Liquidity Traps, Prudential Policies, and International Spillovers*

Javier Bianchi
Federal Reserve Bank of Minneapolis

Louphou Coulibaly
Federal Reserve Bank of Minneapolis,
University of Wisconsin-Madison & NBER

May 2023

Abstract

We investigate optimal monetary and macroprudential policies in an open economy with aggregate demand externalities and an occasionally binding zero lower bound constraint. The optimal policy balances output stabilization and capital flow management, potentially requiring lower or higher nominal interest rates. We show how international spillovers operate through the world real rate and call for macroprudential policies. Finally, we establish that a world economy with macroprudential policy welfare dominates a laissez-faire regime, in stark contrast with recent concerns.

Keywords: Capital flows, monetary and macroprudential policies, liquidity traps, international spillovers

JEL Classifications: E21, E23, E43, E44, E52, E62, F32

*For useful comments and suggestions, we thank Mick Devereux, Luca Fornaro, Rishabh Kirpalani, Karlye Stedman, and seminar participants at Berkeley, Chicago Fed, Cleveland Fed, Federal Reserve Board, Iowa State, IMF, Minneapolis Fed, Michigan, University of California Santa Cruz, University of Virginia, Wisconsin, Bank of Canada Workshop on Monetary Policy, and the NBER IFM and ME Summer Institute Meeting. The views expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Macroprudential policy has emerged as a new pillar of the macroeconomic toolkit. A key premise is that the use of these policies can help manage capital flows and reduce the vulnerability to deep economic contractions. However, our understanding of how macroprudential policy should be integrated with other macro policies, especially monetary policy, remains limited. Moreover, there is a concern that these policies may backfire when adopted on a global scale.[1]

In this paper, we provide an analytical and quantitative analysis of monetary and macroprudential policies. We consider a dynamic model with incomplete markets, sticky prices, and two sectors (tradable and non-tradable). The central bank is subject to an occasionally binding zero lower bound constraint on nominal interest rates. The presence of a constraint on monetary policy implies that fluctuations in capital flows can affect the degree of slack in the economy and generate scope for macroprudential policy, as in Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016). Our study investigates how the availability of macroprudential policy affects the optimal conduct of monetary policy, assesses potential negative spillovers from foreign prudential policies, and explores the role of macroprudential policies in providing insulation.

We first establish that in the absence of macroprudential policy, monetary policy faces an intertemporal tradeoff that balances the current output gap and stabilizing capital flows. Contrary to a widespread policy view, however, we show that a policy of leaning against the wind (raising the interest rate in booms) to avert a liquidity trap is not necessarily optimal. Given aggregate income, an increase in the nominal interest rate leads to a decline in consumption and borrowing through an intertemporal substitution effect. However, the reduction in consumption and aggregate demand reduces output and leads to a higher need for borrowing to smooth consumption. We show that if the elasticity of substitution across sectors is higher than the elasticity across time, a rise in the interest rate is counterproductive because it increases inefficiently the level of borrowing.

When macroprudential policy is available, we establish that monetary policy is no longer used with a prudential purpose. In this case, the central bank uses monetary policy to stabilize output and taxes inflows when the economy is away from the zero lower bound. Interestingly, the macroprudential tax on debt is positive only if the zero lower bound is likely to bind in the following period, whereas monetary policy is used prudentially in the

[1] For an overview of the policy discussions on international spillovers, see Rey (2013), Rajan (2014), Kalemli-Ozcan (2019), and Gourinchas (2022).

absence of macroprudential policy as long as the zero lower bound is foreseen to bind in some distant future. The key lesson is that because monetary policy is a blunter instrument, it has to be used even more preemptively than macroprudential policy. Furthermore, we show that the central bank may find it optimal to restrict outflows when there is a deep downturn caused by the liquidity trap and that as macroprudential policy becomes more stringent, monetary policy should become more expansionary.

Our quantitative evaluation underscores that optimal macroprudential policy can substantially improve macroeconomic stabilization and alleviate the costs of liquidity traps. In the absence of macroprudential policy, the average unemployment, conditional on a liquidity trap, is about 6%, and the unconditional welfare cost of liquidity traps is 0.5% of permanent consumption. With macroprudential policy, unemployment becomes 1.5%, and the welfare cost falls to 0.1%. In terms of policies, we find that the ex-ante prudential tax on inflows is 0.2%, while the ex-post tax on outflows is -0.05% on average. We also find that while liquidity traps are less frequent and less severe with macroprudential policy, perhaps surprisingly, they tend to last longer.

Our final set of results is concerned with international spillovers and their welfare implications. Our findings reveal that these spillovers primarily operate through a financial channel. Specifically, we demonstrate that assessing the welfare effects of changes in monetary policy or macroprudential policy abroad can be fully determined by evaluating whether an increase or decrease in the real interest rate is desirable. When foreign policies lead to a reduction in the real interest rate, the domestic economy's welfare falls when it is vulnerable to a liquidity trap in the future and macroprudential policy is not available. This is because the reduction in the real rate exacerbates the overborrowing problem emerging from the aggregate demand externality. We also argue that these spillovers can open the door to currency wars, necessitating monetary policy cooperation. However, we show that macroprudential policies can be used to insulate the domestic economy from monetary policy spillovers and eliminate the need for coordination, in line with the arguments raised by Blanchard (2021). Moreover, we demonstrate that macroprudential policies achieve welfare gains even in the absence of coordination, in stark contrast with the results in Fornaro and Romei (2019).

Related literature. Our paper relates to several strands of the literature. First, our paper belongs to the literature on aggregate demand externalities that emerge from nominal rigidities and constraints on monetary policy such as fixed exchange rates or the zero lower bound (Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016; Korinek and Simsek, 2016). These papers focus on the optimal macroprudential policy, given an exogenous monetary policy, or deal with jointly optimal monetary and macroprudential policy. A distinct contribution of our paper is to characterize how the availability of macroprudential policy, or lack thereof, affects the optimal monetary policy, and trace the international spillovers and its welfare implications. Specifically, we demonstrate how macroprudential policy can insulate an economy from international policy spillovers and avoid a currency war.[2]

Our paper is also related to the vast literature on liquidity traps in open economies.[3] A key theme in this literature has to do with the extent to which liquidity traps are transmitted across countries. Caballero, Farhi and Gourinchas (2021) present a model of global liquidity traps where a recession in one block is exported abroad through goods and asset markets. Eggertsson, Mehrotra, Singh and Summers (2016) argue that neo-mercantilist policies in some countries can bring the whole world economy into a state of secular stagnation with a permanently depressed level of output. In these two studies, each country produces a tradable good, which is equally demanded by domestic and foreign households. We consider instead the polar opposite case, in which the goods produced subject to nominal rigidities are consumed exclusively by domestic households. Our analysis uncovers how this feature implies that foreign policies that favor savings actually increase the demand for domestic goods via asset markets and can become stabilizing at the zero lower bound.

In terms of the international spillovers from macroprudential policy, our results contrast to those in Fornaro and Romei (2019). They find that capital account policies may lead to a global paradox of thrift, in which uncoordinated macroprudential policies at the global level lead to worse output and welfare outcomes compared with those of a laissez-faire economy without macroprudential policy. Their argument is that macroprudential policy implemented by countries away from a liquidity trap reduces the world real interest rate and tightens the zero lower bound constraint of those countries in a liquidity trap. We show, however, that a simple macroprudential quantity restriction on capital flows can insulate

[2] Farhi and Werning (2020) is another recent paper that examines monetary and macroprudential policy interactions, but in the context of a two-period closed economy model with behavioral features. See also Coulibaly (2020) and Basu, Boz, Gopinath, Roch and Unsal (2020) for models of optimal policies featuring pecuniary externalities. Several other studies consider monetary and macroprudential interactions but do not characterize optimal policies (e.g., Aoki, Benigno and Kiyotaki, 2016; Van der Ghote, 2021; Rubio and Yao, 2020; Ferrero, Harrison and Nelson, 2022). Collard et al. (2017) studies joint optimal monetary and macroprudential policy in the context of a moral hazard externality.

[3] Examples include Cook and Devereux (2013); Devereux and Yetman (2014); Eggertsson, Mehrotra, Singh and Summers (2016); Caballero, Farhi and Gourinchas (2021); Fornaro and Romei (2019); Acharya and Bengui (2018); Jeanne (2009); Benigno and Romei (2014); Fornaro (2018); Corsetti et al. (2019a); Corsetti et al. (2019b); Kollmann (2021) and Amador, Bianchi, Bocola and Perri (2020). Notable closed economy studies include Krugman (1998), Eggertsson and Woodford (2003), and Werning (2011).

an economy from a lower world real interest rate. Thus, we find that a macroprudential policy regime is superior to the laissez-faire, even without coordination.

Egorov and Mukhin (2023) and Fanelli (2023) also study the interaction between optimal monetary policy and optimal capital controls. Egorov and Mukhin (2023) consider a setup with dollar currency pricing and a general production structure. They show that although monetary policy cannot achieve full insularity, capital controls are not desirable because they fail to affect external aggregate demand. Fanelli (2023) studies optimal monetary policy under commitment with home currency portfolios. In his setup, exchange rates have a shock absorber role and an insurance role, thus calling for capital controls. He also shows that to a second-order approximation, the government chooses the same capital control tax for all assets, irrespective of their currency. Our analysis focuses instead on a framework with monetary policy constraints and highlights the transmission of international spillovers.

Outline. Section 2 presents the model. Section 3 studies optimal monetary and macroprudential policy. Section 5 analyzes international spillovers. Section 6 concludes.

2 Model

We consider a small open economy with nominal rigidities and an occasionally binding zero lower bound constraint. There is an infinite horizon and two types of goods: tradables and non-tradables. In this section, we describe the decisions of households and firms and the general equilibrium.

2.1 Households

There is a continuum of identical households of measure one. Households' preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=0}^{t} \delta_k \right) \Big[U(c_t) - v(h_t) \Big], \tag{1}$$

where $\mathbb{E}_t$ denotes the time t expectation operator, $\beta\delta_t$ is the discount factor at time t and δ_t represents a discount factor shock. The utility function over consumption $u(\cdot)$ is strictly increasing and concave, and $v(\cdot)$ denotes an increasing and convex disutility function of labor. We assume that these functions are isoelastic of the form

$$U(c_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad v(h_t) = \frac{h_t^{1+\phi}}{1+\phi},$$

where σ is the elasticity of intertemporal substiution and ϕ is the inverse of the Frisch elasticity. The consumption good c_t is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a constant elasticity of substitution aggregator:

$$c_t = \left[\omega (c_t^T)^{1-\frac{1}{\gamma}} + (1-\omega)(c_t^N)^{1-\frac{1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \quad \text{where } \omega \in (0,1).$$

The elasticity of substitution between tradables and non-tradables is γ. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods.

In each period t, households supply h_t units of labor and are endowed with y_t^T units of tradable goods. We assume that y_t^T is stochastic and follows a first-order Markov process. Households receive a wage rate, W_t, collect profits, ϕ_t^N, all expressed in terms of domestic currency, which serves as the numeraire, and receive government transfers T_t. Households trade two types of one-period non-state-contingent bonds in credit markets: a real bond b_{t+1}^*, which pays a gross return R_t^* units of tradables, and a nominal bond b_{t+1}, which pays R_t in units of domestic currency. The domestic government controls the nominal rate R_t. Both bonds are potentially subject to a tax/subsidy τ_t. When $\tau_t > 0$, households face a tax on debt issuance and a subsidy on savings. Conversely, when $\tau_t < 0$, households face a subsidy on debt issuance and taxes on savings.

The budget constraint of the representative household is therefore given by

$$P_t^N c_t^N + P_t^T c_t^T + \frac{1}{1+\tau_t}\left[\frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*}\right] = \phi_t^N + W_t h_t + P_t^T (y_t^T + T_t) + b_t + P_t^T b_t^\star, \quad (2)$$

where P_t^N and P_t^T denote respectively the price of non-tradables and tradables (in terms of the domestic currency). The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds while the right-hand side represents total income, including the returns from bond holdings.

Optimality conditions. The households' problem consists of choosing sequences of $\{c_t^N, c_t^T, h_t, b_{t+1}, b_{t+1}^*\}$ to maximize the expected present discounted value of utility (1), subject to (2) and taking as given the sequence of tradable endowments $\{y_t^T\}$, profits $\{\phi_t^N\}$, transfers $\{T_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}$.

The first-order conditions for consumption and labor yield

$$\frac{W_t}{P_t^N} = \frac{v'(h_t)}{u_N(c_t^T, c_t^N)} \tag{3}$$

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\gamma}} \tag{4}$$

where u_N denotes the marginal utility of non-tradable consumption in period t. Condition (3) is the labor supply optimality condition equating the marginal rate of substitution between leisure and non-tradable consumption with the wage rate in terms of non-tradables. Condition (4) equates the marginal rate of substitution between tradables and non-tradables to the relative price.

The first-order conditions for the nominal and real bond holdings yield

$$u_T(c_t^T, c_t^N) = \beta R_t^* (1+\tau_t) \mathbb{E}_t \left[\delta_{t+1} u_T(c_{t+1}^T, c_{t+1}^N)\right] \tag{5}$$

$$\frac{u_T(c_t^T, c_t^N)}{P_t^T} = \beta R_t (1+\tau_t) \mathbb{E}_t \left[\delta_{t+1} \frac{u_T(c_{t+1}^T, c_{t+1}^N)}{P_{t+1}^T}\right]. \tag{6}$$

where u_T denotes the marginal utility of tradable consumption. Households equate the marginal benefit from saving in nominal or real bonds to the marginal costs of cutting tradable consumption today to buy the bonds.

2.2 Firms

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a production technology given by $y_t^N = n_t^\alpha$ and perceives profits given by

$$\phi_t^N = P_t^N n_t^\alpha - W_t n_t. \tag{7}$$

We assume that prices are perfectly rigid, $P_t^N = \bar{P}^N$, and that firms produce goods to satisfy demand. That is, labor demand in equilibrium is given by $n = \left(c^N\right)^{1/\alpha}$. In our quantitative analysis, we extend the model to allow for partial price adjustments.

