Sustainable Exchange Rates:
Currency Pegs and the Central Bank's Balance Sheet

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Abstract

Central Banks of small integrated economies often have an exchange rate objective. We argue that achieving this objective is challenging when the Central Bank is also concerned about the size of its balance sheet. In addition, a world economy operating at the zero lower bound magnifies these difficulties. Specifically, in this paper we developed a framework where (i) the fear of future losses can rationalize why a Central Bank is worried about expanding the size of its balance sheet; (ii) this fear will impose limits on the exchange rate policies that a Central Bank will choose to implement; (iii) news about the future foreign interest rates can have significant effects on the domestic policy in a ZLB environment; (iv) commitment to future policies improves the Central Bank ability to sustain its exchange rate objectives.

Keywords: Currency crises, Exchange rates, Zero Lower Bound
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1 Introduction

In January 2015, in the face of sustained capital inflows, the Swiss National Bank (henceforth SNB) decided to abandon the floor for the Swiss Franc against the Euro, a decision which led to a sudden 20% appreciation of the Swiss Franc. Following Cochrane (2015) we name such an event a "Reverse Speculative Attack".[1] This decision by the SNB had a significant effect on financial markets, which seemed to have been surprised by the move. An article on the January 2015 edition of the Economist Magazine suggests that "The doffing of the cap surprised and upset the foreign exchange markets, hobbling several currency brokers," , while Brunnermeier and James (2015) state that "The risks created by the SNB's decision – as transmitted through the financial system – have a fat tail."

The decision by SNB is also surprising when seen from the lenses of standard speculative attack models. It is well known that a Central Bank may be forced to abandon a peg when its foreign currency reserves get depleted, and it no longer has the ability of preventing its currency from depreciating. That is, maintaining the peg eventually becomes unfeasible.[2] However, the case of Switzerland in January 2015 does not fit this narrative. In principle, it could have been feasible for SNB to increase its domestic liabilities (i.e., currency) while acquiring the foreign currency assets necessary to maintain the peg. The SNB decided to do otherwise.

In this paper, we develop a general framework for analyzing the sustainability of exchange rate objectives when a monetary authority is concerned about losses in its balance sheet. The key idea is that maintaining an exchange rate objective might lead to large undesirable fluctuations in the central bank's balance sheet. When reserves are low, there is a risk that they can be depleted, and the central bank might need to depreciate its currency more than it desires (as in the classic speculative attack case). There is also risk when foreign reserves are large. Because of the mismatch between the currency of denomination of the central bank's assets (foreign currency) and its liabilities (domestic currency), a growing balance sheet may constraint *future* exchange rate policies of the central bank because an appreciation would lead to unsustainable capital losses. The central bank may therefore decide to deviate from its preferred exchange rate *today*, and appreciate the currency, because this facilitates its

[1] Switzerland is not the only example of this. In May 1971, the Bundesbank decided to abandon the peg against the U.S. dollar, which also led to an appreciation of the German currency (see Brunnermeier and James, 2015).

[2] There exists a very large literature on standard speculative attacks, i.e. when a central bank abandons a peg, and lets its currency depreciate, as its foreign reserves are drained. See, among others, the seminal papers by Krugman (1979) and Flood and Garber (1984), or the very recent survey by Lorenzoni (2014). However, to our knowledge, there is much less analysis on reverse speculative attacks, which seemed to be quite different in nature. A notable exception is Grilli (1986), which we discuss below.

future exchange rate plans. The goal of the present paper is to apply a simplified version of that environment to the Swiss case, in order to better understand the timing of the peg's abandonment, and how changes in fundamentals, such as international interest rates, affected its likelihood.

We also show that the likelihood of attacks if higher when the economy operates close to or at the lower bound on interest.[3]. To understand this result recall that a reverse attack is a situation in which, because of future expected appreciation, the domestic currency is attractive relative to the foreign. If the domestic rate is far from the bound the central bank can make its currency less attractive by lowering its rate. But when the domestic rate is close to its bound, this is no longer a possibility for the central bank, and attacks can no longer be defended against, i.e. the only possibility of making the currency less attractive is appreciate instantly so future appreciation is reduced.

After reviewing some basic data about the Swiss experience we start by developing a simple theory of a central bank's objective. We assume that the central bank would like, for reason we do not model, to maintain a peg with a foreign currency. Were the central bank not to intervene, the currency would appreciate, as we assume, consistently with the Swiss situation after the Great Recession, that the central bank faces an increasing demand for domestic currency; thus maintaining the peg involves expanding its reserve holdings and its liabilities. We then make two key assumptions for our results. First we assume that reserves are risky, in the sense that they are subject to future loss in value (relative to the monetary liabilities issued) that the central bank cannot control. The second is that the central bank wants to keep its losses below a threshold value. Our first result is that the fear of future losses leads the central bank to an early abandonment of the peg and a currency revaluation. The idea is that be letting the currency appreciate the central bank realizes some losses today, but in doing so it reduces future appreciations and thus larger future losses.

The paper is organized as follows. In section 2 we present some data that characterize the Swiss experience with the peg to the Euro, section 3 presents the model, section 4 contains our main results, and section 5 concludes.

2 Evidence on the Swiss experience

In this section we briefly provide some evidence on the experience of the Swiss National Bank with its peg and subsequent abandonment, as these events are the main motivation of our work. In September 2011 the SNB, mentioning overvaluation of the Swiss franc and its

[3]See the contributions of McCallum (2000) and Svensson (2003) for an analysis of the zero lower bound in open economies

