

A THEORY OF FEAR OF FLOATING*

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Abstract

Many central banks whose exchange rate regimes are classified as flexible are reluctant to let the exchange rate fluctuate. This phenomenon is known as “fear of floating”. We present a simple theory in which fear of floating emerges as an optimal policy outcome. The key feature of the model is an occasionally binding borrowing constraint linked to the exchange rate that introduces a feedback loop between aggregate demand and credit conditions. Contrary to the Mundellian paradigm, we show that a depreciation can be contractionary, and letting the exchange rate float can expose the economy to self-fulfilling crises.

Keywords: Exchange rates, self-fulfilling financial crises

JEL Codes: E44, E52, F33, F34, F36, F41, F45, G01

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1 Introduction

According to the Mundell-Fleming paradigm, a floating exchange rate plays a pivotal role in stabilizing economic fluctuations in an open economy. By depreciating the exchange rate when a negative shock hits the economy, a central bank can shift demand towards domestic goods and help mitigate the recession. Yet, contrary to this policy prescription, many central banks are often reluctant to let the currency float, particularly when facing turbulence in financial markets (Calvo and Reinhart, 2002).^{1,2} Why do central banks experience a “fear of floating”? Moreover, can a nominal exchange depreciation be contractionary? Despite extensive policy debates, these issues remain unresolved.

This paper explores the idea that letting the exchange rate float may expose the economy to a self-fulfilling financial crisis and therefore rationalize the fear of floating. We provide a simple theory featuring household deleveraging and nominal rigidities. For given fundamentals, we show how a depreciation of the nominal exchange rate and a contraction in output can mutually reinforce each other, giving rise to multiple equilibria. In contrast to the Mundell-Fleming paradigm, letting the exchange rate float does not serve as a shock absorber. Crucially, the central bank can prevent deleveraging and avoid a self-fulfilling financial crisis by anchoring the nominal exchange rate.

The model has two key elements, which can account for central features of emerging market crises, also called “sudden stops”. First, households face a borrowing limit linked to the value of their income. Second, nominal wages are downwardly rigid. The first element implies that a reduction in household borrowing can lead to a deterioration of the market value of collateral and can induce self-fulfilling fluctuations (Schmitt-Grohé and Uribe, 2021). The second element implies that absent borrowing frictions, letting the exchange rate depreciate can help to switch demand towards domestically produced goods and help alleviate a recession in response to a negative shock.

We show that the interaction of these two frictions gives rise to two novel insights. First, a nominal exchange rate depreciation can be contractionary. While the classic expenditure-switching channel generates an expansionary effect by shifting demand towards domestically produced goods, a depreciation also lowers the relative value of collateral, leading to a contraction in demand for non-tradable goods. We show that this

¹Calvo and Reinhart (2002) state, “We find that countries that say they allow their exchange rate to float mostly do not—there seems to be an epidemic case of fear of floating.”

²As Ilzetzki, Reinhart and Rogoff (2019) show, the fear of floating phenomenon is as pervasive today. They classify only 10% of the world economy as pure floaters and 51% of the countries as fixed exchange rates. The remaining 39% are classified as managed floaters.

second channel may dominate and cause a depreciation to turn contractionary. Second, anchoring the nominal exchange rate may help the economy avoid a self-fulfilling crisis. The logic for this result is that a self-fulfilling crisis equilibrium emerges when pessimism by households leads them to reduce borrowing and consumption, which in turn generates a real exchange rate depreciation that further tightens borrowing constraints and validates the initial pessimism. A policy that stabilizes the nominal exchange rate can eliminate multiplicity by stabilizing the real exchange rate and breaking the self-fulfilling nature of deleveraging.

In our baseline analysis, we contrast a fixed exchange rate regime with a flexible exchange rate regime where the central bank keeps monetary aggregates constant. In this setting, we show that the region where the economy is vulnerable to a self-fulfilling crisis is larger under a flexible exchange rate. In addition, we also study optimal policy. We first show that when the central bank lacks commitment, there is a wide range of Markov perfect equilibria, which differ markedly in terms of the level of the exchange rate, output, and capital flows. We then show that in the region in which a fixed exchange rate cannot uniquely implement the good equilibrium, there exists a sophisticated monetary policy, along the lines of [Bassetto \(2005\)](#) and [Atkeson, Chari and Kehoe \(2010\)](#), that can help eliminate self-fulfilling crises. The policy prescribes a form of *crawling peg* by which the central bank allows deviations of the exchange rate within a floating band in such a way that it discourages deviations by households from the desired borrowing levels.

Related literature. This paper relates to a vast literature on optimal monetary policy in open economies. A fundamental theme in the literature, going back to [Mundell \(1960\)](#) and [Friedman \(1953\)](#), is that a flexible exchange rate regime can insulate the economy from domestic and external shocks.³ The overarching principle is that by varying the exchange rate—in particular, by depreciating during a recession—the central bank can adjust relative prices and stabilize output at the efficient level. In this paper, we present a model in which letting the exchange rate float may exacerbate inefficient economic fluctuations and establish that depreciations can be contractionary.

Our paper is also related to the literature on monetary policy with credit frictions in open economies. An important finding in one strand of this literature is that weak firms' currency mismatches can magnify the effects of foreign shocks.⁴ However, exchange rates

³Modern treatments of this theme include [Schmitt-Grohé and Uribe \(2011\)](#), among others.

⁴Examples include [Aghion, Bacchetta and Banerjee \(2000; 2004\)](#), [Caballero and Krishnamurthy \(2001\)](#), [Gertler, Gilchrist and Natalucci \(2007\)](#), [Lahiri and Végh \(2001\)](#), [Céspedes, Chang and Velasco \(2004\)](#), [Cook \(2004\)](#), [Christiano, Gust and Roldos \(2004\)](#), [Braggion, Christiano and Roldos \(2009\)](#), [Fornaro \(2015\)](#), [Gourin-](#)

remain useful to provide insulation against adverse shocks.⁵ our paper instead develops a framework where a depreciation can be contractionary and self-fulfilling, providing a rationale for fixing the exchange rate.

Another strand of the literature studies optimal monetary policy in models with households' deleveraging. Our model is closest to [Ottonello \(2021\)](#). He studies Ramsey optimal policy and finds that the interaction between nominal rigidities and households' borrowing limits generates a tradeoff between credit market access and unemployment, or as put by [Farhi and Werning \(2016\)](#) between financial and macroeconomic objectives (see also, [Coulibaly, 2020](#); [Basu, Boz, Gopinath, Roch and Unsal, 2020](#)).⁶ Our analysis in Section 4.3 uncovers that a policy-induced nominal exchange rate depreciation may actually be *contractionary*. This occurs when the reduction in demand due to the financial channel offsets the expenditure-switching channel. More broadly, the central difference in our analysis is that we examine the possibility of multiple equilibria and show that in line with the fear of floating phenomenon, a commitment to keep the exchange rate fixed can rule out self-fulfilling crisis equilibrium.

On the empirical side, our paper casts light on the unresolved question of whether depreciations are contractionary or recessions are devaluatory (see, e.g, [Frankel, 2005](#); [Uribe and Schmitt-Grohé, 2017](#)). Our model articulates both views depending on the configuration of parameters and whether the source of the shock is fundamental or non-fundamental.⁷

We are also related to a literature on financial fragility emerging from multiple equilibria, including [Chang and Velasco \(2000, 2001\)](#) and [Aghion et al. \(2000\)](#). Our framework is

[Chas \(2018\)](#), [Du and Schreger \(2022\)](#), [Devereux and Yu \(2017\)](#); [Devereux, Young and Yu \(2019\)](#), [Cavallino and Sandri \(2022\)](#).

⁵In addition, depreciation typically remains expansionary and desirable in response to negative shocks (see, e.g., [Gertler, Gilchrist and Natalucci, 2007](#) and [Céspedes, Chang and Velasco, 2004](#)). [Cook \(2004\)](#) provides an example in which a depreciation reduces investment, but still, output expands. [Cavallino and Sandri \(2022\)](#) is a notable recent exception where a monetary stimulus can be contractionary—in their case through a reduction in the interest on reserve.

⁶[Ottonello \(2021\)](#) solves directly for the Ramsey optimal allocations, which in general is muted about implementation and, in effect, selects the good equilibrium in cases of indeterminacy. In his simulations, the optimal policy significantly reduces the volatility of consumption relative to a full-employment exchange rate policy.

⁷The idea that depreciations may be contractionary goes back to early work by [Diaz-Alejandro \(1963\)](#) in the context of domestic redistributive effects. Several studies have also studied the possibility of beggar-thy-self depreciation through terms of trade effects when elasticities of substitution are low (e.g., [Tille, 2001](#); [Corsetti, Dedola and Leduc, 2022](#)). [Auclert, Rognlie, Souchier and Straub, 2021](#) consider a rich model with these two channels to show how depreciations triggered by foreign monetary policy shocks can be contractionary. [Adrian, Erceg, Kolasa, Lindé and Zabczyk \(2022\)](#) generates contractionary depreciation through an adaptive expectation mechanism involving staggered wage contracts. See also [De Ferra, Mitman and Romei \(2020\)](#) and [Blanchard, Ostry, Ghosh and Chamon, 2016](#) for other related work.

most closely related to [Schmitt-Grohé and Uribe \(2021\)](#).⁸ In contrast to their work, ours considers a monetary model with nominal rigidities, which allows us to speak about how different exchange rate regimes affect the vulnerability to self-fulfilling financial crises.

This paper is related to the literature on aggregate demand externalities in the presence of constraints on monetary policy. In [Schmitt-Grohé and Uribe \(2016\)](#) and [Farhi and Werning \(2016\)](#), a fixed exchange rate plays a key role in preventing the central bank from stabilizing macroeconomic fluctuations and generates scope for macroprudential policy.⁹ However, these papers abstract from the source of the rigidities in monetary policy. Our paper complements these studies by providing a theory of why the central bank finds it optimal to keep the exchange rate fixed.

Finally, the paper is also related to a few papers that presented models in which giving up monetary independence may be desirable. These benefits may emerge from a reduction in the inflationary bias generated by the time inconsistency problem of monetary policy ([Alesina and Barro, 2002](#); and [Chari, DAVIS and Kehoe, 2020](#)), a reduction in transaction costs ([Mundell, 1961](#)), or larger risk sharing ([Neumeyer, 1998](#), [Arellano and Heathcote, 2010](#), [Fornaro, 2022](#)). Our contribution is to provide a distinct rationale for stabilizing the exchange rate, one that puts the lower vulnerability to financial crises at the center stage.

Outline. Section 2 presents the model. Sections 3 and 4 present the theoretical results on how the exchange rate regime affects the vulnerability to self-fulfilling financial crises. Section 5 analyzes optimal policy and Section 6 concludes.

2 Model

We consider a small open economy with two types of goods: tradables and non-tradables. Time is discrete and infinite. The economy features nominal rigidities and constraints on households' borrowing.

⁸The role of multiple equilibria is also highlighted by [Krugman \(1999\)](#) in the context of the East Asian crisis and by [Bocola and Lorenzoni \(2020\)](#), among many others. Earlier work by [Sachs \(1984\)](#), [Calvo \(1988\)](#), and [Obstfeld \(1984\)](#) displayed a different form of multiple equilibria that emerges instead from a strategic game between the central bank and private agents. In this vein [Corsetti, Pesenti and Roubini \(1999\)](#) and [Schneider and Tornell \(2004\)](#) emphasize bailout expectations, and [Cole and Kehoe \(1996\)](#) emphasize roll over problems (see also [Bianchi and Mondragon, 2022](#)).

⁹Relatedly, in [Bianchi and Lorenzoni \(2021\)](#), the central bank has a flexible exchange rate regime, but there is a utility cost from exchange rate fluctuations. [Acharya and Bengui \(2018\)](#), [Fornaro and Romei \(2019\)](#), and [Bianchi and Coulibaly \(2021\)](#) consider instead a zero lower bound constraint constraints (see also [Korinek and Simsek \(2016\)](#) for a closed economy analysis).

2.1 Households

There is a continuum of identical households of measure one. Households have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \chi \log \left(\frac{M_{t+1}}{\mathcal{P}_t} \right) \right], \quad (1)$$

where $\chi \geq 0$ and $\beta \in (0, 1)$ is the discount factor. The consumption good c_t is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N , according to a constant elasticity of substitution aggregator:

$$c_t = \left[\phi (c_t^T)^{\frac{\gamma-1}{\gamma}} + (1-\phi) (c_t^N)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad \text{where } \phi \in (0, 1).$$

For the most part, we will focus on an elasticity of substitution between tradable and non-tradable consumption, γ , below one, which is the empirically relevant case. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods. The real money holdings, M_{t+1}/\mathcal{P}_t , provide liquidity services to households that enter the utility function, where M_{t+1} is the end-of-period money holdings and \mathcal{P}_t is the ideal price index in period t . Denoting by P_t^N and P_t^T the price of non-tradables and tradables (in terms of the domestic currency) respectively, the ideal price index satisfies

$$\mathcal{P}_t = \left[\phi^\gamma (P_t^T)^{1-\gamma} + (1-\phi)^\gamma (P_t^N)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}.$$

We assume that the law of one price holds for the tradable good and normalize the price of the tradable good in units of foreign currency to unity. This implies that $P_t^T = e_t$, where e_t is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency.