2.3 Government

The government sets a nominal interest rate $R_t \geq 1$ and a tax on all forms of bond issuances τ_t. As is common in the literature, this tax can be interpreted as a capital control or as a macroprudential policy (see e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2016; and Fornaro and Romei, 2019). The tax is assumed to be rebated lump-sum to households, an assumption that is without loss of generality given that Ricardian equivalence holds.[4] That is, the government budget constraint is

$$T_t = -\frac{\tau_t}{1+\tau_t}\left[\frac{b_{t+1}}{P_t^T R_t} + \frac{b_{t+1}^*}{R_t^*}\right]. \tag{8}$$

2.4 Prices, Interest Parity, and Exchange Rates

We assume that the law of one price holds for the tradable good, that is, $P_t^T = e_t P_t^{T*}$, where e is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency, and P^{T*} is the price of the tradable good denominated in foreign currency.

Using the Euler equations for international bond (5) and domestic bond (6), we can equate the marginal benefits from buying the real and nominal bond. Together with the law of one price, this implies that the nominal exchange rate must satisfy the risk-adjusted uncovered interest parity condition:

$$R_t^* = R_t \, \mathbb{E}_t\left[\Lambda_{t+1}\frac{e_t}{e_{t+1}}\frac{P_t^{T,*}}{P_{t+1}^{T,*}}\right], \tag{9}$$

where $\Lambda_{t+1} \equiv \delta_{t+1} u_T(c_{t+1}^T, c_{t+1}^N)/\mathbb{E}_t\left[\delta_{t+1} u_T(c_{t+1}^T, c_{t+1}^N)\right]$ represents a stochastic discount factor. Condition (9) is a standard condition that relates the foreign real interest rate and the domestic nominal interest rate to the expected depreciation of the domestic currency.

[4] We abstract from other the so-called unconventional fiscal policies that can relax the zero lower bound (see e.g. Correia, Farhi, Nicolini and Teles, 2013). We also abstract from differential taxes on domestic and foreign currency bonds. As examined in Acharya and Bengui (2018), differential taxes on bonds across currencies can also help relax the zero lower bound. As long as there are some limitations on the use of these policies (either political or economic), the first best cannot be implemented and our key results would remain.

2.5 Competitive Equilibrium

Market clearing for labor requires that the units of labor supplied by households equal the aggregate labor demand by firms:

$$h_t = n_t. \tag{10}$$

Market clearing for the non-tradable good requires that output be equal to non-tradable consumption:

$$y_t^N = c_t^N. \tag{11}$$

We assume that the bond denominated in domestic currency is traded only domestically. We make this assumption to abstract from portfolio problems and from the possibility of inflating away external debt.[5] Market clearing therefore implies

$$b_{t+1} = 0. \tag{12}$$

Combining the budget constraints of households, firms, and the government, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y_t^T = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{13}$$

which says that the trade balance must be financed with net bond issuances.

An equilibrium, given government policies, is defined as follows.

Definition 1. Given an initial condition b_0^*, exogenous process $\{R_t^*, y_t^T, \delta_t\}_{t=0}^{\infty}$, a rigid price $\bar{P}^N$, and government policies $\{R_t, \tau_t\}_{t=0}^{\infty}$, an equilibrium is a stochastic sequence of prices $\{e_t, P_t^{T*}, W_t\}$ and allocations $\{c_t^T, c_t^N, b_{t+1}^*, n_t, h_t\}_{t=0}^{\infty}$ such that

(i) households optimize, and hence the following conditions hold: (3), (4), (5), (6);

(ii) firms choose hours to meet demand, $h_t^{\alpha} = c_t^N$;

(iii) labor market clears (10) and the domestic currency bond is in zero net supply (12);

(iv) the government budget constraint (8) is satisfied;

(v) the law of one price holds: $P_t^T = e_t P_t^{T*}$.

[5]See Fanelli (2023) for an interesting study of optimal monetary policy with nominal external debt and incomplete markets. In his model, the government can commit to future policies and uses monetary policy to improve risk-sharing in addition to the standard objectives.

Notice that the ideal price index (i.e., the minimum expenditure, denominated in units of tradables, required to buy one unit of the composite good c_t) is given by:

$$\mathcal{P}_t \equiv \left[\omega^{\gamma} + (1-\omega)^{\gamma}\left(\frac{\bar{P}^N}{e_t P_t^{T*}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{14}$$

In addition, for future reference, the share of expenditures in tradables is denoted by

$$\tilde{\omega}_t \equiv P_t^T c_t^T / (P_t^T c_t^T + \bar{P}^N c_t^N). \tag{15}$$

2.6 First-Best Allocation

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the small open economy who chooses allocations subject to resource constraint. The planner's problem can be written as

$$\max_{\left\{b_{t+1}^*, c_t^N, c_t^T\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty}\left(\prod_{k=0}^{t}\beta\delta_k\right)\left[u\left(c_t^T, c_t^N\right) - v\left(\left(c_t^N\right)^{1/\alpha}\right)\right], \tag{16}$$

subject to

$$c_t^T = y_t^T + b_t^* - \frac{b_{t+1}^*}{R_t^*}.$$

The first-best allocation equates the value of one additional employed unit of labor to the marginal cost of leisure

$$\alpha h_t^{\alpha-1} u_N(c_t^T, c_t^N) = v'(h_t) \tag{17}$$

It also equates the marginal utility of current consumption to the marginal utility of saving one extra unit and consuming in the next period:

$$u_T(c_t^T, c_t^N) = \beta R_t^* \mathbb{E}_t\left[\delta_{t+1} u_T(c_{t+1}^T, c_{t+1}^N)\right]. \tag{18}$$

It should be clear that the allocations in a competitive equilibrium with flexible prices would coincide with the first best. This can be seen by noting that if firms could adjust prices, we would have $\alpha h_t^{\alpha-1} = W_t / P_t^N$, which, combined with households' labor supply decision (3), would yield (17). [6] Moreover, as we will see, with sticky prices, a government that can choose monetary policy without any constraints would choose to replicate the

[6] In addition, notice that (18) coincides with households' optimality (5) when $\tau_t = 0$.

flexible price allocation and hence implement the first-best allocations. We note that often New Keynesian open-economy models often feature monopolistic competition and terms of trade externalities which create an additional wedge between competitive equilibrium with nominal rigidities and flexible price equilibria. Our framework allows us to focus squarely on aggregate demand management considerations.

The departure of the equilibrium allocations from the first best can be conveniently summarized in the labor wedge, defined below:

$$\psi_t \equiv 1 - \frac{1}{\alpha h_t^{\alpha-1}} \frac{v'(h_t)}{u_N(c_t^T, c_t^N)}. \tag{19}$$

A positive labor wedge, $\psi_t > 0$, reflects a recession, whereas a negative labor wedge, $\psi_t < 0$, reflects overheating.

3 Optimal Monetary and Macroprudential Policies

In this section, we study optimal monetary and macroprudential policies. To shed light on the policy interactions, we first study optimal macroprudential policy given monetary policy, we then study joint optimal monetary and macroprudential policy, and finally, we study optimal monetary policy given a macroprudential policy.

3.1 Macroprudential Policy

We consider a generic monetary policy that depends on the history of all shocks. We use $\{e_t\}$ to denote the nominal exchange rate policy sequence chosen by the government. An advantage of this formulation is that we are able to provide a general characterization of macroprudential policy encompassing multiple monetary policy regimes. This will set the stage to analyze the interactions between optimal monetary and macroprudential policies.

Under an arbitrary monetary policy, the production of non-tradable goods is, in general, inefficient. For example, given the sticky price $\bar{P}^N$, a low exchange rate implies a high relative price for non-tradables, in turn generating lower household demand for non-tradable goods and leading firms to reduce production and generating a positive labor wedge.

Given a sequence of $\{e_t\}$, the government chooses the state-contingent tax on debt $\{\tau_t\}$ that maximizes private agents' welfare among the set of competitive equilibria. The problem can be written as

$$\max_{\{b^*_{t+1}, c^N_t, c^T_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta \delta_k \right) \left[u\left(c^T_t, c^N_t \right) - v\left((c^N_t)^{1/\alpha} \right) \right], \tag{20}$$

subject to

$$c^T_t = y^T_t + b^*_t - \frac{b^*_{t+1}}{R^*_t},$$

$$c^N_t = \left[\frac{1-\omega}{\omega} \frac{P^{T*}_t}{\bar{P}^N} e_t \right]^{\gamma} c^T_t.$$

The last constraint in problem (20) relates non-tradable consumption to tradable consumption and the relative price of non-tradables. More tradable resources increase aggregate demand for both goods. Given a fixed price for non-tradables, higher resources translate into more demand for non-tradable goods, to which firms respond by raising employment. This general equilibrium feedback is key for the characterization of the optimal macroprudential tax presented below.

Proposition 1 (Optimal macroprudential policy given $\{e_t\}$)**.** *Consider an exogenous exchange rate policy $\{e_t\}$. The optimal tax on borrowing* (20) *satisfies*

$$\tau_t = \frac{1}{\beta R^*_t \mathbb{E}_t \delta_{t+1} u_T(t+1)} \left\{ -\frac{1-\tilde{\omega}_t}{\tilde{\omega}_t} u_T(t) \psi_t + \beta R^*_t \mathbb{E}_t \delta_{t+1} \left[\frac{1-\tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} u_T(t+1) \psi_{t+1} \right] \right\}, \tag{21}$$

where ψ is the labor wedge, defined in (19)*, and $\tilde{\omega}$ is the share of tradable expenditures defined in* (15)*.*

Proof. In Appendix A.1. □

Equation (21) provides an analytical characterization of the optimal tax that emerges to correct the aggregate demand externality at work in the model. When households make savings decisions, they do not internalize that redirecting consumption over time affects firms' demand for non-tradable goods and can move production closer or further away from the first best.

These results are related to the aggregate demand externality emphasized in Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2012) in an economy with a fixed exchange rate. Our analytical characterization uncovers that the sign of τ_t is in principle ambiguous and depends, in particular, on the relative importance of the aggregate demand externality in periods t and $t+1$. When the current labor wedge is zero, the tax on debt takes the sign of the expected risk-adjusted labor wedge. The intuition for the analytical expression is that the government internalizes that an increase in one unit of savings today is associated with an increase in aggregate demand tomorrow, which stimulates employment and reduces the labor wedge. When the labor wedge today and tomorrow are both positive, the government trades-off the marginal benefits from stimulating future demand and easing the recession tomorrow with the marginal costs from reducing current demand and deepening the recession today. On the other hand, if the labor wedge is negative, taxing borrowing and postponing consumption helps to reduce overheating.

We turn next to analyze the interaction between monetary policy and macroprudential policy, which is our main focus.

3.2 Joint Monetary and Macroprudential Policies

We now consider a government that jointly conducts macroprudential and monetary policy. The government chooses τ and R to maximize households' welfare. Importantly, the government is subject to a zero lower bound that restricts its ability to achieve the first-best allocations.

In contrast to the previous section, here, the optimal policy for the government is subject to a time inconsistency problem, common in environments with a zero lower bound (e.g., Eggertsson and Woodford, 2003). We examine the optimal policy without commitment, which we see as the one that is practically most relevant. In particular, we study Markov perfect equilibrium in which the policies of the government at each point in time depend on the relevant payoff states. We use $s_t \equiv \{R_t^*, P_t^{T^*}, y_t^T, \delta_t\}$ to denote the date-t realizations of exogenous shocks, $\mathcal{E}(b^{*\prime}, s^\prime), \mathcal{C}^{\mathcal{T}}(b^{*\prime}, s^\prime), \mathcal{C}^{\mathcal{N}}(b^{*\prime}, s^\prime)$ to denote the stationary policy functions for the exchange rate and tradable and non-tradable consumption followed by future governments, and $V(b^*, s)$ to denote the value function for the government.

To set up the optimal time-consistent problem of the government, we use that by setting τ, the government can control borrowing decisions, and therefore (5) is not a binding implementability constraint. We can thus write the problem as follows:

$$V\left(b^{*},s\right)=\max_{R,e,b',c^{N},c^{T}}u\left(c^{T},c^{N}\right)-v\left((c^{N})^{1/\alpha}\right)+\beta\mathbb{E}_{s'|s}\delta'V\left(b^{*\prime},s'\right), \tag{22}$$

subject to

$$c^{T}=y^{T}+b^{*}-\frac{b^{*\prime}}{R^{*}}$$
$$c^{N}=\left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^{N}}e\right]^{\gamma}c^{T}$$
$$R^{*}=R\,\mathbb{E}_{s'|s}\left[\Lambda\left(\mathcal{C}^{T}(b^{*\prime},s'),\mathcal{C}^{N}(b^{*\prime},s')\right)\frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime},s')}\right]$$
$$R\geq 1.$$

The key difference compared with problem (20) is that now the exchange rate and the nominal interest rate are choices for the government.

In a Markov perfect equilibrium, as defined below, the conjectured policies for future governments have to be consistent with the actual policies chosen.

Definition 2. A Markov perfect equilibrium is defined by policies $\mathcal{R}(b^{*},s),\tau(b^{*},s),\mathcal{E}(b^{*},s)$, $\mathcal{B}^{*}(b^{*},s),\mathcal{C}^{T}(b^{*},s),\mathcal{C}^{N}(b^{*},s)$ and a value function $V(b^{*},s)$ that solve the government problem (22) given future policies for $\mathcal{E}(b^{*},s),\mathcal{C}^{T}(b^{*},s),\mathcal{C}^{N}(b^{*},s)$.

The following proposition characterizes the optimal policy of the government.