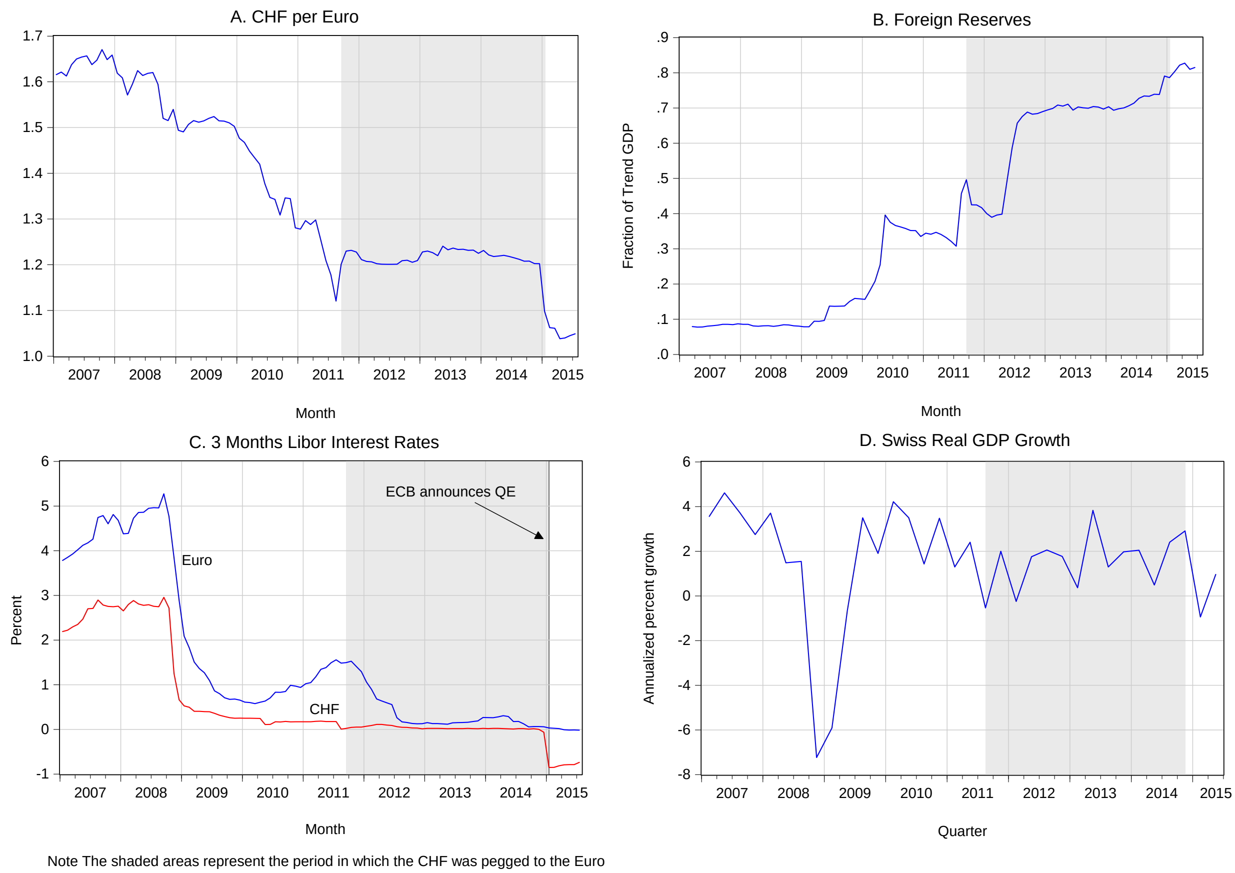

Figure 1: The Swiss experience: before, during and after the peg

negative effect on the Swiss economy announced a peg with the Euro, stating that:

> "With immediate effect, it will no longer tolerate a EUR/CHF exchange rate below the minimum rate of CHF 1.20.The SNB will enforce this minimum rate with the utmost determination and is prepared to buy foreign currency in unlimited quantities."

In January 2015 the SNB abandoned the peg, which resulted in a substantial devaluation of the Euro with respect to the CHF. Panel A of Figure 1 shows the path of the CHF/Euro exchange rate in the years preceding the peg, during the peg (the shaded area) and after the abandonment of the peg. Panel B shows instead the amount of foreign currency reserves held by the SNB (expressed as a fraction of trend GDP).[4]

[4]We normalized reserves by trend GDP (as opposed to actual GDP) in order to isolate the fluctuations in reserve holdings. We computed a linear trend using GDP data from 2007Q1 to 2015Q2.

Notice how in the first part of the sample (pre-peg) the CHF has appreciated quite substantially relative to the Euro, and the SNB has at the same time accumulated foreign reserves. During the peg the CHF has remained stable, while the SNB has continued accumulating reserves at a rapid pace. Panel C plots 3 months LIBOR rates on Swiss Franc and on Euros. Notice how, throughout the whole period, the CHF interest rate has been below the Euro rate, suggesting that, even during the peg, there were expectations that the CHF was going to appreciate gainst the Euro, i.e. that the peg was not going to last.[5] Notice also that the abandonment of the Swiss peg coincides with the announcement by the ECB of the quantitative easing program. Later we will argue that these changes in Euro interest rate policy might be very important to understand the abandonment of the peg.

Finally panel D provides some evidence on the background macroeconomic conditions in which the SNB has been operating. Notice that the peg has been introduced at a time when real GDP growth was slowing down markedly, while the peg has been abandoned at a time in which Swiss growth was mildly accelerating.

3 The model

Let us consider a world composed of a small open economy, which uses a local currency (Swiss Francs) and a large trading partner, which has a different currency (Euros). We will denote by s_t the state of the economy at time t, and with s^t the history of states up to time t, i.e. $s^t = \{s_0, s_1, ..., s_t\}$. We let state to be such that $s \in S$ in a finite set S, and assume that it follows Markov chain, with a transition probability given by the function $\pi(s'|s)$.

3.1 The Central Bank

The key agent of our economy is the domestic central bank. The central bank is the monopolist supplier of domestic currency, which can hold foreign currency reserves and make transfers to the central government. We denote by $m(s^t)$ the supply of domestic currency issued by the central bank in state s^t and by $f(s^t), \tau(s^t)$ foreign currency (Euros) reserves held and transfers to the central government made in state s^t.

The budget constraint (denominated in local currency) of the central bank is then given by

$$e(s^t)\left(f(s^t) - f(s^{t-1})\right) = f(s^{t-1})i^{\star}(s^{t-1})e(s^t) + m(s^t) - m(s^{t-1}) - \tau(s^t) \tag{1}$$

where $e(s^t)$ denotes the nominal exchange rate of the economy i.e. the amount of local

[5] Jermann (2015) which uses option prices to back out probabilities of abandonment and found that over the duration of the peg probability of abandonment averaged 20%

currency necessary to acquire 1 Euro, and $i^\star(s^{t-1}) \geq 0$ represents the foreign interest rate earned on reserves accumulated by the central bank in the previous period. This equation just states that the accumulation of foreign reserves, $e(s_t)[f(s_t) - f(s_{t-1})]$, is given by the income on accumulated foreign reserves $f(s^{t-1})i^\star(s^{t-1})e(s^t)$, plus the increase in money liabilities $m(s^t) - m(s^{t-1})$ minus the transfer to the central government $\tau(s^t)$.

We define the profits (or losses if negative) of the central bank $g(s^t)$, as the sum of the earned interest income on foreign reserves $f(s^{t-1})e(s^t)i^\star(s^{t-1})$ plus the changes in valuation in foreign reserves $f(s^{t-1})(e(s^t) - e(s^{t-1}))$

$$\pi(s^t) \equiv \left((1 + i^\star(s^{t-1}))e(s^t) - e(s^{t-1})\right) f(s^{t-1}). \tag{2}$$

Note that when the local currency appreciates, i.e. $e(s^t)$ falls below $E(s^{t-1})$, the central bank suffers a reduction in profits due to the fact that its existing reserves lose value. Equations (1) and (2) are an accounting relation and a definition, which lead us to a key restriction on central bank actions:

Assumption 1. *The transfer policy is such that*

$$\tau(s^t) = \pi(s^t) \tag{3}$$

In words, we assume that when the central bank makes positive profits, all of its profits are rebated to the treasury. If the bank makes losses then, the treasury recapitalizes the central bank. We postpone the discussion of this assumption until later, as we will impose additional limits on the ability of the Treasury to recapitalize the Central Bank.