Households supply \bar{h} units of labor inelastically. Because of the presence of downward wage rigidity and rationing (to be described below), each household's actual hours worked are given by $h_t \leq \bar{h}$, which is taken as given by the household. Each period households receive a wage rate, W_t , and central bank transfers, T_t , all expressed in terms of domestic currency, which serves as the numeraire. Households also receive y_t^T units of tradable goods. Households trade one-period non-state-contingent nominal bonds in domestic and foreign currency. The foreign currency bond has an exogenous return R . The domestic currency bond is assumed to be traded only within domestic market and pays a return

\tilde{R}_t , determined endogenously.¹⁰ The budget constraint of the representative household is therefore given by

$$P_t^T c_t^T + P_t^N c_t^N + M_{t+1} + \tilde{b}_t + e_t b_t = P_t^T y_t^T + W_t h_t + M_t + \frac{\tilde{b}_{t+1}}{\tilde{R}_t} + \frac{e_t b_{t+1}}{R} + T_t, \quad (2)$$

where \tilde{b}_t and b_t denote respectively the amount of domestic currency debt and foreign currency debt assumed in period $t - 1$ and due in period t . The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds, while the right-hand side represents total income, including the returns from bond issuance.

Households face a borrowing constraint that limits foreign currency debt to a fraction κ of their individual current income:

$$\frac{e_t b_{t+1}}{R} \leq \kappa \left[P_t^T y_t^T + W_t h_t \right]. \quad (3)$$

This borrowing constraint captures the idea that current earnings are a critical factor determining credit market access (see, e.g. Jappelli, 1990, Lian and Ma, 2020, Drechsel, 2022, Greenwald, 2018) and has been shown to be important for accounting for the dynamics of capital flows in emerging markets (e.g., Mendoza, 2002, Bianchi, 2011).^{11,12} To ensure that the borrowing constraint is tighter than the natural debt limit, we assume $0 < \kappa < R/(R - 1)$.

Optimality conditions. Optimality with respect to c_t^T and c_t^N implies that

$$\frac{P_t^N}{e_t} = \frac{1 - \phi}{\phi} \left(\frac{c_t^N}{c_t^T} \right)^{-\frac{1}{\gamma}} \quad (4)$$

Let $\lambda_t \geq 0$ denote the Lagrange multiplier on the budget constraint (2), $\lambda_t \mu_t \geq 0$ the multiplier on the borrowing constraint (3) and u_T the marginal utility of tradable consumption. Households' optimal borrowing choices for foreign currency is determined

¹⁰The assumption that domestic currency bonds are traded only domestically can be easily generalized. We can show, in particular, that if the initial debt in domestic currency is zero, the equilibrium allocations are the same regardless of whether domestic currency bonds are traded only domestically or not.

¹¹The credit constraint can be derived endogenously from a problem of limited enforcement under the assumption that household default occurs at the end of the current period and that upon default, households lose a fraction κ_t of the current income. The borrowing limit could also depend on future income or other variables. What is crucial for our results is that higher current income relaxes the borrowing limit.

¹²The collateral constraint assumes that only foreign debt can be collateralized. We can show that all the results hold when a fraction domestic bonds must be collateralized as well.

by the following Euler equation and complementary slackness

$$(1 - \mu_t)u_T(c_t^T, c_t^N) = \beta R u_T(c_{t+1}^T, c_{t+1}^N) \quad (5)$$

$$\mu_t \times \left[\kappa \left(y_t^T + \frac{W_t}{e_t} h_t \right) - \frac{b_{t+1}}{R} \right] = 0 \quad (6)$$

Similarly, for domestic currency bonds,

$$u_T(c_t^T, c_t^N) = \beta \tilde{R}_t \frac{e_t}{e_{t+1}} u_T(c_{t+1}^T, c_{t+1}^N). \quad (7)$$

Households' optimality condition for money balances yields the following money demand equation decreasing in the nominal interest rate:

$$\frac{M_{t+1}}{\mathcal{P}_t} = \chi \frac{\tilde{R}_t}{U'(c_t)(\tilde{R}_t - 1)}, \quad (\text{MD})$$

Using the Euler equations for real bonds and nominal bonds, and the law of one price, we obtain an interest parity condition, which relates the return on domestic currency bonds to the return on foreign currency bonds and the expected depreciation of the domestic currency

$$\frac{R}{1 - \mu_t} = \tilde{R}_t \frac{e_t}{e_{t+1}}. \quad (8)$$

When the borrowing constraint binds, we have an endogenous deviation from uncovered interest parity.

2.2 Firms and Nominal Rigidities

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a linear production technology given by $y_t^N = n_t$ and obtains profits given by $\phi_t^N = P_t^N n_t - W_t n_t$. Given the linear production function, we obtain that in equilibrium,

$$P_t^N = W_t. \quad (9)$$

An individual firm is therefore indifferent between any level of employment.

We assume there exists a minimum wage in nominal terms. Following [Schmitt-Grohé and Uribe \(2016\)](#), we assume that the current nominal wage is bounded below by the previous period nominal wage; that is, $W_t \geq W_{t-1}$. The labor market is such that aggregate

hours worked are the minimum between labor demand and labor supply, following disequilibrium models with rationing: $h_t = \min\{n_t, \bar{h}\}$. If the market clearing wage satisfies $W_t > W_{t-1}$, the aggregate number of hours worked equals the aggregate endowment of labor. Otherwise, $h_t < \bar{h}$ and $W_t = W_{t-1}$. These conditions can be summarized as

$$(W_t - W_{t-1})(h_t - \bar{h}) = 0. \quad (10)$$

2.3 Monetary Policy

The central bank sets the money supply, M_t^s , and rebates all revenues from the net increase in money supply to the public in the form of lump-sum transfers (lump-sum tax if negative). The central bank's budget constraint at any point in time is given by

$$T_t = M_{t+1}^s - M_t^s.$$

2.4 Recursive Competitive Equilibrium

Market clearing for money requires that the supply of money by the central bank equals the demand for money by households: $M_{t+1}^s = M_{t+1}$. Market clearing for labor requires (10) and that the aggregate labor demand by firms equals the units of labor supplied by households:

$$h_t = n_t. \quad (11)$$

Market clearing for the non-tradable good requires that output equals consumption:

$$y_t^N = c_t^N. \quad (12)$$

We assume that the bond denominated in domestic currency is traded only domestically. Market clearing therefore implies

$$\tilde{b}_{t+1} = 0. \quad (13)$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y_t^T = \frac{b_{t+1}}{R} - b_t, \quad (14)$$

which says that the trade balance must be financed with net bond issuances.

Combining firms' optimality condition, $W_t = P_t^N$, with households, optimality condition (4), we arrive at an equation determining the aggregate demand for non-tradables as a function of the real wage, W_t/e_t , and the level of tradable consumption c^T

$$c_t^N = \left(\frac{1 - \phi}{\phi} \frac{e_t}{W_t} \right)^\gamma c_t^T. \quad (15)$$

Equations (14) and (15) will play a central role in the model dynamics. In the event of a deleveraging episode triggered by a binding credit constraint, the small open economy will have fewer tradable resources available. For a given relative price of non-tradables, this will lead to a reduction in the demand for non-tradable goods. With flexible wages, W_t would fall until $h_t = c^N = \bar{h}$. But if the downward wage rigidity becomes binding, the economy will feature involuntary unemployment, which will in turn feed into consumption and the borrowing capacity.

To define the decentralized equilibrium in recursive form, we separate individual debt under the household's control, b , from the economy's aggregate debt position, B , on which prices depend. Hence, the state variables for a household's problem are the individual state b , the aggregate state B , and the previous period market clearing wage W_{-1} . The optimization problem of the household in recursive form is then given by

$$V(b, B, W_{-1}) = \max_{c^T, c^N, b'} u(c^T, c^N) + \beta V(b', B', W), \quad (16)$$

subject to

$$c^T + \frac{W(B, W_{-1})}{e(B, W_{-1})} c^N + b = y^T + \frac{W(B, W_{-1})}{e(B, W_{-1})} h(B, W_{-1}) + \frac{b'}{R}$$

$$\frac{b'}{R} \leq \kappa \left[y^T + \frac{W(B, W_{-1})}{e(B, W_{-1})} h(B, W_{-1}) \right]$$

where $B' = \Gamma(B, W_{-1})$ describes the perceived the law of motion for aggregate debt.

We can now define a recursive competitive equilibrium

Definition 1 (Recursive Competitive Equilibrium). For a given central bank's monetary policy, a recursive competitive equilibrium is defined by pricing functions $\{W(B, W_{-1}), P^N(B, W_{-1}), e(B, W_{-1})\}$, a perceived law of motion for aggregate debt $\Gamma(B, W_{-1})$, firms' decision rule for hours $\hat{h}(B, W_{-1})$, households' decision rules $\{\hat{b}'(b, B, W_{-1}), \hat{c}^T(b, B, W_{-1}), \hat{c}^N(b, B, W_{-1})\}$ with associated value function $V(b, B, W_{-1})$ such that:

1. $\left\{ \hat{b}'(b, B, W_{-1}), \hat{c}^T(b, B, W_{-1}), \hat{c}^N(b, B, W_{-1}) \right\}$ and $V(b, B, W_{-1})$ solve households' recursive optimization problem (16), taking as given $e(B, W_{-1})$, and $\Gamma(B, W_{-1})$;
2. firms optimize;
3. labor market conditions (10), (11) and $W(B, W_{-1}) \geq W_{-1}$ hold;
4. the government budget constraint holds and the resource constraint for tradables holds: $\hat{c}^T(B, B, W_{-1}) + B = y^T + \frac{\Gamma(B, W_{-1})}{R}$;
5. the perceived law of motion for aggregate debt is consistent with the actual law of motion based on the individual policy. That is, $\Gamma(B, W_{-1}) = \hat{b}'(B, B, W_{-1})$;

2.5 Steady State Equilibrium

We assume now that tradable output is constant, $y_t^T = y^T$ for all t and restrict our attention to the case in which $\beta R = 1$. We define a steady-state equilibrium as a competitive equilibrium where all allocations are constant.

Definition 2 (steady-state equilibrium). A steady-state equilibrium is a competitive equilibrium in which allocations are constant for all $t \geq 0$.

Notice that a constant consumption allocation under $\beta R = 1$ implies that the borrowing constraint is not binding. From the tradable resource constraint, using $B_{t+1} = B_0$, we obtain $c_t^T = y^T - (1 - \beta)B_0$.

In the absence of a borrowing constraint, any initial values of debt lower than the natural debt limit would be consistent with a steady-state equilibrium. Our goal next is to define the range of values of initial debt that are consistent with a steady-state equilibrium in the presence of borrowing constraints. Towards this goal, we use private agents' optimality conditions (4) and (15) and the market clearing condition for non-tradables (12) to define the individual borrowing limit in period t as

$$\bar{b}(B_{t+1}; B_t) = \kappa R \left[y^T + \frac{1 - \phi}{\phi} \left(y^T - B_t + \frac{B_{t+1}}{R} \right)^{\frac{1}{\gamma}} (h_t)^{1 - \frac{1}{\gamma}} \right]. \quad (17)$$

Equation (17) describes how a household's maximum borrowing capacity $\bar{b}(B_{t+1}; B_t)$ depends on aggregates (B_t, B_{t+1}) . We can observe that $\bar{b}(B_{t+1}; B_t)$ is decreasing in B_t and increasing in B_{t+1} , reflecting that higher aggregate consumption appreciates the real exchange rate and relaxes individuals' borrowing constraints.

We let \hat{B} denote the unique value of debt such that $\bar{b}(\hat{B}; \hat{B}) = \hat{B}$ when $h_t = \bar{h}$. The lemma below characterizes when a steady-state equilibrium exists.

Lemma 1 (Steady-state equilibrium). *If $B_0 \leq \hat{B}$, we have that the steady-state equilibrium exists. Moreover, at the steady-state equilibrium the optimal allocations satisfy $h_t = \bar{h}$. Moreover, a constant exchange rate such that*

$$e_t \geq W_{t-1} \frac{\phi}{1-\phi} \left[\frac{y^T - (1-\beta)B_0}{\bar{h}} \right]^{-\frac{1}{\gamma}}$$

implements the optimal allocations.

Proof. In Appendix [A.1](#) □

That is, when the initial and end-of-period debt level equals \hat{B} , we have that the borrowing constraint holds with equality. It follows then that for any level of debt $B_0 < \hat{B}$, the borrowing constraint is satisfied and a steady-state equilibrium exists. Moreover, at the steady-state equilibrium, the optimal monetary policy implements full employment.

In a steady-state equilibrium, we have that the borrowing constraint is slack. Thus, there is only one potential departure from the first-best allocation, the possibility of unemployment. It then follows that the optimal monetary policy achieves full employment, as item (ii) of the lemma shows. Full employment is achieved in this case by depreciating the currency enough so that the real wage falls and the nominal wage rigidity is not binding. Clearly, there is a wide range of monetary policies that deliver such an outcome. We focus on a policy that delivers zero inflation for $t = 0, 1, \dots$. We do this partly for simplicity and partly to capture the traditional price stability objective of central banks. The policy implies that the central bank sets the exchange rate at a constant level given by

$$\bar{e} = W_{-1} \frac{\phi}{1-\phi} \left(\frac{c^T}{\bar{h}} \right)^{-\frac{1}{\gamma}}. \quad (18)$$

To ensure consistency with a constant path for the exchange rate, the central bank needs to set a constant money supply \bar{M} . Using [\(MD\)](#), we have that level of nominal money supply is given by

$$\frac{\bar{M}}{\chi} = \frac{W_{-1}}{u_N(c^T, \bar{h})} \frac{R}{R-1}. \quad (19)$$

Notice that the value of \bar{e} and \bar{M} depend on B_0 . Namely, a higher B_0 implies a lower steady

level of consumption and therefore requires a higher \bar{e} for given W_{-1} . Intuitively, when the level of consumption is lower, the real exchange rate is also lower, and achieving a reduction in the real wage requires a higher nominal exchange rate. By condition (8) and the fact that the borrowing is not binding, it follows that the interest rates on the two bonds have to be equal:

$$\tilde{R} = R.$$

In the next section, we study how this policy does not guarantee that the steady-state equilibrium is uniquely implemented.