Proposition 2 (Optimal monetary and macroprudential policy)**.** *Consider the optimal monetary and macroprudential policy. We have that the labor wedge satisfies $\psi_t\geq 0$ for all t and $\psi_t=0$ if the zero lower bound (ZLB) does not bind at date t. Moreover, e_t is given by*

$$e_{t}=\frac{\omega}{1-\omega}\frac{\bar{P}^{N}}{P_{t}^{T*}}\left[\alpha^{\frac{\sigma}{\gamma}}(1-\omega)\left(\frac{e_{t}P_{t}^{T*}}{\bar{P}^{N}}\mathcal{P}(e_{t})\right)^{\frac{\gamma-\sigma}{\gamma}}\right]^{\frac{\alpha}{(1-\alpha+\phi)\sigma+\alpha}}(c_{t}^{T})^{-\frac{1}{\gamma}}. \tag{23}$$

In addition, the optimal tax on debt is given by

$$\tau_{t}=\frac{1}{\beta R^{*}\mathbb{E}_{t}\delta_{t+1}\left[u_{T}\left(c_{t+1}^{T},c_{t+1}^{N}\right)\right]}\left\{-(1+\Theta)\frac{\xi_{t}}{\gamma c_{t}^{T}}+\beta R^{*}\mathbb{E}_{t}\delta_{t+1}\left[\frac{\xi_{t+1}}{\gamma c_{t+1}^{T}}\right]\right\}, \tag{24}$$

where $\Theta\equiv\gamma c_{t}^{T}\frac{\partial}{\partial b_{t+1}^{*}}\mathbb{E}_{t}\left[\Lambda_{t+1}\frac{P_{t}^{T}}{P_{t+1}^{T}}\right]$ and ξ is the non-negative Lagrange multiplier on the ZLB constraint.

Proof. In Appendix A.2. □

Proposition 2 uncovers several lessons. First, the government implements an allocation with a zero labor wedge whenever the zero lower bound constraint is not binding. To see this, notice that if the ZLB constraint is slack, we can drop all constraints but the resource constraint. Thus, we obtain a static condition that delivers a zero labor wedge and back out R and e that implement those allocations. In particular, we obtain that the nominal interest rate is such that

$$R_t = \frac{R_t^*}{e_t} \left\{ \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\mathcal{E}_{t+1}} \frac{P_t^{T*}}{P_{t+1}^{T*}} \right] \right\}^{-1}. \tag{25}$$

A second lesson is that the economy never experiences overheating (i.e., a negative labor wedge). Intuitively, the zero lower bound imposes a constraint on the ability to depreciate the exchange rate, but the government can always appreciate the exchange rate and reduce the demand of non-tradables by raising the nominal interest rate. On the other hand, if the zero lower bound binds, the government is unable to depreciate the exchange rate by lowering the nominal interest rate and faces a positive labor wedge.

Regarding macroprudential policy, equation (24) shows that the tax on debt crucially depends on the current and future Lagrange multipliers on the zero lower bound constraints, denoted by ξ. Because $\xi \geq 0$, it follows that in a state in which the zero lower bound is not currently binding, the tax on debt is always positive. On the other hand, if the zero lower bound is currently binding but is not expected to bind next period with positive probability, the tax is negative.

To shed further light on these results, we can use the first-order conditions for c_t^N and e_t in (22) and obtain the following relationship between the labor wedge and the Lagrange multiplier on the zero lower bound:

$$\frac{\xi_t}{\gamma c_t^T} = \frac{1 - \tilde{\omega}_t}{\tilde{\omega}_t} u_T(c_t^T, c_t^N) \psi_t. \tag{26}$$

Notice that if we replace (26) into (24), we arrive at an equation analogous to the one characterizing the optimal tax under an arbitrary exchange rate policy (21). That is, it is possible to write the tax as a function of how savings affect the next-period labor wedge or as a function of how savings affect the tightness of the zero lower bound. The two expressions are linked by the government optimization and are, in fact, equivalent. Notice, however, that one difference between the two tax expressions (21) and (24) is that the latter carries an additional term, Θ, related to the restriction that the policy is consistent with an optimal time-consistent equilibrium. The additional term captures that an increase in savings alters both next-period consumption and the exchange rate followed by the next government.

In Figure 1, we illustrate numerically the tax on debt.[7] The figure shows that the tax on debt is non-monotonic on the current level of bond holdings. (In a different axis, the figure also shows the current labor wedge.) There are three distinct regions. For low bond holdings, the economy is in a liquidity trap region in which $R = 1$ and $\psi > 0$. In this region, the tax is increasing in bond holdings. It is initially negative, as the current labor wedge exceeds the expected future ones, and eventually becomes positive once bonds increase sufficiently, at which point the planner finds optimal to tax rather than subsidize inflows. For intermediate levels of bond holdings, the economy is in a fragile region in which $R > 1$ but the zero lower bound constraint may become binding in the next period. In this region, the tax is positive and increasing in bond holdings. Intuitively, in this region, the planner wants to shift resources to the future when the economy may face a recession and a binding zero lower bound. Moreover, as current bond holdings increase, this leads to higher bond holdings tomorrow and, thus a lower tax on debt. For sufficiently high bond holdings, the economy is in a safe region in which the tax becomes zero because there is a zero probability of a binding ZLB in the next period.

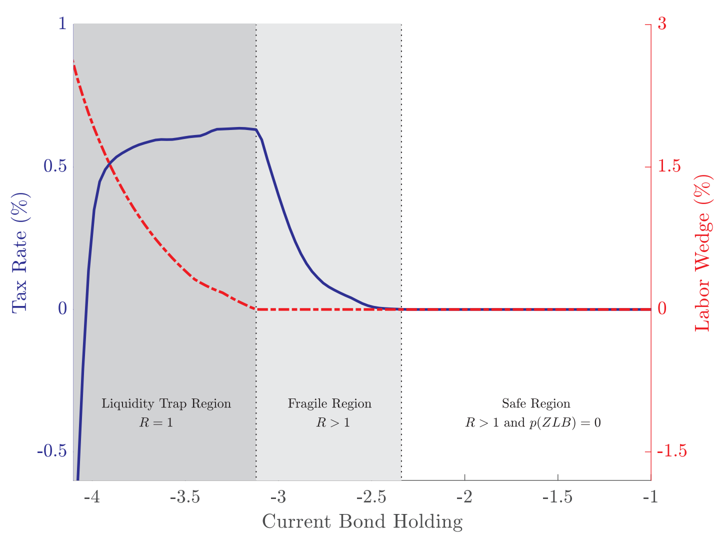

Figure 1: Optimal Macroprudential Policy

[7] The figure considers values of the shocks equal to the mean values. The calibration will be described below. The overall pattern, however, is general and does not hinge on specific parameters.

3.3 Optimal Monetary Policy without Macroprudential Policy

In the previous section, we analyzed the joint use of monetary and macroprudential policy. We saw the optimal policy implies a zero labor wedge whenever the zero lower bound is not binding. We now study optimal monetary policy when the government *does not* have access to macroprudential policy. The key question that emerges is whether the government should use monetary policy prudentially as a substitute for macroprudential policy and, if so, what this implies for the choice of the interest rate. In particular, does a prudential monetary policy call for higher or lower interest rates?

We consider as before the optimal problem under lack of commitment. Relative to problem (22), the government now faces (5) as a binding implementability constraint. This distinction will generate notable differences in the optimal policy, as characterized in the proposition below:

Proposition 3 (Optimal monetary policy without macroprudential policy)**.** *When the government does not have access to macroprudential policy, the optimal monetary policy satisfies*

$$u_T(t)\psi_t = \frac{\tilde{\omega}_t(\sigma-\gamma)}{(1-\tilde{\omega}_t)\sigma+\tilde{\omega}_t\gamma}\mathbb{E}_t\sum_{k=1}^{\infty}\left(\prod_{j=0}^{k-1}\delta_{t+j+1}\frac{\beta R^*_{t+j}}{1+\bar{\Theta}_{t+j}}\right)\frac{\xi_{t+k}}{\gamma c^T_{t+k}}, \tag{27}$$

where $\bar{\Theta}_t \equiv \beta R^*_t \frac{1}{u_{TT}(t)}\mathbb{E}_t\delta_{t+1}\frac{\partial u_T(\mathcal{C}^T(b^*_{t+1},s_{t+1}),\mathcal{C}^N(b^*_{t+1},s_{t+1}))}{\partial b^*_{t+1}}$ *and* ξ *is the Lagrange multiplier on the ZLB constraint.*

Proof. In Appendix A.3. □

Whether monetary policy is used prudentially and whether it leans *with* or *against* the wind turns out to depend on the elasticities of substitution. In the absence of macroprudential policy, monetary policy can potentially be used as a prudential tool to stimulate precautionary savings and reduce the likelihood of future liquidity traps. However, when $\sigma=\gamma$, saving does not respond to a change in the nominal interest rate. In that case, monetary policy focuses solely on stabilizing output and is not used prudentially. When $\sigma>\gamma$, the government optimally raises the nominal interest rate to stimulate savings and reduce the likelihood of future liquidity traps at the expense of a recession today. The optimal monetary policy leans against the wind. Conversely, when $\sigma<\gamma$, the government optimally cuts down the nominal interest rate to reduce the likelihood of future liquidity traps at the expense of an overheating economy. The optimal monetary policy leans with the wind.

The idea is that whether an increase in the interest rate mitigates excessive borrowing ahead of a liquidity trap episode, in the absence of macroprudential policy, depends on two opposing forces. First, there is an intertemporal substitution effect by which given prices and income, households save more externally, tilting consumption towards the future with magnitude σ. Second, there is an income general equilibrium effect by which the resulting contraction in current aggregate demand reduces output and leads to higher external borrowing with magnitude γ. When the elasticity substitution over time σ exceeds the elasticity of substitution across goods γ, raising the interest rate can indeed help reduce borrowing and indirectly mitigate the aggregate demand externality. Otherwise, it aggravates excessive borrowing.[8]

Notice also that the optimal monetary policy—except in the knife-edge case of equal intra and inter-temporal elasticities—is used prudentially as long as the zero lower bound binds in some distant future state. This contrasts with the optimal macroprudential policy, in which a tax is imposed only if the zero lower bound binds in the next period. To put it differently, monetary policy needs to act even more preemptively than macroprudential policy. The reason for this result is that monetary policy is a blunter instrument than macroprudential policy. A binding zero lower bound in some future state k implies that the government needs to reduce overborrowing at $k-1$. With macroprudential policy, the government introduces a tax on borrowing at $k-1$ while preserving a zero labor wedge. On the other hand, without macroprudential policy, the government must introduce a labor wedge at $k-1$. Doing so implies that from the perspective of $k-2$, the government also needs to deviate from a zero labor wedge. Proceeding backwards, this implies a strong history-dependent result: as long as there is a binding ZLB in some future state, the government will deviate from full employment at any period before.

In the preceding analysis, we consider the optimal monetary policy in the absence of macroprudential policy. However, it is also interesting to study optimal monetary policy in response to exogenous changes in macroprudential policies. To do so, we allow for any arbitrary tax in the optimal monetary policy problem and solve for the optimal exchange rate policy, given future policies and values and the arbitrary tax. We can show that for $\tau_t \in [0, \tau_t^*]$ where τ_t^* is the optimal macroprudential policy (24), an increase in τ_t leads to an increase in the labor wedge ψ_t under optimal monetary policy. Whether this implies a lower or higher nominal interest rate depends, in general, on the elasticities. A sufficient condition for a higher tax to call for a lower nominal interest rate is $\gamma \leq \sigma$. Intuitively, a macroprudential tax on debt contracts aggregate demand, and so it is optimal to offset

[8]We provide a formal decomposition of these channels in Bianchi and Coulibaly (2022). See also Auclert, Rognlie, Souchier and Straub (2021).

those effects by lowering the nominal interest rate.

4 Quantitative Results

We evaluate in this section the quantitative implications of the prudential use of monetary policy absent macroprudential policy and the benefits from using macroprudential policy optimally. We start by describing the calibration of the model.

4.1 Calibration

The time period is one-quarter, and data are calibrated using United Kingdom data between 1980 and 2019 as an example of an advanced small open economy.[9] The labor supply elasticity is set to one-third, as in Gali and Monacelli (2005) and α is set to one.

The stochastic processes for $\{y_t^T\}$, $\{R_t^*\}$ and $\{\delta_t\}$ are assumed to be independent and specified as follows. The tradable output y_t^T is measured with the cyclical component of value added in agriculture, mining, fishing, and manufacturing from the World Development Indicators. The world interest rate R_t^* is measured by the U.S. real interest rate, which corresponds to the U.S. federal funds rate deflated with the expected US. CPI inflation. Each process is assumed to be a first-order univariate autoregressive process. The estimated processes are, $\ln y_t^T = 0.6771 \ln y_{t-1}^T + \varepsilon_t^y$ with $\varepsilon_t^y \sim N(0, 0.0377^2)$ and $\ln(R_t^*/R^*) = 0.9173 \ln(R_{t-1}^*/R^*) + \varepsilon_t^{R*}$ with $R^* = 1.0036$ and $\varepsilon_t^{R*} \sim N(0, 0.0026^2)$.

Table 1: Calibration

Description	Parameter Value	Source/Target
Intertemporal elasticity	$\sigma = 1$	Standard value
Technology	$\alpha = 1$	Standard value
Frisch elasticity parameter	$\phi = 3$	Gali and Monacelli (2005)
Weight on tradables in CES	$\omega = 0.25$	Share of tradable output = 24%
Discount factor (long-run)	$\beta = 0.995$	Average NFA-GDP ratio = -17.4%
Transition prob. δ^L to δ^H	$P(\delta^L \vert \delta^H) = 0.20$	4 liquidity traps every century
Transition prob. δ^H to δ^H	$P(\delta^L \vert \delta^L) = 0.39$	2 years duration of liquidity traps

[9] We note that the problem of the zero lower bound has indeed been more pervasive for advanced economies although a side effect of the recent increase in central bank credibility in emerging markets appears to be the increase in vulnerability to liquidity traps, as can be seen from the recent experiences of countries such as Chile and Peru (see Matthew Bristow "Paul Krugman Says the Liquidity Trap Has Spread to Emerging Markets" Bloomberg May 12, 2020).

We set the elasticities of substitution to $\sigma = 1$ and $\gamma = 1$, but consider alternative values of in our analysis. To set the long-run value of the discount factor β, we target the historical average net foreign asset position (NFA) as a share of GDP of -17.4%. This calibration results in a value of $\beta = 0.995$. The discount factor shock δ_t follows a two-state regime-switching Markov process, that is $\delta_t \in \{\delta^L, \delta^H\}$ with $\delta^L < \delta^H$ and ergodic mean equals 1. We set $\delta^L = 0.985$, which represents the normal regime in which households discount the future at a rate of 0.99. The discount factor heightens with probability $P(\delta^H|\delta^L)$ and returns to its normal value with probability $P(\delta^L|\delta^H)$. The transition probability matrix P is set to target the frequency and average duration of liquidity trap episodes. The resulting values are presented in Table 1. The weight on tradable consumption in the CES function ω is calibrated to match a 24% share of tradable output in the total value of production observed in the data over the period 1980-2019, implying that $\omega = 0.252$.