Note that substituting equation (3) into (1) and defining net worth of the central bank $nw(s^t)$ as the difference between its assets $e(s^t)f(s^t)$ and its liabilities $m(s^t)$ yields

$$nw(s^t) \equiv e(s^t)f(s^t) - m(s^t) = e(s^{t-1})f(s^{t-1}) - m(s^{t-1}) \equiv nw(s^{t-1}) \tag{4}$$

showing that under the assumed transfer policy the net worth of the central bank is constant.

3.2 Money demand, exchange rates and interest rates

Another key ingredient of the model is the real (i.e. denominated in Euro) demand for the domestic currency, which we indicate as

$$L(i(s^t), s^t)$$

where $i(s^t)$ is the small open economy's nominal interest rate between period t and period $t+1$.

Equilibrium in the money market requires that

$$m(s^t) = e(s^t)l(i(s^t), s^t)$$

together with the restriction that local nominal interest rates cannot be below a fixed lower bound, which we set to zero:[6]

$$i(s^t) \geq 0$$

We assume that the trading partner is sufficiently large, that the following uncovered interest parity condition connects exchange rates and interest rates across the two countries:

$$1 + i(s^t) = \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)(1 + i^*(s^t))\frac{e(s^{t+1}, s^t)}{e(s^t)}$$

3.3 Central Bank Preferences and Constraints

We now move on to specify the Central Bank preferences, as well as a key additional constraint.

First, we assume that the Central Bank have preferences regarding the level for the exchange rate:

Assumption 2. *The Central Bank evaluates allocations according to the following objective function:*

$$v(s_0) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) u\big(e(s^t) - \bar{e}(s_t)\big) \tag{5}$$

for some strictly positive function $\bar{e}(.)$, $\beta \in (0, 1)$, *and a function* u *which is strictly concave and differentiable with a maximum at* 0.

Note that $\bar{e}(s)$ represents the Central Bank's preferred *nominal* exchange rate if the current state is s.

Second, we impose the following constraint on the Central Bank's actions:

Assumption 3. *The Central Bank cannot make losses bigger than* $\bar{g}$*:*

$$g(s^t) \geq -\bar{g}, \tag{6}$$

[6] In reality, Central Banks around the world have been able to reduce interest rates below zero. We could easily modify the model to allow for this, but for the sake of clarity maintain the zero lower bound as the relevant constraint.

for some $\bar{g} \geq 0$ *and for all* s^t *such that* $\pi(s^t) > 0$.

This assumption implies that the central bank is not allowed to have losses that exceed a fixed limit $\bar{g}$. The justification for this assumption is as follows. Since central banks are not for profit institutions, it seems reasonable to assume that when they make positive profits those are rebated to the treasury. When profits become losses, then they can significantly impact the net worth of the central bank, and that might make it impossible for the central bank to buy back (part of) its cash liabilities, and thus to conduct monetary policy. In this case the treasury would need to recapitalize the central bank; we assume that a recapitalization that is too large would not be politically feasible, hence we impose constraint (6), which limits the transfer the central bank can receive from the Treasury.

We can rewrite (6) as:

$$\big((1+i^\star(s^{t-1}))e(s^t) - e(s^{t-1})\big) f(s^{t-1}) \geq -\,\bar{g}, \text{ for all } s^t \text{ s.t. } \pi(s^t|s^{t-1}) > 0 \tag{7}$$

Now, recalling that $e(s^t)f(s^t) = n_0 + m(s^t)$ and letting $a(s^t) \equiv (1+i^\star(s^t))f(s^t)$, we can write the loss constraint (7) as

$$\big(a(s^{t-1})e(s^t) - m(s^{t-1}) - n_0\big) \geq -\,\bar{g}, \text{ for all } s^t \text{ s.t. } \pi(s^t|s^{t-1}) > 0 \tag{8}$$

We will restrict attention to Markov equilibria, and to do so, we first need to clarify the state space. First, we will assume that the exogenous state, s^t, follows a first-order Markov process. That is, $\pi(s^t|s^{t-1}) = \pi(s_t|s_{t-1})$. The Central Bank, at a state s^t, inherits a balance sheet position from the past given $m(s^{t-1})$ and $a(s^{t-1})$. Its choice of the exchange rate today determines the level of the profits that the bank realizes this period (according to the left-hand side of equation (8). It follows that the state of the economy is then the triplet $\{s, a, m\}$. We define a Markov equilibrium as follows:

Definition 1. *A* Markov equilibrium *is a set of functions for the exchange rate, E, the domestic interest rate, r, the money supply, M, the level of reserves plus interest, A, and the value to the Central Bank, V; as functions of the state $\{s, a, m\}$ such that for all s, a, m:*

- *the money market clears:* $M(s,a,m) = L(r(s,a,m), s)$*;*
- *the net-worth of the central bank is constant:*

$$E(s,a,m)\frac{A(s,a,m)}{1+i^\star(s)} - M(s,a,m) = n_0;$$

- *the uncovered interest parity condition and the zero-lower bound constraint hold :*

$$1 + r(s,a,m) = (1 + i^\star(s)) \sum_{s'} \pi(s'|s) E(s', A(s,a,m), M(s,a,m))/E(s,a,m),$$
$$r(s,a,m) \geq 0;$$

- *and the Central Bank makes decisions about its balance sheet and the exchange rate to maximize its objective subject to the loss constraint, the zero lower bound and the UIP; that is V solves:*

$$V(s,a,m) = \max_{e \geq 0, a' \geq 0, i \geq 0, m'} \left\{ u(e - \bar{e}(s)) + \beta \sum_{s'} \pi(s'|s) V(s', a', m') \right\} \tag{9}$$

subject to:

$$\begin{aligned} &1 + i = (1 + i^\star(s)) \sum_{s'} \pi(s'|s) E(s', a', m')/e \\ &e \frac{a'}{1 + i^\star(s)} - m' = n_0 \\ &ea - m - n_0 + \underline{g} \geq 0 \\ &m' = l(i,s) \end{aligned}$$

where the policies e, a', m' and i correspond to the equilibrium functions E, A, M and r.