3 Self-Fulfilling Crises

In our economy, the amount that households can borrow is increasing in the price of non-tradable goods. Because the price of non-tradables is in turn increasing in the aggregate amount of borrowing, this implies that the borrowing capacity of an individual agent is increasing in the aggregate amount of borrowing. As shown formally [Schmitt-Grohé and Uribe \(2021\)](#), when this complementarity is strong enough, there is a possibility of multiple equilibria.¹³ That is, for a range of initial debt values, a steady-state equilibrium may coexist with another equilibrium in which households reduce their demand for borrowing, the real exchange rate depreciates, and tradable consumption falls.

In our model with monetary non-neutrality because of nominal rigidities, monetary policy may affect the vulnerability to self-fulfilling crises. Our goal is to characterize precisely how the exchange rate regime determines this vulnerability and how this affects the choice of the optimal monetary policy.

We assume that the economy starts period 0 with an initial debt position $B_0 < \hat{B}$. As shown in [Lemma 1](#), one possible competitive equilibrium in this case is the steady-state equilibrium in which $B_{t+1} = B_0$ for all t , consumption is constant, and the borrowing constraint does not bind. In addition, another equilibrium may also exist. We refer to a self-fulfilling crisis equilibrium as a competitive equilibrium featuring deleveraging and lower consumption in period 0. To facilitate the analysis, we focus on a situation in which allocations are constant after period 1.¹⁴

Definition 3 (Self-Fulfilling Crisis Equilibrium). A self-fulfilling crisis equilibrium is a

¹³See also [Mendoza \(1995\)](#) for an early discussion of this possibility. For related mechanisms in closed economy models leading to multiplicity see [Stein \(1995\)](#) and [Brunnermeier and Pedersen \(2009\)](#).

¹⁴This is without loss of generality, in the absence of uncertainty (see [Schmitt-Grohé and Uribe, 2021](#)).

competitive equilibrium in which $B_1 < B_0$.

The possibility of multiplicity of equilibria depends on the strength of the complementarity between aggregate borrowing decisions and the individual borrowing limit. As we will show formally below, the following assumption will be sufficient to guarantee this possibility.

Assumption 1. *The set of parameters satisfies*

$$\kappa \frac{1 - \phi}{\phi} \left[\frac{y^T - \frac{R-1}{R} \hat{B}}{\bar{h}} \right]^{\frac{1}{\gamma} - 1} > 1$$

We assume that Assumption 1 is satisfied in the rest of the paper. This assumption is consistent with a range of plausible parameter values from the data.¹⁵ Even though the model is stylized, it is worth highlighting that it has been shown to be able to replicate important regularities of emerging market business cycles and financial crises (Mendoza, 2002; Bianchi, 2011; Schmitt-Grohé and Uribe, 2021; Ottonello, 2021).

Roadmap In the next section, we will study how monetary policy affects the vulnerability to self-fulfilling financial crisis equilibria. In particular, we consider two monetary policy regimes: a fixed exchange rate and a flexible one. In a fixed exchange rate regime, monetary policy sets $e_t = \bar{e}$ and lets the money supply be determined endogenously. In a flexible exchange rate regime, monetary policy sets money supply $M_t = \bar{M}$, or alternatively commits to set M_t to achieve full employment $h_t = \bar{h}$, and let the exchange rate be determined endogenously.

In the absence of the borrowing constraint, either of these policies would uniquely implement the steady-state equilibrium for any $B_0 < \hat{B}$. However, we will show how anchoring the money supply or the exchange rate will have different implications for the existence of self-fulfilling crises.

3.1 Fixed Exchange Rate

We first focus on the vulnerability of the economy to self-fulfilling crisis equilibria under a fixed exchange rate regime in which the central bank sets $e_t = \bar{e}$, where \bar{e} corresponds to

¹⁵For example, if we take $\phi = 0.2$, in line with a 20% share of tradable-output to GDP, and $\kappa = 0.3$, in line with observed debt levels, an annual interest rate of $R = 1.04$, we obtain multiplicity for values of the elasticity γ between 0.5 and 1.

the efficient steady state level given by (18). We first establish that in a self-fulfilling crisis equilibrium, the economy experiences unemployment.

Lemma 2 (Unemployment in Self-Fulfilling Crisis). *In a self-fulfilling crisis, there is involuntary unemployment.*

Proof. In Appendix A.2 □

Under a fixed exchange rate, the downward nominal wage rigidity translates into a downward rigidity on the real wage. When households become unexpectedly pessimistic and increase their savings, the contraction in demand for non-tradables translates one-to-one to a fall in production, causing involuntary unemployment. Given that households work fewer hours than their aggregate endowment of hours, equilibrium in the labor market requires the downward nominal wage rigidity to be binding; this is, $W_0 = W_{-1}$. Notice that the relative price of non-tradables remains fixed at W_{-1}/e_0 , and so the borrowing capacity becomes

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right]. \quad (20)$$

The next proposition provides shows that for values of debt $B_0 \in ((1+\kappa)y^T, \hat{B})$, a self-fulfilling crisis exists.

Proposition 1 (Crises under Fixed Exchange Rate). *Suppose Assumption 1 holds and $\gamma < 1$. Under a fixed exchange rate policy with \bar{e} given by (18), we have that*

- i. there is a non-empty region of debt levels $B_0 \in ((1+\kappa)y^T, \hat{B})$ for which a unique self-fulfilling crisis equilibrium coexists with the steady-state equilibrium;*
- ii. for $B_0 < (1+\kappa)y^T$, we have a unique equilibrium, and this equilibrium is the steady-state equilibrium.*

Proof. In Appendix A.3 □

Figure 1 illustrates the existence of multiple equilibria. The downward-sloping solid line represents the steady-state borrowing limit of a household $\bar{b}(B, B)$, i.e. the individual borrowing limit when aggregate debt is constant over time. We can see that at the point where this line intersects the 45-degree line, we reach the point \hat{B} . If the initial debt were to start at that point, the borrowing constraint would hold with equality at steady date.

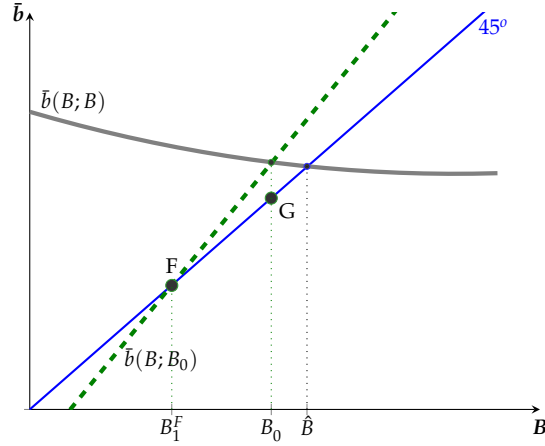


Figure 1: Equilibria under fixed exchange rate

The upward-sloping dashed line represents the individual borrowing limit $\bar{b}(B, B_0)$ for a given initial debt level B_0 , detailed in the figure. When this line intersects the downward-sloping line, we reach an equilibrium with a level of borrowing equal to the initial level B_0 . This is the good equilibrium represented by point G . To see that this is an equilibrium, notice that the borrowing capacity (represented by the intersection between the downward-sloping line and the upward-sloping dashed line) exceeds the actual level of borrowing (represented by the 45-degree line). When the upward-sloping dashed line intersects the 45-degree line, we have another equilibrium. This is the self-fulfilling crisis equilibrium represented by point F . Indeed, at that intersection, the amount of borrowing coincides with the borrowing limit, consumption falls and households are borrowing constrained.

Let us discuss the role of Assumption 1 in the proposition. Assumption 1 says that when evaluated at \hat{B} , the derivative of $\bar{b}(B, B_0)$ with respect to B is larger than one (i.e., an increase in aggregate borrowing expands the individual borrowing capacity by more than one unit). By continuity, this implies that the slope of the dashed line evaluated at a B_0 sufficiently close to \hat{B} is larger than one, as illustrated in the figure. Thus, in addition to the equilibrium point G , there exists another equilibrium point F at which the dashed line crosses the 45-degree line, in which case the borrowing constraint becomes binding.

Finally, given the possibility of multiplicity, it is important to discuss how the central bank is able to implement the target exchange rate \bar{e} . In our model, this is guaranteed by the fact that the central bank has access to lump-sum taxes and transfers. Thus, by accommodating any changes in money demand by injecting or withdrawing currency, it can promise to buy/sell foreign currency at the announced exchange rate and implement the desired level.

3.2 Flexible Exchange Rate

We now turn to analyze the possibility of a flexible exchange rate regime in which the central bank lets the nominal exchange rate fluctuate freely.

We focus first on a flexible exchange rate regime in which the central bank sets money supply $M_t = \bar{M}$, where \bar{M} is the efficient steady state level given by (19).¹⁶ We first establish that in a self-fulfilling crisis equilibrium, the economy experiences a nominal exchange rate depreciation and unemployment.

Lemma 3 (Unemployment Under Fixed Money Supply). *In a self-fulfilling crisis, the exchange rate depreciates at $t = 0$, and there is unemployment.*

Proof. In Appendix A.4 □

For a given nominal wage and price of non-tradables, a reduction in borrowing implies a reduction in the demand for non-tradable goods and output. In this case, the borrowing capacity becomes

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1 - \phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right], \quad (21)$$

where the equilibrium exchange rate e_0 is such that the demand for money (MD) equals the fixed supply of money \bar{M} . The difference relative to (20) is that now the exchange rate will adjust endogenously. The proposition below characterizes how the region of multiple equilibria expands relative to the economy with a fixed exchange rate.

Proposition 2 (Crises under Fixed Money Supply). *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate with \bar{M} given by (19),*

- i. if $B_0 \in ((1 + \kappa)y^T, \hat{B})$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium. Moreover, we have that $\hat{B} > (1 + \kappa)y^T$, and thus the interval is non-empty;*
- ii. if $B_0 \in [\underline{B}^m, (1 + \kappa)y^T)$, there exist two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, where \underline{B}^m is given by (A.18).*
- iii. if $B_0 < \underline{B}^m$, we have one and only one equilibrium (which corresponds to the steady-state equilibrium).*

¹⁶Given our assumption on the separability between consumption and money balances, we can guarantee that the steady state equilibrium has a unique price level (Benhabib, Schmitt-Grohé and Uribe, 2001).

Proof. In Appendix A.5 □

When $B_0 \in ((1 + \kappa)y^T, \hat{B})$, we have again a unique self-fulfilling crisis equilibrium coexisting with the steady-state equilibrium. However, when the initial debt level belongs to the interval $[\underline{B}^m, (1 + \kappa)y^T)$, we now have that two self-fulfilling crisis equilibria can emerge under flexible exchange rates. The set of possible equilibria are illustrated in Figure 2. Again, the downward-sloping solid line represents the steady state borrowing limit of a household $\bar{b}(B, B)$. Under $\gamma < 1$, the upward-sloping dashed line is now convex, giving rise to two self-fulfilling crisis equilibrium, represented by points F and F'.

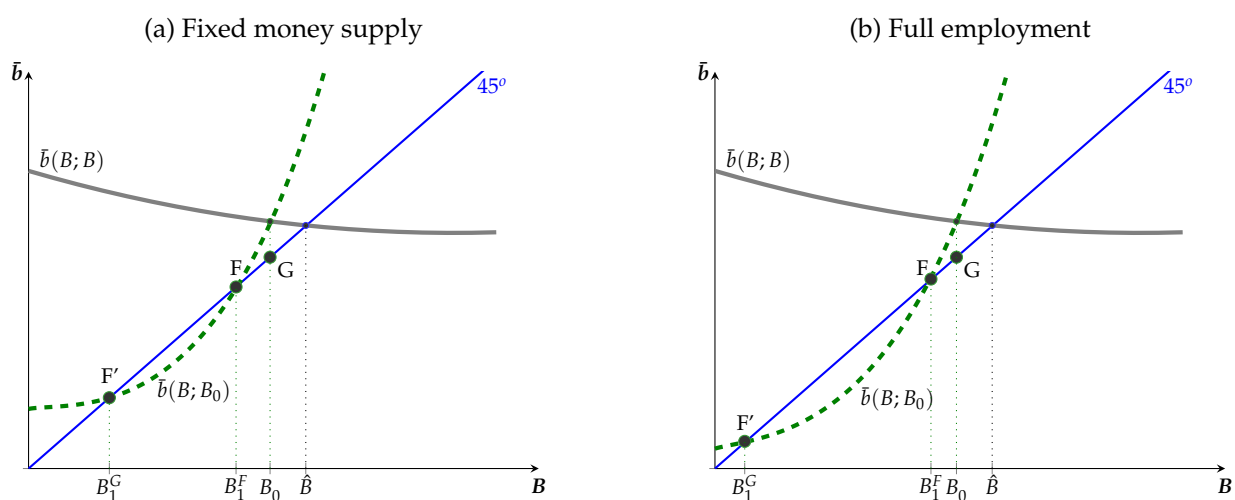


Figure 2: Equilibria under flexible exchange rates

Full employment. We now consider a regime in which the central bank adjusts the money supply to implement full employment. We first show that this is indeed feasible for the central bank.

