Financial Integration and Monetary Policy Coordination

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Abstract

Financial integration generates macroeconomic spillovers that may require international monetary policy coordination. We show that individual central banks may set nominal interest rates too low or too high relative to the cooperative outcome. We identify three sufficient statistics that determine whether the non-cooperative equilibrium exhibits under-tightening or over-tightening: the output gap, sectoral differences in labor intensity, and the response of the trade balance to changes in nominal rates. In the case of higher labor intensity in the non-tradable sector, nominal interest rates are too low under two conditions: when the economy is in a recession and lower nominal rates increase the trade surplus, or when the economy is overheated and lower nominal rates reduce the trade surplus.

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Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is not so much that current and planned moves will fail eventually to quell inflation. It is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account.

Maurice Obstfeld “Uncoordinated monetary policies risk a historic global slowdown,” blog post, Peterson Institute, 09/12/2022

1 Introduction

After a prolonged period of expansionary monetary policy, central banks around the world have shifted to a tightening cycle to tame rising inflation. However, the rapid pace and synchronous nature of the increase in interest rates have raised concerns that the unprecedented monetary tightening could lead to a severe global economic downturn. In this context, there has been a renewed discussion on the necessity of cooperation to avert a global recession and achieve a soft landing (Obstfeld, 2022).

Does cooperative monetary policy prescribe lower interest rates compared to the non-cooperative scenario? Or is it possible that countries may insufficiently tighten monetary policy relative to the social optimum? In a broader sense, what are the benefits from international coordination of monetary policy and how do they depend on the degree of financial integration?

The study of international monetary policy cooperation has a long history in international macro literature, dating back to Hamada (1976), and Canzoneri and Henderson (1991). Early studies argued that countries have incentives to weaken their currencies to gain a trade advantage, leading to concerns about competitive devaluations and widespread inflation. On the contrary, modern international macro-models, as exemplified by Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2005) predict that countries have incentives to appreciate their currencies to improve their terms of trade and extract more rents from foreign countries. From this perspective, dealing with the strategic manipulation of terms of trade call for cooperation towards more expansionary monetary policies.

1 See the Peterson Institute blog post quoted at the beginning of the paper and Figure 1.

2 From a quantitative standpoint, however, the consensus in the literature following Obstfeld and Rogoff (1995) is that the gains from cooperation due to this trade channel are negligible.
Moreover, the gains from cooperation in this literature emerge purely from trade flows and are present even in the absence of financial flows.

In this paper, we approach the questions on international monetary coordination from a different, intertemporal perspective. Central to our model is the notion that monetary policy has effects on an intertemporal price, namely, the world real rate, and through this channel, a central bank’s policy affects the ability of other central banks to achieve their output and inflation stability objectives. Our analysis builds on the analysis of international spillovers in Bianchi and Coulibaly (2021) where we show how countries use monetary policy to raise their net foreign asset position and reduce their vulnerability to liquidity traps. When the world interest rate is lower, this results in larger incentives for households’ borrowing, driving the central bank to deviate further from its efficient level of output in an attempt to raise its net foreign asset position. Given the intertemporal nature of this mechanism, distinct from the static terms of trade manipulation, we refer to it as the financial channel of international spillovers. Here, we compare the Nash equilibrium without monetary policy cooperation with the equilibrium under the optimal cooperative monetary policy and provide a general characterization of whether this financial channel requires coordinating on a more restrictive or expansionary monetary policy.\footnote{3}

Our main result is that the Nash equilibrium may feature nominal rates that are too low or too high relative to the cooperative outcome and to elucidate how the outcome depends on a small set of sufficient statistics, specifically, the output gap, the difference in labor intensity across sectors, and the response of the trade balance to changes in nominal rates. In particular, we establish that when the economy is undergoing a recession, the cooperative monetary policy prescribes lower interest rates relative to the Nash equilibrium (i.e., there is over-tightening) if the product of the difference between the labor intensity in the non-tradable sector and the tradable sector and the response of the trade balance to a domestic monetary policy expansion is positive. Conversely, if the sign of the product is the same, the cooperative monetary policy prescribes lower interest rates relative to the Nash equilibrium (i.e., there is under-tightening) when the economy is facing overheating.

The intuition for our results is as follows. Consider a global economy facing a recessionary shock. To the extent that wages are rigid and inflation is costly, central banks in individual countries expand monetary policy to help reduce the output gap and face an increase in inflation. In such a scenario, we argue that a reallocation of employment from a low labor-intensity sector to a high labor-intensity sector helps mitigate inflation. This

\footnote{Fornaro and Romei (2022) tackles the problem of monetary policy coordination from a similar perspective. We discuss below how our framework and conclusions differ from theirs.}
is because, in sectors with higher labor intensity, prices are less responsive to changes in production under wage stickiness. Or put it differently, the elasticity of the marginal cost with respect to output is decreasing in the labor intensity when wages are sticky. Consequently, to the extent that the non-tradable sector is more labor intensive than the tradable sector, a shift in employment towards non-tradables would lead to an overall reduction in inflation.

In turn, the allocation of employment across sectors depends crucially on financial flows. When households borrow more from abroad, they increase their demand for consumption of both tradable and non-tradable goods. But employment of tradables remains fixed for a given monetary policy. Therefore, higher capital inflows result in more employment in the non-tradable sector (to satisfy the increase in demand) and help reduce inflation.

Even though the central bank cannot control directly financial flows, it can use monetary policy to steer them to help control inflation. In the case where an increase in the interest rate decreases the trade surplus (i.e., Marshall Lerner holds), a central bank thus perceives that raising interest rates helps to contain inflation for given total employment, as the increase in capital inflows generates in equilibrium a reallocation of employment towards non-tradables. However, as all economies seek to run a trade deficit, this pushes up real interest rates keeping capital flows unchanged. The increase in the world real rate then feeds back into each central bank’s problem. Notably, for an individual central bank facing a recession, the increase in the real interest rate induces households to reduce their demand, pushing the economy further down into the recession. Because individual countries do not internalize this pecuniary externality, they tend to over-tighten in this situation. Under cooperative monetary policy, it is optimal to lower nominal interest rates because this helps central banks improve their macroeconomic tradeoffs.

In a situation where the economy is overheating instead, the above conclusions reverse for the same sign of the difference in labor intensities and the same response of the trade balance to a monetary policy expansion. In this case, now, a lower world real rate worsens the macroeconomic tradeoffs faced by central banks, and therefore, central banks tighten too little monetary policy relative to the cooperative outcome (i.e., there is under-tightening).

When a monetary expansion in one country leads to a decrease in its trade surplus instead (i.e., Marshall-Lerner fails), we obtain that the Nash equilibrium displays under-tightening even if the global economy faces a recession, insofar as the non-tradable sector is more labor-intensive than the tradable sector. The wedge changes sign because now an individual central bank is hurt by a more expansionary monetary policy abroad, as this
Figure 1: Synchronous Monetary Policy Tightening

raises the world real rate and worsens its monetary tradeoffs.

When the non-tradable sector is more labor intensive than the tradable sector, we also obtain under-tightening if the economy is in a recession and the Marshall-Lerner condition holds. This is because central banks facing a recession now benefit from a higher world real rate.

To summarize, whether cooperation calls for lower or higher rates can be framed entirely in terms of the sign of the output gap and the sign of the product of the differences in labor intensity between the tradable sector and the non-tradable sector and the response of the trade balance to a monetary expansion. Our quantitative analysis shows that the differences between cooperative and non-cooperative equilibrium can be quite substantial for moderately large shocks.

In one extension, we allow for the anticipation of future shocks. In this case, the comparison between cooperative and non-cooperative outcomes is quite stark. While the cooperative solution maintains zero inflation and zero output in response to the news shock, the Nash equilibrium exhibits either one of these scenarios overheating and inflation or a recession and deflation. Moreover, the sign of the output gap, the differences in labor intensity, and the response of the trade balance to a monetary expansion remain the three key sufficient statistics. These results also hold when we allow for costly labor reallocation or oil price shocks.

pioneer papers adopting a microfounded approach to cooperative monetary policy are Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2005) (see also, e.g., Tille, 2001; Obstfeld and Rogoff, 2002 Clarida, Gali and Gertler, 2002; Canzoneri, Cumby and Diba, 2005; Devereux and Engel, 2003; Benigno, 2009; Egorov and Mukhin, 2020; and Bodenstein, Corsetti and Guerrieri, 2020). A key theme in this literature is that individual countries have incentives to reduce their own production to change terms of trade in their favor at the expense of other countries. According to this optimal tariff argument, central banks generally over-tighten monetary policy relative to the socially optimal level, a result that is independent of the degree of financial integration. Instead, we highlight a financial channel, involving an intertemporal price (i.e., the world real interest rate) and show that this generates the possibility of under-tightening.

Our paper is most closely related to Fornaro and Romei (2022) who also consider the scope for monetary policy cooperation in a two-sector New Keynesian model with tradables and non-tradables. They show how an increase in the preference for tradable goods leads to inflation and a negative output gap in equilibrium. Moreover, they find that cooperative monetary policy prescribes higher output levels relative to the Nash equilibrium. In terms of the model, we differ by considering a more general structure with elastic labor supply, diminishing returns in labor, and non-unitary elasticities of substitution. Our analysis shows that the Nash equilibrium may also exhibit under-tightening and elucidates how this outcome depends on a set of sufficient statistics. Namely, we establish analytically that, independently of the shocks, whether cooperation calls for lower or higher rates depends on the degree of slack in the economy, the differences in labor intensities across sectors, and the response of the trade balance to a monetary expansion.

Our paper is also related to the literature that examines the potential for international coordination in the context of various government policies. Chang (1990) and Kehoe (1987) study the coordination of fiscal policies when fiscal deficits in some countries make it more costly for others to finance their deficit (see also Azzimonti, De Francisco and

\textsuperscript{4}In their setup, a fixed endowment of hours implies that overheating cannot occur, linear production for non-tradables rules out inflation in non-tradables and unitary elasticities imply that the trade balance always increases in response to a depreciation.

\textsuperscript{5}As mentioned earlier, we also draw from our previous work on international spillovers (Bianchi and Coulibaly, 2021) which focuses on a prudential aspect of monetary policy. Another recent paper is Caldara, Ferrante, Iacoviello, Prestipino and Queralto (2023), which studies non-linear effects from monetary spillovers in a model with global banks. Previous work by Acharya and Bengui (2018), Eggertsson, Mehrotra, Singh and Summers (2016), Caballero, Farhi and Gourinchas (2021), and Fornaro and Romei (2019) study the propagation of liquidity traps across countries, but did not consider the scope for monetary policy cooperation. For the empirical literature on international monetary policy spillovers, see, for example, Rey (2013) and Kalemli-Ozcan (2019).
Quadrini, 2014). In Halac and Yared (2018), governments exhibit present bias and fiscal rules are slacker under coordination. Obstfeld and Rogoff (1996) and Costinot, Lorenzoni and Werning (2014) consider the case for excluding capital controls when countries are large and have market power over the world interest rate. In our case, countries are infinitesimal, and the case for coordination is grounded in a pecuniary externality, where the world interest rate influences monetary policy tradeoffs.

The key mechanism at play in our model is also related to the literature on aggregate demand externalities. In Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), nominal rigidities and constraints on monetary policy create a rationale for capital controls. In our model, monetary policy faces no constraints, but inflation is costly, and divine coincidence fails, generating aggregate demand externalities. Crucially, the scope for monetary policy cooperation emerges because of the interaction between this aggregate demand externality and a pecuniary externality operating through the world real rate.

Finally, there has been an active recent literature on the rise of inflation following the Covid-19 pandemic and the connection with sectoral reallocation. Besides our open economy focus, we also contribute to this literature by highlighting, for the first time, to the best of our knowledge, the importance of differences in labor intensity across sectors for the determination of inflation and output.

2 Model

Time is discrete and infinite. We model the world economy as a continuum of identical small open economies indexed by $k \in [0, 1]$. There are two consumption goods, a tradable good and a non-tradable good, which are produced in each economy using labor in a competitive market with nominally rigid wages. For simplicity, we focus on a deterministic environment.

We first describe the problem faced by households and firms in each economy $k$, and then describe the competitive equilibrium. The notation does not index variables in each country by $k$ to avoid clutter. We will use $\{x_t\}$ to refer to the sequence $\{x_{k,t}\}_{t=0}^{\infty}$ for some variable $x$ and country $k$.

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6Other recent examples are Clayton and Schaab (2022) on macroprudential policy with multinational banks and Chari, Nicolini and Teles (2023) on fiscal and trade policies in a multi-country business cycle model.

7See, for example, Rubbo (2020), Guerrieri, Lorenzoni, Straub and Werning (2021), di Giovanni, Kalemli-Özcan, Silva and Yildirim (2022, 2023), and Baqaee and Farhi (2022).
2.1 Households

Each economy is populated by a continuum of identical households of measure one. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[ U(c_t) - \kappa_t n_t + \frac{\chi}{2} (\pi_t - \bar{\pi}_t)^2 \right],$$

(1)

where $\beta \in (0, 1)$ is the discount factor and $U$ is a strictly increasing and concave utility function over a consumption good $c_t$, which is a composite of tradable consumption $c^T_t$ and non-tradable consumption $c^N_t$, according to a Cobb-Douglass aggregator

$$c_t = \left( c^T_t \right)^{\phi^T} \left( c^N_t \right)^{\phi^N},$$

with $\phi^T \in (0, 1)$ and $\phi^N = 1 - \phi^T$. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods and $\sigma_t \equiv \frac{-c_t U''(c_t)}{U'(c_t)}$ to denote the inverse of the intertemporal elasticity of substitution. Households face a linear disutility from working given by $\kappa$. Aggregate hours $n_t$ is the sum of hours worked in the tradable sector $n^T_t$ and in the non-tradable sector $n^N_t$, that is $n_t = n^T_t + n^N_t$. Implicit in the formulation is that labor is perfectly mobile across sectors, which, in turn, implies that in equilibrium the wage is equated in both sectors. In Section 5, we generalize preferences by allowing for imperfect labor mobility and a CES composite for consumption.