4.2 Long-Run Moments

Table 2 displays the likelihood and duration of liquidity trap episodes in an economy in which monetary policy is set optimally both with and without macroprudential policy. An important lesson is that macroprudential policy are effective at reducing the likelihood of a liquidity trap. By taxing borrowing when the economy is vulnerable, the government is successful at reducing the frequency of liquidity traps. Macroprudential policy implies an average tax rate on inflows of 0.2% percent with a correlation between the tax and the nominal interest rate between -0.4 and -0.6. The negative correlation reflects that during positive income shocks, it is optimal to raise the nominal interest rate to stabilize output and to lower the tax on borrowing given that the positive income shock leads to a trade surplus.

Table 2: Frequency and duration of liquidity traps

	Monetary Policy Only		Monetary & Macroprudential			
	Frequency	Duration	Frequency	Duration	mean(τ)	corr(R, τ)
$\gamma = 0.5$	3.5%	7.8	2.9%	11.4	0.2%	-0.4
$\gamma = 1.0$	4.0%	7.8	3.5%	9.3	0.2%	-0.6
$\gamma = 1.5$	4.3%	8.4	3.9%	8.9	0.2%	-0.6

Note: Duration expressed in quarters.

A second lesson is that, perhaps surprisingly, liquidity traps last longer when macroprudential policy are used jointly with monetary policy. This occurs because in a liquidity trap, the government may tax outflows, which implies that the deleveraging process is slowed down. In fact, the average tax during a liquidity trap is -0.05%. Notice that because taxes on inflows are more frequent, macroprudential policy generate a reduction in external debt of about 7 percentage points of GDP.

Table 3 examines the average welfare cost of the ZLB and the unemployment rate during a liquidity trap under optimal monetary policy with and without macroprudential policy.[10] For a given state (b^*, s), the welfare cost of the ZLB under a policy regime is calculated as the compensating consumption variations that equalize the expected utility of a household living in an economy under that policy regime and the expected utility in the efficient allocation (without ZLB).[11]

Table 3: Unemployment rate and welfare costs of the ZLB

	Monetary Policy Only		Monetary & Macroprudential	
	Welfare costs	Unemployment*	Welfare costs	Unemployment*
$\gamma = 0.5$	0.60%	7.98%	0.10%	1.48%
$\gamma = 1.0$	0.50%	6.14%	0.11%	1.51%
$\gamma = 1.5$	0.52%	5.43%	0.14%	1.58%

Note: Unemployment is the average unemployment rate conditional on a liquidity trap.

Table 3 reports an average unemployment rate of about 1.5% with macroprudential policy versus 6.0% when the government refrains from using macroprudential policy. The significant reduction in both the frequency and the severity of liquidity trap episodes points toward substantial quantitative gains from macroprudential policy. macroprudential policy cut the welfare cost of the liquidity traps by more than fourfold. That is, the average welfare cost of the liquidity traps falls from 0.5 percentage points of permanent consumption to 0.1 percentage points when monetary policy is supplemented with macroprudential policy.

[10] The unemployment rate is defined as the gap between the current level of employment and the efficient employment level (that is, the level that would equate the marginal value of employment to the marginal cost of providing an extra unit of labor).

[11] Formally, the welfare cost associated with a policy regime G, for a given state (b^*, s), corresponds to the value of $q(b^*, s)$ that satisfies

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=0}^{t} \delta_k \right) \left[\log((1+q)c_t^G) - v(h_t^G) \right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=0}^{t} \delta_k \right) \left[\log(c_t^E) - v(h_t^E) \right],$$

where c^E and h^E denote consumption and hours worked in the efficient allocation.

4.3 The Prudential Role of Monetary Policy

We examine here the gains from conducting prudential monetary policy in the absence of macroprudential policy. To do so, we compare the frequency and duration of liquidity trap episodes under the optimal discretionary monetary policy against a policy in which the government closes the labor wedge and replicates flexible price allocation as long as the economy is away from a liquidity trap.

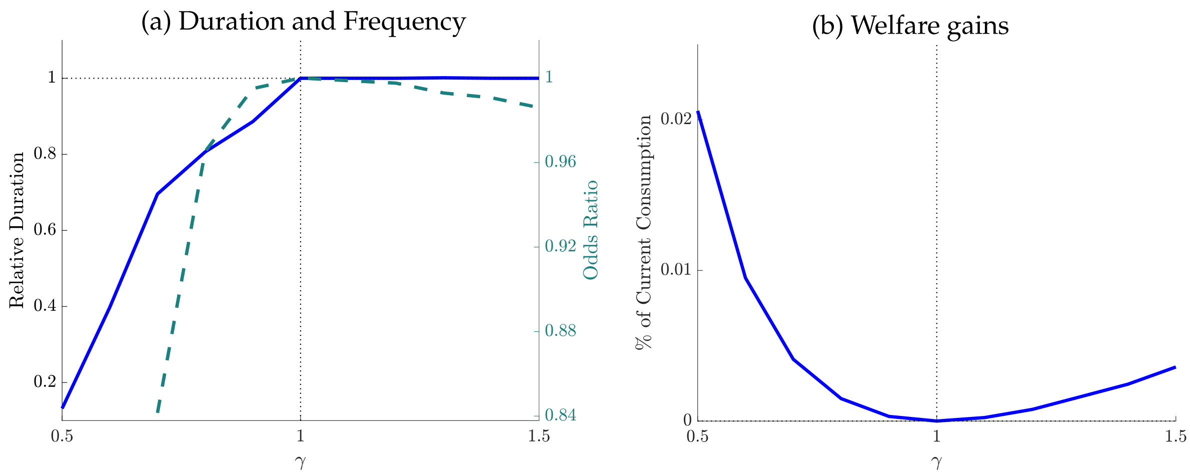

Figure 2: Gains from discretionary monetary policy relative to a full employment policy

The left panel of Figure 2 shows the results. We keep all parameters constant except for γ, the elasticity of substitution between tradables and non-tradables. The intertemporal elasticity of substitution is set to one. As the figure shows, when $\gamma = 1$, there are no gains from prudential monetary policy, in line with Proposition 3. When $\gamma < 1$, we observe larger benefits from prudential monetary policy. In particular, we see a significantly lower duration and frequency of liquidity traps. The welfare gains in terms of current consumption reach 0.02 percentage points when $\gamma = 0.5$. These gains are significantly lower than the ones from using capital controls we documented above.

4.4 Sensitivity with Partial Price Adjustments

We assumed for our baseline analysis that prices were perfectly sticky. We now consider the case with partial price adjustments, as in Rotemberg (1982). Firms are assumed to be monopolistic competitive producers of non-tradable good varieties that face a quadratic price adjustment cost in units of the final non-tradable good $\frac{\varphi}{2}\left(\frac{P_{jt}}{P_{jt-1}} - 1\right)^2 y_t^N$ where

$j \in [0, 1]$ is the index of the variety produced by a firm.[12] The elasticity of substitution between two varieties ε is calibrated is set to 7.66, corresponding to a 15% net markup. The price adjustment cost parameter φ_P, which determines the degree of price stickiness, is set to $\varphi_P = 120$. This implies that all prices would adjust on average after 3.2 quarters, which is in the range of the estimates of Nakamura and Steinsson (2008). We keep all the rest of the parameters constant and focus on the case $\gamma = 1$.

Table 4 shows that the overall results are similar in the model with costly price adjustments as in the case with perfectly sticky prices. Interestingly, the duration and frequency of the zero lower bound are somewhat shorter, but the welfare cost of the zero lower bound is about 0.1 percentage points larger in the absence of macroprudential policy. This result can seem surprising but is in line with the notion present in closed economies that higher price flexibility can be detrimental in an economy subject to a zero lower bound (e.g. Werning, 2011; Galí, 2013). On the other hand, we find that the welfare costs of liquidity traps become smaller in the economy with costly price adjustments when the government also has access to macroprudential policy.

Table 4: Sensitivity with Costly Price Adjustments

	Frequency	Duration	Unemployment	Welfare
Monetary Policy Only	2.7%	3.3	4.12%	0.63%
Monetary & Macroprudential	2.5%	5.4	1.09%	0.07%

Notes: The parameter values are the same as in Table 2 except $\gamma = 1$. Duration expressed in quarters.

5 International Spillovers

So far, we have considered a small open economy. We now extend our framework to tackle how monetary and macroprudential policies abroad affect welfare at home and the extent to which this leads to spillovers. We will first show that when there are international spillovers at play, they operate through a *financial channel*. That is, policies abroad affect domestic welfare to the extent that they alter the world real rate. In addition, we will show that the use of macroprudential policies allows a country to remain insulated from foreign policies and helps prevent currency wars.

[12] See Appendix C for a full description of the firm problem.

5.1 World Economy and Financial Spillovers

We consider a world economy that is composed of a continuum of small open economies with measure one, of the type described in Section 2. For simplicity, we abstract from uncertainty. We use $R_{H,t}$ and $R_{F,t}$ to denote the nominal rates of two countries H and F and $e^H_{F,t}$ to denote their bilateral exchange rate. We obtain the following arbitrage condition between bonds in different currencies:

$$R_{H,t} = R_{F,t}(e^H_{F,t}/e^H_{F,t+1}).$$

The definition of competitive equilibrium extends the definition of Section 2. We now have a sequence of prices and allocations, one for each country. In addition, the real interest rate R^*_t is endogenous, and we have that aggregate savings equal zero at the world level,

$$\int_0^1 b^*_{i,t+1} di = 0. \tag{28}$$

What are the effects of foreign policies on domestic welfare? The next proposition characterizes these effects.

Proposition 4 (Financial Channel of International Spillovers)**.** *Consider small changes $\{d\tau_{k,0}, dR_{k,0}\}_{k\in\Omega_k}$ in monetary and macroprudential policies by a set of countries $\Omega_k \subset [0,1]$. Starting from a symmetric equilibrium with zero net positions, the effect on country i's welfare, $dV_{i,0}$ for $i \notin \Omega_k$, is given by*

$$dV_{i,0} = u_T\left(c^T_{i,0}, c^N_{i,0}\right)\left[\frac{1-\tilde{\omega}_{i,0}}{\tilde{\omega}_{i,0}}\gamma c^T_{i,0}\psi_{i,0} + \left(\tilde{\omega}_{i,0} + (1-\tilde{\omega}_{i,0})\frac{\gamma}{\sigma}\right)v_{i,0}\right]\frac{dR^*_0}{R^*_0}, \tag{29}$$

where $v_{i,0}$ the Lagrange multiplier on the households' Euler equation (18) and the change in the world real rate satisfies

$$\frac{dR^*_0}{R^*_0} = -\int_{\Omega_k} \frac{(1-\mu_{k,0})c^T_{k,0}}{\int_0^1(\sigma\tilde{\omega}_{i,0} + \gamma(1-\tilde{\omega}_{i,0}))(1-\mu_{i,0})c^T_{i,0}di}\left[\sigma d\tau_{k,0} + (\sigma-\gamma)(1-\tilde{\omega}_{k,0})\frac{dR_{k,0}}{R_{k,0}}\right], \tag{30}$$

with $1-\mu_{i,0}$ representing the marginal propensity to save of households in country i.

Proof. In Appendix B.1 □

The proposition provides several lessons. First, the proposition underscores that from the perspective of a small open economy, the relevant spillover is through the changes in the world real interest rate. That is, changes in foreign policies affect the domestic economy only to the extent that they alter the world real rate. Even though a reduction in foreign nominal rates may lead to an appreciation of the nominal exchange rate through the UIP condition (9), the foreign price of tradables also increases, and so the price of tradables in domestic currency (and therefore allocations) remains unaltered.

A second lesson is that the effects of changes in the world real rate on welfare are determined by two sufficient statistics: the labor wedge ($\psi_{i,0}$), and the wedge between private borrowing and the socially desirable level of borrowing captured by the multiplier ($v_{i,0}$) on private agents' Euler equation (18). If both of these wedges were zero, changes in the world real rate would have no effects on welfare. Because the economy is neither a net borrower nor a net saver, a marginal change in the world real rate does not affect the budget constraint. If we start from a current allocation that is efficient (i.e., when these two wedges are zero), this implies that the marginal effects on welfare are zero. As we will see below, this case applies in particular when the economy has macroprudential policy and it is away from the zero lower bound.

On the other hand, when these wedges are different from zero, changes in the world real rate have in general effects on domestic welfare. In particular, when the economy is at the zero lower bound, an increase in the world real rate $dR_0^* > 0$ leads to an increase in the price of tradables through (9) and an expenditure switching effect towards non-tradables. Because $\psi_{i,0} > 0$ at the ZLB, the increased demand for non-tradables improves welfare by bringing output closer to the efficient level. When the domestic government has access to macroprudential policy, we also have that $v_{i,0} = 0$, and so the overall effect is a positive effect on welfare. When the domestic government does not have access to capital control, the overall effect is ambiguous because a higher real interest rate reduces borrowing and this contributes to mitigate a potential liquidity trap in the future, as captured by the second term in (30).

Next, we specialize the implications of Proposition 4 to examine monetary and macroprudential policy spillovers.

5.2 Monetary Spillovers

We start by examining the spillover from changes in policy rates abroad. Notice from (30) that a monetary tightening in a subset of countries abroad $dR_{0,k} > 0$ leads to an increase in the world real rate under $\gamma > \sigma$ and a decrease under $\sigma < \gamma$. In the case in which $\sigma = \gamma$, therefore, there are no spillovers.