Lemma 4. *Given a competitive equilibrium with flexible wages, there exists a nominal exchange rate policy under sticky wages that implements the flexible wage allocation.*

Because the economy under flexible wages is always at full employment, we can refer interchangeably to this economy as one with flexible wages or one with a full employment policy. Interestingly, as we will show below, achieving an increase in employment may require an appreciation of the exchange rate, rather than a depreciation.

In this case, the borrowing capacity becomes given by

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B_0 + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right]$$

Echoing the results in [Schmitt-Grohé and Uribe \(2021\)](#), we have the following proposition

Proposition 3 (Crises Under Full Employment Policy). *Suppose Assumption 1 holds and $\gamma < 1$. Then, under flexible wages,*

- i. if $B_0 \in ((1 + \kappa)y^T, \hat{B})$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium. Moreover, we have that $\hat{B} > (1 + \kappa)y^T$, and thus the interval is non-empty;*
- ii. if $b_0 \in [\underline{B}, (1 + \kappa)y^T)$, there exist two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, where \underline{B} is given by*

$$\underline{B} \equiv (1 + \kappa)y^T - (1 - \gamma) \left[\frac{1}{\gamma} \frac{\kappa(1 - \phi)}{\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h} < \underline{B}^m;$$

- iii. if $B_0 < \underline{B}$, we have one and only one equilibrium (which corresponds to the steady-state equilibrium).*

Proof. In Appendix [A.6](#) □

The proposition establishes that an economy under a full employment policy features two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium for an intermediate level of debt $\underline{B} < B_0 < (1 + \kappa)y^T$. Relative to the economy with a fixed money supply, there is now a lower threshold for the existence of multiplicity.¹⁷

4 To Fix or to Float?

Having characterized the outcomes under fixed and flexible exchange rates in the previous section, we now inspect how they compare them in terms of output and welfare.

¹⁷In Appendix [B](#), we also consider two other cases, an interest rate peg and a Taylor rule. The characterization is similar to that of Proposition 2 (with fixed money supply) and features a smaller region of multiplicity relative to the full employment case.

4.1 Comparison of Crisis Region

We start by comparing the “crisis regions” under flexible and fixed exchange rates (i.e., the debt levels for which a self-fulfilling crisis equilibrium exists). In light of Propositions 1, 2, and 3, we have the following corollary.

Corollary 1 (Exchange Rate Regimes and Vulnerability to Crises). *Suppose Assumption 1 holds and $\gamma < 1$. Let $\Omega_{\bar{e}}, \Omega_{\bar{M}}, \Omega_{\bar{h}}$ be the regions of debt levels for which self-fulfilling crisis equilibria coexist with the steady-state equilibrium under fixed exchange rate, flexible exchange rate with fixed money supply and full employment regimes. We have that the crisis region is smaller under fixed exchange rates:*

$$\Omega_{\bar{e}} \subset \Omega_{\bar{M}} \subset \Omega_{\bar{h}}$$

The results uncover a paradox of exchange rate flexibility. When the central bank fixes the exchange rate, it is able to shrink the region in which it is vulnerable to a self-fulfilling crisis. The logic is that under a floating regime when households become pessimistic and decide to cut consumption, the feedback loop between the depreciation of the nominal exchange rate and the fall in the borrowing capacity shrinks the ability of households to borrow. As they are forced to deleverage and cut consumption, this validates the initial pessimism by tightening in general equilibrium borrowing limits of all households.

To shed further light on how the exchange rate regime affects the vulnerability, we numerically solve the model and present the policy functions in Figure 3 under the different regimes. The dotted line illustrates the steady-state equilibrium, which exists for all debt levels below \hat{B} . The red. The blue broken line indicates the self-fulfilling crisis equilibrium under a fixed exchange rate. The red solid line indicates the equilibria under fixed money supply and the dashed green one represents the equilibria under full employment policy.

For sufficiently low debt levels, all regimes feature the steady state equilibrium. In this region, tradable consumption is decreasing in the debt level, but the economy remains at full employment.¹⁸ For debt levels higher than $(1 + \kappa)y^T$ and lower than \hat{B} , a self-fulfilling crisis equilibrium emerges for all policy regimes considered. For intermediate debt levels, we have a unique equilibrium under fixed exchange rates while we have two self-fulfilling crisis equilibria under flexible exchange rates. In this region, we can see that within the two self-fulfilling crisis equilibria under flex, the one with lower borrowing (indicated with the lighter shade) also features lower tradable consumption, lower employment, and a more depreciated exchange rate.

¹⁸Notice that even though allocations are the same under the different regimes in this region, exchange rates still differ, but for reasons of scale, this is not apparent in the plot.

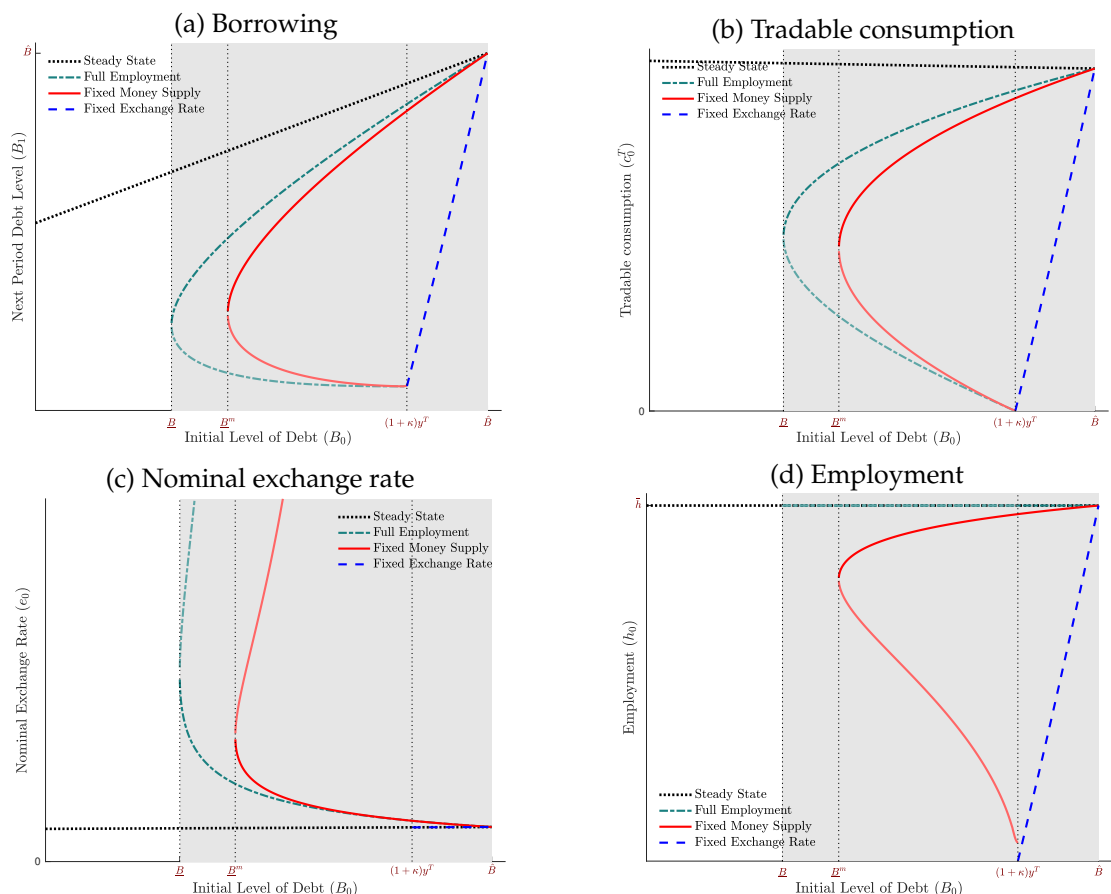


Figure 3: Policy functions

Note: Parameter values are $\phi = 0.2$, $\kappa = 0.3$, $W_{-1} = 1$, $R = 1.04$, $\beta = 1/R$, $\gamma = 0.4$.

Notice also that the nominal exchange is more appreciated under a full employment policy that under a flexible exchange rate with fixed money supply. This suggests the possibility that a depreciation may be contractionary, a point to which we will return below. The key takeaway is that flexible exchange rates, rather than working as a shock absorber, may actually exacerbate economic fluctuations by exposing the economy to self-fulfilling crises.

4.2 Welfare

The fact that the exchange rate regime affects the crisis region has clear implications for welfare. Suppose that the economy starts with $B_0 < \hat{B}$ in period 0. Let π denote the probability that the economy ends in a self-fulfilling crisis equilibrium when the economy is in the crisis region. Abstracting from the utility of money balances to compute welfare,

we obtain the following result.¹⁹

Corollary 2 (Fear of Floating). *Suppose Assumption 1 holds and $\gamma < 1$. Let W_{fix} and W_{flex} be the welfare under fixed and flexible exchange rate regimes, respectively. Then, for any $\pi > 0$ and any initial debt level $B_0 \leq (1 + \kappa)y^T$, we have $W_{fix} > W_{flex}$.*

Proof. In Appendix A.7 □

When a fixed exchange rate ensures a unique equilibrium, it becomes optimal to fix the exchange rate. Letting the nominal exchange rate fluctuate leads to perverse movements in prices and output that make the economy vulnerable to a self-fulfilling financial crisis.

4.3 Contractionary Depreciations

We have shown above that for a given money supply, a self-fulfilling crisis generates a depreciation of the nominal exchange rate and an economic contraction. A related yet different question is whether a depreciation chosen by policy can be contractionary. The following proposition shows that this is a possibility.

Proposition 4 (Contractionary Depreciations). *Assume a level of debt B_0 such that the borrowing constraint binds. Let $y^N(B_0, e_0)$ be the equilibrium level of output as a function of the initial exchange rate e_0 for an initial debt level B_0 . Then, we have that*

- i) If $B_0 < (1 + \kappa)y^T$ and $\gamma < 1$, $y^N(B_0, e_0)$ is decreasing in e_0 if $e_0 \in [\bar{e}, \underline{e}]$ where \bar{e} is given by (18) and \underline{e} is defined in (A.22). Moreover, it is strictly decreasing if $e_0 \in [\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e}]$.*
- ii) If $B_0 > (1 + \kappa)y^T$ and $\gamma > 1$, $y^N(B_0, e_0)$ is decreasing in e_0 if $e_0 \in [\bar{e}, \underline{e}]$ where \bar{e} is given by (18) and \underline{e} is defined in (A.22). Moreover, it is strictly decreasing if $e_0 \in [\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e}]$.*

Finally, if (i) and (ii) are not satisfied, a depreciation is expansionary.

Proof. In Appendix A.8 □

To understand why a depreciation may be contractionary, let us substitute the borrowing constraint with equality in (15) and totally differentiate to obtain

$$\frac{dc_0^N}{de_0} = \frac{c_0^N}{e_0} \left[\gamma + \frac{e_0}{c_0^T} \cdot \frac{1}{R} \frac{d\bar{b}(e_0, B_1; B_0)}{de_0} \right], \quad (22)$$

¹⁹Formally, one can also follow the standard cashless limit.

where

$$d\bar{b}_1 = \kappa \frac{W_{-1}}{e_0} \left[dy_0^N - \frac{y_0^N}{e_0} de_0 \right] \quad (23)$$

Expression (22) spells out two channels by which a nominal exchange depreciation affects demand for non-tradable consumption. First, given a level of resources, a depreciation shifts expenditure toward domestically produced goods. This is the standard expenditure-switching channel that makes depreciations expansionary.

Second, through general equilibrium effects, a depreciation also alters the resources available through a collateral channel, a term characterized in (23). When there is a depreciation, the number of hours that firms demand changes. At the same, given the number of hours (and non-tradable output), depreciation reduces the value measured in units of tradables. Depending on which of these two effects dominates, the collateral channel can be expansionary or contractionary. If it is expansionary (i.e., the number of hours effect dominates the relative price effect), then, the overall effect of a depreciation is to contract output. On the other hand, if the collateral channel is contractionary, the overall effect depends on the strength of this channel relative to the expenditure-switching channel. Which effect dominates can be determined by using that in equilibrium $dy_0^N = dc_0^N$ and combining (22) and (23) to solve for dy_0^N and $d\bar{b}$. The proposition uses these relationships to characterize when a depreciation is contractionary.

Notice that the proposition does not use Assumption 1. This implies that our results on contractionary depreciation apply also in a configuration with a unique equilibrium in which the economy faces a binding borrowing constraint. The general result is that when an economy faces a binding borrowing constraint, it is possible that a depreciation leads to a contraction in aggregate demand and output.

The results above can help understand the connection with the “credit access unemployment tradeoff” in Ottonello (2021). A central result in his model is that the optimal exchange rate policy does not necessarily implement the full employment allocation. This is because a departure from full employment can be associated with a more appreciated real exchange rate and a more relaxed borrowing limit, *for given tradable consumption*.²⁰ Our results are consistent with his, but on the other hand, demonstrate that it is possible that an appreciation is actually expansionary (in which case there is no tradeoff at play as

²⁰This can be seen from the fact that the borrowing capacity can be written as $\kappa \left[y^T + \frac{1-\phi}{\phi} (c^T)^{\frac{1}{\gamma}} h^{\frac{\gamma-1}{\gamma}} \right]$, which is decreasing in h if $\gamma < 1$. This mechanism is also present in Coulibaly (2020), Basu et al. (2020), and one of the applications in Farhi and Werning (2016).

an appreciation is unambiguously welfare improving). When an appreciation expands the borrowing capacity, this raises demand for consumption, and this collateral channel can offset the expenditure-switching channel, in line with eqs. (22) and (23). Therefore, as highlighted in Proposition 4, an appreciation can be expansionary (and a depreciation contractionary).