The last term in (1) represents a utility cost of inflation, which represents standard losses from price adjustments that emerge in models with costly price adjustments à la Rotemberg or staggered prices à la Calvo. This cost is assumed to be quadratic in the deviations of the inflation rate of the consumer price index $\pi_t \equiv P_t / P_{t-1} - 1$ from the central bank target $\bar{\pi}_t$. Given a unitary elasticity of substitution between tradables and non-tradables, the consumer price index $P_t$ satisfies

$$P_t = \left( \frac{P^T_t}{\phi^T} \right)^{\phi^T} \left( \frac{P^N_t}{\phi^N} \right)^{\phi^N}$$

(2)

where $P^N_t$ and $P^T_t$ denote respectively the price of non-tradables and tradables in terms of the domestic currency.

We assume that the law of one price holds for the tradable good. Let us denote by $P^T_{jt}$ the price of the tradable good in terms of the country $j$ currency. Thus, it follows that

$$P^T_{kt} = P^T_{jt} e^j_{kt}$$

for any pair of countries $k$ and $j$, where $e^j_{kt}$ is the nominal exchange rate defined
as the price of the country \( j \)’ currency in terms of the country \( k \)’ currency.

In each period, households receive their labor income, \( W_t(n_T^t + n_N^t) \). They also collect profits \( \varphi_t \) from domestic firms. Households have two assets available, a real international bond that pays \( R_t^* \) units of tradables and a nominal domestic bond that pays \( R_t \) in units of the domestic currency. These assets are referred to as \( b_t^* \) and \( b_t \) respectively. The budget constraint is, therefore, given by

\[
P_T^t c_T^t + P_N^t c_N^t + \frac{b_{t+1}}{R_t} + \frac{P_T^t b_{t+1}^*}{R_t^*} = W_t(n_T^t + n_N^t) + \varphi_t + b_t + P_T^t b_t^*.
\]  \hspace{1cm} (3)

We assume that wages are rigid in period 0 at a given value \( W_0 \). For \( t = 0 \), households are off their labor supply and hours worked are determined by firms’ labor demand. For \( t > 0 \), we assume that wages are flexible.

The problem of the household consists of choosing a sequence of consumption \( \{c_T^t, c_N^t\}_{t=0}^{\infty} \), asset positions \( \{b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty} \), and hours \( \{n_T^t, n_N^t\}_{t=1}^{\infty} \), to maximize the expected present discounted value of utility (1), subject to (3) and taking as given profits \( \{\varphi_t\} \), and prices \( \{W_t, P_N^t, P_T^t, R_t, R_t^*\}_{t=0}^{\infty} \).

The optimality condition with respect to \( c_T^t \) and \( c_N^t \) equates the marginal rate of substitution between the two goods to the relative price. Given the Cobb-Douglas aggregator, households allocate a constant share of their expenditures to each good, implying that

\[
\phi_T^t P_N^t c_N^t = \phi_T^t P_T^t c_T^t.
\]  \hspace{1cm} (4)

The linearity of the disutility from working implies that for \( t > 0 \) (when wages are flexible) the wage in both sectors must satisfy

\[
\frac{W_t}{P_N^t} = \frac{\kappa_t}{u_N(c_T^t, c_N^t)}, \quad \frac{W_t}{P_T^t} = \frac{\kappa_t}{u_T(c_T^t, c_N^t)}.
\]  \hspace{1cm} (5)

where we use \( u_T \) and \( u_N \) to denote the respective partial derivatives.

Finally, the optimality condition with respect to asset holdings yield

\[
u_T(c_T^t, c_N^t) = \beta R_t^* u_T(c_T^t, c_N^t) \quad \text{and} \quad R_t^* = R_t \frac{P_T^t}{P_{t+1}^t},
\]  \hspace{1cm} (6)

(7)

Condition (6) is the Euler equation for the real bond. Condition (7) is a no-arbitrage
condition that equates the return on real international bonds and domestic currency bonds, both expressed in units of tradables.

2.2 Firms

There is a continuum of firms of measure one producing tradable goods and non-tradable goods. Output of the two goods $i = \{T, N\}$ is produced using labor with a production function $F$ such that

$$y_i = F_i(h_i, A_i)$$

We assume an isoelastic production function such that $F_i(h_i, A_i) = A_i^{\alpha_i}(h_i)$, We refer to $\alpha_i$ as the labor intensity parameter.

Profits are given by $P_i^T F^T(h_i^T, A_i^T) - W_i h_i^T$. At the optimum, firms equate the marginal product of labor to the nominal wage in the two sectors:

$$P_i^T F^T(h_i^T, A_i^T) = W_i,$$  \hspace{1cm} (8)

$$P_i^N F^N(h_i^N, A_i^N) = W_i.$$  \hspace{1cm} (9)

Given competitive markets, the labor intensity equals the labor share for each sector in equilibrium. As we will see, differences in labor intensity across sectors, $\alpha^N - \alpha^T$, will play an important role in the analysis. We note that the fact that labor is the only factor of production or that the production function exhibits decreasing returns to scale is not restrictive.\footnote{Adding a factor in fixed unit of supply with a flexible factor price does not alter allocations. In Section 5, we incorporate oil as an additional factor of production.}

2.3 Monetary Policy

In each small open economy, there is a central bank that chooses nominal interest rates $\{R_i\}$. Because of the assumption that prices are flexible for $t > 0$, monetary policy is neutral starting from period 1. Therefore, we assume that monetary policy implements a strict inflation targeting regime such that $\pi_t = \bar{\pi}_t$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.
2.4 Competitive equilibrium

In each country, the market for non-tradable goods must clear. That is,

\[ c_i^N = F_i^N(h_i^N, A_i^N). \]  \hspace{1cm} (10)

At \( t = 0 \), households in each country supply hours in the tradable and non-tradable sectors to meet the demand by firms. For \( t > 0 \), the labor clears the labor market. That is, \( n_i^T = h_i^T \) and \( n_i^N = h_i^N \).

We assume without loss of generality that the bond denominated in domestic currency is only traded domestically in each country. Market clearing therefore implies

\[ b_{t+1} = 0. \]  \hspace{1cm} (11)

Finally, at the world level, real bonds are in zero net supply. To account for market clearing at the world level, we now explicitly index the policies of each country by \( k \). We have that

\[ \int b^*_{k,t+1} dk = 0. \]  \hspace{1cm} (12)

We now define a competitive equilibrium in the global economy.

**Definition 1** (Competitive Equilibrium). Given initial positions \( b^*_{k,0} \), a sticky wage \( W \), and a sequence of central bank policies \( \{ R_t \} \) in each country \( k \), an equilibrium is a sequence of world real rates \( \{ R^*_t \} \), prices \( \{ P^T_k, P^N_k, W_t, e^J_{k,t} \} \) and allocations \( \{ c^T_i, c^N_i, h^T_i, h^N_i, b_{t+1} = b^*_{t+1} \} \) in each country \( k \) such that

(i) Households optimize, and hence the following conditions hold: (4), (6), (7) for all \( t \geq 0 \) and (5) holds for all \( t \geq 1 \);

(ii) Firms optimize, implying (8) and (9) hold for all \( t \geq 0 \);

(iii) The law of one price holds for tradables: \( P^T_{k,t} = P^T_{j,t} e^J_{k,t} \) for any country-pair \( k \) and \( j \),

(iv) The market for non-tradables (10) and domestic bonds (11) clears; moreover, the labor market clears for \( t \geq 1 \).

(v) Globally, the market for the real bond clears. That is, (12) holds.

If we combine the budget constraints of households and firms as well as market clearing conditions, we arrive at the country budget constraint for tradables, or the balance of payment condition:

\[ c_i^T - F_i^T(h_i^T, A_i^T) = b^*_i - \frac{b^*_{t+1}}{R^*_i}, \]  \hspace{1cm} (13)
which says that if a country runs a trade deficit, it accumulates net debt and if it runs a trade surplus, it accumulates net external assets.

We assume that all countries start at \( t = 0 \) with zero net foreign asset position. To the extent that all countries follow the same policies, we can therefore restrict the analysis to symmetric competitive equilibrium.

2.5 Efficient allocation, output gaps, and the natural wage

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations to maximize welfare subject to a resource constraint. The planner’s problem can be written as

\[
\max_{\{h^N_t, h^T_t\}} \sum_{t=0}^{\infty} \beta^t \left[ u \left( F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t) \right) - \kappa_t \left( h^T_t + h^N_t \right) \right].
\]

First-order conditions with respect to tradable and non-tradable employment yield

\[
F^T_h(h^T_t, A^T_t) u_T \left( F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t) \right) = \kappa_t, \tag{14}
\]

\[
F^N_h(h^T_t, A^T_t) u_N \left( F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t) \right) = \kappa_t. \tag{15}
\]

These conditions imply zero labor wedges for the tradable and non-tradable sectors. Let us denote by \( \bar{h}^T_t \) and \( \bar{h}^N_t \) the employment levels in the two sectors in the first-best allocation. We obtain the following lemma characterizing the ratio of employment levels solely as the product of the relative weights in preferences and the relative labor intensities:

**Lemma 1 (First-best).** The optimal ratio of hours in the first-best allocation is given by

\[
\frac{\bar{h}^N_t}{\bar{h}^T_t} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T}. \tag{16}
\]

**Proof.** In Appendix A.1

We highlight that the first-best allocations coincide with those in a competitive equilibrium in a flexible wage version of our model. This can be seen by noting that if the nominal wage were flexible, we would arrive at (14) and (15) by combining firms’ demand for labor (8) and (9) with households’ labor supply decisions (5). This result will provide a clear benchmark for the normative analysis.
Output gaps. To characterize the central banks’ tradeoff and to highlight the differences between the competitive equilibrium and the first-best allocation, we define a measure of output gaps as the deviations of employment relative to the first-best levels

\[ \hat{h}_i^N \equiv \frac{h_i^N}{\bar{h}_i^N} - 1, \quad \hat{h}_i^T \equiv \frac{h_i^T}{\bar{h}_i^T} - 1 \]

In addition, we define the labor wedges in the tradable and non-tradable sector as:

\[ \tau^T \equiv F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N) - \kappa, \quad \tau^N \equiv F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N) - \kappa \]  

The assumption that good prices are flexible and the fact that wages are equalized across sectors owing to perfect labor mobility implies that in any competitive equilibrium

\[ F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N) = F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N) \]  

and thus the labor wedges are equated. Accordingly, we use \( \tau = \tau^T = \tau^N \). In addition, the next lemma shows that output gaps are equated in any symmetric equilibrium given any monetary policy.

**Lemma 2.** In any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are equalized \( \hat{h}_i^T = \hat{h}_i^N \). Furthermore, the employment ratio \( h_i^N/h_i^T \) in the first-best allocations coincides with those in a competitive symmetric equilibrium for any monetary policy.

**Proof.** In Appendix A.2

The fact that output gaps are equated across sectors also implies that the ratio of hours in the competitive equilibrium coincides with the ratio in the first-best. The levels of course do not. Depending on the parameters and the stance of monetary policy, output gaps can be negative (implying a recession) or positive (implying overheating).

**The natural wage.** We define the natural wage as the nominal wage that would prevail in equilibrium if wages were flexible and the central bank stabilized inflation at \( \bar{\pi}_t \). Denoting variables without a subscript as \( t = -1 \) variables, we can write the natural wage at date \( t = 0 \) as described in the following lemma:

**Lemma 3 (Natural Wage).** The natural wage at date \( t \) is given by

\[ W_t^n = (1 + \pi_t) P_{-1} \left[ \prod_{i=T,N} \left( \alpha^i A_i \right)^{\phi^i} \left( \bar{h}_i \right)^{-\frac{(1-\alpha^i)\phi^i}{\phi^i}} \right] \]
Proof. In Appendix A.3 □

Equation (19) characterizes the natural wage as a function of parameters for productivity, preferences, and the inflation target. In particular, the natural wage falls in period 0 when there is a decline in productivity for tradables or non-tradables, when there is a positive labor supply shock, or when there is a negative shock to the inflation target. In what follows, we assume that in period $-1$ the nominal wage is at its natural level, but not in period 0 where the nominal wage is fixed at an arbitrary value $W$.

3 Monetary Policy in a Nash Equilibrium

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad.

3.1 Optimal Monetary Policy for a Single Country

We first study the individual problem of a central bank that takes as given $\{R_t^*\}$ and policies conducted in other countries.

3.1.1 Time $t > 0$ problem

Recall that because prices are flexible for $t \geq 1$, we can focus on a situation where the central bank implements the flexible price allocation with $\pi_t = \bar{\pi}$ for all $t \geq 1$. The lifetime welfare for a central bank with net foreign asset $b_1^*$ in period 1 is given by

$$V_1(b_1^*) = \max_{\{c_t^T, c_t^N, h_t^T, h_t^N\}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^T, c_t^N) - \kappa(t)(h_t^T + h_t^N)\right]$$

subject to (10), (14), (15) and

$$b_1^* = -\sum_{t=1}^{\infty} \frac{F^T(h_t^T, A_t^T) - c_t^T}{\prod_{j=1}^{t-1} R_j^*}$$

9This is clearly without loss of generality when the central bank optimizes at $t > 0$. Moreover, we can also show that with commitment at $t = 0$, the central bank would also choose $\pi_t = \bar{\pi}$. 

13
where the last constraint says that the present discounted value of future trade balances must be consistent with the initial level of debt.

### 3.1.2 Time 0 problem

Turning to the date-0 problem, the central bank’s policy choice is the nominal interest rate, anticipating that the continuation value is given by (20). The central bank’s objective is to choose $R_0$ that maximizes the welfare of the domestic household subject to domestic allocations and prices consistent with a competitive equilibrium given policies $\{R_{k,t}\}$ conducted in other countries.

**Implementability constraints.** Following a primal approach, we proceed to combine constraints to express the problem in terms of allocations to derive the implementability constraints. First, combining the optimality condition of households (4) with the one for firms (8) and (9), we arrive at an equation that determines the relative demand for hours in the two sectors.

$$
\frac{h^N_0}{h^T_0} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} (1 - \bar{z}_0) \tag{21}
$$

where $\bar{z}_0 \equiv (y^T_0 - c^T_0) / F^T(h^T_0, A^T_0)$ represents the trade balance to output ratio.