Let us examine the case in which the home country is *away from a liquidity trap*. Using the results from Proposition 3 on and eq. (30), we have that the welfare effects of a monetary expansion abroad reduce to[13]

$$dV_{i,0} = -\left[u_T(c^T_{i,0}, c^N_{i,0})v_{i,0}\right] \\ \times (\sigma - \gamma) \int_{\Omega_k} \frac{(1-\mu_{k,0})c^T_{k,0}}{\int_0^1 (\sigma\tilde{\omega}_{i,0} + \gamma(1-\tilde{\omega}_{i,0}))(1-\mu_{i,0})c^T_{i,0}di}(1-\tilde{\omega}_{k,0})\frac{dR_{k,0}}{R_{k,0}} \quad (31)$$

The sign of the welfare effect depends on the interaction between $v_{i,0}$, the Lagrange multiplier on the households' Euler equation (18), and the direction of the world interest rate in response to the monetary policy abroad. There are two important cases to consider, one in which macroprudential policy is available and another in which it is not. Let us analyze each case.

Without macroprudential policy. From the analysis in Section 3.3, we can see that $v_{i,t} > 0$ when the ZLB is not currently binding but is expected to bind in the future. Formally, we have[14]

$$v_{i,0} = \frac{1}{-u_{TT}(c^T_{i,0}, c^N_{i,0})} \sum_{t=1}^{\infty} \left(\prod_{j=0}^{t-1} \frac{\beta R^*_j}{1+\bar{\Theta}_{i,j}}\right) \frac{\xi_{i,t}}{\gamma c^T_{i,t}}. \quad (32)$$

A strictly positive Lagrange multiplier reflects that agents tend to overborrow relative to the constrained-efficient benchmark. When the monetary policy abroad generates a reduction in the world real interest rate, this causes a reduction in welfare in the home country. Intuitively, the reduction in the real interest rate generates incentives for households to borrow even more—from an already inefficiently high level—and increases the vulnerability to a liquidity trap.

As shown in Proposition 3, individual economies have incentives to increase the net foreign asset position so as to become less vulnerable to a binding zero lower bound, thus, leading to a decrease in the world real rate. If $\sigma > \gamma$, the central bank achieves the increase in savings by raising the nominal interest rate, while if $\sigma < \gamma$, the central bank achieves the increase in savings by *lowering* the nominal interest rate.

These results suggest that when all countries are pursuing a prudential monetary policy, doing so ends up backfiring at the aggregate level, generating a form of currency war. This

[13] Away from the ZLB, as shown in (A.28) in Appendix A.3, the optimal monetary policy in its target form (27) can be rewritten as $\psi_{i,0} = \tilde{\omega}_{i,0}(\gamma^{-1} - \sigma^{-1})v_{i,0}/c^T_{i,0}$. Combining this with (29), we obtain (31).

[14] The expression of the Lagrange multiplier $v_{i,0}$ is obtained by substituting the labor wedge away from the ZLB, (27), into the SOE government's optimality condition for bonds. See Appendix A.3 for more details.

is because central banks are deviating from the efficient allocation today with the goal of increasing the net foreign asset position. However, in general equilibrium, this pushes down the world real rate, leading to welfare losses, as analyzed before.[15]

We summarize this result in the corollary below.

Corollary 1 (Currency wars)**.** *When a government does not use macroprudential policy, a prudential monetary intervention abroad lowers home welfare, strictly so if the zero lower bound binds in the future.*

Proof. In Appendix B.2. □

We proceed to analyze the monetary policy spillover effects when central banks also use macroprudential policy. We will argue that the use of macroprudential policy will prevent the eruption of currency wars.

With macroprudential policy. In the presence of macroprudential policy, the crucial difference is that there is no inefficiency stemming from households' saving decisions (i.e, $v_{i,0} = 0$). As (31) shows, a foreign monetary policy intervention has no effect on domestic households' welfare. The joint optimal monetary and macroprudential policy response, therefore, renders the domestic country insulated from foreign monetary policy. Intuitively, when the central bank is away from a liquidity trap, monetary policy optimally closes the labor wedge ($\psi_{i,0} = 0$), while macroprudential policy ensures that the level of borrowing is inefficient.

Corollary 2 (No currency wars with macroprudential policy)**.** *When a government uses macroprudential policy, a monetary policy intervention abroad does not affect home welfare away from the zero lower bound.*

Proof. In Appendix B.3 □

5.3 Macroprudential Policy Spillovers

We argued in the previous section that macroprudential policy can serve to insulate a country from foreign monetary policy spillovers. But what happens when all countries use macroprudential policy? Is it possible that the use of this policy backfires at a global scale?

[15] In the case of symmetric countries, in particular, the net foreign asset positions are always zero in equilibrium. Therefore, the effect of central banks following a prudential monetary policy is a reduction in the world real rate (and the output distortion). See Fornaro and Romei (2022) and Bianchi and Coulibaly (2023) for recent work on the optimal cooperative monetary policy in a related environment.

Fornaro and Romei (2019) argue that it is indeed possible that a global economy with macroprudential policies would be Pareto-dominated by an economy *without* macroprudential policies.[16] Their argument is that when countries impose macroprudential policy to reduce their vulnerability to a future liquidity trap, this lowers the world real rate, making the zero lower bound more binding for other economies. They dub this phenomenon a "paradox of global thrift."

We argue, however, that it is possible to design a macroprudential policy that eliminates the possibility of a paradox of global thrift. In particular, if we allow the government to restrict capital flows in a liquidity trap, the government can alter the domestic equilibrium real rate and ensure that welfare does not fall in response to foreign macroprudential policies. In particular, if the government restricts the level of capital flows at the same level as the laissez-faire equilibrium, it can maintain the same real rate as in the laissez-faire equilibrium, averting the potential adverse effects from macroprudential policies abroad. The key result is that the adoption of a quantity restriction on capital flows enables the government to achieve the same real rate as in the laissez-faire economy and deliver at least the same level of welfare. The proposition below formalizes this result.

Proposition 5 (Welfare dominance of quantity-macroprudential policy)**.** *Consider the welfare of the home country under laissez-faire versus the welfare under a macroprudential policy regime in which the government controls directly the country's capital account, starting from a symmetric equilibrium with zero net positions. Then, home welfare is weakly higher in the macroprudential policy regime.*

Proof. In Appendix B.4 □

The key difference between our analysis and Fornaro and Romei's is that in their case, the domestic real rate corresponds to the world real rate. This is because they assume that the government in a liquidity trap does not have access to the macroprudential quantity restrictions highlighted above. In their model, an economy experiences a liquidity trap when its tradable endowment is low and the borrowing constraint binds—specifically, under their zero liquidity assumption, asset holdings are zero. When other countries away from a liquidity trap try to raise their net foreign asset position, this leads to a lower world real rate. In the absence of macroprudential quantity restrictions, the domestic real rate also falls, and this makes the zero lower bound constraint more binding. Our argument, however, is that if the government can shut down the capital account, the domestic real rate

[16] They consider an economy with "zero liquidity," in which this result is demonstrated analytically, and then present results from quantitative simulations.

would necessarily increase and raise welfare. That is, by following this macroprudential intervention, the economy can achieve a higher welfare than a laissez-faire equilibrium without macroprudential policies, eliminating the global paradox of thrift.[17]

6 Conclusion

We provide an integrated analysis of monetary and macroprudential policies in an open economy subject to an occasionally binding zero lower bound constraint. In the absence of macroprudential policy, monetary policy faces a tradeoff between stabilizing output today and reducing capital inflows to reduce the vulnerability to a liquidity trap. However, the optimal monetary policy may call for lower or higher nominal interest rates. Our analysis also provides a more benign perspective on international spillovers in contrast to widespread concerns. We show that to the extent that economies can deploy macroprudential policies in response to foreign policies, currency wars can be prevented. Finally, a world economy where countries use macroprudential policy in an uncoordinated welfare dominates a Nash equilibrium without macroprudential policy.

[17] In a previous version of the paper, we also provide general conditions under which taxes on capital flows, as opposed to quantity restrictions, can generate a Pareto improvement relative to a laissez-faire regime. See also the previous version for a characterization of the optimal macroprudential policy under cooperation.

References

Acharya, Sushant and Julien Bengui, "Liquidity Traps, Capital Flows," *Journal of International Economics*, 2018, *114*, 276–298.

Amador, Manuel, Javier Bianchi, Luigi Bocola, and Fabrizio Perri, "Exchange Rate Policies at the Zero Lower Bound," *Review of Economic Studies*, 2020, *87* (4), 1605–1645.

Aoki, Kosuke, Gianluca Benigno, and Nobuhiro Kiyotaki, "Monetary and Financial Policies in Emerging Markets," Technical Report, mimeo 2016.

Auclert, Adrien, Matt Rognlie, Martin Souchier, and Ludwig Straub, "Monetary Policy and Exchange Rates with Heterogeneous Agents: Sizing up the Real Income Channel," 2021. Mimeo, MIT.

Basu, Suman Sambha, Emine Boz, Gita Gopinath, Francisco Roch, and Filiz Unsal, "A Conceptual Model for the Integrated Policy Framework," 2020. Mimeo, IMF.

Benigno, Pierpaolo and Federica Romei, "Debt Deleveraging and the Exchange Rate," *Journal of International Economics*, 2014, *93* (1), 1–16.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

_ **and Louphou Coulibaly**, "Monetary Policy and Capital Flows: Dissecting the Transmission Mechanism," 2022. University of Wisconsin Mimeo.

_ **and** _ , "Demand Imbalances, Global Inflation and Monetary Policy Coordination," 2023. Mimeo, Minneapolis Fed.

Blanchard, Olivier, "Currency wars, coordination, and capital controls," in "The Asian Monetary Policy Forum: Insights for Central Banking" World Scientific 2021, pp. 134–157.

Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas, "Global Imbalances and Policy Wars at the Zero Lower Bound," *Review of Economic Studies*, 2021.

Collard, Fabrice, Harris Dellas, Behzad Diba, and Olivier Loisel, "Optimal monetary and prudential policies," *American Economic Journal: Macroeconomics*, 2017, *9* (1), 40–87.

Cook, David and Michael B Devereux, "Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap," *American Economic Journal: Macroeconomics*, 2013, *5* (3), 190–228.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles, "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, 2013, *103* (4), 1172–1211.

Corsetti, Giancarlo, Eleonora Mavroeidi, Gregory Thwaites, and Martin Wolf, "Step Away from the Zero Lower Bound: Small Open Economies in a World of Secular Stagnation," *Journal of International Economics*, 2019, *116*, 88–102.

_, Gernot Mueller, and Keith Kuester, "The Case for Flexible Exchange Rates after the Great Recession," 2019. Mimeo, Cambridge.

Coulibaly, Louphou, "Monetary Policy in Sudden Stop-prone Economies," 2020. Forthcoming, American Economic Journal: Macroeconomics.

der Ghote, Alejandro Van, "Interactions and Coordination between Monetary and Macroprudential Policies," *American Economic Journal: Macroeconomics*, 2021, *13* (1), 1–34.

Devereux, Michael B and James Yetman, "Capital controls, global liquidity traps, and the international policy trilemma," *The Scandinavian Journal of Economics*, 2014, *116* (1), 158–189.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *34* (1), 139–235.

_, Neil R. Mehrotra, Sanjay R. Singh, and Lawrence H. Summers, "A Contagious Malady? Open Economy Dimensions of Secular Stagnation," *IMF Economic Review*, 2016, *64* (4), 581–634.

Egorov, Konstantin and Dmitry Mukhin, "Optimal Policy under Dollar Pricing," 2023. Forthcoming, American Economic Review.

Fanelli, Sebastian, "Monetary Policy, Capital Controls, and international portfolios," 2023. Mimeo, MIT.

Farhi, Emmanuel and Iván Werning, "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates," 2012. NBER Working Paper 18199.

_ and Iván Werning, "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

_ and Iván Werning, "Taming a Minsky Cycle"," 2020. Mimeo, MIT.

Ferrero, Andrea, Richard Harrison, and Benjamin Nelson, "House price dynamics, optimal ltv limits and the liquidity trap," 2022. Bank of England, Working Paper.

Fornaro, Luca, "International Debt Deleveraging," *Journal of the European Economic Association*, 2018, *16* (5), 1394–1432.

— **and Federica Romei**, "The Paradox of Global Thrift," *American Economic Review*, 2019, *109* (11), 3745–3779.

— **and** — , "Monetary policy during unbalanced global recoveries," 2022.

Galí, Jordi, "Notes for a New Guide to Keynes (I), Wages, Aggregate Demand and Employment," *Journal of the European Econmic Association*, 2013, *11*, 973–1003.

Gali, Jordi and Tommaso Monacelli, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 2005, *72* (3), 707–734.

Gourinchas, Pierre Olivier, "International Macroeconomics: From the Great Financial Crisis to COVID-19, and Beyond," *IMF Economic Review*, 2022, pp. 1–34.

Jeanne, Olivier, "The Global Liquidity Trap," 2009. Mimeo, John Hopkins University.

Kalemli-Ozcan, Sebnem, "US monetary policy and international risk spillovers," 2019. Jackson Hole Symposium Proceedings 2019.

Kollmann, Robert, "Liquidity Traps in a World Economy," 2021. CAMA Working Paper.

Korinek, Anton and Alp Simsek, "Liquidity Trap and Excessive Leverage," *American Economic Review*, 2016, *106* (3), 699–738.

Krugman, Paul, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, *1998* (2), 137–205.

Nakamura, Emi and Jón Steinsson, "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.

Rajan, Raghuram, "Containing competitive monetary easing," *Project Syndicate*, 2014, *28.*

Rey, Helene, "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence," Federal Reserve Bank of Kansas City Economic Policy Symposium 2013.

Rotemberg, Julio, "Sticky prices in the united states," *Journal of Political Economy*, 1982, *90*, 1187–1211.

Rubio, Margarita and Fang Yao, "Macroprudential policies in a low interest rate environment," *Journal of Money, Credit and Banking*, 2020, *52* (6), 1565–1591.

Schmitt-Grohé, Stephanie and Martin Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

Werning, Ivan, "Managing a liquidity trap: Monetary and fiscal policy," Technical Report, NBER Working Paper No. 17344 2011.

