Banks, Liquidity Management
and Monetary Policy

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Abstract

We develop a tractable model of banks’ liquidity management and the credit channel of monetary policy. Banks finance loans by issuing demand deposits. Loans are illiquid, and deposit transfers across banks must be settled with reserves in a frictional over-the-counter market. To mitigate the risk of large withdrawals of deposits, banks hold a precautionary buffer of liquid assets (reserves). We show how monetary policy affects the banking system by altering the trade-off between profiting from lending and incurring greater liquidity risk. In a quantitative application, we study the driving forces behind the decline in bank lending during the 2008 financial crisis. Our analysis underscores the importance of a combination of disruptions in interbank markets early during the crisis, followed by a persistent decline in credit demand.

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1 Introduction

Banks’ liquidity management is central to the transmission and implementation of monetary policy. By affecting the trade-off between lending and holding liquid assets, central banks affect the supply of loanable funds, and through this channel they affect the real economy. Understanding how banks manage liquidity is, thereby, paramount to understanding the transmission and implementation of monetary policy. However, macroeconomic models used for monetary policy analysis abstract from banks’ liquidity management and are therefore silent about how monetary policy is transmitted through the banking system.

This paper provides a tractable general equilibrium model of the banking system, featuring a liquidity management problem and a credit channel of monetary policy. In the theory, banks operate in competitive markets for loans and deposits and trade in an over-the-counter (OTC) interbank market. A spread between loan and deposit rates leads banks to engage in maturity transformation, but at the same time, exposes them to idiosyncratic liquidity risk. To mitigate this risk, banks hold a buffer of liquid assets (central bank reserves). The central bank affects the return differential between reserves and loans by setting a corridor on interest rates and conducting open market operations. This ability of the central bank to alter banks’ trade-off between lending profits and liquidity risk gives rise to a credit channel of monetary policy.

The first building block of our model is the liquidity management problem of an individual bank. When a bank grants a loan, it simultaneously creates demand deposits—or credit lines. These deposits can be used by the borrower to perform transactions at any time. When deposits are transferred out of a bank, that bank must transfer an asset to the bank that absorbs the liabilities. Because loans cannot be sold immediately, the bank needs to transfer reserves. If the bank receives a large withdrawal of deposits, it may fall short of the reserves needed to settle the transaction. Being short of reserves, that bank must incur expensive borrowing, either from other banks or from the central bank’s discount window. By holding a large precautionary buffer of reserves or, more generally, liquid assets, the bank can reduce this liquidity risk. The opportunity cost of this buffer is that it reduces the profits from intermediation. This is the classic bank liquidity management problem.

In the second building block, we embed this liquidity management problem into a dynamic equilibrium model of industry dynamics. In the model, there is a continuum of banks that are subject to idiosyncratic shocks to withdrawals of deposits. Following the realization of these shocks, banks trade in an OTC market for reserves to accommodate their reserve surpluses or
deficits. The probability of finding a counterpart, as well as the interbank market rate (i.e., the federal funds rate), depends on the abundance or scarcity of reserves. When few banks have a surplus of reserves, being short of reserves becomes increasingly expensive for a bank. The abundance of reserves, as well as the efficiency of the interbank market, therefore plays a key role in determining banks' individual lending decisions. Together with the amount of aggregate bank equity, these portfolio decisions determine the supply of bank lending, the demand for reserves and deposits. The banking system also faces a demand schedule for loans and a supply schedule for deposits.

The third and final building block of the model is the central bank. In the theory, the central bank has access to various tools. A first set of instruments includes reserve requirements, discount window rates, and interest payments on reserves. This first set of instruments affects the demand for reserves by altering the return on reserves relative to loans. A second set of instruments includes open market operations (OMO) that involve an exchange between liquid and illiquid assets. This second set of instruments alters the volume of reserves in the system. Both sets of instruments carry real effects by tilting the liquidity management trade-off and affecting the aggregate supply of bank lending.

Overall, the model has several features both theoretically and quantitatively: it introduces an analytically tractable OTC market for assets into a general equilibrium theory of banking; it delivers an endogenous liquidity premium and money multiplier; it allows for a tractable solution of stationary equilibrium and transitional dynamics; it allows for interaction between liquidity and the real economy; and it allows us to analyze how monetary policy affects the banking sector and the real economy through the credit channel.

Despite the richness of bank portfolio decisions, idiosyncratic withdrawal risk, and an OTC interbank market, we are able to reduce the state space into a single aggregate endogenous state: the aggregate value of bank equity. Moreover, the bank’s problem satisfies portfolio separation. In turn, this allows us to analyze the liquidity management problem through a portfolio problem with non-linear returns that depend only on aggregate market conditions. Being analytically tractable, this makes the analysis of the model transparent and amenable to various applications both theoretically and quantitatively.

The model yields several insights regarding how monetary policy affects bank lending. We first show that the wedge between the return on loans and liquid assets, which we refer to as the liquidity premium, can be decomposed into two terms. The first term constitutes the expected marginal benefits from reducing the costs of illiquidity. The second term is a risk premia term
that captures that liquid assets tend to have a higher payoff more when the bank faces higher withdrawals and the return on equity is low. Both terms depend on the width of the corridor rate (i.e., the difference between the discount window rate and the interest on reserves) and the efficiency of the interbank market. Moreover, we characterize how the return on loans varies with policy rates and show the power of monetary policy.

A key result is that the return on loans, and hence the volume of loans, reacts non-monotonically to changes in the interest paid on reserves by the central bank. For low interest on reserves, issuing deposits is very costly for banks because holding liquidity (low-interest-bearing assets) to deal with deposit withdrawals leads to significant reductions in the expected return on equity. This implies that for low interest on reserves, the capital requirement is not binding. In this case, a rise in the interest on reserves leads to an increase in bank leverage and a higher supply of loans, as well as a low equilibrium return on loans. As the interest on reserves rises and the effective costs of deposits fall, the capital requirement constraint becomes binding. At the point in which the capital requirement constraint becomes binding, further increases in the interest on reserves leads to reductions in the supply of bank loans and increases in the return on loans. Higher interest on reserves crowds out lending. Finally, when the interest on reserves is as high as the discount window rate, banks become satiated with reserves, which eliminates the liquidity premium. Other policies that can lead to satiation are large purchases of loans by the central bank, or remunerating reserves at the same rate as loans. In contrast, remunerating reserves at the same rate as deposits is not sufficient to ensure that banks are satiated with reserves.

**Quantitative Application.** As an application of our model, we qualitatively and quantitatively investigate the explanation for the deep and protracted decline in bank lending during the recent financial crisis. Through the lens of the model, we evaluate the plausibility of the following hypotheses, which have been much discussed in policy circles as possible explanations for the decline in bank lending

**Hypothesis i  Low Bank Equity**

Large losses in banks’ asset portfolios during the crisis due to subprime mortgages eroded bank capital and caused a reduction in bank lending (Gertler and Kiyotaki, 2010). Because of constraints on new equity issuances and binding capital requirements, the decline in bank equity and credit was persistent.

**Hypothesis ii  Increased Precautionary Holdings of Reserves**
Banks cut back on lending because of higher liquidity risk (Lucas and Stokey, 2011). As emphasized by Brunnermeier (2009), the crisis was characterized by a substantial risk of a freezing of market liquidity: assets that used to be liquid before the crisis experienced large increases in bid-ask spreads, banks became reluctant to lend to each other, and concerns arose about runs on financial institutions.

**Hypothesis iii**  
*Interest on Reserves and Federal Reserve Policy*

Interest payments on excess reserves have led banks to scale back on lending.¹ As the Federal Reserve started paying interest on reserves, reserves became relatively more attractive compared with loans.²

**Hypothesis iv**  
*Weak Credit Demand*

Banks faced a weaker demand for loans. For various reasons, including the increase in uncertainty and reductions in aggregate consumption, firms have scaled down investment and reduced their demand for loans, leading naturally to reductions in bank lending.

Our model provides a unified framework in which to address these different hypotheses. Relative to many studies investigating the decline in bank lending during the recent financial crisis, liquidity risk and the interaction with monetary policy take center stage in our environment. These features are indeed key to studying hypotheses ii and iii.

To quantitatively evaluate these hypotheses, we first calibrate our model in a stationary environment for “normal times” using Federal funds data. We then conduct numerical simulations, feeding into the model the shocks associated with each hypothesis. To weigh on hypotheses i and iii, we feed into the model shocks to equity losses and a Federal Reserve policy that we observe directly from the data. To weigh on hypotheses ii and iv, we introduce shocks to the dispersion of deposit withdrawals and the efficiency of the interbank market (hypothesis ii) and to loan demand (hypothesis iv), calibrated to match certain patterns in the data. Specifically, we extract the path of these shocks, which can replicate the behavior of bank credit, discount window loans, and the volume of the interbank market.


²It is well understood that interest payment on reserves lead to large substitution within liquid assets (i.e., Treasuries were substituted with excess reserves). What is less clear, which is the subject of our application, is the extent to which this policy has lead to reductions in bank lending at the expense of liquid assets overall.
To see the logic for the identification of these unobserved shocks, consider first a negative shock to loan demand. Given banks’ supply schedule, the decline in loan demand causes a decline in loan returns, which in turn leads banks to substitute reserves for loans. As a result, banks resort to the discount window less often. Consider instead a reduction in the efficiency of the interbank market or an increase in the volatility of deposit withdrawals. Both of these shocks also lead to a contraction in banks’ supply of loans but, at the same time, lead banks to resort to the discount window more frequently. In turn, reductions in interbank market efficiency can be disentangled from increases in withdrawal volatility because lower efficiency depresses interbank market lending volumes, whereas increases in volatility produce the opposite effect.

Finally, we provide a decomposition to measure the relative importance of each shock. Our decomposition favors a combination of an early disruption in the interbank market—concentrated during September 2008—followed by a substantial and persistent shock to loan demand as main drivers of the decline in credit. Although not targeted, the model also explains the persistent increase in the liquidity ratio and is consistent with a higher liquidity premium around Lehman Brothers’ bankruptcy in September 2008 relative to the post crisis period.

**Related Literature.** A tradition in macroeconomics dating back to at least Bagehot (1873) stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework in which to study policy with a full description of households, firms, and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was abandoned from macroeconomics for many years, and the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.

In the aftermath of the global financial crisis, numerous calls have been made for the development of macroeconomic models with an explicit role for banks (see, e.g., Woodford, 2010). Some early steps were taken by Gertler and Karadi (2011) and Curdia and Woodford (2009), who show how shocks that disrupt financial intermediation can have important effects on the real economy. Following the models in these papers, a large literature has studied how various policies affect macroeconomic outcomes. Our model also belongs to the banking channel view, but instead it emphasizes how monetary policy affects the trade-off banks face in holding assets of different liquidity. In turn, this approach relates our model to classic models of bank liquidity
management and monetary policy. Our contribution to this literature is to bring the classic insights from the liquidity management literature into a modern dynamic general equilibrium model that can be used for policy analysis and the study of banking crises.

Our paper also builds on the search theoretic literature of monetary exchange (see the survey by Williamson and Wright, 2010). Williamson (2012) studies an environment in which assets of different maturity have different properties as mediums of exchange. In Cavalcanti, Erosa, and Temzelides (1999), reserves emerge as a disciplining device to sustain credit creation under moral hazard and to guarantee the circulation on deposits. Atkeson, Eisfeldt, and Weill (2015) present a tractable model to study trading decisions in an OTC market where agents have different credit risk exposures. Afonso and Lagos (2015) develop an OTC model of the federal funds market and use it to study the intraday evolution of the distribution of reserve balances and the dispersion in loan sizes and federal funds rates. Our market for reserves is a simplified version of that model, which we embed in a fully dynamic general equilibrium monetary model. Armenter and Lester (2015) provide, as we do, a quantitative analysis of the interbank market. They use a static search model that incorporates new features of the landscape of the interbank market and study the abundance of excess reserves, whereas our focus is on the accumulation of liquid assets more broadly.

We share common elements with recent work at the intersection of money and banking. Brunnermeier and Sannikov (2017) introduce inside and outside money into a dynamic macro model and study the real effects of monetary policy through the redistributive effects of inflation. In contrast to their work, reserves and deposits are not perfect substitutes in our model, giving rise to a liquidity management problem. Piazzesi and Schneider (2016) study the link between the

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3Classic papers that study static liquidity management—also called reserve management—by individual banks in static settings are Poole (1968) and Frost (1971). Bernanke and Blinder (1988) present a reduced-form model that blends reserve management with an IS-LM model. Many modern textbooks for practitioners deal with liquidity management. For example, Saunders and Cornett (2010) and Duttweiler (2009) provide managerial and operations research perspectives. Many modern banking papers have focused on bank runs. See, for example, Diamond and Dybvig (1983), Allen and Gale (1998), Ennis and Keister (2009), or Holmstrom and Tirole (1998). Gertler and Kiyotaki (2015) is a recent paper that incorporates bank runs into a dynamic macroeconomic model. Del Negro et al. (2017) study a rich dynamic stochastic general equilibrium model with shocks to the resaleability of assets, as in Kiyotaki and Moore (2008).

4A large empirical literature provides the underpinnings for the monetary policy transmission mechanism that we study here. Bernanke and Blinder (1988) and Kashyap and Stein (2000) are early studies on the bank lending channel of monetary policy. In recent work, Nagel (2016) documents significant time variation in the liquidity premium and how it relates to monetary policy. Jiménez, Ongena, Peydró, and Saurina (2012) and Jiménez et al. (2014) exploit both firm heterogeneity in loan demand and variation in bank liquidity ratios to identify the presence of the bank lending channel in Spain. Chodorow-Reich (2014) analyzes the effects of credit contractions on employment outcomes.

5Ashcraft and Duffie (2007) and Afonso and Lagos (2014) provide empirical support for search frictions in the federal funds market and the presence of substantial liquidity costs.
payments system and securities markets with a focus on asset pricing. One important consider-
eration in their work that is not present here is that interbank market loans require collateral
assets.

Outline. The paper is organized as follows. Section 2 presents the model, and Section 3
provides theoretical results. Section 4 presents the calibration of the model. Section 5 and
Section 6 present the application. Section 7 presents extensions of our baseline model, and
Section 8 concludes. All proofs are in the appendix.

2 The Model

We propose a dynamic equilibrium model of the banking system, in which banks issue deposits
that are subject to idiosyncratic withdrawal risk. There is a single final consumption good and
no aggregate uncertainty. Banks face a portfolio choice between liquid and illiquid assets. Liquid
assets are monetary liabilities issued by the central bank, which we denote as reserves. Illiquid
assets correspond to loans. A central feature of the model is that the risk of deposit withdrawals
generates a precautionary buffer stock of liquid assets.

To present the model, we first describe in detail the dynamic portfolio problem of an individual
bank, followed by the description of the interbank market. The model is closed by considering the
policies of the central bank—which we refer to as the Fed—and introducing a demand schedule
for loans and a supply schedule for deposits.

2.1 Banks: Preferences and Budgets

Preferences. There is a unit-mass continuum of heterogeneous banks indexed by \( j \). Banks’
preferences over a stochastic stream of dividend payments \( \{c^j_t\} \) are given by

\[
E_0 \sum_{t \geq 0} \beta^t u(c^j_t),
\]

where \( \beta < 1 \) is the time discount factor, and \( u(c) \equiv \frac{c^{1-\gamma}}{1-\gamma} \) is the period utility function with
\( \gamma \geq 0 \). A strictly positive value of \( \gamma \) contributes to generate dividend smoothing and slow-
moving bank equity, as observed empirically, and can be rationalized by assuming undiversified
bank owners or other frictions on equity financing.\(^6\)

\(^6\)Considering CRRA or more generally homothetic preferences is convenient because it facilitates aggregation.
Timing. Time is discrete, indexed by $t$, and there is an infinite horizon. Each period is divided into two stages: the lending stage, $(l)$, and the balancing stage, $(b)$. At the lending stage, banks make portfolio decisions and solve a liquidity management problem. At the balancing stage, banks are subject to random idiosyncratic withdrawals of deposits. A deposit withdrawn from one bank is transferred to another bank. That transaction is settled with reserves. Reserves are issued by the central bank and serve as the numeraire. If banks lack the reserves to settle that transaction, they can borrow them from other banks in the interbank market or from the central bank.

Lending stage. Banks enter the lending stage with a portfolio of assets and liabilities and collect or make associated interest payments. On the asset side of their balance sheet, banks hold loans, $b$, and liquid assets, $m$. We will often refer to liquid assets as reserves, but the reader should keep a broader interpretation in mind. What is important is that liquid assets will correspond to monetary liabilities issued outside the banking system, in particular by the Fed. On the liability side, they issue demand deposits, $d$, discount window loans, $w$, and net interbank loans, $f$. If the bank has borrowed funds, $f$ is positive, and if it has lent reserves, $f$ is negative. Loans and deposits are denominated in nominal terms and pay respectively $1 + i^b$ and $1 + i^d$ in units of reserves. We denote by $P$ the price level (the price of consumption goods in terms of reserves).

During the lending stage, banks choose real dividends, $c_t$, and a new portfolio of loans, reserves, and deposits. Their portfolio is the triplet $\{\tilde{b}_{t+1}^j, \tilde{m}_{t+1}^j, \tilde{d}_{t+1}^j\}$: we use $\tilde{x}_{t+1}$ to denote a portfolio variable chosen in the lending stage and $x_{t+1}$ to denote the end-of-period portfolio variable in the balancing stage (i.e., the beginning-of-period portfolio variable for $t+1$). Likewise, we follow the convention that $i_t$ denotes the interest rate for an asset held between period $t-1$ and $t$. The bank’s nominal budget constraint in the lending stage is

$$P_t c^j_t + \tilde{b}_{t+1}^j + \tilde{m}_{t+1}^j - \tilde{d}_{t+1}^j = (1 + i^{bh}_t)b_t^j + (1 + i^{l}_{t}+i^{or}_{t})m_t^j - (1 + i^{d}_{t})d_t^j - (1 + i^{f}_{t})f_t^j - (1 + i^{dw}_{t})w_t^j - P_t T_t^j.$$ (2)

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7One can also interpret liquid assets as securities issued by the government. This makes little difference in the theory. When we turn to our quantitative application in Section 7, we will consolidate all liquid assets when mapping the model to the data.

8Absent aggregate shocks, having assets denominated in units of reserves or denominated in units of consumption are equivalent. This is because there is no difference between ex-ante and ex-post interest rates. However, in the presence of unanticipated shocks, the denomination of debt matters because as the initial price level changes, this affects the transitional dynamics through valuation effects.
The nominal rates $i^{	ext{ior}}_t$ and $i^{	ext{dw}}_t$ are, respectively, the interest rates on reserves and discount window loans. These rates are set by the Fed, and can vary over time. These rates must satisfy $i^{	ext{dw}}_t \geq i^{	ext{ior}}_t$, so that there is a positive spread between the discount window rate and the interest on reserves, as occurs in practice.\footnote{If $i^{	ext{dw}}_t < i^{	ext{ior}}_t$ for some $t$, banks would obtain an arbitrage by borrowing from the discount window and holding reserves at the Fed.} Outstanding interbank market loans earn a weighted average return $\bar{r}^j_t$. This rate is the average rate among multiple transactions in the interbank market, which will be described in the next section. Finally, $T^j_t$ represents bank-specific taxes, which we will assume to be proportional to bank equity.

Banks are subject to a capital requirement constraint,

$$\bar{d}^j_{t+1} \leq \kappa \left( \bar{b}^j_{t+1} + \bar{m}^j_{t+1} - \bar{d}^j_{t+1} \right),$$

which imposes an upper bound, $\kappa$, on the stock of deposits relative to the value of equity at the end of the lending stage.\footnote{Through the definition of net worth (or capital) $n \equiv b + m - d$, we can re-express (3) as $n \geq \frac{1}{1 + \kappa} (\bar{b} + \bar{m})$, which can be reinterpreted more directly as a requirement on bank capital. Standard motivations for capital requirements have to do with regulatory constraints or to address moral hazard or limited commitment problems.}

The problem of the bank in the lending stage is to choose the portfolio $\{\bar{b}^j_{t+1}, \bar{m}^j_{t+1}, \bar{d}^j_{t+1}\}$ and dividend payments $c_t$, subject to the budget constraint (2) and the capital requirement (3). Notice that for an individual bank, it is budget feasible to increase one unit of loans by issuing one unit of deposits, as long as the capital requirement does not bind. When it binds, a bank can only finance a fraction $\kappa/(1 + \kappa)$ of loans with deposits. The residual fraction needs to be financed by cutting dividends or by a reduction in reserves. The latter will expose the bank to higher liquidity risk in the balancing stage.

**Balancing stage.** Banks enter the balancing stage with the portfolio $\{\bar{b}^j_{t+1}, \bar{m}^j_{t+1}, \bar{d}^j_{t+1}\}$ chosen at the lending stage. At the beginning of the balancing stage, banks face a withdrawal shock $\omega^j_t$. The shock induces a random inflow/withdrawal of deposits $\omega^j_t \bar{d}^j_{t+1}$. Given this shock, the end-of-balancing-stage deposits, $d^j_{t+1}$, are

$$d^j_{t+1} = \bar{d}^j_{t+1} (1 + \omega^j_t).$$

When $\omega^j_t$ is positive, the bank receives deposits from other banks, and when $\omega$ is negative, the bank loses deposits to other banks. The $\omega$ shock has a cumulative distribution $\Phi(\cdot)$ common
to all banks.\footnote{We could allow $\Phi$ to be a function of the bank’s liquidity or leverage ratio. This would add complexity to the bank’s decisions but would not break aggregation. What is crucial for tractability is that $\Phi_t$ is not a function of the bank’s size.} The support of $\Phi$ is $[\omega_{\text{min}}, \infty)$ where $\omega_{\text{min}} \geq -1$ and $\int_{\omega_{\text{min}}}^{\infty} \omega_t \Phi_t(\omega) = 0, \forall t.\footnote{Setting $\omega_{\text{min}} > -1$ captures that not all liabilities can be withdrawn immediately from the bank—for example, because a fraction of the deposits are term deposits. In this case, $i^d_t$ would correspond to the weighted average of the demand and term deposits.} Because the withdrawal shock is idiosyncratic and has a zero mean, deposits are only reshuffled across banks and hence preserved within banks.\footnote{Adding a non-zero mean to the deposit withdrawal shock is straightforward in a stationary equilibrium. Everything else held constant, banks would issue more (fewer) deposits if the shock has a negative (positive) mean. The mean of the withdrawal shock could also be time varying introducing aggregate fluctuations in outside liquidity. In section 7.3, we explore an extension of the model that introduces a shock to outside liquidity in the balancing stage that operates similar to a withdrawal shock with time varying mean.}

Withdrawal shocks capture an essential element of the payment system: the circulation of deposits. Deposit circulation is the fundamental feature that enables banks to facilitate transactions between third parties. When a bank issues a loan, a borrower is credited with deposits at the issuing bank. As the borrower makes payments to third parties, those deposits may end up being transferred to other banks. In turn, the withdrawal of a deposit from one bank is an inflow of that deposit to another bank. Because the receptor bank absorbs the liabilities of the bank that issued the deposit, an asset needs to be transferred to settle the transaction. We assume that loans cannot be sold during the balancing stage—that is, loans are illiquid.\footnote{The lack of a liquid market for loans during the balancing stage can be explained by several market frictions discussed by the literature. For example, loans may be illiquid assets if banks specialize in particular customers, if they face agency frictions, if there is asymmetric information, or if these transfers take time. Reserves, instead, are special assets issued by the central bank and are fully liquid in the balancing stage.} Therefore, the bank that faces a withdrawal of deposits transfers reserves to the receptor bank, as occurs in practice. In this environment, the randomness in $\omega$ captures the complexity of the payment system. A deposit withdrawal captures a negative payment shock. An alternative interpretation of a deposit withdrawal is a negative confidence shock, leading depositors to switch accounts to a different bank.

We adopt the convention that the bank that issues deposits pays for the interest on those deposits. Thus, a transfer of one unit of deposits is settled with $(1 + i^d_{t+1}) / (1 + i^{ior}_{t+1})$ reserves, which guarantees that the bank that receives the deposit is compensated by the interest it will pay the depositor at $t+1$ and does not earn the interest on the reserves used in the settlement.

By the end of the balancing stage, banks must maintain a minimum of reserve balances. Specifically, banks must satisfy

$$m^j_{t+1} \geq \rho d^j_{t+1}.$$
with $\rho_t \in [0, 1]$. The case in which $\rho = 0$ corresponds to a system without reserve requirements, in which case banks are solely required to finish with a positive balance of reserves; banks cannot issue reserves. An alternative regulatory constraint links a minimum amount of liquid assets (i.e., reserves) to illiquid assets (i.e., loans) rather than to deposits. As we show in Section 7, this has very similar implications. What is key for the bank’s portfolio management problem is that there is a lower bound on reserve holdings.

Because of the withdrawal shock $\omega$, banks will have uncertain reserve balances after the shock. This shock will map into a reserve surplus (or deficit), which is the final reserve balance in excess of required reserves:

$$s^j_t = s(m^j_{t+1}, d^j_{t+1}, \omega^j_t) \equiv \left( m^j_{t+1} + \left( \frac{1 + i^d_{t+1}}{1 + i^{ior}_{t+1}} \right) \omega^j_t d^j_{t+1} \right) - \rho \tilde{d}^j_{t+1} \left( 1 + \omega^j_t \right).$$

(6)

The surplus, $s^j_t$, depends on the lending-stage choices of $\tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}$, and $\omega^j_t$. The first term in (6) is the reserve balance, given by the initial reserve position plus the reserves transferred from/to other banks. The second term is the required reserves given by (4): the product of the reserve requirement, $\rho_t$, and the outstanding amount of deposits after the withdrawal shock. It follows that if a bank faces a large withdrawal, the level of reserves falls below the reserve requirement. In fact, shocks $\omega < \omega^* \equiv [\rho_t - (m/d)] / [(1 + i^d_{t+1})/(1 + i^{ior}_{t+1}) - \rho_t]$ translate into a reserve deficit. Moreover, if a bank accumulates few reserves in the lending stage or, likewise, issues many deposits, it is more likely to incur a reserve deficit.

Banks with a positive $s^j_t$ will try to lend their excess reserves. Banks with a negative $s^j_t$ must obtain reserves to satisfy the reserve requirements (5). Banks in deficit obtain reserves by borrowing from surplus banks in the interbank market or by ultimately borrowing from the Fed’s discount window. Considering the interbank and discount window loans, reserves evolve from the balancing stage to the next lending stage as follows:

$$m^j_{t+1} = \tilde{m}^j_{t+1} + \left( \frac{1 + i^d_{t+1}}{1 + i^{ior}_{t+1}} \right) \omega^j_t \tilde{d}^j_{t+1} + f^j_{t+1} + w^j_{t+1}.$$  

(7)

This law of motion states that the end-of-period reserves, $m^j_{t+1}$, are the reserves left after the withdrawal shock plus reserves borrowed in the interbank market and from the discount window.\footnote{Notice that in order to satisfy (5), the sum of funds borrowed in the interbank market loans and at the discount window must satisfy $f^j_{t+1} + w^j_{t+1} \geq \rho_t \tilde{d}^j_{t+1} - \tilde{m}^j_{t+1} - \omega^j_t \tilde{d}^j_{t+1} \left( \frac{1 + i^d_{t+1}}{1 + i^{ior}_{t+1}} \right)$}. Next, we describe how the interbank market operates.
**Interbank market.** At the beginning of the balancing stage, the realization of idiosyncratic withdrawal shocks generates a distribution of banks with reserve surpluses and deficits, $s^j_t$. Banks with a shock $\omega > \omega^*$ have a reserve surplus and therefore want to lend reserves; banks with $\omega < \omega^*$ are in deficit and must borrow reserves. Because there are matching frictions in the interbank market, banks on either side of the market may be unable to lend/borrow all of their surplus/deficit. If a bank in deficit cannot obtain enough funds in the interbank market, it can borrow the difference from the discount window as the last resort. Similarly, if a bank in surplus is unable to lend all of its surplus, it can keep its balance at the central bank and earn the interest on reserves. Because in equilibrium the interbank market rate lies strictly between the interest on reserves and the discount window rates, banks seek to trade in the interbank market before trading with the Fed. Interbank market and discount window loans are repaid in the next lending stage.

The interbank market is an OTC market, as advocated by Ashcraft and Duffie (2007). We follow Afonso and Lagos (2015) but make departures to obtain a closed-form solution for the interbank rate. Because the complete description of the interbank market is intensive in notation, we only provide a brief description here; all details can be found in a companion paper (Bianchi and Bigio, 2017).

The interbank market operates in a sequential way. At the beginning of the balancing stage, each bank instructs a continuum of traders with a trading order. Each trader must close an infinitesimal position, as in Atkeson et al. (2015). There are $N$ trading rounds. Matches are formed at random according to a matching process in each round, and a number of trading positions close accordingly. The probability of a match at a given round is determined by an efficiency parameter $\lambda$ and a matching function, which in turn depends on the aggregate amount of surplus and deficit positions that remain open at that round. When traders meet, they bargain over the rate used and split the dynamic surplus according to Nash bargaining. The bargaining power for borrowers is denoted by $\eta$.

This OTC market generates a sequence of volumes of interbank market loans and terms of trade throughout the trading rounds within each balancing stage. In the following proposition, we present a formula for the pair of interbank market and discount window loans $\{f^j, w^j\}$ as a function of market tightness (i.e., the relative magnitudes of banks in deficit and sur-

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16Using the large family assumption of Atkeson et al. (2015) simplifies the analytical solution substantially. The assumption makes matching probabilities linear in the deficit or surplus position, regardless of the size of the surplus or deficit of a bank. This also avoids having match-specific terms in the bargaining problem. Without this assumption, the combinatorial problem of determining the distribution of matches becomes intractable.
plus). Let the aggregate amounts of surplus and deficit be denoted respectively by 

\[ S_t^+ \equiv \int_0^1 \max \{ s_t^i, 0 \} \, dj \] and 

\[ S_t^- \equiv \int_0^1 \min \{ s_t^i, 0 \} \, dj. \] Then, the market tightness is 

\[ \theta_t \equiv S_t^- / S_t^+. \]

**Proposition 1.** Given \( \theta_t \), the amount of interbank market loans and discount window loans for a bank of surplus \( s_t^j \) is

\[
(f_{t+1}^j, w_{t+1}^j) = \begin{cases} 
-s_t^j \cdot (\Psi^- (\theta_t), 1 - \Psi^- (\theta_t)) & \text{for } s_t^j < 0 \\
-s_t^j \cdot (\Psi^+ (\theta_t), 0) & \text{for } s_t^j \geq 0 
\end{cases}
\]

and the average interbank market rate is

\[
\bar{i}_t^j (\theta) = i_{\text{ior}} + (1 - \phi_t (\theta))(i_{\text{dw}} - i_{\text{ior}}),
\]

where the formulas for \{\( \Psi^+ (\theta_t), \Psi^- (\theta_t), \phi_t(\theta_t) \}\} are described in Appendix F.

According to Proposition 1, a bank short of reserves \((s_t^j < 0)\) is able to patch the fraction, \( \Psi^- \), of its deficit with interbank market loans and the rest, \( 1 - \Psi^- \), from the Fed. Similarly, a bank with surplus lends its fraction \( \Psi^+ \) in the interbank market and keeps the difference in an account at the Fed. These fractions depend endogenously on the abundance of reserves. If there are many banks in deficit (surplus), the probability that a bank in deficit finds a match is low (high). In addition, the federal funds rate is a weighted average of the \( i_{\text{ior}} \) and \( i_{\text{dw}} \) with endogenous weights as a function of the abundance of reserves. As in Afonso and Lagos (2015), the weight \( \phi_t (\theta_t) \) can be interpreted as an effective bargaining power for borrower banks in a one-time match. The federal funds rate is closer to \( i_{\text{dw}} \) if more banks are in deficit because this lowers the bargaining power of the borrowers. Conversely, the federal funds rate is closer to \( i_{\text{ior}} \) if more banks are in surplus.\(^{17}\) As shown in Appendix F, the functional forms for \( \phi (\theta_t) \) and \( \{\Psi^- (\theta_t), \Psi^+ (\theta_t)\} \) depend on the two deep parameters of the matching market: efficiency of matching \( \lambda \) and the bargaining power \( \eta \). In particular, a higher efficiency leads to higher fractions of matches \( \Psi^-, \Psi^+ \) in the interbank market. Likewise, \( \phi \) increases with the bargaining power of borrowers \( \eta \), which makes the federal funds rate closer to the lower band of the corridor. Naturally, the volume of interbank market loans (interbank market) must satisfy

\[
\Psi^+ (\theta_t) S_t^+ = -\Psi^- (\theta_t) S_t^-.
\]

\(^{17}\) The Federal funds rate is always within the corridor. Notice that one recent anomaly that has been observed in the interbank market is that the Federal funds rate has often traded below interest on reserves, suggesting a violation of arbitrage: a depositary institution in principle could borrow in the interbank market and lend to the Fed at a higher rate. A common explanation for this pattern is that the gains from the interest rate differential do not compensate for the costs from leverage, deposit insurance premiums, and so on (Williamson, 2015; Martin et al., 2013). It is possible to expand the model to incorporate these issues, but we abstract from them for reasons of simplicity.
To summarize the costs/benefits of being short/long in reserves, we introduce the liquidity yield function. This object is useful once we express the law of motion for equity in the recursive formulation.

**Definition 1.** The liquidity yield function for a bank with a surplus $s$ is

$$
\chi_t(s) = \begin{cases} 
\chi^+_t s & \text{if } s \geq 0 \\
\chi^-_t s & \text{if } s < 0
\end{cases},
$$

(10)

The term $\chi_t$ is precisely the marginal cost/benefit of having a reserve deficit/surplus. An important observation is that interbank market frictions will produce a positive wedge between the marginal cost of reserve deficits and the marginal benefit of surpluses. Implementing this wedge is endogenous and depends on the relative abundance of reserves and monetary policy. In frictionless environments, this wedge disappears because there is no kink in $\chi$. For example, in a Walrasian interbank market, when $\lambda \to \infty$, $\tilde{\ell}_t^f$ equals $i_{ior}$ if $S^+ > S^-$ or equals $i_{dw}$ if $S^+ < S^-$. At the other extreme in which there is no interbank market, the wedge becomes such that $\chi^- = i_{dw} - i_{ior}$ and $\chi^+ = 0$. This kink is critical for the liquidity management problem and the transmission of monetary policy, as we will argue in the policy analysis section. An exact formula for $\chi$ as a function of policy rates $\{\lambda, \eta\}$ and $\theta_t$ is presented in Appendix F. Figure A.1 summarizes the timing of events.