A key implication of (21) is that when a country runs a larger trade balance surplus, it will display in equilibrium a lower amount of hours in the non-tradable sector relative to the tradable sector. The logic is as follows. Running a larger trade surplus requires an accumulation of net foreign assets and lower resources available for consumption. Because preferences are homothetic, this implies lower consumption for both tradables and non-tradables. As non-tradable goods are produced domestically, the decline in non-tradable consumption must be associated with lower hours worked in the non-tradable sector.

Second, the level of inflation can be expressed as

$$
\frac{\dot{\pi}_0}{1 + \pi_0} = \frac{W}{W^r_0} \left( \frac{h^T_0}{h^N_0} \right)^{(1 - \alpha_T) \phi^T} \left( \frac{h^N_0}{h^T_0} \right)^{(1 - \alpha_N) \phi^N} - 1 \tag{22}
$$

This condition is an open economy version of the Phillips curve that relates employment in both sectors to inflation (22).\(^\text{10}\) For a given wage, higher employment in tradables or

---

\(^\text{10}\)To obtain (22) we use the definition of the price index (2) at dates $t = 0$ and $t = -1$ and combine with firms’ optimality (8) and (9), and the definition of the natural wage (19).
non-tradables requires higher prices in the respective sectors, as can be seen from (8) and (9). Aggregating prices yields (22)

An important implication from (22) is that the labor intensities of the sectors play a crucial role in determining the extent to which higher employment in each sector raises inflation. To see this more clearly, we can totally differentiate firms’ first-order conditions, and using that the wage is constant we obtain

\[ d \log p_i^t = \frac{1}{\alpha^i} d \log y_i^t. \]

The higher is the labor intensity in each sector, the lower is the rise in prices needed to achieve a certain increase in output. Crucial for this result is that wages are sticky. Thus, if a good is more labor intensive, this means that firms can scale up production without significant raises in prices. As the curvature in the production becomes lower, an increase in employment leads to a faster decline in the marginal product, thus necessitating a larger increase in prices to induce higher employment to be optimal for firms. Put differently, a higher labor intensity implies a lower elasticity of marginal cost. To our knowledge, this role of labor intensity in shaping the response of inflation to a monetary expansion is a channel that has not received attention in the literature.

Third, using that the initial net foreign asset position is zero, the country budget constraint for tradables (13) can be expressed as

\[ \frac{b^*_1}{R^*_0} = F^T(h^T_0, A^T_0) \tilde{z}_0, \] (23)

Finally, in addition to (21)-(23), the government is also subject to the household intertemporal Euler equation (7).

Lagrangian. We can then write the Lagrangian for the government problem as:

\[
\begin{aligned}
&u \left( (1-\tilde{z}_0)F^T(h^T_0, A^T_0), F^N(h^N_0, A^N_0) \right) - \kappa_0(h^T_0 + h^N_0) - \frac{X}{2}(\tilde{\pi}_0)^2 + \beta V_1 \left( R^*_0 F^T(h^T_0, A^T_0) \tilde{z}_0 \right) \\
&+ \theta \left[ \frac{\tilde{\pi}_0}{1+\tilde{\pi}_0} - \frac{W}{W^T} \left( \frac{h^T_0}{h^T_0} \right)^{(1-\alpha^T)\phi^T} \left( \frac{h^N_0}{h^N_0} \right)^{(1-\alpha^N)\phi^N} + 1 \right] + \eta \left[ (1-\tilde{z}_0) \frac{\alpha^N \phi^N h^T_0}{\alpha^T \phi^T h^N_0} - 1 \right] \\
&+ \mu \left[ u_T \left( (1-\tilde{z}_0)F^T(h^T_0, A^T_0), F^N(h^N_0, A^N_0) \right) - \beta R^*_0 u_T \left( C^T(R^*_0 F^T(h^T_0, A^T_0) \tilde{z}_0), C^N(R^*_0 F^T(h^T_0, A^T_0) \tilde{z}_0) \right) \right]
\end{aligned}
\]
where we have replaced (23) in the objective function and kept the remaining implementability constraints with multipliers $\vartheta, \eta$ and $\mu$.

Two important observations from this problem are worth making. First, the only foreign variable that appears is the world real rate. The reason is that although foreign monetary policies can alter the exchange rate vis-à-vis the domestic country, the domestic central bank can alter these movements by varying the nominal rate. Because the presence of the world real rate reflects an intertemporal channel, we refer to it as the “financial channel.”

Second, the trade balance not only changes resources today versus tomorrow but also affects the last two implementability constraints. Taking first-order condition with respect to $\tilde{z}_0$ we obtain:

$$\eta = \left[ \delta_0 - \phi^T + \sigma_0 \phi^T \right] u_T(c^T_0, c^N_0) \mu$$

where $\delta_0$ is given by (A.7) in Appendix A.4 and satisfies $\delta_0 > 1$.

Condition (25) implies that the Lagrange multipliers on households’ Euler equation (6) and households’ intra-temporal allocation of hours worked (21) have the same sign. The intuition is as follows. Suppose the central bank perceives a positive shadow value from raising the ratio of non-tradable employment to tradable employment (that is, $\eta > 0$). Notice that if households were to borrow more, the increase in consumption would lead to higher demand for tradables and non-tradables. But for given monetary policy, employment of tradables remains fixed.\footnote{This is because tradable employment depends only on the wage in units of tradables, and the price of tradables in the small open economy $k$ is given by $P_{j,t}^T h_{j,t}^T$.} Therefore, a higher level of borrowing would result in more hours in the non-tradable sector relative to the tradable sector. From the perspective of the government of the small open economy, this implies that a positive shadow value from higher non-tradable to tradable hours is associated with a positive shadow value from higher household borrowing.

Optimality with respect to $h_0^T$ and $h_0^N$ delivers a targeting rule for the small open economy.\footnote{See A.4 for a derivation.}

$$\sum_{i=T,N} \delta_0^i \alpha^i \phi^i (1 + \psi_z \tilde{z}_0) \chi (1 + \tau_0) \tilde{\tau}_0$$

where $\delta_0^T, \delta_0^N$ and $\psi_z$ are positive coefficients defined in (A.5), (A.6) and (A.8) and recall
that $\tau_0$ stands for the labor wedge.

Equation (26) equates the weighted average of the net marginal utility benefits from raising employment in both sectors and the marginal cost of higher prices. Relative to closed economy targeting rules, a novel consideration that emerges here is the trade balance. We next delve into the incentives for an individual central bank to manage the trade balance.

**Trade-balance-management.** When households borrow, they equate the marginal benefits of consuming today to the marginal costs of repaying tomorrow, as given by (6). However, a central bank also perceives that changes in international borrowing (and thus changes in the trade balance) affect the reallocation of hours worked across sectors, by (21), which in turn affects inflation. By combining the optimality conditions for $h^T$ and $h^N$ with (21), we obtain In particular, the perceived social benefit of the reallocation of hours worked $\eta$ across sectors is, to a first order, given by

$$\eta_0 = \frac{\phi^N \phi^T}{\sum_i \delta_i \alpha_i \phi_i} (\alpha^N - \alpha^T) \chi \hat{\tau}_0, \quad (27)$$

An important takeaway from condition (27) that the sign of $\eta$ (and thus $\mu$) depends on the difference in labor intensity across sectors $\alpha^N - \alpha^T$ and the sign of the inflation gap. When the economy has high inflation, the central bank in the small open economy would like to redistribute labor toward the more labor-intensive sector (i.e., $\eta > 0$). When a sector is more labor intensive, this means that prices respond less to a change in production in that sector. Therefore, starting from a situation with high inflation, the central bank can achieve a reduction in inflation by shifting employment towards the more labor-intensive sector, for a given monetary policy.

When the two sectors are equally labor intensive $\alpha^N = \alpha^T$, households’ borrowing choices are optimal from the perspective of the central bank, as it does not perceive any social benefit from changing the composition of hours between the tradable sector and non-tradable sector.

When $\alpha^N > \alpha^T$, if the inflation gap is positive, the central bank internalizes that a reallocation of hours away from the less labor-intensive sector (tradables) toward the most labor-intensive sector (non-tradables) would help bring down inflation and improve social welfare. Recall that by (25), the Lagrange multipliers $\mu$ and $\eta$ have the same sign. This means that in this situation, individual households under-borrow relative to the central

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$^{13}$ (27) is obtained by combining (25) with the optimality conditions for $h^T_0$ and $h^N_0$ and using (18).
bank’s desired level of borrowing. On the other hand, if the inflation gap is negative, the central bank internalizes that a reallocation of hours away from the more labor-intensive sector (non-tradables) toward the less labor-intensive sector (tradables) would help raise inflation towards the target and improve welfare. In this case, households overborrow.

When $\alpha^N < \alpha^T$, the signs of both $\eta$ and $\mu$ are reverted. In that case, if the inflation gap is positive, the economy features over-borrowing and when the inflation gap is negative, the economy displays under-borrowing.

Given that the trade balance affects inflation, the key question then is how monetary policy affects households’ borrowing choices, and thus the trade balance in the small open economy. There are three distinct forces that determine the sign of the response of the trade balance. First, there is an expenditure switching effect by which a lower nominal rate depreciates the exchange rate, raises the price of tradables by the interest parity condition, and leads households to shift expenditures towards non-tradables and to an increase in non-tradable output. Second, the higher price for tradables leads to higher tradable output. Third, there is an intertemporal substitution effect by which, given prices and income, households borrow more externally, tilting consumption towards the present. Finally, there is a general equilibrium effect by which the resulting increase in current aggregate demand increases output and translates into higher external savings.

The following lemma characterizes the effect of an infinitesimal change in the nominal interest rate on the trade balance.

**Lemma 4 (Generalized Marshall-Lerner Condition).** Starting from a zero net asset position, the response of the trade balance to a domestic monetary expansion satisfies

\[
-\frac{d\bar{z}_0}{d \log R_0} > 0 \iff \sigma_0 > \bar{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \alpha^N \phi^N}
\]

**Proof.** In Appendix A.5

The lemma generalizes existing results in the literature to a situation with multi-sector production.\(^{14}\) Whether an expansionary monetary policy expands the trade balance depends on the elasticities of substitution and labor intensities in the two sectors. If the tradable sector were an endowment, $\alpha^T = 0$, we would obtain the familiar result that the trade balance increases in response to a fall in the nominal rate (i.e., $d\bar{z}_0/dR_0 < 0$).

\(^{14}\)The classic Marshall-Lerner condition depends on static elasticities of exports and imports, but as is well-understood, in dynamic general equilibrium model, the effects depend on intertemporal considerations (see e.g., Lane, 2001; Bianchi and Coulibaly, 2021).
0) if and only if the intertemporal elasticity of substitution was lower than the intratemporal elasticity of substitution between tradables and non-tradables (which in this case is assumed to be one). In our model with endogenous production in the tradable sector, the lower interest rate expands tradable output and thus is an additional force toward a trade surplus. Therefore, to obtain a decrease in net exports in response to a lower nominal interest rate, the intertemporal elasticity of substitution must be lower. In addition, it also follows that if $\alpha^T \geq \alpha^N$, a monetary expansion increases the trade surplus for any intertemporal elasticity of substitution. Intuitively, a higher $\alpha^T$ implies that tradable output responds more to an increase in the price of tradables (for a given wage), and through consumption smoothing, this means a higher trade surplus.

**Takeaway.** To summarize, the key takeaway of this section is that by influencing the trade balance, the central bank can improve its output-inflation tradeoff when labor intensities differ between the tradable and non-tradable sectors. Moreover, the sign of the response of the trade balance to changes in monetary policy depends on elasticities of substitution and labor intensities.

### 3.2 Nash Equilibrium

In the previous section, we characterized the optimal policy for the central bank of a small open economy for an arbitrary world real rate. We can now define a Nash equilibrium as the outcome when all central banks are simultaneously maximizing the welfare of their representative household and the market for the global real asset clears. Because all countries are identical, we can restrict to symmetric Nash equilibrium.

We let $U(R_0, R^*_0)$ denote the lifetime utility of the representative household in a competitive equilibrium where the central bank sets the nominal rate to $R_0$ and the world real rate is $R^*_0$. In addition, we let $R^*(R_0)$ the equilibrium world real rate when all countries set $R_0$.

**Definition 2** (Nash Equilibrium). The nominal interest rate in the Nash equilibrium is such that

$$R_0 = \arg\max_x U(x, R^*(R_0))$$

By symmetry, in any Nash equilibrium there are no capital flows and exchange rates

---

15 When the elasticities are equal, changes in the nominal rate do not affect capital flows. The Cole-Obstfeld parameterization, which is adopted in much of the literature, is a special case.
are constant. Replacing \( \hat{z}_0 = 0 \) in the targeting rule (26), we arrive at

\[
\tau_0 = \chi \psi^{NE} \frac{1 + \pi_0}{h_0^N} \hat{\tau}_0, \quad \text{with} \quad \psi^{NE} \equiv \alpha^N \phi^N \frac{\sum_{i=T}^{T} \delta_0^i (1 - \alpha^i) \phi^i}{\sum_{i=T}^{T} \delta_0^i \alpha^i \phi^i}
\]  

(28)

This expression reveals that under optimal policy, only one of the two scenarios can emerge in the Nash equilibrium: either the economy is overheating, \( \hat{h}_0^N > 0 \) (or \( \tau_0 < 0 \)), and inflation is below target or there is a recession \( \hat{h}_0^N < 0 \) (or \( \tau_0 > 0 \)) and inflation is above target. To understand the intuition, consider the possibility that in a Nash equilibrium, there is a negative output gap in the tradable sector (and the non-tradable) and inflation is below the target. In that case, by lowering the nominal interest rate and allowing for higher prices, the central bank can narrow the output gap and inflation gap. By the same token, if there is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. From these conditions, it is also clear that if the inflation cost is zero \( \chi = 0 \), central banks can implement the first-best allocation for any shocks.