We have the following (weak) monotonicity result:

Lemma 1. *In any equilibrium, the value function $V(s,a,m)$ is weakly increasing in the assets, a, and weakly decreasing in the liabilities, m.*

In any Markov equilibrium consider the following relaxed Central Bank problem (one without the loss constraint):

$$\hat{V}(s) \equiv \max_{e \geq 0, a' \geq 0, i \geq 0, m'} \left\{ u(e - \bar{e}(s)) + \beta \sum_{s'} \pi(s'|s) V(s', a', m') \right\} \tag{10}$$

subject to:

$$\begin{aligned} &1 + i = (1 + i^\star(s)) \sum_{s'} \pi(s'|s) E(s', a', m')/e \\ &e \frac{a'}{1 + i^\star(s)} - m' = n_0 \\ &m' = l(i,s) \end{aligned}$$

and let $\hat{E}(s)$ denote the associated argmax with respect to e. Note that in the above problem, the Central Bank is using the equilibrium value function in the future to evaluate its continuation payoff. Note also that without the loss constraint, the above problem is not any longer a affected by the balance sheet position of the Central Bank, and thus the solution and its policy are just functions of the exogenous state s. We have then the following result:

Lemma 2. *In any Markov equilibrium,*

- *for any (s,a,m) such that $a\hat{E}(s)-m-n_0+\underline{g}\geq 0$, the value function $V(s,a,m)=\hat{V}(s)$ and $E(s,a,m)=\hat{E}(s)$;*
- *for any (s,a,m) such that $a\hat{E}(s)-m-n_0+\underline{g}<0$, the value function $V(s,a,m)<\hat{V}(s)$ and is strictly increasing in a and strictly decreasing in m. And the equilibrium is such that $E(s,a,m)=(m+n_0-\underline{g})/a>\hat{E}(s)$.*

4 A Numerical Analysis

For now, we abstract from the ZLB and study the following special case

- Absorbing Appreciation Risk $A(s)$: From each s, economy moves, with fixed prob. λ, to s' s.t. $A(s'')=1$ for all $s''\succeq s'$ and

$$\begin{aligned} u(E(s)-\bar{E}(s),s) &= \begin{cases} -(E-1)^2 & \text{if } A(s)=0 \\ -\xi(E-\bar{E})^2 & \text{if } A(s)=1 \end{cases} \\ \text{with } 0 &< \bar{E}<1, \xi>0 \text{ arbit. large} \end{aligned}$$

- In normal times CB likes to peg at 1. If appreciation risk hits tomorrow, CB forced to appreciate at $\bar{E}$ (otherwise infinite losses). This implies CB today will never leave the future CB an exchange rate/balance sheet that makes it impossible to appreciate, i.e. violates loss constraint under appreciation.

In order to characterize the dynamics surrounding the reverse speculative attack, we first impose more structure on the states of the economy and their evolution, as well as specify numerical values for the parameters of the model. We then numerically solve for Markov equilibria and finally characterize the patterns of key variables along the Markov equilibria described above. We would like to stress that, given the highly stylized model we are using, the goal of this exercise is just to provide the reader with some simple qualitative and quantitative insights on reverse speculative attacks; we will surely not provide a comprehensive quantitative evaluation on the issue.

4.1 States of the economy

As we discussed previously, our economy is going to be subject to three exogenous disturbances.

For the exchange rate shock, we assume that the economy starts initially at $A_0 = 0$, and we let λ denote the probability that $A_t = 1$ next period if $A_t = 0$ today. The state $A_t = 1$ is assumed to be absorbing.

We assume that the level of money demand (B_t in equation **??**) obeys the following process. At any t,

$$B_t = e^{g \times b_t} \tag{11}$$

where $b_t \in \{0, 1, ..., N\}$ and represents possible shocks to money demand. The parameter $g > 0$ is a fixed and determines by how much money demand increases when a money demand shock hits. We assume that the state $b_t = N$ is absorbing, i.e. that money demand shocks are bounded, and that once money demand shock reaches its maximum level, it will stay there. For all $b_t < N$, b_{t+1} will stay constant with probability $1 - \gamma > 0$ or increase by 1 with probability $\gamma > 0$. In words, γ represents the probability that the economy is hit by a shock that increases money demand by g. This probability is assumed to be independent from other events in the economy.

The third and final source of uncertainty in the economy regards the foreign interest rates. Our modeling of the foreign interest rates is loosely motivated by panel C in figure 1, where we observe that, during the period of the Swiss peg, Euro interest rates fell initially (in late 2011), and did not move much subsequently. As a consequence, we assume that the foreign interest rate can take two possible states: high (i_h) or low (i_l), with $i_h > i_l$. The probability of transiting from the high to the low interest rate state is denoted by θ_{hl}; and from the low to the high, θ_{lh}. As with the previous shocks, we assume that these transition probabilities are independent from the realization of the other shocks.

To sum-up, figure 2 shows possible paths for the three sources of uncertainty. The value $\hat{T}$ on the x axis represents the time in which the economy switches from $A_t = 0$ to $A_t = 1$. After $\hat{T}$ our model economy is not interesting, as by assumption exchange rate will be constant at $\bar{E} < 1$. Before $\hat{T}$ the economy faces a period of stochastically increasing demand for its own currency (due for example to global increased risk aversion, or fears of inflation in the trading partner) represented by the line labelled b_t and/or stochastic international rates,

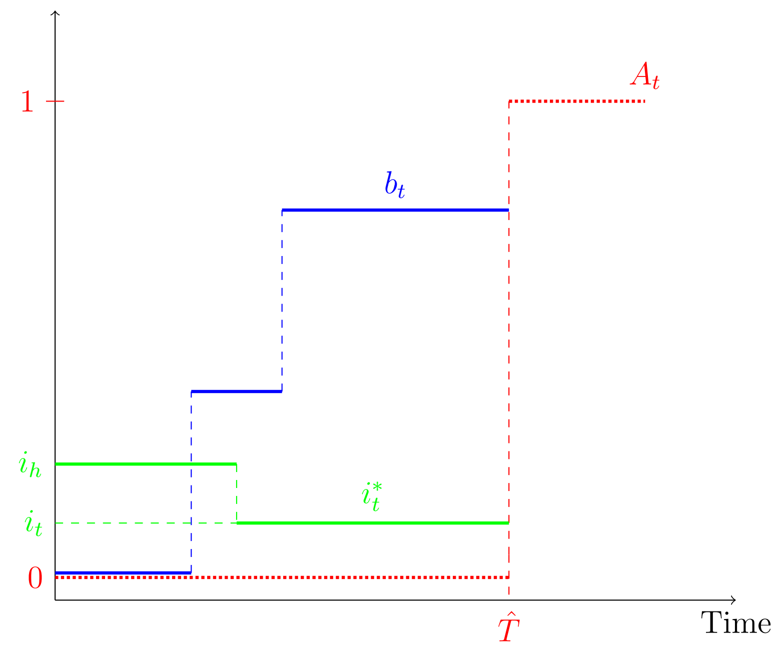

Figure 2: Possible paths for the exogenous stochastic variables

represented by the line labelled i^*_t. Our goal in the reminder of the paper is to analyze the central bank behavior, and to analyze its decision whether to keep a peg (i.e. keep $E_t = 1$) or not, when $t < \hat{T}$.