### 2.2 Central Bank Policies

The Fed issues reserves, sets corridor rates, and purchases private loans. We denote by $\tilde{M}_{Fed}$ and $M^{Fed}$, the amount of reserves issued by the Fed in the lending and balancing stage. On the

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18 The wedge is positive since $\chi^-_t$ is a weighted average between $(i_{dw} - i_{ior})$ and $(\tilde{\ell}_t^f - i_{ior})$, and $\chi^+_t$ is a weighted average between $(\tilde{\ell}_t^f - i_{ior})$ and $0$. Since $\tilde{\ell}_t^f \in [i_{ior}, i_{dw}]$, the wedge is weakly positive.

19 We note that there are potentially other frictions that can create a wedge between lending and borrowing rates in the interbank market related (e.g. default risk). What is key for our liquidity management problem is the existence of such a wedge, and hence our analysis also applies to these other contexts.
asset side, the Fed holds discount window loans, $W^{Fed}$, and private loans, $B^{Fed}$.$^{20}$

The Fed budget constraints in the lending and balancing stage are respectively

$$
\tilde{M}_{t+1}^{Fed} + B_{t+1}^{Fed} + (1 + i_{t}^{dw})W_{t+1}^{Fed} = (1 + i_{t}^{ior})M_{t}^{Fed} + B_{t+1}^{Fed} + P_{t}T_{t}^{Fed},
$$

$$
M_{t+1}^{Fed} = \tilde{M}_{t+1}^{Fed} + W_{t+1}^{Fed}
$$

Combining these constraints we arrive at

$$
(1 + i_{t}^{ior})M_{t}^{Fed} + B_{t+1}^{Fed} + W_{t+1}^{Fed} = M_{t+1}^{Fed} + (1 + i_{t}^{b})B_{t}^{Fed} + (1 + i_{t}^{dw})W_{t}^{Fed} + P_{t}T_{t}^{Fed}. \quad (11)
$$

The Fed generates operating profits/losses from its balance sheet position and from activity in the interbank market: it pays interest on reserves, $i_{t}^{ior}$, and collects revenues from the interest from discount window loans, $i_{t}^{dw}$, and from holdings of private loans, $i_{t}^{b}$. To fund new discount window loans, $W_{t+1}^{Fed}$, and purchases of loans, $B_{t+1}^{Fed}$, the Fed issues new reserves, $M_{t+1}^{Fed}$. We also endow the Fed with taxes/transfers, $T_{t}^{Fed}$. $^{21}$

Monetary policy can be specified in various ways. We focus on a monetary-dominant regime in which $T_{t}^{Fed}$ is a choice for the Fed to achieve a monetary policy objective. In a stationary equilibrium, the inflation rate will be constant and equal to $g$, the nominal growth rate of reserves set by the Fed. At a steady state with $g = 0$, we have that the tax at steady state is given by

$$
T_{ss}^{Fed} = \frac{i_{ss}^{ior}M_{ss}^{Fed} - i_{ss}^{b}B_{ss}^{Fed} - i_{ss}^{dw}W_{ss}^{Fed}}{P_{ss}},
$$

where we used subscript $ss$ to denote steady-state variables. That is, at steady state, taxes finance the interest payment of reserves net of the return on the stock of discount window and private loans. Away from a stationary equilibrium, we will consider two classes of policies. In the first class of policies, we keep the nominal balance sheet of the Fed growing at the long-run value. In this economy, movements in the real demand for reserves are accommodated by the

$^{20}$Incorporating Treasury bills (T-bills) and conventional open market operations into our model is relatively straightforward. If T-bills are illiquid in the balancing stage, T-bills and loans become perfect substitutes from a bank’s perspective and the model becomes equivalent to our baseline model—with an additional market clearing condition for T-bills. If T-bills are perfectly liquid but are not counted as part of the reserve balance, we can show that banks that have a deficit in reserves will first sell their holdings of T-bills in exchange for reserves to banks that have surplus and only then go to the federal funds market. An extension with T-bill would be useful, for example, to study the composition of liquid assets after the Fed started paying interest on excess reserves. Because our focus is more broadly on liquidity management rather than on the composition of liquid assets, we prefer to keep only one type of liquid assets in the model.

$^{21}$We completely abstract in the paper from frictions between the monetary authority and fiscal authority by considering a consolidated budget constraint.
price level, following a quantity theory relationship, as we will explain below. Taxes follow from the operating profits/losses of the Fed and the balance sheet policies, according to (11). In a second class of policies, we consider an inflation-target regime, in which the Fed alters policies, either the nominal supply of reserves or corridor rates, to keep the price level at a stationary path.

2.3 Loan Demand and Deposit Supply

To close the model, we need a loan demand and deposit supply schedule. We assume that there is a downward-sloping loan demand and an upward-sloping deposit supply. In both cases, we consider constant elasticity functions. The loan demand schedule is

\[ \frac{B_{t+1}^d}{P_t} = \Theta_b^t \left( \frac{1}{1 + i_{t+1}^b} \frac{P_{t+1}}{P_t} \right)^\epsilon, \epsilon > 0, \Theta_b^t > 0, \]  

(12)

where \( \epsilon \) is the semi-elasticity of credit demand with respect to the real return on loans. The intercept \( \Theta_b^t \) captures possible credit demand shifts.

The deposit supply schedule is

\[ \frac{D_{t+1}^S}{P_t} = \Theta_d^t \left( \frac{1}{1 + i_{t+1}^d} \frac{P_{t+1}}{P_t} \right)^-\varsigma, \varsigma > 0, \Theta_d^t > 0, \]  

(13)

where \( \varsigma \) is the semi-elasticity of the deposit supply with respect to the real return.

Given these schedules for loan demand and deposit supply, and using the bank’s optimal portfolios, we can solve for market clearing returns for loans and deposits. In Appendix C, we endogenize the demand schedule for loans and supply for deposits. These schedules are static and thus do not impose additional dynamic restrictions on the model.\(^{22}\) Throughout the paper, we will work directly with the exogenous schedules.

2.4 Competitive Equilibrium

The initial conditions for an equilibrium are a distribution of \( \{d_{0,j}^i, m_{0,j}^i, b_{0,j}^i, f_{0,j}^i, w_{0,j}^i\} \) over banks and a balance sheet for the Fed, \( \{B_{0,Fed}^0, M_{0,Fed}^0, W_{0,Fed}^0\} \). Taking as given returns and Fed policies, banks choose \( \{\tilde{d}_{t+1,j}^i, \tilde{b}_{t+1,j}^i, \tilde{m}_{t+1,j}^i, \tilde{c}_{t+1,j}^i, \tilde{f}_{t+1,j}^i, \tilde{w}_{t+1,j}^i\} \) contingent on their history of idiosyncratic shocks to maximize expected lifetime utility. Macroeconomic aggregates are deterministic since there is

\(^{22}\)In other words, once we solve the model with exogenous schedules, we infer the equilibrium values for employment, household consumption and wages from the conditions in the Appendix C.
no aggregate risk. We adopt the convention of denoting aggregate variables in uppercase letters, and they are defined as

\[ B_{t+1} \equiv \int b_{t+1}^j dj, \quad M_{t+1} \equiv \int m_{t+1}^j dj, \quad D_{t+1} \equiv \int d_{t+1}^j dj, \quad W_{t+1} \equiv \int w_{t+1}^j dj. \]

The competitive equilibrium is formally defined below.

**Definition 2.** Given a distribution \( \{d_0^j, m_0^j, b_0^j, f_0^j, w_0^j\} \) and a deterministic sequence of government policies \( \{B_{Fed}^t, M_{Fed}^t, W_{Fed}^t, T_{Fed}^t, T_{ior}^t, i_{Fed}^t\}_{t \geq 0} \), a **competitive equilibrium** is a deterministic sequence of interest rates \( \{i_b^t, i_d^t, i_f^t\}_{t \geq 0} \), a price level \( \{P_t\} \), a deterministic sequence of matching probabilities \( \{\Psi_t^+, \Psi_t^-\}_{t \geq 0} \), a deterministic path for aggregates \( \{D_t, B_t, M_t, W_t\}_{t \geq 0} \), and a stochastic sequence of bank policy variables \( \{b_t^j, \tilde{m}_t^j, \tilde{d}_t^j, c_t^j, f_t^j, w_t^j, m_{t+1}\}_{t \geq 0} \) such that

(i) Banks’ policies \( \{b_t^j, \tilde{m}_t^j, \tilde{d}_t^j, c_t^j\}_{t \geq 0} \) solve the banks’ optimization problem, \( \{f_t^j, w_t^j\}_{t \geq 0} \) are given by the formula in Proposition 1 and \( m_{t+1} \) satisfy 7

(ii) The central bank’s budget constraint (11) is satisfied and \( \int T_t dj = T_{Fed}^t \).

(iii) Aggregate loans are consistent with (12), and aggregate deposits are consistent with (13).

(iv) Markets clear \( \forall t \geq 0: \)

\[
\begin{align*}
\int b_{t+1}^j dj + B_{Fed}^t &= B_{t+1}^d \quad \text{(loan market clearing)} \\
\int d_{t+1}^j dj &= D_{t+1}^S \quad \text{(deposit market clearing)} \\
\int m_{t+1}^j dj &= M_{Fed}^t \quad \text{(reserve market clearing)} \\
\int f_{t+1}^j dj &= 0 \quad \text{(interbank market clearing)} \\
\int w_{t+1}^j dj &= W_{Fed}^t \quad \text{(discount window clearing)}
\end{align*}
\]

(v) The matching probabilities \( \{\Psi_t^+, \Psi_t^-\}_{t \geq 0} \) and the federal funds rate \( \tilde{i}_t^f \) are consistent with the surplus and deficit masses \( S_t^- \) and \( S_t^+ \), as given by Proposition 1.

A definition of equilibrium that considers the non-financial side of the economy, as in Definition 5) in Appendix C, would also include optimization for households and firms, market clearing in labor markets (C.3) and an additional price, the wage, as part of the equilibrium definition.
Before proceeding, it is useful to define a stationary equilibrium. That is, an equilibrium in which all nominal variables grow at a constant rate, and real variables remain constant.

**Definition 3** (Stationary equilibrium). A competitive equilibrium where $P_t$ and $\{D_{t+1}, B_{t+1}, M_{t+1}, W_{t+1}\}_{t \geq 0}$ grow at a constant rate $g$ is called a stationary equilibrium. A steady-state equilibrium is a stationary competitive equilibrium with $g = 0$.

### 3 Theoretical Analysis

We start the theoretical analysis with a description of the features that make the model tractable. In particular, we show aggregation and how the bank’s problem features portfolio separation. The portfolio problem, in turn, can be reduced to a choice of liquidity and leverage ratio for banks. Following this, we study a number of theoretical properties of the model including existence, uniqueness, and price level determination. We finish the section by analyzing how monetary policy affects the economy.

#### 3.1 Recursive Bank Problems

It is convenient to describe the banks’ optimization problem in recursive form. Denote by $V^l$ and $V^b$ the banks’ value functions during the lending and balancing stages, respectively. To keep track of aggregate states, which follow a deterministic path, we index policy functions, value functions, and prices by $t$. To ease notation, we omit superscript $j$ in the Bellman equations.

At the beginning of each lending stage, the individual states are $\{b, m, d, f, w\}$. Choices in the lending stage are consumption, $c$, and portfolio variables $\{\tilde{b}, \tilde{m}, \tilde{d}\}$. These portfolio variables, together with the idiosyncratic shock, $\omega$, become the initial states in the balancing stage. The continuation value is given by the expected value at the balancing stage $V^b_t$, under the probability distribution of $\omega$. The bank problem for the lending stage is:

**Problem 1** (Lending Stage Bank Problem).

\[
V^l_t (b, m, d, f, w) = \max_{\{c, b, m, d\} \geq 0} \{u(c) + \mathbb{E}[V^b_t(\tilde{b}, \tilde{m}, \tilde{d}, \omega)]\} 
\]

\[
P_t c + \tilde{b} + \tilde{m} - \tilde{d} = (1 + i^b_t)b - (1 + i^d_t)d + (1 + i^{i, or}_t)m - (1 + i^f_t)f - (1 + i^{dw}_t)w - P_t T_t, \quad \text{(Budget Constraint)}
\]

\[
\tilde{d} \leq \kappa (\tilde{b} + \tilde{m} - \tilde{d}), \quad \text{(Capital Requirement)}
\]
and the bank problem at the balancing stage is:

**Problem 2 (Balancing Stage Bank Problem).**

\[ V^b_t(b, \bar{m}, \bar{d}, \omega) = \beta V^l_t(b', m', d', f', w') \]  \hspace{1cm} (15)

\[
\begin{align*}
    b' & = \bar{b} \quad \text{(Evolution of Loans)} \\
    d' & = \bar{d} + \omega \bar{d} \quad \text{(Evolution of Deposits)} \\
    m' & = \bar{m} + \left( \frac{1 + i^d_{t+1}}{1 + i^{ior}_{t+1}} \right) \omega \bar{d} + f' + w' \quad \text{(Evolution of Reserves)} \\
    s & = \bar{m} + \left( \frac{1 + i^d_{t+1}}{1 + i^{ior}_{t+1}} \right) \omega \bar{d} - (1 + \omega) \rho \bar{d} \quad \text{(Reserve Balance)} \\
    m' & \geq \rho d' \quad \text{(Reserve Requirement)} \\
    (f', w') & = \begin{cases} 
    -s(\Psi_t^-, 1 - \Psi_t^-) & \text{for } s_t < 0 \\
    -s(\Psi_t^+, 0) & \text{for } s_t \geq 0 
    \end{cases} \quad \text{(Interbank Market Transactions)}
\end{align*}
\]

One can observe that there is no maximization in Problem 2 because the optimal choices of interbank loans and discount window loans have already been taken into account, following Proposition 1. In addition, the continuation value in the balancing stage is the value of the bank in the lending stage, which depends on the end-of-period portfolio variables.

### 3.2 Model Solution

Here we summarize the bank’s problem in one Bellman equation with a single individual state variable. The single state is real equity, which we denote by \( e \). The first step is to substitute \( V^b \) defined in (15) into (14). The second step is to express the right-hand side of the bank’s budget constraint as a function of \( e \). For this, we allow taxes on banks to be proportional to bank equity.\(^{23}\) Thus, define

\[ e_t \equiv \frac{(1 + i^{ior}_t) m_t + (1 + i^d_t) b_t - (1 + i^{d}_t) d_t - (1 + i^{f}_t) f_t - (1 + i^{dw}_t) w_t}{P_t}(1 - \tau_t), \]  \hspace{1cm} (16)

where \( \tau \) is the tax rate on bank equity. For future reference, we denote by \( E_t \) the aggregate level of real equity, \( E \equiv \int_j e^j_t dj \).

The third step is to construct a law of motion for the individual equity of a bank. For that,

\(^{23}\)A linear tax on bank equity is key to keep the bank problem homogeneous.
we update (16) one period forward and use the definition of the liquidity yield function (10) and
the laws of motion of deposits and reserves, (4) and (7), to express
\[ e_{t+1} = \frac{(1 + i_{t+1}^{\text{ior}}) \tilde{m}_{t+1} + (1 + i_{t+1}^{b}) \tilde{b}_{t+1} - (1 + i_{t+1}^{d}) \tilde{d}_{t+1} + \chi_{t+1}(s_t)(1 - \tau_{t+1})}{P_{t+1}}. \] (17)

This law of motion for equity is a function of portfolio choices in the lending stage, portfolio
returns, and the withdrawal shock. Define the gross real returns of all assets held between \( t \) and
\( t + 1 \) as:
\[ R_{t+1}^{w} \equiv \frac{1 + i_{t+1}^{daw}}{1 + \pi_{t+1}}, \quad R_{t+1}^{m} \equiv \frac{1 + i_{t+1}^{ior}}{1 + \pi_{t+1}}, \quad R_{t}^{b} \equiv \frac{1 + i_{t+1}^{b}}{1 + \pi_{t+1}}, \quad R_{t}^{d} \equiv \frac{1 + i_{t+1}^{d}}{1 + \pi_{t+1}}, \] (18)
and \( \bar{\chi}_{t+1}(\tilde{m}, \tilde{d}, \omega) \equiv \frac{\chi_{t+1}(s_t(\tilde{m}, \tilde{d}, \omega))}{1 + \pi_{t+1}} \)

where \( 1 + \pi_{t+1} \equiv P_{t+1}/P_{t} \) is the gross inflation rate between \( t \) and \( t + 1 \).

Proposition 2 makes use of the law of motion for future equity, the definition of real returns
and shows that we can summarize the value functions in (14) and (15) in a single Bellman
equation written in real terms.

**Proposition 2 (Single State Representation).** Problems 1 and 2 can be combined into a single
Bellman equation with equity as the only individual state variable
\[ V_{t}(e) = \max_{\{c, \tilde{m}, \tilde{b}, \tilde{d}\} \geq 0} u(c) + \beta \mathbb{E}_{\omega} [V_{t+1}(e')] , \] (19)
\[ e = \frac{\tilde{m}}{P_t} + \frac{\tilde{b}}{P_t} - \frac{\tilde{d}}{P_t} + c, \]
\[ e' = \left[ R_{t+1}^{m} \frac{\tilde{m}}{P_t} + R_{t+1}^{b} \frac{\tilde{b}}{P_t} - R_{t+1}^{d} \frac{\tilde{d}}{P_t} + \bar{\chi}_{t+1} \left( \frac{\tilde{m}}{P_t}, \frac{\tilde{d}}{P_t}, \omega \right) \right] (1 - \tau_{t+1}), \]
\[ \tilde{d} \leq \kappa \left( \tilde{b} + \tilde{m} - \tilde{d} \right). \]

This problem is a portfolio savings problem with a leverage constraint. The bank starts with
equity, \( e \), that can be allocated into dividends or investments. In turn, investments can be
allocated into loans, \( \tilde{b} \), and reserves, \( \tilde{m} \), and the bank can leverage its position by issuing deposits
\( \tilde{d} \). Next period equity \( e' \) depends on the return realizations of the portfolios. Leverage is limited
by the capital requirement. A non-standard feature is the presence of a non-linear return given
by the kink in \( \chi \). Next, we show that despite the kink, we can still aggregate banks into a
representative bank.
Proposition 3 (Homogeneity and Portfolio Separation). The bank Bellman equation (19) can be characterized as follows:

(ii) The certainty equivalent of the bank’s equity return solves the following problem:

\[
\Omega_t \equiv (1 - \tau_t) \max_{\{b, m, d\} \geq 0} \left\{ \mathbb{E}_\omega \left[ R_{t+1}^b \bar{b} + R_{t+1}^m \bar{m} - R_{t+1}^d \bar{d} + \bar{\chi}_{t+1}(\bar{m}, \bar{d}, \omega) \right] \right\}^{1 - \gamma},
\]

where

\[
\bar{b} + \bar{m} - \bar{d} = 1, \\
\bar{d} \leq \kappa(\bar{b} + \bar{m} - \bar{d}).
\]

The value function \( V_t(e) \) is

\[
V_t(e) = v_t(e)^{1 - \gamma} - 1/(1 - \beta)(1 - \gamma),
\]

where \( v_t \) is

\[
v_t = \frac{1}{1 - \gamma} \left[ 1 + \beta(1 - \gamma)\Omega_t^{1 - \gamma} v_{t+1}^{1/\gamma} \right]^{\gamma}.
\]

(iii) The optimal bank dividend–equity ratio \( \bar{c} \equiv c/e \) is

\[
\bar{c}_t = \frac{1}{1 + [\beta(1 - \gamma)\Omega_t^{1 - \gamma}]^{1/\gamma}}.
\]

(iv) Policy functions for \( \{\bar{b}, \bar{m}, \bar{d}\} \) from (19) can be recovered from the optimal portfolio weights \( \{\bar{b}, \bar{m}, \bar{d}\} \) obtained in (20) and consumption decisions \( \{\bar{c}\} \) obtained in (23):

\[
\bar{b}_{t+1}(e_t) = P_t \bar{b}_t (1 - \bar{c}_t) e_t, \\
\bar{m}_{t+1}(e_t) = P_t \bar{m}_t (1 - \bar{c}_t) e_t, \\
\bar{d}_{t+1}(e_t) = P_t \bar{d}_t (1 - \bar{c}_t) e_t.
\]

Key to this proposition is that the budget constraint is linear in \( e \) and the objective is homothetic. Alvarez and Stokey (1998) show that the standard properties of dynamic programming on bounded spaces apply to homogeneous dynamic programming problems such as the ones here. This implies that the solution here is unique and policy functions are linear. Although there is a kink in the liquidity yield function, the bank’s problem is homothetic and thus satisfies these properties.
Equation (20) represents the liquidity management problem. This problem consists of the choice of portfolio weights that maximize the risk-adjusted return on equity. The certainty equivalent portfolio value of the bank can be expressed as the sum of the returns of the individual assets plus the liquidity cost that depends on the surplus of reserves. The portfolio of the bank pins down a real demand for reserves in addition to a real demand for deposits and a real supply of loans. In the next section, we present properties of this portfolio problem.

An important implication of Proposition 3 item (iv) is that policy functions are linear in equity. As a result, two banks with different levels of equity are scaled versions of a bank with one unit of equity. This implies that the distribution of equity is not a state variable. Rather, the aggregate equity is a sufficient state for the banking side of the model.24 As we show in Appendix D, aggregate bank equity evolves according to

\[
E_{t+1} = (R^b_{t+1}b_t - P^d_{t+1}d_t)E_t(1 - c_t) + \frac{\hat{M}^Fed_{t+2} - (B^Fed_{t+2} - B^Fed_{t+1}(1 + i^b_{t+1}))}{P_{t+1}}.
\]  

This law of motion is obtained after combining the banks’ and the Fed’s budget constraints, and using market clearing for reserves and interbank market loans. This law of motion states that the equity of the bank tomorrow is given by the claims against firms (loans) net of the claims owed to households (deposits), and the net claims with the Fed, which are determined by next-period balance sheet policies. Notice that liquidity costs captured by \( \chi \) do not appear because payments within banks cancel on the aggregate while discount window payments cancel with the Fed transfers. Liquidity costs, however, distort the portfolio choices and affect the evolution of equity through this means.

### 3.3 Portfolio Management and Liquidity Premia

We analyze the bank’s portfolio problem. As outlined in Proposition 3 (item i), the portfolio management problem consists of choosing portfolio weights on loans \( b \), reserves \( m \), and deposits \( d \) to maximize the risk-adjusted return on equity:

\[
R^e \equiv R^b b + R^m m - R^d d + \bar{\chi}(m, d, \omega),
\]

24 Studying differences between large and small banks is beyond the scope of this paper. See Corbae and D’Erasmo (2014) and Corbae and D’Erasmo (2013) for a model where bank size plays an important role in shaping aggregates.
where the real returns were defined in (18). If we substitute out loans, $\bar{b}$, from the budget constraint and suppress time subscripts, we have that the bank’s objective is

$$
\max_{\bar{d} \in [0,\kappa], \bar{m} \in [0,1+\bar{d}]} \left( \mathbb{E}_{\omega} \left[ \left( \frac{R^b}{\text{return on loans}} - (R^b - R^m) \bar{m} + (R^b - R^d) \bar{d} + \bar{x}(\bar{m}, \bar{d}, \omega) \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}. \quad (25)
$$

If the bank invests only in loans and $\bar{d} = \bar{m} = 0$, it obtains a return on equity equal to the return on loans, $R^e = R^b$. If the return on loans exceeds the return on deposits, issuing deposits provides an external finance premium of $R^b - R^d$. However, it also exposes the bank to greater liquidity risk. To mitigate that risk, banks will hold reserves. The cost of that insurance is the liquidity premium (LP), the spread $R^b - R^m$. We explain how both premia relate to the solution portfolio problem and the conditions of the interbank market captured by $\bar{x}$.

**Liquidity decision and liquidity premium.** When portfolio weights on loans and reserves are strictly positive, we have an expression for the LP, the spread between loans and reserves:

$$
\frac{R^b - R^m}{\text{liquidity premium}} = \frac{\mathbb{E}_{\omega} \left[ (R^e)^{-\gamma} \cdot \frac{\partial \bar{x}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right]}{\mathbb{E}_{\omega} \left[ (R^e)^{-\gamma} \right]}.
$$

$$
= \mathbb{E}_{\omega} \left[ \frac{\partial \bar{x}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] + \text{COV}_{\omega} \left[ \frac{(R^e)^{-\gamma} \cdot \frac{\partial \bar{x}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}}}{\mathbb{E}_{\omega} \left[ (R^e)^{-\gamma} \right]} \right]. \quad (26)
$$

This condition follows the first-order condition derived in Appendix G.3. By convention, the expectation operator $\mathbb{E}_{\omega}$ in this condition excludes the zero-measure point where $\frac{\partial \bar{x}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}}$ is not defined, the point $\omega = \omega^*$. The liquidity premium, can be decomposed into two components.

The first component, interbank market return, represents the expected marginal benefit of holding an extra unit of reserves in the interbank market. Using the definitions for the liquidity yield and the reserve surplus, this term can be expressed as

$$
\mathbb{E}_{\omega} \left[ \frac{\partial \bar{x}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] = \frac{1}{1 + \pi_t} \left[ \chi^+(1 - F(\omega^*(\bar{m}, \bar{d}))) + \chi^- F(\omega^*(\bar{m}, \bar{d})) \right],
$$

As $\omega = \omega^*$ has zero measure, we obtain can take first-order condition by partitioning the bank’s objective into the regions ($\omega < \omega^*$) and ($\omega > \omega^*$). Within those regions, the bank’s objective is smooth.

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[25] Since $\omega = \omega^*$ has zero measure, we obtain can take first-order condition by partitioning the bank’s objective into the regions ($\omega < \omega^*$) and ($\omega > \omega^*$). Within those regions, the bank’s objective is smooth.
where $\omega^*(\bar{m}, \bar{d})$ is the threshold at which the reserve balance changes from surplus to deficit for a given portfolio $\{\bar{m}, \bar{d}\}$. Given a withdrawal shock $\omega$, an additional unit of reserves allows the bank to save $\chi^-$ if the bank is in deficit, which occurs with probability $F(\omega^*)$, and allows the bank to earn $\chi^+$ if the bank is in surplus, which occurs with probability $1 - F(\omega^*)$.

The second term in the LP is the liquidity-risk premium. Appendix G.3 establishes that this term is positive; this follows because $\partial \bar{X}/\partial \bar{m}$ is decreasing in $\omega$ and $R^e$ is increasing in $\omega$. Intuitively, reserves provide a higher return when the bank suffers adverse withdrawal shocks. This premium disappears if banks are risk neutral or if there is no kink in the liquidity yield function. In the latter case, $\partial \bar{X}_t/\partial \bar{m}$ is a constant, and hence the covariance vanishes.

The decomposition of the liquidity premium clarifies the role of the friction in the interbank market: In a Walrasian limit, $\chi^+ = \chi^-$, so the kink and therefore the risk premium disappear. Different from a Walrasian limit, with trading frictions in the interbank market, changes in the volatility of $\omega$—which capture instability in the interbank market—do have effects on the LP and, thus, on the volume of loans.26

**Leverage decision and credit spread.** When portfolio weights on loans and deposits are strictly positive, the spread between the rates on loans and deposits is:

$$R^b - R^d \geq - \frac{E_{\omega}\left[(R^e_{\omega})^{-\gamma} \cdot \frac{\partial \chi(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}}\right]}{E_{\omega}\left[(R^e)^{-\gamma}\right]} \quad \text{with equality if } \bar{d} < \kappa. \quad (27)$$

The term on the right-hand side of (27) is the liquidity component of the credit spread. It is is the risk-weighted marginal cost of resorting to the interbank market or the discount window to face withdrawals associated with one more unit of deposits. This term captures how raising deposits increase expected payments in the interbank market. Absent this liquidity costs, the credit spread $R^b - R^d$ can also be strictly positive if there is a binding capital requirement and is always positive. The second reason why there can be a positive credit spread is if capital requirements bind.

The next proposition, presents additional properties of both premia and the portfolio solutions

**Proposition 4 (Premiums and Portfolios).** The liquidity premium (26) and the liquidity component of the credit spread (27) are positive. The liquidity premium is decreasing $\bar{m}$ and the liq-

---

26See Bianchi and Bigio (2017) for a discussion of the Walrasian limit of the interbank market. In a Walrasian limit, the federal funds rate is either $i^{tor}$ when there is an aggregate surplus of reserves, or $i^{disc}$ when there is an aggregate deficit of reserves.
uidity component of the finance premium is increasing in \( \bar{d} \). In addition, the portfolio has positive weights in all assets \( \{ \bar{m}, \bar{d}, \bar{b} \} > 0 \) if \( R^w > R^b > R^m \) and \( R^b - R^d > \min \{ (R^b - R^m) L_{\text{min}}, \rho (R^w - R^m) \} \) where \( L_{\text{min}} \equiv \rho + \omega_{\text{min}} \left( \rho - \frac{R^d}{R^m} \right) \).

The proposition establishes the sign of the premiums defined in (26) and (25) and how they change with the portfolio. These results follow directly from the definition of the liquidity yield \( \chi \). The conditions for positive portfolio weights are natural: \( R^b > R^m \) guarantees that loans are not dominated by loans while \( R^w > R^b \) guarantees that reserves are not dominated by a policy of holding only loans and obtaining reserves at the discount window. The lower bound on \( R^b - R^d \) guarantees that it is not too expensive to borrow deposits by either fully insuring against all \( \omega \) shocks—setting \( \bar{m}/\bar{d} = L_{\text{min}} \)—or by issuing deposits and setting \( \bar{m} = 0 \).

### 3.4 Existence, Uniqueness and Price Level Determinacy

Assume for simplicity log utility and \( B^{Fed} = 0 \). The former simplifies the analysis because it delivers a constant consumption equity ratio of \( \beta \). In a stationary equilibrium, the law of motion for aggregate equity (24) translates into the following equation:

\[
1 = \left( R^b_{ss} + (R^b_{ss} - R^d_{ss}) \bar{d}_{ss} - (R^b_{ss} - 1) \bar{m}_{ss} \right) \cdot \frac{\beta}{\text{return on equity (after tax)}} \cdot \frac{\beta}{\text{fraction of equity invested}}.
\]

where \( \{ \bar{m}_{ss}, \bar{d}_{ss} \} \) are solutions to the bank’s problem given policy rates \( \{ R^w_{ss}, R^m_{ss} \} \) and steady state returns \( \{ R^b_{ss}, R^d_{ss} \} \) that clear the loan market and deposit markets. Intuitively, if the fraction of equity invested \( \beta \) times the after-tax return on equity is unity, equity will remain constant.

The next proposition shows a sufficient condition for existence of a stationary equilibrium. The proposition guarantees the existence of equilibria for any policy choice, with the only restriction that \( \{ R^w_{ss} \geq R^m_{ss} \} \). That is, banks cannot make profits by borrowing from the Fed and holding reserves.

**Proposition 5** (Existence and Uniqueness of Stationary Equilibrium). Consider a special case where there is no trade among banks \( (\lambda = 0) \), banks have log utility \( (\gamma = 1) \) and the Fed holds no loans \( (B^{Fed}_{ss} = 0) \). In addition, consider an infinitely elastic deposit supply such that for any \( D_{ss} \), \( R^d \) is constant and satisfies:

\[
1 + \kappa \left( 1 - R^d \right) < \frac{1}{\beta}.
\]
Then, there always exists a stationary equilibrium. If in addition \( R^m > R^d \), the equilibrium is unique.

The proof of Proposition 5 is presented in Appendix G.4. The proof consists of showing that, given values for \( \{ R^d_{ss}, R^m_{ss}, R^w_{ss} \} \), there exists a value for \( R^b_{ss} \) that produces bank portfolio weights that satisfy, (28) and are consistent with all market clearing conditions. The condition on the deposit rate, guarantees that the bank cannot get returns above \( 1/\beta \) simply by borrowing at rate \( R^d \) and holding all its assets at a rate of \( R^m \). When \( R^m > R^d \), we know, by Proposition 4, that capital requirements are binding and that \( \bar{b} \) is increasing in \( R^b \). Thus, it is immediate to verify that there is a unique solution to (28) and hence, the equilibrium is unique.

Once we have the return on loans in the stationary equilibrium, the steady-state value of equity can be determined as follows: Given the stationary returns, we can obtain all portfolio weights. In turn, given portfolio weights, the market clearing condition for loans determines steady-state equity through \( (1 - c_{ss})e_{ss}\bar{b}_{ss} = \Theta_t^b (R^b_{ss})^{-\epsilon} \). Notice that in a stationary equilibrium with \( R^b = R^m > R^d \) or \( R^b = R^m = R^d \), the portfolio is indeterminate at the individual level but the aggregate amount of loans and deposits (and therefore equity) are still determinate. Using that portfolios must also solve (28) for \( R^b_{ss} = R^m_{ss} \) and market clearing for loans and reserves, we have then three equations and three unknowns \( \{ e_{ss}, \bar{b}, \bar{d} \} \).

Further conditions that guarantee a unique monotonic path to a unique stationary equilibrium are presented in Appendix J. That special case restricts \( R^m_{ss} = R^w_{ss} \). However, we expect the general model to inherit similar properties as long as parameters are not far from the requirements in Appendix J. Indeed, we verify all these properties numerically for a wide range of parameter values.

**Price-Level Determination.** The equilibrium price level \( P_t \) is determined by a classic quantity-theory equation that equates the demand and supply of reserves:

\[
P_t \cdot (\bar{m}_t + \bar{w}_t) (1 - \bar{c}_t) E_t = M^{Fed}_{t+1} \tag{29}
\]

The real demand for reserves is given by the value of equity after paying dividends, \( (1 - \bar{c}_t) E_t \),
times the sum of the portfolio weight on reserves, $\bar{m}_t$ and discount window loans $\bar{w}$.\textsuperscript{27} These are the demands for reserves at the lending stage and for reserve loans from the discount window. These demands encode all future information on returns and policies.

Given $M_{t+1}^{Fed}$, the nominal supply of reserves set by the Fed, this quantity-theory equation determines the price level $P_t$—as long as $\bar{m}_t + \bar{w}_t > 0$.\textsuperscript{28} Given the price level, one can further determine $\bar{M}_{t+1}^{Fed} = \bar{m}_t P_t$, and $W_{t+1} = \bar{w}_t P_t$. Notice also that if the Fed chooses a policy sequence $\{R^w, R^m\}$ that produces $\bar{m} = 0$, we have that $\bar{M}_{t+1}^{Fed} = 0$ and the price level is determined by $P_t = W_{t+1}/\bar{w}_t$. Finally, it is worth pointing out that the price level remains determined even under equilibria in which the portfolio is indeterminate at the individual level. As discussed above, all aggregates are still pinned down from market clearing conditions, and hence, this implies that there is a unique price level consistent with (29).\textsuperscript{29}

The price level determination is connected with classic cash-in-advance and money-search frameworks. Those models feature a demand for real balances due to transaction motives. Here instead, the demand for money emerges from a bank liquidity management problem in an environment in which reserves serve as a settlement instrument for the interbank market.

