



# A Fisherian approach to financial crises: Lessons from the Sudden Stops literature <sup>☆</sup>



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## ABSTRACT

Sudden Stops are financial crises defined by a large, sudden current-account reversal. They occur in both advanced and emerging economies and result in deep recessions, collapsing asset prices, and real exchange-rate depreciations. They are preceded by economic expansions, current-account deficits, credit booms, and appreciated asset prices and real exchange rates. Fisherian models (i.e. models with credit constraints linked to market prices) explain these stylized facts as an outcome of Irving Fisher's debt-deflation mechanism. On the normative side, these models feature a pecuniary externality that provides a foundation for macroprudential policy (MPP). We review the stylized facts of Sudden Stops, the evidence on MPP use and effectiveness, and the findings of the literature on Fisherian models. Quantitatively, Fisherian amplification is strong and optimal MPP reduces sharply the size and frequency of crises, but it is also complex and potentially time-inconsistent, and simple MPP rules are less effective. We also provide a new MPP analysis incorporating investment. Using a constant debt-tax policy, we construct a crisis probability-output frontier showing that there is a tradeoff between financial stability and long-run output (i.e., reducing the probability of crises reduces long-run output).

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## 1. Introduction

The Mexican crisis of 1994 was a harbinger of a series of financial crises that have since affected both emerging and advanced economies. The defining feature of these crises is a large, sudden reversal in the current account, which is a country's broadest measure of net foreign financing. Because of the sudden loss of access to international capital markets, these events are known as *Sudden Stops*.<sup>1</sup> As we document in the next section, by the end of 2016 there had been 58 Sudden Stop events worldwide, 35 in emerging markets and 23 in advanced economies.

Sudden Stops have been the focus of a large theoretical and empirical literature since the mid-1990s. More recently, following the 2008 Global Financial Crisis (GFC), a growing literature has been studying macroprudential regulation as a way

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<sup>1</sup> Oral tradition has it that this nickname originated in a comment from the audience at a presentation by the late Rudi Dornbusch on the Mexican crisis, making the point that like in the familiar Douglas Adams quote, with the sharp current-account reversals "it is not the fall that kills you, it is the sudden stop at the end."

to avert financial crises. In this article, we start by reviewing the stylized facts of Sudden Stops, with a new event analysis spanning the 1979–2016 period and including advanced and emerging economies. Then, we provide a short survey of the findings of the empirical literature on the use and effectiveness of macroprudential policies to date. The paper then moves on to review the literature that examines the quantitative implications, both positive and normative, of a class of Sudden Stops models labeled *Fisherian models*. In these models, Sudden Stops are the result of financial amplification driven by the debt-deflation mechanism proposed in the seminal work on the Great Depression by Fisher (1933) and also emphasized in the classic studies by Minsky (1992) and Kiyotaki and Moore (1997).

The central element of a Fisherian model is an occasionally binding credit constraint that limits borrowing capacity to a fraction of the market value of the goods or assets pledged as collateral. This constraint is essential to the Fisherian debt-deflation mechanism: when the constraint binds, agents fire-sale the goods or assets that serve as collateral, and as they do so, they drive down the value of those goods or assets, which tightens the constraint further and forces further fire sales. From a normative perspective, the constraint entails a pecuniary externality that creates scope for macroprudential policy (MPP): individual borrowers do not internalize how their borrowing decisions made in “good times” affect the size of the deflation in collateral values and the reduction in the economy’s borrowing capacity during a Sudden Stop (Bianchi, 2011; Bianchi and Mendoza, 2018). As a result, private agents undervalue the social marginal cost of borrowing, and hence they “overborrow.” Optimal MPP thus calls for tightening access to credit in a procyclical fashion, a feature consistent with newly introduced regulation such as the Countercyclical Capital Buffer (CCyB).

In addition to reviewing the findings of the existing literature, we conduct a new analysis examining MPP tradeoffs involving capital accumulation. To date, most studies quantifying MPP tradeoffs have focused only on consumption-smoothing tradeoffs. Filling this gap is important because policy discussions on MPP emphasize the potentially costly tradeoff between capital accumulation, long-run output, and financial stability.

We propose a new version of a widely used Fisherian two-sector model with tradable and nontradable goods, extended to introduce production of investment goods using both tradables and non-tradables as inputs. In the model, households consume tradables and non-tradables, make investment decisions, and face a Fisherian constraint that limits their debt to a fraction of the market value of the capital stock. Both the real exchange rate and the market price of capital (i.e., the value of collateral) are determined by the relative price of non-tradables to tradables and are central for the financial amplification mechanism.

We show that by raising the cost of borrowing above the risk-free rate, MPP also raises the effective opportunity cost of capital and reduces investment. Introducing a simple MPP rule in the form of a constant debt tax, allows regulators to reduce leverage and hence the magnitude and frequency of crises, but it also reduces output and investment in a manner akin to a capital income tax. In some states, reducing both consumption and investment is efficient, but in others it is not. Using simple MPP rules therefore induces costly investment and output tradeoffs of over- or under-regulating credit.

We calibrate the model to data and examine the model’s quantitative implications. First, we study long-run cyclical properties and Sudden Stop dynamics in the absence of regulation. Then, we illustrate the costly tradeoffs of simple rules governing debt taxes. This includes a long-run crisis probability-output loss frontier that shows pairs of losses in long-run output and reduction in crisis probability attained with different constant debt tax rates. In addition, we examine the incentives of optimal policy to use debt taxes or adjust debt decisions in the short run.

We also use the model to analyze regulatory loan-to-value (LTV) ratios as an alternative MPP tool. We show that their incidence on incentives to borrow and invest is similar to that of debt taxes but they are not equivalent instruments. In particular, there is no LTV regulation policy that can replicate the equilibrium of a given debt tax policy. At the same levels of credit and consumption obtained with a given debt tax at some date  $t$ , regulatory LTVs support higher collateral values, which also imply different prices and allocations in previous periods. This result is important because to date there is little research analyzing the macro implications of LTV regulation as an MPP tool, and yet, as we document in Section 2, empirical studies show that borrower-targeted instruments (i.e., LTVs) are an effective MPP tool, while evidence is at best mixed for lender-targeted instruments akin to debt taxes.<sup>2</sup>

The rest of the paper is organized as follows. Section 2 conducts an empirical analysis of Sudden Stops using a cross-country dataset and reviews the evidence on MPP use and effectiveness. Section 3 reviews the main elements and findings of the quantitative literature on Fisherian models of Sudden Stops, both positive and normative. Section 4 proposes the new model we use to analyze MPP tradeoffs related to capital accumulation and to compare LTV regulation with debt taxes. Section 5 examines the quantitative implications of the model. Section 6 concludes.

## 2. Empirical evidence

This Section of the paper examines the stylized facts of Sudden Stops and reviews the empirical literature on the use and effectiveness of macroprudential policies. The goals are to provide a quantitative characterization of the movements in macroeconomic variables that define Sudden Stops and to summarize the existing evidence on the performance of macroprudential policies aimed at preventing financial crises, many of which, since the 1980s, have been Sudden Stops.

<sup>2</sup> Debt taxes in Fisherian models are equivalent to lender-based regulation using capital requirements or capital controls (e.g., Bianchi, 2011; Mendoza and Rojas, 2019).