Appendix to "Liquidity Traps, Prudential Policies and International Spillovers"

A Proofs for Section 3

A.1 Proof of Proposition 1

Proof. The problem of the government consists in choosing τ to maximize households' welfare subject to the equilibrium conditions (3), (4), (5), (6) and (13). For a given exogenous path of the nominal exchange rate $\{e_t\}$, we solve the relaxed problem of the government:

$$\max_{\{b^*_{t+1}, c^N_t, c^T_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\prod_{k=0}^{t} \delta_k \right) \left[u\left(c^T_t, c^N_t\right) - v\left((c^N_t)^{1/\alpha} \right) \right] \tag{A.1}$$

subject to

$$c^T_t = y^T_t + b^*_t - \frac{b^*_{t+1}}{R^*_t} \tag{A.2}$$

$$c^N_t = \left[\frac{1-\omega}{\omega} \frac{P^{T*}_t}{\bar{P}^N} e_t \right]^{\gamma} c^T_t \tag{A.3}$$

After solving (A.1), we back out τ_t using (5). Taking the first-order conditions, we arrive at

$$c^T_t: \quad \lambda_t = u_T(t) + \vartheta_t \frac{c^N_t}{c^T_t} \tag{A.4}$$

$$c^N_t: \quad \vartheta_t = u_N(t) - \frac{1}{\alpha}(h_t)^{1-\alpha} v'\left((c^N_t)^{1/\alpha} \right) \tag{A.5}$$

$$b^*_{t+1}: \quad \frac{z_t \lambda_t}{R^*_t} = \beta \mathbb{E}_t z_{t+1} \lambda_{t+1} \tag{A.6}$$

where $\lambda_t \geq 0$ and ϑ_t are the Lagrange multipliers on constraints (A.2) and (A.3) respectively. Combining (A.4) and (A.5), we have

$$\lambda_t = u_T(t) + u_T(t) \frac{\bar{P}^N c^N_t}{P^T_t c^T_t} \psi_t \tag{A.7}$$

where we use $\frac{\bar{P}^N}{P^T_t} = \frac{u_N(t)}{u_T(t)}$. We then substitute (A.7) into (A.6) to get

$$u_T(t) \left[1 + \frac{\bar{P}^N c^N_t}{P^T_t c^T_t} \psi_t \right] = \beta R^*_t \mathbb{E}_t \left\{ \delta_{t+1} u_T(t+1) \left[1 + \frac{\bar{P}^N c^N_{t+1}}{P^T_{t+1} c^T_{t+1}} \psi_{t+1} \right] \right\} \tag{A.8}$$

We can now derive the optimal tax rate on debt by plugging (5) into (A.8) which leads to

$$\tau_t = \frac{1}{\beta R_t^* \mathbb{E}_t \delta_{t+1} u_T(t+1)} \left\{ -\frac{1-\tilde{\omega}_t}{\tilde{\omega}_t} u_T(t)\psi_t + \beta R_t^* \mathbb{E}_t \delta_{t+1} \left[\frac{1-\tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} u_T(t+1)\psi_{t+1} \right] \right\}.$$

□

A.2 Proof of Proposition 2

Proof. The problem of the government in recursive form is given by:

$$V\left(b^*, s\right) = \max_{R,\tau,e,b^{*\prime},c^N,c^T} u\left(c^T, c^N\right) - v\left(\left(c^N\right)^{1/\alpha}\right) + \beta \mathbb{E}_{s'|s} \delta' V\left(b^{*\prime}, s'\right)$$

subject to

$$c^T = y^T + b^* - \frac{b^{*\prime}}{R^*} \tag{A.9}$$

$$c^N = \left[\frac{1-\omega}{\omega} \frac{P^{T*}}{\bar{P}^N} e\right]^{\gamma} c^T \tag{A.10}$$

$$u_T(c^T, c^N) = \beta R^* (1+\tau)\, \mathbb{E}_{s'|s} \left[\delta' u_T\left(\mathcal{C}^T(b^{*\prime}, s'), \mathcal{C}^N(b^{*\prime}, s')\right)\right] \tag{A.11}$$

$$R^* = R\mathbb{E}_{s'|s}\left[\Lambda\left(\mathcal{C}^T(b^{*\prime}, s'), \mathcal{C}^N(b^{*\prime}, s')\right) \frac{P^{T*}}{P^{T*\prime}} \frac{e}{\mathcal{E}(b^{*\prime}, s')}\right] \tag{A.12}$$

$$R \geq 1 \tag{A.13}$$

Because τ only appears in (A.11), it is immediate that (A.11) does not bind. Let $\lambda \geq 0$, ϑ, v, χ and $\xi \geq 0$ be the Lagrange multiplier on (A.9), (A.10), (A.11), (A.12) and (A.13) respectively. The optimality conditions, after substituting for χ, are

$$\xi = \gamma c^N \vartheta \tag{A.14}$$

$$\vartheta = u_N(c^T, c^N) - \frac{v'(h)}{\alpha h^{\alpha-1}} \tag{A.15}$$

$$\lambda = u_T(c^T, c^N) + \frac{c^N}{c^T}\vartheta \tag{A.16}$$

$$\lambda = -\xi\, \mathbb{E}_{s'|s} \frac{\partial}{\partial b^{*\prime}} \left[\frac{\Lambda(b^{*\prime}, s')}{\mathcal{E}(b^{*\prime}, s')} \frac{eP^{T*}}{P^{T*\prime}}\right] + \beta R^* \mathbb{E}_{s'|s} \delta' \lambda' \tag{A.17}$$

We combine (A.14) and (A.15) to obtain

$$\xi = \gamma c^T \frac{1-\tilde{\omega}}{\tilde{\omega}} u_T(c^T, c^N)\psi \tag{A.18}$$

This corresponds to (26) in the text.

Next, we determine the optimal exchange rate when the ZLB is not binding. When the ZLB is not binding $\xi = 0$ which implies that $\psi = 0$, and by (19) we have

$$
\begin{aligned}
\psi = 0 \Leftrightarrow (c^N)^{\frac{1-\alpha+\phi}{\alpha}+\frac{1}{\sigma}} &= \alpha(1-\omega)\left(\frac{c}{c^N}\right)^{\frac{1}{\gamma}-\frac{1}{\sigma}} \\
\Leftrightarrow (c^N)^{\frac{(1-\alpha+\phi)\sigma+\alpha}{\alpha\sigma}} &= \alpha(1-\omega)^{\frac{\gamma}{\sigma}}\left[\omega^{\gamma}\left(\frac{eP^{T*}}{\bar{P}^N}\right)^{1-\gamma}+(1-\omega)^{\gamma}\right]^{\frac{1}{\gamma-1}\frac{\sigma-\gamma}{\sigma}}
\end{aligned}
$$

where the second equality uses (A.10). Using again (A.10) and $\mathcal{P}$ defined in (14), we simplify both sides of the equation and we arrive at

$$
\left[\left(\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^N}e\right)^{\gamma} c^T\right]^{\frac{(1-\alpha+\phi)\sigma+\alpha}{\alpha\sigma}} = \alpha(1-\omega)^{\frac{\gamma}{\sigma}}\left[\frac{eP^{T*}}{\bar{P}^N}\mathcal{P}\right]^{\frac{\sigma-\gamma}{\sigma}}
$$

which implies that

$$
e = \frac{\omega}{1-\omega}\frac{\bar{P}^N}{P^{T*}}\left[\alpha^{\frac{\sigma}{\gamma}}(1-\omega)\left(\frac{eP^{T*}}{\bar{P}^N}\mathcal{P}\right)^{\frac{\gamma-\sigma}{\gamma}}\right]^{\frac{\alpha}{(1-\alpha+\phi)\sigma+\alpha}}(c^T)^{-\frac{1}{\gamma}}
$$

Finally, we turn to deriving the optimal tax. Defining $\Theta \equiv \gamma c^T \frac{\partial}{\partial b^{*\prime}}\mathbb{E}_{s'|s}\left[\frac{\Lambda(b^{*\prime},s')}{\mathcal{E}(b^{*\prime},s')}\frac{eP^{T*}}{P^{T*\prime}}\right]$ and plugging (A.16) into (A.17), we get

$$
u_T(c^T,c^N)+(1+\Theta)\frac{\xi}{\gamma c^T} = \beta R^*\mathbb{E}_{s'|s}\delta'\left[u_T\left(c^{T\prime},c^{N\prime}\right)+\frac{\xi'}{\gamma c^{T\prime}}\right] \tag{A.19}
$$

Then, we substitute (A.11) into (A.19) and obtain

$$
\tau = \frac{1}{\beta R^*\mathbb{E}_{s'|s}\delta'\left[u_T\left(c^{T\prime},c^{N\prime}\right)\right]}\left\{-(1+\Theta)\frac{\xi}{\gamma c^T}+\beta R^*\mathbb{E}_{s'|s}\delta'\left[\frac{\xi'}{\gamma c^{T\prime}}\right]\right\}
$$

□

A.3 Proof of Proposition 3

Preliminaries. Absent capital controls, the government sets its policy $\{R\}$ to maximize households' welfare subject to resource and implementability constraints, and a zero lower bound constraint on nominal interest rate. The government problem is given by:

$$V\left(b^{*},s\right)=\max_{R,e,b^{*\prime},c^{N},c^{T}} u\left(c^{T},c^{N}\right)-v\left((c^{N})^{1/\alpha}\right)+\beta\mathbb{E}_{s'|s}\delta' V\left(b^{*\prime},s'\right) \tag{A.20}$$

subject to

$$c^{T}=y^{T}+b^{*}-\frac{b^{*\prime}}{R^{*}} \tag{A.21}$$

$$c^{N}=\left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^{N}}e\right]^{\gamma}c^{T} \tag{A.22}$$

$$u_{T}(c^{T},c^{N})=\beta R^{*}\,\mathbb{E}_{s'|s}\delta'\left[u_{T}\left(\mathcal{C}^{T}(b^{*\prime},s'),\mathcal{C}^{N}(b^{*\prime},s')\right)\right] \tag{A.23}$$

$$R^{*}=R\mathbb{E}_{s'|s}\left[\Lambda\left(\mathcal{C}^{T}(b^{*\prime},s'),\mathcal{C}^{N}(b^{*\prime},s')\right)\frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime},s')}\right] \tag{A.24}$$

$$R\geq 1 \tag{A.25}$$

We let $\lambda\geq 0$ be the Lagrange multiplier on (A.21), ϑ, v and χ the multiplier on (A.22), (A.23) and (A.24) respectively, and $\xi\geq 0$ the multiplier on (A.25).

We proceed by first defining a Markov perfect equilibrium in the absence of capital controls and then characterizing the optimal monetary policy.

Definition A.1 (Markov perfect equilibrium absent capital controls)**.** A Markov perfect equilibrium is defined by the current government policy functions $R(b^{*},s)$, $\mathcal{E}(b^{*},s)$ with associated decision rules $c^{T}(b^{*},s)$, $b'(b^{*},s)$ $c^{N}(b^{*},s)$, and value function $V(b^{*},s)$, and the conjectured function characterizing the decision rule of future governments $\mathcal{R}(b^{*},s)$, $\mathcal{B}(b^{*},s)$ and the associated decision rules $\mathcal{C}^{T}(b^{*},s)$, $\mathcal{C}^{N}(b^{*},s)$, such that: (i) given the conjecture of future policies, the value function and the policy functions solve the government problem (A.20); and (ii) The conjectured policy rules that represent optimal choices of future governments coincide with the solutions to (A.20).

Optimal monetary policy. The first-order conditions with respect to e and c^{N} are

$$\xi=\gamma c^{N}\vartheta \tag{A.26}$$

$$\vartheta=u_{N}(c^{T},c^{N})-\frac{v'(h)}{\alpha h^{\alpha-1}}-u_{TN}(c^{T},c^{N})v \tag{A.27}$$

where we use the optimality condition for R to substitute for χ. To derive the optimal monetary policy when the ZLB is not binding, we substitute (A.26) into (A.27) to get

$$\begin{aligned}\xi&=\gamma c^{N}\left[u_{N}(c^{T},c^{N})\psi-u_{TN}(c^{T},c^{N})v\right]\\&=\gamma c^{N}u_{N}(c^{T},c^{N})\left[\psi-\frac{\tilde{\omega}(\sigma-\gamma)}{\sigma\gamma}\frac{v}{c^{T}}\right]\end{aligned}$$

Thus, when the ZLB does not bind

$$\psi = \frac{\tilde{\omega}(\sigma - \gamma)}{\sigma\gamma} \frac{v}{c^T} \tag{A.28}$$

We now need to determine v. Using the first order conditions with respect to tradable consumption c^T and foreign bonds b'^*

$$\lambda = u_T(c^T, c^N) - u_{TT}(c^T, c^N)v + \frac{c^N}{c^T}\vartheta \tag{A.29}$$

$$\begin{aligned} \lambda = {} & \beta R^* \mathbb{E}_{s'|s}\delta'\lambda' - \xi\, \mathbb{E}_{s'|s}\frac{eP^{T*}}{P^{T*\prime}}\frac{\partial}{\partial b^{*\prime}}\left[\frac{\Lambda(b^{*\prime}, s')}{\mathcal{E}(b^{*\prime}, s')}\right] \\ & + v\, \beta R^* E_{s'|s}\delta' \frac{\partial}{\partial b^{*\prime}}\left[u_T\left(\mathcal{C}^T(b^{*\prime}, s'), \mathcal{C}^N(b^{*\prime}, s')\right)\right] \end{aligned} \tag{A.30}$$

and plugging (A.29) into (A.30), we get

$$u_T(c^T, c^N) - (1 + \bar{\Theta})u_{TT}(c^T, c^N)v = \beta R^* \mathbb{E}_{s'|s}\delta'\left[u_T(c^{T\prime}, c^{N\prime}) - u_{TT}(c^{T\prime}, c^{N\prime})v' + \frac{\xi'}{\gamma c^{T\prime}}\right]$$

where $\bar{\Theta} \equiv \frac{1}{u_{TT}(c^T, c^N)}\beta R^* \mathbb{E}_{s'|s}\delta' \frac{\partial}{\partial b^{*\prime}} u_T\left(c^{T\prime}, c^{N\prime}\right) > 0$. Then, substituting the implementability constraint (A.23) into this equation leads to

$$-u_{TT}(c^T, c^N)v = \beta R^*(1 + \bar{\Theta})^{-1}\mathbb{E}_{s'|s}\delta'\left[-u_{TT}(c^{T\prime}, c^{N\prime})v' + \frac{\xi'}{\gamma c^{T\prime}}\right] \tag{A.31}$$

Iterating forward (A.31) and using the transversality condition, we obtain (for convenience, the equations are written in their sequential form)

$$\begin{aligned} v_t &= \frac{1}{-u_{TT}(t)}\mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{k|0}\frac{\xi_{t+k}}{\gamma c^T_{t+k}} \\ \frac{v_t}{c^T_t} &= \frac{1}{u_T(t)}\frac{\sigma\gamma}{(1 - \tilde{\omega}_t)\sigma + \tilde{\omega}_t\gamma}\mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{t+k|t}\frac{\xi_{t+k}}{\gamma c^T_{t+k}} \end{aligned} \tag{A.32}$$

where $\bar{Q}_{t+k|t} = \beta^k \prod_{j=0}^{k-1}\left(\delta_{t+j+1}\frac{R^*_{t+j}}{1+\bar{\Theta}_{t+j}}\right)$. Finally, we substitute (A.32) into (A.28) to get the optimal monetary policy in its target form

$$u_T(t)\psi_t = \frac{\tilde{\omega}_t(\sigma - \gamma)}{(1 - \tilde{\omega}_t)\sigma + \tilde{\omega}_t\gamma}\mathbb{E}_t \sum_{k=1}^{\infty} \bar{Q}_{t+k|t}\frac{\xi_{t+k}}{\gamma c^T_{t+k}}.$$

□

B Proofs for Section 5

B.1 Proof of Proposition 4

Proof. The proof has two parts. In the first part, we derive the effects of changes in external prices on domestic welfare. In the second part, we derive the effect of foreign government policies on external prices.