### 3.5 Monetary Policy Analysis

This section analyzes the transmission of monetary policy and highlights the central role played by the liquidity premium. We show how Fed policies alter the liquidity premium and carry real effects, even in the long run. In addition, we provide conditions under which certain Fed policies reproduce classic neutrality results.

We will employ the following definition.

**Definition 4 (Satiation).** Banks are satiated with reserves if $m > 0$ and the liquidity premium $R^b - R^m$ is zero.

Satiation describes a situation in which banks do not face strictly positive liquidity costs in equilibrium, and hence there is no return differential between loans and reserves.

\textsuperscript{27}The portfolio weight $\bar{w}$ is technically an average of the portfolio weights for banks. Recall that Proposition 3 establishes that portfolio weights $\{\bar{b}, \bar{m}, \bar{d}\}$ are the same for all banks. The ratio of discount loans to equity depends on the idiosyncratic withdrawal of each bank as given by $\Psi - s^j$. The explicit formula for $\bar{w}$ is:

$$\bar{w} = (1 - \Psi^{-} (\theta (\bar{m}, \bar{d}))) \left( \bar{m} - \rho \bar{d} + \left( \frac{R^d}{R^d - \rho} \right) \mathbb{E}_\omega [\omega | \omega < \omega^* (\bar{m}, \bar{d})] \bar{d} \right).$$

\textsuperscript{28}Notice that for any strictly positive level of deposits, there is always a strictly positive demand for reserves, either at the lending stage or at the balancing stage.

\textsuperscript{29}For related results on price level determination with interest on reserves see Ennis (2014).
Proposition 6 (Conditions for Satiation). Banks are satiated with reserves at \( t \) if either (i) \( i^d_t = i^i_t \) or (ii) \( \bar{m}_t > \rho \omega_{\min} \tilde{l}_t \).

Under condition (i), the Fed lends reserves at the discount window at the same rate that it remunerates reserves, \( i^i_t \). This means that the cost of the reserve deficit is zero, and in that case there is no liquidity risk and hence no premium. Moreover, the volume of the interbank market is indeterminate: banks are indifferent between trading with the Fed or trading with each other. Remunerating reserves at the same rate of deposits, rather than the return on loans, is not sufficient to ensure satiation. In the presence of a binding capital requirement, the return on loans would exceed the return on deposits, and hence, a wedge would remain between the return on loans and reserves, as implied by (26) and (27). Under condition (ii), banks hold sufficient reserves to be in surplus for any withdrawal shock. In this case, the interbank market is not active.

Before considering policies that deliver real effects via changes in the liquidity premium, we establish a classic neutrality result.

Proposition 7 (Conditions for Policy Neutrality). Consider an equilibrium sequence induced by a Fed balance sheet policy \( \{M_{t+1}, W_{t+1}, B_{t+1}\} \) that grows at rate \( g_t \) and a sequence of nominal policy rates \( \{i^d_t, i^i_t\} \). Then,

i) Consider the case where that policy sequence induces a stationary equilibrium. Consider an alternative sequences where the Fed balance sheet is scaled by a multiple \( k > 0 \). Then, that alternative policy increases a stationary equilibrium where the price level is scaled by \( k \) and all real variables are the same as in the original stationary equilibrium.

ii) An increase in \( g_t \) for some \( t \), has no real effects if and only if the Fed alters its nominal policy rates to keep \( \left\{ \frac{1+i^i_t}{1+g_t}, \frac{1+i^d_t}{1+g_t} \right\} \) constant.

Part i) establishes long-run money neutrality. A qualification for this neutrality result is that it applies only to the stationary equilibrium only. The reason that neutrality does not hold in the short run is a valuation effect. Loans and deposits are denominated in nominal terms, and thus, the change in policy affects the initial price level. Through this, it affects real equity. In the long run, however, a change in the nominal balance sheet of the Fed by a multiple \( k \) leads to a scaled stationary equilibrium. Part ii) is concerned with the issue of super-neutrality along any equilibrium sequence. If the Fed adjusts its nominal policy rates by inflation, so as to keep those rates constant in real terms, variations in the growth rate of its nominal balance sheet only translate into changes in inflation, again without any real effects.
The next proposition establishes the effects of open market operation that exchange reserves for loans.

**Proposition 8** (Real Effects of Open Market Operations). Consider a competitive equilibrium induced by balance sheet policies \(\{M_{t+1}, W_{t+1}, B_{t+1}\}\) and policy rates \(\{i^d_t, i^o_t\}\). Consider also a time-zero operation of size \(\Delta B^F_0\) reversed the following period, that is, a policy sequence \(\{\tilde{M}_{t+1}, \tilde{W}_{t+1}, \tilde{B}_{t+1}\}\), such that

1. \(\tilde{B}^F_1 = B^F_1 + \Delta B^F_1\), \(\tilde{M}^F_1 = M^F_1 + \Delta M^F_1\), and \(\Delta M^F_1 = \Delta B^F_1 \geq 0\).
2. \(\{M_{t+1}, W_{t+1}, B_{t+1}\} = \{\tilde{M}_{t+1}, \tilde{W}_{t+1}, \tilde{B}_{t+1}\}\) for all \(t > 1\).

The operation is neutral if and only if banks are satiated with reserves at \(t = 0\) in the equilibrium induced by \(\{M_{t+1}, W_{t+1}, B_{t+1}\}\).

When banks are satiated with reserves, open market operations are irrelevant. For every unit of loans the Fed purchases, the banks reduce their holdings of loans by one unit. Away from satiation, however, open market operations alter the liquidity premium and induce a change in the total amount of loans. We examine this below.

**Lower bound on Loans Rate.** One important question is what are the loan rates that the Fed can implement. The next proposition establishes a lower bound on real loans rate in a stationary equilibrium.

**Proposition 9** (Lower bound on loan rates). Let \(\gamma = 1\) and \(B^F_0 = 0\). Fed policies that can generate any loan rate in the stationary equilibria such that:

\[
R^b_{ss} \geq \max \left\{ R^m_{ss}, \min \left\{ \frac{1}{\beta}, \frac{1}{1 + \kappa} \right\} \right\},
\]

where \(\bar{R}^d\) is either of the following:

1. When \(\varsigma \to \infty\) and the deposit rate meets the condition in Proposition 5, \(\bar{R}^d = R^d\).
2. If \(\varsigma\) is finite, then \(\bar{R}^d\) is the solution to:

\[
(1 + \kappa) \left( \frac{\Theta^d}{\Theta^b} \frac{\kappa}{1 + \kappa} \right)^{1/\epsilon} \left( \bar{R}^d \right)^{-\frac{\varsigma}{\epsilon}} = \frac{1}{\beta} + \kappa \bar{R}^d.
\]

Furthermore, the lower bound is attainable by setting \(R^m_{ss} = R^m_{ss}\). 

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The Proposition establishes a lower bound for the return on loans. In other words, once the loan rate reaches this bound, the Fed is unable to achieve further increases in bank lending. The lower bound is the maximum of two bounds. The first bound is straightforward. If $R_{ss}^b < R_{ss}^m$, loans would be dominated by reserves, and $\bar{b} = 0$, which would produce contradiction. This bound applies to stationary equilibrium and away from it. The second bound is the minimum between two numbers, $\min \left\{ \frac{1}{\beta}, \frac{\frac{1}{\bar{m}} + \kappa R_d}{1 + \kappa} \right\}$. The min of these two numbers is the lowest return on loans that is consistent with the stationarity condition (28) and the feasibility of the portfolio, $\bar{d} \leq \kappa$ and $\bar{b} \leq 1 + \bar{d}$. That lower bound is reached when $\bar{m} = 0$. The min operator appears because if the return $\bar{R}^d \geq 1/\beta$, the lowest feasible return on loans consistent with stationarity is $1/\beta$—with $\bar{d} = 0$. If $\bar{R}^d \leq 1/\beta$, the return on loans can be even smaller than $1/\beta$. It is the return on loans consistent with the stationarity condition when the portfolio is $\{\bar{b}, \bar{m}, \bar{d}\} = \{1 + \kappa, 0, \kappa\}$. Overall then, the return on loans is bounded by the policy rate $R^m$ and the lowest return on loans in absence of real balances.

The proposition further establishes that the lower bound is attained when monetary policy that eliminates the spread in the corridor rates. This proposition also implies that there is no upper bound on $R^b$, because by offering a high $R^m$ the Fed can always raise $R^b$. There are no fiscal costs because the Fed taxes banks to finance the interest on reserves, and although $R^b$ is arbitrarily high, the return on the bank’s portfolio remains at $1/\beta$ because of the tax. The converse is not true, the Fed cannot induce a loans rate below $\min \left\{ \frac{1}{\beta}, \frac{\frac{1}{\bar{m}} + \kappa R_d}{1 + \kappa} \right\}$. If for example, the Fed sets $R^m = R^w < \min \left\{ \frac{1}{\beta}, \frac{\frac{1}{\bar{m}} + \kappa R_d}{1 + \kappa} \right\}$, the equilibrium adjusts to a situation where $R^b = \min \left\{ \frac{1}{\beta}, \frac{\frac{1}{\bar{m}} + \kappa R_d}{1 + \kappa} \right\} > R^m$ but $\bar{m} = 0$ and the non-negativity constraint on reserves binds. Since the system in a stationary equilibrium is highly non-linear, we cannot go analytically beyond a description of this lower bound, but we can characterize stationary equilibria numerically.

**Characterizing stationary equilibria as a function of policies.** Next, we describe how the stationary value of the loan rate $R^b$ varies with policy instruments. We first compute the set of stationary equilibria for combinations of $i^{\text{dw}} \geq i^{\text{ior}}$. The vertical axis of Figure 1 (panel a) shows loans rate as function of the policy rates. For all these plots we use the parameter values described in the calibration section with the exception of the policy rates that are varied

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30 We do not model why the Fed might choose to induce a positive liquidity premium. We simply take as given that this is a standard policy instrument to affect credit creation. This can be motivated by a fire sale externality that arises because of a marked-to-market capital requirement constraint (see, e.g., Stein (2012) and Bianchi and Mendoza (2018)). Another natural motivation for a policy that induces a positive premium is aggregate demand management for macroeconomic and price stability.
to trace the effects and $\kappa$ that we set it so that the capital requirement marginally binds given all baseline parameter values. We do so to illustrate an important interaction that will emerge between capital requirements and policies. A key result is that the return on loans is non-monotonic in $i^{ior}$ holding $i^{dw}$ fixed, as observed more clearly in panel (b) of Figure 1. For low $i^{ior}$, increases in $i^{ior}$ lead to a decline in the loan rate while for high $i^{ior}$, increases in $i^{ior}$ lead to an increase in the loan rate. This pattern emerges because of the capital requirements—the red dashed line is placed at the threshold at which the capital requirement binds. When $i^{ior}$ is low, raising deposits is less attractive for banks. To insure against negative withdrawals, banks must hold liquid assets. But because the real return of these assets is very low, raising deposits is very costly as banks must hold more reserves. As a result, banks issue few deposits, and the capital requirement does not bind. In this context, an increase in $i^{ior}$ reduces the costs of holding reserves, and increases the incentives to issue deposits and provide more loans. In equilibrium, this imply that the real loans rate must fall. At some point, once $i^{ior}$ is high enough, the incentives to raise deposits is high enough so that the capital requirement becomes binding. At that point, once the funding capacity becomes limited, further increases in $i^{ior}$ lead banks to require a higher rate on loans in order to lend.

These results are relevant for the policy discussion on whether payment on reserves have been holding back lending. Our results suggest that the answer is a complex one, which depends on whether the capital requirements bind or not.

Panels (c) and (d) of Figure 1 show how loan rates vary with the discount window rate and with the real stock of Fed loans. As expected, the loan rate is increasing in the former and decreasing in the latter. An increase in the discount window rate raises the liquidity premium, and hence for a given $R^{m}$, this raises the loan rate. An increase in the amount of loans provided by the Fed increases the overall credit supply, and hence requires a decline in the loan rate to clear the credit market. Effectively, this intervention partially crowds out the banking system.

It is worth to highlight that open market operations and policy rates are two separate instruments with which the Fed can affect bank lending. Keeping policy rates constant, a permanent increase in the real holdings of loans by the Fed leads to a reduction in the lending rate. Likewise, keeping Fed loans constant, changes in policy rates also have effects on lending rates.

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31 See footnote 3 for references on this debate.

32 Another relevant distinction here with respect to the canonical New-Keynesian model is that the long-run real rate in NK models is pinned down by the inverse of the rate of time preference to a first order. Here, however, Fed policy alters long-run real returns.
Figure 1: Stationary equilibria as a function of policies

Note: Real rates $r_b, r_m, r_w$ are in net terms. The red dashed line denotes the threshold at which the capital requirement becomes binding.
4 Quantitative Analysis

The previous section articulated a tractable model of banks’ liquidity management and showed how monetary policy affects the banking system by altering the liquidity premium. In this section, we show how to calibrate the model in a stationary equilibrium during “normal” times.

4.1 Calibration

We calibrate the model so that its steady state fits regularities of the pre-crisis US financial system. We take 2006-2007, the last two years before the recent US financial crisis, as the reference period.

Model period. We define the time period to be a month. In the United States, the Federal funds market operates daily, and reserve requirements are computed by averaging end-of-day balances over a two-week window. Bank portfolio decisions and loan sales, however, are likely to take longer than two weeks. Capturing these institutional details would require a more complex structure with multiple balancing stages and more complicated reserve requirements. We view a monthly model as a reasonable middle ground between the daily nature of the Federal funds market and the lower frequency of bank decisions. The choice of a monthly model is also practical once we turn to the application in Section 6: data are available only monthly, and having a longer time period would complicate the numerical implementation. As long as interbank market positions are sufficiently persistent within lending stages, a higher frequency of interbank trades will not lead to quantitatively significant differences in our model.

Distribution of withdrawal shocks. For the distribution of the withdrawal shocks to deposits, $\Phi_t$, we assume $1 + \omega$ follows a log-normal distribution with standard deviation $\sigma$ and where the mean is chosen so that given $\sigma$, $\omega$ has a zero mean. The calibration of $\sigma$ is explained below. A log-normal distribution is convenient because it delivers a distribution of excess reserves that fits the empirical counterpart.

Parameter values. The values of all parameters are listed in Table 1. In summary, we need to assign values to 15 parameters that we divide into two subsets $\{g, i_{ior}, i_{iow}, B^{Fed}, \kappa, \rho, \beta, \gamma, \Theta^b, \Theta^d, \zeta, \eta\}$ and $\{\lambda, \sigma, \epsilon\}$. The parameters in the first subset are chosen independently of model simulations. We set $g$ the nominal growth of reserves to obtain a steady-state inflation rate of 2 percent. We set the interest on reserves $i_{ior}$ to 0 percent because the Fed did not pay interest on reserves...
prior to 2008. Accordingly, we also set $B_{Fed}^t = 0$, in line with the close to nil holdings of private securities by the Fed before the crisis. The discount window rate, $d_{dw}$, is set to 6 percent in annualized terms, which was the nominal primary credit discount rate during 2006. The bargaining parameter takes the value $\eta = 0.5$ as the baseline value. An equal bargaining power to banks in surplus and deficit leaves the federal funds rate in the middle of the corridor when the market tightness is close to one. We set $\rho$, the reserve requirement, to 0.10 which is the reserve requirement that applies to roughly the entire banking system. We set $k$, the capital requirement, to 10 to have a capital adequacy ratio of 9 percent, in line with Basel regulation. Two parameters, $\{\gamma, \beta\}$, govern banks’ preferences. We choose a value of $\gamma = 1$ for the value of the intertemporal elasticity of substitution. A unit value has the advantage of simplifying the computations by making dividend payments only a function of the level of equity, as substitution and income effects of changes in the return on equity cancel out. As we discussed earlier, when $\gamma = 1$, the discount factor $\beta$ determines the stationary after-tax return on equity. A condition for stationarity is given by $\beta R^e_{ss} = 1$, which corresponds to condition (28). As a source for the return on equity, we use the estimation in Atkeson et al. (2018) (Table 2) of the return on equity for banks with the highest rating of bank assets, which is 8 percent. Accordingly, we set $\beta$ so that the sum of the annual return on equity is 8 percent: $\beta = (1/1.08)^{1/12} = 0.993$.

The calibration of the loan demand and deposit supply schedules requires four parameter values: two scale and two elasticity parameters. The scale of the deposit supply, $\Theta^d$, is set to obtain an annualized real deposit interest rate of 1 percent, which is in the range of the interest rate on deposits, according to balance sheet data (see Drechsler, Savov, and Schnabl, 2017). The scale of the loan demand, $\Theta^b$, is normalized so that the level of equity is 1 at steady state. Neither elasticity plays a role at steady state because the rates are constant by construction, but these elasticities do matter in the transitions. Because the capital requirement is binding at steady state, the deposit demand is insensitive to the interest rate on deposits, and hence, the elasticity of the deposit supply does not affect how banks’ portfolios react on impact after a relatively small shock. Changes in the elasticity of the deposit supply still affect the speed of convergence of

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33 For banks with net transactions over USD 48.3 million as of 2006, the reserve requirement is 10 percent (see Federal Reserve Bulletin, Table 1.15).

34 Basel regulation features various capital requirements that banks simultaneously need to satisfy, some of which feature different risk weights when computing the value of banks’ assets. We see 9 percent as appropriate given these different requirements. Notice that implicitly we are applying the same risk weights to loans and reserves, which is sensible in our model because both reserves and loans are risk-free. Below, we discuss an extension of the model with risky loans.

35 In simulations in which the capital requirement constraint does not bind, there are effects on banks’ portfolios on impact, as variations in the interest rate on deposits affect banks’ willingness to leverage up.
bank equity to its steady-state level. If equity is below steady state, a relatively lower elasticity leads to a bigger contraction in the interest rate of deposits, which increases banks’ profits and speeds up convergence. The elasticity of loan demand also affects the speed of convergence in a similar way, but in addition is more important for the response in the volume of loans after a shock. We set the elasticity of deposit supply equal to the loan demand elasticity and follow the calibration described below.\textsuperscript{36}

The second set of parameters, $\{\lambda, \sigma, \epsilon\}$, is chosen to match empirical features of the federal funds market. The three moments we target are: (i) the ratio of discount window loans to reserves; (ii) the distribution of excess reserves at the beginning of a trading session; and (iii) the response of bank credit to an increase in the federal funds rate. While this is a joint calibration exercise, each moment is particularly sensitive to a certain parameter, as we explain below (see also Figures 2 and 3).

The parameter $\lambda$ governs the efficiency of the interbank market and hence is set to match the fraction of discount window loans given by the Fed as a fraction of the total amount of reserves. In 2006, this ratio was equal to 2 percent, which is obtained by setting $\lambda = 2.1$.

Next, we describe the choice of the volatility parameter $\sigma$. Afonso and Lagos (2014) describe how the distribution of excess reserve balances evolves throughout a typical federal funds trading session. Because their data are daily, we implicitly assume that the distribution within a business day is the same as the distribution within a month, the model frequency. The volatility of the withdrawal shock is set to minimize the discrepancy between the distribution of excess reserves at the beginning of each trading session in the model vis-à-vis the empirical counterpart. To achieve this, we follow a two-step iterative procedure. First, given $\sigma$, we set the value of $\lambda$ that delivers the targets for discount window loans. Second, we compute the mean squared difference between the distribution of excess reserves in the model and the data, and pick the value of $\sigma$ that minimizes this discrepancy. The resulting value is $\sigma = 5$ percent. The resulting distribution of the withdrawal shock and the equilibrium excess reserves vis-à-vis the empirical distribution is presented in panel (b) of Figure 2.\textsuperscript{37}

Finally, the elasticity of loan demand is set to be consistent with vector autoregression ev-

\textsuperscript{36}We have also experimented with different elasticities of deposit supply in the transitional dynamics, but this did not have significant effects on the results.

\textsuperscript{37}In a previous version of the paper, we used a different approach to calibrate the volatility of withdrawals using the cross-sectional dispersion of deposits. That calibration deliver a volatility of withdrawal shocks that was relatively close to the one in the current calibration.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital requirement</td>
<td>$\kappa = 10$</td>
<td>Regulatory parameter</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.993$</td>
<td>Return on equity = 8%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma = 1$</td>
<td>Constant dividend-equity ratio</td>
</tr>
<tr>
<td>Reserve requirement</td>
<td>$\rho = 0.1$</td>
<td>Regulatory parameter</td>
</tr>
<tr>
<td>Deposit supply intercept</td>
<td>$\Theta_d = 9.6$</td>
<td>Annual deposit rate = 1%</td>
</tr>
<tr>
<td>Loan demand intercept</td>
<td>$\Theta_b = 10.4$</td>
<td>Unit steady-state equity</td>
</tr>
<tr>
<td>Discount window rate (annual)</td>
<td>$i_{\text{dw}} = 6%$</td>
<td>2006 value</td>
</tr>
<tr>
<td>Interest on reserves (annual)</td>
<td>$i_{\text{i.or}} = 0%$</td>
<td>2006 value</td>
</tr>
<tr>
<td>Fed holdings of loans</td>
<td>$B^{\text{Fed}} = 0$</td>
<td>Baseline value</td>
</tr>
<tr>
<td>Bargaining parameter</td>
<td>$\eta = 0.5$</td>
<td>Baseline value</td>
</tr>
<tr>
<td>Inflation</td>
<td>$g = 0.085%$</td>
<td>Annual inflation target = 2%</td>
</tr>
<tr>
<td>Matching friction</td>
<td>$\lambda = 2.1$</td>
<td>DW to reserves W/M = 2%</td>
</tr>
<tr>
<td>Volatility</td>
<td>$\sigma = 0.05$</td>
<td>Reserve-balance distribution</td>
</tr>
<tr>
<td>Loan demand deposit supply elasticities</td>
<td>$\zeta = -\epsilon = 25$</td>
<td>Bank credit response to policy rate</td>
</tr>
</tbody>
</table>

idence on the response of bank credit to a monetary policy shock.\(^{38}\) In the classic Bernanke and Blinder (1988) study, a 1 percentage point increase in the nominal policy rates produces a decline in bank credit of 2 percent within a one-year horizon. We replicate this response in our model by setting $\zeta = 25$. Given the monthly frequency, this implies that a $1/12$ percentage point increase in the rate of loans reduces the stock of loan demand by $1/12 \times 25 \sim 2\%$.

4.2 Sensitivity and Model Fit

We first show that the model fits the targeted moments in steady state. Panel (a) of Figure 2 shows the calibrated distribution of withdrawal shocks. Panel (b) shows the fit to the distribution of excess reserves in the data in Afonso and Lagos (2014)—recall that volatility is calibrated to minimize the distance between the model and the empirical distributions. Panel (b) also shows how increasing or reducing the volatility of the shock by 50 percent leads to a distribution of excess reserves that departs further from the data. Panel (c) shows the mapping between the severity of the friction in the interbank market, parameterized by $\lambda$, and the amount of discount window loans as a fraction of total reserves—our first target. As $\lambda$ rises, the interbank market becomes more efficient and there is less use of the discount window. At $\lambda = 2.1$, the model

\(^{38}\)An alternative to our macro approach to discipline the elasticity of loan demand would be to use loan-level demand elasticities. The challenge is to disentangle supply from demand effects and to infer the aggregate responses.
matches the empirical target of 2 percent.

Note: For panel (a), the distribution of excess reserves in the model and the data has been normalized by the mean levels. Data source: Afonso and Lagos (2014).
Finally, we show how monetary policy shocks and withdrawal volatility shocks affect the economy as a function of the magnitude of the friction of the interbank market, as captured by $\lambda$. We compute transitional dynamics away from steady state, in response to a one-period shock, and report the impact effects on the quantity of loans and the liquidity premium.\(^{39}\) Panels (a) and (b) of Figure 3 show how the effects on the liquidity premium vary for a range of $\lambda$ for the two shocks. This figure shows how the response to these shocks is amplified as the value of $\lambda$ is reduced (i.e., the friction in the interbank market becomes stronger).

## 5 Dynamic Responses

This section studies the economy’s response to shocks that are associated with the hypotheses described in the introduction. In addition, the shock to equity losses illustrates the convergence properties of the model. The goal of these exercises is to show how the model works and to provide a basis for the identification that will be examined in more detail when we turn to infer several shocks from the data in Section 6. The results in this section are meant to qualitatively illustrate the responses of the model. Quantitative results will be discussed in Section 6.

### General details of the experiments.

All the shocks are unanticipated and arrive at $t = 0$. Their paths are deterministic thereafter (i.e., we consider transitional dynamics). All shocks, except the equity loss that occurs only at $t = 0$, follow an auto-regressive process $\varepsilon_t = \varrho \varepsilon_{t-1}, \forall t \geq 1$, where $\varepsilon_0$ is the log deviation of a shock from the steady state. We set $\varrho = 0.8$ so that the half-life of all shocks is three years. The size of the shocks we consider is in the range of those we infer from the data in Section 6. The initial value of equity is taken to be the steady state value (for all shocks other than the shock to equity losses). Therefore, the economy eventually reverts back after the shocks expire to the initial value, and the transitional dynamics are entirely due to the shocks.

To compute the transitional dynamics, we also need to specify the Fed policy away from the stationary equilibrium. We assume that the Fed keeps the growth rate for reserves, $M^{Fed}$, and $i^{lor}$ at their steady-state values. In addition, the Fed adjusts the nominal discount window rate $i^{du}$ to keep a fixed real discount window equal to the steady state value, an assumption that is useful for avoiding variations in the real cost of resorting to the discount window as a result

\(^{39}\)In these exercises, we assume that the Fed’s policy is to keep inflation at its target value by varying the nominal quantity of reserves. This implies that the real return on reserves stays constant and that variations in the liquidity premium are accounted for exclusively by variations in the real return on loans.
of changes in inflation, therefore isolating the transmission mechanism to different shocks. We report in Figures 4-6 transitions for real equity and loans (in percentage deviations from the steady state), discount-window and interbank market loans, the liquidity premium and liquidity ratio (all in levels), and inflation (expressed as a simple deviation from their steady state) for all shocks.  

5.1 Equity Losses

We begin with the analysis of a transition to steady state when the initial level of equity is 1 percent below steady state. This shock captures an unexpected rise in non-performing loans. Because equity is the only endogenous state, these transitional dynamics are important in understanding the model’s internal dynamics after all other shocks. The responses of some key variables are reported in Figure 4.

How does the economy return to steady state when equity is below steady state? To understand these dynamics, recall that Proposition 3 demonstrated that bank policies are linear in equity. This means that if portfolios weights are kept constant, a 1 percent drop in equity translates into a 1 percent contraction in the supply of loans, the demand for deposits, and the demand for reserves. For the loans market to clear after a contraction in the loan supply, an increase in the return for loans is needed. Similarly, the deposit rate must fall in order to clear the deposit market. Finally, for the reserve markets to clear, the initial price level needs to jump above steady state. With this increase in the price level, the real supply of reserves falls and equilibrium is restored.

Over time, bank equity increase because the real lending rate rises as the deposit rate falls. This greater spread makes intermediation more profitable. Over time, we can also see that the price level reverts to its stationary path and deflation keeps the real return on reserves relatively high. The overall effect of the shock to bank equity on the portfolio weights for reserves and

\[\text{In Figure A.11 in the Appendix we also report the convergence properties of the model when the Fed sets the interest rate on reserves to stabilize the price level at the stationary level.}\]

\[\text{In terms of the endogenous modelling of the loan demand and deposit supply schedules in Appendix C, the shock can be modelled as a loss in the firms' output that would have been devoted to debt repayment. Alternatively, we can simply assume that firms default on the debt and pay higher profits to shareholders. Given the simple two-period OLG structure, these two possibilities only translates into differences in the consumption level of the old generation without any other changes in prices or allocations. The total loss in equity we consider is then given by these losses due to default plus the valuation effects that arise because of assets and liabilities are denominated in nominal terms. It is straightforward to decompose these two.}\]

\[\text{Appendix J provides parameter conditions that guarantee monotone convergence to a unique steady state for a policy where the Fed induces satiation and earns no profits from its portfolio. We expect that for small distortions and shocks, the exercises has the same properties, a behavior that we verified numerically.}\]
loans—and thus the liquidity ratio—depends on the elasticities of the loan demand and deposit supply. For our baseline calibration, the real quantity of reserves falls less than the real quantity of loans. This, in turn, results in a higher liquidity ratio and a lower liquidity premium. The latter result is inconsistent with the observed patterns during the crisis (see Appendix L).

For lower loan demand elasticities, we can produce a decrease in the liquidity ratio. However, the liquidity ratio and the liquidity premium always move in opposite directions, which again is inconsistent with the observed patterns during the Great Recession. For example, consider the extreme case in which loan demand is perfectly inelastic at some quantity $B$. Then, the real volume of loans should be unaltered at equilibrium by the decline in equity. Since $B = \beta b_t E_t / P_t$, the equilibrium can only be restored with a decrease in the liquidity ratio, with an increase in $b_t$. This in turn would lead to an increase in the liquidity premium. For higher elasticities, the phenomenon is reversed.

5.2 Shocks to Interbank Markets

We consider two sources of disruptions in the interbank market: a shock to the volatility of withdrawals and a shock to the matching efficiency $\lambda$. Figure 5 presents the transitions to these two shocks.
Increased Volatility. For a given portfolio, the increase in volatility generates an increase in liquidity risk. In response, banks increase their buffer of reserves and reduce the supply of lending. Since the Fed keeps the nominal supply growing at a constant rate, the higher demand for reserves is accommodated with a drop in the price level. Likewise, the decline in the loan supply generates an increase in the loan rate. In the aftermath of the shock, equity declines because banks allocate a lower fraction of their portfolio to loans, the high-return asset. We also observe that banks borrow more from each other and from the Fed, as a result of the larger withdrawal shocks that are realized (panels f and g in Figure 5). As the shock dissipates, the dynamics become similar to the case when equity is below steady state.

One important takeaway is that the increase in deposit instability can explain the effect on the liquidity premium and liquidity hoarding, but in contrast to the crisis, it leads to a counterfactual increase in the activity of the federal funds market. According to these predictions of the models, the observed pattern in the crisis appears to call for a shock to the efficiency in the interbank market, $\lambda$, which we explore below.

Matching Frictions. Like the increase in volatility, more frictions in the interbank market generate hoarding of reserves. Qualitatively, the responses are therefore similar with an important difference: while an increase in volatility raises trade in the interbank market, the reduction in $\lambda$ has the opposite effect (panel g in Figure 5). In the following section, we calibrate paths of shocks to $\sigma$ and $\lambda$ to match the observed features of the interbank market and show their contribution.
to the decline in lending.

5.3 Shocks to Fed Policies

As in Section 3.5, we analyze the effects of policies, but now consider transitional dynamics rather than permanent changes. Figures A.6 and A.7 in Appendix A.1 illustrate respectively the dynamics in response to an increase in the nominal interest on reserves and an open market operation. A temporary increase in the nominal interest on reserves leads banks to respond by allocating a larger fraction of their portfolio to reserves. Since the nominal supply of reserves is fixed, the price level declines. Simultaneously, because of this desired substitution and the fact that the capital requirement constraint binds—recall Figure 1—the supply of loans contracts. To restore market clearing in the loans market, the return on loans increases. As the liquidity ratio increases, there are fewer interbank market loans and discount window loans.

The open market operation is an exchange of reserves for loans between the Fed and the banking system. In essence, this operation exchanges illiquid assets for liquid assets. Because the Fed acquires assets that deliver higher returns than the liabilities, this means the Fed makes profits, which in turn are rebated back to banks. The overall effect is to increase the total outstanding loans in the economy but there is a crowding out effect on bank lending. Bank equity increases, reflecting the larger overall investment in loans. Finally, this policy generates inflationary effects from resulting from the increase in the supply of reserves.

5.4 Shocks to Credit Demand

Finally, we consider the effects of a negative credit demand shock, as captured by a decline in $\Theta^b_t$. Figure 6 illustrates the effects of a negative temporary shock to loan demand. The effects contrast sharply with the effect of the shocks considered above because, there, the supply rather than the demand for loans contracts. As a result, a key difference is that the demand shocks produce a decline in the return on loans. Since loans become less attractive, this leads to a higher liquidity ratio and lower discount window and interbank market loans.
5.5 Summary and Discussion of Transitional Dynamics

Table 2 summarizes the response of the economy to different shocks considered. For reference, we also include a row with the movements observed in the data during the crisis. As the table shows, several of these shocks are candidates for explaining the decline in lending during the crisis. However, these shocks have different implications for other key variables. In the next section, we exploit the lessons from these transitional dynamics to examine the shocks that led to the collapse in lending during the crisis.

Table 2: Summary of Responses to Shocks

<table>
<thead>
<tr>
<th></th>
<th>Loans</th>
<th>Reserves</th>
<th>Interbank Loans</th>
<th>DW Loans</th>
<th>LP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Equity loss</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Capital requirement</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>Volatility</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Interest on reserves</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>Credit demand</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>

Note: The table lists the effects of shocks on different variables on impact.
6 Inspecting the Decline in Lending

We now turn to the quantitative application. We draw on the lessons from the transitional dynamics in Section 5 to quantitatively examine the causes of the contraction in bank lending during the 2008 crisis in the United States. We consider the four hypotheses discussed in the introduction: (i) low bank equity, (ii) increased precautionary holdings of reserves, (iii) interest on reserves and Federal Reserve policies, and (iv) weak credit demand. We discuss other alternatives in Section 7. Details on the motivating facts are presented in Appendix L and data sources are described in Appendix K.

Before we delve into the analysis, we would like to clarify important aspects of hypotheses ii and iii. First, it is common in policy discussions to argue that Fed policies mechanically determine the amount of reserves by conducting open market operations. In a monetary model like ours, however, this claim only applies to the nominal amount of reserves, and hence it does not follow that recent quantitative easing policies affected mechanically the real amount of reserves and bank credit.\(^44\) Second, when we refer to precautionary holding of reserves (hypothesis ii), we are referring more broadly precautionary holdings of “liquid assets.” That is, the goal is to consider the extent to which a higher demand for liquid assets, such as US Treasuries, agency securities, and central bank reserves, had a negative impact on bank lending.\(^45\) This is important because when the Fed started paying interest on reserves in October 2008, banks substituted holdings of US Treasuries and other liquid assets with reserves (see Ennis and Wolman, 2015), which resulted in large amounts of “excess reserves.” Because the total supply of liquid assets provided by the consolidated US government did not nearly increase in the same amount, the issues of liquidity management remained at the core of banks’ decision problems.

6.1 Data and Empirical Strategy

To weigh on these hypotheses, we feed a sequence of shocks to parameters of the model starting at the stationary equilibrium. We should note that while these are transitional dynamics (i.e., not shocks in the technical sense) we still refer to “shocks” for ease of exposition. We divide these shocks into two sets. The first set of shocks are fed directly into the model. These shocks are an initial drop in equity of 2 percent, in line with the losses from subprime mortgages, as well as Fed

\(^{44}\) Presumably, this view comes from standard models of monetary policy implementation, which are technically real models without nominal or monetary considerations.