## 2.1. Stylized facts of Sudden Stops

Sudden Stops (SS) are economic fluctuations defined by a set of empirical regularities associated with a large, sudden reversal of capital inflows (i.e., a sudden “loss of access” to international financial markets). Empirical studies on this subject originated in the seminal contributions by Milesi Ferretti and Razin (2000), Calvo et al. (2004), and Calvo et al. (2006), which motivated several authors to conduct further studies, resulting in a large empirical literature that includes many well-known contributions (e.g., Edwards, 2004; Rothenberg and Warnock, 2006; Forbes and Warnock, 2012; Calvo et al., 2013; Eichengreen et al., 2017). Most empirical studies apply event analysis tools to cross-country panel datasets, using one or more filters to identify Sudden Stop events. A sufficiently large increase in the current account GDP ratio ( $ca/y$ ) is widely used as the main identification filter, because the current account is the broadest measure of the flow of credit of an economy vis-a-vis the rest of the world, and hence a large increase in  $ca/y$  indicates a sharp contraction in credit (both private and public) from abroad. This filter is often used together with a second filter that detects if the Sudden Stop is systemic across countries (e.g., using the EMBI+ index for emerging markets), and in some instances, other filters, such as large output drops to capture Sudden Stops with deep recessions (e.g., Calvo et al., 2013) are added.

We revisit here the analysis of the empirical regularities of Sudden Stops with a similar methodology but apply it to a panel dataset that includes data up to 2016 and covers both emerging and advanced economies. We adopt the same event analysis specification as in Korinek and Mendoza (2014), who used data ending in 2012. This specification defines Sudden Stops as year-on-year increases in  $ca/y$  that are in the 95th percentile of the frequency distribution of annual changes in  $ca/y$  of a particular country. It also includes a filter for systemic Sudden Stops, which is the market-wide EMBI for emerging markets or the VIX index for advanced economies. We present the results using both current-account and systemic filters for consistency with the more recent literature, but the event plots characterizing the stylized facts of Sudden Stops in macro data are very similar if we use only the  $ca/y$  filter. We use annual data for 35 emerging market (EM) economies and 23 advanced economies (AE) covering the 1979–2016 period, and construct five-year event windows for the HP-filtered cyclical components of macroeconomic data centered on the date of Sudden Stops. The plots show as separate curves the median for EMs and AEs.

The event analysis yields five key stylized facts, which are consistent with the findings from the existing literature:

1. Across all countries, a typical SS event features a current-account reversal of 3.7 percentage points of GDP. The reversals are larger in EMs (4.4 percentage points) than in AEs (2.7 percentage points), as shown in panel (a) of Fig. 1. Moreover, SS events are preceded by growing current-account deficits and followed by surpluses of about 0.7 percent of GDP that persist two years later. Hence, the credit inflow from the rest of the world expands in the run-up to a Sudden Stop, then it sharply reverts when a Sudden Stop hits, and two years later it is still depressed.
2. SS events are infrequent, but they are twice as likely to occur in emerging than in advanced economies. We found 51 Sudden Stops in total (2.4 percent frequency), of which 36 occurred in EMs (2.9 percent frequency) v. 15 in AEs (1.7 percent frequency). Hence, Sudden Stops are rare events that co-exist with typical business cycles. Current accounts are countercyclical over the business cycle but with much smaller increases than what we observe in Sudden Stops.
3. SS events are clustered around “big events.” They are not randomly distributed over time. As Fig. 2 shows, there are several years in which no Sudden Stops occur, while we observe 14 Sudden Stops in 1982 and 1983, when the Sovereign Debt Crisis of the early 1980s exploded, 13 in 1998 and 1999, when the Asian crisis occurred, and seven in 2009, with the Global Financial Crisis.
4. SS events are associated with sharp economic downturns, preceded by expansions, and followed by protracted recessions (see panels (b) and (c) of Fig. 1). For all countries combined, GDP and consumption are 2.5 and 1.6 percent below trend respectively. In EMs (AEs), GDP and consumption are 3.6 (1.1) and 1.5 (1.6) percent below trend, respectively. Moreover, compared with the expansions that precede Sudden Stops, these downturns represent sharp reversals. Relative to the year before a Sudden Stop hits, the deviations from trend in GDP and consumption for all countries fall by 4.4 and 4 percentage points, respectively, and again the reversals are larger for EMs than AEs. Keep in mind, however, that business cycles are also larger in EMs, so relative to the standard deviations of cyclical components, they are comparable. Investment (panel (d) of Fig. 1) shows a similar pattern, but with larger changes, since investment is also more volatile over the business cycle. For all countries, investment is nearly 11 percentage points below trend when a Sudden Stop hits, and this represents a reversal of nearly 19 percentage points relative to the year before. Two years after SS events, all three macro-aggregates remain significantly below trend. Across EMs and AEs, GDP and consumption are 1.5 to 2 percent below trend and investment 3.3 to 5.5 percent below trend. This result is in line with the findings from Reinhart and Rogoff (2009) indicating that recoveries from recessions triggered by financial crises are slow.
5. SS are associated with falling real equity prices and depreciated real exchange rates (again with larger declines in EMs), and they are preceded (followed) by higher (lower) equity prices and real exchange rates (see panels (e) and (f) of Fig. 1). Hence, the sharp credit reversal reflected in the current-account reversal coincides with deflation in relative prices, both equity prices and relative consumer prices. This is an important observation, for it indicates that relative price deflation is an empirical regularity of Sudden Stops and hence should be considered in the design of models aiming to explain this phenomenon.