In the section below, we leverage these insights to study the optimal exchange rate policy when taking into account the multiplicity from policy instruments to equilibrium outcomes and highlighting the role of commitment.

5 Optimal Policy

Until now, we consider the equilibrium outcomes when the central bank sets an instrument once and for all at the beginning of time. In particular, we consider a flexible exchange rate regime where the central bank sets the money supply and lets the exchange rate float, and a fixed exchange rate where the central bank sets the exchange rate and lets the money supply adjust. Moreover, in both cases, the instrument is set so that there is no inflation in the good equilibrium. We now examine the optimal policy when the central bank chooses the monetary policy stance optimally, first under discretion, and then under commitment.

5.1 Markov Perfect Equilibrium

In this section, we show that when the central bank chooses monetary policy optimally without commitment, it ends up magnifying the vulnerability to self-fulfilling crises.

Because policies can lead to multiple outcomes, analyzing the optimal policy requires being specific about the precise timing of actions. We consider the following timing within the period: (i) households choose b' ; (ii) the central bank chooses e ; ²¹ (iii) households choose c^T, c^N and firms choose h .

We solve for the Markov perfect equilibrium (MPE) by backward induction. For any initial value of debt B and any possible B' chosen by households, we can express the

²¹It is equivalent to formulate the problem as the government choosing M instead of e .

problem of the central bank as follows:

$$\max_{c^T, e, h \leq \bar{h}, W \geq W_{-1}} u(c^T, h) + \frac{\beta}{1 - \beta} u \left[y^T - \frac{R - 1}{R} B', \bar{h} \right], \quad (24)$$

subject to

$$\begin{aligned} c^T &= y^T - B + \frac{B'}{R} \\ h &= \left(\frac{1 - \phi}{\phi} \frac{e}{W} \right)^\gamma c^T \\ \frac{B'}{R} &\leq \kappa \left[y^T + \left(\frac{1 - \phi}{\phi} \right)^\gamma \left(\frac{W}{e} \right)^{1 - \gamma} c^T \right] \end{aligned}$$

where we have used that the continuation value is such that the economy is in a stationary equilibrium with debt level B' . An inspection of this problem reveals that the central bank must choose a level of employment and associated level of the exchange rate that induces a feasible level of borrowing for the household. Moreover, the central bank finds it optimal to choose the highest level of employment consistent with a valid continuation equilibrium. We summarize this result in the following proposition.

Proposition 5 (Optimal Policy in a MPE). *For any B' , the optimal monetary policy $\mathcal{E}(B'; S)$ in a Markov perfect equilibrium implements*

$$\mathcal{H}(B'; S) = \min \{ \tilde{h}(B'; S), \bar{h} \} \quad (25)$$

where

$$\tilde{h}(B'; S) = \left[y^T - B + \frac{B'}{R} \right]^{\frac{1}{1 - \gamma}} \left[\frac{\kappa(1 - \phi)}{\phi \left(\frac{B'}{R} - \kappa y^T \right)} \right]^{\frac{\gamma}{1 - \gamma}}$$

Proof. In Appendix [A.9](#) □

From Proposition 5, we obtain the central bank policy for e with implied c^T, h, W for any B' chosen by households. Letting $S \equiv (B, W_{-1})$ summarize the aggregate state of the economy at the beginning of the period, the problem of an individual household is given

by

$$\max_{c^T, c^N, b'} u(c^T, c^N) + \beta V(b', S') \quad (26)$$

subject to

$$\begin{aligned} \mathcal{E}(B'; S)c^T + W(B'; S)c^N + b &= \mathcal{E}(B'; S)y^T + W(B'; S)\mathcal{H}(B'; S) + \mathcal{E}(B'; S)\frac{b'}{R} \\ \frac{b'}{R} &\leq \kappa \left[y^T + \frac{W(B'; S)}{\mathcal{E}(B'; S)}\mathcal{H}(B'; S) \right] \end{aligned}$$

In this problem, households choose b' taking as given the aggregate choice B' and the government policy. In a Markov perfect equilibrium as defined below, the conjectured decisions for aggregate debt and exchange rate policy have to be consistent with the actual choices made by households and the central bank.

Definition 4 (Markov Perfect Equilibrium). A Markov perfect equilibrium is defined by central bank policy $\mathcal{E}(B'; S)$, policy functions $\mathcal{H}(B'; S)$ and $\mathcal{C}^T(B'; S)$, and decision rules $b'(b, B'; S)$ such that

1. Households' optimization: $b'(b, B'; S)$ solve the household's problem (26) given $\mathcal{E}(B'; S)$ and $\mathcal{H}(B'; S)$;
2. Central bank's optimization: $\{\mathcal{E}(B'; S), \mathcal{H}(B'; S), \mathcal{C}^T(B'; S)\}$ solve the central bank's optimal policy problem (24) given B' ;
3. Consistency: The conjectured aggregate debt matches individual household's optimal borrowing decision rule $b'(B, B'; S) = B'$.

The next Lemma establishes that there is a continuum of Markov perfect equilibrium and the set of equilibria is convex.

Lemma 5 (Convexity). *The set of debt levels B' that constitute a Markov perfect equilibrium is convex.*

Proof. In Appendix A.10 □

Given two levels of debt that constitute a Markov perfect equilibrium, a convex combination of those debt levels is also a Markov perfect equilibrium. Therefore, determining the lowest and highest debt level suffices to characterize the set of debt levels that constitute a Markov perfect equilibrium. The following proposition characterizes these bounds.

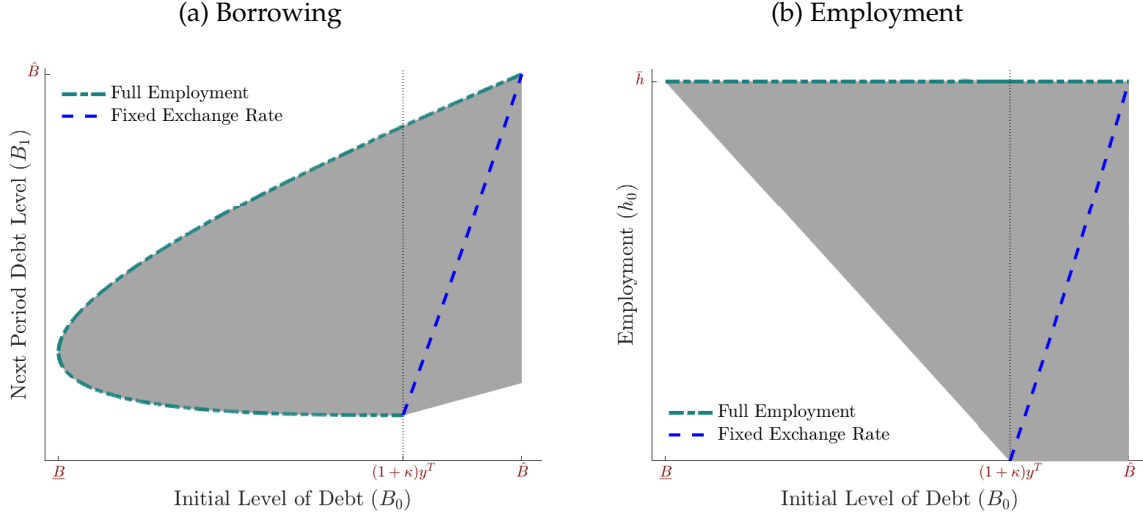


Figure 4: Set of Markov perfect equilibria

Note: parameter values are $\phi = 0.2$, $\kappa = 0.3$, $W_{-1} = 1$, $R = 1.04$, $\beta = 1/R$, $\gamma = 0.4$.

Proposition 6 (Worst and Best MPE). *The range of MPE is such that:*

- i. *If $B_0 < (1 + \kappa)y^T$, the lowest B' in the set of Markov perfect equilibria corresponds to B' in the worst of the two crisis equilibria under full employment policy;*
- ii. *If $B_0 < \hat{B}$, the highest B' in the set of Markov perfect equilibria corresponds to B' in the best of the two crisis equilibria under full employment policy.*

Proof. In Appendix [A.11](#) □

The proposition establishes that the highest possible value of debt in the MPE corresponds to the level of debt in the best of the two self-fulfilling crisis equilibrium under full employment. Meanwhile, we show that for $B_0 < (1 + \kappa)y^T$, the lowest possible value of debt in the MPE corresponds to the level of debt in the worst of the two self-fulfilling crisis equilibrium under full employment. These results are illustrated in Figure 4.

5.2 Monetary Policy under Commitment

In this section, we show how the ability to commit to monetary policy can help avert self-fulfilling financial crises.

Our approach follows [Bassetto \(2005\)](#) and [Atkeson et al. \(2010\)](#) in that we allow the central bank to commit to a strategy that depends upon the choices of households. We assume that now the central bank announces a commitment to the state contingent exchange rate policy that $e(B_1, B_0)$ before households choose the level of borrowing.²² Individual households choose their individual level of borrowing b_1 based on their belief about aggregate borrowing B_1 , after which the central bank sets M_0 to implement the exchange rate $e(B_1, B_0)$ to which it committed. Finally, households choose their level of consumption, firms choose employment and markets clear.²³

The next proposition describes the monetary policy strategy that can avert self-fulfilling crises.

Proposition 7 (Unique implementation with sophisticated monetary policy). *There exists an exchange rate rule $e(B_1, B_0)$ that rules out the possibility of self-fulfilling crises equilibria. Given the initial debt-to-tradable output ratio of the economy, this rule can be described as follows:*

$$e(B_1, B_0) = \begin{cases} \bar{e} & \text{if } B_0 \leq (1 + \kappa)y^T \\ \bar{e} \left[\frac{B_1}{B_0} + \left(1 - \frac{B_1}{B_0}\right) \Phi(B_1, B_0) \right] & \text{otherwise} \end{cases} \quad (27)$$

where \bar{e} is given by [\(18\)](#) and

$$\Phi(B_1, B_0) \equiv \left[\frac{y^T - (1 - \beta) B_0}{\bar{h}} \right]^{\frac{1}{\gamma}} \left[\kappa \frac{1 - \phi R(y^T - B_0) + B_1}{B_1 - R\kappa y^T} \right]^{\frac{1}{1-\gamma}}$$

Proof. In [Appendix A.12](#) □

When the economy starts with a relatively low level of debt, $B_0 \leq (1 + \kappa)y^T$, an announcement by the central bank to commit to stabilizing the exchange rate at its natural level is sufficient to guarantee the implementation of the desirable outcome, in line with [Proposition 1](#). When the initial debt exceeds that amount, a non-state contingent commitment is not enough to uniquely implement the good equilibrium. However, the proposition presents a sophisticated policy that can rule out a self-fulfilling crisis. As shown in [\(27\)](#), the exchange rate turns out to be a combination of the desired exchange rate level and the exchange rate that the government chooses in the Markov perfect equilibrium for given B_1 , with weights that depend on the deviation of the net foreign asset position

²²Notice that we do not need to specify policies in response to non-degenerate actions by households, because the household optimum is unique and thus government responses to non-degenerate actions are irrelevant for the game (see [Bassetto, 2005](#) for a discussion).

²³[Schmitt-Grohé and Uribe \(2016\)](#) provide a feedback rule for capital control that can also implement the good equilibrium. Methodologically, we follow more closely [Bassetto \(2005\)](#) and [Atkeson et al. \(2010\)](#).

relative to the efficient one. This policy can therefore be interpreted as a *flotation band*. Notice that this rule implements the first-best allocation, $e(B_1, B_0) = \bar{e}$, if aggregate borrowing coincides with the desired level of borrowing $B_1 = B_0$. However, when aggregate borrowing falls below the desirable level, the central bank tolerates exchange rate depreciations but commits to appreciating it below its level under free floating. At the expense of creating involuntary unemployment, the appreciation of the nominal exchange rate relaxes the individual household's borrowing constraint, making $b_1 = B_1$ suboptimal from the individual household's perspective. The policy rule (27) thus ensures the uniqueness of the steady-state equilibrium by making the best response of each household different from the average choice whenever $B_0 < B_1$, and hence discouraging deviations from the desired level of borrowing.²⁴

A key takeaway from this section is that an active central bank policy can backfire in the absence of government commitment. However, a commitment to a policy resembling a crawling peg can ensure the unique implementation of the good equilibrium and thus rule out self-fulfilling financial crises.

6 Conclusion

We provide a theory of fear of floating, the ubiquitous policy among central banks of preventing large fluctuations in exchange rates. In our model, an exchange rate depreciation does not play the role of a shock absorber, in contrast to the Mundell-Fleming paradigm. Instead, it can be contractionary and make the economy more vulnerable to a self-fulfilling financial crisis.

The central mechanism in the paper that gives rise to financial fragility emerges from the interaction between a feedback loop between relative prices and borrowing conditions and the lack of commitment of monetary policy to keeping the exchange rate stable. While the precise details involve external borrowing and a relative price between tradables and non-tradables, the key insight can be extended to other frameworks involving domestic borrowing and asset prices. In addition, we have only considered non-fundamental uncertainty. However, our analysis can be extended to allow for the coexistence of fundamental and non-fundamental uncertainty. We leave an exploration of these channels for future work.

²⁴Recent work by *Itskhoki and Mukhin (2022)* point out a different mechanism that can justify a managed peg. In their model, intermediaries are exposed to exchange rate risk and stabilizing the exchange rate can contribute to increase risk-sharing.