\(^{45}\) As mentioned in footnote 20, one could also expand the model to allow for different types of liquid assets, but we abstract from this to keep the model simpler and focus more broadly on the management between liquid and illiquid assets.
policies for interest on reserves and asset purchases. The second set of shocks are calibrated to match a series of observables in the data. These shocks are to the withdrawal volatility, matching efficiency, and credit demand, which are set to reproduce the time series for the volumes of discount window loans, interbank market loans, and bank credit (linearly detrended). In terms of monetary policy, we consider a constant inflation rate equal to the stationary equilibrium and a nominal supply of reserves that achieve this target.\footnote{It should be clear that in a stationary equilibrium, reserves and the price level increase at the same rate, but not away from the stationary equilibrium.} This assumption is numerically convenient because, with it, we do not need to solve for the initial price level as we search for shocks.\footnote{Appendix M.3 provides the details on how the Fed accomplishes this in the model and the numerical algorithm used to compute the transitional dynamics.} This policy is also consistent with the stable inflation path we observe in the data.

We consider the period February 2008 through February 2010, which centers on the Lehman Brothers’ bankruptcy. Data for discount window loans and interbank market volume are obtained from the Board of Governors of the Federal Reserve (respectively releases H.3 Aggregate Reserves of Depository Institutions and the Monetary Base and H.8 Assets and Liabilities of Commercial Banks in the United States). For the estimation, we normalize these variables by the amount of total deposits of commercial banks and take the deviations relative to the value in January 2008. To construct the analogues in the model, we consider the deviations of the ratios of discount window loans to deposits and interbank market loans to deposits relative to the stationary values. Credit corresponds to Commercial and Industrial Loans for all commercial banks. We fit a log-linear trend between 2000 and 2007 and project it through 2008 and 2009. For the estimation, we use the actual deviation from that trend in the data and consider the percentage deviation of the path of credit relative to the stationary value in the model. In terms of the Fed asset purchases that we feed into the model, we consider the sum of securities, unamortized premiums and discounts, repo, and loans (WRSL) minus US Treasury securities (WSHOTS), which we normalize by the total amount of credit. The other policy shocks we feed are the interest on reserves and the amount of $M^{Fed}$ that delivers a price level within the stationary path. The latter imply an increase by more than twofold in the nominal amount, given the substantial increase in the real demand for reserves inferred in the estimation.

Given the sample, we have 25 months and three shocks (withdrawal volatility, matching efficiency, and credit demand), so overall there are $25 \times 3 = 75$ shock values to back out from a numerical procedure. Because we have as many shocks as observables, the model can, in principle, perfectly reverse-engineer the sequence of values that replicate the path of the series.
in the data. This is not guaranteed, however, because there is limited support for the realization of the endogenous variables that the model can match. For example, as $\lambda$ becomes large, the interbank market behaves similarly to a Walrasian market, in which case volatility shocks do not affect credit. Below we explain how the model indeed renders identification.

6.2 Identification

To explain how we can identify the matching friction, volatility, and loan demand shocks, we take two shocks at a time and plot the combinations of the value of the shocks that deliver the same data target. When we produce plots, we fix the third shock at its steady-state value. Our goal is to show how the two shocks uniquely pin down two moment targets.

Let us use panel (a) in Figure 7 as a first example. Panel (a) shows combinations of matching friction $\lambda$ and volatility $\sigma$ that deliver the same volumes of trade in the discount window (the constant DW curve) and interbank market (the Constant Interbank curve).\footnote{Recall that volumes are expressed relative to deposits, that is, we target $W/D = 0.2$ percent and $\min\{S^-, S^+\}/D = 2$ percent.} As we can see, increasing the matching efficiency requires an increase in volatility to keep the discount window loans constant, which reflects an upward-sloping Constant DW curve. The opposite relation holds for the constant interbank curve: since an increase in matching efficiency increases the interbank market volume, volatility needs to decrease to keep the interbank market constant. As a result, the two curves cross only one. A single shock combination produces those two moments.

Panels (b) and (c) show that, similarly, one can separate the loan demand and volatility and the loan demand and matching efficiency. As loan demand decreases, pushing down the equilibrium level of loans, one requires a reduction in volatility and an increase in matching efficiency to keep the level of credit constant. This is reflected in an upward-sloping curve for

\hspace{1cm} Figure 7: Identification for Quantitative Application

48
the constant loans curve in panel (b) and a downward-sloping one in panel (c). At the same
time, discount window and interbank market loans are relatively less sensitive to changes in loan
demand. Hence, the constant DW and constant interbank curves are relatively flat in panels (b)
and (c). Based on this finding, we conclude that discount window loans and interbank loans
are more informative about \( \lambda \) and \( \sigma \) than about \( \Theta^b \).

6.3 Results

Figure 8 reports the results of our main experiment. The three panels in the upper row report
the data series that we use as a target for the estimation and the model analogue, under four
different simulation scenarios. Panels d-e in the middle row report two additional data series: the
liquidity premium and the liquidity ratio. In the lower row, we present the series for \( \{\lambda, \sigma, \Theta^b\} \)
that the model needs to fit the three targets in the data.

To evaluate each hypothesis, we evaluate the importance of each shock by turning off one
shock at a time. The figure reports the simulations with all shocks (straight line) and when
we turn off interbank market shocks (dash-dotted line), demand shocks (dashed line), and open
market operations (dotted line). Neither equity losses nor the increase in interest on reserves
plays a quantitatively important role. The fact that equity losses do not play an important role
should be clear from Figure 4, in which we analyze the transition to an a drop in equity. Interest
on reserves are around 0.5 percent in these period and hence they play a modest role as well.

By construction, the benchmark case that includes all shocks (the matching friction, with-
drawal volatility and loan demand) matches the three data targets perfectly (in the upper panels).
As an external validation, we also plot the model’s fit to the liquidity premium in the data (Panel
d) and the liquidity ratio (Panel e). Quantitatively, the model tracks the liquidity ratio well. As
in the data, the liquidity premium reaches a relatively higher value around Lehman compared
to the post-crisis period, but the value is not as high or as volatile as in the data.

The shocks that the model needs to fit the data: the model needs a gradual decline in
matching efficiency, an increase in withdrawal volatility that spikes around September 2008 and

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49 Qualitatively, a decrease in loan demand also increases the level of reserves, and through this effect, it reduces
discount window loans, hence, the negative slope in the constant DW curve in panel (b). The effect of a decrease in
loan demand over the interbank market depends on whether the market has a deficit or an excess of reserves. For
the relevant case during the crisis period, in which banks had excess reserves, one obtains a positive relationship
between the loan demand and the matching efficiency.

50 Again, as we noted above, if one were to look at the composition of liquid assets, interest on reserves would
probably play a more quantitatively important role.

51 As explained above we measure the liquidity premium as the difference between the return of the three-month
general collateral repurchase agreements and the interest on reserves.
partially reverts afterward, and a decline in the loan demand that escalates starting in 2009. The counterfactual simulations show that both the interbank market and demand shocks had a prevalent role and their timing is instructive. Before the Lehman bankruptcy in September 2008, neither shock produces a substantial decline in credit. However, to match interbank market features, the model needs a significant matching shock to produce the early pronounced decline in the volume of interbank credit. In the run-up to the crisis, we also observe a substantial increase in deposit volatility, consistent with the spike in discount-window loans. Absent these interbank market shocks, the access to the discount-window and the volume of interbank market loans would have remained essentially flat throughout the period. After the Lehman crisis, both shocks contributed to the decline in lending, as Panel (c) shows. The credit demand shock produces the lion’s share of the impact, except for the period around September 2008 in which the interbank market is more important role.

As a policy counterfactual, we can observe that open market operations were important to mitigate the collapse in total credit. While they certainly had a crowding out effect on bank credit—the amount of loans purchased by the Fed reaches about 10 percent of the stock—one can see that credit would have fallen about 2 percent more absent this intervention. In effect, the model suggests that the negative effect of the interbank market shock was about the same magnitude as the positive effect of the open market operations.

One possible interpretation of the reduction in the interbank market efficiency that were inferred as hitting early in the 2008 crisis is an episode of increased counterparty risk in the interbank market and increased instability in the deposit base, consistent with the narrative of the financial crisis in Brunnermeier (2009). Furthermore, in the extension section 7.3, we explain how withdrawal volatility shocks are similar to outside funding shocks once we provide some liquidity value to loans. Finally, the timing of the shocks is suggestive of a deeper economic phenomenon in which an initial contraction in the supply of loans, produced by disruptions in the interbank market, eventually led to a collapse in credit demand. More research is needed to shed light on these interactions.
Figure 8: All Experiments

Note: all model variables are expressed as deviations from the steady state except credit which is expressed in percentage deviations from steady state. Data series are reported in deviations relative to the value in January 2008. Discount window loans and interbank market loans are normalized by deposits, in the model and in the data. In the model, discount window loans and interbank market loans are expressed as simple deviations from steady state, while loans are expressed as a fraction of the steady state level of loans. Interbank market loans are expressed in percentage terms. In the data, variables have been smoothed using a 3-month rolling window. Credit in the data corresponds to C&I loans. Credit is linearly detrended between 2000-2007 and we project a linear trend through 2008 and 2009. Data sources are in Appendix K.
7 Extensions of the Baseline Model

7.1 Liquidity Coverage Ratio

The new regulatory framework following the recent financial crisis requires banks to hold a minimum fraction of total assets in reserves, the so-called liquidity coverage ratio (LCR). In contrast with the reserve requirement in the baseline model, the required reserve holdings are tied to the amount of assets rather than liabilities. We show next how this feature is easily accommodated into our model, and how the introduction of this new regulatory tool affects the banking system.

In addition to the reserve requirement (5), banks are subject to

\[ m^j_{t+1} \geq \rho^{lcr} b^j_{t+1}. \]  

Here, \( \rho^{lcr} \) is a parameter that works analogously to reserve requirements but is applied to loans instead of deposits.\(^52\) Since banks need to satisfy both the reserve requirement (RR) and the LCR, we can redefine the surplus function as

\[ s(\omega) \equiv \min \{ s^{rr}(\omega), s^{lcr}(\omega) \}, \]  

where \( s^{lcr}(\omega_j) = \rho^{lcr} \tilde{b}^j_{t+1} - \left( \tilde{m}^j_{t+1} + \frac{\omega_j \tilde{d}^j_{t+1}(1+i^k)}{1+i^{ior+1}} \right) \), and \( s^{rr}(\omega) \), as defined in (6). The characterization of the individual bank problem and the determination of equilibrium remain essentially unchanged except that the surplus function in Proposition 3 has to be redefined following (31).\(^53\)

The key difference in the portfolio problem is that reserves carry an additional premium over loans because an increase in loans tightens the LCR constraint in states where the surplus is given by \( s(\omega) = s^{lcr}(\omega) \).

As Figure A.8 shows, the introduction of an LCR increases the demand for reserves and reduces bank lending. While the LCR was not active throughout the crisis, and hence we do not include it as one of the hypotheses for the contraction in bank lending, this regulatory tool will

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\(^52\) The parameter \( \rho^{lcr} \) could be linked to the volatility of deposit withdrawals, so as to capture constraints related to the net stable funding ratio (NSFR). This constraint could also be imposed in the lending stage instead. If the constraint binds at the lending stage, shocks need to be large enough to have effects on the reserve-to-loans ratio.

\(^53\) Because \( s^{RR} \) and \( s^{LCR} \) are linear in \( \omega \), the surplus function is characterized simply as

\[ s(\omega) = \begin{cases} 
    s^{LCR}(\omega) & \text{if } \omega < \bar{\omega} \\
    s^{RR}(\omega) & \text{if } \omega \geq \bar{\omega},
\end{cases} \]

where \( \bar{\omega} \) is the value of \( \omega \) such that \( s^{LCR}(\omega) = s^{RR}(\omega) \).
play an important role going forward.

7.2 Credit Risk and Risk Aversion

We introduce two other sources of time-varying risk: idiosyncratic shock to loan returns and variations in risk aversion. On the former, we assume that each bank’s portfolio of loans is subject to an idiosyncratic “repayment shock” at the beginning of each period. As is the case for the withdrawal shock, the shock to loan returns is assumed to have zero mean. Besides the modification to the portfolio problem of the bank, which now incorporates an additional source of uncertainty, the model remains essentially the same.\(^{54}\) In particular, because the shock has mean zero, the law of motion for aggregate equity continues to be determined by (24). Figure A.9 (solid line) shows the effects of an increase in the variance of this shock: it goes from zero, as in our benchmark, to 20 bps. Because of banks’ risk aversion, the increase in the risk of loan returns leads to a decline in total lending and to a concomitant increase in the liquidity ratio. The liquidity premium also rises on impact, reflecting that banks require a higher premium on loans to be willing to absorb the increase in risk. As banks have higher reserves, discount window loans and interbank market loans are reduced on impact. Finally, as equity falls because of the lower investment in loans, the impact of credit risk on the liquidity premium is eventually reversed.

To analyze the case of an increase in risk aversion, we consider Epstein-Zin preferences, keeping the intertemporal elasticity of substitution equal to unity, and consider a level of risk aversion equal to 100.\(^ {55}\) Figure A.9 (dashed line) shows that the dynamics after an increase in risk aversion are similar to the dynamics resulting from an increase in credit risk.

In terms of the hypothesis for the contraction in bank lending analyzed in Section 5, the response of the banking system to an increase in credit risk or risk aversion delivers outcomes that are qualitatively fairly consistent with various banking variables in the crisis. To the extent that the financial crisis was characterized by increased uncertainty about loan repayments by

\[\Omega_t \equiv (1 - \tau_t) \max_{\{\bar{b}, \bar{m}, \bar{d}\} \geq 0} \left\{ \mathbb{E}_{\omega, z} \left[ R^b_t (1 + z) \bar{b} + R^m_t \bar{m} - R^d_t \bar{d} + \chi_t (\bar{m}, \bar{d}, \omega) \right]^{1-\gamma} \right\} \frac{1}{1-\gamma}, \tag{32} \]

\[\bar{b} + \bar{m} - \bar{d} = 1, \]

\[\bar{d} \leq \kappa (\bar{b} + \bar{m} - \bar{d}).\]

\(^{54}\)Let \(z\) be the idiosyncratic default rate on bank loans, with zero mean and variance \(\sigma_z\). The portfolio problem now becomes

\[^{55}\text{A version of Proposition 3 applies with Epstein-Zin preferences, and hence tractability is not lost in this case.}\]
borrowers at the cross section of banks and an increase in risk aversion, this is a shock that could have played an active role in the crisis.

7.3 Outside Funding

A feature of the US financial crisis was a serious disturbance in short-term funding, which originated from withdrawals of funds from wholesale finance arrangements, a run on the sale and repurchase market (the repo market), and unprecedented increases in haircuts (see e.g., Gorton and Metrick, 2012). While our model allows for disturbances within the interbank market and to the volatility of withdrawal from deposits, so far it fails to capture that shocks from the rest of the financial system could affect bank liquidity.

To incorporate this possibility into the analysis, we extend the model by assuming that a fraction \( \xi_t \leq \tilde{\xi}_t \) of loans can be sold in the balancing stage to outside investors. The sales of loans capture more broadly securitization or collateralized (Repo) lending. With this access to outside liquidity, the evolution of reserves follow

\[
m^j_{t+1} = \tilde{m}^j_{t+1} + \left( \frac{1 + i^d_{t+1}}{1 + i^{or}_{t+1}} \right) \omega^j_t \tilde{d}^j_{t+1} + f^j_{t+1} + u^j_{t+1} + \xi_t \tilde{b}_{t+1} \left( \frac{1 + i^i_{t+1}}{1 + i^{or}_{t+1}} \right).
\]

(33)

Implicit in the latter term is that banks have full bargaining power, and therefore sell loans at a price \( (1 + i^i_{t+1})/(1 + i^{or}_{t+1}) \) to outside investors. With this assumption, banks sell \( \tilde{\xi}_t \) every period, and their surplus is given by (6) subtracted by the sale of loans term. We can thus readily solve the extended model by adjusting the surplus function in banks’ liquidity problem accordingly.

Our experiment, following the previous ones conducted, consists of considering a shock to \( \tilde{\xi}_t \) and examine the response of the economy. Figure A.10 shows how this shock raises the liquidity premium and reduces the amount of bank credit. Through the lens of our model, this shock is another plausible explanation for the decline in credit.

8 Conclusion

Historically, the topics of money and banking have been studied and taught together. Despite this historical connection, modern monetary models developed independently from banking. The financial crises of the last decades in the United States, Europe, and Japan, however, have revealed the need for a unified framework.
This paper presents a tractable quantitative model of banks’ liquidity management and the credit channel of monetary policy. In the model, banks engage in maturity transformation, which exposes them to liquidity risk. To insure against unexpected deposit withdrawals, banks hold reserves as a precautionary buffer. Banks that face large withdrawals deplete their reserves and resort to a frictional OTC interbank market and discount window borrowing. Monetary policy has the power to alter the liquidity premium and, in that way, to affect real economic activity. As an application, we study the driving forces behind the decline in bank lending and liquidity hoarding by banks during the 2008 financial crisis. We argue that this pattern was the result of an early disruption in the interbank market, followed by a persistent decline in credit demand.

The application we carried out in the paper is one of many possible ones. Our model could be used to shed light on classic historical debates. For example, it could be used to evaluate the hypothesis in Friedman and Schwartz (2008) that an increase in the deposit-to-currency ratio was responsible for the colossal credit crunch during the Great Depression. Also, Friedman and Schwartz (2008) argued that the Fed’s increase in reserve requirements in 1937 was a serious policy mistake, but Tobin (1965) opposed that view, arguing that banks held considerable excess reserves—such as during the episode we studied here. One could also use the model to study how different monetary policy and regulatory regimes affect the stability of bank credit. For example, monetary policy regimes around the world have evolved from a gold standard system to one that targets monetary aggregates and, more recently, to one that targets interest rates. One could also use the model to evaluate the desirability and implementation of liquidity regulations for financial stability, such as LCR. Along similar lines, Safonova (2017) studies how the network of the interbank market affects the transmission of monetary policy. Chen, Ren, and Zha (2017) use a similar model to study whether tight monetary policy in China induced shadow banking activities. Exploring these kinds of applications is a future task.

\[56\] We thank an anonymous referee for providing a rich list of possible extensions and applications.
References


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Online Appendix to
“Banks, Liquidity Management and Monetary Policy”

July 2018
A Additional Figures

A.1 Illustration Figures

Figure A.1: Timeline diagram and banks’ balance sheet. For illustration purposes, it is assumed that the portfolio chosen in the lending stage is $\tilde{m} = \rho \tilde{d}$. 
Figure A.2: Description of the liquidity management problem

Note: This figure illustrates the key trade-off underlying liquidity management. The $x$-axis in the figure corresponds to the reserve balance $s$ of a given bank. Above the $x$-axis we plot $\chi(s)$, the total interest earned or paid in the interbank market as a function of the reserve surplus. The slope of this liquidity yield is given by the aggregate market conditions and the Fed’s policy. The reserve surplus depends on its portfolio choices in the lending stage and the realization of the withdrawal shock. Below the $x$-axis we plot the probability distributions for $s$ for two different choices of $\{\bar{m}, \bar{d}\}$. The region to the left of the $y$-axis represents the probability of ending up in deficit. The figure depicts how, by increasing $m$, the distribution, depicted with a dashed red line, shifts to the right, meaning that surpluses are more likely. When a bank chooses between lending more or holding more reserves, it compares the spread between the return on loans and reserves with the increase in the probability of ending in a surplus. With risk aversion, probabilities are weighted by marginal utilities. This is precisely the trade-off encapsulated in condition (26). In summary, the LP indicates that banks are willing to sacrifice the premium on illiquid assets to insure against the possibility of adverse withdrawal shocks.
A.2 Figures on Policies and Stationary Equilibrium

Figure A.3: Stationary equilibria for different policy rates
Figure A.4: Stationary equilibria for different Fed loan holdings

(a) Loans portfolio $\bar{b}$

(b) Deposits $\bar{d}$

(c) Market tightness $\theta$

(d) Threshold $\omega^*$
A.3 Figures on Transitional Dynamics

Figure A.5: Transition after tightening of capital requirement

(a) Equity ($E$)  
(b) Total Loans ($B$)  
(c) Liquidity Ratio ($\bar{m}$)  
(d) LP ($R^b - R^m$)  
(e) Price Level  
(f) Discount Window ($W$)  
(g) Interbank Market ($\Psi^+ S^+$)  
(h) Shock leverage ($\kappa$)

Note: The figure considers a reduction in $\kappa$ of 10 percent. The shock produces an immediate 10 percent decrease in bank leverage, which, like the equity shock, reduces the funds available to the bank. On impact, the general equilibrium effects are similar to those after equity losses and produce the same effects: a reduction in the supply of loans, a loan rate increase, and a deposit rate decline, as well as a contraction in the demand for reserves, which again produces a jump in the price level, an increase in the liquidity ratio, and a fall in the liquidity premium. Immediately after the shock, equity grows beyond its steady-state value. This happens because the borrowing-lending spread increases. Eventually, the increase in equity overcomes the tightening of capital requirements, and the sign of the effects reverses before the economy returns to steady state.
Figure A.6: Transition after increase on interest on reserves

(a) Equity ($E$)  
(b) Total Loans ($B$)  
(c) Liquidity Ratio ($\bar{m}$)  
(d) LP ($R^b - R^m$)

Figure A.7: Transition after open-market operations

(a) Equity ($E$)  
(b) Total Loans ($B$)  
(c) Liquidity Ratio ($\bar{m}$)  
(d) LP ($R^b - R^m$)

(e) Inflation ($\pi$)  
(f) Discount Window ($W$)  
(g) Interbank Market ($\Psi^+ S^+$)  
(h) Interest on Reserves ($i^{or}$)

(e) Price Level ($P$)  
(f) Discount Window ($W$)  
(g) Interbank Market ($\Psi^+ S^+$)  
(h) OMO Shock ($B^{FED}$)
A.4 Extensions of the Baseline Model

Figure A.8: Liquidity coverage ratio
Figure A.9: Credit risk and risk aversion

(a) Equity ($E$)

(b) Total Loans ($B$)

(c) Liquidity Ratio ($\bar{m}$)

(d) LP ($R^b - R^m$)

(e) Price Level ($P$)

(f) Discount Window ($W$)

(g) Interbank Market ($\Psi^+ S^+$)

(h) Risk aversion/credit risk

Figure A.10: Shock to outside funding

(a) Equity ($E$)

(b) Total Loans ($B$)

(c) Liquidity Ratio ($\bar{m}$)

(d) LP ($R^b - R^m$)

(e) Price Level ($P$)

(f) Discount Window ($W$)

(g) Interbank Market ($\Psi^+ S^+$)

(h) Outside Funding ($\xi$)
A.5 Comparison of monetary policy rules

Figure A.11: Transition after equity losses for different monetary policy rules.

- (a) Equity ($E$)
- (b) Total Loans ($B$)
- (c) Liquidity Ratio ($\overline{m}$)
- (d) LP ($R^b - R^m$)
- (e) Price Level ($P$)
- (f) Discount Window ($W$)
- (g) Interbank Market ($\Psi^+ S^+$)

Note: The solid line corresponds to the baseline policy that keeps the nominal supply of reserves constant with an alternative rule that adjusts $i^o$ to keep the price level in the stationary path. The solid blue line is the baseline. The broken line is the inflation-targeting regime. The figure shows that under the inflation-targeting regime, the convergence to the stationary equilibrium is slower under the inflation targeting regime. The reason is that to keep the price level from jumping at $t = 0$, the Fed pays interest on reserves, which leads to a rise in the rate on loans and contracts lending. Because of lower lending, equity takes longer to recover.
## B List of Variables

### Table B.1: List of variables

<table>
<thead>
<tr>
<th>Interest rates</th>
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<tbody>
<tr>
<td>$i^f$</td>
<td>average interbank market rate</td>
<td></td>
</tr>
<tr>
<td>$i^b$</td>
<td>nominal interest rate on loans</td>
<td></td>
</tr>
<tr>
<td>$i^d$</td>
<td>nominal interest rate on deposits</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Individual bank variables</th>
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</thead>
<tbody>
<tr>
<td>$b$ bank loans in lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{m}$ reserves held by banks at end of lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$ bank deposits at end of lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$ loans at beginning of lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$ reserves held at beginning of lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$ deposits owed at beginning of lending stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{b}$ portfolio share in loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{m}$ portfolio share in reserves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d}$ portfolio share in deposits</td>
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<table>
<thead>
<tr>
<th>Others</th>
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<tbody>
<tr>
<td>$c$ consumption</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega$ risk-adjusted value of bank equity</td>
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<td></td>
</tr>
<tr>
<td>$R^e$ return on equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$ real equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$ Lagrange multiplier on capital requirement constraint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V^l, V^b$ value of the bank at lending/balancing stage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V$ value of the bank as a function of bank equity</td>
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</table>

<table>
<thead>
<tr>
<th>Interbank market</th>
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</thead>
<tbody>
<tr>
<td>$\omega$ withdrawal shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$ surplus at beginning of balancing stage after shock $\omega$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$ market tightness in interbank market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$ interbank market loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w$ discount window loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi^+$ probability that a bank with surplus finds a match</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi^-$ probability that a bank with deficit finds a match</td>
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<table>
<thead>
<tr>
<th>Aggregates</th>
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<tbody>
<tr>
<td>$E$ bank equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Theta^e$ intercept loan demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$ elasticity loan demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Theta^d$ intercept deposit supply</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varsigma$ elasticity deposit supply</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B, B^d$ loan supply/demand</td>
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<td></td>
</tr>
<tr>
<td>$D, D^e$ deposits supply/demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ price level</td>
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<td></td>
</tr>
<tr>
<td>$\pi$ inflation</td>
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<table>
<thead>
<tr>
<th>Fed policies</th>
<th></th>
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<tbody>
<tr>
<td>$i^{tor}$ nominal interest rate on reserves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i^{dw}$ nominal interest rate on discount window loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M^{Fed}$ supply of reserves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^{Fed}$ Fed holdings of private loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W^{Fed}$ discount window loans</td>
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</tr>
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</table>
C Microfoundations: Loan Demand/Deposit Supply

In this section, we provide a microfoundation for the loan demand and deposit supply schedule for the interested reader. In this extension, in addition to bankers, the economy is populated by overlapping generations of workers and entrepreneurs. Each group has a unit measure, and agents live for two periods.

**Workers.** Workers belong to a single family and maximize

\[
\max \{ c_{w,t} \geq 0 \} \quad \frac{(h_{t+1}^{w})^{1+\nu}}{1 + \nu} + \beta \left( \frac{(c_{w,t+1})^{1-1/(\varsigma + 1)}}{1 - 1/ (\varsigma + 1)} \right) \tag{C.1}
\]

subject to

\[
d_{t+1} + P \epsilon c_{w,t+1} = z_{t} h_{t} \quad \text{and} \quad D_{t+1} = B_{t+1} - z_{t} h_{t}. \tag{C.2}
\]

The objective is to maximize \( t + 1 \) profits to maximize consumption when old. Profits are the sum of sales, minus financial expenses, plus earnings on deposits. The entrepreneur borrows \( B_{t+1}^{d} \) at an interest rate cost \( (1 + i_{t+1}^{d}) \) and uses these funds to finance payroll, \( z_{t} h_{t} \), or save in a deposit account. To finance this payroll, entrepreneurs borrow from banks. In particular, they obtain a loan from the banks in the form of a number of deposits that can be used to pay workers. The loan is a promise to repay \( (1 + i_{t+1}^{d}) \) by \( t + 1 \).

When we consider the non-financial side of the model, there are now two additional market clearing conditions: labor market clearing and goods market clearing. The goods market clearing condition is

\[
A_{t} h_{t-t-1}^{\alpha} = c_{t}^{t-1} + c_{t}^{w,t} + c_{t}^{w,t-1} + c_{t}. \tag{C.2}
\]

This equation states that output is used either as intermediate inputs or for consumption of old entrepreneurs, young and old workers, and bankers. The market clearing condition in the labor market is given by

\[
h_{t} = h_{t}^{w}. \tag{C.3}
\]
Equilibrium in the labor market and the optimal policy functions of entrepreneurs and workers yield an autonomous system of demand and supply equations for loans and deposits.

**Proposition 10.** The equilibrium loan demand and deposit supply take the form of (12) and (13), and the reduced-form parameters are given by

\[ Θ^b_t = (α A_{t+1})^ε, \epsilon = \left( \frac{α}{(ν + 1)} - 1 \right)^{-1} \text{ and } Θ^d_t = β^{1+ε}. \]

Output and labor are decreasing functions of \( (1 + i^b_t) / (1 + π_{t+1}) \).

In the paper we do not make reference to this microfoundation and work with the exogenous demand schedule for loans (12) and exogenous supply schedule for deposits (13). However, the definition of equilibria is consistent with the general equilibrium version of the model where we consider this non-financial sector. Note that Proposition 10 uses a labor market clearing condition. Then, clearing in the loans and deposit markets, by Walras’s law, implies clearing in the goods market. Once we compute equilibria taking the schedules as exogenous in the bank’s problem, it is possible to obtain output and household consumption from the equilibrium rates. One important observation is that an equilibrium real rate for loans, \( R^b_t \), immediately maps into an equilibrium output. In particular, the higher the rate, the lower the output. When monetary policy induces a higher rate, it has the power to reduce output in this environment. This is a notion of the credit channel.

**General Equilibrium.** If we consider the non-financial side of the economy, the equilibrium is given by the following.

**Definition 5.** Given a distribution of bank portfolios \( \{d^j_{0t}, m^j_{0t}, b^j_{0t}, f^j_{0t}, w^j_{0t}\} \), initial deposits for households \( D^0_t \), firms’ debt \( B^0_t \) and a deterministic sequence of government policies \( \{B^Fed_t, M^Fed_t, W^Fed_t, T^Fed_t, i^ior_t, i^dw_t\}_{t≥0} \), a competitive equilibrium is a deterministic sequence of interest rates \( \{i^b_t, i^d_t, i^f_t\}_{t≥0} \), a price level \( \{P_t\} \), a deterministic sequence of matching probabilities \( \{Ψ^+_t, Ψ^-_t\} \), a deterministic path for aggregates \( \{D_{t+1}, B_{t+1}, M_{t+1}, W_{t+1}, h_t\}_{t≥0} \), a stochastic sequence of bank policy variables \( \{v^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, c^j_t, f^j_{t+1}, w^j_{t+1}, m_{t+1}\}_{t≥0} \), firms’ employment and credit decisions \( \{B_{t+1}, h_t\}_{t≥0} \), and households’ consumption, deposits, and labor supply \( \{h^{wt-1}_t, D^S_{t+1}, c^{wt}_t, c^{et}_t\}_{t≥0} \) such that

(i) Banks’ policies \( \{h^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, c^j_t\}_{t≥0} \) solve the banks’ optimization problem, \( \{f^j_{t+1}, w^j_{t+1}\}_{t≥0} \) are given by the formula in Proposition 1, and \( m_{t+1} \) satisfy 7.

(ii) The central bank’s budget constraint (11) is satisfied and \( \int J T^j dj = T^Fed_t \).

(iii) The quantity of loans \( B_{t+1} \) and labor \( h_t \) solve the entrepreneur’s problem.

(iv) The quantity of deposits \( D_{t+1} \) and labor \( h_t \) solve the young worker’s problem.
(v) Markets clear $\forall t \geq 0$:

$$\int_j b^j_{t+1} dj + B^Fed_{t+1} = B^d_{t+1}$$  \hspace{1cm} \text{(loan market clearing)}

$$\int_j d^j_{t+1} dj = D^S_{t+1}$$  \hspace{1cm} \text{(deposit market clearing)}

$$\int_j m^j_{t+1} dj = M^Fed_{t+1}$$  \hspace{1cm} \text{(reserve market clearing)}

$$\int_j f^j_{t+1} dj = 0$$  \hspace{1cm} \text{(interbank market clearing)}

$$\int_j w^j_{t+1} dj = W^Fed_{t+1}$$  \hspace{1cm} \text{(discount window clearing)}

$$c^e_{t-1} + c^w_{t} + c^{w,t-1} + c_t = A_t h^w_{t-1}$$  \hspace{1cm} \text{(goods clearing)}

(vi) The matching probabilities $\{\Psi^+_t, \Psi^-_t\}_{t \geq 0}$ and the federal funds rate $i^f_t$ are consistent with the surplus and deficit masses $S^-_t$ and $S^+_t$, as given by Proposition 1.

The definition in the body of the paper is consistent with this definition. Solving the equilibrium in the financial market delivers the solution to the goods and labor market.
D Law of Motion for Aggregate Equity

To derive the law of motion for aggregate equity, we combine the Fed’s budget constraint with the bank’s budget constraint and use the market clearing conditions. The budget constraint for the Fed during the balancing and lending stages is

\[
\tilde{M}_t^{Fed}(1 + \bar{i}_t^{ior} + B_t^{Fed} = \tilde{M}_t^{Fed} + W_t^{Fed}
\]

\[
M_t^{Fed}(1 + i_t^{ior}) + B_t^{Fed} = \tilde{M}_t^{Fed} + W_t^{Fed}(1 + i_t^{dw}) - W_t^{Fed}(1 + i_t^{ior}) + B_t^{Fed}(1 + i_t^b) + P_t T_t. \tag{D.2}
\]

Combining these two constraints, we obtain

\[
(1 + \bar{i}_t^{ior}) M_t^{Fed} - (1 + \bar{i}_t^{ior}) W_t^{Fed} + B_t^{Fed} - W_t^{Fed}(1 + \bar{i}_t^{ior}) + P_t T_t = \ldots \tag{D.3}
\]

\[
M_{t+1}^{Fed} + (1 + i_t^b) B_t^{Fed} + (1 + i_t^{dw}) W_t^{Fed} - W_t^{Fed}(1 + \bar{i}_t^{ior}) + P_t T_t = \ldots \tag{D.4}
\]

The last line is the law of motion presented in the body of the paper.