To understand what kind of shocks can lead the economy to a recession or overheating, it is useful to define the wage gap as the deviation of the market wage relative to the nominal wage that would ensure full employment \( \hat{w}_0 \equiv \frac{W_0}{W_0^n} - 1 \). It thus follows that the sign of the output gap in the Nash equilibrium is the opposite of the wage gap. Intuitively, when the market wage is above the natural wage, countries face a recession whereas when the market wage is below the natural wage, countries face overheating. That is,

\[
\hat{w}_0 \cdot \hat{h}_0^N \leq 0.
\]

4 Monetary Policy under Cooperation

We now turn to analyze the optimal monetary policy under cooperation. The key question we will tackle is whether coordination calls for tighter or looser monetary policy relative to the Nash equilibrium.

We define the optimal cooperative monetary policy as the outcome of a planner’s problem that chooses the interest rates on behalf of all countries to maximize average welfare. Because all countries are identical, this means it maximizes the welfare of any given country and the nominal interest rate and the allocations are the same for all countries.
4.1 Optimal Policy Problem

The global planning problem consists of choosing \(\{h_0^N, h_0^T, \hat{\pi}_0\}\) to maximize current utility. Relative to the problem for a small open economy (24), the planner now internalizes that in equilibrium the market for the global asset must clear implying that \(c_T = F_T(h_T^0, A_T^0)\).

We can write the associated Lagrangian as follows:

\[
\begin{align*}
&u \left( F_T(h_T^0, A_T^0), F_N(h_N^0, A_N^0) \right) - \kappa_0(h_T^0 + h_N^0) - \frac{X}{2}(\hat{\pi}_0)^2 \\
&+ \theta \left[ \frac{\hat{\pi}_0}{1+\hat{\pi}_0} - W \left( \frac{h_T^0}{h_0^T} \right)^{(1-\alpha_T)\phi_T} \left( \frac{h_N^0}{h_0^N} \right)^{(1-\alpha_N)\phi_N} + 1 \right] + \eta \left[ \frac{\alpha_N \phi_N h_T^0}{\alpha_T \phi_T} h_0^N - 1 \right] \\
&= u \left( F_T(h_T^0, A_T^0), F_N(h_N^0, A_N^0) \right) - \kappa_0(h_T^0 + h_N^0) - \frac{X}{2}(\hat{\pi}_0)^2 \\
&+ \theta \left[ \frac{\hat{\pi}_0}{1+\hat{\pi}_0} - W \left( \frac{h_T^0}{h_0^T} \right)^{(1-\alpha_T)\phi_T} \left( \frac{h_N^0}{h_0^N} \right)^{(1-\alpha_N)\phi_N} + 1 \right] + \eta \left[ \frac{\alpha_N \phi_N h_T^0}{\alpha_T \phi_T} h_0^N - 1 \right]
\end{align*}
\]

Optimality with respect to \(h_T^0\) and \(h_N^0\) implies that\(^{16}\)

\[
\tau_0 = X\psi^{GP} \frac{1+\pi_0}{h_0^N} \hat{\pi}_0, \quad \text{with} \quad \psi^{GP} = \frac{\psi^{NP}}{1+(\alpha_N - \alpha_T)(\sigma_0 - \hat{\sigma})\Delta}
\]

Equation (30) shows that independently of the shocks whether the planner puts more weight on inflation or output than individual central banks depends on the product of two sufficient statistics, the difference in labor intensities, \(\alpha_N - \alpha_T\) and the response of the trade balance to an expansionary policy, i.e. the sign of \(\sigma_0 - \hat{\sigma}\). For example, when \(\alpha_N > \alpha_T\) and \(\sigma_0 > \hat{\sigma}\), the planner puts relatively more weight on output compared to the Nash equilibrium. When the global planner increases output, it internalizes that this has effects on the world real rate and in turn, the world real rate alters the output-inflation tradeoff faced by individual countries. If \(\sigma_0 > \hat{\sigma}\), an expansionary monetary policy, raises output and also raises demand for global real assets and lowers the world real rate. If the economy is in a recession and if \(\alpha_N > \alpha_T\), this helps the world economy.

The next Proposition formalizes the comparison between the levels of employment in the Nash equilibrium and in the cooperation equilibrium.

**Proposition 1.** The output gaps in the Nash equilibrium \(\hat{h}_0^{NE}\) and in the cooperative equilibrium \(\hat{h}_0^{GP}\) have the same sign. Moreover, we have that

\[
\hat{h}_0^{GP} < \hat{h}_0^{NE} \iff \left( \alpha_N - \alpha_T \right) (\sigma_0 - \hat{\sigma})\hat{\omega}_0 > 0
\]

**Proof.** In Appendix A.9

We now turn to comparing the choice of nominal interest rates in the Nash equilibrium \(^{16}\)See Appendix A.8 for details.
and in the cooperative equilibrium. The following proposition summarizes our results.

**Corollary 1 (Sufficient statistics).** Denote by $\hat{h}_{0}^{NE}$ the output gap in the Nash equilibrium. Then, we have that

$$R_{0}^{NE} < R_{0}^{GP} \iff (\alpha^{N} - \alpha^{T})(\sigma_{0} - \bar{\sigma})\hat{h}_{0}^{NE} > 0.$$ 

**Proof.** In Appendix A.10

The result on whether the Nash equilibrium displays over-tightening ($R_{0}^{NE} > R_{0}^{GP}$) or under-tightening ($R_{0}^{NE} < R_{0}^{GP}$) can be understood by tracing the difference in the targeting rules highlighted above and the sign of the spillover effects. Namely, depending on the sign of the differences $\alpha^{N} - \alpha^{T}$, and $\sigma_{0} - \bar{\sigma}$, the planner puts more or less weight on inflation relative to the Nash equilibrium. In the case where both of these differences are positive, the planner puts more weight on output. As Proposition 1 shows, this means that if the economy faces overheating $\hat{h}_{0}^{N} > 0$, the planner keeps nominal rates higher compared to the Nash equilibrium. Conversely, if the economy faces a recession $\hat{h}_{0}^{N} < 0$, the planner keeps nominal rates lower compared to the Nash equilibrium. That is, the extent of over-tightening or under-tightening is state-dependent.

In the case where $\alpha^{N} < \alpha^{T}$, in which case $\sigma_{0} - \bar{\sigma} > 0$, the state dependence is reversed. That is, facing overheating, the planner reduces the interest rate relative to the Nash equilibrium whereas when facing a recession, the planner raises the interest rate relative to the Nash equilibrium.

The result in Proposition 1 generalizes and clarifies results in the literature. In particular, **Fornaro and Romei (2022)** considers $\alpha^{N} = 1$, $\sigma_{0} = 1$ and $\hat{h}_{0}^{N} \leq 0$. Thus countries benefit from lower world real rates and monetary policy expansions lower world real rates. This implies over-tightening.\(^{17}\) In **Bianchi and Coulibaly (2021)** $\alpha^{T} = 0$, $\chi = 0$ and countries benefit from higher world real rates as this reduces their vulnerability to a liquidity trap. If the intertemporal elasticity of substitution is lower than the intratemporal elasticity, then lower nominal rates lower real rates and this reduces welfare.

**Illustration.** Figure 2 presents a graphical illustration of the cooperative and non-cooperative equilibrium. The x-axis denotes the output gap (which recall is the same for tradables and non-tradables) and the y-axis denotes the inflation gap. The downward sloping curves

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\(^{17}\)Over-tightening also emerge in the game theoretic approach of **Canzoneri and Henderson (1991)** who postulate reduced-form relationship for output and inflation.
represent the targeting rules for the cooperative and non-cooperative equilibrium, respectively (28) and (30), which we label inflation-output tradeoff (IO). The Phillips curve is represented by the green upward sloping curve and is common to the cooperative and non-cooperative equilibrium. This curve is given by (22) (using again that \( \hat{h}^T = \hat{h}^N \)). The intersection of the two curves represents the equilibrium.

The plot considers the case of \( \alpha^N > \alpha^T \) and \( \sigma_0 > \overline{\sigma} \). Notice that the slope of the inflation-output curve for the planner is steeper and intersects with the Nash at the ideal point (0,0). The figure displays three panels depending on the sign of the wage gap: negative wage gap (panel [a]), zero wage gap (panel [b]), and negative wage gap (panel [c]). Starting from the middle, we can see that the allocations under cooperative and non-cooperative monetary policy coincide and equal the first-best allocation. That is, the intersection of the two curves, goes through the ideal point (0,0). When the wage gap is negative (panel [a]), both economies feature a recession. Because the planner puts more weight on output and less weight on inflation, the planner allows for more inflation and faces a small recession. Finally, when the wage gap is negative (panel [c]), the planner allows for a bigger inflation gap and reduces the degree of overheating.

![Figure 2: Nash equilibrium vs under cooperation for \( \alpha^N > \alpha^T \) and \( \sigma_0 > \overline{\sigma} \)](image)

Note: IO stands for Inflation-Output trade-off. IO (Nash) and IO (Planner) correspond respectively to (28) and (30). Phillips curve corresponds to (??).

### 4.2 Inspecting the Mechanism

An alternative formulation of the planner’s problem is

\[
\max_{R_0} \mathcal{U}(R_0, \mathcal{R}_0(R_0))
\]
The optimality condition for the nominal rate for the planner yields

\[
\frac{\partial U(R_0, R_0^*)}{\partial R_0} + \frac{dR_0^*}{dR_0} \frac{\partial U}{\partial R_0^*} = 0 \quad (32)
\]

In contrast to the Nash equilibrium where each country sets the nominal rate to maximize their welfare implying that \(\frac{\partial U_0}{\partial R_0} = 0\), the social planner instead realizes that changing nominal rates alters the real rate and in turn changes in the real rate affect welfare. To understand how the planner would deviate from the non-cooperative equilibrium, there are therefore two crucial considerations: how welfare changes with \(R_0^*\) and how \(R_0^*\) changes with \(R_0\). We proceed now to analyze these spillover effects.

Consider an infinitesimal change in the world real rate. We have that evaluated at the Nash equilibrium, the welfare effects are given by

\[
\left. \frac{\partial U}{\partial R_0^*} \right|_{R_0^* = R_0^{*, NE}} = -\frac{\Delta}{R_0^*} (\alpha^N - \alpha^T) \tau_0 \frac{h_N^N}{\alpha^N \phi_N} \quad (33)
\]

with \(\Delta \equiv \phi^T \phi^N [(\delta_0 - \phi^T + \sigma_0 \phi^T) \sum_i \delta_i (1 - \alpha^i) \phi^i]^{-1} > 0\).

This expression follows from an envelope condition. It shows that the first-order effects of changes in the world real rate on welfare are determined by the output gap and the differences in labor intensity. In particular, welfare goes up when interest rates rise if the sign of the product of the output gap and the difference in labor intensity \(\alpha^N - \alpha^T\) is positive.

When the output gap is zero or labor intensities are equal across sectors, changes in the world real rate have no effect on welfare. Notice that by the targeting rule (26), if the output gap is zero, the inflation gap is also zero, and so the economy is at the first-best allocation. Notice that when \(\alpha^N = \alpha^T\), a change in the world real rate has no first-order effects on welfare, regardless of the sign of the output gap. The intuition for this result is that when the two sectors have the same labor intensity, the social and private marginal benefits of borrowing are aligned. This can be seen by combining (25) and (27), which

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18 Appendix A.7 provides the derivation. See also Bianchi and Coulibaly (2021).
19 Notice that because countries are neither net borrowers nor net savers, a marginal change in the world real rate does not affect the country’s resource constraint.
20 Crucial for this result is that countries are neither net borrowers nor net savers. If countries were asymmetric, there would be winners and losers.
yields

\[ \mu_T(c^T_0, c^N_0) \mu_0 = \Delta \frac{\alpha^N - \alpha^T}{\alpha^N} \tau_0 h^N_0 \]

which implies that \( \mu_0 = 0 \) when \( \alpha^T = \alpha^N \). In this case, even though central banks may not be achieving the efficient allocation, changes in \( R^*_0 \) are irrelevant for welfare.

Consider instead the case where \( \alpha^N > \alpha^T \). Recall that if the economy is in a recession \( \hat{h}^N_0 < 0 \), it also faces positive inflation. As explained in Section 3.1, the central bank from every small open economy would like to relocate employment towards non-tradables and induce more households’ borrowing (i.e., \( \eta > 0 \) and \( \mu > 0 \)). A reduction in the real rate of the world, therefore, stimulates more borrowing, relaxes the implementability constraints (6) and (21), and improves social welfare.

If instead the economy faces overheating \( \hat{h}^N_0 > 0 \) and the inflation gap is negative, the reduction in the world real rate now reduces welfare. That is, households are borrowing too much in the competitive equilibrium, and a reduction in the world real rate exacerbates overheating.

The key takeaway is that when countries are not at the first-best allocation and labor intensities are different across sectors, changes in the world real rate have first-order effects on welfare. However, when individual countries set their monetary policy, they do not internalize the potential effects on the world real rate and how this affects welfare in other countries.

Consider now how the real rate changes with the nominal rate. Following the results from Lemma 4 and using market clearing \( \tilde{z}_0 = 0 \), we have\(^{21}\)

\[ \sigma_0 \frac{dR^*_0}{R^*_0} = (\sigma_0 - \bar{\sigma}) \frac{dR_0}{R_0}, \tag{34} \]

When \( \sigma_0 > \bar{\sigma} \), a monetary policy expansion in one country raises its trade balance. When all countries simultaneously expand their monetary policy, the real rate must fall to clear the asset market.

Putting together (33) and (34), we can now trace the sign of the second term in the planner’s optimality (32). In particular, suppose the non-tradable good is more labor intensive \( \alpha^N > \alpha^T \). Then, starting from the Nash equilibrium, a monetary policy expansion will improve welfare when \( \sigma_0 > \bar{\sigma} \) and \( \hat{h}^N_0 < 0 \) or when \( \sigma_0 < \bar{\sigma} \) and \( \hat{h}^N_0 > 0 \). In addition,\(^{21}\) Equation (34) uses (6), (7), (8), (21), with market clearing for global assets \( \tilde{z}_0 = 0 \).
a monetary policy contraction will improve welfare when $\sigma_0 > \bar{\sigma}$ and $\hat{h}_0^N > 0$ or when $\sigma_0 < \bar{\sigma}$ and $\hat{h}_0^N < 0$.