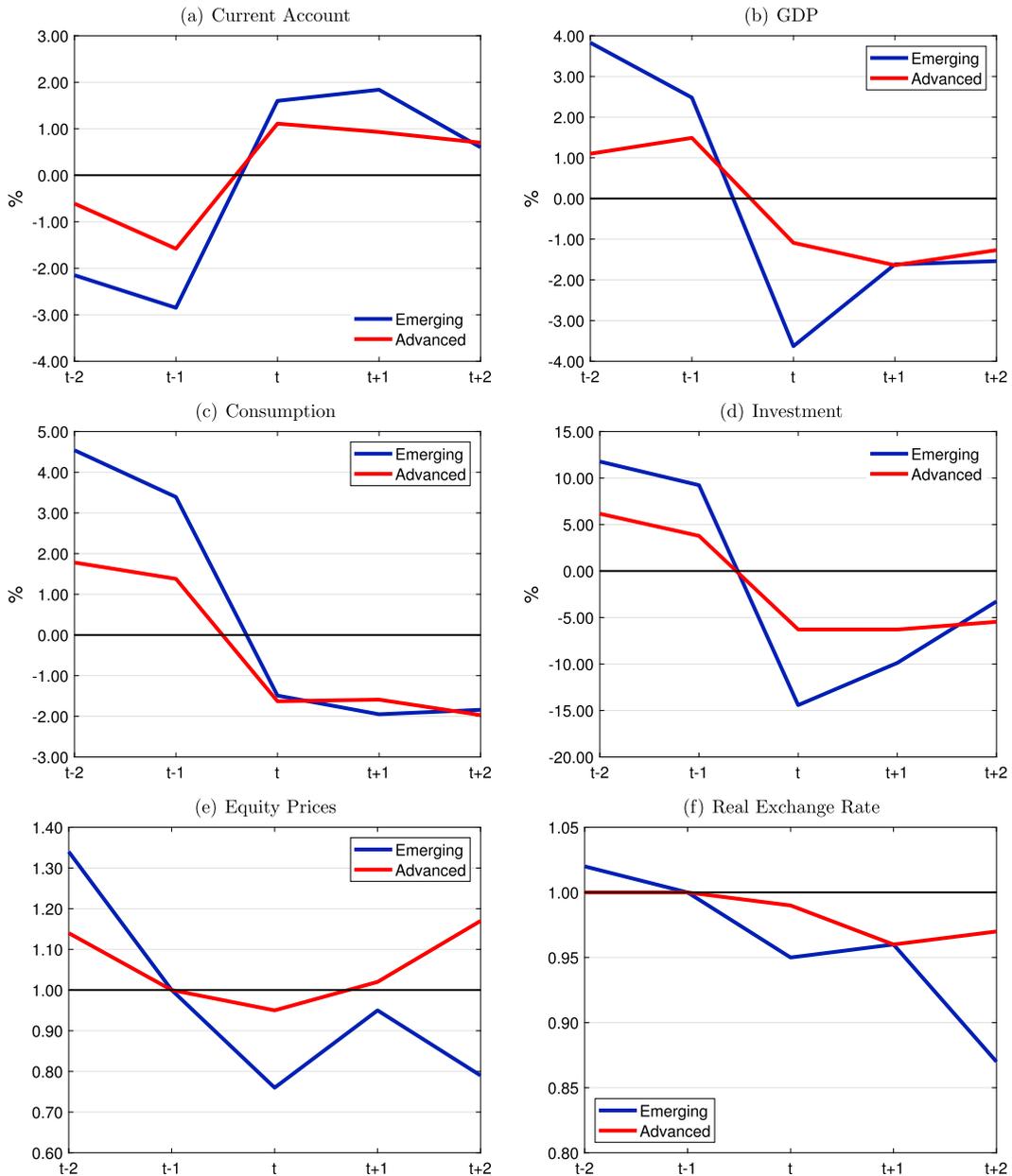


Fig. 1. Sudden Stops dynamics.

It is worth noting also that growth accounting of the recessions associated with Sudden Stops shows that conventional measures of capital and labor account for a small fraction of the large declines in GDP (Mendoza, 2010; Meza, 2008; Calvo et al., 2006). Hence, the output drops would seem to be mostly due to large technology shocks as measured by standard Solow residuals. Those studies showed also, however, that sharp declines in factor utilization and large increases in the relative prices of imported inputs can explain the large decline in Solow residuals.

## 2.2. Macroprudential policy use and effectiveness

Empirical studies of Sudden Stops complement a related empirical literature documenting the deep recessions and price corrections that follow the collapse of credit booms (e.g., Mendoza and Terrones, 2008; Mendoza and Terrones, 2012; Schularick and Taylor, 2009) and the historical characteristics of financial crises (Reinhart and Rogoff, 2009). Together with the recent GFC experience, the evidence documented in all of these studies highlights the importance of implementing policies aimed at reducing the frequency and magnitude of financial crises, which are now grouped under the label of *macroprudential policy*.

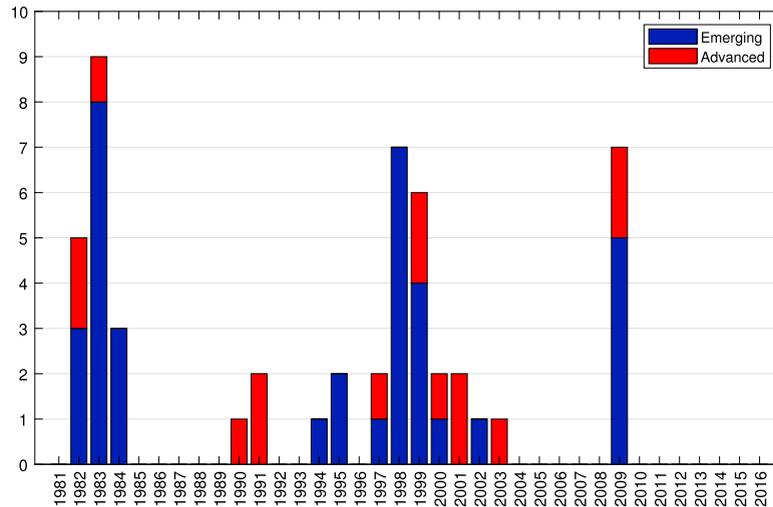


Fig. 2. Number of Sudden Stops per year.

To be sure, financial policies that can be regarded as macroprudential are not new. There are examples of policies aimed at managing aggregate credit dating back to the 1930s. In addition, since the aftermath of the Emerging Markets Crisis in the late 1990s, several emerging economies introduced financial regulation that is now considered macroprudential. Yet it was only in the aftermath of the 2008–2009 GFC that a wide consensus was reached on the desirability of using macroprudential policies. Several countries accordingly adopted new financial policies and institutional changes, international organizations endorsed their use and contributed to their implementation (e.g., the countercyclical capital buffer, known as CCyB, included in the BIS's Basel III regulatory framework), and new institutions were created to implement and coordinate them (e.g., the Financial Stability Board and the European Systemic Risk Board).<sup>3</sup>

In the paragraphs that follow, we summarize the macroprudential policies that have been more commonly used since the 1990s and the existing evidence on their effectiveness so far. The aim is to provide an empirical background for the normative analysis that we conduct later in this paper. A number of comprehensive empirical studies have analyzed macroprudential policies and their effectiveness in emerging and advanced economies (e.g., Cerutti et al., 2015; Buch et al., 2018; Ahnert and Kakhbod, 2017; Cordella et al., 2014; Acharya et al., 2017). We base the description provided here on the work of Cerutti et al. (2015) and the survey by Galati and Moessner (2018).

Using data from two IMF surveys, Cerutti et al. (2015) constructed time-series, cross-sectional indexes of 12 macroprudential policy instruments. They grouped the instruments into borrower-targeted (BTI) and financial intermediary-targeted (FIT). The former consist of regulatory loan-to-value (LTV) and debt-to-income (DTI) ratios on new loans, and the latter consist of ten instruments, including countercyclical capital buffers, bank leverage ratios and limits on foreign currency loans (see Annex 1 in Cerutti et al., 2015, for full details). The overall macroprudential index (MPI) is defined as the sum of the scores on all twelve instruments in a given year and country (120 countries from 2000 to 2013). For a given country, it assigns a value of 1 to each instrument that the country has, from the year the instrument was introduced to the year it was removed. Similar indexes are defined separately for the BTI and FIT.

The evolution of the MPI since 2000 highlights the growing importance of macroprudential policy. The average MPI across all countries rose from 1.1 in 2000 to nearly 2.5 in 2013. The increase has been common to advanced, emerging, and developing economies, although emerging markets show higher average MPIs than advanced economies throughout the sample period (1.3 v. 0.8 in 2000, 2.6 v. 1.9 in 2013). By 2013, 90 percent of the 120 countries in the sample had implemented at least one macroprudential policy instrument. Instruments targeting financial institutions are the most common, but since the mid-2000s borrower-targeted instruments have gained relevance. By 2013, 35 percent of the countries had adopted borrower-oriented LTV and/or DTI regulations. As we review below, this is perhaps in response to the stronger evidence on the effectiveness of borrower-oriented instruments.

While the data studied by Cerutti et al. (2015) shows that macroprudential instruments are now widely used, the survey by Galati and Moessner (2018) documents that the empirical literature studying their effectiveness has produced mixed results. On one hand, there is strong evidence across several studies on the effectiveness of LTVs and DTIs for moderating credit and asset price growth, particularly in housing markets, in emerging and advanced economies and across exchange rate regimes (see, for example, Cerutti et al., 2015; Claessens and Ayhan Kose, 2013; Kuttner and Shim, 2016). On the other hand, evidence on the effectiveness of financial institutions-based instruments is less conclusive. For instance,

<sup>3</sup> The first widely cited use of the term “macroprudential” is generally attributed to the discussion by Borio (2003) on the need to revamp financial regulation with a macroprudential perspective because of the financial turbulence observed during the 1990s emerging markets crisis.