Iterating forward one period (D.2) and using (D.2), we obtain

\[
P_t T_t = \tilde{M}_t^{Fed}(1 + i_t^{ior}) + B_t^{Fed} - \tilde{M}_t^{Fed} - W_t^{Fed}(1 + i_t^{dw}) + W_t^{Fed}(1 + \bar{i}_t^{ior}) - B_t^{Fed}(1 + i_t^b).
\]

Shift forward one period to obtain

\[
T_{t+1} = \frac{\tilde{M}_t^{Fed}(1 + i_{t+1}^{ior}) + B_{t+1}^{Fed} - \tilde{M}_{t+1}^{Fed} - W_{t+1}^{Fed}(i_{t+1}^{dw} - \bar{i}_{t+1}^{ior}) - B_{t+1}^{Fed}(1 + i_{t+1}^b)}{P_{t+1}}.
\]

On the bank’s side, recall that individual equity is defined as

\[
e_t^j = \left( \frac{\tilde{m}_t^j(1 + i_t^{ior}) + \tilde{b}_t^j(1 + i_t^b) - \tilde{d}_t^j(1 + i_t^d) + w_t^j(1 + i_t^{dw}) + f_t^j(\gamma_{t+1} - \bar{i}_{t+1}^{ior}) - P_t T_t^j}{P_t} \right). \tag{D.5}
\]

Iterating one period forward, integrating across banks, and the market clearing for reserves, discount window, and interbank market loans, we obtain:

\[
E_{t+1} = \frac{\tilde{M}_{t+1}(1 + i_{t+1}^{ior}) + \tilde{B}_{t+1}(1 + i_{t+1}^b) - \tilde{D}_{t+1}(1 + i_{t+1}^d) + W_{t+1}(i_{t+1}^{dw} - \bar{i}_{t+1}^{ior}) - P_{t+1} T_{t+1}^j}{P_{t+1}}. \tag{D.6}
\]

Multiplying and dividing by \(P_t\) in the denominator, and using definition of real returns, we obtain

\[
E_{t+1} = \left( R_{t+1}^b \tilde{b}_t + R_{t+1}^m \tilde{m}_t - R_{t+1}^d \tilde{d}_t \right) E_t(1 - \bar{c}_t) - \frac{W_{t+1}^{Fed}(i_{t+1}^{dw} - \bar{i}_{t+1}^{ior}) - T_{t+1}}{P_{t+1}} \tag{D.7}
\]

\[
= \left( R_{t+1}^b \tilde{b}_t - R_{t+1}^d \tilde{d}_t \right) E_t(1 - \bar{c}_t) + \left( 1 + \bar{i}_{t+1}^{ior} \right) \frac{P_t \tilde{m}_t E_t(1 - \bar{c}_t) - W_{t+1}^{Fed}(i_{t+1}^{dw} - \bar{i}_{t+1}^{ior}) - T_{t+1}}{P_{t+1}} \tag{D.7}
\]

\[
= \left( R_{t+1}^b \tilde{b}_t - R_{t+1}^d \tilde{d}_t \right) E_t(1 - \bar{c}_t) + \left( 1 + \bar{i}_{t+1}^{ior} \right) \frac{\tilde{M}_{t+1}^{Fed}}{P_{t+1}} - \frac{W_{t+1}^{Fed}(i_{t+1}^{dw} - \bar{i}_{t+1}^{ior}) - T_{t+1}}{P_{t+1}}. \tag{D.7}
\]
If we replace this condition by (D.2), we obtain
\[
E_{t+1} = (R_{t+1}^b \tilde{b}_t - R_{t+1}^d \tilde{d}_t)E_t(1 - \tilde{c}_t) + \frac{\bar{M}_{t+2}^{Fed} - B_{t+2}^{Fed} + B_{t+1}^{Fed}(1 + i_{t+1}^b)}{P_{t+1}}, \tag{D.8}
\]
which is equation (24).

**Stationarity Condition.** Following Proposition 6, consider a Fed policy of \(i_{t+1}^{dw} = i_{t+1}^{ior}\) to generate satiation, and in addition, assume \(\bar{M}_t = B_{t}^{Fed}\), for any \(t\). We must then have that \(T_t = 0\). Thus, by (D.7) we have that
\[
E_{t+1} = (R_{t+1}^b \tilde{b}_t + R_{t+1}^m \bar{m}_t - R_{t+1}^d \tilde{d}_t)E_t(1 - \tilde{c}_t).
\]
Thus, under satiation we have that \(R_{t+1}^b = R_{t+1}^m\), and hence we obtain
\[
E_{t+1} = (R_{t+1}^b (\tilde{b}_t + \bar{m}_t) - R_{t+1}^d \tilde{d}_t)E_t(1 - \tilde{c}_t),
\]
and because \(\tilde{b}_t + \bar{m}_t = \bar{1} + \bar{d}\), we have that
\[
E_{t+1} = (R_{t+1}^b + \bar{d}(R_{t+1}^b - R_{t+1}^d))E_t(1 - \tilde{c}_t).
\]
However, we know that under satiation, either \(R_{t+1}^b = R_{t+1}^d\) or capital requirements bind and \(\bar{d} = \kappa\). Thus, this law of motion is written as
\[
E_{t+1} = \left( R_{t+1}^b + \kappa \max \left\{ R_{t+1}^b - R_{t+1}^d, 0 \right\} \right) E_t(1 - \tilde{c}_t). \tag{D.9}
\]

**Law of Motion under Satiation and \(\bar{M}_t = B_{t}^{Fed}\).** Following Proposition 6, consider a Fed policy of \(i_{t+1}^{dw} = i_{t+1}^{ior}\) to generate satiation, and in addition, assume \(\bar{M}_t = B_{t}^{Fed}\) (for any \(t\)). Thus, by (D.7) we have that
\[
E_{t+1} = (R_{t+1}^b \tilde{b}_t + R_{t+1}^m \bar{m}_t - R_{t+1}^d \tilde{d}_t)E_t(1 - \tilde{c}_t).
\]
Thus, under satiation we have that \(R_{t+1}^b = R_{t+1}^m\), and hence we obtain
\[
E_{t+1} = (R_{t+1}^b (\tilde{b}_t + \bar{m}_t) - R_{t+1}^d \tilde{d}_t)E_t(1 - \tilde{c}_t),
\]
and because \(\tilde{b}_t + \bar{m}_t = \bar{1} + \bar{d}\), we have that
\[
E_{t+1} = (R_{t+1}^b + \bar{d}(R_{t+1}^b - R_{t+1}^d))E_t(1 - \tilde{c}_t).
\]
However, we know that under satiation, either \(R_{t+1}^b = R_{t+1}^d\) or capital requirements bind and \(\bar{d} = \kappa\). Thus, this law of motion is written as
\[
E_{t+1} = \left( R_{t+1}^b + \kappa \max \left\{ R_{t+1}^b - R_{t+1}^d, 0 \right\} \right) E_t(1 - \tilde{c}_t). \tag{D.10}
\]


E Equilibrium Conditions

E.1 Transitions

We characterize the set of equilibrium conditions in the main paper. Here, we present a summary of the conditions in one place. Given a sequence of government policy \( \{ \rho_t, \kappa, i_{t+1}^{or}, i_t^{dw}, W_t, B_t^{Fed}, M_t^{Fed}, T_t \} \) that satisfies the Fed’s budget constraint, the system that characterizes equilibrium yields a solution for individual bank variables, \( \{ \bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t \} \), aggregate variables, \( \{ B_t, M_t, D_t, E_t \} \), and a system of prices and real returns \( \{ P_t, R_b, R_m, R_d, \bar{\chi}_t \} \). The system features 15 unknowns to be determined for all \( t \). However, there is only one endogenous state variable. The Fed’s budget constraint adds one restriction to the set of policy sequences. Other variables that follow from definitions are described in Appendix B.

Individual Bank Variables

The portfolio solution to \( \{ \bar{b}_t, \bar{m}_t, \bar{d}_t \} \) and the value of \( \Omega_t \) are the solutions and value of the following problem:

\[
\Omega_t \equiv \max_{\{ \bar{b}, \bar{m}, \bar{d} \} \geq 0} \left\{ \mathbb{E}_\omega \left[ R_t^{\bar{b}} \bar{b} + R_t^{\bar{m}} \bar{m} - R_t^{\bar{d}} \bar{d} + \bar{\chi}_t(\bar{m}, \bar{d}, \omega) \right]^{1-\gamma} \right\}^{1/(1-\gamma)}, \tag{E.1.1}
\]

\[
\bar{b} + \bar{m} - \bar{d} = 1,
\]

\[
\bar{d} \leq \kappa_t (\bar{b} + \bar{m} - \bar{d}).
\]

The value of the bank’s problem is

\[
v_t = \frac{1}{1-\gamma} \left[ 1 + (\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1})^{\frac{1}{\gamma}} \right]^{\frac{1}{\gamma}}. \tag{E.1.2}
\]

Dividends depend on \( \{ \Omega_t, v_t \} \) via

\[
\bar{c}_t = \frac{1}{1 + [\beta(1-\gamma)v_{t+1}\Omega_{t+1}^{1-\gamma}]^{1/\gamma}}. \tag{E.1.3}
\]

This block of equations yields the equations needed to obtain \( \{ \bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t \} \) for a given path for real rates \( \{ R_t^b, R_t^m, R_t^d, \bar{\chi}_t \} \).

Aggregate Banking Variables

Next, homogeneity in policy functions gives us the aggregate bank portfolio:

\[
B_{t+1} = P_t \bar{b}_t (1 - \bar{c}_t) E_t \tag{E.1.4}
\]

\[
M_{t+1} = P_t \bar{m}_t (1 - \bar{c}_t) E_t \tag{E.1.5}
\]

\[
D_{t+1} = P_t \bar{d}_t (1 - \bar{c}_t) E_t. \tag{E.1.6}
\]

Real aggregate equity evolves according to

\[
E_{t+1} = \frac{P_t \left( (1 + \rho_{t+1}) \bar{b}_t + (1 + i_{t+1}^{or}) \bar{m}_t - (1 + i_{t+1}^{dw}) \bar{d}_t \right) (1 - \bar{c}_t) E_t - (1 + i_{t+1}^{dw}) W_{t+1} - PT_t}{P_{t+1}}. \tag{E.1.7}
\]
This block of equations determines \( \{ B_t, M_t, D_t, E_t \} \) given a path for inflation and nominal rates—which together determine real rates—and transfers.

**Market Clearing Conditions**

The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

\[
\frac{B_{t+1} + B_{t}^{FED}}{P_t} = \Theta_t^b (P_t) \epsilon, \quad (E.1.8)
\]

\[
\frac{D_{t+1}}{P_t} = \Theta_t^d (P_t) \zeta, \quad (E.1.9)
\]

\[M_{t}^{Fed} = M_t, \quad (E.1.10)\]

\[R_m t = 1 + i_{ior} t P_t + 1 / P_t. \quad (E.1.11)\]

The last term is the definition of \( R_m t \). This block determines \( \{ P_t, R_b t, R_m t, R_d t \} \) given aggregate bank variables. Notice that \( M_{t}^{Fed} = M_t \) pins down the price level using \( P_t m_t (1 - c_t) E_t \). To close the system, we need the equations that determine \( \chi_t \).

**Interbank Market Block**

We need to determine \( \bar{\chi}_t \). This follows from the conditions obtained from Proposition 1:

\[
S_t^- = \int_{1}^{n/d-\rho} s(\omega) d\Phi \quad \text{and} \quad S_t^+ = \int_{\omega/d - \rho}^{\infty} s(\omega) d\Phi.
\]

The market tightness is defined as

\[\theta_t = S_t^- / S_t^+.\]

From here, discount window loans are

\[W_t = (1 - \Psi^{-} (\theta_t)) S_t^- \quad (E.1.12)\]

and the average interbank market rate, \( \bar{i}_t^f \), is

\[\bar{i}_t^f = \phi(\theta_t) i_{ior}^t + (1 - \phi(\theta_t)) i_{dw}^t.\]

This system of equations gives us

\[\chi_t^- = \Psi_t^- \left( \bar{i}_t^f - i_{ior}^t \right) + (1 - \Psi_t) \left( i_{dw}^t - i_{ior}^t \right) \quad \text{and} \quad \chi_t^+ = \Psi_t^+ \left( \bar{i}_t^f - i_{ior}^t \right). \quad (E.1.13)\]

Note that here we take the probabilities \( \Psi_t^- \) and \( \Psi_t^+ \) as given functions of market tightness, as in the main text. This block determines \( \bar{\chi}_t \) and the amount of discount window loans, \( W_t \). Note that so far, we have provided enough equations to solve for \( \{ b_t, \bar{m}_t, \bar{d}_t, c_t, \Omega_t, v_t \} \), \( \{ B_t, M_t, D_t, E_t \} \), and \( \{ P_t, R_b t, R_m t, R_d t, \bar{\chi}_t \} \). The value of \( W_t \) enters in the Fed’s budget constraint.

**Fed Budget Constraint**

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The government’s budget policy sequence \( \{ \rho_t, \kappa_t, \theta^i_t, \theta^d_t, \theta^w_t, W_t, B_t^{Fed}, M_t^{Fed}, T_t \} \) satisfies the following constraint:

\[
M_t(1 + \theta^i_t) + B_t^{Fed} + W_{t+1} = M_{t+1} + D_t^{Fed}(1 + \theta^d_t) + B_t^{Fed}(1 + \theta^i_t) + W_t(1 + \theta^d_t) + P_t T_t.
\]

**Law of Motion for Aggregate Equity**

A useful expression is obtained combining the individual laws of motion with the Fed’s budget constraint:

\[
E_{t+1} = \left( R_t^b \bar{b}_t + R_m^m \bar{m}_t - R_d^d \bar{d}_t \right) E_t (1 - \bar{c}_t) - \frac{B_t^{Fed} - \bar{M}_{t+2} - \left( B_t^{Fed}(1 + \theta^b_t) - \bar{M}_{t+1}^{Fed}(1 + \theta^i_t) \right)}{P_{t+1}}.
\]  
(E.1.14)

Equation (E.1.14) shows that portfolio choices, market returns, and next-period Fed policies and price level determine next-period aggregate real equity.

**E.2 Stationary Equilibrium**

Consider now the equilibrium conditions for a stationary equilibrium. These are summarized by replacing time subscripts for steady state subscripts \( ss \).

**Individual Bank Variables**

For the individual bank variables, we have

\[
\begin{align*}
\bar{c}_s &= 1 - \beta \Omega_s^{1/\gamma - 1} \quad \text{(E.2.1)} \\
\bar{v}_s &= \frac{1}{1 - \gamma} \left( \frac{1}{1 - \left( \beta \Omega_s^{1-\gamma} \right)^{\frac{\gamma}{\gamma}}} \right) \quad \text{(E.2.2)}
\end{align*}
\]

\[
\Omega_s \equiv \max_{b,m,d} \left\{ \mathbb{E}_\omega \left[ \left( \frac{(1 + \theta^b)\bar{b} + (1 + \theta^{ior})\bar{m} - (1 + \theta^d)\bar{d} + \chi(\bar{m}, \bar{d}) (1 - \tau^{ss})}{1 + \pi} \right)^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}, \quad \text{(E.2.3)}
\]

\( \bar{b} + \bar{m} - \bar{d} = 1, \)

\( \bar{d} \leq \kappa \left( \bar{b} + \bar{m} - \bar{d} \right), \)

where \( \{ \bar{b}_{ss}, \bar{m}_{ss}, \bar{d}_{ss} \} \) are the optimal choices of \( \{ \bar{b}, \bar{m}, \bar{d} \} \) in the problem above.

**Aggregate Bank Variables and Market Clearing Conditions**
The nominal rates and price sequences are given by
\[
(1 - c_{ss})\hat{b}_{ss} E_{ss} = \Theta^b \left(1 + \frac{i^b}{1 + \pi}\right)^{\epsilon} - B_{t+1}^{Fed} / P_t
\]  
(E.2.4)
\[
(1 - c_{ss})\bar{d}_{ss} E_{ss} = \Theta^d \left(1 + \frac{i^d}{1 + \pi}\right)^{\zeta}
\]  
(E.2.5)
\[
(1 - c_{ss})\bar{m}_{ss} E_{ss} = M_{t}^{Fed} / P_t.
\]  
(E.2.6)

Interbank Market Block

The interbank market tightness is
\[
\theta_{ss} \equiv S_t^- / S_t^+, 
\]  
(E.2.7)

and the interbank market conditions are
\[
W_t^{Fed} = \Psi^- S_t^-, \quad \Psi^- = \Psi^- (\theta_t) \text{ and } \Psi^+ = \Psi^+ (\theta_{ss})
\]  
(E.2.8)
\[
\bar{S}_t^- = \int_1^{m_{ss}/d_{ss}-\rho} s(\omega) d\Phi 
\]  
\[
\bar{S}_t^+ = \int_1^{\infty} s(\omega) d\Phi
\]
\[
\bar{z}_t^f = \phi(\theta_{ss}) \bar{i}_{ss}^f + (1 - \phi(\theta_{ss})) \bar{i}_{t}^{dw},
\]  
(E.2.10)
\[
\chi_t^- = \Psi^- \left(\bar{z}_t^f - \bar{i}_{ss}^f\right) + (1 - \Psi^-) \left(\bar{i}_{t}^{dw} - \bar{i}_{ss}^f\right) \text{ and } \chi_t^+ = \Psi^+ \left(\bar{z}_t^f - \bar{i}_{ss}^f\right).
\]  
(E.2.11)

Government Budget Constraint and Aggregate Equity

The government budget constraint and the law of motion for equity are given by
\[
B_{t+1}^{Fed} \left(1 + \frac{i^b}{1 + \pi}\right) + W_t^{Fed} \left(1 + \frac{i^{dw}}{1 + \pi}\right) = M_{t}^{Fed} \left(1 + \frac{i^{ior}}{1 + \pi}\right) + P_t E_{ss} \frac{\tau_{ss}}{1 - \tau_{ss}}, \forall t
\]  
(E.2.12)
\[
E_{ss} = \frac{((1 + i^b)\hat{b}_{ss} + (1 + i^{ior})\bar{m}_{ss} - (1 + i^d)\bar{d}_{ss})}{1 + \pi} \left(1 - \bar{c}_{ss}\right) E_{ss} - \frac{1 + \bar{i}_{t}^{dw}}{1 + \pi} W_t^{Fed} / P_t - E_{ss} \frac{\tau_{ss}}{1 - \tau_{ss}}
\]  
(E.2.13)
\[
1 + \pi = P_t / P_{t-1}.
\]  
(E.2.14)
F  Expressions for \{\Psi^{+}, \Psi^{-}, \phi, \bar{\iota}_{t}^{f}, \chi^{+}, \chi^{-}\} in Proposition 1

Here we reproduce formulas derived from Proposition 1 in the companion paper, Bianchi and Bigio (2017). The companion paper includes the market structure that delivers these functional forms. This proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market. The formulas are the following.

Given \( \theta \), the market tightness after the federal funds trading session is

\[
\bar{\theta} = \begin{cases} 
1 + (\theta - 1) \exp(\lambda) & \text{if } \theta > 1 \\
1 & \text{if } \theta = 1 \\
(1 + (\theta^{-1} - 1) \exp(\lambda))^{-1} & \text{if } \theta < 1
\end{cases}
\]

Trading probabilities are given by

\[
\Psi^{+} = \begin{cases} 
1 - e^{-\lambda} & \text{if } \theta \geq 1 \\
\theta (1 - e^{-\lambda}) & \text{if } \theta < 1
\end{cases}, \quad \Psi^{-} = \begin{cases} 
(1 - e^{-\lambda}) \theta^{-1} & \text{if } \theta > 1 \\
1 - e^{-\lambda} & \text{if } \theta \leq 1
\end{cases}
\]

The reduced-form bargaining parameter is

\[
\phi \equiv \begin{cases} 
\frac{\theta}{\bar{\theta}^{-1}} \left( \frac{\bar{\theta}}{\theta} \right)^{\eta} - 1 \left( \exp(\lambda) - 1 \right)^{-1} & \text{if } \theta > 1 \\
\eta & \text{if } \theta = 1 \\
\frac{\theta(1-\theta)-\bar{\theta}}{\theta(1-\bar{\theta})} \left( \frac{\bar{\theta}}{\theta} \right)^{\eta} - 1 \left( \exp(\lambda) - 1 \right)^{-1} & \text{if } \theta < 1
\end{cases}
\]

and \( \bar{\iota}_{t}^{f} = (1 - \phi)i^{dw} + \phi i^{ior} \). The slopes of the liquidity yield function are given by

\[
\chi^{+} = (i^{dw} - i^{ior}) \left( \frac{\bar{\theta}}{\theta} \right)^{\eta} \left( \frac{\theta^{\eta} \bar{\theta}^{1-\eta} - \theta}{\theta - 1} \right) \quad \text{and} \quad \chi^{-} = (i^{dw} - i^{ior}) \left( \frac{\bar{\theta}}{\theta} \right)^{\eta} \left( \frac{\theta^{\eta} \bar{\theta}^{1-\eta} - 1}{\theta - 1} \right).
\]

G  Proofs of Propositions 2 and 3 and Liquidity Premium

The proofs of Proposition 2 and Proposition 3 make use of the following two lemmata. First, we establish the homogeneity in \( \chi \):

Lemma 1. The function \( \bar{\chi}_{t} \) is homogeneous of degree 1 in \( (m, d) \).

**Proof.** We need to show \( \bar{\chi}_{t} (am, ad, \omega) = a \bar{\chi}_{t} (m, d, \omega) \) for any \( a > 0 \). By definition:

\[
\bar{\chi}_{t}(am, ad, \omega) = \begin{cases} 
\chi_{t}^{+} s & \text{if } s \geq 0 \\
\chi_{t}^{-} s & \text{if } s < 0
\end{cases},
\]

\[
s = am + a\omega d \frac{1 + i^{d}_{t+1}}{1 + i^{ior}_{t+1}} - \rho d \left(1 + \omega\right), \quad \text{(G.1)}
\]

where \( \chi_{t}^{-} \) and \( \chi_{t}^{+} \) are functions of \( \{\Psi_{t}^{-}, \Psi_{t}^{+}, \bar{\iota}^{f}_{t}, \theta_{t}\} \) and independent of \( m \) and \( d \). We can factor the constant \( a \) from the right-hand side of (G.1) and obtain

\[
s = a \left( m + \omega d \frac{1 + i^{d}_{t+1}}{1 + i^{ior}_{t+1}} - \rho d \left(1 + \omega\right) \right).
\]

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Define the position without the scaling factor $a$ as $\tilde{s}$ given by

$$\tilde{s} = \left( m + \omega d \left[ \frac{1 + \bar{i}_{t+1}^d}{1 + \bar{i}_{t+1}^{ior}} - \rho d (1 + \omega) \right] \right).$$

Observe that $(s > 0) \iff (\tilde{s} > 0)$, $(s < 0) \iff (\tilde{s} < 0)$ and $(s = 0) \iff (\tilde{s} = 0)$. Thus,

$$\check{\chi}_t(a, m, d, \omega) = \left\{ \begin{array}{ll} \chi_t^+ s & \text{if } s \geq 0 \\ \chi_t^- s & \text{if } s < 0 \end{array} \right. = \left\{ \begin{array}{ll} \chi_t^+ a \tilde{s} & \text{if } s \geq 0 \\ \chi_t^- a \tilde{s} & \text{if } s < 0 \end{array} \right. = a \check{\chi}_t(m, d, \omega).$$

The last line verifies that $\chi$ is homogeneous of first degree. QED.

The next lemma establishes that an increase in the (gross) nominal policy rates by a constant scales $\chi_t$ by that constant. We use this lemma in the policy analysis results when we discuss the neutrality of inflation.

**Lemma 2.** Let $\chi_t$ be given by two policy rates, $\{i_{t, t}^{ior}, i_{t, t}^{dw}\}$, given $\theta_t$. Consider alternative rates $\{i_{t, t}^{ior}, i_{t, t}^{dw}\}$ such that they satisfy $(1 + i_{t, t}^{ior}) \equiv k(1 + i_{t}^{ior})$ and $(1 + i_{t, t}^{dw}) \equiv k(1 + i_{t}^{dw})$ for some $k$. Then, the $\check{\chi}_{a,t}$ associated with $\{i_{t, t}^{ior}, i_{t, t}^{dw}\}$ for the same $\theta_t$ satisfy $\check{\chi}_{a,t} = k\check{\chi}_t$.

**Proof.** Observe that $\chi_t$ in Definition 1 (which follows from Proposition 1) is a function scaled by the width of the corridor system $(i_{t, t}^{dw} - i_{t}^{ior})$. Then,

$$i_{t, t}^{dw} - i_{t, t}^{ior} = (1 + i_{t}^{dw}) - (1 + i_{t}^{ior}) = k((1 + i_{t}^{dw}) - (1 + i_{t}^{ior})) = k(i_{t}^{dw} - i_{t}^{ior}).$$

Then the result follows immediately from the functional form of $\chi_t$ in Proposition 1. QED.

**G.1 Proof of Proposition 2**

We have to show that the recursive problems of banks during the lending and balancing stages can be summarized as a single Bellman equation $V_t(e)$ where $e$ is a single state variable and $V_t$ the value at the lending stage. To show this, define the after-tax real value of equity at the start of a lending stage:

$$e_t \equiv \frac{(1 + \bar{i}b_t + (1 + \bar{i}^{ior})m_t - (1 + \bar{i}^d)d_t - (1 + \bar{i}^f)f_t - (1 + \bar{i}^{dw})w_t - P_tT_t}{P_t}.$$

This term is the right-hand side of equation (14) in Problem 1 over the price level. If we use this definition, the budget constraint of a given bank satisfies

$$c_t + \frac{\tilde{b}_t + \bar{m}_t - \bar{d}_t}{P_t} = e_t. \quad (G.1)$$

The capital requirement constraint only depends on $\{\tilde{b}_t, \bar{m}_t, \bar{d}_t\}$, and the budget constraint is independent of the composition of real equity; the value $V_t^l(b, m, d, f, w)$ depends only on $e$, not its composition. Therefore, this implies the relation $V_t(e) \equiv V_t^l(b, m, d, f, w)$. This shows that $V_t$ is the value at the lending stage. Next, we try to find a recursive expression for $V_t$.

The next step shows that, indeed, $V_t(e)$ can be written recursively. The value of real equity at $t + 1$ can be written in terms of variables determined at the lending stage of period $t$ and the shock
\[ e_{t+1} = \frac{(1 + i^b_{t+1})b_{t+1} + (1 + i_{t+1}^{ior})m_{t+1} - (1 + i_{t+1}^d)d_{t+1} - (1 + i_{t+1}^f)f_{t+1} - (1 + i_{t+1}^{dw})w_{t+1} - P_{t+1}T_{t+1}}{P_{t+1}}. \]

By assumption, the tax \( T_t \) is proportional to real equity. Thus, we can write \( T_{t+1} = \frac{\tau_{t+1}}{1-\tau_{t+1}}e_{t+1} \) for some convenient choice of \( \tau_{t+1} \). Multiplying both sides by \( 1 - \tau_{t+1} \) and rearranging, we obtain

\[ e_{t+1} = \frac{(1 + i^b_{t+1})b_{t+1} + (1 + i_{t+1}^{ior})m_{t+1} - (1 + i_{t+1}^d)d_{t+1} - (1 + i_{t+1}^f)f_{t+1} - (1 + i_{t+1}^{dw})w_{t+1}}{P_{t+1}} (1 - \tau_{t+1}). \]

(G.2)

Now, observe that by definition of \( \tilde{m}_{t+1} \) and \( \tilde{d}_{t+1} \),

\[ m_{t+1} = \tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \left( \frac{1 + i_{t+1}^{ior}}{1 + i_{t+1}^{ior}} \right) + f_{t+1} + w_{t+1}, \]

and

\[ d_{t+1} = \tilde{d}_{t+1} + \omega_t \tilde{d}_{t+1}. \]

Substituting these last two expressions in the evolution of equity (G.2), we obtain the after-tax value of equity \((1 + \tau_{t+1}/(1 - \tau_{t+1})) P_{t+1} e_{t+1} \):

\[
\begin{align*}
&= (1 + i^b_{t+1})b_{t+1} + (1 + i_{t+1}^{ior}) \left( \tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \frac{1 + i_{t+1}^d}{1 + i_{t+1}^{ior}} + f_{t+1} + w_{t+1} \right) \\
&\quad - (1 + i_{t+1}^d)(1 + \omega_t)\tilde{d}_{t+1} - (1 + i_{t+1}^f)f_{t+1} - (1 + i_{t+1}^{dw})w_{t+1} \\
&= (1 + i^b_{t+1})b_{t+1} + (1 + i_{t+1}^{ior}) \left( \tilde{m}_{t+1} + \omega_t \tilde{d}_{t+1} \frac{1 + i_{t+1}^d}{1 + i_{t+1}^{ior}} \right) - (1 + i_{t+1}^d)(1 + \omega_t)\tilde{d}_{t+1} \\
&\quad - \left( i_{t+1}^f - i_{t+1}^{ior} \right) f_{t+1} - (i_{t+1}^{dw} - i_{t+1}^{ior}) w_{t+1} \\
&= (1 + i^b_{t+1})b_{t+1} + (1 + i_{t+1}^{ior})\tilde{m}_{t+1} - (1 + i_{t+1}^d)\tilde{d}_{t+1} - \left( i_{t+1}^f - i_{t+1}^{ior} \right) f_{t+1} - (i_{t+1}^{dw} - i_{t+1}^{ior}) w_{t+1}.
\end{align*}
\]

By Proposition 1 and by the definition of \( \chi(s), (10) \), the law of motion for \( e_{t+1} \) satisfies

\[ e_{t+1} = \frac{\tilde{b}_{t+1}(1 + i^b_{t+1}) + \tilde{m}_{t+1}(1 + i_{t+1}^{ior}) - \tilde{d}_{t+1}(1 + i_{t+1}^d) + \tilde{\chi}_{t} \left( \tilde{m}_{t+1}, \tilde{d}_{t+1}, \omega \right)}{P_{t+1}} (1 - \tau_{t+1}). \]

(G.3)

Now, since we already showed that there exists a function \( V_t(e) = V_t(b, m, d, f, w), \) the value function at the balancing stage can be written in terms of the single state—future equity—as

\[ V^b_t(\tilde{b}, \tilde{m}, \tilde{d}, \omega) = \beta V_{t+1}(e') \]

\[ e' = \frac{\tilde{b}(1 + i^b_{t+1}) + \tilde{m}(1 + i_{t+1}^{ior}) - \tilde{d}(1 + i_{t+1}^d) + \tilde{\chi}_{t} \left( \tilde{m}, \tilde{d}, \omega \right)}{P_{t+1}} (1 - \tau_{t+1}). \]

(G.4) (G.5)

From here, we substitute this value function at the balancing stage into the value at the lending
stage and obtain

$$E_t \left[ V_{t+1}^{b}(\tilde{b}, \tilde{m}, \tilde{d}, \omega) \right] = E_t \left[ V_{t+1}(e') \right],$$

and thus, we have that

$$V_t(e) = \max_{c, \tilde{m}, \tilde{b}, \tilde{d}} u(c) + \beta E_t \left[ V_{t+1}(e') \right],$$

$$e = \frac{\tilde{b} + \tilde{m} - \tilde{d}}{P_t} + c,$$

$$\tilde{d} \leq \kappa \left( \tilde{b} + \tilde{m} - \tilde{d} \right)$$

$$e' = \frac{\tilde{b}(1 + i_{t+1}^b) + \tilde{m}(1 + i_{t+1}^{ior}) - \tilde{d}(1 + i_{t+1}^d) + \tilde{\chi}_{t+1} \left( \tilde{m}, \tilde{d}, \omega \right)}{P_{t+1}} \left( 1 - \tau_{t+1} \right).$$

Using the definitions of real returns in the main text is enough to establish the claim in the proposition. QED.

**G.2 Proof of Items (i)-(iv) in Proposition 3**

This section presents a proof of Proposition 3. Here we show that the single state representation satisfies homogeneity. We follow the guess-and-verify approach, common to all dynamic programming models. Our guess is that the value function satisfies $V_t(e) = v_t e^{1-\gamma} - 1/(1 - \beta) (1 - \gamma)$, where $v_t$ is a time-varying scaling factor in the value function, common to all banks. From Proposition 1, the bank’s problem is summarized by

$$V_t(e) = \max_{c, \tilde{m}, \tilde{b}, \tilde{d}} u(c) + \beta E_t \left[ V_{t+1}(e') \right],$$

subject to

$$c + \frac{\tilde{b} + \tilde{m} - \tilde{d}}{P_t} = e,$$

$$\tilde{d} \leq \kappa \left( \tilde{b} + \tilde{m} - \tilde{d} \right)$$

$$e' = \left( 1 + i_{t+1}^b \right) \tilde{b} + \left( 1 + i_{t+1}^{ior} \right) \tilde{m} - \left( 1 + i_{t+1}^d \right) \tilde{d} + \tilde{\chi}_{t+1} \left( \tilde{m}, \tilde{d}, \omega \right) \left( 1 - \tau_{t+1} \right).$$

Note that multiplying and dividing by $P_t$, we have that $e'$ can also be written as

$$e' = \frac{\left( \tilde{b}(1 + i_{t+1}^b) + \tilde{m}(1 + i_{t+1}^{ior}) - \tilde{d}(1 + i_{t+1}^d) + \tilde{\chi}_{t+1} \left( \tilde{m}, \tilde{d}, \omega \right) \right) \left( 1 - \tau_{t+1} \right)}{P_{t+1}} \left( 1 + \pi_{t+1} \right),$$

where $(1 + \pi_{t+1}) = P_{t+1}/P_t$. 

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If the conjecture for the value function is correct, then this condition satisfies
\[
v_t e^{1-\gamma} - \frac{1}{(1 - \beta)(1 - \gamma)} = \max_{c,\bar{b},\bar{m},\bar{d}} \frac{c^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E}_t \left[ v_{t+1} (e')^{1-\gamma} - \frac{1}{(1 - \gamma)(1 - \beta)} \right],
\]
subject to
\[
c + \frac{\bar{b} + \bar{m} - \bar{d}}{P_t} = e,
\]
\[
\bar{d} \leq \kappa \left( \bar{b} + \bar{m} - \bar{d} \right)
\]
\[
e' = \frac{\bar{b}(1 + i^b_{t+1}) + \bar{m}(1 + i^m_{t+1}) - \bar{d}(1 + i^d_{t+1}) + \bar{\chi}_{t+1} \left( \bar{m}, \bar{d}, \omega \right)}{P_t} \left( 1 - \tau_{t+1} \right) \left( 1 + \pi_{t+1} \right).
\]

Observe that we can factor out constants from the objective:
\[
\frac{c^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E}_t \left[ v_{t+1} (e')^{1-\gamma} - \frac{1}{(1 - \gamma)(1 - \beta)} \right] = \frac{c^{1-\gamma}}{1 - \gamma} + \beta \mathbb{E}_t \left[ v_{t+1} (e')^{1-\gamma} \right] - \frac{1}{(1 - \beta)(1 - \gamma)}.
\]

Then, if we substitute the evolution of \(e'\) in (G.1), we obtain
\[
v_t e^{1-\gamma} = \max_{c,\bar{b},\bar{m},\bar{d}} \frac{c^{1-\gamma}}{1 - \gamma} + \beta \mathbb{E}_t \left[ v_{t+1} \left( \frac{\bar{b}(1 + i^b_{t+1}) + \bar{m}(1 + i^m_{t+1}) - \bar{d}(1 + i^d_{t+1}) + \bar{\chi}_{t+1} \left( \bar{m}, \bar{d}, \omega \right)}{P_t} \right) \left( 1 - \tau_{t+1} \right) \left( 1 + \pi_{t+1} \right) \right]^{1-\gamma},
\]
subject to
\[
e = \frac{\bar{b} + \bar{m} - \bar{d}}{P_t} + c
\]
\[
\bar{d} \leq \kappa \left( \bar{b} + \bar{m} - \bar{d} \right).
\]

Let us define variables in terms of equity, \(\bar{c} = c/e\). Also, define \(\bar{b} = \bar{b}/ ((1 - \bar{c})eP_t)\), \(\bar{m} = \bar{m}/((1 - \bar{c})eP_t)\), and \(\bar{d} = \bar{d}/((1 - \bar{c})eP_t)\), as in the statement of Proposition 3. By Lemma 1, we can factor constants \((1 - \bar{c})eP_t\) from \(\bar{\chi}_t\) and express it as
\[
(1 - \bar{c})eP_t\bar{\chi}_t \left( \frac{\bar{m}}{P_t(1 - \bar{c})e}, \frac{\bar{d}}{P_t(1 - \bar{c})e}, \omega \right) = P_t(1 - \bar{c})e\bar{\chi}_t (\bar{m}, \bar{d}, \omega).
\]
Using this observation, we can replace $\bar{c}$ in the value function to obtain

$$v_t e^{1-\gamma} = \max_{c,\bar{m},\bar{d},\bar{\tau}} e^{1-\gamma} \frac{c^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} ((1-\bar{c})e)^{1-\gamma} \mathbb{E}_\omega \ldots \quad (G.3)$$

subject to:

$$\begin{align*}
\bar{b} + \bar{m} - \bar{d} \\
\frac{\bar{d}/P_t}{(1-\bar{c})e} \leq \kappa \left( \frac{\bar{b}/P_t}{(1-\bar{c})e} + \frac{\bar{m}/P_t}{(1-\bar{c})e} - \frac{\bar{d}/P_t}{(1-\bar{c})e} \right)
\end{align*}$$

From this expression, we can cancel out $e^{1-\gamma}$ from both sides of (G.3), which verifies that the objective is scaled by $e^{1-\gamma}$. Thus, we verify the guess that $V_t(e) = v_t e^{1-\gamma} - ((1-\beta)(1-\gamma))^{-1}$.