To summarize, in a Nash equilibrium, central banks use monetary policy to steer capital flows to improve their output-inflation stability tradeoff. In general equilibrium, however, capital flows are zero and the global economy ends up with a distorted inflation-output outcome. Whether the planner would choose a lower or higher nominal interest rate depends on whether individual central banks benefit from lower or higher real interest rates.

### 4.3 The Need for Cooperation

In this section, we delve into the importance of cooperation by inspecting how individual countries would unilaterally deviate from the coordinated solution.

Linearizing the equilibrium conditions for a single small open economy, we obtain the following system:

\[
\tilde{z} = a_1 \left[ - (\sigma - \bar{\sigma}) \hat{R} + \sigma \hat{R}^* \right] \tag{MP}
\]

\[
\hat{\pi} = \tilde{\omega} + \sum_{i=T,N} (1 - \alpha^i) \phi^i \hat{h}^N + a_2 \left[ (\sigma - \bar{\sigma}) \sum_{i=T,N} \alpha^i \phi^i \hat{h}^N + \hat{R}^* \right] \tag{AS}
\]

Given a nominal interest rate $\hat{R}$ and a world real rate $\hat{R}^*$, (MP), (AS) and

\[
a_2 (\sigma - \bar{\sigma}) \sum_{i=T,N} \alpha^i \phi^i \hat{h}^N = \tilde{z} - a_2 \hat{R}^* \tag{35}
\]

fully determine the endogenous outcomes $(\tilde{z}, \hat{h}^N, \hat{\pi})$.\(^{23}\) To illustrate the tradeoff faced by the central bank in the choice for $R$, we derive a second-order approximation of the objective function around the efficient allocation. This gives rise to the following welfare-based loss function:

\[
L \equiv \frac{1}{2} \left\{ \frac{\kappa \hat{h}^N}{\alpha N \phi N} \left[ 1 + (\sigma - 1) \sum_{i=T,N} \alpha^i \phi^i \right] \sum_{i=T,N} \alpha^i \phi^i \left( \hat{h}^N \right)^2 + \chi \left( \hat{\pi} \right)^2 + \left( \delta - \phi^T + \sigma \phi^T \right) \phi^T \left( \tilde{z} \right)^2 \right\} \tag{36}
\]

On the other hand, for the world general equilibrium, the aggregate supply is instead

\(^{22}\) (MP) combines linearized (6) and (7) while (AS) combines linearized (21) and (?). Moreover, we have that $a_1 \equiv \sum_{i=T,N} \alpha^i \phi^i \left[ (\delta - \alpha^T) \left( 1 + (\sigma - 1) \sum_{i=T,N} \alpha^i \phi^i \right) \right]^{-1} > 0$ and $a_2 \equiv \left[ \delta + (\sigma - 1) \left( \phi^T + \alpha^N \phi^N \right) \right]^{-1} > 0$.

\(^{23}\) (35) is obtained by linearizing the Euler equation (6) and uses (21).
given by

\[ \hat{\pi} = \hat{\omega} + \sum_{i=T,N} (1 - \alpha^i) \phi^i \hat{h}^N \]  

(AS\_W)

Because changes in output affect the world real interest rate, the aggregate supply faced by central banks is different than the one faced by the small open economy. In particular, it could be steeper or flatter depending on the sign of \( \sigma - \bar{\sigma} \). As described above, when \( \sigma - \bar{\sigma} > 0 \), the slope of the aggregate supply faced by a central bank in the small open economy is steeper than the one faced by a global planner who internalizes the effect of output on the world real interest rate.

Figure 3 presents a graphical illustration. The analysis assumes \( \sigma < \bar{\sigma} \), \( \alpha^N > \alpha^T \) and an initial rigid wage \( W \) such that \( \hat{h}^N < 0 \) and \( \hat{\pi} > 0 \).

The circled line in Panel (b) represents the indifference curve, as given by (36), where we replace \( \hat{z} \) with (35). Notice that the slope of the indifference curves change sign when the inflation gap or output gap change sign. As the indifference curves get closer to the \((0,0)\) point, the level of utility goes up.

The blue-dashed lines represent the pair of inflation and output gap that lies on the AS curve for world economy. The point \( G \) in this curve represents the point chosen by the global planner. The green-solid line represents the AS curve for an individual central bank that takes as given \( R^* \). Because this curve is flatter, this implies that the optimum as reflected by the tangency point \( E' \) features higher employment and higher inflation (and thus a lower interest rate than the one chosen under cooperation).

However, the point \( E' \) captures a situation where only an individual central bank deviates. As panel (a) shows, the lower nominal interest rate for an individual central bank gives rise to a trade deficit. When all central banks deviate by lowering the nominal interest rate, this implies that the world real rate must go up. Graphically, this means the curve MP in panel (a) shifts to the right until the point where \( \hat{z} \). In addition, once the world real rate goes up, the AS curve faced by an individual central bank shifts to up and to the left. As a result the Nash equilibrium ends up at a point \( E \) which moves further away from the ideal point.
4.4 Anticipated Shocks: A Case of Prudential Undertightening

Until now, we considered an economy that faces a sudden shock that creates an output-inflation tradeoff at $t=0$. In this section, we consider the possibility of a future shock. This extension allows us to examine a situation where central banks may be using monetary policy to affect their net foreign asset position and improve their output-inflation tradeoff in the future.

We assume that the economy is initially at period $t=-1$. The wage is rigid at a value $W$ such that $W = W_{-1}$. We assume that agents suddenly anticipate a shock to the economy at period $t=0$.

Let us start with the analysis of the non-cooperative solution. The problem the central bank faces at $t=-1$ is analogous to the one described in (24) with the difference that now the continuation value is not the one associated with the flexible wage allocation. The individual central bank at period can still achieve the efficient allocation at $t=-1$, given that the shock will hit at $t=0$. However, the central bank perceives that by changing its net foreign asset position, it will improve the output-inflation tradeoff at $t=0$ when the shock hits. In particular, the central bank wants to run a trade deficit at $t=-1$ when a higher net foreign asset position at $t=0$ would help improve domestic policy tradeoff. By Lemma 4, this may require a monetary expansion or contraction depending on the sign of $\sigma_{-1} - \bar{\sigma}$.

Under the assumption that $\alpha^N > \alpha^T$, the central bank would try to boost its net foreign asset position if the shock tomorrow leads to a recession (and reduce its net foreign asset position if the shock tomorrow leads to overheating). In turn, to the extent that $\sigma_{-1} > \bar{\sigma}$,
the central bank would cut the nominal rate if the shock tomorrow will lead to a recession
(and increase the nominal interest rate if the shock tomorrow will lead to overheating).

On the other hand, the anticipation of the shock has no effect on the optimal monetary
policy under cooperation at period \( t = -1 \). That is, the planner sets the nominal rate
to achieve the efficient allocation. Intuitively, the desire to accumulate net foreign asset
position for individual countries is a zero-sum game. When central banks depart from the
efficient allocation at \( t = -1 \), they end up worsening the allocation without any future gains.

These insights are summarized in the next proposition.

**Proposition 2.** Consider \( \hat{w}_{-1} = 0 \). Then,

1. the optimal monetary policy under cooperation features \( \hat{h}_{-1}^N = \hat{\pi}_{-1} = 0 \)

2. the Nash equilibrium features
   
   \[ \begin{align*}
   & i) \quad \hat{h}_{-1}^N > 0 \text{ and } \hat{\pi}_{-1} > 0 \text{ if } (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T)\hat{h}_0^N > 0 \\
   & ii) \quad \hat{h}_{-1}^N < 0 \text{ and } \hat{\pi}_{-1} < 0 \text{ if } (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T)\hat{h}_0^N < 0 
   \end{align*} \]

**Proof.** In Appendix A.11

A feature of our environment with anticipated shocks is that countries can now expe-
rience both overheating labor markets and high inflation. This is an interesting feature
because a common characteristic of New Keynesian models is that the central bank faces
unemployment and high inflation or overheating and low inflation.

The implications of cooperation for policy rates is summarized in the following corol-
lary.

**Corollary 2** (Prudential under-tightening). Suppose countries anticipate a recession at \( t = 0 \). Then,

\[ R_{-1}^{NE} < R_{-1}^{GP} \iff (\alpha_N - \alpha_T)(\sigma_{-1} - \bar{\sigma})\hat{h}_{-1}^{NE} > 0. \]

**Proof.** In Appendix A.12

Or sufficient statistic results therefore remain valid in the presence of anticipated shocks.
That is, the extent to which there is over or under-tightening depends on the product of
the difference in labor intensity \( \alpha^N - \alpha^T \) and the difference \( \sigma_{-1} - \bar{\sigma} \), and the sign of the
output gap.
The inefficiency of the non-cooperative outcome can be referred to as a problem of “prudential under-tightening”. That is, by attempting to increase the future net foreign asset position, with a prudential goal, central banks will conduct a monetary policy that inefficiently boosts output when there is an expected recession (and inefficiently depresses output when there is an expectation of overheating).

4.5 Quantitative Gains from Monetary Policy Coordination

We evaluate in this section the quantitative gains from monetary policy coordination. The time period is a year. We calibrate the economy using standard parameters from the literature. Households’ utility function has the constant relative risk-aversion form, \( U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \) and we set \( \sigma = 5 \).

Following Schmitt-Grohé and Uribe (2016), we set the labor intensity in the non-tradable sector to \( \alpha_N = 0.75 \), and the weight on tradable consumption in the CES function \( \phi_T = 0.26 \). The labor intensity in the tradable sector is set to ensure an aggregate labor share of \( \frac{2}{3} \), which implies \( \alpha_T = 0.43 \). Notice that given the calibrated parameters, \( \sigma_0 = \sigma_{-1} > \tilde{\sigma} \). The discount factor \( \beta \) is set to 0.96 which ensures a steady state value of 4 percent for the risk-free world real interest rate. We set the parameter governing the inflation cost \( \chi \) to 10. With this value, we obtain using a second-order approximation of the objective function, an effective weight on the output and inflation of 20% and 80%, which are in the range of those used in standard New Keynesian models. \( ^{25} \)

The experiment we consider is a labor disutility shock \( \kappa_0 \), which is anticipated at \( t = -1 \). Notice that a shock to \( \kappa \) does not affect actual allocations in the Nash equilibrium but they do affect the efficient allocation and the optimal policy under cooperation. Figure 4 plots the output gap and the inflation rate under cooperation and in the Nash equilibrium in periods \( t = -1 \) and \( t = 0 \) and the welfare gains for a range of values of \( \kappa \).

Let us discuss first the effects at \( t = 0 \) (panels [c] and [d]) If \( \kappa \) falls at \( t = 0 \), the efficient level of output increases which implies that the natural wage falls below the sticky wage and the economy faces an inefficiently low level of output given the initial monetary policy. In the Nash equilibrium, central banks respond by loosening monetary policy in order to mitigate the recession and this gives rise to inflation. Under the constellation of parameters considered, individual central banks do not lower enough interest rates relative to the

\[^{24}\text{The aggregate labor share is given by } \frac{W_t h_t^T + W_t h_t^N}{P_t y_t^T + P_t y_t^N} = \alpha^T \phi^T + \alpha^N \phi^N.\]

\[^{25}\text{The relative weights are obtained by deriving a second-order approximation of the households’ welfare. In a symmetric equilibrium, this corresponds to (36) with } \tilde{\xi}_0.\]
cooperative solution. As a result, countries face a deeper recession in the Nash equilibrium and a lower inflation rate. As shown in panels (c) and (b) of Figure 4, the difference in output gaps can reach about 2 percentage points the difference in inflation gaps can reach about 1.5 percentage points.

Let us discuss now the effects at $t = -1$. At $t = -1$, central banks seek to raise their trade surplus so as to have a higher NFA position and be in a better position to manage the recession at $t = 0$. Under the value of $\sigma$ considered, this implies that central banks cut the nominal rate at $t = -1$ in the Nash equilibrium giving rise to an overheated economy. As the figure shows, inflation can reach 2% and the output gap 6%. Meanwhile, as discussed above, under cooperation, the planner keeps policy rates unchanged and continues to stabilize the output gap and the inflation gap at $t = -1$.

Finally, we assess the welfare implications of the lack of cooperation in panel (e). The welfare gain from cooperation is calculated as the compensating consumption variation at date $t = -1$ that equalizes the welfare of a household in the Nash equilibrium and the utility of a household under the cooperative monetary policy. This panel shows a significant welfare gain from cooperation when there are moderately large shocks.

Figure 4: Cooperation versus Nash Equilibrium

Note: The parameter values are $\sigma = 5$, $\alpha^N = 0.75$, $\alpha^T = 0.43$, $\phi^T = 0.26$, $\beta = 0.96$, $\chi = 10$. 
5 Extensions

In this section, we extend our baseline model and show how our results can be generalized.

5.1 CES aggregate

In our baseline analysis, we consider a unitary elasticity of substitution between tradables and non-tradables. We now generalize the consumption of the composite to allow for a CES aggregator with elasticity $1/\gamma$.

Relative to our baseline analysis, the only difference is in the condition for the trade balance to increase in response to a monetary expansion. In particular, the Marshall-Lerner condition is such that the trade balance increases in response to a monetary expansion if and only if $\sigma_0 > \gamma \bar{\sigma}$. That is, the lower is the elasticity of substitution across goods, the lower is the elasticity of intertemporal substitution that delivers an increase in the trade balance in response to a monetary expansion. Intuitively, as the intra-temporal elasticity of substitution increases, a depreciation leads to larger expenditure switching from tradables towards non-tradables. Consuming less tradables therefore implies that more tradable output can be exported and the trade balance increases.

5.2 Imperfect Labor Mobility

We now relax the assumption of perfect labor mobility. We assume that aggregate hours worked is a composite of hours worked in the tradable sector and in the non-tradable sector according to the following CES aggregator:

$$n_t = \left[ \frac{1}{2} \left( n_t^T \right)^{1+\frac{1}{\xi}} + \frac{1}{2} \left( n_t^N \right)^{1+\frac{1}{\xi}} \right]^{\frac{\xi}{\xi+1}}, \quad (37)$$

where $\xi \geq 0$ measures the degree of labor mobility, that is how easy it is for a household to substitute hours worked in the tradable sector for hours worked in the non-tradable sector. When $\xi \to \infty$, there is perfect labor mobility and the aggregate hours worked reduce to $2n_t = n_t^T + n_t^N$ as in Section 2. For $\xi = 0$ labor is perfectly immobile across sectors.