4.2 Functional forms and parameter values

Our baseline parameters values are reported in Table 2 below. We now briefly describe how we pick those values. We start with estimation of money demand elasticity and money demand shocks. In order to do so we first construct a measure of money demand. The measure that is more consistent with our stylized model is monetary base, which is a measure of the monetary liabilities of the central bank. We construct this by adding currency in circulation plus deposits of domestic and foreign banks at the central bank (as reported in the balance sheet of the SNB) all converted in Euros.[7]. Panel A in figure 3 plots the log of monetary base along with the Swiss Franc 3 months Libor rate. The panel shows an overall negative correlation between the two series, but also suggests that it is difficult to separately identify the impact of interest rate change from the impact of exogenous (positive) shock to the demand for Swiss francs, that are also correlated with reduction of the Swiss rates (the shocks in the figure are marked by the solid vertical lines). To see this consider the Euro

[7]This measure is narrower than more traditional measures of money demand such as M1 or M2, but is highly correlated with those

crisis of 2011. Panel A of the figure shows that during the crisis there was, at the same time, a large increase in the demand for Swiss currency and a small reduction in the Swiss interest rates. If one estimated a money demand without shocks, one would attribute the whole increase in money demand to the reduction in interest rate, and would come up with a very large estimated elasticity. In reality a large fraction of the increase in money demand came from exogenous reasons (i.e. a sharp recession in the Euro area) that at the same time increased the demand for Swiss francs and induced the SNB to lower its rate.

Our (admittedly simplistic) attempt to separately identify the impact of shocks from the impact of interest rates on money demand is to specify the log money demand as the following linear function:

$$\begin{aligned} \log L(i) &= \sum_{j=1}^{S} D_j \phi_j - l(i) \\ &= \sum_{j=1}^{S} D_j \phi_j - \psi i \end{aligned} \tag{12}$$

where S is the number of permanent shocks to money demand (to be specified below), D_j is a dummy variable that takes the value of 0 for all the months before shock j hits, and 1 for the month in which the shock hits and for all subsequent months. The parameters to be estimated in the equations are the ϕ_j is the percentage increase in money demand caused by shock j,, which pin down the parameter g in equation 11 , and the constant $\psi > 0$, which captures the elasticity of money demand to the interest rate. Note that our specification of the functional form of interest elastic portion of money demand is the commonly used Cagan specification.[8]

Guided by the evidence from panel A, we consider 5 possible specifications of the shocks in equation (12).

[8] See Lucas Jr (2000) for different specifications.

Figure 3: Money Demand in Switzerland: 2007-2015

Table 1. Estimation of Swiss Money Demand, 2007-2015

	No Shocks	1 shock	2 Shocks	3 Shocks	4 Shocks
ψ	0.72 (0.05)	0.36 (0.02)	0.35 (0.02)	0.34 (0.02)	0.17 (0.06)
ϕ_1, Euro Crisis		1.33 (0.04)	1.00 (0.04)	1.01 (0.04)	1.04 (0.04)
ϕ_2, Draghi Speech			0.44 (0.04)	0.43 (0.04)	0.43 (0.04)
ϕ_3, ECB QE				0.09 (0.05)	0.22 (0.04)
ϕ_4, Great Recession					0.41 (0.04)
Avg. ϕ	-	1.33	0.72	0.51	0.53
Adj. R^2	0.66	0.97	0.98	0.98	0.99
Obs.	103	103	103	103	103

The first one includes no shock, the second one includes one shock (the onset of the Euro crisis dated August 2011), the third one includes two shocks (the Euro crisis plus the Draghi "Whatever it takes" Speech, dated July 2012), the third one includes three shocks (the Euro crisis, the Draghi Speech, and the announcement of ECB QE, dated on January 2015) and the final one includes 4 shocks (the Euro crisis, the Draghi Speech, the ECB QE and the Great Recession, dated on December 2008). Table 1 reports the estimate of the interest elasticity for these 4 specifications along with estimates of the impact of each shock on Swiss money demand

Note that as we include more and more shocks the interest elasticity ϕ falls, reflecting that more movements of money demand are explained by shocks, and not by changes in the Swiss interest rates. As our baseline specification we use the three shocks case, and the fit of that specification is visualized in panel B of figure 3. The thick lines in the figure all have slope equal to ψ (the estimated interest elasticity) while the difference in the intercept of the lines represents the shock. The specification implies an interest elasticity (ψ) of 0.34 and an average shock size (g) of 51%. Since the elasticity of money demand is an important parameter, in section 5 we analyze how our results change when vary it.

In order to specify the probability of a shock to money demand we we note that in our baseline specification we observe 3 such shocks in a period of seven years, so we set the monthly probability of such a shock equal to 3.5% which implies, on average, one shock every 28 months.

We move next to the values for the foreign interest rates. Figure 1 shows how in the early phase of the peg, Euro rates moved from 1.5% to about 0%. For this reason we set $i_h = 1.5\%$ and $i_l = 0\%$. To specify the transition probabilities for interest rates we think of transition from high to low as a standard transitions associated to changes of monetary policy over the business cycle, so we set the monthly transition probability $\theta_{hl} = 1\%$, which translates into an expected duration of a high interest rate period (expansion phase) of 8 years and the probability $\theta_{lh} = 1.7\%$, which roughly translates in to a duration of the low interest period of six years. The latter is consistent with the data, if we project that Euro interest rates will stay at 0 throughout 2016.

The next parameters are related to the appreciation risk (i.e. the A shock), which are the value of the currency in case of appreciation $\bar{E}$ and the probability of such an appreciation λ. Note that these two parameters jointly determine the minimum expected appreciation of the domestic currency during the peg, but are very hard to pin down as such event is not observed in our sample. We set the probability of appreciation to 0.4%, which implies that this event is rare (one every 20 years), and we set the value of the currency in case of appreciation to 0.7, which implies a minimum expected annual appreciation during the peg of about 1%. In section 5 we explore how our results change when we change these parameters.

The final set of parameters concern the balance sheet of the central bank. Figure 4 plots monetary base and it shows that in September 2011 (the month in which the peg was introduced) the difference between foreign reserves and monetary base (which in our model corresponds to net worth) was about 20% of monetary base. So we set the value of the initial net worth, NW_0 so that the model matches that ratio in the first period of the simulation, i.e. when the peg starts. Note from the figure that net worth of the central SNB stays fairly constant, despite large fluctuations in the monetary base and in reserves. This pattern is consistent with our modelling of the transfer policy of the central bank, that implies a constant net worth.

Finally we set the maximum value of losses that can be sustained by the Central Bank (Π) equal to 1.6 the value of the monetary base at the start of the peg. This value is chosen so that the expected duration of the peg, under a constant high foreign rate, is approximately 7 years. Again the exact value of the parameter is hard to pin down, as it is hard to quantify exactly what is the maximum size of balance sheet losses a central bank is willing to take. In section 5 we assess how are results change with different values for Π.