Next, we derive the policies that attain $V_t(e)$ and the value of $v_t$. If the conjecture is correct, using the definition of $\bar{b}$, $\bar{m}$, and $\bar{d}$, we obtain

$$v_t = \max_{\{\bar{b}, \bar{m}, \bar{d}\}} \frac{c^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} \ldots \quad (G.4)$$

$$\mathbb{E}_\omega \left( \left( (1+\bar{\tau}_{t+1}^b)\bar{b} + (1+\bar{\tau}_{t+1}^m)\bar{m} - (1+\bar{\tau}_{t+1}^d)\bar{d} + \chi_{t+1}(\bar{m},\bar{d},\omega) \right) \frac{(1-\tau_{t+1})}{(1+\tau_{t+1})} \right)^{1-\gamma}$$

subject to:

$$\begin{align*}
\bar{b} + \bar{m} - \bar{d} &= 1 \\
\bar{d} &\leq \kappa
\end{align*}$$

Thus, any solution to $V_t(e)$ must be consistent with the solution of $v_t$ if the conjecture is correct.

Define real return on equity as follows:

$$R_{t+1}^E(\bar{b}, \bar{m}, \bar{d}, \omega) \equiv (R_{t+1}^b \bar{b} + R_{t+1}^m \bar{m} - R_{t+1}^d \bar{d} + \chi_{t+1}(\bar{d}, \bar{m}, \omega)) (1-\tau_{t+1}).$$

Then, the value function can be written as

$$v_t = \max_{\{\bar{b}, \bar{m}, \bar{d}\}} \frac{c^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-\bar{c})^{1-\gamma} \mathbb{E}_\omega \left( \left( R_{t+1}^E(\bar{b}, \bar{m}, \bar{d}, \omega) \right)^{1-\gamma} \right).$$

We now use the principle of optimality. Let $\Omega_t$ be the certainty equivalent of the bank’s optimal portfolio problem, that is,

$$\Omega_t \equiv \max_{\{\bar{b}, \bar{m}, \bar{d}\}} \left[ \mathbb{E}_\omega \left[ \left( R_{t+1}^E(\bar{b}, \bar{m}, \bar{d}, \omega) \right)^{1-\gamma} \right] \right]^{1/(1-\gamma)}$$

subject to $\bar{b} + \bar{m} - \bar{d} = 1$ and $\bar{d} \leq \kappa$. Assume $\bar{c}$ is optimal. If $\gamma < 1$, the solution that attains $v_t$ must maximize $\mathbb{E}_\omega \left[ R_{t+1}^E(\bar{b}, \bar{m}, \bar{d}, \omega)^{1-\gamma} \right]$ if $v_{t+1}$ is positive. If $\gamma > 1$, the solution that attains $v_t$ must minimize $\mathbb{E}_\omega \left[ R_{t+1}^E(\bar{b}, \bar{m}, \bar{d}, \omega)^{1-\gamma} \right]$ if $v_{t+1}$ is negative. We guess and verify that when $\gamma < 1$, the term $v_{t+1}$ is positive and $v_{t+1}$ is negative when $\gamma > 1$. Under this assumption, if $\gamma < 1$, we have that
\( v_{t+1} > 0 \), so \( 1 - \gamma > 0 \). Thus, by maximizing \( \Omega_t \), we are effectively maximizing the right-hand side of \( v_t \). Instead, when \( \gamma > 1 \), we have that \( v_{t+1} < 0 \), so \( 1 - \gamma < 0 \). Thus, by maximizing \( \Omega_t \), we are minimizing \( \Omega_t^{1-\gamma} \), which multiplied by a negative number—\( v_{t+1} \)—maximizes the right-hand side of \( v_t \).

Hence, the Bellman equation becomes

\[
v_t = \max_{\{e,b,m,d\} \geq 0} \frac{e^{1-\gamma}}{(1-\gamma)} + \beta v_{t+1} (1-e)^{1-\gamma} \Omega_t^{1-\gamma}.
\]

This yields the statements in items (i) and (ii), provided that \( v_t \) inherits the sign of \( (1 - \gamma) \).

To prove item (iii), we take the first-order conditions with respect to \( \bar{c} \), and raising both sides to the \( -\frac{1}{\gamma} \) power, we obtain

\[
\bar{c} = (\bar{v}_{t+1})^{-\frac{1}{\gamma} \Omega_t^{-(1-\gamma)\gamma}} (1 - \bar{e}) (1 - \gamma)^{-\frac{1}{\gamma}}.
\]

We can rearrange terms to obtain

\[
\bar{c} = \frac{1}{1 + [\beta v_{t+1} (1 - \gamma) \Omega_t^{1-\gamma}]^{1/\gamma}}.
\]

Define \( \xi_t = (1 - \gamma) \beta v_{t+1} \Omega_t^{1-\gamma} \). Under the conjectured sign of \( v_t \), the term \( \xi_t \) is always positive. Substituting this expression for dividends, we obtain a functional equation for the value function

\[
v_t = \frac{1 + \xi_t^{1/\gamma}}{(1-\gamma)} + \beta v_{t+1} \Omega_t^{1-\gamma} \left[ \frac{\xi_t^{1/\gamma}}{1 + \xi_t^{1/\gamma}} \right]^{(1-\gamma)} = \frac{1 + \xi_t^{1/\gamma}}{(1-\gamma)} + \frac{\xi_t}{(1-\gamma)} \left[ \frac{\xi_t^{1/\gamma}}{1 + \xi_t^{1/\gamma}} \right]^{(1-\gamma)}
\]

and finally,

\[
v_t = \frac{1}{(1-\gamma)} \left[ (1 + \xi_t^{1/\gamma})^{-(1-\gamma)} + \xi_t \left[ \frac{\xi_t^{1/\gamma}}{1 + \xi_t^{1/\gamma}} \right]^{(1-\gamma)} \right].
\]

Thus, we obtain

\[
v_t = \frac{1}{(1-\gamma)} \left[ \frac{1}{(1 + \xi_t^{1/\gamma})^{(1-\gamma)}} + \frac{\xi_t^{1/\gamma}}{1 + \xi_t^{1/\gamma}} \right]^{(1-\gamma)} = \frac{1}{(1-\gamma)} \frac{1 + \xi_t^{1/\gamma}}{(1 + \xi_t^{1/\gamma})^{(1-\gamma)}} = \frac{1}{(1-\gamma)} \left( 1 + \xi_t^{1/\gamma} \right)^{\gamma}.
\]

This verifies that \( v_t \) inherits the sign of \( (1 - \gamma) \). Thus, we can use \( \Omega^* \) directly in the value function. Furthermore, \( v_t \) satisfies the following difference equation:

\[
v_t = \frac{1}{1 - \gamma} \left[ 1 + \left( \beta (1 - \gamma) \Omega_t^{1-\gamma} v_{t+1} \right)^{\frac{1}{\gamma}} \right].
\]  

(this verifies that \( v_t \) inherits the sign of \( (1 - \gamma) \). Thus, we can use \( \Omega^* \) directly in the value function. Furthermore, \( v_t \) satisfies the following difference equation:

\[
v_t = \frac{1}{1 - \gamma} \left[ 1 + \left( \beta (1 - \gamma) \Omega_t^{1-\gamma} v_{t+1} \right)^{\frac{1}{\gamma}} \right].
\]

We can treat the right-hand side of this functional equation, solved independently of consumption.
If we solve for this equation independently of the banker’s consumption, we can obtain a solution to the banker’s consumption policy via

$$\bar{c} = \frac{1}{1 + [\beta v_{t+1}(1 - \gamma)\Omega_t^{1-\gamma}]^{1/\gamma}}.$$

This concludes the proof of items (i)-(iv), for all cases except $\gamma \to 1$. We work out that case next.

**Log-Case.** Observe that as $\gamma \to 1$, then $v_t$ in (G.5) explodes. However, we can guess and verify that

$$\lim_{\gamma \to 1} v_t(1 - \gamma) = 1 / (1 - \beta).$$

This assumption can be verified in equation (G.5). In this case,

$$\lim_{\gamma \to 1} (1 - \gamma) v_t = \lim_{\gamma \to 1} \left[1 + (\beta(1 - \gamma)\Omega_t^{1-\gamma}v_t)^{1/\gamma}\right] = 1 + \beta / (1 - \beta) = 1 / (1 - \beta).$$

Thus, as $\gamma \to 1$, we have that $c = (1 - \beta)$. Thus,

$$\Omega_t \equiv \max_{\{b, m, d\}} \exp \left(\mathbb{E}_\omega [\log (R_t^E(b, m, d, \omega))]\right).$$

QED.

**G.3 Derivation of Premia and Proof of Proposition 4**

In this section, we avoid all time subscripts and study the properties of the static portfolio problem (25):

$$\Omega \equiv (1 - \tau) \max_{d \in [0, k], m \in [0, 1 + s]} \left\{ \mathbb{E}_\omega \left[R^b + (R^m - R^b) \bar{m} + (R^b - R^d) \bar{d} + \bar{\chi}(m, d, \omega)\right]^{1-\gamma} \right\}^{1/\gamma}.$$

Consider the optimal choice of $\{\bar{d}, \bar{m}\}$. The optimal choice leads to a threshold condition $\omega^*(\bar{d}, \bar{m})$.

Partition the expectation inside the objective into two terms:

$$\int_{-1}^{\omega^*(\bar{d}, \bar{m})} \left[R^b + (R^m - R^b) \bar{m} + (R^b - R^d) \bar{d} + \bar{\chi}(m, d, \omega)\right]^{1-\gamma} f(\omega) d\omega +$$

$$\int_{\omega^*(\bar{d}, \bar{m})}^{\infty} \left[R^b + (R^m - R^b) \bar{m} + (R^b - R^d) \bar{d} + \bar{\chi}(m, d, \omega)\right]^{1-\gamma} f(\omega) d\omega.$$

**Derivation of the Liquidity Premium.** Assuming the solution is interior in $\bar{m}$, we can take the derivative with respect to $\bar{m}$ to obtain

$$\mathbb{E}_\omega \left[\left(R^b_\omega\right)^{-\gamma} \left(R^m - R^b\right)\right] + \int_{-1}^{\omega^*(\bar{d}, \bar{m})} \left(R^b_\omega\right)^{-\gamma} \bar{\chi}(m, d, \omega) f(\omega) d\omega + \int_{\omega^*(\bar{d}, \bar{m})}^{\infty} \left(R^b_\omega\right)^{-\gamma} \bar{\chi}(m, d, \omega) f(\omega) d\omega = 0.$$

The terms corresponding to the limits of integration do not appear because $s(\bar{m}, \bar{d}, \omega^*(\bar{d}, \bar{m})) = 0$. 

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Hence, the terms associated with Leibniz’s rule vanish. Rearranging terms leads to

\[ R^b - R^m = \frac{\mathbb{E}_\omega \left( (R^e_\omega)^{-\gamma} \frac{\partial \bar{\chi}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right)}{\mathbb{E}_\omega \left( (R^e_\omega)^{-\gamma} \right)}, \]  

where the expectation operator excludes the point \( \omega = \omega^* \)—since this is a zero probability event, we simply exclude the point where the derivative is not included in the notation.

**Derivation of the External Financing Premium.** The derivation of the external financing premium follows the same steps. However, the presence of the capital requirement constraint implies that

\[ R^b - R^d \geq -\frac{\mathbb{E}_\omega \left( (R^e_\omega)^{-\gamma} \frac{\partial \bar{\chi}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right)}{\mathbb{E}_\omega \left( (R^e_\omega)^{-\gamma} \right)}, \]  

where the inequality is strict if the capital requirement binds.

**Factorization of Portfolio Problem.** We derive a useful decomposition. Consider any solution with \( \bar{d} > 0 \). We can write the portfolio problem as:

\[
\max_{\bar{d} \in [0, \kappa]} \left( \mathbb{E}_\omega \left[ (R^b + \bar{d} [(R^b - R^d) - (R^b - R^m) L + \bar{\chi}(L, 1, \omega)])^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}
\]  

where \( L \equiv \bar{m}/\bar{d} \). Observe that

\[
 s = \left( \bar{m} + \left( \frac{R^d}{R^d} \right) \omega \bar{d} \right) - \rho \left( \bar{d} (1 + \omega) \right)
 = \bar{d} \left( L + \left( \frac{R^d}{R^d} \right) \omega - \rho ((1 + \omega)) \right).
\]

The threshold shock that triggers a negative balance is given by

\[
\omega^*(\bar{m}, \bar{d}) \equiv \frac{\bar{m}/\bar{d} - \rho}{\rho - \left( \frac{R^d}{R^m} \right)} = \frac{L - \rho}{\rho - \left( \frac{R^d}{R^m} \right)}.
\]

Thus,

\[
\bar{\chi}(\bar{m}, \bar{d}, \omega) = \bar{d} \bar{\chi}(L, 1, \omega),
\]

and the factorization follows. If \( \bar{d} > 0 \), the \( L \) if \( L \geq L_{min} \) such that

\[
\omega_{min} = \frac{L_{min} - \rho}{\rho - \left( \frac{R^d}{R^m} \right)},
\]

the bank is always in surplus. To see this, notice that

\[
L_{min} = \rho + \omega_{min} \left( \rho - \left( \frac{R^d}{R^m} \right) \right) = \rho (1 + \omega_{min}) - \omega_{min} \left( \frac{R^d}{R^m} \right),
\]

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which is positive since

\[-1 < \omega_{\text{min}} < 0.\]

In equilibrium, when all banks chose \( L \geq L_{\text{min}} \) we have

\[\bar{\chi}(L_{\text{min}}, 1, \omega) = 0, \text{ for any } \omega.\]

**Derivatives of the Liquidity Cost Function.** Recall that the surplus function is given by

\[s = \bar{m} - \rho \bar{d} + \bar{d} \left( \frac{R^d}{R^m} - \rho \right) \omega.\]

Hence, we have that

\[
\frac{\partial \bar{\chi}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} = \frac{1}{1 + \pi} \begin{cases} 
\chi^+ & \text{if } \omega > \omega^* \\
\chi^- & \text{if } \omega < \omega^*
\end{cases}
\]

and

\[
\frac{\partial \bar{\chi}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} = \frac{1}{1 + \pi} \begin{cases} 
\chi^+ \left( -\rho + \left( \frac{R^d}{R^m} - \rho \right) \omega \right) & \text{if } \omega > \omega^* \\
\chi^- \left( -\rho + \left( \frac{R^d}{R^m} - \rho \right) \omega \right) & \text{if } \omega < \omega^*
\end{cases}
\]

**First-Order Terms of the Liquidity Premium.** The first-order term of the liquidity premium is

\[
\mathbb{E}_\omega \left[ \frac{\partial \bar{\chi}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] = \frac{1}{1 + \pi} \mathbb{E}_\omega \left[ \frac{\partial \chi(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right]
\]

\[
= \frac{1}{1 + \pi} \mathbb{E}_\omega \left[ \frac{\partial (\chi^+ s \mathbb{I}[s > 0] + \chi^- s \mathbb{I}[s < 0])}{\partial \bar{m}} \right]
\]

\[
= \frac{1}{1 + \pi} \mathbb{E}_\omega \left[ \frac{\partial s}{\partial \bar{m}} \left( \chi^+ \mathbb{I}[s > 0] + \chi^- \mathbb{I}[s < 0] \right) \right]
\]

\[
= \frac{1}{1 + \pi} \left[ \chi^+ (1 - F(\omega^*)) + \chi^- F(\omega^*) \right] \geq 0. \tag{G.4}
\]

**Risk-Neutral Probability.** Note that given \( \{\bar{m}, \bar{d}\} \),

\[\tilde{f} \equiv \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}_\omega [(R^e)^{-\gamma}]}\]

defines a probability density in the space of \( \omega \). Define the risk-neutral probability as \( \tilde{f} \), and let \( \tilde{F} \) be its cumulative distribution function:

\[\tilde{F}(\omega') = \int_{-1}^{\omega'} \frac{(R^e_\omega)^{-\gamma} f(\omega)}{\mathbb{E}[(R^e)^{-\gamma}]} d\omega.\]

Also, let \( \tilde{\mathbb{E}}_\omega \) be the expectation operator under \( \tilde{F} \)—excluding the measure zero point \( \omega = \omega^* \).
Sign of the Liquidity Premium. The liquidity premium satisfies

\[
\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial \tilde{X} (\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] = \tilde{\mathbb{E}}_\omega \left[ \frac{\partial \tilde{X} (\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] = \frac{1}{1 + \pi} \left[ \chi^+ (1 - \tilde{F} (\omega^*)) + \chi^- \tilde{F} (\omega^*) \right]
\]

\[
\geq \frac{1}{1 + \pi} \left[ \chi^+ (1 - F (\omega^*)) + \chi^- F (\omega^*) \right]
\]

\[
\geq \frac{1}{1 + \pi} \chi^+ \geq 0.
\]

(G.5)

The first equality follows from the definition of \( \tilde{f} \). The second line follows the same steps as (G.4). The first inequality holds because the physical probability is dominated (in the first-order stochastic sense) by the risk-neutral probability and \( \chi^- > \chi^+ \). This establishes the following remark.

Remark 1. The liquidity premium is always positive and strictly positive unless \( \chi^+ = 0 \). That is

\[
\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial \tilde{X} (\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] \geq 0.
\]

Liquidity Component of the External Financing Premium. Following similar arguments as in the derivation of (G.5), we obtain

\[
\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial \tilde{X} (\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right] = -\rho \frac{1}{1 + \pi} \left[ \chi^+ (1 - \tilde{F} (\omega^*)) + \chi^- \tilde{F} (\omega^*) \right]
\]

\[
+ \left( \frac{R^d}{R^m} - \rho \right) \frac{1}{1 + \pi} \tilde{\mathbb{E}}_\omega \left[ \omega \chi^+ \mathbb{I}_{[\omega > \omega^*]} + \omega \chi^- \mathbb{I}_{[\omega < \omega^*]} \right].
\]

(G.6)

Let’s examine the second term. Since \( \omega \chi^+ \mathbb{I}_{[\omega > \omega^*]} + \omega \chi^- \mathbb{I}_{[\omega < \omega^*]} \) is a concave function, by Jensen’s inequality, we have that

\[
A1 \equiv \tilde{\mathbb{E}}_\omega \left[ \omega \chi^+ \mathbb{I}_{[\omega > \omega^*]} + \chi^- \omega \mathbb{I}_{[\omega < \omega^*]} \right] \leq \tilde{\mathbb{E}}_\omega \left[ \omega \left( \chi^+ \mathbb{I}_{[\omega > \omega^*]} + \chi^- \mathbb{I}_{[\omega < \omega^*]} \right) \right] \cdot
\]

We also know that \( \tilde{\mathbb{E}}_\omega [\omega] \leq \mathbb{E}_\omega [\omega] = 0 \) because the risk-neutral probability first-order stochastically dominates the physical probability. The condition becomes an equality if there is no risk. Consider the sum

\[
A2 \equiv \left[ \chi^+ (1 - \tilde{F} (\omega^*)) + \chi^- \tilde{F} (\omega^*) \right] + A1
\]

\[
= \left[ \chi^+ (1 - \tilde{F} (\omega^*)) + \chi^- \tilde{F} (\omega^*) \right] + \left( \chi^+ \mathbb{E} [\omega | \omega > \omega^*] (1 - \tilde{F} (\omega^*)) + \chi^- \mathbb{E} [\omega | \omega > \omega^*] \tilde{F} (\omega^*) \right).
\]

Since both \( \mathbb{E} [\omega | \omega > \omega^*] \) and \( \mathbb{E} [\omega | \omega < \omega^*] \) are greater than 1, \( A2 \geq 0 \).

We note then that

\[
\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial \tilde{X} (\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right] = -\rho \frac{1}{1 + \pi} A2 + \frac{R^d}{R^m} \frac{1}{1 + \pi} A1.
\]

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Since \( A_1 \leq 0 \) but \( A_2 \geq 0 \), we established the following remark.

**Remark 2.** The liquidity component of the external financing premium (G.2) is always positive and strictly positive unless \( \chi^+ = 0 \). That is:

\[
-\mathbb{E}_\omega \left[ (R_e^c)^{-\gamma} \cdot \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial d} \right] \geq 0.
\]

**Liquidity Finance Cost.** Now consider the relationship the an the liquidity benefit of an equal increase in \( \{\tilde{d}, \bar{m}\} \). The impact in the portfolio is

\[
A_3 \equiv \mathbb{E}_\omega \left[ (R_e^c)^{-\gamma} \cdot \left( \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial m} + \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial \bar{m}} \right) \right].
\]

Adding (G.5) and (G.2) we obtain

\[
A_3 = (1 - \rho) \mathbb{E}_\omega \left[ \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial \bar{m}} \right] + \left( \frac{R_d^d}{R_m^d} - \rho \right) \frac{1}{1 + \pi} A_1,
\]

which follows immediately from (G.2). Assume that \( R_m^m > R_d^d \), then we have that

\[
A_3 \geq (1 - \rho) \left( \mathbb{E}_\omega \left[ \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial \bar{m}} \right] + A_1 \right)
= (1 - \rho) A_2 \geq 0.
\]

The first inequality follows because \( A_1 \leq 0 \) and \( R_m^m > R_d^d \). The second line follows because \( A_2 \geq 0 \).

**Binding Capital Requirements.** Assume that \( R_m^m - R_d^d > 0 \) and that the capital requirement does not bind. Then, the premium conditions hold with equality:

\[
R_b^b - R_m^m = \mathbb{E}_\omega \left[ (R_e^c)^{-\gamma} \cdot \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial \bar{m}} \right] \quad \text{and} \quad R_b^b - R_d^d = -\mathbb{E}_\omega \left[ (R_e^c)^{-\gamma} \cdot \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial d} \right].
\]

Adding both conditions yields

\[
R_m^m - R_d^d = -A_3 \leq 0,
\]

a contradiction. We established the following property:

**Remark 3.** If capital requirements do not bind, then \( R_d^d \geq R_m^m \)—with strict inequality if \( R^u > R_m^m \). If \( R_m^m - R_d^d > 0 \), the capital requirements bind.

**Derivative of the Liquidity Premium.** Recall the identity (G.5). Take derivatives with respect to \( \bar{m} \) to obtain

\[
\frac{\partial}{\partial \bar{m}} \left[ \mathbb{E}_\omega \left[ \frac{(R_e^c)^{-\gamma}}{\mathbb{E}[(R_e^c)^{-\gamma}]} \cdot \frac{\partial \chi(m, \tilde{d}, \omega)}{\partial m} \right] \right] = \left( \chi^+ - \chi^- \right) \left[ \tilde{f}((\omega^*; \bar{m}) \frac{\partial \omega^*}{\partial \bar{m}} + \frac{\partial \tilde{F}(\omega^*; \bar{m})}{\partial \bar{m}} \right].
\]

We know that \( \frac{\partial \omega^*}{\partial \bar{m}} \leq 0 \) because with higher liquidity in the portfolio, we need a more adverse
shock to produce a reserve deficit:

$$\text{and } \frac{\partial F(\omega^*; \bar{m})}{\partial \bar{m}} \leq 0.$$  

The first term is negative. The second inequality follows because the increase in \( \bar{m} \) makes the portfolio safer. Thus, \( F(\omega^*; \bar{m}) \) puts more weight on lower states. As a result,

$$\frac{\partial}{\partial \bar{m}} \left[ \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] \right] \leq 0. \quad (G.7)$$

Now we consider the derivative with respect to the deposit share of the portfolio, at an interior solution:

$$\frac{\partial}{\partial \bar{d}} \left[ \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right] \right] = (G.8)$$

$$(R^b - R^d) - (R^b - R^m) \cdot L + \bar{X}(L, 1, \omega) - L \frac{\partial}{\partial L} \left[ \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(L, 1, \omega)}{\partial L} \right] \right].$$

The equivalence follows from the equivalence of the portfolio. Since if \( \bar{d} \) requires the first-three terms to be positive, the fourth term is weakly negative because \( \frac{\partial \omega^*}{\partial \bar{m}} \leq 0 \). As a result,

$$\frac{\partial}{\partial \bar{d}} \left[ \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(\bar{m}, \bar{d}, \omega)}{\partial \bar{d}} \right] \right] \geq 0. \quad (G.9)$$

We conclude with the following observation.

**Remark 4.** At an interior portfolio, the following inequalities (G.8) and (G.9) hold.

**Conditions for an Interior Portfolio.** Consider an equilibrium. If \( \bar{d} = 0 \), then: \{\( \bar{b}, \bar{m} \) = \{1, 0\} for any \( R^b > R^m \) and \{\( \bar{b}, \bar{m} \) = \{0, 1\} \}. If \( R^b = R^m \), the portfolio is indeterminate and any \{\( \bar{b}, \bar{m} \) \} in the constraint set is a solution.

Consider the solutions where \( \bar{d} > 0 \). By the Principle of Optimality, the portfolio problem (G.3) is equivalent to

$$\Omega = \max_{L \in [0, (1+\bar{d})/\bar{d}]} \max_{d \in [0, \bar{d}]} \left( \mathbb{E}_\omega \left[ \left( R^b + \bar{d} \left[ (R^b - R^d) - (R^b - R^m) \cdot L + \bar{X}(L, 1, \omega) \right] \right)^{\gamma} \right] \right)^{1/\gamma}. $$

Then, the derivative of the portfolio with respect to \( L \) is given by

$$\left( R^b - R^m \right) - \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(L, 1, \omega)}{\partial L} \right].$$

We have the following results for \( L \in \{0, L_{\min}\} \).

For \( L = 0 \),

$$\mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot \frac{\partial \bar{X}(L, 1, \omega)}{\partial L} \right]_{L=0} = \mathbb{E}_\omega \left[ \frac{(R^e_\omega)^{-\gamma}}{\mathbb{E}[(R^e_\omega)^{-\gamma}]} \cdot (R^w - R^m) \right] = (R^w - R^m),$$

where the equality follows from the equilibrium condition that all banks have the same portfolio and that \( L = 0 \rightarrow \chi^- = (1 - \pi) (R^w - R^m) \).
If \( L = L_{\text{min}} \),

\[
\mathbb{E}_{\omega} \left[ \frac{(R_0^c)^{-\gamma}}{\mathbb{E}[(R_0^c)^{-\gamma}]} \cdot \frac{\partial \tilde{\chi}(L, 1, \omega)}{\partial L} \right]_{L = L_{\text{min}}} = \mathbb{E}_{\omega} \left[ \frac{(R_0^c)^{-\gamma}}{\mathbb{E}[(R_0^c)^{-\gamma}]} \cdot 0 \right] = 0,
\]

where the equality follows from: \( L = L_{\text{min}} \rightarrow \chi^+ = 0 \). Hence, considering the sign of the portfolio, and that the liquidity premium is decreasing in \( \tilde{m} \), we have the following.

**Remark 5.** If \( \tilde{d} > 0 \). Then, \( \tilde{m} \in (0, 1 + \tilde{d}) \) if \( R^w > R^b > R^m \).

Let’s consider the conditions for positive deposits. The derivative of the portfolio problem with respect to \( \tilde{d} \) is given by

\[
(R^b - R^d) - (R^b - R^m)L + \mathbb{E}_{\omega} \left[ \frac{(R_0^c)^{-\gamma}}{\mathbb{E}[(R_0^c)^{-\gamma}]} \cdot \tilde{\chi}(L, 1, \omega) \right].
\]

Now, observe that if \( L = 0 \), the bank is in deficit in all states and \( \tilde{\chi} = R^w - R^m \). Thus, the bank will hold a positive amount of deposits if

\[
(R^b - R^d) + \mathbb{E}_{\omega} [(R^w - R^m) s] = (R^b - R^d) - \rho [(R^w - R^m)] > 0.
\]

and \( L = 0 \). If \( L = L_{\text{min}} \), then all banks are in surplus. Following the same logic,

\[
(R^b - R^d) - (R^b - R^m)L_{\text{min}} > 0.
\]

Thus, we have the following.

**Remark 6.** If

\[
R^b > R^d + \min \left\{ (R^b - R^m)L_{\text{min}}, \rho (R^w - R^m) \right\},
\]

where \( L_{\text{min}} = \rho + \omega_{\text{min}} \left( \rho - \left( \frac{R^d}{R^m} \right) \right) \), then \( \tilde{d} > 0 \).

Combining remarks 1-6, we obtain the proof of Proposition 4. QED.

### G.4 Proof of Proposition 5

**Existence.** Consider the specialization in the statement of the proposition. That is, let \( \lambda = 0 \), \( \gamma = 1 \), and \( B_{\text{Fed}}^{fs} = 0 \). In addition, let the deposit rate converge in the following sense: let \( \zeta \rightarrow \infty \) and \( \lim_{\zeta \rightarrow \infty} \Theta^d(\zeta) = 0 \) such that \( R^d \) converges to a constant for any quantity of deposits. Let the limit \( R^d \) satisfy

\[
1 + \kappa \left( 1 - R^d \right) < \frac{1}{\beta}. \tag{G.1}
\]

Now, consider the correspondence

\[
G(R_b) = \beta(R_b \tilde{b}(R_b) - R^d d(\tilde{R}_b) + \tilde{m}) - 1.
\]

\[
= \beta((R_b^b - 1)\tilde{b}(R_b) - (R^d - 1)\tilde{d} + 1) - 1,
\]

where the latter uses \( \tilde{b} + \tilde{m} + \tilde{d} = 1 \). By (28), a stationary equilibrium is characterized by \( R^{bs} \) such that \( G(R^{bs}) = 0 \).
To prove existence, we need to show that there exists $R^{b*}$ such that $G(R^{b*}) = 0$. To do so, we use the intermediate value theorem. First, we evaluate $G$ at two extremes. First, we evaluate $G$ at a value of $R^b = 0$. Note that

$$G(0) = \beta((0 - 1)\bar{b}(0) - (R^d - 1)\bar{d} + 1) - 1$$

$$= -r^d\beta\bar{d} - (1 - \beta) < 0,$$

where the second equality follows from $\bar{b}(0) = 0$ when $R^b = 0 < R^m$. The inequality is a rearrangement of (G.1).

Now consider, $\bar{R}^b > \max \{R^w, 1/\beta\}$. In that case, we have

$$\lim_{R^b \to \bar{R}^b} G(R^b) = \lim_{R^b \to \bar{R}^b} \beta(R^b\bar{b}(R^b) - R^d) - 1$$

$$= \lim_{R^b \to \bar{R}^b} \beta(R^b + \bar{d}(R^b)(R^b - R^d)) - 1$$

$$\geq \lim_{R^b \to \bar{R}^b} \beta R^b \geq 1.$$

By Proposition 4, $\bar{m} = 0$ since $R^b > R^w$. The second line follows from the budget constraint of the bank. Since $\bar{m} = 0$, the bank bears risk. Thus, to have a strictly positive amount of deposits, the expected return from borrowing deposits must be positive and thus, $\bar{d}(R^b) (R^b - R^d) \geq 0$. The inequalities are immediate from $\bar{R}^b > \max \{R^w, 1/\beta\}$.

Portfolios are hemi-continuous in $R^b$ since the portfolio problem (25) satisfies the conditions of the Theorem of the Maximum—{$\bar{b}, \bar{d}$} live in a compact space and the objective is continuous. Hence, there must exist $R^{b*}$ such that $G(R^{b*}) = 0$.

**Uniqueness.** We now consider the uniqueness of the fixed point. We have

$$G(R^b) = \beta \left( (R^b - 1)\bar{b}(R^b) - (R^d - 1)\bar{d}(R^b) \right) - 1.$$

Consider the additional restriction that $R^m > R^d$. By Proposition 4, the capital requirement is binding. Hence, the condition is equivalent to

$$G(R^b) = \beta \left( (R^b - 1)\bar{b}(R^b) - (R^d - 1)\kappa \right) - 1.$$

Consider the first-order condition that follows from the liquidity premium

$$R^b - R^m = \frac{\mathbb{E}_\omega \left[ (R^b_\omega)^{-\gamma} \frac{\partial \bar{h}(\bar{m}, \kappa, \omega)}{\partial \bar{m}} \right]}{\mathbb{E}_\omega \left[ (R^b_\omega)^{-\gamma} \right]}.$$

By Proposition 4 again, the right-hand side is decreasing in $\bar{m}$. Hence, we conclude that $\partial \bar{h}(R^b)/\partial R^b \geq 0$. This is enough to guarantee that

$$G'(R^b) = \bar{b}(R^b) + (R^b - 1)\partial \bar{h}(R^b)/\partial R^b > 0 \text{ when } \bar{b}(R^b) > 0.$$

However, since at $\bar{b}(R^b) = 0$, $G(R^b) < 0$, the solution must be unique. **QED.**

**H Proofs for the Monetary Policy Analysis of Section 3.5**

To present formal proofs, we define two important concepts: reserve satiation and neutrality.
Definition 6 (Satiation). Banks are satiated with reserves at period t if the liquidity premium is zero, that is, if \( R^b_t = R^m_t \).