Given that hours worked in the tradable sector and in the non-tradable sector are not perfect substitutes, wages need not be equal across the two sectors. We denote by $W^N$ and $W^T$ the prevailing sticky wages at date $t = 0$ in the tradable and the non-tradable sector,
respectively. The ratio of hours in a small open economy is given by
\[
\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N} \frac{\alpha^N \phi^N}{\alpha^T \phi^T} (1 - \tilde{z}_0),
\]
from which, it follows that, in any symmetric competitive equilibrium, the output gaps in the two sectors remain proportional. The optimal targeting rule under cooperation continues to be given by (28) and (30), but now the relative weights on the output gap satisfy
\[
\frac{\psi_{NE}}{\psi_{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \tilde{\sigma}) \Delta(\xi), \quad \text{with} \quad \Delta(\xi) > 0 \tag{38}
\]
Equation (38) shows that relative to individual central banks in the Nash equilibrium, the global planner puts more weight puts on closing the output gaps if and only if the product of \(\alpha^N - \alpha^T\) and \(\sigma_0 - \tilde{\sigma}\) is positive. In other words, our key sufficient statistic results from Proposition 1 continues to hold.

5.3 Oil shocks

Our baseline model assumes that labor is the sole factor of production. In this section, we incorporate oil as intermediate input and show how our results extend to this case.

We assume that households in each country are endowed with \(M_t\) units of oil which are used as intermediate inputs for production and can be exchanged with the rest of the world without any trade costs. The endowment \(M_t\) is potentially time-varying and thus can give rise to “oil shocks.”\(^{26}\) The production functions, now given by \(F_i(h_t^i, m_t^i, A_t^i)\), are differentiable, strictly increasing, concave, isoelastic with intensity parameters
\[
\alpha^i \equiv \frac{d \log F_i(h_t^i, m_t^i, A_t^i)}{d \log h_t^i} \quad \text{and} \quad \zeta^i \equiv \frac{d \log F_i(h_t^i, m_t^i, A_t^i)}{d \log m_t^i}.
\]
The aggregate demand for oil in the domestic country is \(m_t = m_t^T + m_t^N\). The optimal policy problem of individual central banks in the Nash equilibrium and the problem under cooperation are presented in Appendix B.2. The optimal targeting rules are still given by

\(^{26}\)Recent work by Auclert, Monnery, Rognlie and Straub (2023) focus on an energy price shock and show that from the perspective of the oil importer countries, coordinating on a tighter monetary policy is desirable to reduce import prices. These terms of trade manipulation motives are absent in our setup.
but now the relative weights on inflation are given by

$$\frac{\psi_0^{NE}}{\psi_0^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \bar{\sigma}) \Delta m, \quad \text{with} \quad \Delta m > 0.$$  

The difference in labor intensity across sectors remains as in the baseline a key sufficient statistic. On the other hand, the difference in the intensity of oil in production across sectors is irrelevant to whether central banks over- or under-tighten in the Nash equilibrium. The takeaway is that the relevant factor intensity is the one corresponding to the sticky price factor.

6 Conclusion

We presented a simple general theory of monetary policy coordination under financial integration. Instead of terms of trade externalities as in the classic approach, we emphasize a pecuniary externality operating through the global capital market. Individual countries do not internalize how their monetary policy decisions affect the world real interest rate and alter the ability of foreign central banks to stabilize output and inflation.

We identify three sufficient statistics that determine whether the Nash equilibrium exhibits over or under-tightening: the output gap, sectoral differences in labor intensity, and the response of the trade balance to changes in nominal rates. In particular, under the assumption that non-tradables are more labor intensive, we find that when the economy faces a recession, the Nash equilibrium displays over-tightening (under-tightening) relative to the cooperative outcome if a monetary expansion generates a positive (negative) response of the trade balance. Conversely, when the economy faces overheating, the Nash equilibrium displays over-tightening (under-tightening) relative to the cooperative outcome if a monetary expansion generates a negative (positive) response of the trade balance. The characterization is independent of the specific shocks driving the economy and provides general guidelines for concrete policy discussions on monetary policy coordination.
References


APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (14) and (15) and the specification of the utility function.

A.2 Proof of Lemma 2

The mix of hours coincides with the competitive equilibrium. This follows from combining the optimality conditions of firms (8) and (9), with the optimality condition of households (4). Using the functional forms for \( F(\cdot) \) and \( u(\cdot) \) in (18), we obtain

\[
\frac{h_t^N}{h_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} (1 - \bar{z}_t), \quad (A.1)
\]

Combining (A.1) with \( \bar{z}_t = 0 \) and (16), we obtain

\[
\frac{h_t^N}{\bar{h}_t^N} = \frac{h_t^T}{\bar{h}_t^T}
\]

Rearranging this equation and using \( 1 + \bar{h}_t = \frac{h_t^i}{\bar{h}_t^i} \), we arrive at \( h_t^N = \bar{h}_t^T \). □

A.3 Proof of Lemma 3

We combine (2) with (9) and (8) to get

\[
P_t = W_t \left[ \prod_{i=1}^{T,N} \left( F_h \left( h_{0,i}^i, A_{0,i}^i \right) \right)^{-\phi^i} \right]
\]

Assume flexible wage \( h_t^i = \bar{h}_t^i \) and zero inflation gap \( \frac{P_i}{\pi_{t-1}} - 1 = \bar{\pi}_t \), then we get

\[
(1 + \bar{\pi}_t) P_{t-1} = W_t^i \left[ \prod_{i=1}^{T,N} \left( F_h \left( h_{0,i}^i, A_{0,i}^i \right) \right)^{-\phi^i} \right] \quad (A.2)
\]

Rearranging (A.2) we obtain (19). □
A.4 Derivation of (26)

The first-order conditions of the central bank’s problem (24) with respect to \( h_0^N \) and \( h_0^T \) are respectively given by

\[
F_h(h_0^N, A_0^N)u_N(c_0^T, c_0^N) - \kappa_0 = \frac{\phi_h^N}{h_0^N}(1 - \alpha^N)\chi(1 + \tilde{\pi}_0)\tilde{\pi}_0 + \delta_0^T\eta \tag{A.3}
\]

\[
F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N) - \kappa_0 = \frac{\phi_h^T}{h_0^T}(1 - \alpha^T)\chi(1 + \tilde{\pi}_0)\tilde{\pi}_0 - \delta_0^N\eta \tag{A.4}
\]

where \( \delta_0, \delta_0^T \) and \( \delta_0^N \) are given by

\[
\delta_0^T \equiv 1 - \alpha^N + \alpha^N \frac{(\delta_0 - 1) + \sigma_0}{\delta_0 - \phi^T + \sigma_0\phi^T} \tag{A.5}
\]

\[
\delta_0^N \equiv 1 - \alpha^T + \alpha^T \frac{(\delta_0 - 1)(1 - \tilde{z}_0)^{-1}}{\delta_0 - \phi^T + \sigma_0\phi^T} \tag{A.6}
\]

\[
\delta_0 \equiv 1 + R_0^c \frac{1}{u_T} \frac{-du_T(C^T(b^*_1), C^N(b^*_1))}{\sum_i \phi^i} \tag{A.7}
\]

Note that \( \delta_0^T > 0 \) and \( \delta_0^N > 0 \) follows directly from \( \delta_0 > 1 \). Combining (A.3) and (A.4) and using (21), we arrive at

\[
\frac{h_0^N}{\alpha^N \phi^N} \sum_{i = T, N} \delta_0^i \phi^i \left[ F_h(h_0^N, A_0^N)u_i(c_0^T, c_0^N) - \kappa_0 \right] = \sum_{i = T, N} \delta_0^i (1 - \alpha^i) \phi^i \chi(1 + \tilde{\pi}_0)\tilde{\pi}_0. \tag{A.8}
\]

and where

\[
\psi_z \equiv \frac{\psi_1 + \psi_2}{1 - \psi_2 \tilde{z}_0} \quad \text{with} \quad \psi_1 \equiv \frac{(1 - \alpha^T)\alpha^N \phi^N}{\sum_i \delta_0^i \phi^i} \quad \text{and} \quad \psi_2 \equiv \frac{\alpha^T (1 - \alpha^N) \phi^N}{\delta_0 - \phi^T + \sigma_0\phi^T} \frac{\delta_0 (1 - \tilde{z}_0)^{-1}}{\sum_i \delta_0^i (1 - \alpha^i) \phi^i}. \tag{A.9}
\]

A.5 Proof of Lemma 4

The change in \( \tilde{z}_0 \) induced by the change in \( R_0 \) can be decomposed as

\[
\frac{db^*_1}{dR_0} = \frac{db^*_T}{dh_0^T} \frac{dh_0^T}{dR_0}. \tag{A.9}
\]

Using (6) and (21), we obtain

\[
u_T \left( (1 - \tilde{z}_0)F^T(h_0^T, A_0^T), F^N \left( \frac{\alpha^N \phi^N}{\alpha^T \phi^T} (1 - \tilde{z}_0)h_0^T, A_0^N \right) \right) = \beta R_0^* \nu_T \left( C^T(R_0^*F^T(h_0^T, A_0^T)\tilde{z}_0), C^N(R_0^*F^T(h_0^T, A_0^T)\tilde{z}_0) \right). \tag{A.10}
\]
Totally differentiating this expression with respect to $h_0^T$ and $\tilde{z}_0$, we obtain

\[ [1+(\sigma_0-1)(\phi^T+\alpha^N\phi^N)]d\tilde{z}_0 - [\alpha^T+(\sigma_0-1)(\alpha^T\phi^T+\alpha^N\phi^N)]\frac{d h_0^T}{h_0^T} = (1-\delta_0) \left[ d\tilde{z}_0 + \tilde{z}_0 \cdot \alpha^T \frac{dh_0^T}{h_0^T} \right] \]

Assuming that the economy starts with $b_1^* = 0$ by which $\tilde{z}_0 = 0$ from (23), we arrive at

\[ [\delta_0+(\sigma_0-1)(\phi^T+\alpha^N\phi^N)]d\tilde{z}_0 = [\alpha^T+(\sigma_0-1)(\alpha^T\phi^T+\alpha^N\phi^N)]\frac{d h_0^T}{h_0^T} \] (A.11)

Next, we use (8) to express (7) as $R_0 = R_0 \frac{W_1 F_0(h_0^T, A_1^T)}{F_0(h_1^T, A_1^T) w}$. We then use $\frac{\kappa_1}{F_0(h_1^T, A_1^T)} = u_T(c_1^T, c_1^N)$ and differentiate this equation to obtain

\[ \frac{d R_0}{R_0} = -(1-\alpha^T) \frac{d h_0^T}{h_0^T} + (1-\delta_0)d\tilde{z}_0 \] (A.12)

Finally, we substitute (A.11) and (A.12) into (A.9) and get

\[ \frac{-d\tilde{z}_0}{d \log R_0} = \frac{\alpha^T + (\sigma_0-1)(\alpha^T\phi^T + \alpha^N\phi^N)}{[1+(\sigma_0-1)(\phi^T + \alpha^N\phi^N)](\delta_0 - \alpha^T)} \]

The result in the lemma then follows directly from this expression.

\[ \square \]

### A.6 Proof of Lemma ??

We combine (28) and (??) to obtain that the level of employment in the Nash equilibrium, which we denote for convenience $h_{0,NE}^N$ satisfies $\mathcal{T}(h_{0,NE}^N; \tilde{w}_0) = 0$ with

\[ \mathcal{T}(h_{0}^N; \tilde{w}_0) \equiv \left[ \tau(h_0^N,0) - \tau(h_0^N,0) \right] - \chi \psi^N (1+\bar{\pi}_0)^2 (1+\tilde{w}_0) \left( \frac{h_0^N}{\bar{h}_0^N} \right)^{\sum_i (1-\alpha^i)\phi^i} \left( 1+\tilde{w}_0 \right) \left( \frac{h_0^N}{\bar{h}_0^N} \right)^{\sum_i (1-\alpha^i)\phi^i} - 1 \]

We have that

\[ \frac{d \mathcal{T}(h_{0,NE}^N; \tilde{w}_0)}{dh_{0}^N} < 0 \] (A.13)

and $\mathcal{T}(h_0^N; \tilde{w}_0) = -\chi \psi^N (1+\bar{\pi}_0)^2 (1+\tilde{w}_0)\tilde{w}_0$. Therefore $h_{0,NE}^N = \bar{h}_0^N$, that is there is no output gap in the Nash equilibrium, if and only if $\tilde{w}_0 = 0$. Furthermore, if $\tilde{w}_0 < 0$ then $\mathcal{T}(h_0^N; \tilde{w}_0) > 0$ and by (A.13) $h_{0,NE}^N > \bar{h}_0^N$, that is $h_{0,NE}^N > 0$. By the same token, if $\tilde{w}_0 > 0$ then $\mathcal{T}(h_0^N; \tilde{w}_0) < 0$ and by (A.13) $h_{0,NE}^N < \bar{h}_0^N$, that is $h_{0,NE}^N < 0$. The sign of the output gap is therefore the opposite of the sign of the wage gap in the Nash equilibrium.  \[ \square \]
A.7 Derivation of (33) and (??)