Cerutti et al. (2015) find that taken as a group, these policies reduce credit growth in emerging economies, but in advanced economies the effect is not statistically significant. Similarly, a variety of studies looking at specific instruments using different methodologies and/or sample periods often find conflicting results. Moreover, they generally find that while in some cases the financial institution-targeted instruments are effective in expansions, they are much less helpful at dampening credit contractions in bad times. In particular, Dell’Ariccia et al. (2012) found that macroprudential instruments reduce both the overall frequency of credit booms and the frequency with which booms end up in crises. Kuttner and Shim (2016) found that hiking housing taxes reduces the growth of house prices, but a tax cut does not have a statistically significant effect. These results are interesting because, as we show later in the paper, the normative analysis of Sudden Stops models calls for the use of macroprudential policy during credit expansions as a way to reduce the frequency and magnitude of financial crises.

One of the most widely studied financial institution-targeted macroprudential policy instruments is capital controls. This is natural because, as we documented earlier, since the 1980s financial crises characterized by large reversals in international credit flows (i.e., Sudden Stops) have affected a large number of countries. The large current-account deficits that precede Sudden Stops indicate that a rapid expansion in foreign capital inflows plays an important role in these crises, hence the interest in using capital controls as a macroprudential policy instrument. Galati and Moessner (2018) report that the empirical evidence on the effectiveness of macroprudential capital controls is also inconclusive and mixed.<sup>4</sup> Ostry et al. (2012) find that they affect the *composition* of capital flows, while Beirne and Friedrich (2014) find only a limited effect on *aggregate* capital inflows. Forbes and Warnock (2012) find that macroprudential capital controls weaken some indicators of financial fragility such as leverage, credit growth, and gross inflows in the banking sector, but they do not have a significant effect on net capital inflows.<sup>5</sup>

These empirical findings are interesting in light of some of the findings of theoretical work showing that domestic debt taxes are equivalent to capital controls in workhorse models with Fisherian constraints, and hence the effectiveness of macroprudential policy does not hinge on discriminating foreign from domestic credit sources (Bianchi, 2011; Mendoza and Rojas, 2019).<sup>6</sup> It is worth noting also that leakages in capital controls do not necessarily call for softer regulation, and in fact, optimal policy may call for even stronger regulation (Bengui and Bianchi, 2018).

Looking forward, the CCyB introduced with the Basel III Global Regulatory Framework will also become a widely-studied macroprudential policy tool. The CCyB calls for an “add-on” capital buffer during large credit expansions, as defined by a “reference guide” suggested to be the deviation from the Hodrick-Prescott trend in aggregate credit to the private sector as a share of GDP.<sup>7</sup> The CCyB is activated when this deviation from trend rises above a given activation threshold, then it is tightened linearly if credit continues to expand up to a maximum, and then it is removed linearly as the deviation from trend reverts to the activation threshold. The activation threshold and the maximum are left to the discretion of country regulators, but 2 and 10 percentage points are recommended. Similarly, the values that the add-on buffer can take and the rate at which they are tightened in the upswing and weakened in the downswing are left to the discretion of country authorities. Implementation by BIS member countries was to be completed by the end of 2018. As of February 2018, nine countries had active CCyBs or announced the forthcoming activation of their CCyBs, including the Czech Republic, Denmark, Hong Kong, Iceland, Lithuania, Norway, the Slovak Republic, Sweden and the United Kingdom. We will have to wait a few years to have enough data for conducting a systematic evaluation of the CCyB’s effectiveness. Some quantitative experiments of procyclical MPP rules linked to the credit-GDP ratio suggest, however, that while these rules contribute to dampen credit cycles, they are much less effective than optimal policy rules at reducing the magnitude and frequency of crises (see Bianchi and Mendoza, 2018; Mendoza and Rojas, 2019).

In summary, the empirical literature to date on the effectiveness of macroprudential policy instruments shows generally mixed results. The only instruments for which the evidence is unambiguous are borrower-targeted instruments (e.g., LTVs, DTIs, housing taxes). In addition, macroprudential instruments are effective at hampering credit growth in booms but less effective at containing credit collapse in bad times. Macroprudential capital controls alter the composition of capital inflows and weaken some measures of banking fragility but do not affect aggregate and net capital inflows. We also acknowledge, however, that endogeneity and identification issues are major challenges for empirical work in this area. To the extent that MPP is deployed to curb what would be an excessive credit expansion, as theoretical models predict, this is likely to bias the results toward ineffectiveness.<sup>8</sup> Moreover, while the empirical literature typically identifies the effects of financial regulation instruments used with a systemic focus (i.e., across the financial system as a whole), it does not identify whether the policy instruments are used in a prudential form (i.e., in ex-ante fashion relating the current value of the instrument to potential future outcomes). In the normative theory discussed later, MPP instruments explicitly aim to make credit more expensive

<sup>4</sup> Not all forms of capital controls are macroprudential. As Galati and Moessner (2018) explain, capital controls are macroprudential only if their governance rules state the aim of reducing system-wide financial vulnerabilities.

<sup>5</sup> An interesting point made by Yeyati (2011) is that a skeptical but inconsistent argument regarding capital controls is that: “Not only are they ineffective but, in addition, they raise domestic interest rates.” As he points out, it is precisely the fact that domestic interest rates rise that makes capital controls effective.

<sup>6</sup> In the analysis of these authors, a reserve requirement or a tax on foreign borrowing by banks is equivalent to a tax on households borrowing from domestic banks.

<sup>7</sup> See <https://www.bis.org/bcb/ccyb/> for full details.

<sup>8</sup> For a forceful argument of the empirical challenges that need to be overcome, see Werning (2012).

in “good times” so as to induce agents to internalize the social cost of future financial crises associated with increased borrowing ex-ante. We see the growing widespread use of macroprudential policies in line with the shifting paradigm and the availability of richer micro data as particularly fertile ground for more research on these issues.

### 3. The Fisherian approach to Sudden Stops

The Fisherian theory of financial crises provides a quantitative framework for explaining the stylized facts of Sudden Stops and for designing and assessing macroprudential policies. This Section of the paper provides a brief, generic description of Fisherian models. The goal is to describe the essential elements that drive the transmission mechanism of financial crises in these models and their normative implications. The next Section will discuss in detail the specifics and quantitative implementation of a particular model aimed at studying the tradeoffs of macroprudential policy in a setup with capital accumulation.

The defining element of Fisherian models of Sudden Stops is an occasionally binding collateral constraint that limits borrowing capacity not to exceed a measure of pledgeable collateral that depends on market prices.<sup>9</sup> The constraint is occasionally binding because whether it binds is a state-contingent equilibrium outcome that depends on the optimal plans formulated by the models’ agents, the realizations of exogenous shocks and the values of aggregate variables, particularly equilibrium prices. Pledgeable collateral is defined as a (potentially time-varying) fraction  $\kappa_t$  of the market value of an agent’s income or assets. Most models assume a representative-agent small open economy in which debt takes the form of a negative position in an internationally-traded one-period bond  $b_{t+1}$  sold at a world-determined (possibly stochastic) price  $q_t^b$  (i.e., the gross real interest rate is  $R_t \equiv 1/q_t^b$ ).<sup>10</sup>

Most of the literature uses collateral constraints that are directly imposed on the optimization problems of agents, rather than an endogenous outcome of an explicitly-modeled contract. This is common practice in a branch of the macro literature on financial frictions, as in the seminal studies by Aiyagari and Gertler (1999) and Kiyotaki and Moore (1997). There are, however, studies of Fisherian models in which the collateral constraint is explicitly derived from a contractual setup, typically as a result of limited enforcement or costly state verification (e.g., Bianchi and Mendoza, 2018; Mendoza and Quadrini, 2010).