To discuss policy effects, we compare an original policy sequence—with sub-index o—with an alternative (shocked) policy—sub-index s in all of the exercises. We mean that a policy is neutral relative to the other in the following sense.

Definition 7 (Neutrality). Consider original and alternative policy sequences:

\[
\{ \rho_{o,t}, B^F_{o,t}, M_{o,t}, W_{o,t}, T_{o,t}, \kappa_{o,t}, i^i_{o,t}, i^d_{o,t} \} \quad \text{and} \quad \{ \rho_{s,t}, B^F_{s,t}, M_{s,t}, W_{s,t}, T_{s,t}, \kappa_{s,t}, i^i_{s,t}, i^d_{s,t} \}.
\]

Policy s is neutral—relative to o—if the induced equilibria satisfy

\[
\{ e_{s,t}, c_{s,t}, \bar{b}_{s,t}, \bar{d}_{s,t}, \bar{m}_{s,t} \} = \{ e_{o,t}, c_{o,t}, \bar{b}_{o,t}, \bar{d}_{o,t}, \bar{m}_{o,t} \} \quad \text{for all } t \geq 0.
\]

When the condition holds, real aggregate loans and deposits are also determined. The rest of this appendix shows the proofs for the classic exercises in monetary policy analysis that we studied in the main text.

H.1 Proof of Proposition 6 (Conditions for Satiation)

By definition of satiation, the right-hand side of (26) equals zero under satiation:

\[
R^b_t - R^m_t = 0 = \frac{\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial R^x(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right]}{\mathbb{E}_\omega (R^e)^{-\gamma}}.
\]

Since \((R^e)^{-\gamma}\) is a strictly positive function for any \(\omega\) and \(\frac{\partial R^x(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}}\) is weakly positive, we must show that \(\frac{\partial R^x(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} = 0\). By definition,

\[
\frac{\partial R^x(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} = \frac{1}{1 + \pi_t} [(\chi^- \mathbb{I} [\omega < \omega^*] + \chi^- \mathbb{I} [\omega > \omega^*])], \text{for any } \omega \neq \omega^*.
\]

Since, \(\omega \neq \omega^*\) is a zero-probability event,

\[
\mathbb{E}_\omega \left[ (R^e_\omega)^{-\gamma} \cdot \frac{\partial R^x(\bar{m}, \bar{d}, \omega)}{\partial \bar{m}} \right] = \frac{1}{1 + \pi_t} \left[ \chi^- \mathbb{E}_\omega [(R^e)^{-\gamma} | \omega < \omega^*] \Pr [\omega < \omega^*] + \chi^+ \mathbb{E}_\omega [(R^e)^{-\gamma} | \omega > \omega^*] \Pr [\omega > \omega^*] \right].
\]

This expression equals zero in two cases:

**Case 1.** If \(i^d_t = i^i_t\), then the condition holds immediately since \(\chi^- = \chi^+ = 0\). This case is condition (i) in the proposition.

**Case 2.** If \(i^d_t > i^i_t\), then it must be that no bank can have a reserve deficit with positive probability. Recall that \(\{\chi^-, \chi^+\}\) satisfy

\[
\chi^- = \Psi_t \left( i^d_t - i^i_t \right) + (1 - \Psi_t) \left( i^d_t - i^i_t \right)
\]

\[
\chi^+ = \Psi_t \left( i^d_t - i^i_t \right).
\]
Suppose that $\omega < \omega^*$ in an event with non-zero mass. Then, it must be the case that $\chi_t = 0$. A necessary condition is that $\Psi_t = 1$ because $(i_{t}^{dw} - i_{t}^{ior}) > 0$. This is not possible since $\Psi_t < 1$ when market tightness $\theta_0 > 0$ and $\lambda$ is finite. Hence, $[\omega > \omega^*]$ should occur with probability 1. Thus, we need to argue that the aggregate conditions must guarantee that there are enough reserves so that all banks can have a positive balance of reserves, for any $\omega$. Clearly, in that case, $\omega$ must be bounded below. Let that lower bound be $\omega_{min}$, as described in the main text. Under condition (ii) of the proposition, no bank is in deficit even for the worst shock. QED.

### H.2 Proof of Proposition 7 Item (i)

Consider a policy sequence $\{o\}$ and an alternative policy $\{s\}$ such that

1. $X_{s,t} = kX_{o,t}$ for some $k > 0$ for the balance sheet variables $X \in \{B_{Fed}^{o}, M, W\}$,

2. policies are identical for non-balance-sheet variables $\{\rho_{o,t}, \kappa_{o,t}, \bar{i}_{t}^{ior}, i_{t}^{dw}\} = \{\rho_{s,t}, \kappa_{s,t}, \bar{i}_{t}^{ior}, i_{t}^{dw}\}$.

The proposition states that the stationary equilibrium induced by either policy features identical real asset positions and price levels that satisfy $P_{s,t} = kP_{o,t}$.

The proof is by construction and requires us to verify that the equilibrium conditions that determine $\{b_{ss}, m_{ss}, d_{ss}, c_{ss}, E_{ss}\}$ in Section E.2 are satisfied by any pair of policy sequences $\{M_{o,t}, B_{o,t}^{Fed}, W_{o,t}\}_{t \geq 0}$ and $\{M_{s,t}, B_{s,t}^{Fed}, W_{s,t}\}_{t \geq 0}$ that satisfies the relationship above. We proceed to check that $\{b_{ss}, m_{ss}, d_{ss}, c_{ss}, E_{ss}\}$ solves the set of equilibrium equations in Section E.2 in both cases.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both satisfy

$$M_{a,t} = M_{a,t-1}(1+\pi_{ss}), \quad B_{a,t}^{Fed} = B_{a,t-1}^{Fed}(1+\pi_{ss})$$

and $W_{a,t} = W_{a,t-1}(1+\pi_{ss})$ for some $\pi_{ss}$ and $a \in \{o, s\}$.

By hypothesis, inflation and nominal rates are equal under both policies. Thus, the real interest rate on reserves is equal under both policies. We check the equilibrium conditions in the order in which they appear in Section E.1.

First, we guess and verify that the real returns on loans and deposits are also equal under both policies. If both policies yield the same real rates, the solution for bank portfolios (the solution for $\Omega_t$) must also be equal in both equilibria:

$$\{b_{o,ss}, m_{o,ss}, d_{o,ss}, c_{o,ss}\} = \{b_{ss}, m_{ss}, d_{ss}, c_{ss}\}.$$

Consider now the aggregate supply of loans and reserves under either policy:

$$(1 - c_{ss})b_{ss}E_{ss} = \Theta^b (R_{ss}^b)' - B_{t+1}^{Fed}/P_t.$$

That equation can be satisfied under both policies because $B_{o,t+1}/P_{o,t} = (1+g)B_{o,t+1}/(1+g)P_{o,t} = B_{s,t+1}/P_{s,t}$. This verifies that the real rate on loans is equal under both policies.

The same steps verify that $R_{ss}^d$ is the same under both policies. Similarly, the demand for reserves can be satisfied in both equations because

$$(1 - c_{ss})m_{ss}E_{ss} = M_{o,t}/P_{o,t} = M_{a,t}/P_{a,t}.$$

This verifies market clearing for reserves. Thus, asset market clearing is satisfied under both policies.

Now, the ratio of surpluses to deficits is also equal under both policies:
\[ \theta_{ss} \equiv \frac{S_{a,t}^-}{S_{a,t}^+} \text{ for } a \in \{o, s\}. \]

Because \( \theta \) and policy rates are equal, the liquidity cost function \( \chi \) is also equal under both policies. Observe that \( \chi \) is a function of \( \theta \) only. With equal inflation under both policies, the liquidity return \( R^\chi \) must also be equal. This verifies that all the real rates in both equilibria are the same under both policies.

We now need to verify that the steady-state condition for bank equity is also satisfied and that both policies are feasible. Observe that \( \chi \) is a function of \( \theta \) only. With equal inflation under both policies, the liquidity return \( R^\chi \) must also be equal. This verifies that all the real rates in both equilibria are the same under both policies.

We now need to verify that the steady-state condition for bank equity is also satisfied and that both policies are feasible. Observe that the interbank market loans under both policies satisfy \( W^{Fed}_{o,t}/P_{o,t} = W^{Fed}_{s,t}/P_{s,t} \). Therefore, the law of motion for steady-state equity is given by

\[
E_{ss} = \frac{(1 + i_b^t)\bar{b}_{ss} + (1 + \bar{d}_{ss})\bar{c}_{ss} E_{ss} - 1 + i^{dw}_t W^{Fed}_{t}/P_t}{1 + \pi_{ss}}.
\]

Since the nominal rates, portfolios, inflation, and real discount loans, \( W^{Fed}_{t}/P_t \), are the same under both policies, the equation must yield the same solution \( E_{ss} \) under both policy \( o \) and policy \( s \). Finally, we verify that the government budget constraint is satisfied under both policies. In any steady state,

\[
B^{Fed}_t \left( \frac{1 + i^b_{ss}}{1 + \pi_{ss}} \right) + W^{Fed}_t \left( \frac{1 + i^{dw}_t}{1 + \pi_{ss}} \right) = M^{Fed}_t \left( \frac{1 + i^{ior}_t}{1 + \pi_{ss}} \right) + P_tE_{ss} \frac{\tau_{ss}}{1 - \tau_{ss}}.
\]

Once we divide both sides of the equation by \( P_t \), we verify that the budget constraint is satisfied by the second sequence if it is satisfied by the first sequence. QED.

**H.3 Proof of Proposition 7 Item (ii)**

This statement of the proposition regards superneutrality and non-superneutrality. The proof closely follows the proof of Proposition 7, item (i). The main difference is that we prove neutrality along an equilibrium sequence, not only in stationary equilibrium. The proof is again by construction and only requires that we verify that the equilibrium conditions that determine \( \{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{c}_t, E_t\} \) in Section E.1 lead to the same values under both policies. Let \( \{M_{o,t}, B^{Fed}_{o,t}, W_{o,t}\}_{t \geq 0} \) and \( \{M_{a,t}, B^{Fed}_{a,t}, W_{a,t}\}_{t \geq 0} \) be two policy sequences. Again, to ease notation, we follow the order of the equations in Section E.1.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both equilibria satisfy

\[
X^{Fed}_{a,t} = M^{Fed}_{a,t-1}(1+g_a), \quad B^{Fed}_{a,t} = B^{Fed}_{a,t-1}(1+g_a), \quad \text{and} \quad W^{Fed}_{a,t} = W^{Fed}_{a,t-1}(1+g_a) \text{ for some } g_a \text{ and for } a \in \{o, s\}.
\]

Also, let both economies feature identical time-zero conditions: \( X_{s,0} = X_{o,0} \) for \( X \in \{M^{Fed}, B^{Fed}, W^{Fed}\} \).

Then, the condition for the Fed’s balance sheet implies that

\[
X_{s,t+1} = (1 + g_s)^t X_{s,0} = (1 + g_s)^t X_{o,0},
\]

\[
X_{o,t+1} = (1 + g_o)^t X_{o,0}.
\]
for $X \in \{M^{Fed}, B^{Fed}, W^{Fed}\}$. Thus, we can relate both balance sheet variables via

$$X_{s,t+1} = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t X_{o,t+1}.$$ 

Through the proof, we guess and verify the following:

A.1 $\{\bar{R}_o,t, \bar{R}_d,t, \bar{R}_m,t, \bar{R}_c,t\} = \{\bar{R}_s,t, \bar{R}_d,s,t, \bar{R}_m,s,t, \bar{R}_c,s,t\}$. 

A.2 $P_{o,0} = P_{s,0} = P_0$.

A.3 $(1 + \pi_{s,t}) = (1 + \pi_{o,t}) \left(1 + \frac{g_s - g_o}{1 + g_o}\right)$.

First, we verify (A.1). Under the conjecture that real returns are the same along a sequence, we have that

$$\{\bar{b}_{o,t}, \bar{m}_{o,t}, \bar{d}_{o,t}, \bar{c}_{o,t}\} = \{\bar{b}_{s,t}, \bar{m}_{s,t}, \bar{d}_{s,t}, \bar{c}_{s,t}\},$$

so the optimality conditions are satisfied in both cases.

Next, consider the aggregate supply of loans and reserve demand. Equilibrium in the loans market requires

$$(1 - c_t)\bar{b}_t E_t = \Theta^b \left(R^b_t\right)^t - B^{Fed}_{t+1}/P_t.$$ 

If the equation is satisfied under both policies, then we must verify that $B^{Fed}_{o,t+1}/P_{o,t} = B^{Fed}_{s,t+1}/P_{s,t}$. To see that this condition holds, recall that

$$B^{Fed}_{s,t+1} = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t B^{Fed}_{s,0}.$$ 

Now, if $\pi_{s,t} - \pi_{o,t} = (g_s - g_o)/(1 + g_o)$, by (A.2) we have that

$$P_{a,t} = \prod_{\tau=1}^{t} (1 + \pi_{a,\tau}) P_0 \text{ for } a \in \{o, s\}.$$ 

Combined with the guess (A.3) above, we obtain

$$P_{s,t} = \prod_{\tau=1}^{t} (1 + \pi_{o,t}) \left(1 + \frac{g_s - g_o}{1 + g_o}\right) P_0 = P_{a,t} \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t.$$ 

Therefore,

$$B^{Fed}_{s,t+1}/P_{s,t} = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t B^{Fed}_{s,0}/P_{s,t} = B^{Fed}_{o,t+1}/P_{o,t},$$

which shows that the real holdings of loans under both policies are equal. This is the condition we needed to verify that under our guess, $R^b_t$ is the same under both policies. That $R^d_t$ is the same under both policies follows immediately using the same steps. Next, by assumption, note that $R^m_t$ is the same under both policies because

$$R^m_{o,t} = (1 + \bar{i}_{o,t+1}) / (1 + \pi_{o,t+1}) = (1 + \bar{i}_{s,t+1}) \left(1 + \frac{g_s - g_o}{1 + g_o}\right) / (1 + \pi_{o,t+1}),$$

and by assumption (A.3), the condition is also equal:

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\[(1 + \bar{i}_{s,t+1}^{\ior}) \left(1 + \pi_{o,t+1}\right)/(1 + \pi_{s,t+1}) = R_{s,t}^m.\]

Note that this condition is true because it is one of the assumptions of Proposition 7 that we are proving.

Next, consider the condition for an equilibrium in the reserve market that gives rise to the quantity. In any equilibrium, it must satisfy

\[(1 - c_t)\bar{m}_t E_t = M_{o,t}/P_{o,t} = M_{s,t}/P_{s,t}.\]

This condition is used to verify our guess (A.3). The condition above requires

\[\frac{P_{s,t+1}}{P_{o,t+1}} = \frac{M_{s,t+1}}{M_{o,t+1}} = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t \frac{M_{o,t+1}}{M_{o,t+1}} = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t.\]

Then, since by Assumption (A.2), initial prices are the same, we have that

\[\frac{P_{s,t+1}}{P_{o,t+1}} = \prod_{\tau=1}^t \left(1 + \pi_{s,t}\right) P_0 = \left(1 + \frac{g_s - g_o}{1 + g_o}\right)^t \prod_{\tau=1}^t \left(1 + \pi_{o,t}\right) = \prod_{\tau=1}^t \left(1 + \pi_{o,t}\right) \left(1 + \frac{g_s - g_o}{1 + g_o}\right).\]

Since the condition holds for all \(t\), then A.3 is deduced from the quantity equation of reserves.

The next step is to verify that \(R_t^\chi\) is constant under both policies. For that, observe that the interbank market tightness is the same under both economies. To see that, simply note that the ratio of reserves to deposits is the same under both policies, and that this is enough to guarantee that \(\theta_t\) is equal under both policies. By Lemma 2 and the condition for policy rates in the proposition—\((1 + i_{o,t}^x) = (1 + i_{s,t}^\ior) \left(1 + \frac{g_s}{1 + g_o}\right)\) for \(x \in \{dw, ior\}\)—in states away from satiation,

\[\chi(\cdot; i_{o,t}^x, i_{o,t}^\ior) = \left(1 + \frac{g_s}{1 + g_o}\right) \chi(\cdot; i_{s,t}^x, i_{s,t}^\ior).\]

Therefore, we have that

\[R_{o,t}^\chi = \frac{\chi(\cdot; i_{o,t}^x, i_{o,t}^\ior)}{1 + \pi_{o,t}} = \frac{\chi(\cdot; i_{s,t}^x, i_{s,t}^\ior)}{1 + \pi_{o,t}} = \frac{\chi(\cdot; i_{s,t}^x, i_{s,t}^\ior)}{1 + \pi_{s,t}} = R_{s,t}^\chi.\]

This step verifies that \(R_{o,t}^\chi = R_{s,t}^\chi\). So far, we have checked the consistency of assumptions (A.1) and (A.3), and that the policy rules for \(\{\bar{b}_t, \bar{m}_t, \bar{d}_t, \bar{e}_t\}\) and the real rates are the same under both equilibria. We still need to show that the sequences for \(E_t\) are the same under both policies, that the initial price level is the same, and that the Fed’s budget constraint is satisfied under both policies. We verify these conditions jointly. Consider the law of motion for aggregate real equity,

\[E_{t+1} = (R_{t+1}^b \bar{b}_t + R_{t+1}^m \bar{m}_t - (1 + R_{t+1}^d) \bar{d}_t) (1 - \bar{c}_t) E_t + \frac{1 + \bar{i}_{t+1}^\ior}{1 + \pi_{t+1}} W_t^{Fed} \frac{P_{t+1}^{Fed}}{P_t} - E_t \bar{\tau}_t \frac{\tau_t}{1 - \tau_t},\]

and the Fed’s budget constraint in real terms is
\[
\frac{B_{Fed}^t}{P_t} \Delta R_{t+1}^b + \frac{W_{Fed}^t}{P_t} \left( \frac{1+\pi_{dw}}{1+\tau_t} \right) = \frac{M_{Fed}^t}{P_t} \Delta R_{t+1}^m + E_t \frac{\tau_t}{1-\tau_t}.
\]

We have already verified that \( B_{Fed}^{s,t+1}/P_{s,t} = B_{Fed}^{o,t}/P_{o,t} \). Following the same steps, we can show that real reserves \( M_{t}^{Fed}/P_{t} \) and discount loans \( W_{t}^{Fed}/P_{t} \) are identical in both equilibria. Away from satiation, \( R_{s,t}^h = R_{s,t}^o \), so that means that real income from the discount window, \( \frac{W_{Fed}^t}{P_t} \left( \frac{1+\pi_{dw}}{1+\tau_t} \right) \), is constant under both policies. Provided that \( \tau_t \) is constant under both sequences, the Fed’s budget constraint is satisfied under both policies. Similarly, the law of motion for real aggregate equity is the same, provided that \( E_o \) is the same under both policies. Consider now \((B_0, D_0, M_0, W_0)\), the initial condition under both policies. If \( P_0 \) is same initial price under both policies, \( E_{o,0} = E_{s,0} \). This is precisely the pair of initial conditions that we need to confirm our guess \( E_{o,0} = E_{s,0} \) and \( P_{o,0} = P_{s,0} \). This finalizes the proof that equilibria are the same along both policies. \( \text{QED.} \)

### H.4 Proof of Proposition 8

Formally, we prove the following result. Consider two policies, \( o \) and \( s \), and let the alternative policy feature an open-market operation performed at \( t = 0 \) and reverted at \( t = 1 \) in the sense that

1. \( B_{s,0}^{Fed} = B_{o,0}^{Fed} + \Delta B_{Fed} \), \( M_{s,0}^{Fed} = M_{o,0}^{Fed} + \Delta M_{Fed} \), for some loan purchase where \( \Delta M_{Fed} = \Delta B_{Fed} \geq 0 \).
2. \( \{ \rho_{o,t}, \kappa_{o,t}, i_{o}^{ior}, i_{o}^{dw} \} = \{ \rho_{s,t}, \kappa_{s,t}, i_{s}^{ior}, i_{s}^{dw} \} \) for all \( t \geq 0 \).
3. \( \{ \rho_{o,t}, B_{o,t}^{Fed}, M_{o,t}^{Fed}, \ldots \} = \{ \rho_{s,t}, B_{s,t}^{Fed}, M_{s,t}^{Fed}, \ldots \} \) for all \( t > 1 \).

The statement of the proposition says that the operation is neutral only if banks are satiated with reserves at time zero. Away from satiation, the policy has real effects.

The proof is in two steps. First, we show that if the policies induce identical allocations, the equilibrium prices \( P_{o,0} = P_{s,0} \) must also be equal. Then, we show by contradiction that if the price is constant, the open-market operation must have real effects away from satiation. If banks are satiated, the policy has no effect.

Next, we prove the auxiliary lemma corresponding to the first step of the proof.

**Lemma 3.** Consider two arbitrary policy sequences \( o \) and \( s \) as described above. If real loans, deposits, dividends, reserves, and bank equity are the same in all periods in both equilibria, then \( P_{o,0} = P_{s,0} \).

**Proof.** We proceed by contradiction. Let us avoid the use of the time subscript and remember that the policy change is at time zero. Without loss of generality, normalize the price in the original equilibrium to \( P_o = 1 \). If both policies have no effects on real loans and real deposits, then, given the demand and supply functions, \( R_{o,t}^h = R_{s,t}^h \) and \( R_{o,t}^d = R_{s,t}^d \). These rates are consistent with a real quantity of loans \( B \) and deposits \( D \).

Consider now the representative bank. By hypothesis, real dividends are equal in both equilibria \( c_o = c_s \) and \( e_o = e_s \). Let \( \{ b_{o}^{Fed}, b_{o} \} \) and \( \{ b_{s}^{Fed}, b_{s} \} \) be the holding of real loans by the Fed under the original policy and the alternative policy. Also, let \( \{ m_o, m_s \} \) be the real balances under both policies. Then, we know that by market clearing in the loans market,

\[
B = b_{o}^{Fed} + b_{o} = b_{s}^{Fed} + b_{s} \tag{H.1}
\]
Since equity, dividends, and real deposits are constant, from the bank’s budget constraints we obtain

\[ b_o - b_s = m_s - m_o. \] (H.2)

From the quantity equation we obtain

\[ M_o + \Delta M = P_s m_s = P_s (b_o + m_o - b_s) = P_s (b_o + m_o - (B - b_s^{Fed})). \] (H.3)

The second equality follows from (H.2) and the third from (H.1). Now consider the definition of real loans held by the Fed in the alternative policy:

\[ b_{Fed}^s = \frac{b_{Fed}^o + \Delta B}{P_s} = \frac{b_{Fed}^o + \Delta M}{P_s} = \frac{b_{Fed}^o}{P_s} + \frac{\Delta M}{P_s}. \]

Substituting the last term into (H.3), we obtain

\[ M_o + \Delta M = P_s \left( b_o + m_o - B + \frac{b_{Fed}^o}{P_s} + \frac{\Delta M}{P_s} \right) \]

Thus, we have that the price under the alternative policy satisfies

\[ m_o = P_s \left( b_o + m_o - \bar{B} + \frac{b_{Fed}^o}{P_s} \right). \]

Because this equation is independent of \( \Delta M \), it must hold for any open-market operation, in particular, for \( \Delta M = 0 \). Therefore, it must be that \( P_o = P_s = 1 \). QED.

Next, we establish that if the policy is neutral, we reach a contradiction. To reach that contradiction, assume that the policy is neutral. First, observe that if policy \( s \) is neutral with respect to policy \( o \), real assets and bank equity must be equal in both equilibria. For that to hold, portfolios must be the same in both equilibria. Thus, consider the first-order condition for loans and reserves. Then, under the original policy,

\[ R^b - R^m = \frac{\mathbb{E}_\omega \left[ (R^c)^{-\gamma} \frac{\partial \bar{X}(m, D, \omega)}{\partial m} \bigg| m_o \right]}{\mathbb{E}_\omega (R^c)^{-\gamma}} \equiv \Lambda \left( m_o, \bar{D} \right). \]

Assume the economy is away from satiation, and assume the false hypothesis that the two policy sequences lead to the same real loans, deposits, and dividend sequences. Now, since banks are away from satiation but hold different portfolios, there are differences in the discount window loans. Since the policy is identical from \( t = 1 \) onward, any difference in Fed income from discount window loans must be offset with \( t = 1 \) transfers to banks under the alternative policy. That means that if the hypothesis is right, the policy leads to the same equity growth sequence. Since the increase in discount loans is rebated to banks, the return on bank equity is the same if the bank’s portfolio is the same. By Lemma 3, the price sequence is the same under both policies. This implies that \( R^m \) is the same in both equilibria. Since the aggregate amount of loans is constant, the liquidity premia
must be constant. This is where the contradiction appears: away from satiation \( \frac{\partial \Lambda (m, \bar{D})}{\partial m} \leq 0 \), and \( m_s > m_o \), the first-order condition cannot hold under both policies. This contradiction proves that away from satiation, the policy must have real effects.

Next, we verify that under satiation, the policy change has no effects. We verify that under satiation, the quantity and real rate of deposits are invariant under both policies. Under satiation, \( \Lambda (m_o, \bar{D}) - \frac{\partial \Lambda (m_o, \bar{D})}{\partial m_o} = 0 \) and \( R^b = R^m = (1 + i^{or}) \). Since \( R^b \) depends on real quantities of loans, we have that

\[
R^b = \Theta^b \left( b_o + b_{o fed} \right) \epsilon.
\]

Now, consider the alternative policy. Under the hypothesis that the policy has no effects, the price is constant and equal to 1. Thus, \( \Delta \bar{b}_{fed} = \Delta \bar{m} \). Then, the balance sheet changes to

\[
b_{o fed} - m_o = b_{o fed} + \Delta b_{fed} - (m_o + \Delta \bar{m}).
\]

Since banks are indifferent between holding loans and reserves, as long as \( R^b = R^m \), we must verify that

\[
R^b = \Theta^b \left( b_o + b_{o fed} \right) \epsilon = \Theta^b \left( b_s + b_{o fed} + \Delta b_{fed} \right) \epsilon.
\]

From the budget constraint of the bank,

\[
b_s = b_o - \Delta m = b_o - \Delta b_{fed}.
\]

Thus, \( \Theta^b \left( b_s + b_{o fed} + \Delta b_{fed} \right) \epsilon = \Theta^b \left( b_o - \Delta b_{fed} + b_{o fed} + \Delta b_{fed} \right) \epsilon \), which is precisely what we needed to show to verify that loans remain constant. Since under both policies, real asset returns are the same, real deposit rates are also the same. The same is true about dividends. Finally, since the Fed earns zero profits from the discount window under both policies and the Fed buys assets with equal returns, the operation leads to the same transfers. This verifies that the policy has no effects under satiation. QED.

H.5 Proof of Proposition 9

We first recall some observations that result from Proposition 4. We have (i) \( R^b \geq R^m \), (ii) that if \( \bar{m} > 0 \), we have that \( R^b \geq R^m \) unless banks are satiated with reserves, (iii) if \( R^w = R^m \) and \( R^b > R^m \) if and only if \( \bar{m} = 0 \), and (iv) \( R^b < R^d \) implies \( \bar{d} = 0 \).

**Establishing the Lower Bound.** Now, we prove the bound:

\[
R^b \geq \max \left\{ R^m, \min \left\{ \frac{1}{\beta}, \kappa R^d \frac{1}{(1 + \kappa)} \right\} \right\}.
\]

From the stationary equilibrium condition, (28), we obtain

\[
R^b = 1/\beta - \bar{d} \left( R^b - R^d \right)^+ + r^b \bar{m},
\]

where the term \( \bar{d} \left( R^b - R^d \right)^+ \) is without loss of generality. We discuss the bound in three cases.

**Case 1.** Let \( R^d \geq \frac{1}{\beta} \), then

\[
\min \left\{ \frac{1}{\beta}, \kappa R^d \frac{1}{(1 + \kappa)} \right\} = \frac{1}{\beta}.
\]
Assume by contradiction that \( R^b < 1/\beta \). This condition implies that (H.2)
\[
R^b = 1/\beta - \bar{d} (R^b - R^d)^+ + r^b \bar{m} = 1/\beta + r^b \bar{m},
\]
and thus
\[
1 + (R^b - 1) (1 - \bar{m}) = 1/\beta \rightarrow (R^b - 1) = \frac{(1/\beta - 1)}{1 - \bar{m}}.
\]
Since \( \bar{m} > 0 \), this condition implies that \( R^b \geq 1/\beta \). Hence, we have a contradiction. Since by (i), \( R^b \geq R^m \), this establishes condition (H.1) when \( R^d \geq 1/\beta \).

**Case 2.** \( R^d < \frac{1}{\beta} \leq R^m \). Since \( R^m > \frac{1}{\beta} \), we have
\[
R^m > \frac{1}{\beta} > \frac{1}{\beta} + \kappa R^d \left( \frac{1}{1 + \kappa} \right).
\]
This establishes condition (H.1) immediately from (i).

**Case 3.** \( R^m, R^d < \frac{1}{\beta} \). Note that the stationary equilibrium condition (28) can also be written as
\[
1/\beta = (R^b - 1) \bar{b} - \bar{d} (R^d - 1) + 1
\]
by substitution of the portfolio. We look for the lowest budget feasible return on loans—not necessarily consistent with an equilibrium choice:
\[
\min_{\{b, d\}} R^b - 1
\]
subject to
\[
(R^b - 1) \bar{b} - \bar{d} (R^d - 1) = 1/\beta - 1,
\]
and
\[
0 \leq \bar{b} \leq 1 + \bar{d}, 0 \leq \bar{d} \leq \kappa.
\]
Transforming the problem, we have
\[
\min_{\{b, d\}} \frac{(1/\beta - 1) + \bar{d} (R^d - 1)}{b}.
\]
Assume that \( R^d > 1 \). Then, the minimizing portfolio is \( \{\bar{b}, \bar{m}, \bar{d}\} = \{1, 0, 0\} \) and the minimal return on loans is \( 1/\beta \). If \( R^d \leq 1 \), then the minimizing portfolio is \( \{\bar{b}, \bar{m}, \bar{d}\} = \{1 + \kappa, 0, \kappa\} \) and the return on loans is at least greater than
\[
R^b - 1 = \frac{(1/\beta - 1) + \kappa (R^d - 1)}{1 + \kappa} \rightarrow R^b = \frac{1/\beta + \kappa R^d}{1 + \kappa} > \frac{R^d + \kappa R^d}{1 + \kappa} = R^d.
\]
Thus, combined with (i), this establishes condition (H.1) again.

**Attaining the bound.** Next we show that the lower bound is attainable by setting \( R^m = R^w \). We know that \( R^b = R^m \) in the case of satiation, unless \( \bar{m} = 0 \). Assume that
\[
R^m \geq \min \left\{ \frac{1}{\beta}, \frac{1}{\beta} + \kappa R^d \left( \frac{1}{1 + \kappa} \right) \right\}.
\]
Then, the condition for stationarity is (28) with $R^m = R^b$, and thus, the bound is attainable if we can find a portfolio such that

$$1/\beta = R^m + \bar{d} \left( R^m - R^d \right)^+ - r^m \bar{m}.$$ 

If $R^d \geq R^m$, the portfolio solves

$$\bar{m} = \frac{R^m - 1/\beta}{R^m - 1}.$$ 

If $R^d < R^m$, the portfolio solves

$$\bar{m} = \frac{R^m - 1/\beta - \kappa \left( r^m - r^d \right)}{R^m - 1},$$

which is also feasible from the condition.

Consider now the case in which

$$R^m < \min \left\{ \frac{1}{\beta}, \frac{1/\beta + \kappa R^d}{1 + \kappa} \right\}.$$ 

Then, since the bound (H.1) must be satisfied, by (iii), we have that $\bar{m} = 0$. Hence, we then have that the condition for stationarity is given by

$$1/\beta = R^b + \kappa \left( R^b - R^d \right)^+.$$ 

If $R^d \geq 1/\beta$, then $R^b = 1/\beta$ and the portfolio $\{\bar{b}, \bar{d}, \bar{m}\} = \{1, 0, 0\}$ is optimal and the lower bound is attained. If $R^d < 1/\beta$, then, the portfolio return solves

$$R^b = \frac{1/\beta + \kappa R^d}{1 + \kappa} > R^d$$

and the portfolio $\{\bar{b}, \bar{d}, \bar{m}\} = \{1 + \kappa, \kappa, 0\}$ is optimal. This shows that the lower bound is attainable.

**The bound with an elastic rate.** So far, we have treated the deposit rate as exogenous. Note that the bound $R^b = \frac{1/\beta + \kappa R^d}{(1 + \kappa)}$ applies when the portfolio is $\{\bar{b}, \bar{d}, \bar{m}\} = \{1 + \kappa, \kappa, 0\}$. With those portfolios, the loans and deposit rates satisfy the following relation, by definition and clearing in the loans market:

$$R^d = \left( \frac{\Theta^d}{\Theta^b} \frac{\kappa}{1 + \kappa} \right)^{1/\epsilon} (R^b)^{-\frac{\epsilon}{\kappa}}.$$ 

The condition for stationarity is, in that case, given by the solution to

$$(1 + \kappa) \left( \frac{\Theta^d}{\Theta^b} \frac{\kappa}{1 + \kappa} \right)^{1/\epsilon} (R^d)^{-\frac{\epsilon}{\kappa}} = \frac{1}{\beta} + \kappa R^d,$$

which solves the term in the statement of the proposition. **QED.**
I Proof of Proposition 10

Worker’s Problem. We substitute $t + 1$ consumption of worker $t$’s problem into his objective to obtain that his objective equals

$$c_t^{w,t} - \frac{h_{t}^{1+\nu}}{1+\nu} + \beta U\left(\frac{(1+i_{t+1}^d)}{P_{t+1}}\right).$$

Then, taking first-order conditions with respect to $c_t^{w,t}$, we obtain

$$1 = P_t \zeta_t^w,$$  \hspace{1cm} (I.1)

where $\zeta_t^w$ is a Lagrange multiplier on the worker’s time $t$ budget constraint. Then, the first-order condition with respect to labor supply yields a labor supply that only depends on the real wage:

$$h_t^\nu = z_t \frac{\zeta_t^w}{P_t} = z_t / P_t.$$  \hspace{1cm} (I.2)

Next, we take the first-order condition with respect to deposits:

$$\zeta_t^w = \beta U'\left(\frac{(1+i_{t+1}^d)}{P_{t+1}/P_t}\right)\left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t}\right).$$

We rewrite this condition as

$$P_t \zeta_t^w = \beta U'\left(\frac{(1+i_{t+1}^d)}{P_{t+1}/P_t}\right)\left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t}\right).$$

Thus, noticing that $P_t \zeta_t^w = 1$, from (I.1) this expression becomes

$$\left(\frac{d_{t+1}}{P_t}\right)^{1/(\varsigma+1)} = \beta \left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t}\right)^{1-1/(\varsigma+1)}.$$

Clearing $d_{t+1}$ we obtain

$$\frac{d_{t+1}}{P_t} = \beta^{\varsigma+1} \left(\frac{1+i_{t+1}^d}{P_{t+1}/P_t}\right)^{\varsigma}.$$  \hspace{1cm} (I.3)

Thus, setting $\beta^{\varsigma+1} = \Theta_t^d$, we obtain the functional form in Proposition 10.