The Lagrangian associated with the central bank’s problem is as follows:

\[ V_0 = u\left( (1-\tilde{z}_0)F^T(h_0^T, A_0^T), A_0^N F^N(h_0^N) \right) - \kappa_0(h_0^T+h_0^N) - \frac{\chi}{2}(\tilde{\pi}_0)^2 + \beta V_1 \left( R_0^* A_T^T F^T(h_0^T) \tilde{z}_0 \right) \]

\[ + \theta \left[ \frac{\tilde{\pi}_0}{1+\tilde{\pi}_0} - \frac{W}{W_0} \left( \frac{h_0^T}{h_0^N} \right) (1-\alpha)^\phi \left( \frac{h_0^N}{\tilde{h}_0^N} \right) (1-\alpha^N)^\phi^N + 1 \right] + \eta \left[ (1-\tilde{z}_0) \frac{\alpha^N \phi^N h_0^T}{\alpha^T \phi^T h_0^N} - 1 \right] \]

\[ + \mu \left[ u_T \left( (1-\tilde{z}_0)F^T(h_0^T, A_0^T), A_0^N F^N(h_0^N) \right) - \beta R_0^* u_T \left( C^T(b_1^*), C^N(b_1^*) \right) \right] \]

Recalling that in the Nash equilibrium \( \tilde{z}_0 = 0 \), by the envelope theorem we have

\[
\frac{dV_0}{dR^*_0} \bigg|_{R^*_0=\tilde{R}^*_0} = \beta \tilde{z}_0 F^T(h_0^T) \frac{dV_1(b_1^*)}{db_1^*} - \left[ \frac{u_T(c_0^T, c_0^N)}{R^*_0} + \beta \tilde{z}_0 F^T(h_0^T) \frac{dV_1(b_1^*)}{db_1^*} \right] \mu \]

\[ = - \frac{1}{R^*_0} u_T(c_0^T, c_0^N) \mu \]

(A.14)

Combining (A.3) and (A.4), we get

\[ \eta = \frac{\phi^T \phi^N}{\sum_i \delta^i_0 \alpha^i} (\alpha^N - \alpha^T) \chi (1+\pi_0) \tilde{\pi}_0 \]

(A.15)

Using the targeting rule (28) to express \( \eta \) in (A.15) in terms of \( \tilde{h}_0^N \) and substituting into (25) we arrive at (??). Finally, (33) is obtained by substituting (??) into (A.14).

A.8 Derivation of (30)

Combining the first-order conditions of the global planning problem (29) with respect to \( h_0^N \) and \( h_0^T \) along with (18) we arrive at

\[ F^N(h_0^N, A_0^N) \mu_N(c_0^T, c_0^N) - \kappa_0 = \psi^GP \chi (1+\pi_0) \tilde{\pi}_0, \quad \text{with} \quad \psi^GP \equiv \frac{\alpha^N \phi^N \sum_i (1-\alpha^i) \phi^i}{\kappa_0 h_0^N} \]

(A.16)

We then take the ratio \( \psi^NE / \psi^GP \) where \( \psi^NE \) is defined in (28) to obtain the expression of \( \psi^GP \) in (30).

A.9 Proof of Proposition 1

Proceeding similarly as in Appendix (A.6), we have the sign of \( \tilde{h}_0^N \) in the cooperative solution is the opposite of \( \tilde{\omega}_0 \). By Proposition ??, the sign of the output gap is the same
in the Nash and under cooperation. Let \( h_{0,NE}^N \) and \( h_{0,GP}^N \) denote the level of non-tradable employment in the Nash equilibrium and in the cooperative equilibrium, and let us define

\[
\mathcal{T}(h_0^N; \psi_0) \equiv \left[ \tau(h_0^N, 0) - \tau(h_0^N, 0) \right] - \chi \psi_0 (1 + \overline{\pi}_0)^2 (1 + \overline{\omega}_0) \left( \frac{h_0^N}{h_0^N} \right)^{\Sigma_i(1-\alpha^i)} (1-\alpha^i) \psi^i
\]

Notice that \( \mathcal{T}(h_{0,NE}^N; \psi_0^{NE}) = 0 \) and \( \mathcal{T}(h_{0,GP}^N; \psi_0^{GP}) = 0 \). Moreover, we have that

\[
\mathcal{T}(h_{0,NE}^N, \psi_0^{GP}) = \mathcal{T}(h_{0,NE}^N, \psi_0^{NE}) + \Delta(\sigma_0 - \overline{\sigma})(\alpha^N - \alpha^T) \left[ \tau(h_0^N, 0) - \tau(h_0^N, 0) \right] \frac{\psi_0^{GP}}{\psi_0^{NE}}
\]

which implies that \( \mathcal{T}(h_{0,NE}^N, \psi_0^{GP}) < 0 \iff (\sigma_0 - \overline{\sigma})(\alpha^N - \alpha^T)h_0^N > 0 \). In addition, we have \( \frac{dT(h,\psi)}{dh} < 0 \) by \( \tau'(h, 0) < 0 \). Therefore, \( \mathcal{T}(h_{0,NE}^N, \psi_0^{GP}) < 0 \iff h_{0,GP}^N < h_{0,NE}^N \) and thus

\[
h_{0,GP}^N < h_{0,NE}^N \iff (\alpha^N - \alpha^T)(\sigma_0 - \overline{\sigma})h_{0,NE}^N > 0. \quad (A.17)
\]

### A.10 Proof of Proposition 1

Using (2) and (4) and market clearing (10) and (13), we get in any symmetric equilibrium

\[
P_1 = \frac{F(h_1^T, A_1^T)}{F_0(h_0^N, A_0^N)} \psi^N \left( \frac{h_0^N}{h_1^N} \right)^{\alpha^T \phi^N} F_h(h_1^T, A_1^T) u_T \left( F \left( \alpha^T \phi^T, h_0^N \right), F_0(h_0^N) \right)
\]

We next use \( P_1 = (1 + \overline{\pi}_1)P_0 \) since monetary policy stabilizes prices for \( t \geq 1 \) and then substitute \( (A.18), (A.10) \) and (21) with \( \overline{\omega}_0 = 0 \) into (7) to arrive at

\[
R_0 = \frac{1 + \overline{\pi}_1}{\beta k_0} \left( \begin{array}{cc}
A_0^T & A_1^T \\
A_0^N & A_1^N
\end{array} \right)^{\phi^N} \left( \begin{array}{cc}
\frac{h_0^N}{h_1^N} \\
\frac{h_0^N}{h_1^N}
\end{array} \right)^{(\alpha^T \phi^N)} \left( \frac{h_0^N}{h_1^N} \right) F_h(h_1^T, A_1^T) u_T \left( \frac{\alpha^T \phi^T}{\alpha^N \phi^N}, F_0(h_0^N) \right)
\]

Totally differentiating this equation we obtain

\[
\frac{dR_0}{dh_0^N} = -\sigma_0 \left[ \alpha^T \phi^T + \alpha^N \phi^N \right] \frac{R_0}{h_0^N} < 0 \quad (A.19)
\]

from which it follows that \( R_0^{GP} > R_0^{NE} \iff h_0^{GP} < h_0^{NE} \). Combined with \( (A.17) \) we get

\[ R_0^{GP} > R_0^{NE} \iff (\alpha^N - \alpha^T)(\sigma_0 - \overline{\sigma})h_0^{NE} > 0. \]
A.11 Proof of Proposition 2

Given that the global planning problem is static, the solution to the problem date \( t = -1 \) (the targeting rule) is given by (A.16) where variables at \( t = 0 \) are replaced with variables at \( t = -1 \). Combining this rule with the aggregate supply curve (??) we arrive at

\[
\left[ \tau(h_{\bar{N}}^{N}, 0) - \tau(h_{\bar{N}}^{N}, 0) \right] - \chi \psi^{GP} (1 + \bar{\pi}_0)^2 \left( \frac{h^{N}_{-1}}{h^{N}_{-1}} \right)^\Sigma (1-\alpha^i) \phi^i \left[ \left( \frac{h^{N}_{-1}}{h^{N}_{-1}} \right)^\Sigma (1-\alpha^i) \phi^i \right] = 0
\]

which implies that \( h_{\bar{N}}^{N} = \bar{h}_{\bar{N}}^{N} \). Next, we turn to the solution under Nash. Combine (??) and (??), and use (??) to obtain

\[
T(h_{\bar{N}}^{N}) = -\beta \Theta \frac{\Delta}{\alpha N} (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T) \left[ \tau(h_{0}^{N}, 0) - \tau(h_{0}^{N}, 0) \right] h_{0}^{N} \quad (A.20)
\]

where

\[
T(h_{\bar{N}}^{N}) \equiv \left[ \tau(h_{\bar{N}}^{N}, 0) - \tau(h_{\bar{N}}^{N}, 0) \right] - \chi \psi^{GP} (1 + \bar{\pi}_0)^2 \left( \frac{h^{N}_{-1}}{h^{N}_{-1}} \right)^\Sigma (1-\alpha^i) \phi^i \left[ \left( \frac{h^{N}_{-1}}{h^{N}_{-1}} \right)^\Sigma (1-\alpha^i) \phi^i \right] - 1
\]

The left-hand side of (A.20), that is \( T(h_{\bar{N}}^{N}) \), is decreasing in \( h_{\bar{N}}^{N} \) with \( T(\bar{h}_{\bar{N}}^{N}) = 0 \). Therefore, \( h_{\bar{N}}^{N} > \bar{h}_{\bar{N}}^{N} \) if and only if \( (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T) \bar{h}_{0}^{N} > 0 \). Moreover, for \( h_{\bar{N}}^{N} > \bar{h}_{\bar{N}}^{N} \), we have by (??) that \( \tilde{\pi}_{-1} > 0 \). Conversely when \( h_{\bar{N}}^{N} < \bar{h}_{\bar{N}}^{N} \) we have that \( \tilde{\pi}_{-1} < 0 \). \( \square \)

A.12 Proof of Corollary 2

Suppose \( \bar{h}_{0}^{N} < 0 \). Note that in cooperation solution features \( \hat{h}_{\bar{N}}^{N} = 0 \). By (A.20) the Nash equilibrium coincides with the cooperation solution if and only if \( (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T) = 0 \). Furthermore, the Nash equilibrium features under-tightening \( \bar{h}_{\bar{N}}^{N} > 0 \) if and only if \( (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T) > 0 \) or equivalently if and only if \( (\sigma_{-1} - \bar{\sigma})(\alpha^N - \alpha^T) \bar{h}_{\bar{N}}^{N} > 0 \). \( \square \)
B Proofs of Extensions

B.1 Elasticity of Substitution

This section extends the baseline model with CES aggregators. Households preferences are still described by (1) where the consumption good $c_t$ is now a composite of tradable consumption $c^T_t$ and non-tradable consumption $c^N_t$, according to a CES aggregator

$$c_t = \left[ \sum_{i \in S} \phi^i (c^i_t)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

The budget constraint of households is identical to the one in the baseline model. The household’s optimality condition with respect to $c^T_t$ and $c^N_t$ (4) is now given by

$$\frac{p^N_t}{p^T_t} = \frac{\phi^N}{\phi^T} \left( \frac{c^T_t}{c^N_t} \right)^{\gamma}$$ (A.21)

Using (A.21), we can express the share of expenditures in tradables $\tilde{\phi}^T_t \equiv P^T_t c^T_t / (P_t c_t)$ as $\tilde{\phi}^T_t = \phi^T (c^T_t / c_t)^{1-\gamma}$. and the share of expenditures in non-tradables is $\tilde{\phi}^N_t = 1 - \tilde{\phi}^T_t$. The remaining optimality conditions of the household’s problem are given by (6), (7) (and (5) for $t > 0$) while for firms, (8), (9) continue to hold. Combining (A.21) with (8) and (9) condition we arrive at

$$\frac{h^N_t}{h^T_t} = \frac{\alpha^N \tilde{\phi}^N_t}{\alpha^T \tilde{\phi}^T_t} (1 - \tilde{z}_t)$$ (A.22)

While using (14) and (15), the optimal ratio of hours in the first-best allocation becomes

$$\frac{\tilde{h}^N_t}{\tilde{h}^T_t} = \frac{\alpha^T \tilde{\phi}^T_t}{\alpha^N \tilde{\phi}^N_t}$$

which corresponds to the employment ratio in a competitive symmetric equilibrium for any monetary policy. Therefore, in any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are proportional, and to a first-order

$$\tilde{h}^N_t = \Gamma \tilde{h}^T_t, \quad \text{where } \Gamma \equiv \frac{1 - \alpha^T + \alpha^T \gamma}{1 - \alpha^N + \alpha^N \gamma} > 0.$$ (A.23)

In the lemma below, we summarize the effects of monetary policy on the trade balance.

**Lemma B.1** (Generalized Marshall-Lerner Condition). *The response of the trade balance to a domestic monetary expansion satisfies $\frac{d\varepsilon_0}{dR_0} > 0 \iff \sigma_0 > \gamma \tilde{\sigma}$ where $\tilde{\sigma} \equiv 1 - \frac{\alpha^T \tilde{\phi}^T_t + \tilde{\alpha}^N \tilde{\phi}^N_t}{\alpha^T \phi^T_t + \tilde{\alpha}^N \phi^N_t}$.\*
Proof. Proceeding similarly as in Appendix A.5 by combining (6), (7), (8), (21) we arrive at
\[
\left[ \delta_0 + (\sigma_0 - 1)(\alpha^T \phi^T + \Gamma \alpha^N \phi^N) \right] (\delta_0 - \alpha^T) d\hat{z}_0 = - \left[ \alpha^T + (\sigma_0 - \gamma)(\alpha^T \phi^T + \Gamma \alpha^N \phi^N) \right] dR_0 \tag{A.24}
\]
Thus \( \frac{d\hat{z}_0}{dR_0} > 0 \) if and only if \( \alpha^T + (\sigma_0 - \gamma)(\alpha^T \phi^T + \Gamma \alpha^N \phi^N) > 0 \). Defining \( \tilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \Gamma \alpha^N \phi^N} \), we obtain that \( \frac{d\hat{z}_0}{dR_0} > 0 \) if and only if \( \sigma_0 > \gamma \tilde{\sigma} \).