It is worth noting also that both the financial-amplification mechanism and the pecuniary-externality argument underpinning the normative implications of these models apply to a wider class of financial-frictions models in which market prices determine borrowing capacity. For instance, the classic financial accelerator model of Bernanke and Gertler (1989), in which an external financing premium as a function of net worth endogenously emerges as an outcome of an optimal contract, features a similar pecuniary externality because net worth is valued at market prices and borrowers do not internalize the effect of their actions on those prices. Models like those studied by Lorenzoni (2008), Stein (2012), Gertler and Kiyotaki (2010) and Brunnermeier and Sannikov (2014) also feature credit frictions and related pecuniary externalities affecting the efficiency of competitive equilibria.

The majority of the existing research focuses on two types of constraints:

1. *Loan-to-value (LTV) or stock constraints:* Pledgeable collateral is an asset  $k$  (e.g., land, equity, housing, etc) with a market price  $q_t$  and the borrowing constraint is:

$$q_t^b b_{t+1} \geq \kappa_t q_t k_{t+j}, \quad j = 0, 1. \quad (1)$$

The timing of the assets posted as collateral in the right-hand-side of the constraint depends on assumptions about the nature of credit contracts and their enforcement. For instance, Mendoza and Smith (2006) use  $k_{t+1}$  based on the model of margin loans proposed by Aiyagari and Gertler (1999), in which the assets that an agent buys at date  $t$  are used as collateral. Bianchi and Mendoza (2018) use  $k_t$  based on the incentive-compatibility constraint of an optimal debt contract with limited enforcement that is set at the beginning of each date  $t$  before the asset market opens (see Appendix A5 of Bianchi and Mendoza, 2018 for details). Constraints with either timing specification have been extensively used, including in models with capital accumulation and working-capital financing added to the credit constraint (Mendoza, 2010), models of learning and financial innovation (Boz and Mendoza, 2014), exchange-rate policy (Fornaro, 2018), studies on the effects of financial policies and financial integration (Durdu and Mendoza, 2006; Mendoza and Smith, 2014; Jeanne and Korinek, 2018; Reyes-Heroles and Tenorio, 2017), and models of self-fulfilling Sudden Stops (Schmitt-Grohé and Uribe, 2017).

2. *Debt-to-income (DTI) or flow constraints:* Debt cannot exceed a given fraction of income, including income from tradable and non-tradable sectors, denoted  $y^T$  and  $y^N$ , respectively:

$$q_t^b b_{t+1} \geq \kappa_t \left( y_t^T + p_t^N y_t^N \right). \quad (2)$$

<sup>9</sup> Chapter 12 of Uribe and Schmitt-Grohé (2017) provides a useful textbook presentation of these models.

<sup>10</sup> See Mendoza and Quadrini (2010) for an example of a Sudden Stops model with heterogeneous agents.

This DTI constraint was proposed by Mendoza (2002) in a study examining the feedback between real-exchange-rate movements and borrowing capacity. This type of income-based constraint is typical in household financing (e.g., DTI's on mortgages required by entities like Fannie Mae or Freddie Mae or those used for credit scoring for auto and credit-card loans). The constraint limits debt denominated in units of tradables to the value of pledgeable collateral expressed in the same units.<sup>11</sup> From a contractual point of view, the reason why non-tradable goods enter as collateral is that foreigners can seize them from a defaulting borrower and sell them in the domestic market in exchange for tradable goods. The literature has extensively used this formulation of the credit constraint in models of reserve accumulation (Durdu et al., 2009; Arce et al., 2019), macroprudential policy (Bianchi, 2011), real-exchange-rate stabilization policies (Benigno et al., 2013), ex-post intervention with industrial policy (Hernandez and Mendoza, 2017), self-fulfilling crises (Schmitt-Grohé and Uribe, 2018), noisy news and regime-switching shocks (Bianchi et al., 2016), trend shocks (Flemming et al., 2019; Seoane and Yurdagul, 2019), imperfect enforcement in capital-flow management policies (Bengui and Bianchi, 2018), models with banks intermediating capital inflows in  $T$  units to fund domestic loans in units of the domestic CPI (Mendoza and Rojas, 2019), and models of exchange-rate policy with nominal rigidities and credit frictions (Ottonello, 2015; Coulibaly, 2018; Farhi and Werning, 2016).

### 3.1. Fisherian debt-deflation amplification mechanism

The models we study here are labeled “Fisherian” because when the collateral constraint binds, they display dynamics driven by the classic debt-deflation mechanism first proposed in the work of Fisher (1933): agents fire sale goods and/or assets in order to meet their obligations, but as they do so the prices of those goods or assets fall. Since the Fisherian constraint links borrowing capacity to prices, the decline in prices further tightens the constraint, forcing further fire sales. This feedback mechanism amplifies the effects of shocks relative to states of nature in which the credit constraint does not bind. These models differ from other models with credit constraints, because while a binding credit constraint always implies a negative effect on aggregate demand, due to the direct (or balance-sheet) effect of the constraint on demand for consumer goods, only models in which market prices enter in the credit constraint feature Fisherian amplification. Moreover, some Fisherian models also include adverse supply-side effects due to declining values of marginal products of inputs in response to price deflation (e.g., Durdu et al., 2009), binding credit limits for working capital (e.g., Bianchi and Mendoza, 2018), and falling investment in response to collapsing equity prices (e.g., Mendoza, 2010). In addition, some models include international spillovers driven by international asset trading and short-selling constraints or mark-to-market capital requirements (e.g., Mendoza and Smith, 2006; Mendoza and Quadrini, 2010).

A tractable analytic characterization of Fisherian amplification is difficult to obtain in general, because of the lack of closed-form solutions typical of dynamic general-equilibrium models and the non-linearities implied by the occasionally binding credit constraint. One exception is the perfect-foresight analysis of the DTI model with endowment incomes examined in Mendoza (2005) and extended to examine equilibrium multiplicity in Schmitt-Grohé and Uribe (2018) and intermediation of capital inflows in Mendoza and Rojas (2019). In these models, a representative agent consumes a CES composite commodity  $(c^T, c^N)$  that combines tradables and non-tradables. The agent chooses consumption and optimal bond holdings so as to maximize a standard intertemporal utility function subject to the DTI constraint and a budget constraint with a time-varying income of tradables and a fixed endowment of non-tradables. In Mendoza (2005), the competitive equilibrium of the economy is given by sequences of allocations  $\{c_t^T, c_t^N, b_{t+1}\}_{t \geq 0}$  and prices  $\{p_t^N\}_{t \geq 0}$  that satisfy the following conditions:

$$p_t^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\eta+1} \tag{3}$$

$$u_T(t) = \beta R [u_T(t+1)] + \mu_t \tag{4}$$

$$q^b b_{t+1} \geq -\kappa [y_t^T + p_t^N \bar{y}^N], \quad \text{with equality if } \mu_t > 0, \tag{5}$$

$$c_t^N = \bar{y}^N \tag{6}$$

$$c_t^T = y_t^T - q^b b_{t+1} + b_t, \tag{7}$$

where  $\mu_t$  is the non-negative Lagrange multiplier on the credit constraint,  $u_T(t) \equiv u'(c_t) \partial c_t / \partial c_t^T$ , and  $\omega$  and  $\eta$  are parameters of the CES aggregator  $c(\cdot)$  such that  $\omega$  is the tradables share parameter and  $1/(1 + \eta)$  is the elasticity of substitution between  $c_t^T$  and  $c_t^N$ .<sup>12</sup>

The above equilibrium conditions have two immediate implications that are important for the analysis that follows: First, (3) and (6) imply that at equilibrium, the price of non-tradables is an increasing function of the allocation of tradables

<sup>11</sup> Assuming the law of one price holds and no foreign inflation, this is equivalent to assuming the debt is in hard currency but backed partially by income in domestic currency, and hence the setup captures in part the liability dollarization phenomenon.