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APPENDIX TO “A THEORY OF FEAR OF FLOATING”

A Proofs

A.1 Proof of Lemma 1

We start by showing that the steady-state equilibrium exists if $B_0 \leq \hat{B}$. At the steady-state equilibrium $B_{t+1} = B_0$ for all t and by (14) $c^T = y^T - \frac{R-1}{R}B_0$. The equilibrium exists if the collateral constraint is satisfied. That is, if

$$B_0 \leq \bar{b}(B_0; B_0) = \kappa R \left[y_t^T + \frac{1-\phi}{\phi} \left(y_t^T - \frac{R-1}{R}B_0 \right)^{\frac{1}{\gamma}} (h)^{\frac{\gamma-1}{\gamma}} \right].$$

where $h \leq \bar{h}$ is the steady-state level of employment. Because $\bar{b}(\hat{B}; \hat{B}) = \hat{B}$ (by definition of \hat{B}) and $\frac{\partial \bar{b}(B_0; B_0)}{\partial B_0} < 0$, it follows that for any $B_0 \leq \hat{B}$ we have $\bar{b}(B_0; B_0) \geq B_0$. Moreover, $\kappa < \frac{R}{R-1}$ ensures that $c^T > 0$.

The second part of the proof requires showing that it is optimal for the government to implement a full-employment allocation. Because allocations are constant at the steady-state equilibrium, from (5) we have

$$(1 - \mu)u_T(c^T, h) = u_T(c^T, h) \Rightarrow \mu = 0$$

The borrowing constraint does not bind. The problem of the central bank then reduces to

$$\max_{c^T, h, e, W \geq W_{-1}} \frac{1}{1 - \beta} u(c^T, h),$$

subject to

$$c^T = y^T - \frac{R-1}{R}B_0 \tag{A.1}$$

$$h = \left(\frac{1-\phi}{\phi} \frac{e}{W} \right)^{\gamma} c^T \tag{A.2}$$

$$h \leq \bar{h} \tag{A.3}$$

Because e only appears in (A.2), it is immediate that (A.2) does not bind. Since the objective is strictly increasing in h , it must be that (A.3) binds, and thus $h = \bar{h}$.

Finally plugging $h = \bar{h}$ and (A.1) into (A.2) and using $W \geq W_{-1}$ we get

$$e \geq W_{-1} \frac{\phi}{1-\phi} \left[\frac{y^T - (1-\beta)B_0}{\bar{h}} \right]^{-\frac{1}{\gamma}} \tag{A.4}$$

□

A.2 Proof of Lemma 2

The proof is by contradiction. Suppose that $h_0 = \bar{h}$. From (15), we have

$$W_0 = \bar{e} \frac{1 - \phi}{\phi} \left(\frac{y^T - B_0 + \frac{B_1}{R}}{\bar{h}} \right)^{\frac{1}{\gamma}} \quad (\text{A.5})$$

By definition of \bar{e} , we also have that wages in a steady state equilibrium satisfies

$$W_{-1} = \bar{e} \frac{1 - \phi}{\phi} \left(\frac{y^T - \frac{R-1}{R}B_0}{\bar{h}} \right)^{\frac{1}{\gamma}} \quad (\text{A.6})$$

Because $B_1 < B_0$, (A.5) and (A.6) imply that $W_0 < W_{-1}$ which violates downward wage rigidity. Therefore, $h_0 < \bar{h}$ in a self-fulfilling crisis equilibrium. □

A.3 Proof of Proposition 1

The maximum borrowing of an individual household under fixed exchange rates is

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1 - \phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right]$$

and we have that

$$\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} = \left(\frac{1 - \phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma}$$

Notice that B_1 is part of an equilibrium if $\bar{b}(B_1; B_0) = B_1$, $B_1 < B_0$, and $\frac{B_1}{R} > B_0 - y^T$. The first condition is such that the constraint holds with equality. The second condition ensures that $\mu > 0$ and the last condition ensures that $c_0^T > 0$. Because $\bar{b}(B_0; B_0) > B_0$, that is the borrowing constraint does not bind in the stationary equilibrium, a sufficient condition for non-existence B_1 that satisfies the first two conditions is $\partial \bar{b}(B_1; B_0) / \partial B_1 < 1$.

Using (18) to substitute for W_{-1} / \bar{e} leads to

$$\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} = \kappa \frac{1 - \phi}{\phi} \left(\frac{y^T - \frac{R-1}{R}B_0}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} > \kappa \frac{1 - \phi}{\phi} \left(\frac{y^T - \frac{R-1}{R}\hat{B}}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} > 1$$

where the first inequality uses $B_0 < \hat{B}$ and the last inequality uses Assumption 1. Given

that $\bar{b}(B_0; B_0) > B_0$ and $\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} > 1$, it follows by continuity of the function $\bar{b}(B_1; B_0) - B_1$ that there exists $B_1 < B_0$ such that $\bar{b}(B_1; B_0) - B_1 = 0$. Next, we need to check condition under which $c_0^T > 0$ in the self-fulfilling crises equilibrium. Using $\bar{b}(B_1; B_0) = B_1$ and the resource constraint (14) we get

$$c_0^T = \frac{B_0 - (1 + \kappa)y^T}{\kappa \left(\frac{1-\phi}{\phi}\right)^\gamma \left(\frac{W_{-1}}{\bar{\varepsilon}}\right)^{1-\gamma} - 1}$$

Hence, $c_0^T > 0$ if and only if $B_0 > (1 + \kappa)y^T$. Moreover, because $B_0 < \hat{B}$ it follows that a self-fulfilling crisis equilibrium coexists with the stationary equilibrium under $e_0 = \bar{\varepsilon}$ for any $B_0 \in ((1 + \kappa)y^T, \hat{B})$. It remains to show that $((1 + \kappa)y^T, \hat{B})$ is non-empty. Recall that $\bar{B}(\hat{B}, \hat{B}) = 0$, that is

$$\begin{aligned} \frac{\hat{B}}{R} &= \kappa y^T + \kappa \frac{1-\phi}{\phi} \left(y^T - \frac{R-1}{R} \hat{B}\right)^{\frac{1}{\gamma}} (\bar{h})^{\frac{\gamma-1}{\gamma}} \\ &= \kappa y^T + \hat{c}^T \kappa \frac{1-\phi}{\phi} \left[\frac{y^T - (1-\beta)\hat{B}}{\bar{h}}\right]^{\frac{1-\gamma}{\gamma}} \end{aligned} \quad (\text{A.7})$$

Using the resource constraint $\hat{c}^T = y^T - \hat{B} + \frac{\hat{B}}{R}$ and substituting (A.7), we get

$$\left[1 - \kappa \frac{1-\phi}{\phi} \left(\frac{y^T - (1-\beta)\hat{B}}{\bar{h}}\right)^{\frac{1-\gamma}{\gamma}}\right] \hat{c}^T = (1 + \kappa)y^T - \hat{B} \quad (\text{A.8})$$

Assumption 1 implies that the left-hand side of equation (A.8) is negative. Therefore, $(1 + \kappa)y^T < \hat{B}$. The interval $((1 + \kappa)y^T, \hat{B})$ is thus non-empty. \square

A.4 Proof of Lemma 3

Unemployment. Assume by contradiction that $h_0 = \bar{h}$. The demand for money in the steady state equilibrium and in period 0 are given by

$$\frac{\chi W_{-1}}{\bar{M}} = \left[1 - \frac{1}{R}\right] u_N \left(y^T - \frac{R-1}{R} B_0, \bar{h}\right) \quad (\text{A.9a})$$

$$\frac{\chi W_0}{\bar{M}} = \left[1 - \frac{1}{\tilde{R}_0}\right] u_N \left(y^T - B_0 + \frac{B_1}{R}, \bar{h}\right) \quad (\text{A.9b})$$

it must be that $\tilde{R}_0 \geq R$. This is because if it were that $\tilde{R}_0 < R$ then by (A.9a) and (A.9b) we have that $W_0 < W_{-1}$ which violates the constraint on the nominal wage. Given that

$\tilde{R}_0 \geq R$, from the (8) condition $e_0 \leq \frac{e_1}{1-\mu_0}$. Then, using (15) we get

$$h_0 = \left[\frac{1-\phi}{\phi} \frac{e_0}{W_0} \right]^\gamma c_0^T \leq \left[\frac{1-\phi}{\phi} \frac{e_1}{W_0} \frac{1}{1-\mu_0} \right]^\gamma c_0^T$$

Using (5) to substitute for μ_0 we arrive to

$$h_0 \leq \left[\frac{1-\phi}{\phi} \frac{e_1}{W_0} \right]^\gamma c_1^T \left(\frac{c_0}{c_1} \right)^{1-\gamma} < \left[\frac{1-\phi}{\phi} \frac{e_1}{W_0} \right]^\gamma c_1^T = \bar{h} \quad (\text{A.10})$$

where the second inequality uses $c_0 < c_1$ by $c_0^T < c_1^T$. (A.10) contradicts $h = \bar{h}$. Therefore, $h_0 < \bar{h}$ in a self-fulfilling crises equilibrium when it exists.

Exchange depreciation. Using (8) and (5) to substitute for \tilde{R}_0 and μ_0 , (MD) becomes

$$\frac{\chi e_0}{\bar{M}} = \left[1 - \frac{e_0}{R e_1} \frac{u_T(c_1^T, \bar{h})}{u_T(c_0^T, c_0^N)} \right] u_T(c_0^T, c_0^N) \quad (\text{A.11})$$

Using $\frac{e_1}{W_0} = \frac{u_T(c_1^T, \bar{h})}{u_N(c_1^T, \bar{h})}$ and plugging it into (A.11) we get

$$\frac{\chi}{\bar{M}} = \left[\frac{1}{e_0} u_T \left(c_0^T, \left(\frac{1-\phi}{\phi} \frac{e_0}{W_{-1}} \right)^\gamma c_0^T \right) - \frac{1}{R W_{-1}} u_N(c_1^T, \bar{h}) \right] \quad (\text{A.12})$$

Note that $W_0 = W_{-1}$ because $h_0 < \bar{h}$. We use $c_0^T = y^T - B_0 + \frac{B_1}{R}$ and $c_1^T = y^T + (1-\beta)B_1$ and totally differentiate (A.12) with respect to B_1 to obtain

$$[\gamma + (1-\gamma)\tilde{\phi}_0] \frac{R c_0^T}{e_0} \frac{de_0}{dB_1} = - \left[1 + \tilde{\phi}_0 \frac{(1-\gamma)(R-1)}{\gamma(1-\mu_0)\tilde{R}_0} \right] < 0 \quad (\text{A.13})$$

where $\tilde{\phi}_0 \equiv e_0 c_0^T / (e_0 c_0^T + W_0 c_0^N) \in (0, 1)$. It follows that in a self-fulfilling crisis equilibrium ($B_1 < B_0$), the exchange rate depreciates. \square

A.5 Proof of Proposition 2

Under flexible exchange rates with fixed money supply, the maximum borrowing of an individual household is given by

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right]$$

where e_0 is determined by the implicit function (A.12). Letting $\xi_{B_1} \equiv \frac{Rc_0^T}{e_0} \frac{de_0}{dB_1}$ denote the elasticity of the nominal exchange rate with respect to B_1 (A.13), we have

$$\begin{aligned}\frac{\partial \bar{b}(B_1; B_0)}{dB_1} &= \kappa \frac{1-\phi}{\phi} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} [1 - (1-\gamma)\xi_{B_1}] \\ \frac{\partial^2 \bar{b}(B_1; B_0)}{dB_1^2} &= (1-\gamma) \left[-\frac{\partial \bar{b}(B_1; B_0)}{dB_1} \xi_{B_1} - \kappa \frac{1-\phi}{\phi} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \frac{d\xi_{B_1}}{dB_1} \right]\end{aligned}$$

Owing to $\xi_{B_1} < 0$, we have that $\frac{\partial \bar{b}(B_1; B_0)}{dB_1} > 0$. Differentiating (A.13), we obtain after some algebraic manipulation,

$$\begin{aligned}\frac{d\xi_{B_1}}{dB_1} &= -\frac{(1-\gamma)^2(1-\tilde{\phi}_0)^2}{1-(1-\gamma)(1-\tilde{\phi}_0)} \frac{1}{Rc_0^T} (\xi_{B_1})^2 \\ &\quad - (1-\gamma) \frac{(1-\beta)\tilde{\phi}_1}{\gamma \tilde{R}_0 c_1^T} \left[\frac{2+(R-1)c_0^T/c_1^T}{\gamma+(1-\gamma)\tilde{\phi}_0} - \xi_{B_1} \right] < 0\end{aligned}\quad (\text{A.14})$$

It follows from (A.14) and $\xi_{B_1} < 0$ that $\frac{\partial^2 \bar{b}(B_1; B_0)}{dB_1^2} > 0$.