Next, we move to derive the demand for loans. From the entrepreneur’s problem, we have that

$$\max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - (1+i_t^b) B_{t+1}^d + (1+i_t^d) (B_{t+1}^d - z_t h_t)$$

subject to

$$z_t h_t \leq B_{t+1}^d.$$ 

First, observe that

$$P_{t+1} A_{t+1} h_t^\alpha - (1+i_t^b) B_{t+1}^d + (1+i_t^d) (B_{t+1}^d - z_t h_t) = P_{t+1} A_{t+1} h_t^\alpha - z_t h_t - (i_t^b - i_t^d) (B_{t+1}^d + z_t h_t).$$

Since $i_t^b \geq i_t^d$, without loss of generality, $z_t h_t = B_{t+1}^d$. Thus, the objective is
$$P_{t+1}A_{t+1}h_t^\alpha - (1 + i_t^b) B_{t+1}^d$$

with \(z_t h_t = B_{t+1}^d\). Suppose \(h_t\) was already chosen by the entrepreneur. Thus, back in the objective function, we have that

$$P_{t+1}A_{t+1}h_t^\alpha - (1 + i_t^b) z_t h_t.$$ 

The first-order condition in \(h_t\) yields

$$P_{t+1}\alpha A_{t+1}h_t^\alpha - (1 + i_t^b) z_t h_t.$$ 

Dividing both sides by \(P_t\), we obtain

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1}h_t^\alpha = (1 + i_t^b) \frac{z_t}{P_t} h_t.$$ 

Now, employing the labor supply function (I.2), we have

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1}h_t^\alpha = (1 + i_t^b) h_t^{\nu+1} \rightarrow \frac{1 + i_t^b}{P_{t+1}/P_t} = \frac{\alpha A_{t+1}h_t^\alpha}{h_t^{\nu+1}}.$$ 

Now, we finally deduce that

$$\frac{B_{t+1}^d}{P_t} = \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{1}{\nu+1}}.$$ 

Combining, we obtain

$$\frac{1 + i_t^b}{P_{t+1}/P_t} = \alpha A_{t+1} \left(\frac{B_{t+1}^d}{P_t}\right)^{-1} \left(\frac{B_{t+1}^d}{P_t}\right)^{\alpha/\nu+1} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t \left(1 + i_t^b\right)^{\epsilon} \left(\frac{P_{t+1}/P_t}{P_t}\right).$$

Thus, the microfoundation yields

$$\Theta_t = (\alpha A_{t+1})^{\epsilon} \text{ and } \epsilon = \left(\frac{\alpha}{\nu+1} - 1\right)^{-1}.$$
J Dynamical Properties

In this section, we study the dynamical properties of the model. We fully characterize these dynamics when banks have log utility and the Fed carries out a policy of no distortions in the interbank market. Both assumptions simplify the analysis. Although the results are not general, for small deviations around that policy, the dynamic properties should be similar.

Stationary Equilibrium and Policy Effects with Satiation. We begin describing the transitional dynamics of the model when the Fed carries out a policy that satiates the market with reserves and sets $M_t = B_t^{Fed}$. By satiating the market for reserves and maintaining an equal amount of reserves as Fed loans, the Fed induces no distortions in the credit market. A spread between loans and deposits only results from capital requirements. This characterization is useful because it describes the dynamics of the model in absence of any distortions. As long as the Fed does not deviate too much from this policy, the properties should go through.

For this section, it is useful to define the inverse demand elasticity of loans and supply elasticity of deposits, $\tilde{\epsilon} \equiv \epsilon^{-1}$ and $\tilde{\zeta} \equiv \zeta^{-1}$, respectively, as well as the intercept of the inverse demand for loans and supply of deposits, $\tilde{\Theta}^b \equiv (\Theta^b)^{1/\epsilon}$ and $\tilde{\Theta}^d \equiv (\Theta^d)^{1/\zeta}$. We obtain the following characterization for a transition:

**Proposition 11.** [Transitions under Satiation] Consider a policy sequence where the Fed induces satiation at all $t$ and satisfies $M_t = B_t^{Fed}$. Then:

1. **Dynamics.** Real aggregate bank equity follows:
   $$E_{t+1} = (R^b_t + \kappa \min\{(R^b_t - R^d_t), 0\}) \beta E_t, \text{ with } E_0 > 0 \text{ given.}$$

2. **Existence and uniqueness.** There $\exists!$ steady state level of $E_{ss} > 0$. The steady state features binding capital requirements if and only if
   $$\left(\tilde{\Theta}^b\right)^{\frac{\zeta}{\epsilon + \zeta}} \left(\tilde{\Theta}^b\right)^{\frac{\tilde{\epsilon}}{\epsilon + \zeta}} \kappa \frac{\zeta}{\epsilon + \zeta} (1 + \kappa)^{(\tilde{\epsilon} - \tilde{\zeta})} (\frac{\zeta}{\epsilon + \zeta}) < \beta. \quad (J.1)$$

3. **Sufficient condition for monotone convergence.** If $(1 - \tilde{\epsilon}) / (1 + \tilde{\zeta}) > \kappa / (\kappa + 1)$, then $E_t$ converges to $E_{ss}$ monotonically.

In the paper, we satisfy the parameter restrictions of items 2 and 3 in Proposition 11.

**J.1 Proof of Proposition 11**

The proof of the proposition is presented in three steps. First, we derive a threshold equity level where capital requirements are binding. Second, we prove that there can be at most one steady state. Third, we provide conditions such that the equilibrium features binding reserve requirements. Finally, we derive the sufficient condition for monotone convergence. We then establish the result for the rate of inflation and the determination of the price level.

**Part 1 - Law of Motion of Bank Equity.** As shown in the Proof of Proposition 3, under log utility $\bar{c}_t = (1 - \beta)$. Then, the law of motion in (D.10) becomes

$$E_{t+1} = (R^b_t + \kappa \min\{(R^b_t - R^d_t), 0\}) \beta E_t.$$

This shows that the law of motion of bank equity satisfies the differential equation in the proposition. Thus, we have obtained a law of motion for bank equity in real terms. We use this to establish
convergence. Consider now the condition such that capital requirements are binding for a given \( E_t = E \). For that we need that \( R^b_t > R^d_t \). Using the inverse of the loan demand function, \( R^b_t \) can be written in terms of the supply of loans using the market clearing condition:

\[
R^b_t = \Theta^b \left( b \beta E_t + \frac{B^{Fed}_t}{P_t} \right)^{-\bar{\epsilon}},
\]

but since \( B^{Fed}_t = \bar{M}_t \),

\[
R^b_t = \Theta^b (\beta E_t (1 + \kappa))^{-\bar{\epsilon}}.
\]

Using the result that capital requirements are binding, \( R^b_t > R^d_t \), we obtain

\[
\Theta^b (\beta E_t (1 + \kappa))^{-\bar{\epsilon}} > \Theta^d (\beta E_t \kappa)^{\bar{\xi}}.
\]

Clearing \( E \) at equality delivers a threshold,

\[
E_\kappa = \frac{1}{\beta} \left[ \frac{\Theta^b / \Theta^d}{(1 + \kappa)^{\kappa \bar{\xi}}} \right]^{\frac{1}{\bar{\epsilon}}},
\]

such that for any \( E < E_\kappa \), capital requirements are binding. Thus, the law of motion of capital is broken into a law of motion for the binding and non-binding capital requirements regions.

We obtain

\[
E_{t+1} = \Theta^b (\beta E_t (1 + \kappa))^{1-\bar{\epsilon}} - \Theta^d (\beta E_t \kappa)^{1+\bar{\xi}} \text{ for } E_t \leq E_\kappa
\]

and

\[
E_{t+1} = \Theta^b ((1 + \bar{d}_t) \beta E_t)^{-\bar{\epsilon}} \beta E_t \text{ for } E_t > E_\kappa.
\]

Here, we substituted \( \bar{d} = \kappa \) in (D.10) for the law of motion in the constrained region and \( \bar{d}_t \left( R^b_t - R^d_t \right) = 0 \) in the second region.

**Part 2 - Uniqueness of Steady State.** Here we show that there cannot be more than one steady state level of real bank equity. We prove this in a couple of steps. First, we ask whether there can be more than one steady state in each region—in the binding and non-binding regions. We show that there can be only one steady state in each region. Then, we ask whether two steady states can coexist, given that they must lie in separate regions. The answer is no.

To see this, define

\[
\Gamma \left( E \right) \equiv \Theta^b (\beta (1 + \kappa))^{1-\bar{\epsilon}} E^{-\bar{\epsilon}} - \Theta^d (\beta (1 + \kappa))^{1+\bar{\xi}} E^\bar{\xi}.
\]

If a steady state exists in the binding region, it must satisfy the following condition:

\[
1 = \Gamma \left( E_{ss} \right) \text{ and } E_{ss} \leq E_\kappa.
\]

It is straightforward to verify that

\[
\Gamma' \left( E \right) < 0, \quad \lim_{E \to 0} \Gamma \left( E \right) \to \infty, \text{ and } \lim_{E \to \infty} \Gamma \left( E \right) \to -\infty.
\]

Since the function is decreasing and starts at infinity, and the function ends at minus infinity, there can be at most one steady state—with positive \( E \)—in the constrained region, \( E_{ss} < E_\kappa \).

In the unconstrained region, \( E_{ss} \geq E_\kappa \) a steady state occurs only when

\[
1 = R^b_t \beta.
\]
We need to find the level of equity that satisfies that condition. Also, we know that $R^d = R^b$ in the unconstrained region. Thus, the supply of loans in the unconstrained region is given by

$$\beta E_t + (R^d)^- (R^b),$$

the sum of real bank equity plus real deposits. Thus, we can define the equilibrium rate on loans through the implicit map, $\tilde{R}^b (E)$, that solves

$$\tilde{R}^b (E) \equiv \left\{ \tilde{R} | \tilde{R} = \Theta^b \left( \beta E + (R^d)^- \left( \tilde{R} \right) \right)^{-\epsilon} \right\}.$$ 

If we can show that $\tilde{R}^b (E)$ is a function and $\tilde{R}^b (E) = \beta^{-1}$ for only one $E$, then we know that there can be at most one steady state in the unconstrained region. To show that $\tilde{R}^b (E)$ is a function, we must show that there is a unique value of $\tilde{R}^b$ for any $E$. Note that $\tilde{R}^b (E) = \tilde{R}$ for $\tilde{R}$ that solves

$$\left( \tilde{R} \right)^{-\frac{1}{\epsilon}} - (R^d)^- \left( \tilde{R} \right) = \beta E.$$

Thus, since $(R)^{-\frac{1}{\epsilon}}$ is decreasing and $-(R^d)^- (R)$ is decreasing, $\tilde{R}^b (E)$ is a function. Observe that

$$\lim_{\tilde{R} \to 0} \left( \tilde{R} \right)^{-\frac{1}{\epsilon}} - (R^d)^- \left( \tilde{R} \right) = \infty, \quad \text{and} \quad \lim_{\tilde{R} \to \infty} \left( \tilde{R} \right)^{-\frac{1}{\epsilon}} - (R^d)^- \left( \tilde{R} \right) = -\infty,$$

so $\tilde{R}^b (E)$ exists for any positive $E$. Since $\tilde{R}^b$ is decreasing in $E$ and defined everywhere, there exists at most one value for $E$ such that $\tilde{R}^b (E) = (\beta)^{-1}$. This shows that there exists at most one steady state in the unconstrained region.

Next, we need to show that if there exists a steady state where $E_{ss} \leq E_\kappa$, there cannot exist another steady state where $E_{ss} \geq E_\kappa$. To see this, suppose that there $\exists$ a steady state in the unconstrained region. Thus, there exists some value $E_u > E_\kappa$ such that

$$\tilde{R}^b (E_u) = 1/\beta.$$ 

Since $\tilde{R}^b$ is decreasing and $E_u > E_\kappa$, by assumption we obtain that

$$1/\beta < \tilde{R}^b (E_\kappa) = R^b (\beta E_\kappa (1 + \kappa)), \quad \text{(J.1)}$$

where the equality follows from the definition of $E_\kappa$.

As a false hypothesis, suppose that there is another steady state where $E_c < E_\kappa$. Then, using the law of motion for equity in the constrained region,

$$R^b (\beta E_c (1 + \kappa)) = 1/\beta - \kappa \left( R^b (\beta E_c (1 + \kappa)) - R^d (\beta E_c \kappa) \right),$$

$$R^b (\beta E_c (1 + \kappa)) < 1/\beta, \quad \text{(J.2)}$$

where the second line follows from $R^b > R^d$ for any $E_c < E_\kappa$. Thus,

$$R^b (\beta E_\kappa (1 + \kappa)) < R^b (\beta E_c (1 + \kappa)) < \beta^{-1}$$

because $R^b$ is decreasing. However, (J.2) and (J.1) cannot hold at the same time. Thus, there $\exists!$ steady state with positive real equity.

**Part 3 - Conditions for Capital Requirements Binding at Steady State.** Since $\tilde{R}^b$ is
decreasing, it suffices to show that if
\[ R^b_t (E_\kappa) < \beta, \]
there exists no steady state with \( E_\kappa > E \). This condition is guaranteed if
\[
\Theta^b \left( \left[ \frac{\Theta^b / \Theta^d}{(1 + \kappa)^{\xi \kappa}} \right] \frac{1}{\xi^{\frac{\kappa}{\xi}} (1 + \kappa)} \right)^{-\xi} < \beta \rightarrow

\left( \Theta^b \right)^{\frac{\xi}{\kappa}} (\Theta^b)^{\xi \kappa} (1 + \kappa)^{(\xi \kappa)} < \beta,
\]
which is the condition in the statement of the proposition.

**Part 4 - Conditions for Monotone Convergence.** Assume that parameters satisfy the conditions for a steady state with binding capital requirements. Observe that if \( E_t > E_\kappa \), then \( E_{t+1} < E_t \) since \( R^b_t < (\beta)^{-1} \) for all \( E > E_\kappa \). Thus, any sequence that starts from \( E_0 > E \) eventually abandons the region. Thus, without loss of generality, we only need to establish monotone convergence within the \( E < E_\kappa \) region.

Now consider \( E_t < E_{ss} \). We must show that \( E_{t+1} \) also satisfies \( E_{t+1} < E_{ss} \) if that is the case. Employing the law of motion of equity in the constrained region, notice that
\[
E_{t+1} - E_{ss} = \Theta^b (\beta E_t (1 + \kappa))^{1-\xi} - \Theta^d (\beta E_t \kappa)^{1+\xi} - E_{ss}.
\]
Define \( g(E) \equiv \Gamma(E) E \). Thus,
\[
E_{t+1} - E_{ss} = \Gamma(E_t) E_t - E_{ss} = - \int_{E_t}^{E_{ss}} g'(e) \, de.
\]
It is enough to show that \( g'(e) > 0 \) for any \( e \). We verify that under the parameter assumptions, this is indeed the case. Note that
\[
g'(e) = (1 - \bar{\epsilon}) \Theta^b (\beta (1 + \kappa))^{1-\xi} e^{-\xi} - (1 + \varsigma) \Theta^d (\beta \kappa)^{1+\xi} e^\xi
\]
\[
= (1 - \bar{\epsilon}) R^b (\beta (1 + \kappa) e) \beta (1 + \kappa) > (1 + \varsigma) R^d (\beta \kappa e) \beta \kappa,
\]
where the second line follows from the definition of \( R^b \) and \( R^d \) and the result that capital requirements are binding in \( E < E_{ss} \). Furthermore, since in this region, \( R^b > R^d \) for all \( E < E_\kappa \), then a sufficient condition for \( g'(E) > 0 \) is simply
\[
(1 - \bar{\epsilon}) \beta (1 + \kappa) \geq (1 + \varsigma) \beta \kappa.
\]
Thus, a sufficient condition for monotone convergence is
\[
\frac{(1 - 1/\epsilon)}{(1 + 1/\varsigma)} \geq \frac{\kappa}{(1 + \kappa)}.
\]
K Data Sources

Except for the liquidity premium, all data series are obtained from the Federal Reserve Bank of St. Louis Economic Research Database (FRED ©) and are available at the FRED ©website. The original data sources for each series are collected by the Board of Governors of the Federal Reserve System (US). We use the following series corresponding to:

- the volume of interbank market loans:
  - Board of Governors of the Federal Reserve System (US), Interbank Loans, All Commercial Banks [IBLACBW027NBOG], H.8 Assets and Liabilities of Commercial Banks in the United States,
    
    https://fred.stlouisfed.org/series/IBLACBW027NBOG

- the volume of discount window loans:
  - Discount Window Borrowings of Depository Institutions from the Federal Reserve [DISCBORR], H.3 Aggregate Reserves of Depository Institutions and the Monetary Base,
    
    https://fred.stlouisfed.org/series/DISCBORR

- the interest on discount window loans:
  - Board of Governors of the Federal Reserve System (US), Primary Credit Rate [DPCREDIT],
    
    https://fred.stlouisfed.org/series/DPCREDIT

- the interest on reserves:
  - Board of Governors of the Federal Reserve System (US), Interest Rate on Required Reserves [IORR],
    
    https://fred.stlouisfed.org/series/IORR

- bank deposits:
  - Board of Governors of the Federal Reserve System (US), Deposits, All Commercial Banks [DPSACBM027NBOG],
    
    https://fred.stlouisfed.org/series/DPSACBM027NBOG

- the T-Bill rate is:
  - Board of Governors of the Federal Reserve System (US), 3-Month Treasury bill: Secondary Market Rate [TB3MS],
https://fred.stlouisfed.org/series/TB3MS

- bank loans:
  - Board of Governors of the Federal Reserve System (US), Commercial and Industrial Loans, All Commercial Banks [BUSLOANS],

https://fred.stlouisfed.org/series/BUSLOANS

The series that corresponds to open-market operations is the ratio of a measure of the Fed’s assets, normalized by total bank credit. The references for these series are:

- total bank credit
  - Board of Governors of the Federal Reserve System (US), Bank Credit of All Commercial Banks [TOTBKCR],

https://fred.stlouisfed.org/series/TOTBKCR

- The Fed’s assets are the sum of (WSRLL) securities, unamortized premiums and discounts, repo, and loans held by the fed minus treasury securities (WSHOTS)
  - Board of Governors of the Federal Reserve System (US), Assets: Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans [WSRLL],

https://fred.stlouisfed.org/series/WSRLL

  - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: U.S. Treasury Securities [WSHOTS],

https://fred.stlouisfed.org/series/WSHOTS

The return on the illiquid bond is obtained from Nagel (2016), which uses the return of three-month general collateral repurchase agreements. The liquidity premium corresponds to the difference between the return of this asset and the interest on reserves.

- liquidity ratio
  - Board of Governors of the Federal Reserve System (US), (Cash Assets, All Commercial Banks/Total Assets, All Commercial Banks)*100,

https://fred.stlouisfed.org/graph/?g=IW4
Background Facts

To frame the discussion in the quantitative application, we collect some features of the data recorded during the crisis. The data for the period June 2007 to February 2010 are presented in Figure L.1

**Fact 1: Depressed Lending.** Panel (a) presents the series for commercial and industrial (C&I), linearly detrended. We observe a decline in lending that begins around in late 2007. The decline continues through mid-2008 until there’s a partial recovery during the last quarter of 2008, possibly accounted for the drawdown of bank credit lines. Nevertheless, the decline in lending accelerates dramatically from January 2009 until the end of our sample in mid 2009.

**Fact 2: Deposit Expansion.** Panel (b) presents the series for deposits. We can observe that throughout the period, banks continued to issue deposits, suggesting that there was not a systemic problem of bank funding.

**Fact 3: Increased Discount Window Loans.** Panel (c) plots the monthly series for the ratio of Fed discount window loans relative to total deposits. The figure shows an increase in discount loans by the Fed that began in early 2008. The volume of discount window loans jumps rapidly around the Lehman Brothers crisis in September 2008 and then reverts to pre-crisis levels.

**Fact 4: Depressed Federal Funds Market Borrowing.** Panel (d) plots the monthly series for total Federal Funds market loans relative to deposits. The figure shows a continuous decline that begins in early 2007. The decline persists throughout the entire sample, as emphasized in Afonso and Lagos (2015) and Afonso and Lagos (2014). Together with Fact 3, this figure shows a substitution away from interbank lending to discount window lending. The figure also shows that the decline in interbank lending precedes the expansion of the Fed’s balance sheet observed in Panel (f). This and the series in Panel (c) are used to reproduce a series for the withdrawal volatility of deposits in the application of Section 6.

**Fact 5: Increased Interest on Reserves.** Panel (e) plots the monthly series for the Fed’s interest payment on reserves. The figure shows how the Fed began paying interest on reserves since October 2008. Following an initial increment, the rate on reserves dropped to a floor of 25 bps.

**Fact 6: Fed Balance Sheet Expansion.** Panel (f) shows the large-scale open-market operations of the Fed. We can see how since the beginning of our sample, the Fed carried out a sequence of expansions of its balance sheet. The series begins with a balance sheet that amounts to 0.5 percent of total bank loans, but by the end of the sample, the size of the Fed’s balance sheet is 12 percent of private loans.

**Fact 7: Increase of Liquidity Ratio.** Panel (g) shows the substantial increase in the liquidity ratio, which is the counterpart of the decline in bank lending.

**Fact 8: Increased Liquidity Premium.** Panel (h) presents the liquidity premium. We compute the liquidity premium as the difference in returns between the three-month general collateral repurchase agreements (GC repo), and reserves at the Federal Reserve. The GC repo is effectively an interbank loan collateralized with Treasury securities. This definition of liquidity premium follows from Nagel (2016), except that we use the interest on reserves as opposed to the return on T-bills. We observe that following an initial decline in the first quarter of 2008, the premium spikes during the third quarter of that year and begins a decline in 2009. For more details, see Nagel (2016) or Del Negro et al. (2017).
Figure L.1: Data
M Algorithms

This appendix presents the numerical algorithms that we use to solve the model. We first present the algorithm to solve the stationary equilibrium in which the Fed’s nominal balance sheet grows at rate $g_m$. We then present the algorithm to solve for transitional dynamics. In both cases, we assume that $\sigma = 1$ to simplify the dividend decision.

M.1 Computation: Stationary Equilibrium

We describe the algorithm to solve for the stationary equilibrium. In a stationary equilibrium, all nominal variables grow at rate $g_m$ and real variables are constant. We also set a value for $R^d_{ss}$ based on the calibration target and infer the intercept term $\Theta^d$ that is consistent with that value using (13).

1. Set the growth rate of the Fed’s nominal balance sheet to $g_m$ and fix a level for $B^{Fed}$, $M^{Fed}$. Assume for simplicity that $B^{Fed} = 0$.

2. Guess a stationary value for the return on loans $R^b_{ss}$ and market tightness $\theta_{ss}$.

3. Given market tightness, policy rates, and $R^d_{ss}$, compute the liquidity yield function using (10).

4. Solve banks’ optimization problem:

   (a) Compute portfolio weights $\{\bar{b}, \bar{d}, \bar{m}\}$ and certainty equivalent value $\Omega$ with (20) with conjectured real return on loans and liquidity yield function computed in step 3.

   (b) Compute value of the bank $v$ and consumption using (E.2.2) and (E.2.1), respectively.

5. Check whether banks’ policies are consistent with steady state:

   (a) Compute aggregate gross equity growth as

   $$(1 - \bar{c}) \mathbb{E}_\omega (R^b_{ss} \bar{b} + \bar{m} - R^d_{ss} \bar{d}).$$

   (b) Compute implied market tightness:

   $$S^- = \int_{1}^{\frac{\bar{m}/(1-\rho)}{\bar{d}/(1-\rho)}} s(\omega) d\Phi$$
   and
   $$S^+ = \int_{\frac{\bar{m}/(1-\rho)}{\bar{d}/(1-\rho)}}^{\infty} s(\omega) d\Phi.$$  

   Market tightness is defined as
   $$\tilde{\theta} = \frac{S^-}{S^+}.$$  

6. If the equity growth rate is zero and $\tilde{\theta} = \theta_{ss}$, move to step 7. Otherwise, adjust the guess for $R^b_{ss}$ and $\theta_{ss}$ and go to step 3.

7. Compute the nominal amount of reserves and the intercepts of the loan demand and deposit
supply functions using that real equity and the initial price level are normalized to one and

\[
\tilde{M}^{Fed} = E(1 - \tilde{c})\bar{m}EP,
\]

\[
\Theta_b\left(\frac{1}{R^b}\right)^\epsilon = \left(E\tilde{b}(1 - \tilde{c})\right) - \frac{B^{Fed}}{P},
\]

\[
\Theta_d\left(\frac{1}{R^d}\right)^{-\varsigma} = E\tilde{d}(1 - \tilde{c}).
\]

8. Compute nominal returns using definitions of real returns and transfers \(T^{Fed}\) from the Fed budget constraint:

\[
W^{Fed}(i_{dw} - \pi) + B^{Fed}(i^b - \pi) + PT^{Fed} = M^{Fed}(i^{ior} - \pi),
\]

where

\[
M^{Fed} = \tilde{M}^{Fed} + W,
\]

\[
W = (1 - \Psi^- (\theta))S^-.
\]

To compute expectations, we use a Newton-Cotes quadrature method. Specifically, we apply the trapezoid rule with a grid of 2,000 equidistant points. To specify the lower and upper boundaries of the grid, we take the shock values that guarantee \(10^{-5}\) mass in the tails of the distribution.

**M.2 Computation: Transitional Dynamics under Baseline Policy**

The algorithm to solve for transitional dynamics starts by conjecturing an initial price level and then solves for all sequences of prices and quantities using market clearing conditions and bank problems. After that, we check that the initial price leads the economy to converge to the stationary equilibrium after many periods. The balance sheet of the Fed grows at rate \(g\) and sets \(1 + i_{t+1} = (1 + r_{t+1})(1 + \pi_{t+1})\). We assume log utility.

1. Establish a finite period \(T \in \mathbb{N}\) for convergence to steady state and convergence criterion \(\varepsilon\).
2. Set initial deviation from real steady-state equity \(\delta \in (0, 1)\) such that \(E_1 = (1 - \delta)E_{ss}\).
3. Guess an initial price level \(P_0\).
4. Set \(t = 1\).
5. Given \(E_t, P_t, \tilde{M}_{t+1}^{Fed}, \forall t = 1, ..., T\), define the real fixed supply of the share of reserves:

\[
\tilde{m} \equiv \frac{\tilde{M}_{t+1}^{Fed}}{\beta P_t E_t}.
\]
6. Given \( \tilde{m} \), compute \((\tilde{b}_t, \tilde{m}_t, \tilde{d}_t, r^b_{t+1}, r^d_{t+1}, i_{t+1})\), which solve
\[
\begin{align*}
\tilde{m}_t &= \tilde{m} , \\
\beta E_t \tilde{d}_t &= \Theta^d_t \left(1 + r^d_{t+1}\right) \zeta , \\
\beta E_t \tilde{b}_t &= \Theta^b_t \left(1 + r^b_{t+1}\right) \epsilon + B^{Fed}_{t+1} P_t ,
\end{align*}
\]
where
\[
\tilde{b}_t + \tilde{m}_t - \tilde{d}_t = 1,
\]
\[
(\tilde{b}_t, \tilde{m}_t, \tilde{d}_t) = \arg \max_{b, m, d} \{ \mathbb{E}_\omega \left[ \ln \left( R^b_{t+1} \tilde{b} + R^m_{t+1} \tilde{m} - R^d_{t+1} \tilde{d} + \chi(s_t) \right) \right] \}
\]
s.t. \( \tilde{b} + \tilde{m} - \tilde{d} = 1 \)
\( \tilde{d} \leq \kappa . \)

This is a system of six equations and six unknowns. Notice that if the capital requirement constraint binds, the system can be reduced to one unknown, \( R^m_{t+1} \), and one equation,
\[
\begin{align*}
\arg \max_{\tilde{m}} \mathbb{E}_\omega \left[ \ln \left( \left( \frac{\beta E_t}{\Theta^b_t} - B^{Fed}_{t+1} \right)^{-\frac{1}{\epsilon}} (1 + \kappa - \tilde{m}) \frac{\epsilon - 1}{\epsilon} + \frac{R^m_{t+1}}{P_t} - \left( \frac{\beta E_t}{\Theta^d_t} \right)^{\frac{1}{\zeta}} \kappa \frac{\zeta - 1}{\zeta} + \chi(s_t) \right) \right] = \tilde{m}.
\end{align*}
\]

If the capital requirement does not bind, the system can be reduced to two unknowns \( \{R^b_{t+1}, R^m_{t+1}\} \) and two equations,
\[
\begin{align*}
\arg \max_{\tilde{m}, \tilde{d}} \mathbb{E}_\omega \left[ \ln \left( R^b_{t+1} (1 + \tilde{d} - \tilde{m}) + R^m_{t+1} \tilde{m} - \left( \frac{\beta E_t}{\Theta^b_t} \right)^{\frac{1}{\epsilon}} \tilde{d} \frac{\epsilon - 1}{\epsilon} + \chi(s_t) \right) \right] = \left( \tilde{m}, \frac{\Theta^b_t (R^b_{t+1})^{-\epsilon} + B^{Fed}_{t+1} P_t}{\beta E_t} + \tilde{m} - 1 \right) .
\end{align*}
\]

7. Given \( R^m_{t+1} \) and the nominal interest on reserves set by the Fed \( i_{t+1} \), compute inflation as
\[
\pi_{t+1} = \left( \frac{1 + i_{t+1}}{R^m_{t+1}} \right) - 1 .
\]

8. Given \( \pi_{t+1} \) and \( P_t \), compute next-period price \( P_{t+1} = (1 + \pi_{t+1}) P_t . \)

9. Compute next-period equity:
\[
E_{t+1} = \beta \left( P^b_{t+1} \tilde{b}_t + \tilde{m}_t - R^d_{t+1} \tilde{d}_t \right) E_t - \left( \frac{B^{Fed}_{t+2}}{P_{t+1}} \right) - \left( \frac{\tilde{m}^{Fed}_{t+2}}{P_{t+1}} \right) .
\]

10. If \( t < T \), return to step 6 with \( t = t + 1 \).
11. Compute criteria for convergence of \( z = P_{T+1} - P_0(1 + \pi_{ss})^T \).

12. If \(|z| < \varepsilon\), exit algorithm. Otherwise, adjust \( P_0 \) and go to step 4.

### M.3 Transitional Dynamics under Inflation Targeting

We describe the transitional dynamics when the Fed adjusts its balance sheet to keep the price level growing at the steady state inflation rate. To do this, we expand the tools of the Fed with deposits on banks, which we denote by \( D_{Fed}^{t+1} \). Given this, the budget constraint of the Fed becomes

\[
M_{Fed}^t(1 + i_{ior}^t) + B_{Fed}^{t+1} + D_{Fed}^t + W_{Fed}^t = M_{Fed}^{t+1} + D_{Fed}^t(1 + i_d^t) + W_{Fed}^{t+1} + P_t T_t,
\]

and we assume the Fed offsets variations in \( M_t \) with \( D_{Fed}^{t+1} \) (i.e., \( \Delta M_{Fed}^t = \Delta D_{Fed}^{t+1} \)). Given this, following the same steps as in (D.8), the law of motion for aggregate equity becomes

\[
E_t^{t+1} = \left( R_b^t (1 - \bar{c}_t) + \tilde{M}_{t+2} + \frac{D_{Fed}^t(1 + i_d^t)}{P_t} \right) E_t^t.
\]

We continue to assume as in our baseline that the Fed sets \( 1 + \tau_{dw}^{t+1} = \frac{1 + \pi_{dw}^{t+1}}{1 + \pi_{ss}} \). Notice that since \( P_{t+1}/P_t = 1 + \pi \), we have that the real return on reserves is entirely determined by policy \( R_{m}^t = (1 + \tau_{ior}^{t})/(1 + \pi) \). We assume log utility.

1. Establish a finite period \( T \in \mathbb{N} \) for steady-state convergence and convergence criterion \( \varepsilon \).

2. Set initial deviation from real steady-state equity \( \delta \in (0, 1) \) such that \( E_1 = (1 - \delta) E_{ss} \).

3. Set \( t = 0 \).

4. Find \((\bar{d}_t, \bar{b}_t, \bar{m}_t, R_{b,t+1}^t, R_{d,t+1}^t, M_{t+1}, D_{t+1}^{Fed})\) that solves

\[
\beta E_t \bar{d}_t = \Theta_d \left( \frac{1}{R_d} \right)^\zeta + D_{t+1}^{Fed}/P_t,
\]

\[
\beta E_t \bar{b}_t + B_{t+1}^{Fed}/P_t = \Theta_b \left( \frac{1}{R_{b,t+1}} \right)^\epsilon,
\]

\[
\bar{m}_t = \frac{\bar{M}_t}{\beta P_t E_t},
\]

\[
\Delta M_{t}^{Fed} = \Delta D_{Fed}^{t+1},
\]

where

\[
(\bar{b}_t, \bar{m}_t, \bar{d}_t) = \arg \max_{\bar{m}, \bar{b}, \bar{d}} \left\{ \mathbb{E}_\omega \left[ \ln \left( \left( R_{b,t+1}^t \bar{b} + (1 + r_{ior}^{t+1}) \bar{m} - (1 + r_d^{t+1}) \bar{d} + \chi(s_t) \right) \right) \right] \right\}
\]

s.t. \( \bar{b} + \bar{m} - \bar{d} = 1 \),

\( \bar{d} \leq \kappa \).

This system of seven equations and seven unknowns can be solved block recursively. If the capital requirement constraint binds, the system can be reduced to one unknown, \( \bar{m} \), and one
equation,
\[
\arg \max_{\tilde{m}} \left\{ \mathbb{E}_\omega \left[ \ln \left( \left( \frac{\beta E_t}{\Theta_t^b} \right)^{\frac{1}{\gamma}} (1 + \kappa - \tilde{m})^{\frac{\gamma-1}{\gamma}} + R_{t+1}^m \tilde{m} - \left( \frac{\beta E_t}{\Theta_t^d} \right)^{\frac{1}{\gamma}} \kappa^{\frac{\gamma+1}{\gamma}} + \chi(s_t) \right) \right] \right\} = \tilde{m}.
\]

If the capital requirement is not binding, the system can be reduced to two equations and two unknowns, $R^b, R^d$. In order to solve this system, conjecture a pair $(R^b, R^d)$, solve the portfolio problem, find the rates that are consistent with market clearing conditions, and then update the guesses for $R^b, R^d$ accordingly.

5. Compute next-period equity using (M.1).

6. If $|E_t - E_{ss}| < \varepsilon$, exit algorithm. Otherwise, $t = t + 1$ and go to step 4.