<sup>12</sup> The CES aggregator is  $c_t = [\omega (c_t^T)^{-\eta} + (1 - \omega) (c_t^N)^{-\eta}]^{-\frac{1}{\eta}}$ ,  $\eta > -1$ ,  $\omega \in (0, 1)$ .

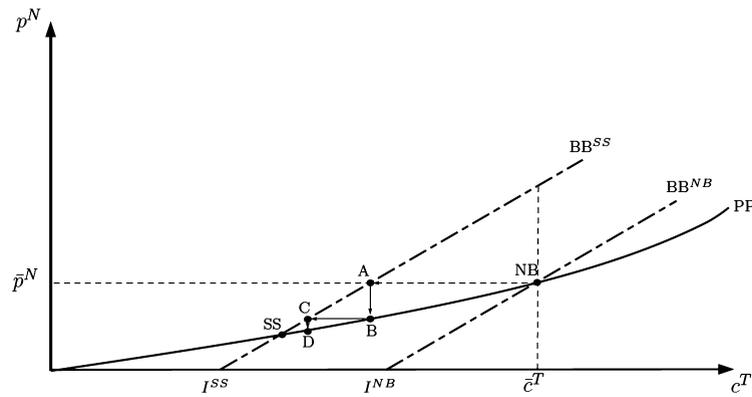


Fig. 3. Sudden Stops under perfect foresight: unique equilibrium.

consumption, so that the equilibrium price can be denoted  $p^N(C_t^T)$ . Second, if the credit constraint binds,  $C_t^T$  is given by the solutions to the following nonlinear equation in  $C_t^T$  formed by conditions (3), (5) holding with equality, (6) and (7):

$$C_t^T = (1 + \kappa)y_t^T + \kappa p^N(C_t^T)\bar{y}^N + b_t. \tag{8}$$

Assuming no-Ponzi-game condition, the intertemporal resource constraint for tradables is

$$\sum_{t=0}^{\infty} R^{-t} c_t^T = \sum_{t=0}^{\infty} R^{*-t} y_t^T + b_0, \tag{9}$$

where  $\sum_{t=0}^{\infty} R^{*-t} y_t^T \equiv W_0$  is the tradables non-financial wealth of the economy. This condition, together with (3)-(6), fully characterizes this model's equilibrium. Following Mendoza (2005), we simplify the analysis by assuming that: a)  $\beta R^* = 1$ ; b)  $b_0 < 0$  (i.e., the economy starts with some debt), c)  $y_0^T$  is lower than in the future, so that agents would want to set  $b_1 < 0$ ; and d) there are wealth-neutral shocks that reduce  $y_0^T$  keeping  $W_0$  constant.<sup>13</sup> These shocks induce agents to borrow more. For a shock large enough to make  $y_0^T$  fall below a threshold value  $\hat{y}_0^T$ , the collateral constraint binds, but for weaker shocks it does not.

For  $y_0^T \geq \hat{y}_0^T$ , the collateral constraint does not bind at  $t = 0$ , and since  $\beta R^* = 1$ , we get the standard result that tradables consumption is a constant fraction of total wealth:

$$\bar{c}^T = (1 - \beta)(W_0 + b_0). \tag{10}$$

Moreover, since consumption of tradables and the non-tradables endowment (and consumption) are constant, the equilibrium price of non-tradables is also constant.

For  $y_0^T < \hat{y}_0^T$ , the collateral constraint binds, and a Sudden Stop occurs. Condition (7) implies that  $c_0^T$  falls, because access to debt is insufficient to sustain  $\bar{c}^T$ . Then, it follows from condition (3) that  $p_0^N$  falls to clear the non-tradables market. This triggers the Fisherian amplification mechanism: the endogenous price drop further tightens the collateral constraint because it reduces the value of collateral provided by the non-tradables endowment in condition (5). Formally, the value of  $c_0^T$  is now determined by condition (8).

Fig. 3 illustrates the determination of unique constrained and unconstrained equilibria in a manner analogous to Figure 2 in Mendoza (2005). The multiple-equilibria case is discussed later in this Section. The PP curve is the  $p^N(C_t^T)$  function, which as noted above is increasing and convex in  $C^T$ . The  $BB^{SS}$  and  $BB^{NB}$  curves plot values of  $p^N$  that correspond to values of  $c^T$  such that the collateral constraint holds with equality and the tradables resource constraint is satisfied (i.e., equation (8) solved for  $p_0^N$  as a function of  $c_0^T$ ) for different values of  $y_0^T$ .<sup>14</sup>  $BB^{NB}$  is for  $y_0^T = \hat{y}_0^T$  so that the constraint is marginally binding: a wealth-neutral shock of this magnitude sustains the exact amount of debt that agents would want to have.<sup>15</sup>  $BB^{SS}$  is for  $y_0^T < \hat{y}_0^T$ , so that the binding constraint does alter allocations and prices.

The competitive equilibrium when the constraint is not binding (binding) is determined at point NB (SS), which is the intersection of the PP curve with the  $BB^{NB}$  ( $BB^{SS}$ ) curve. For any  $y_0^T \geq \hat{y}_0^T$ , the constraint does not bind and the equilibrium remains at point NB with consumption at  $\bar{c}^T$  and the non-tradables price at  $\bar{p}^N$ . Income shocks such that  $y_0^T < \hat{y}_0^T$  shift

<sup>13</sup> A wealth-neutral income shock at  $t=0$  is defined by income levels  $(y_0^T, y_1^T)$  such that  $y_1^T - \bar{y}^T = R(\bar{y}^T - y_0^T)$  and  $y_t^T = \bar{y}^T$  for  $t \geq 2$ . Hence, wealth remains constant at  $W_0 = \bar{y}^T / (1 - \beta)$ .

<sup>14</sup> Mendoza (2005) showed that the BB curves are increasing, linear functions of  $c_0^T$  with a horizontal intercept given by  $I^{SS} \equiv (1 + \kappa)y_0^T + b_0$  and a slope of  $m^{SS} \equiv 1/(\kappa\bar{y}^N)$ .

<sup>15</sup>  $\hat{y}_0^T = \frac{\bar{c}^T - b_0 - \kappa\bar{p}^N\bar{y}^N}{1 + \kappa}$ , where  $\bar{c}^T$  and  $\bar{p}^N$  are the unconstrained equilibrium outcomes.

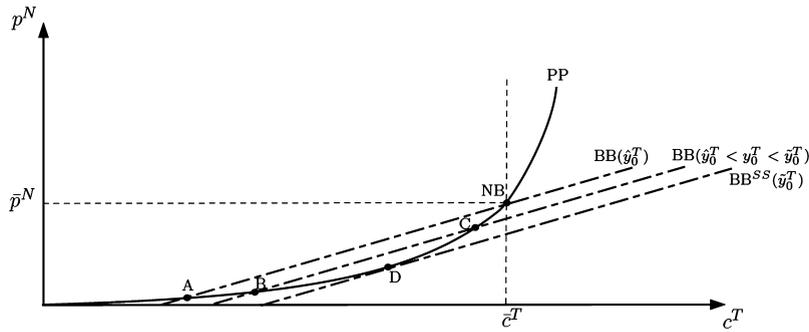


Fig. 4. Sudden Stops under perfect foresight: multiple equilibria.

the BB curve to the left, triggering the credit constraint and causing a Sudden Stop. The credit constraint forces agents to reduce consumption to the value consistent with point A, which is lower than  $\bar{c}^T$ , but at that consumption level, clearing the market of non-tradables makes the price fall to the one consistent with point B. But at this lower price, the credit constraint tightens and forces consumption to fall to the value consistent with point C, and market-clearing then requires the price to fall to point D. This Fisherian debt-deflation feedback loop continues until the Sudden Stop equilibrium is reached at point SS. This point yields the consumption and price values that solve equation (8). Hence, this Figure highlights how the Fisherian mechanism amplifies the effects of income shocks, causing a sharp drop in consumption and the price of non-tradables (i.e., the real exchange rate). Implicit in the consumption reversal is a sharp reversal of the current account, implied by the sudden increase in the NFA position forced by the binding credit constraint at the sharply lower  $p^N$ .