Note again that B_1 is part of a self-fulfilling crises equilibrium if the following conditions are satisfied $\bar{b}(B_1; B_0) = B_1$, $B_1 < B_0$, and $\frac{B_1}{R} > B_0 - y^T$. Because $\bar{b}(B_1; B_0)$ is an increasing and convex function in B_1 with $\bar{b}(B_0; B_0) > B_0$, the equation $\bar{b}(B_1; B_0) = B_1$ has at most two solutions with one solution featuring $\frac{\partial \bar{b}(B_1; \tilde{B}_0)}{dB_1} \geq 1$. Moreover, owing to

$$\begin{aligned}\frac{\partial \bar{b}(B_1; B_0)}{dB_0} &= \kappa R \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left[-1 - (1-\gamma) \frac{c_0^T}{e_0} \frac{\partial e_0}{\partial B_0} \right] \\ &= \kappa R \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left[-1 - \frac{1-\gamma}{\gamma+(1-\gamma)\tilde{\phi}_0} \right] < 0\end{aligned}$$

at the minimum level of initial debt level, \underline{B}^m , for which a crises equilibrium exists, we have $\frac{\partial \bar{b}(B_1; \underline{B}^m)}{dB_1} = 1$. To simplify the algebra, let define $\psi_0 \equiv 1 - (1-\gamma)\xi_{B_1} > 1$. We have

$$\frac{\partial \bar{b}(B_1; \underline{B}^m)}{dB_1} = 1 \Leftrightarrow \psi_0 \kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} = 1 \quad (\text{A.15})$$

$$\Leftrightarrow \psi_0 \kappa \frac{1-\phi}{\phi} \left(\frac{c_0^T}{h_0} \right)^{\frac{1-\gamma}{\gamma}} = 1. \quad (\text{A.16})$$

Using $\underline{B}^m = y^T + \frac{B_1}{R} - c_0^T$ and plugging in $B_1 = \bar{b}(B_1; \underline{B}^m)$, we obtain by (A.15),

$$\underline{B}^m = (1 + \kappa)y^T + \left(\frac{1}{\psi_0} - 1\right) c_0^T \quad (\text{A.17})$$

Using (A.16), one can solve for c_0^T and obtain (recall that $\psi_0 > 1$)

$$\underline{B}^m = (1 + \kappa)y^T - \frac{\psi_0 - 1}{\psi_0} \left[\psi_0 \kappa \frac{1 - \phi}{\phi} \right]^{\frac{-\gamma}{1-\gamma}} h_0 \quad (\text{A.18})$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}^m, \hat{B})$ where \underline{B}^m is given by (A.18) and we use $B_0 < \hat{B}$ by Lemma 1. It can also be shown that $\psi_0 > \frac{1}{\gamma}$. Using $\frac{1}{\psi_0} < \gamma$ and $h_0 < \bar{h}$, it follows from (A.18) that

$$\underline{B}^m > (1 + \kappa)y^T - (1 - \gamma) \left[\frac{1}{\gamma} \kappa \frac{1 - \phi}{\phi} \right]^{\frac{-\gamma}{1-\gamma}} \bar{h} \quad (\text{A.19})$$

Moreover, since $\bar{b}(B_1; B_0)$ is convex in its first argument, the equation $\bar{b}(B_1; B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1; B_0) = B_1$ has a solution and at \tilde{B}_1 such that $\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} \Big|_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1; B_0) > B_0$. Because $\partial \bar{b}(B_1; B_0) / \partial B_1 = 0$ implies that $c_0^T = 0$, it follows that \tilde{B}_1 lowest value in the feasible domain of B_1 , i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1 + \kappa)\kappa y^T > B_0 \Leftrightarrow B_0 < (1 + \kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = B_1$ has two solutions for $B_0 \in [\underline{B}^m, (1 + \kappa)y^T)$ and a unique solution for $B_0 \in [(1 + \kappa)y^T, \hat{B})$. Furthermore, as shown in Appendix A.3, the interval $[(1 + \kappa)y^T, \hat{B})$ is non-empty under Assumption 1. \square

A.6 Proof of Proposition 3

Under full employment policy, the maximum borrowing of an individual household is

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \frac{1 - \phi}{\phi} \left(y^T - B_0 + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{\frac{\gamma-1}{\gamma}} \right]$$

Notice again that B_1 is part of a self-fulfilling crises equilibrium if the following conditions are satisfied: $\bar{b}(B_1; B_0) \geq 0$, $B_1 \leq B_0$, and $\frac{B_1}{R} > B_0 - y^T$. Because $\bar{b}(B_1; B_0)$ is an increasing and convex function in B_1 with $\bar{b}(B_0; B_0) > B_0$, the equation $\bar{b}(B_1; B_0) = B_1$ has at most

two solutions with one solution featuring $\frac{\partial \bar{b}(B_1; \tilde{B}_0)}{\partial B_1} \geq 1$. Moreover, owing to

$$\frac{\partial \bar{b}(B_1; B_0)}{\partial B_0} = -\kappa R \frac{1-\phi}{\phi \gamma} \left(y^T - B_0 + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} < 0$$

at the minimum level of initial debt level, \underline{B} , for which a crises equilibrium exists, we have

$$\frac{\partial \bar{b}(B_1; \underline{B})}{\partial B_1} = 1 \Leftrightarrow \frac{\kappa}{\gamma} \frac{1-\phi}{\phi} \left(\frac{c_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} = 1 \quad (\text{A.20})$$

Using $\underline{B} = y^T + \frac{B_1}{R} - c_0^T$ and plugging in $\bar{b}(B_1; \underline{B}) = B_1$ to substitute for B_1 yields

$$\underline{B} = (1 + \kappa)y^T - (1 - \gamma) \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h} \quad (\text{A.21})$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}, \hat{B})$ where \underline{B} is given by (A.21) and we use $B_0 < \hat{B}$ by Lemma 1.

Moreover, since $\bar{b}(B_1; B_0)$ is convex in its first argument, the equation $\bar{b}(B_1; B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1; B_0) = B_1$ has a solution and at \tilde{B}_1 where $\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} |_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1; B_0) > B_0$. Because $\partial \bar{b}(B_1; B_0) / \partial B_1 = 0$ implies that $c_0^T = 0$, it follows that \tilde{B}_1 lowest value in the feasible domain of B_1 , i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1 + \kappa)\kappa y^T - B_0 > 0 \Leftrightarrow B_0 < (1 + \kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = 0$ has two solutions for $B_0 \in [\underline{B}, (1 + \kappa)\kappa y^T)$ and a unique solution for $B_0 \in [(1 + \kappa)\kappa y^T, \hat{B})$. Furthermore, as shown above the interval $[(1 + \kappa)\kappa y^T, \hat{B})$ is non-empty under Assumption 1. By (A.19), we have $\underline{B} < \underline{B}^m$. \square

A.7 Proof of Proposition 2

Let W_{ss} and W_{crisis} be the welfare in the steady state equilibrium and in the self-fulfilling crisis equilibrium, respectively. We have $W_{ss} > W_{crisis}$. Suppose now that $B_0 < (1 + \kappa)\kappa y^T$ and there is non-zero probability $\pi > 0$ that the economy ends in a self-fulfilling crisis equilibrium when the economy is in the vulnerable region. By Proposition 1, under fixed exchange rate there is a unique equilibrium which implies that $W_{fix} = W_{ss}$. By Proposition 2 and 3, there are two self-fulfilling crisis that coexists with the steady state equilibrium under flexible exchange rate. Thus, $W_{flex} < W_{ss} = W_{fix}$. \square

A.8 Proof of Proposition 4

Let us define

$$\underline{e} = W_{-1} \left[\frac{\kappa}{\gamma} \left(\frac{1-\phi}{\phi} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \quad (\text{A.22})$$

Combining (15) with market clearing for nontradables, $c_0^N = y_0^N$, we have

$$y_0^N = \left(\frac{1-\phi}{\phi} \frac{e_0}{W_{-1}} \right)^\gamma \left[y^T - B_0 + \frac{B_1}{R} \right] \quad (\text{A.23})$$

We also have that if the borrowing constraint holds with equality B_1 is given by (3)

$$B_1 = \kappa y^T + \kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \frac{B_0 - (1+\kappa) y^T}{\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} - 1} \quad (\text{A.24})$$

Substituting b_1 in (A.23) and deriving

$$y_0^N = \left(\frac{1-\phi}{\phi} \frac{e_0}{W_{-1}} \right)^\gamma \frac{B_0 - (1+\kappa) y^T}{\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} - 1} \quad (\text{A.25})$$

We then differentiate (A.25) with respect to e_0 to obtain

$$\frac{dy_0^N}{de_0} = \frac{\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} - \gamma}{\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} - 1} \cdot \frac{y_0^N}{e_0}. \quad (\text{A.26})$$

Let us denote by $y_0^N(\bar{e})$ the level of output for $e_0 = \bar{e}$. We have

$$y_0^N(\bar{e}) = \left(\frac{1-\phi}{\phi} \frac{\bar{e}}{W_{-1}} \right)^\gamma c_0^T < \left(\frac{1-\phi}{\phi} \frac{\bar{e}}{W_{-1}} \right)^\gamma \left[y_0^T - \frac{R-1}{R} B_0 \right] = \bar{h}$$

Case (i). Consider the case of $\gamma < 1$ and $B_0 < (1+\kappa)y^T$. From (A.25), it follows that $y_0^N > 0$ if and only if

$$\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} < 1 \quad (\text{A.27})$$

Moreover, for any $e_0 \in (\bar{e}, \underline{e})$ we have

$$\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} > \kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{\underline{e}}{W_{-1}} \right)^{\gamma-1} = \gamma \quad (\text{A.28})$$

Using (A.27) and (A.28), it follows from (A.26) that $\frac{dy_0^N}{de_0} < 0$. Moreover, notice by (A.27) that y_0^N is well defined, i.e. $y_0^N > 0$ iff $e_0 > \underline{e}\gamma^{\frac{1}{1-\gamma}}$. \square

Case (ii) Consider now that $\gamma > 1$ and $B_0 > (1+\kappa)y^T$. It is then straightforward to see that y_0^N is well defined, that is $y_0^N > 0$, if and only if

$$\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} > 1 \quad (\text{A.29})$$

Moreover, for any $e_0 \in (\bar{e}, \underline{e})$ we have

$$\kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} < \kappa \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{\underline{e}}{W_{-1}} \right)^{\gamma-1} = \gamma \quad (\text{A.30})$$

Using (A.29) and (A.30), it follows from (A.26) that $\frac{dy_0^N}{de_0} < 0$. Moreover, notice by (A.29) that y_0^N is well defined, i.e. $y_0^N > 0$ iff $e_0 > \underline{e}\gamma^{\frac{1}{1-\gamma}}$.

Finally, for the expansionary case, notice that if $\gamma \leq 1$ and $B_0 > (1+\kappa)y^T$ or if $\gamma \geq 1$ and $B_0 < (1+\kappa)y^T$ then we have by (A.25) and (A.26) that $\frac{dy_0^N}{de_0} > 0$.

A.9 Proof of Proposition 5

For any B and any possible B' chosen by households, the central bank solves

$$\max_{c^T, c^N, e, W \geq W_{-1}} u(c^T, h) + \frac{\beta}{1-\beta} u \left[y^T - \frac{R-1}{R} B', \bar{h} \right],$$

subject to

$$c^T = y^T - B + \frac{B'}{R} \quad (\text{A.31})$$

$$h = \left(\frac{1-\phi}{\phi} \frac{e}{W} \right)^\gamma c^T \quad (\text{A.32})$$

$$\frac{B'}{R} \leq \kappa \left[y^T + \left(\frac{1-\phi}{\phi} \right)^\gamma \left(\frac{W}{e} \right)^{1-\gamma} c^T \right] \quad (\text{A.33})$$

$$h \leq \bar{h} \quad (\text{A.34})$$

Notice that if the borrowing constraint (A.33) is not binding, then the optimal monetary policy implies $h = \bar{h}$. To prove this, assume by contradiction that the borrowing constraint is not binding. Because in this case e only appears in (A.32), it is immediate that (A.32) does not bind. Letting $\eta \geq 0$ be the Lagrange multiplier on (A.34), the optimality condition for h requires $\eta = u_N(c^T, h) > 0$ which implies that $h = \bar{h}$. Assuming that the borrowing constraint (A.33) binds, it is possible to combine (A.32) and (A.34), and use (A.31) to get

$$h = \left[y^T - B + \frac{B'}{R} \right]^{\frac{1}{1-\gamma}} \left[\frac{\kappa(1-\phi)}{\phi(\frac{B'}{R} - \kappa y^T)} \right]^{\frac{\gamma}{1-\gamma}} \equiv \tilde{h}(B'; S)$$

If $\tilde{h}(B'; S) \geq \bar{h}$, then (A.34) binds and $h = \bar{h}$. Otherwise $h = \tilde{h}(B'; S)$. The employment policy function $\mathcal{H}(B'; S)$ is therefore given by

$$\mathcal{H}(B'; S) = \min\{\tilde{h}(B'; S), \bar{h}\} \quad (\text{A.35})$$

□

A.10 Proof of Lemma 5

\tilde{B}_1 is part of a Markov perfect equilibrium if \tilde{B}_1 satisfies

$$\tilde{B}_1 = \kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B_0 + \frac{\tilde{B}_1}{R} \right)^{\frac{1}{\gamma}} h(\tilde{B}_1)^{1-\frac{1}{\gamma}} \right] \quad (\text{A.36})$$

where $h(\tilde{B}_1)$ is the solution to

$$\begin{aligned} & \max_{h \leq \bar{h}} h & (\text{A.37}) \\ \text{s.t. } & \tilde{B}_1 \leq \kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B_0 + \frac{\tilde{B}_1}{R} \right)^{\frac{1}{\gamma}} h^{1-\frac{1}{\gamma}} \right] \end{aligned}$$

Given $\gamma < 1$, the constraint set is decreasing in h , this means that the constraint is binding and the borrowing constraint holds with equality provided that $h < \bar{h}$.

Let B_1^i and B_1^j be part of a Markov perfect equilibrium (MPE) and $h(B_1^i), h(B_1^j)$ the associated levels of employment. Assume without loss of generality that $B_1^i > B_1^j$. By continuity of the right-hand side of (A.36), we have that for any $B' \in (B_1^j, B_1^i)$, there exists a level of employment such that (A.36) holds with equality. This proves that the set of MPE is convex. □

A.11 Proof of Proposition 6

Consider that the economy starts with $B_0 < (1 + \kappa)y^T$, then Proposition 3 establishes that there exists two self-fulfilling crises equilibria that coexist with the steady state equilibrium.