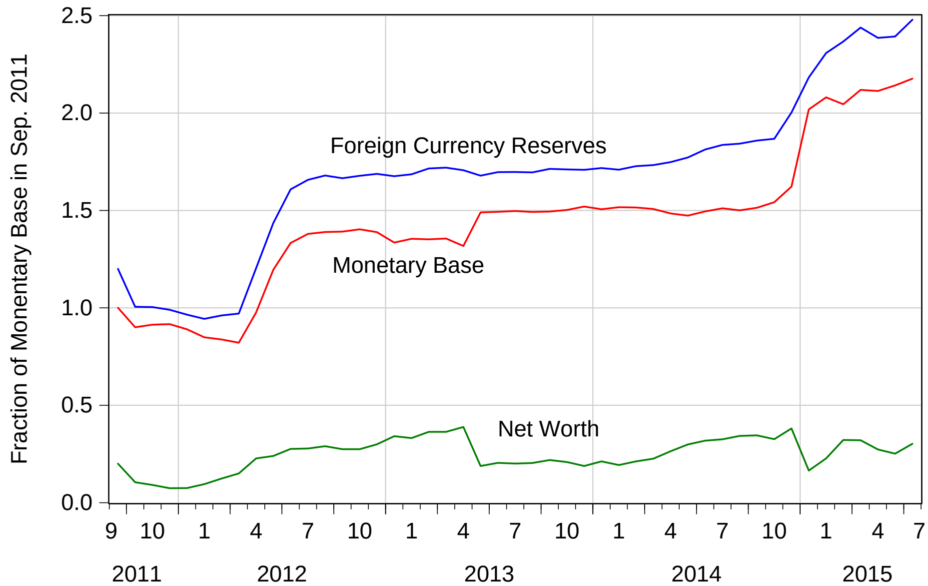

Figure 4: The Balance Sheet of the SNB during the Peg

Table 2. Parameter Values

	Symbol	Name	Value
Money Demand	ψ	Interest Elasticity of Money Demand	0.34
	g	Size of jump in money demand	0.51
	γ	Probability of a jump (monthly)	3.5%
Interest rate	i_h	High foreign interest rate	1.5%
	i_l	Low foreign interest rate	0%
	θ_{hl}	Prob. from high to low	1%
	θ_{lh}	Prob. from low to high	1.7%
Appreciation Risk	$\bar{E}$	Appreciated exchange rate	0.7
	λ	Probability of Appreciation	0.04%
Balance sheet[a]	NW_0	Net Worth of Central Bank	0.2
	Π	Maximum Loss	1.6

[a] NW_0 and Π are expressed as ratio to monetary base at the start of the peg

4.3 Results

Given these parameter values, we can numerically solve for the Markov equilibria We solve for an equilibrium numerically in the following way. We guess an initial exchange rate function, $E_0(.)$. Given this, for every state, we compute the exchange rate that is closest to 1 and and the loss constraint in case of the exogenous appreciation shock in the following period, while assuming that $E_0(.)$ is the equilibrium exchange rate policy the following period. This generates a new equilibrium conjecture $E_1(.)$ for every state. We keep iterating this procedure until the $E_i(.)$ converges. We found that this procedure converges to a unique exchange rate function, for a very large set of of the initial guesses.

Once we have a numerical solution we can characterize the periods that feature reverse speculative attacks, i.e. abandonment of the peg. In particular we focus on two types of abandonments: those driven by shocks to money demand, and those driven by a reduction in the foreign interest rates.

Figure 5 displays the key variables of the economy in all possible states. In each panel, the x-axis represents the increasing permanent shocks to money demand, while the different lines represent different states for the foreign interest rates. For example, the right most square on the top line in panel A represents the equilibrium exchange rate that will prevail when the money demand shock is in fourth largest state value the foreign interest rate is high.

To understand the first type of abandonment, consider an economy that moves along the lines represented by the square markers, i.e. an economy that is facing a high foreign interest rate and experiencing a sequence of increases in money demand. Panel A shows how, for the first two money demand shocks, the central bank keeps the exchange rate at 1. In those states the loss constraint is not binding, and thus the central bank can maintain the exchange rate pegged at parity, its preferred outcome. Panel C shows that maintaining the peg while facing an increasing money demand involves accumulation of reserves. The jump in money demand that takes place when shock 2 hits can possibly capture the experience of the SNB during the second half of 2012, where the peg was maintained through a large accumulation of foreign reserves.

Note however that, as reserves grow, so does the size of the losses of the central bank in case of an exogenous appreciation (i.e., the A shock), and that makes the loss constraint more likely to bind. Indeed, the money demand state 2 is the largest state for which the Central Bank can maintain the peg. Panel A shows that when the next money demand shock (state 3) hits, the central bank will abandon the peg and the exchange rate will appreciate by about 7%. When this appreciation happens the Central Bank experiences losses, while setting an exchange rate away from its preferred target. The benefit of doing so is that the current appreciation prevents a larger appreciation in the future, that would lead to much larger losses.

Panel B plots the domestic interest rates. The top line shows that appreciation in state 3 is anticipated by investors, and that it induces a decline in domestic interest rate in state 2 (a result that follows directly from the UIP condition, equation **??**). The fall in interest rates causes a further increase in money demand (over and above the increase caused directly by the shock) that forces an even larger increase in reserves just before the peg is abandoned (see the steep increase between states 1 and 2 in the bottom line in panel C). It is interesting that before the attack, the model displays patterns that resemble a defense against an attack. That is, as abandonment of the parity becomes more likely (the economy moves to state 2), reserves increase, while domestic interest rates fall.

Note that quantitatively the increase in reserves implied by the model is too large relative to the Swiss data: in the model reserve during the peg increase 4 times, while in the data (see figure 1) reserves roughly doubled. Also domestic interest rates fall to a much lower level (-3%) than what is observed in the data. We conjecture that these discrepancies are due to the fact that we do not explicitly model a lower bound on interest rates, and the patterns of money demand and capital flows around that bound. This is the subject of our current work.

Figure 5 also suggests another possible trigger of the abandonment of the parity. Suppose, for example, that the economy is at state 2 in panel A. Consider now a change in the foreign rate from high to low. In this situation, the central bank abandons the parity while the exchange rate appreciates by about 3%. The logic behind this result is similar to the one described above: the fall in the foreign interest rate causes (should the central bank maintain the parity) a fall in the domestic rate, and this induces an increase in demand for local currency, accompanied by a similar increase in reserves. Panel D shows the increase in reserves that takes place when the economy moves from the high to the low foreign interest rate. The increase in reserves might, in some state, cause the loss constraint to bind, and hence in that state maintaining the parity is no longer feasible for the central bank.

To sum up, we have highlighted two possible causes of an abandonment of the peg. In both of them, the Central Bank abandons the peg because maintaining the exchange rate at parity involves a large reserve accumulation, which coupled with the appreciation risk may lead to losses in the Central Bank's balance sheet that are large, and by assumption, not sustainable by the central bank. By letting the currency appreciate early on, the Central Bank realizes some of these losses when reserves are still low, and in this way, reduces the size of future losses.