How do results change if we consider a credit constraint independent of market prices instead of the Fisherian constraint? For example, debt could be set not to exceed a constant value or set to keep a debt-to-income cap in which non-tradables are valued at a notional “book value.” In this case, the BB curves in Fig. 3 become vertical lines at the level of tradables consumption that the fixed credit constraint supports. The figure determines only the equilibrium price (where the vertical BB curves would intersect with the PP curve), because consumption is exogenously determined by the credit constraint. The constraint still forces consumption to fall relative to the unconstrained outcome, but there is no Fisherian amplification mechanism, as the lower equilibrium price does not cause an endogenous tightening of the credit limit.

It is worth noting also that there is a second intersection of the BB and PP curves in Fig. 3, which would be visible if we extended its domain far enough. This second intersection, however, is not an equilibrium, because  $c_0^T$  would be higher than  $\bar{c}^T$ . Moreover, given the specification of the wealth-neutral shock, the intertemporal resource constraint would imply  $c_1^T < c_0^T$ , which would imply a negative Lagrange multiplier ( $\mu_0 = u'(c_0^T) - u'(c_1^T) < 0$ ). In addition, the unconstrained equilibrium cannot co-exist with the unique Sudden Stops equilibrium, because as Fig. 3 shows, with the credit constraint binding, the value of  $p_0^N$  at which the non-tradables market would clear if tradables consumption is  $\bar{c}^T$  is too low for the resource constraint to be satisfied.

While the constrained and unconstrained equilibria illustrated in Fig. 1 are unique, it is possible to obtain multiple equilibria in both DTI and LTV Sudden Stops models, as Schmitt-Grohé and Uribe (2017) and (2018) showed. This opens up the possibility of Sudden Stops due to self-fulfilling expectations or “sunspots.” In the specific DTI example we are analyzing, Mendoza (2005) noted that a sufficient condition for uniqueness is that the BB curve be flatter than the PP curve around the unconstrained equilibrium (point NB). Formally, the sufficiency condition for uniqueness is:<sup>16</sup>

$$\kappa \leq \hat{\kappa} \equiv \frac{\bar{c}^T}{(1 + \eta)\bar{p}^N \bar{y}^N}. \tag{11}$$

Note, however, that the condition involves not only the parameter  $\kappa$  but also the preference parameters that determine  $\bar{c}^T$  and  $\bar{p}^N$ , which include  $\beta$ ,  $\omega$ , and  $\eta$ . Given these parameters, if  $\kappa$  satisfies this condition, the constrained and unconstrained equilibria are unique. Intuitively, the condition states that the cap on debt relative to income cannot exceed the product of the elasticity of substitution times the ratio of tradables-to-non-tradables expenditures.

Failure of the above condition is necessary but not sufficient for multiplicity. Multiplicity requires also that  $y_0^T$  be in a particular interval. To see this, Fig. 4 shows the equilibrium determination assuming that the sufficiency condition for uniqueness fails. For simplicity, we assume also that the preference parameters are given, and hence the condition is violated because  $\kappa$  is “too large.” As the figure shows, at  $y_0^T = \hat{y}_0^T$  we now have two equilibria. One is the same unconstrained equilibrium as in Fig. 1 (point NB), but now point A is also an equilibrium and is one in which the constraint binds, resulting in sharply lower consumption and prices. Here, Sudden Stops would be the result of self-fulfilling expectations.

<sup>16</sup> This condition follows from noting that the slope of the BB curve is  $1/(\kappa \bar{y}^N)$ , and since the PP curve is given by condition (3), its slope can be expressed as  $\frac{(1+\eta)\bar{p}^N}{c_t^T}$ , which at the unconstrained equilibrium equals  $\frac{(1+\eta)\bar{p}^N}{\bar{c}^T}$ .

But what happens as  $y_0^T$  changes? For  $y_0^T < \hat{y}_0^T$ , the BB curve would shift to the left, and there is a *unique* Sudden Stop equilibrium. But if BB shifts slightly *to the right*, namely, if income rises a little above  $\hat{y}_0^T$ , there are now three equilibria: two Sudden Stop equilibria at the two points where the BB curve intersects with the PP curve (points B and C), as well as the unconstrained equilibrium at NB (because with  $y_0^T > \hat{y}_0^T$ , the unconstrained outcome is also an equilibrium). BB keeps shifting rightward as income rises, and we keep finding three equilibria. But when we reach the income level  $\tilde{y}_0^T$  such that BB is tangent to SS, we have only two equilibria (the tangency point D and the unconstrained equilibrium NB). For  $y_0^T > \tilde{y}_0^T$ , multiplicity disappears, and the unconstrained equilibrium is the unique equilibrium. Hence, multiplicity requires  $\hat{y}_0^T \leq y_0^T \leq \tilde{y}_0^T$ .

Schmitt-Grohé and Uribe (2018) derived a similar necessary condition on  $\kappa$  required for multiplicity, but focused on how multiplicity emerges if initial bond holdings fall within an interval of relatively high values (i.e. multiplicity requires low initial debt), instead of focusing on income shocks. The value of  $b_0$  must be high enough (i.e. debt low enough) so that at the unconstrained equilibrium condition (11) holds, but not so high that the credit constraint does not bind (since higher  $b_0$  also implies higher  $b_1$  at the unconstrained equilibrium). Intuitively, the consistency of the two treatments follows from noticing that by capitalizing initial income differences into changes in bond holdings, differences in  $b_0$  keeping  $y_0^T$  constant can be alternatively represented as differences in  $y_0^T$  keeping  $b_0$  constant. Hence, an interval of relatively high  $b_0$  (i.e., low debt) that sustains multiplicity is equivalent to an interval of relatively *high* initial income values.

The quantitative relevance of multiplicity in Fisherian Sudden Stops models is an open question. It is possible to construct reasonably calibrated models where condition (11) can hold or fail (e.g., Hernandez and Mendoza, 2017 v. Schmitt-Grohé and Uribe, 2018). This depends on assumptions about the relevant values of  $\kappa$  and parameters like the intertemporal elasticity of substitution in consumption, estimates of which are very noisy, as well as the size of debt ratios and the data frequency at which they are being targeted. Moreover, violating condition (11) is necessary but not sufficient for multiplicity. If it fails, the likelihood of multiplicity then depends on the probability of observing income in the relevant interval of relatively high but not too-high values (above  $\hat{y}_0^T$  but below  $\tilde{y}_0^T$ ). But this implies that Sudden Stops should be associated with relatively high income, which is not in line with the stylized facts. Finally, both condition (11) and the multiplicity income interval are model dependent. For instance, if banks intermediate capital inflows in units of  $T$  goods to fund domestic loans in units of the domestic CPI, Mendoza and Rojas (2019) show that multiplicity requires  $\kappa$  to be higher than in condition (11), and the multiplicity income interval narrows. The literature has found, however, that Fisherian amplification in models with a unique equilibrium is quantitatively large, and hence the Fisherian framework can provide a plausible theory of Sudden Stops even without multiplicity and with Sudden Stops co-existing with typical business cycles (Mendoza, 2010). Moreover, Sudden Stops result from financial amplification of shocks of standard magnitudes included in the known set of shock realizations, so this theory of Sudden Stops does not rely on large, unexpected shocks.