Lower bound. Let B_1^{FE} be the smallest of the two level of borrowing associated with a full employment policy characterized in Proposition 3 (point F' in Figure 2). As shown in Appendix A.6, because $\bar{b}(B_1, B_0)$ is convex in B_1 under full employment,

$$\left. \frac{\partial \bar{b}(B_1, B_0)}{\partial B_1} \right|_{B_1^{FE}} < 1 \quad (\text{A.38})$$

Suppose now there is a Markov perfect equilibrium with $h < h_0$ and $B_1 = B_1^{FE} - \varepsilon < B_1^{FE}$ where $\varepsilon > 0$ is arbitrarily small. B_1 is a Markov perfect equilibrium implies

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1^{FE}}{R} \right)^{\frac{1}{\gamma}} (h_0)^{1-\frac{1}{\gamma}} \right] - \frac{B_1^{FE}}{R} = 0 \quad (\text{A.39})$$

with $h_0 < \bar{h}$. However, by (A.38) we have that

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} > 0 \quad (\text{A.40})$$

From (A.40) it is straightforward to see that (A.39) holds if and only if for $h_0 > \bar{h}$. Thus, we reach a contradiction. Because the set of MPE is convex and $B_1^{FE} - \varepsilon$ with $\varepsilon > 0$ is not a MPE, any $B_1 < B_1^{FE}$ is not part of a MPE. Therefore B_1^{FE} is the lowest B' in the set of MPE.

Upper bound. Let B_1^{HE} be the largest of the two level of borrowing associated with a full employment policy characterized in Proposition 3 (point F in Figure 2). As shown in Appendix A.6, because $\bar{b}(B_1, B_0)$ is convex in B_1 under full employment,

$$\left. \frac{\partial \bar{b}(B_1, B_0)}{\partial B_1} \right|_{B_1^{HE}} > 1 \quad (\text{A.41})$$

Suppose now there is an Markov perfect equilibrium with $h_0 < \bar{h}$ and $B_1 = B_1^{HE} + \varepsilon > B_1^{HE}$ where $\varepsilon > 0$ is arbitrarily small. B_1 is a Markov perfect equilibrium implies

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1^{HE}}{R} \right)^{\frac{1}{\gamma}} (h_0)^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} = 0 \quad (\text{A.42})$$

with $h_0 < \bar{h}$. However, by (A.41) we have that

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} > 0 \quad (\text{A.43})$$

From (A.43) it is straightforward to see that (A.42) holds if and only if for $h_0 \geq \bar{h}$. Thus, we reach a contradiction. Because the set of MPE is convex and $B_1^{HE} + \varepsilon$ with $\varepsilon > 0$ is not a MPE, any $B_1 > B_1^{HE}$ is not part of a MPE. Therefore B_1^{HE} is the lowest B' in the set of MPE.

A.12 Proof of Proposition 7

$e(B_1, B_0)$ rules out the possibility of self-fulfilling crisis when $B_0 < (1 + \kappa)y^T$ follows from Proposition 1. For $B_0 \geq (1 + \kappa)y^T$, we start by rewriting the policy rule. We have

$$e(B_1, B_0) = \bar{e} \left[\frac{B_1}{B_0} + \left(1 - \frac{B_1}{B_0} \right) \underbrace{\Phi(B_1, B_0)}_{\text{depreciation in a MPE}} \right], \quad (\text{A.44})$$

where $\bar{e} \Phi(B_1, B_0)$ is the exchange rate level in the Markov perfect equilibrium with

$$\Phi(B_1, B_0) \equiv \left[\frac{y^T - (1 - \beta) B_0}{\bar{h}} \right]^{\frac{1}{\gamma}} \left[\kappa \frac{1 - \phi}{\phi} \frac{R(y^T - B_0) + B_1}{B_1 - R\kappa y^T} \right]^{\frac{1}{1-\gamma}}$$

We want to show that under the policy rule (A.44), if a generic household i believes that all other households will choose B_1 that household will find it optimal to choose a different action, that is $b_1 \neq B_1$.

Assume that $B_1 < B_0$ and the household i chooses $b_1 = B_1$. Then, the household's Euler equation for foreign bonds (5) requires $\mu_0 > 0$. Moreover, notice that for $e_0^{MPE} = \bar{e} \Phi(B_1, B_0)$ that the borrowing constraint is satisfied with equality. Because the government commits to appreciating the exchange rate above this level, $\bar{e} < e(B_1, B_0) < e_0^{MPE}$, the borrowing constraint is relaxed

$$b_1 < \bar{b}(B_1, B_0) = \kappa R \left[y^T + \frac{W_{-1}}{e_0(B_1, B_0)} h_0 \right] \quad (\text{A.45})$$

The complementary slackness condition is not satisfied $\mu_0(b_1 - \bar{b}(B_1, B_0)) > 0$. It is thus not optimal for the household i to choose $b_1 = B_1$. This proves that the set of MPE is convex. \square

B Interest Rate Policy

We consider in this section a flexible exchange rate regime where the central bank controls nominal rates. We consider two cases, an interest rate peg and a Taylor rule. We will establish that just like in the case of a fixed money supply, we have two self-fulfilling crises equilibria and the crisis region expands relative to the fixed exchange rate regime.

B.1 Crises Region under Interest Rate Peg

Let us focus on a regime where the exchange rate tomorrow is given by \bar{e} and the central bank today sets the nominal rate at $\tilde{R} = R$. The current exchange is then determined by

$$e_0 = \frac{\bar{e}}{1 - \mu_0} \quad (\text{B.1})$$

We have the following proposition:

Lemma B.1 (Unemployment under target rate). *In a self-fulfilling crisis, the exchange rate depreciates at $t = 0$ and there is unemployment.*

Proof. To see this why $h_0 < \bar{h}$, combine market clearing $h_0 = y_0^N = c_0^N$ with the demand for non-tradables (15) to obtain

$$h_0 = \left[\frac{1 - \phi}{\phi} \frac{e_1}{W_0} \frac{1}{1 - \mu_0} \right]^\gamma c_0^T$$

Using (5) and $c_0^T < c_1^T$ we arrive to

$$h_0 < \left[\frac{1 - \phi}{\phi} \frac{e_1}{W_0} \right]^\gamma c_1^T = \bar{h} \quad (\text{B.2})$$

Therefore, if a self-fulfilling crisis exists under $\tilde{R}_0 = R$ it has to be that $h_0 < \bar{h}$. □

We turn to showing that the exchange rate depreciates. From (B.1), we have

$$e_0 = W_{-1} \frac{u_T(y^T - B_0 + \frac{B_1}{R}, h_0)}{u_N(y^T - \frac{R-1}{R}B_1, \bar{h})}, \quad \text{with } h_0 = \left(\frac{1 - \phi}{\phi} \frac{e_0}{W_{-1}} \right)^\gamma \left(y^T - B_0 + \frac{B_1}{R} \right) \quad (\text{B.3})$$

Totally differentiating (B.3) yields

$$[\gamma + (1 - \gamma)\tilde{\phi}_0] \frac{Rc_0^T}{e_0} \frac{de_0}{dB_1} = - \left[1 + (R - 1) \frac{-u_{TT}(c_1^T, \bar{h})}{u_N(c_1^T, \bar{h})} \right] \quad (\text{B.4})$$

where $u_{TT}(c^T, h) \equiv \frac{\partial u(c^T, h)^2}{\partial (c^T)^2} < 0$. The exchange rate depreciates in a crisis equilibrium. □

The next proposition characterizes when an economy under an interest rate peg features multiple equilibria:

Proposition B.1 (Crises under target rate). *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate with a target interest rate,*

- i. *if $B_0 \in ((1 + \kappa)y^T, \hat{B}) \neq \emptyset$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium.*
- ii. *if $B_0 \in [\underline{B}^r, (1 + \kappa)y^T)$, there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, with $\underline{B}^r > \underline{B}$.*
- iii. *if $B_0 < \underline{B}^r$, we have one and only one equilibrium (which corresponds to the steady state equilibrium).*

Proof. Note that the maximum borrowing capacity is given by (21) where e_0 is determined by (B.1). Differentiating (21), we obtain

$$\begin{aligned} \frac{\partial \bar{b}(B_1; B_0)}{dB_1} &= \kappa \frac{1 - \phi}{\phi} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} [1 - (1 - \gamma)\xi_{B_1}] \\ \frac{\partial^2 \bar{b}(B_1; B_0)}{dB_1^2} &= (1 - \gamma) \left[-\frac{\partial \bar{b}(B_1; B_0)}{dB_1} \xi_{B_1} - \kappa \frac{1 - \phi}{\phi} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \frac{d\xi_{B_1}}{dB_1} \right] \end{aligned}$$

where $\xi_{B_1} \equiv \frac{Rc_0^T}{e_0} \frac{de_0}{dB_1} < 0$ is the elasticity of the exchange rate with respect to B_1 in (A.13). Differentiating (B.4), we obtain

$$\begin{aligned} \frac{d\xi_{B_1}}{dB_1} &= -\frac{(1 - \gamma)^2(1 - \tilde{\phi}_0)^2}{1 - (1 - \gamma)(1 - \tilde{\phi}_0)} \frac{1}{Rc_0^T} (\xi_{B_1})^2 \\ &\quad - (1 - \gamma) \frac{(1 - \beta)\tilde{\phi}_1}{\gamma \tilde{R}_0 c_1^T} \left[\frac{2 + (R - 1)c_0^T/c_1^T}{1 - (1 - \gamma)(1 - \tilde{\phi}_0)} - \xi_{B_1} \right] < 0 \end{aligned} \quad (\text{B.5})$$

It follows from (B.4) and $\xi_{B_1} < 0$ that $\frac{\partial^2 \bar{b}(B_1; B_0)}{dB_1^2} > 0$. Next, following the same steps as in the proof of Proposition 2, we arrive at

$$\underline{B}^r = (1 + \kappa)y^T - \frac{\psi_0 - 1}{\psi_0} \left[\psi_0 \kappa \frac{1 - \phi}{\phi} \right]^{\frac{-\gamma}{1-\gamma}} h_0 \quad (\text{B.6})$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}^r, \hat{B})$ where \underline{B}^r is given by (B.6) and we use $B_0 < \hat{B}$ by Lemma 1. Using $\frac{1}{\psi_0} < \gamma$

and $h_0 < \bar{h}$, we have follows from (B.6) that

$$\underline{B}^r > (1 + \kappa)y^T - (1 - \gamma) \left[\frac{1}{\gamma} \kappa \frac{1 - \phi}{\phi} \right]^{\frac{-\gamma}{1-\gamma}} \bar{h} \quad (\text{B.7})$$

Moreover, since $\bar{b}(B_1; B_0)$ is convex in its first argument, the equation $\bar{b}(B_1; B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1; B_0) = B_1$ has a solution and at \tilde{B}_1 such that $\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} \Big|_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1; B_0) > B_0$. Because $\partial \bar{b}(B_1; B_0) / \partial B_1 = 0$ implies that $c_0^T = 0$, it follows that \tilde{B}_1 lowest value in the feasible domain of B_1 , i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1 + \kappa)\kappa y^T > B_0 \Leftrightarrow B_0 < (1 + \kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = B_1$ has two solutions for $B_0 \in [\underline{B}^r, (1 + \kappa)y^T)$ and a unique solution for $B_0 \in [(1 + \kappa)y^T, \hat{B})$ which is non-empty under Assumption 1. \square

B.2 Crises Region under a Taylor Rule

Consider a form of Taylor rule where

$$\tilde{R}_0 = R \left(\frac{h_0}{\bar{h}} \right)^{-\phi_h} \quad (\text{B.8})$$

where $\phi_h \geq 0$ is a non-negative coefficient that describes the strength of the interest rate response to deviations of employment from its efficient level. For $\phi_h \rightarrow \infty$, the rule (B.8) corresponds to the full employment policy where monetary policy ensures $h_0 = \bar{h}$ and for $\phi_h = 0$ the rule (B.8) reduces to $\tilde{R}_0 = R$, i.e. the interest rate target policy. Using (8) and substituting for μ_0 using (5) we get

$$\tilde{R}_0 e_0 = e_1 \frac{u_T(y^T - B_0 + \frac{B_1}{R}, h_0)}{u_T(y^T - \frac{R-1}{R} B_1, \bar{h})} \quad (\text{B.9})$$

and totally differentiating it we arrive to

$$[(1 - \phi_h)\gamma + (1 - \gamma)\tilde{\phi}_0] \frac{Rc_0^T}{e_0} \frac{de_0}{dB_1} = - \left[1 + \phi_h + (R - 1) \frac{-u_{TT}(c_1^T, \bar{h})}{u_N(c_1^T, \bar{h})} \right] \quad (\text{B.10})$$

We characterize the crisis region in the proposition below.

Proposition B.2 (Crises under Taylor rules). *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate where monetary policy is set according to the Taylor rule (B.8),*

- i. if $B_0 \in ((1 + \kappa)y^T, \hat{B}) \neq \emptyset$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium.*

ii. if $B_0 \in [\underline{B}^T, (1 + \kappa)y^T)$, there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, with $\underline{B}^T > \underline{B}$.

iii. if $B_0 < \underline{B}^T$, we have one and only one equilibrium (which corresponds to the steady state equilibrium).

Proof. The proof follows the same steps as the proof of Proposition B.1 with $\zeta_{B_1} \equiv \frac{Rc_0^T}{e_0} \frac{de_0}{dB_1}$ now given by (B.10). □