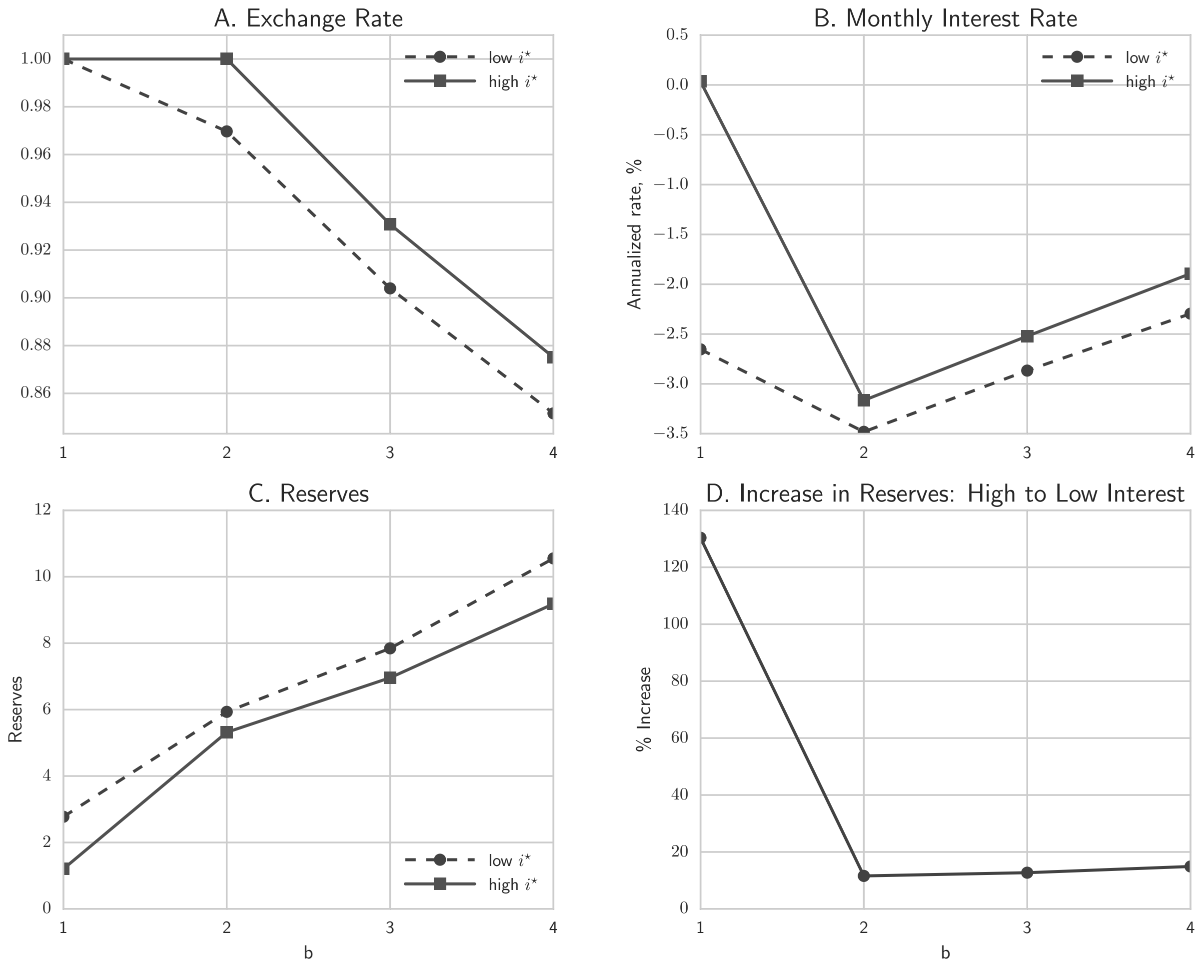

Figure 5: Markov Equilibria

5 Sensitivity Analysis

The objective of this section is to illustrate how the dynamics of reverse speculative attacks change as we change some of the fundamental parameters of the model economy. Figure 6 displays the patterns of exchange rates, while figure 7 displays the pattern of reserves, for Markov equilibria under different parameters specifications.

Panels A in both figures show how the patterns of speculative attack depend on the expected size of the appreciation shock $1 - \bar{E}$. When the size of appreciation shock is larger (the dashed lines) the expected losses of the central bank, in case of appreciation, are larger, and the central bank is willing to tolerate a lower level of reserves; this implies that the peg will be abandoned earlier. Note that the higher probability of abandonment causes a higher expected appreciation of the exchange rate in every state, and thus (through the UIP equation) a lower interest rate; this implies an increase in reserves even in the first state where

