac{\tilde{M}^{FED}_{t+2}}{P_{t+1}} \right).$$

10. If $t < T$ return to step 6 with $t = t+1$.

11. Compute criteria for convergence of $z = P_{T+1} - P_0(1+\pi_{ss})^T$.

12. If $|z| < \varepsilon$, exit algorithm. Otherwise, adjust P_0 and go to step 4.

M.3 Transitional Dynamics under Inflation Targeting

We describe the transitional dynamics when the Fed adjusts the balance sheet to keep the price level growing at the steady state inflation rate. To do this, we expand the tools of the Fed with deposits on banks, which we denote by D^{Fed}_{t+1}. Given this, the budget constraint of the Fed becomes

$$M^{Fed}_t(1+i^{ior}_t) + B^{Fed}_{t+1} + D^{Fed}_{t+1} + W^{Fed}_{t+1} = M^{Fed}_{t+1} + D^{Fed}_t(1+i^d_t) + W^{Fed}_t(1+i^{dw}_t) + P_tT_t,$$

and we assume the Fed offsets variations in M_t with D^{Fed}_t (i.e., $\Delta M^{Fed}_t = \Delta D^{Fed}_t$). Given this, following the same steps as in (I.8), the law of motion for aggregate equity becomes

$$E_{t+1} = (R^b_{t+1}\bar{b_t} - R^d_{t+1}\bar{d}_t)E_t(1-\bar{c}_t) + \frac{\tilde{M}_{t+2} + D^{Fed}_{t+1}(i^d_{t+1} - i^{ior}_{t+1}) + (B^{Fed}_{t+1}(1+i^b_{t+1}) - B^{Fed}_{t+2})}{P_{t+1}}. \quad \text{(M.1)}$$

We continue to assume as in our baseline that the Fed sets $1 + i^{dw}_{t+1} = \frac{1+r^{dw}}{1+\pi_{t+1}}$. Notice that since $P_{t+1}/P_t = 1+\pi$, we have that the real return on reserves is entirely determined by policy $R^m_t = (1+i^{ior}_t)/(1+\pi)$. We assume log utility.

1. Establish a finite period $T \in \mathbb{N}$ for steady state convergence and convergence criterion ε.

2. Set initial deviation from real steady state equity $\delta \in (0,1)$ such that $E_1 = (1-\delta)E_{ss}$.

3. Set $t = 0$.

4. Find $(\bar{d}_t, \bar{b_t}, \bar{m}_t, R^b_{t+1}, R^d_{t+1}, M^{Fed}_{t+1}, D^{Fed}_{t+1})$ that solve

$$\begin{aligned}
\beta E_t \bar{d}_t &= \Theta^d_t \left(\frac{1}{R^d}\right)^{\zeta} + D^{Fed}_{t+1}/P_t, \\
\beta E_t \bar{b_t} + B^{Fed}_{t+1}/P_t &= \Theta^b_t \left(\frac{1}{R^b_{t+1}}\right)^{\epsilon}, \\
\bar{m}_t. &= \frac{\tilde{M}_t}{\beta P_t E_t}, \\
\Delta M^{Fed}_t &= \Delta D^{Fed}_t,
\end{aligned}$$

   where

$$\begin{aligned}
(\bar{b}_t, \bar{m}_t, \bar{d_t}) = \arg\max_{m,\bar{b},\bar{d}} & \left\{\mathbb{E}_\omega \left[\ln\left(\left(R^b_{t+1}\right)\bar{b} + \left(1 + r^{ior}_{t+1}\right)\bar{m} - \left(1 + r^d_{t+1}\right)\bar{d} + \chi(s_t)\right)\right]\right\} \\
\text{s.t. } & \bar{b} + \bar{m} - \bar{d} = 1, \\
& \bar{d} \leq \kappa.
\end{aligned}$$

   This system of seven equations and seven unknowns can be solved block recursively. If the capital requirement constraint binds, the system can be reduced to one unknown $\bar{m}$ and one equation,

$$\arg\max_{\bar{m}} \left\{\mathbb{E}_\omega \left[\ln\left(\left(\frac{\beta E_t}{\Theta^b_t}\right)^{-\frac{1}{\epsilon}} (1+\kappa-\bar{m})^{\frac{\epsilon-1}{\epsilon}} + R^m_{t+1}\bar{m} - \left(\frac{\beta E_t}{\Theta^d_t}\right)^{\frac{1}{\zeta}} \kappa^{\frac{\zeta+1}{\zeta}} + \chi(s_t)\right)\right]\right\} = \tilde{\mathbf{m}}.$$

   If the capital requirement is not binding, the system can be reduced to two equations and two unknowns R^b, R^d. In order to solve this system, conjecture a pair (R^b, R^d), solve the portfolio problem, find the rates consistent with market clearing conditions, and then update the guesses for R^b, R^d accordingly.

5. Compute next period equity using (M.1).

6. If $|E_t - E_{ss}| < \varepsilon$, exit algorithm. Otherwise, $t = t+1$ and go to step 4.