From the perspective of the SOE, we have to infer the effects of the foreign monetary policy shock on $\{P_t^{T*}, R_t^*\}_{t=0}^{\infty}$, which are taken as given by the SOE. Let $V_{i,0}(b_{i,0}^*, \{P_t^{T*}, R_t^*\}_{t=0}^{\infty})$ denote the welfare of households in the SOE at the initial period and $c_{i,0}^N(b_{i,0}^*, \{P_t^{T*}, R_t^*\}_{t=0}^{\infty})$, $c_{i,0}^T(b_{i,0}^*, \{P_t^{T*}, R_t^*\}_{t=0}^{\infty})$, $b_{i,1}^*(b_{i,0}^*, \{P_t^{T*}, R_t^*\}_{t=0}^{\infty})$, $e_{i,0}(b_{i,0}^*, \{P_t^{T*}, R_t^*\}_{t=0}^{\infty})$ the associated policy functions. The effect on welfare is then given by

$$dV_{i,0} = \sum_{t=0}^{\infty} \frac{\partial V_{i,0}}{\partial P_t^{T*}} dP_t^{T*} + \sum_{t=0}^{\infty} \frac{\partial V_{i,0}}{\partial R_t^*} dR_t^* \tag{B.1}$$

We determine $\partial V_{i,0}/\partial P^{T*}$ and $\partial V_{i,0}/\partial R^*$ by applying the envelope theorem to the SOE problem that follows

$$V_{i,0} = \max_{c_{i,0}^N, c_{i,0}^T, b_{i,1}^*, e_{i,0}} u\left[y_{i,0}^T + b_{i,0}^* - \frac{b_{i,1}^*}{R_0^*}, c_{i,0}^N\right] - v\left[(c_{i,0}^N)^{1/\alpha}\right] + \beta \frac{z_{i,1}}{z_{i,0}} V_{i,1}(b_{i,1}^*)$$

subject to

$$c_{i,0}^T = y_{i,0}^T + b_{i,0}^* - \frac{b_{i,1}^*}{R_0^*} \qquad (\times \lambda_{i,0})$$

$$c_{i,0}^N = \left[\frac{1-\omega}{\omega} \frac{P_0^{T*}}{\bar{P}^N} e_{i,0}\right]^{\gamma} c_{i,0}^T \qquad (\times \vartheta_{i,0})$$

$$u_T(c_{i,0}^T, c_{i,0}^N) = \beta R_0^* (1+\tau_{i,0}) \frac{z_{i,1}}{z_{i,0}} \left[u_T\left(\mathcal{C}^T(b_{i,1}^*), \mathcal{C}^N(b_{i,1}^*)\right)\right] \qquad (\times v_{i,0})$$

$$1 \geq \frac{1}{R_0^*} \frac{e_{i,0}}{\mathcal{E}_i(b_{i,1}^*)} \frac{P_0^{T*}}{P_1^{T*}} \qquad (\times \xi_{i,0})$$

where we omitted the arguments for the value function $V_{i,0}$ and policy functions $c_{i,0}^N$, $c_{i,0}^T$, $b_{i,1}^*$, $e_{i,0}$ to simplify the expressions. We therefore have using the envelope condition that the partial derivative of the home households' welfare with respect to P_0^{T*} is given by

$$\frac{\partial V_{i,0}}{\partial P_0^{T*}} = \gamma c_{i,0}^N \vartheta_{i,0} - \xi_{i,0} = 0 \tag{B.2}$$

where the second equality uses the government's first order condition with respect to e_0.

For the derivative with respect to P_1^{T*},

$$\frac{\partial V_{i,0}}{\partial P_1^{T*}} = -\gamma c_{i,0}^N \vartheta_{i,0} + \xi_{i,0} = 0 \tag{B.3}$$

Next, applying the envelope condition to $\partial V_{i,0}/\partial P_t^{T*}$ for $t > 1$, and $\partial V_{i,0}/\partial R_t^*$ for $t \geq 1$, it is straightforward to see that

$$\frac{\partial V_{i,0}}{\partial P_t^{T*}} = 0 \ \text{ for } t > 1, \quad \text{and} \quad \frac{\partial V_{i,0}}{\partial R_t^*} = 0 \ \text{ for } t \geq 1. \tag{B.4}$$

It remains to determine $\partial V_{i,0}/\partial R_0^*$. Use once again the envelope condition to arrive to

$$\frac{\partial V_{i,0}}{\partial R_0^*} = \lambda_{i,0} \frac{b_{i,1}^*}{(R_0^*)^2} + \frac{1}{R_0^*} \left[u_T(c_{i,0}^T, c_{i,0}^N) v_{i,0} + \xi_{i,0} \right] \tag{B.5}$$

Then, combine the government's first order condition with respect to e and c^N to get

$$\begin{aligned} \xi_{i,0} &= \gamma c_{i,0}^N \left[u_N(c_{i,0}^T, c_{i,0}^N) \psi_{i,0} - u_{TN}(c_{i,0}^T, c_{i,0}^N) v_{i,0} \right] \\ &= \gamma c_{i,0}^N u_N(c_{i,0}^T, c_{i,0}^N) \psi_{i,0} - \gamma c_{i,0}^N \frac{\tilde{\omega}}{c_{i,0}^T} u_N(c_{i,0}^T, c_{i,0}^N) \frac{\sigma - \gamma}{\sigma \gamma} v_{i,0} \\ &= u_T(c_{i,0}^T, c_{i,0}^N) \left[\frac{1 - \tilde{\omega}_{i,0}}{\tilde{\omega}_{i,0}} \psi_{i,0} - (1 - \tilde{\omega}_{i,0}) \frac{\sigma - \gamma}{\sigma \gamma} v_{i,0} \right] \end{aligned} \tag{B.6}$$

Plugging (B.6) into (B.5), and given that we start from $b_{i,1}^* = 0$, we get

$$\frac{\partial V_{i,0}}{\partial R_0^*} = \frac{u_T(c_{i,0}^T, c_{i,0}^N)}{R_0^*} \left[v_{i,0} + \frac{1 - \tilde{\omega}_{i,0}}{\tilde{\omega}_{i,0}} \gamma c_{i,0}^T \psi_{i,0} - (1 - \tilde{\omega}_{i,0}) \frac{\sigma - \gamma}{\sigma} v_{i,0} \right] \tag{B.7}$$

Finally, we substitute (B.2), (B.3), (B.4) and (B.7) into (B.1) to obtain

$$dV_{i,0} = \frac{u_T(c_{i,0}^T, c_{i,0}^N)}{R_0^*} \left[\frac{1 - \tilde{\omega}_{i,0}}{\tilde{\omega}_{i,0}} \gamma c_{i,0}^T \psi_{i,0} + \left(\tilde{\omega}_{i,0} + (1 - \tilde{\omega}_{i,0}) \frac{\gamma}{\sigma} \right) v_{i,0} \right] dR_0^*.$$

which corresponds to (29).

We now turn to determine the effects of $d\tau_{k,0}$ and $dR_{k,0}$ in country k on the equilibrium real interest rate. To do so, we start by deriving the effect of $d\tau_{k,0}$, and $dR_{k,0}$, and dR_0^* on bond holdings $b_{k,0}$ in country k.

Let us consider the inter-temporal problem of households in country i. Define Y_t as

$$\mathcal{P}_{k,t} Y_{k,t} \equiv y_{k,t}^T + T_{k,t} + (W_{k,t} h_{k,t} + \phi_{k,t}^N)/P_{k,t}^T$$

Using this definition and the budget constraint (2) then becomes

$$\mathcal{P}_{k,t}c_{k,t} + \frac{1}{1+\tau_{k,t}}\left[\frac{b_{k,t+1}}{R_{k,t}} + P^T_{k,t}\frac{b^*_{k,t+1}}{R^*_t}\right] = \mathcal{P}_{k,t}Y_{k,t} \tag{B.8}$$

Integrating (B.8) forward, using the standard terminal conditions, and plugging the Euler equation for real bond

$$c_{k,t} = \left[\beta R^*_t(1+\tau_{k,t})\frac{\mathcal{P}_{k,t}}{\mathcal{P}_{k,t+1}}\right]^{-\sigma} c_{k,t+1} \tag{B.9}$$

we obtain that, for any arbitrary sequence of the nominal exchange rate, the policy function for bond holdings is given by

$$\frac{b^*_{k,1}}{R^*_0} = \mathcal{P}_{k,0}\left\{Y_{k,0} - \left[\sum_{t=0}^{\infty}\beta^{t\sigma}q^{1-\sigma}_{k,t|0}\right]^{-1}\sum_{t=0}^{\infty}q_{k,t|0}Y_{k,t}\right\} \quad \text{for } i \in \{k,F\} \tag{B.10}$$

where $q_{k,t|0} = \frac{\mathcal{P}_{k,t}}{\mathcal{P}_{k,0}}\prod_{s=0}^{t-1}\left(R^*_s(1+\tau_{k,s})\right)^{-1}$. Totally differentiating (B.10) yields

$$db^*_{k,1} = b^*_{k,1}\frac{dR^*_0}{R^*_0} + b^*_{k,1}\frac{d\mathcal{P}_{k,0}}{\mathcal{P}_{k,0}} + \sum_{t=0}^{\infty}\frac{\partial b^*_{k,1}}{\partial q_{k,t|0}}dq_{k,t|0} + \sum_{t=0}^{\infty}\frac{\partial b^*_{k,1}}{\partial Y_{k,t}}\left[\frac{\partial Y_{k,t}}{\partial e_{k,t}}de_{k,t} + \frac{\partial Y_{k,t}}{\partial y^N_{k,t}}dy^N_{k,t}\right] \tag{B.11}$$

Next, we have

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{\partial b^*_{k,1}}{\partial q_{k,t|0}}dq_{k,t|0} &= -R^*_0\left[(\sigma-1)(1-\mu_{k,0})\mu_{k,0}\sum_{t=0}^{\infty}q_{k,t|0}\mathcal{P}_{k,0}Y_{k,t} + \mu_{k,0}\sum_{t=1}^{\infty}q_{k,t|0}\mathcal{P}_{k,0}Y_{k,t}\right]\frac{dq_{i,1|0}}{q_{i,1|0}} \\ &= -R^*_0\left[\sigma(1-\mu_{k,0})\mathcal{P}_{k,0}c_{k,0} - \mu_{k,0}\frac{b^*_{k,1}}{R^*_0}\right]\frac{dq_{i,1|0}}{q_{i,1|0}}\end{aligned} \tag{B.12}$$

where we use $q_{i,0|0} = 1$ and $q_{k,t|0} = \beta^t q_{i,1|0}$ for any $t \geq 1$ since the economy is at the stationary equilibrium, and with

$$\mu_{k,0} \equiv \left[\sum_{t=0}^{\infty}\beta^{t\sigma}q^{1-\sigma}_{k,t|0}\right]^{-1}.$$

Next, use (B.12) and the change in $q_{t|0}$ (evaluated at the initial equilibrium) $d\log q_{0|0} = 0$, and for $t \geq 1$ we have

$$\begin{aligned}d\log q_{k,t|0} &= -\sum_{s=0}^{t-1}\left[d\tau_{k,s} + \tilde{\omega}_k d\log R^*_k + (1-\tilde{\omega}_k)d\log R_{k,s}\right], \\ &= -d\tau_{k,0} - \tilde{\omega}_k d\log R^*_0 - (1-\tilde{\omega}_k)d\log R_{k,0},\end{aligned} \tag{B.13}$$

and we arrive to

$$\sum_{t=0}^{\infty}\frac{\partial b_{k,1}^*}{\partial q_{k,t|0}}dq_{k,t|0}=R_0^*\left[\sigma(1-\mu_{k,0})\mathcal{P}_{k,0}c_{k,0}-\mu_{k,0}\frac{b_{k,1}^*}{R_0^*}\right]\left[d\tau_{k,0}+\tilde{\omega}_k\frac{dR_0^*}{R_0^*}+(1-\tilde{\omega}_k)\frac{dR_{k,0}}{R_{k,0}}\right] \quad \text{(B.14)}$$

starting from $b_{k,1}^*=0$. Consider now the third term in (B.11). We have

$$\begin{aligned}\frac{\partial b_{k,1}^*}{\partial Y_{k,0}}\left[\frac{\partial Y_{k,0}}{\partial e_{k,0}}de_{k,0}\right]&=-\frac{\partial b_{k,1}^*}{\partial Y_{k,0}}(1-\tilde{\omega}_{k,0})(c_{k,0}-Y_{k,0})\frac{de_{k,0}}{e_{k,0}}\\&=-\frac{\partial b_{k,1}^*}{\partial Y_{k,0}}\frac{1-\tilde{\omega}_{k,0}}{1+\tau_{k,0}}\frac{b_{k,1}^*}{R_0^*}\frac{de_{k,0}}{e_{k,0}}\end{aligned} \quad \text{(B.15)}$$