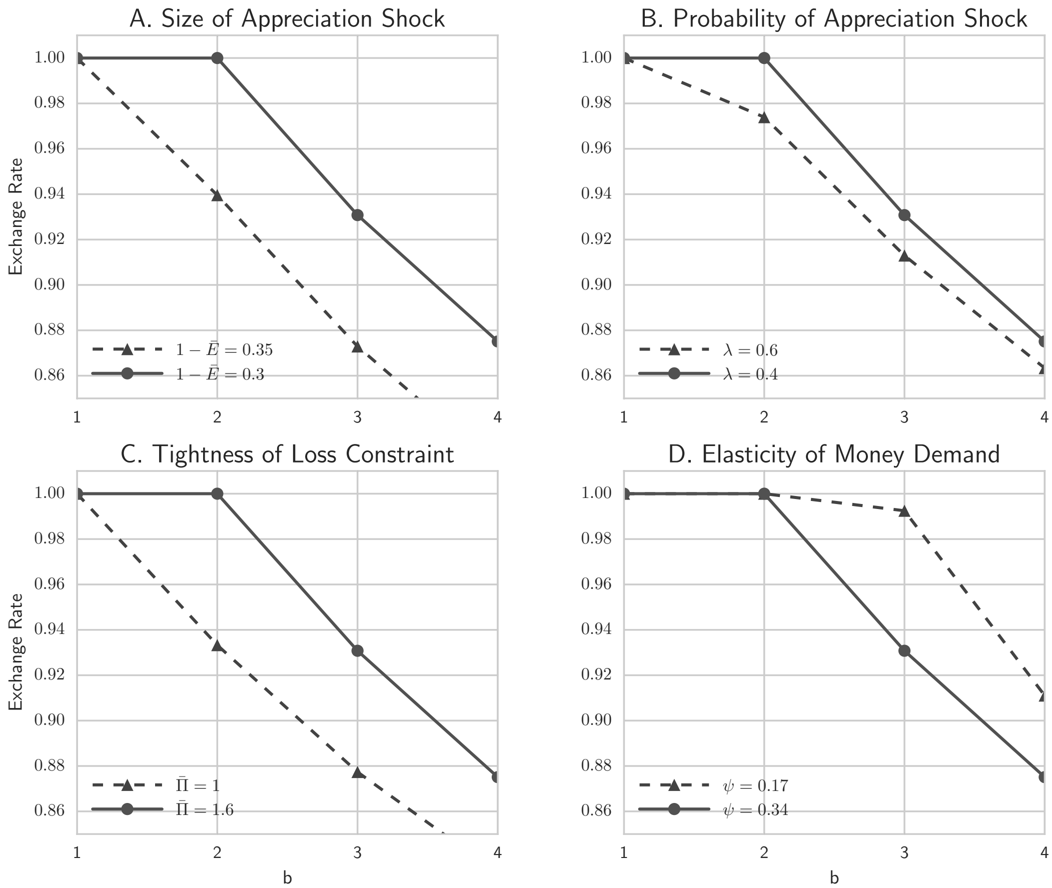

Figure 6: Sensitivity analysis: Exchange Rates

the central bank can maintain the peg (see panel A in figure 7). Panels B in both figures show the impact of a higher probability of the appreciation shock (the dashed lines). Notice that, because the way we have specified the loss constraint of the central bank (equation 8), a higher likelihood of the appreciation shock does not make the loss constraint directly more binding. Yet, a higher probability of appreciation induces an earlier abandonment. The logic is again that a more likely appreciation induces a current lower domestic interest rate, higher money demand and thus more reserves, that make the loss constraint more likely to bind, and thus induce earlier abandonment. Going back to figure 1, we noticed how the the attack that caused the collapse of the Swiss exchange rate floor happened after the Swiss economy experienced two quarters of positive economic growth. The positive news about growth could be interpreted, through the lens of our set-up, as news that the appreciation shock is more likely to happen, and thus could help explain the abandonment of floor, even

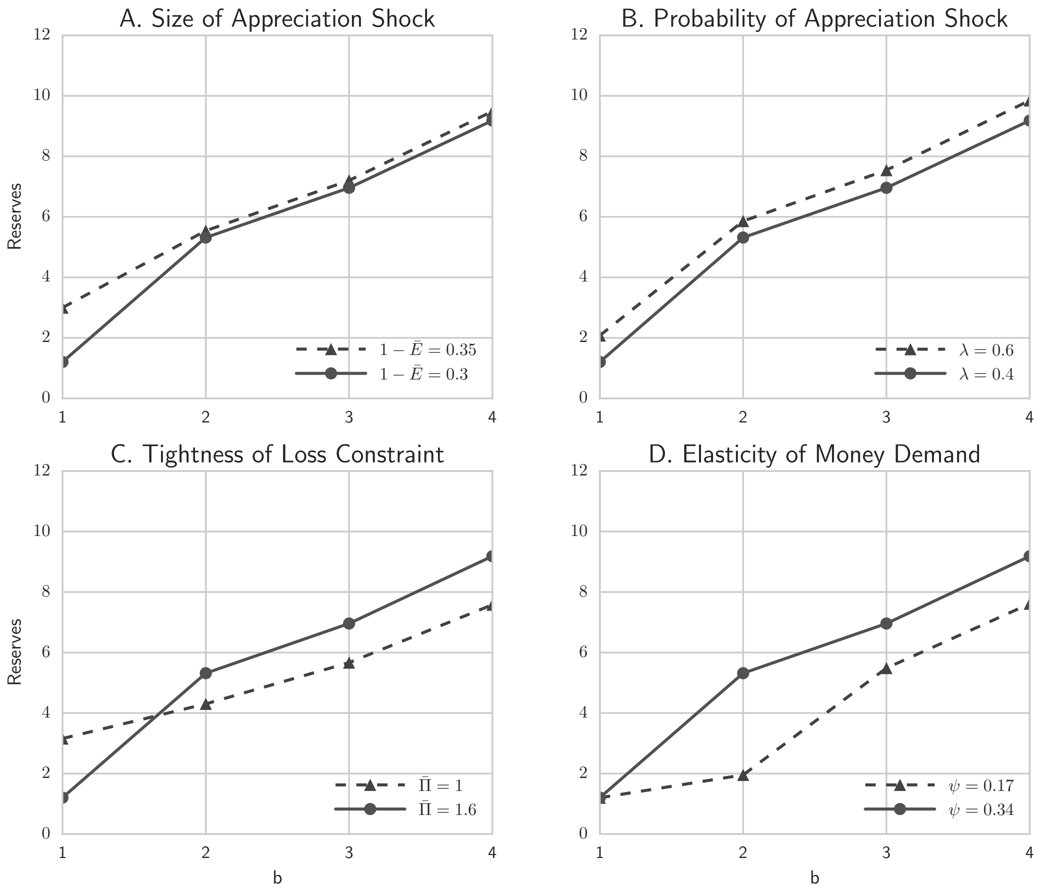

Figure 7: Sensitivity analysis: Reserves

before the shock actually happens.

Panels C in figures 6 and 7 show the impact of having a tighter loss constraint on the central bank (a lower $\bar{\Pi}$). Not surprisingly a tighter loss constraint leads to earlier abandonment. Comparing Panels A and Panels C we notice that, in the first state, the effect of a tighter constraint is very similar to the effect of a larger depreciation shock. However, in subsequent states the tighter loss constraint has a milder impact on exchange rates and on reserves, than the larger appreciation shock. The reason is that the larger appreciation shock also causes a lower interest rate (thought the UIP), higher demand for money and higher reserves. The tighter loss constraint does not have this additional channel, as it does not directly affect the expected exchange rate.

Finally panels D explores the impact of a different elasticity for the demand for money. The dashed lines in panels D in figures 6 and 7 depict the case when the elasticity is lower

($\psi = 0.17$, corresponding to the 4 shocks specification in table 1). Notice that with a lower elasticity of money demand the central bank accumulates less reserves as money demand shock become larger (the dashed line in panel D in figure 7 is below the solid line). As a consequence its loss constraint is less likely to bind, and the bank can keep the exchange rate closer to the peg than in the benchmark case (see panel D in figure 6). To understand why this is the case recall that when money demand increases and domestic appreciation becomes more likely the UIP equation implies that the domestic interest rates fall. When domestic interest rates fall demand for domestic currency increases further, making reserves grow faster, making the loss constraint more likely to bind and appreciation more likely. With lower elasticity this additional increase in money demand is muted, and thus the central bank accumulates less reserves, and can delay appreciation. Another consequence of the lower elasticity is that the exchange rate is less sensitive to foreign interest rate shocks. As we discussed earlier a reduction in foreign interest rate induces lowers domestic rates and increase domestic money demand, which forces the central bank to appreciate the currency. With lower elasticity the increase in money demand stemming from a reduction in foreign rate is lower and hence the appreciation of the exchange rate is also lower, Indeed we find that when the foreign interest rates falls in state 2 and the elasticity is high ($\psi = 0.34$, the benchmark case) the central bank appreciates the exchange rate by 3% (see panel A, figure 5). If instead the elasticity is low ($\psi = 0.17$) we find that the central bank can maintain the parity in state 2, even when the foreign interest rate falls to its low level.

6 Conclusions

This paper has presented a stylized framework to analyze the abandonment of the peg and subsequent appreciation, experienced by the Swiss National Bank in January 2015. We consider a framework in which maintaining a peg involves accumulation of risky foreign reserves, and the central bank might abandon the peg in order to limit its exposure to this risk. We have shown that in this framework shocks to the demand for local currency, and/or to the foreign interest rates can lead to dynamics of reserves and exchange rates that resemble those observed in Switzerland.

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