Note that, given preferences, the consumer price index \( P_t \) now satisfies
\[
P_t = \left[ \sum_{i \in S} (\phi^i) \frac{1}{\gamma} (P^i_t)^{1 - \frac{1}{\gamma}} \right]^{\gamma - 1}
\]
Thus, using the definition of the natural wage we can express the inflation gap as
\[
\hat{\pi}_0 \equiv \frac{W}{W_0} \left[ \frac{\sum_{i=T,N} (\phi^i) \frac{1}{\gamma} (F_i \theta_i, A_i)}{\sum_{i=T,N} (\phi^i) \frac{1}{\gamma} (F_i \theta_i, A_i)} \right]^{\gamma - 1} - 1 \tag{A.25}
\]
The Lagrangian associated with the central bank’s problem can be written as follows
\[
\begin{align*}
&u \left( (1 - \hat{z}_0) A_0^T F^T (h_0^T), A_0^N F^N (h_0^N) \right) - \kappa_0 (h_0^T + h_0^N) - \frac{X}{2} (\hat{\pi}_0)^2 + \beta V_1 \left( R_0^T A_0^T F^T (h_0^T) \hat{z}_0 \right) \\
&+ \theta \left[ \frac{\hat{\pi}_0}{1 + \pi_0} - \frac{W}{W_0} \left( \frac{\sum_{i=T,N} (\phi^i) \frac{1}{\gamma} (F_i \theta_i, A_i)}{\sum_{i=T,N} (\phi^i) \frac{1}{\gamma} (F_i \theta_i, A_i)} \right) \right]^{\gamma - 1} + 1 \eta \left[ (1 - \hat{z}_0) \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \frac{h_0^T}{h_0^N} - 1 \right] \\
&+ \mu \left[ u_T \left( (1 - \hat{z}_0) A_0^T F^T (h_0^T), A_0^N F^N (h_0^N) \right) - \beta R_0 u_T \left( C^T (b_1^T), C^N (b_1^N) \right) \right] \tag{A.26}
\end{align*}
\]
The optimality condition for \( \hat{z}_0 \) yields \( \eta_0 = [\delta_0 + (\sigma_0 - 1) \hat{\phi}_0^T] u_T (c_0^T, c_0^N) \mu_0 \) where \( \delta_0 \) is given by (A.7). Using this equation and combining the first-order conditions for \( h_0^T \) and \( h_0^N \), we obtain the following targeting rule in the Nash equilibrium (where \( \hat{z}_0 = 0 \):
\[
\tau (h_0^N, 0) - \tau (\hat{h}_0^N, 0) = \chi \psi_0^{NE} (1 + \pi_0) \hat{\pi}_0 \quad \text{with} \quad \psi_0^{NE} = \frac{\alpha^N \phi^N}{h_0^N} \sum_{i=T,N} \delta_0^i (1 - \alpha^i) \hat{\phi}_0^i \sum_{i=T,N} \delta_0^i \alpha^i \phi_0^i \tag{A.27}
\]
where as in (??) we have
\[
\tau (h_0^N, 0) \equiv F_h^N (h_0^N, A_0^N) u_N \left( F^T \left( \frac{\alpha^T \hat{\phi}_0^T}{\alpha^N \phi^N} h_0^N, A_0^T \right), F^N \left( h_0^N, A_0^N \right) \right) \tag{A.28}
\]
with \( \tau' (h_0^N, 0) < 0 \) and \( \tau (\hat{h}_0^N, 0) = \kappa_0 \), and where the parameters \( \delta_0^T > 0 \) and \( \delta_0^N > 0 \) are
given by

$$\delta_0^T \equiv 1 + \alpha^N(\gamma - 1) + \frac{(\sigma_0 - \gamma)\alpha^N\phi_0^N}{\delta_0 - \phi_0^T + \sigma_0\gamma^{-1}\phi_0^T}$$

$$\delta_0^N \equiv 1 + \alpha^T(\gamma - 1) - \alpha^T \frac{\gamma + (\sigma_0 - \gamma)\phi_0}{\delta_0 - \phi_0^T + \sigma_0\gamma^{-1}\phi_0^T}$$

To see why $\delta_0^T > 0$ and $\delta_0^N > 0$, notice that for $\sigma > \gamma$ this is trivial. For $\sigma < \gamma$, it can be shown that $\delta_0^T$ and $\delta_0^N$ are increasing in $\gamma$, and we have $\lim_{\gamma \to 0} \delta_0^T > 0$ and $\lim_{\gamma \to 0} \delta_0^N > 0$.

Under cooperation, the Lagrangian associated with the global planning problem can be written as follows

$$u \left( (1-\nu_0)A_0^T F^T(h_0^T), A_0^N F^N(h_0^N) \right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\tilde{\pi}_0)^2$$

$$+ \theta \left[ \frac{\tilde{\pi}_0}{1 + \pi_0} - \frac{W}{W_0} \left( \sum_{i=T,N} \left( \phi_i^{1/\gamma} \left( F_{h_0}(h_i^T, A_i^T) \right) \right) \frac{1}{\gamma} \frac{\gamma}{\gamma - 1} + 1 \right) + \eta \left[ \frac{\alpha^N \phi_0^N h_0^T}{\alpha^T \phi_0^T h_0^N} - 1 \right] \right]$$

The targeting rule, which combined the first-order condition, for $h_0^T$ and $h_0^N$ is given by

$$\tau(h_0^N, 0) - \tau(h_0^N, 0) = \chi \psi_0^{GP} (1 + \pi_0) \tilde{\pi}_0 \quad \text{with} \quad \psi_0^{GP} = \frac{\alpha^N \phi_0^N \sum_{i=T,N} \delta_i^T (1 - \alpha^T) \tilde{\phi}_0^T}{\sum_{i=T,N} \delta_i^T \alpha^T \tilde{\phi}_0^T}$$

(A.29)

with $\delta_i^T = 1 + \alpha^N(\gamma - 1)$ and $\delta_i^N = 1 + \alpha^T(\gamma - 1)$. Taking the ratio between the relative weights in the targeting rules (A.27) and (A.29), we arrive at

$$\frac{\psi_0^{NE}}{\psi_0^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \bar{\sigma}) \Delta_m, \quad \text{with} \quad \Delta_m = \frac{\tilde{\phi}_0^T \tilde{\phi}_0^N}{\phi_0^T \phi_0^N} \sum_{i} \delta_i^T (1 - \alpha^T) \tilde{\phi}_0^T > 0$$

### B.2 Oil Shock

This section extends the model to incorporate oil as an intermediate input. We assume that households receive an endowment of oil which is used by firms as inputs for production and can be exchanged with the rest of the world. The law of one price is assumed to hold in the market for oil, that is $P_{mt} = e_l P_{mt}^*$ where $P_{mt}$ and $P_{mt}^*$ are the domestic and the world price of oil, respectively. Combined with the law of one price for tradables, this implies that $P_i^m = \frac{P_{mt}^*}{p_i^T}$. We can thus express the budget constraint of the household as

$$p_{i}^{T} c_{i}^{T} + p_{i}^{N} e_{i}^{N} + \frac{b_{i}^{T} + 1}{R_i} + \frac{p_{i}^{T} b_{i}^{T}}{R_i} = w_i (n_i^T + n_i^N) + \varphi_i + P_{mt} m_s + P_{mt}(M_t - m_i^S) + b_i + P_{i}^{T} b_{i}^*$$

47
where $m_t^S$ is the domestic supply of oil and $M_t - m_t^S$ is the net export of oil. The production functions are given by $F^i(h^i_t, m^i_t, A^i_t)$ with $\alpha^i$, and $\zeta^i$ denoting the intensity of labor and oil respectively. At the optimum, the demand for labor is analogous to (8)-(9) and given by $P_t^i F^i(h^i_t, m^i_t,A^i_t) = W_t$ for all $i \in S$; while their demand for oil is given by

$$m_t^T = \frac{\zeta^T}{\alpha^T P_{mt}} h_t^T, \quad m_t^N = \frac{\zeta^N}{\alpha^N P_{mt}} h_t^N$$  \hfill (A.30)

The Lemma below describes the allocation of oil in any symmetric competitive equilibrium.

**Lemma B.2.** In any symmetric competitive equilibrium, the allocation of intermediate oil inputs is efficient and given by

$$m_t^N = \frac{\zeta^N \phi^N}{\sum_{i=T,N} \alpha^i_m \phi^i M_t}, \quad m_t^T = \frac{\zeta^T \phi^T}{\sum_{i=T,N} \alpha^i_m \phi^i M_t}$$ \hfill (A.31)

**Proof.** The proof combines the ratio of the two equations in (A.30) with (21), together with $\tilde{z}_0$ and market clearing for oil $m_t^T + m_t^N = M_t$. \qed

Denoting by $\bar{m}_0^T$ and $\bar{m}_0^N$ the allocation in (A.31), the Lagrangian associated with the global planning problem can be expressed as

$$u \left( F^T(h^T_0, m^T_0, A^T_0), F^N(h^N_0, m^N_0, A^N_0) \right) - \kappa_0 (h^N_0 + h^N_0) - \frac{\tilde{\chi}}{2} (\tilde{\pi}_0)^2$$

$$+ \vartheta \left[ \frac{\tilde{\pi}_0}{1 + \tilde{\pi}_0} - \frac{W}{W_0} \left( \frac{F^T(h^T_0, m^T_0, A^T_0)}{F^T(h^T_0, m^T_0, A^T_0)} \right) \phi^T \left( \frac{F^N(h^N_0, m^N_0, A^N_0)}{F^N(h^N_0, m^N_0, A^N_0)} \right) \phi^N + 1 \right] + \eta \left[ \frac{\phi^N h^T_0}{\phi^T h^N_0} - 1 \right]$$

Notice that the allocation of oil is independent of policy. The targeting rule under cooperation, which combines the optimality condition for $h^T_0$ and $h^N_0$, is therefore identical to (30) and given by

$$\tau(h^N_0, \bar{m}^N_0, 0) - \tau(h^N_0, \bar{m}^N_0, 0) = \chi \psi^G_0 (1 + \pi_0) \tilde{\pi}_0 \quad \text{with} \quad \psi^G_0 = \frac{\alpha^N \phi^N}{h^N_0} \sum_i (1 - \alpha^i) \phi^i \hfill (A.32)$$

where

$$\tau(h^N_0, \bar{m}^N_0, 0) \equiv F^N(h^N_0, m^N_0, A^N_0) u_N \left( F^T \left( \frac{\alpha^T \phi^T}{\alpha^N \phi^N} h^N_0, M_0 - \bar{m}^N_0, A^T_0 \right), F^N(h^N_0, m^N_0, A^N_0) \right)$$

We now turn to deriving the targeting rule in the Nash equilibrium. Combining (A.30) with (21), the allocation of oil input across sectors in a small open economy is given by

$$m_t^T(\bar{z}_i) = \frac{\zeta^T \phi^T (1 - \bar{z}_i)}{\zeta^N \phi^N + \zeta^T \phi^T (1 - \bar{z}_i)} m_t^S \quad \text{and} \quad m_t^N(\bar{z}_i) = \frac{\zeta^N \phi^N}{\zeta^N \phi^N + \zeta^T \phi^T (1 - \bar{z}_i)} m_t^S \hfill (A.33)$$
which can be used to express the Lagrangian associated with the central bank’s problem as

\[
\begin{align*}
&u \left( (1-\bar{z}_0)F^T(h_0^T, m^T(\bar{z}_0), A_0^T), F^N(h_0^N, m^N(\bar{z}_0), A_0^N) \right) \\
&- \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2} (\pi_0)^2 + \beta V_1 \left( R_0^T F(h_0^T) \bar{z}_0 \right) + \eta \left[ (1-\bar{z}_0) \frac{\alpha^N \phi^N h_0^T}{\alpha^T \phi^T h_0^N} - 1 \right] \\
&+ \vartheta_0 \left[ \frac{\hat{\pi}_0}{1+\pi_0} - \frac{W}{W_0} \left( F_h (h_0^T, m^T, A_0^T) \right) \frac{\phi^T}{1} \left( F_h (h_0^N, m^N, A_0^N) \right) + 1 \right] \\
&+ \mu \left[ u_T \left( (1-\bar{z}_0)F(h_0^T, m^T(\bar{z}_0), A_0^T), F^N(h_0^N, m^N(\bar{z}_0), A_0^N) \right) - \beta R_0 u_T \left( C^T(b_1^T), C^N(b_1^N) \right) \right]
\end{align*}
\]

The optimality condition with respect to $\bar{z}_0$ is given by

\[
\eta = \left[ \delta^m_0 + (\sigma_0 - 1) \phi^T \right] u_T(c_0^T, c_0^N) \mu \tag{A.34}
\]

where on the competitive equilibrium path, $\delta^m_0$ is given by

\[
\delta^m_0 = \delta_0 + \frac{\bar{c}^T \phi^T \cdot \bar{c}^N \phi^N}{\bar{c}^T \bar{c}^T + \bar{c}^N \phi^N} + \chi \left( \frac{\phi^T F_h^T F_h^m}{F_h^T} + \phi^N F_h^m \frac{F_h^N}{F_h^N} \right) (1 + \pi_0) \bar{\pi}_0 \tag{A.35}
\]

Notice by (A.33) and (A.31) that in the Nash equilibrium where $\bar{z}_0 = 0$, the allocation of oil is optimal. Moreover, the optimality condition for $h_0^N$ and $h_0^T$ are akin to (A.3) and (A.4) where $\delta_0$ is replaced with $\delta^m_0$. As a result, the targeting rule in the Nash equilibrium is

\[
\tau(h_0^N, \bar{m}_0^N, 0) - \tau(\bar{h}_0^N, \bar{m}_0^N, 0) = \chi \psi_{\text{GP}}^N (1 + \pi_0) \bar{\pi}_0 \quad \text{with} \quad \psi_{\text{NE}}^N = \frac{\alpha^N \phi^N \sum_i \delta^l_i (1-\alpha^i) \phi^i}{\bar{h}_0^N} \sum_i \delta^l_i (1-\alpha^i) \phi^i \tag{A.36}
\]

where $\delta_0^T$ and $\delta_0^N$ satisfy (A.5) and (A.6) where $\delta_0$ is replaced with $\delta^m_0$. Taking the ratio of the relative weights on inflation in (A.32) and (A.36), we arrive at

\[
\frac{\psi_{\text{NE}}^N}{\psi_{\text{GP}}^N} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \bar{\sigma}) \Delta_m, \quad \text{with} \quad \Delta_m \equiv \frac{\phi^T \phi^N \left( \delta^m_0 - \phi^T \sigma_0 \phi^T \right) \sum_i \delta^l_i (1-\alpha^i) \phi^i}{(\delta^m_0 - \phi^T \sigma_0 \phi^T) \sum_i \delta^l_i (1-\alpha^i) \phi^i} > 0.
\]