Similarly, we have

$$\frac{\partial b_{k,1}^*}{\partial Y_{k,1}}\left[\frac{\partial Y_{k,1}}{\partial e_{k,1}}de_{k,1}\right]=\frac{\partial b_{k,1}^*}{\partial Y_{k,1}}\frac{1-\tilde{\omega}_{k,1}}{1+\tau_{k,0}}\frac{b_{k,1}^*}{R_0^*}\frac{de_{k,1}}{e_{k,1}} \quad \text{(B.16)}$$

Next, combine the Euler equation (B.9), market clearing condition for non-tradables $y_{k,t}^N=c_{k,t}^N$ and demand for non-tradables $c_{k,t}=(1-\omega)^\gamma\left[\frac{P_{k,t}^{T*}e_{k,t}}{\bar{P}^N}\mathcal{P}_{k,t}\right]^\gamma c_{k,t}$ to get

$$\frac{y_{k,t}^N}{y_{k,t+1}^N}=\left[\beta R_t^*(1+\tau_{k,t})\frac{\mathcal{P}_{k,t}}{\mathcal{P}_{k,t+1}}\right]^{-\sigma}\left[\frac{P_t^{T*}e_{k,t}}{P_{t+1}^{T*}e_{k,t+1}}\frac{\mathcal{P}_{k,t}}{\mathcal{P}_{k,t+1}}\right]^{-\gamma}$$

This is our small open economy version of the New Keynesian dynamic IS curve. Differentiating this equation yields

$$d\log(y_{k,t}^N/y_{k,t+1}^N)=-\sigma\left[d\tau_{k,t}+\tilde{\omega}_k d\log R_t^*+(1-\tilde{\omega}_k)d\log R_{k,t}\right]+\gamma\tilde{\omega}_k(d\log R_t^*-d\log R_{k,t})$$

since we consider temporary changes, we have from (B.17) that

$$d\log y_{0,t}^N-d\log y_{k,t}^N=-\sigma d\tau_{k,0}-(\sigma-\gamma)\tilde{\omega}_k d\log R_0^*-[\sigma(1-\tilde{\omega}_k)+\gamma\tilde{\omega}_k]d\log R_{k,0} \quad \text{(B.17)}$$

for any $t\geq 1$. Moreover, we have

$$\begin{aligned}\Delta\equiv&\sum_{t=0}^{\infty}\frac{\partial\mathcal{B}^*}{\partial Y_{k,t}}\left[\frac{\partial Y_{k,t}}{\partial y_{k,t}^N}dy_{k,t}^N\right]\\=&R_0^*(1-\mu_{k,0})(1-\tilde{\omega}_{k,0})\mathcal{P}_{k,0}c_{k,0}\frac{dy_{k,0}^N}{y_{k,0}^N}-R_0^*\mu_{k,0}\sum_{t=1}^{\infty}q_{k,t|0}\mathcal{P}_{k,0}c_{k,t}(1-\tilde{\omega}_{k,t})\frac{dy_{k,t}^N}{y_{k,t}^N}\\=&R_0^*(1-\mu_{k,0})(1-\tilde{\omega}_{k,0})\mathcal{P}_{k,0}c_{k,0}\frac{dy_{k,0}^N}{y_{k,0}^N}-R_0^*\mu_{k,0}\sum_{t=1}^{\infty}\beta^{t\sigma}q_{k,t|0}^{1-\sigma}\mathcal{P}_{k,0}c_{k,0}(1-\tilde{\omega}_{k,t})\frac{dy_t^N}{y_t^N}\end{aligned}$$

where the second equality uses (B.9). Note that $1-\mu_{k,0}=\mu_{k,0}\sum_{t=1}^{\infty}\beta^{t\sigma}q_{k,t|0}^{1-\sigma}$. Thus, starting from the initial equilibrium and substituting (B.17) into the previous equation yields

$$\Delta=-R_0^*(1-\mu_{k,0})\mathcal{P}_{k,0}c_{k,0}(1-\tilde{\omega}_{k,0})\left[\sigma d\tau_{k,0}+\tilde{\omega}_k(\sigma-\gamma)\frac{dR_0^*}{R_0^*}+((1-\tilde{\omega}_{k,0})\sigma+\tilde{\omega}_{k,0}\gamma)\frac{dR_{k,0}}{R_{k,0}}\right] \quad \text{(B.18)}$$

Finally, using the fact the economy starts from the initial equilibrium with $b_{k,1}^*=0$, and plugging (B.14), (B.15)-(B.16), (B.18) into (B.11), we arrive at

$$db_{k,1}^*=R_0^*(1-\mu_{k,0})c_{k,0}^T\left[\sigma d\tau_{k,0}+(\sigma\tilde{\omega}_{k,0}+\gamma(1-\tilde{\omega}_{k,0}))\frac{dR_0^*}{R_0^*}+(\sigma-\gamma)(1-\tilde{\omega}_{k,0})\frac{dR_{k,0}}{R_{k,0}}\right] \quad \text{(B.19)}$$

Because only countries belonging to the subset Ω_k engineer a change in policies $d\tau_{k,0}\neq 0$ and $dR_{k,0}\neq 0$, we get

$$db_{i,1}^*=R_0^*(1-\mu_{i,0})c_{i,0}^T\left[(\sigma-\gamma)(1-\tilde{\omega}_{i,0})\frac{dR_{i,0}}{R_{i,0}}\right],\ \text{for } i\notin\Omega_k$$
$$db_{k,1}^*=R_0^*(1-\mu_{k,0})c_{k,0}^T\left[\sigma d\tau_{k,0}+(\sigma-\gamma)(1-\tilde{\omega}_k)\frac{dR_{k,0}}{R_{k,0}}+(\sigma\tilde{\omega}_i+\gamma(1-\tilde{\omega}_k))\frac{dR_0^*}{R_0^*}\right], k\in\Omega_k$$

Finally we use market clearing condition for bond, $\int_0^1 db_{i,1}^*=0$, to get

$$\frac{dR_0^*}{R_0^*}=-\int_{\Omega_k}\frac{(1-\mu_{k,0})c_{k,0}^T}{\int_0^1(\sigma\tilde{\omega}_{i,0}+\gamma(1-\tilde{\omega}_{i,0}))(1-\mu_{i,0})c_{i,0}^T di}\left[\sigma d\tau_{k,0}+(\sigma-\gamma)\tilde{\omega}_{k,0}\frac{dR_{k,0}}{R_{k,0}}\right]$$

□

B.2 Proof of Corollary 1

Proof. The proof of the corollary follows from equation (31). Consider a prudential monetary policy intervention that aims at increasing aggregate savings in order to mitigate overborrowing. For a given R_0^*, we have by (B.19) that

$$db_{k,1}^*>0\Rightarrow(\sigma-\gamma)dR_{k,0}>0$$

Moreover by (A.32), $v_{k,0}\geq 0$ with strict inequaality if the ZLB binds in some future (that is $\exists\, k$ such that $\xi_{t+k}>0$) which we plug into (31) to obtain

$$dV_{i,0}=-\left[u_T(c_{i,0}^T,c_{i,0}^N)v_{i,0}\right]\int_{\Omega_k}\frac{(1-\tilde{\omega}_{k,0})(1-\mu_{k,0})c_{k,0}^T}{\int_0^1(\sigma\tilde{\omega}_{i,0}+\gamma(1-\tilde{\omega}_{i,0}))(1-\mu_{i,0})c_{i,0}^T di}\frac{(\sigma-\gamma)dR_{k,0}}{R_{k,0}}\leq 0$$

□

B.3 Proof of Corollary 2

Proof. The proof of the corollary follows from Proposition 4. With capital controls, $v_{k,0} = 0$ and we have

$$dV_{k,0} = -\left[u_T(c^T_{k,0}, c^N_{k,0})v_{k,0}\right] \int_{\Omega_k} \frac{\sigma(1-\tilde{\omega}_{k,0})(1-\mu_{k,0})c^T_{k,0}}{\int_0^1 (\sigma\tilde{\omega}_{k,0} + \gamma(1-\tilde{\omega}_{k,0}))(1-\mu_{k,0})c^T_{k,0}di} \frac{(\sigma-\gamma)dR_{k,0}}{R_{k,0}} = 0$$

□

B.4 Proof of Proposition 5

Consider the welfare of an individual country in a regime where all countries are using macroprudential policy. Let V_{MP} be the welfare and R_{MP} the world real interest rate. If the country is not at the ZLB, it is immediate that welfare cannot be lower, under the assumption that country is neither a net borrower nor a net saver. Consider then the case at the ZLB. Let $\hat{R}$ be the real interest rate in the domestic economy after imposing a quantity control $\hat{b}'$ on capital inflows. The resulting problem is:

$$V_{MP,t}(b^*) = \max_{e, c^N, \hat{R}^*, \hat{b}^{*\prime}} u(c^T, c^N) - v\left((c^N)^{1/\alpha}\right) + \beta V_{MP,t+1}(\hat{b}^{*\prime})$$

subject to

$$\hat{c}^T = y^T_t + b^* - \frac{\hat{b}^{*\prime}}{R^*_{MP}}$$

$$c^N = \left[\frac{1-\omega}{\omega}\frac{P^{T*}}{\bar{P}^N}e\right]^{\gamma}\hat{c}^T$$

$$u_T(\hat{c}^T) = \beta\hat{R}^* u_T\left(\mathcal{C}^T(b^{*\prime}, y^T_{t+1})\right)$$

$$\hat{R}^* = \frac{P^{T*}}{P^{T*\prime}}\frac{e}{\mathcal{E}(b^{*\prime}, y^T_g)}$$

$$\hat{b}^{*\prime} \geq \bar{b}$$

Let $\hat{b}^{*\prime}(b^*, y^T_t)$ be the level of debt from the decentralized equilibrium in the no capital control regime. Assume that in the current period the government chooses directly this level of borrowing and blocks all private inflows or outflows. It thus follows that the level of tradable consumption is given by

$$\hat{c}^T(b^*, y^T_t) = y^T_t + b^* - \frac{\hat{b}^{*\prime}(b^*, y^T_t)}{R^*_{MP}} \quad \text{(B.20)}$$

The resulting equilibrium real interest rate thus satisfies:

$$u_T(\hat{c}^T(b^*, y_t^T)) = \beta \hat{R}^* u_T\left(\mathcal{C}^T\left(\hat{b}^{*\prime}(b^*, y_t^T), y_{t+1}^T\right)\right) \tag{B.21}$$

where $\mathcal{C}^T\left(\hat{b}^{*\prime}(b^*, y_t^T), y_g^T\right)$ is also the equilibrium policies in the laissez-faire equilibrium. This follows from the observation that for $\sigma = \gamma$ the government also closes the labor wedge in the laissez-faire equilibrium in the good state. It is then immediate that $\hat{R}^*$ equals the interest rate in the no capital control regime. Moreover, e and c^N satisfy

$$e = \mathcal{E}(\hat{b}^{*\prime}(b^*, y_t^T), y_g^T)\hat{R}^* \frac{P^{T*\prime}}{P^{T*}},$$
$$c^N = \left[\frac{1-\omega}{\omega}\,\frac{P^{T*}}{\hat{P}^N}\,e\right]^{\gamma} \hat{c}^T,$$

and thus, allocations are identical to the laissez faire equilibrium. Therefore, in a capital control regime governments in the bad state can at least the same welfare as in the no capital control regime.

C Extension with Staggered Pricing

Derivation of the Phillips curve. We describe here the environment with staggered price setting and derive firms' optimal pricing decisions. There is a continuum of firms, each using a constant return to scale technology that uses labor as the sole input to produce a unique variety j, $y_{j,t}^N = n_{j,t}$. Firms are monopolistic competitors and are subject to Rotemberg (1982) price-adjustment costs measured in terms of the final non-tradable good,

$$\frac{\varphi}{2}\left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1\right)^2 y_t^N$$

where φ is an adjustment cost parameter and the final non-tradable good y_t^N is defined as a Dixit-Stiglitz aggregator of the continuum of non-tradable varieties indexed by $j \in [0,1]$

$$y_t^N = \left(\int_0^1 {y_{j,t}^N}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Cost minimization implies that the marginal costs of production are: $MC_t = (1-\tau^n)W_t$ where $\tau^n = \frac{1}{\varepsilon}$ is labor subsidy. Taking as given the sequence for mc_t and y_t^N, a monopolist

j chooses $P_{j,t}^N$ to maximize the stream of its expected discounted profit:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\Theta_t}{\Theta_0} \left[\left(P_{j,t}^N - MC_t \right) \left(\frac{P_{j,t}^N}{P_t^N} \right)^{-\varepsilon} P_t^N y_t^N - \frac{\varphi}{2} \left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1 \right)^2 P_t^N y_t^N \right], \tag{B.22}$$

Θ_t / Θ_0 is the stochastic discount factor and where P_t is the consumer price index. Notice also by households optimality condition $\Theta_t / \Theta_0 = \beta^t \left(\prod_{k=0}^{t} \delta_k \right) \frac{u_N(c_t^T, c_t^N)/P_t^N}{u_N(c_0^T, c_0^N)/P_0^N}$. The first order condition of the firm's problem then yields the following optimal pricing rule or dynamic Phillips curve:

$$\pi_t^N (1 + \pi_t^N) = \frac{\varepsilon - 1}{\varphi} \left[\frac{\varepsilon(1 - \tau^n)}{\varepsilon - 1} \frac{W_t}{P_t^N} - 1 \right] + \mathbb{E}_t \frac{\Theta_{t+1}}{\Theta_t} \left[\frac{y_{t+1}^N}{y_t^N} \pi_{t+1}^N (1 + \pi_{t+1}^N) \right] \tag{B.23}$$

where $1 + \pi_t^N \equiv P_t^N / P_{t-1}^N$ denote the inflation rate in the non-tradable sector. Notice also that, given the labor subsidy, $\frac{\varepsilon(1-\tau^n)}{\varepsilon - 1} = 1$.