### 3.2. Normative implications of Fisherian models

We close this Section by examining the market-failure argument that justifies the use of macroprudential policy in Fisherian models. The competitive equilibrium in these models is generally constrained-inefficient because of a pecuniary externality by which agents do not internalize the effects of their borrowing decisions on collateral values. Pecuniary externalities generally do not distort allocations, but in Fisherian models they do because market prices alter borrowing capacity. In particular, there is scope for implementing policies that reduce borrowing (e.g., debt taxes or equivalent instruments) because agents do not internalize that higher borrowing at time  $t$  contributes to reduce the price of collateral at  $t + 1$  and, in this way, tightens the collateral constraint of all agents in the economy (see Bianchi, 2011). As a result, the private marginal cost of borrowing ignores the response of collateral values at  $t + 1$  if the credit constraint becomes binding. A social planner or financial regulator would factor this in, and hence private and social marginal costs of borrowing differ.

To formalize the above argument in a generic form that applies to both DTI and LTV models, consider that the equilibrium relative prices determining the market value of collateral correspond to marginal rates of substitution in consumption. Because these are general equilibrium outcomes, individual borrowers do not internalize the effects of their own borrowing decisions on the aggregate variables that pin down the value of collateral via these equilibrium conditions, but a social planner does, because it internalizes that prices are determined jointly with allocations. Appendix A provides a full characterization of the constrained-efficient planner's problem illustrating the nature of the pecuniary externality and showing how the planner's allocations can be decentralized with debt taxes.

In DTI models, the relevant marginal rate of substitution for the value of collateral is that between  $c^T$  and  $c^N$ , and in the widely used case in which income from  $N$  goods is an exogenous endowment, we saw before that we can express the price of non-tradables as a function  $p_t^N(C_t^T)$  of aggregate tradables consumption  $C_t^T$ . In the LTV models, the relevant marginal rate of substitution is the entire set of future intertemporal marginal rates of substitution, so the equilibrium value of collateral can be expressed as  $q_t(C_t, C_{t+1} \dots)$ . Notice a key difference between the two cases: in the DTI model, the price depends only on date- $t$  aggregate variables, whereas in the LTV model, it depends on current and future variables. This difference has crucial implications for time consistency of optimal macroprudential policy that we discuss later in this Section.

A property common to all Fisherian models is that in a decentralized equilibrium without policy intervention the households' Euler equation for bond holdings is

$$u'(t) = \beta R_t \mathbb{E}_t [u'(t + 1)] + \mu_t. \quad (12)$$

Here,  $u'(t)$  denotes the marginal utility of consumption, with the caveat that for DTI models like the one examined earlier it denotes the marginal utility of  $c^T$ . The non-negative Lagrange multiplier  $\mu_t$  is again the multiplier on the collateral constraint. Intuitively, when the constraint binds, the marginal cost of borrowing rises because it is as if the effective interest rate rose above  $R_t$  by an amount that depends on the shadow value of the constraint.

Studies of optimal macroprudential policy differ widely, depending on the particular structure of the model and the policy instruments that are available. A general result, however, is the presence of overborrowing, characterized by a wedge in the intertemporal Euler equation of debt that arises in the optimization problem of a constrained social planner who chooses borrowing on behalf of the households. The planner is subject to the implementability constraints associated with the private optimality conditions—except the one for debt, because it is chosen by the planner—in addition to the financial and resource constraints (Appendix A provides a full description of the constrained-efficient planner’s problem). Focusing on a state of nature in which the collateral constraint does not currently bind, the planner’s Euler equation for bonds typically takes this form<sup>17</sup>:

$$u'(t) = \beta R_t \mathbb{E}_t \left[ u'(t) + \mu_{t+1} \kappa_{t+1} \psi_{t+1}^i \right], \quad i = DTI, LTV, \tag{13}$$

where  $\mu_{t+1}$  is the planner’s multiplier on the borrowing constraint, and  $\psi_{t+1}^i$  measures the change in the market value of collateral at  $t + 1$  as the bond holdings chosen by the planner at date  $t$  (i.e.,  $B_{t+1}$ ) change. For the DTI and LTV models, this is given by these expressions:

$$\psi_{t+1}^{DTI} = y_{t+1}^N (\partial p_{t+1}^N / \partial C_{t+1}^T), \tag{14}$$

$$\psi_{t+1}^{LTV} = K_{t+1+j} (\partial q_{t+1} / \partial C_{t+1}), \quad j = 0, 1. \tag{15}$$

The term  $\mu_{t+1} \kappa_{t+1} \psi_{t+1}^i$  represents an externality because it captures the difference in the marginal cost of borrowing faced by the social planner v. the one faced by private agents in the absence of regulation. It is relevant only if the constraint is expected to bind in at least some states of nature at  $t + 1$ . It is a pecuniary externality because it is caused by the price effects that are the aggregate result of individual choices and, as such, are not internalized by private agents. The externality is said to induce overborrowing (underborrowing) if  $\psi_{t+1}^i > 0$  ( $\psi_{t+1}^i < 0$ ), because it implies that private agents undervalue (overvalue) the marginal cost of borrowing relative to what is socially optimal. In turn, the sign of  $\psi_{t+1}^i$  is determined by the sign of the derivative of the collateral price with respect to aggregate consumption:  $\partial p_{t+1}^N / \partial C_{t+1}^T$  and  $\partial q_{t+1} / \partial C_{t+1}$  for the DTI and LTV models, respectively. Notice that these derivatives are an equilibrium object, so establishing their sign, rather than assuming it, is essential (see Appendix K of Bianchi and Mendoza, 2018 for a discussion of this issue).

In the simpler versions of DTI and LTV models, it is relatively straightforward to establish that the price derivatives are indeed positive, because of the concavity of utility. The equilibrium pricing function derivatives in these models are

$$\frac{\partial p_{t+1}^N}{\partial C_{t+1}^T} = \frac{-p_{t+1}^N u''(t+1)}{u'(t+1)} > 0; \quad \frac{\partial q_{t+1}}{\partial C_{t+1}} = \frac{-q_{t+1} u''(t+1)}{u'(t+1)} > 0.$$

As noted earlier, the derivatives of  $u(\cdot)$  for the DTI model are with respect to  $c^T$ , with composite consumption given by a CES aggregator. Those for the LTV model are with respect to the standard aggregate consumption measure.

Since the pricing derivatives are positive, the pecuniary externality is positive, and agents overborrow at date  $t$  when the constraint does not bind, because they fail to internalize that additional debt taken at  $t$  leads to a larger collapse in collateral values if the credit constraints bind at  $t + 1$ . The allocations of the social planner can therefore be decentralized by taxing debt at date  $t$  by the correct amount, rebating the revenue generated by this tax as a lump-sum transfer. The optimal debt tax is the one that leads the private marginal cost of borrowing in a decentralized equilibrium with regulation (i.e., with debt taxes) to equalize the social marginal cost of borrowing. This requires an optimal macroprudential debt tax given by:

$$\tau_t = \frac{\mathbb{E}_t [\mu_{t+1} \kappa_{t+1} \psi_{t+1}^i]}{\mathbb{E}_t [u'(t+1)]} \quad i = DTI, LTV. \tag{16}$$

If the constraint is expected to bind at  $t + 1$  in at least some states of nature, this tax is strictly positive, because it inherits the sign of the pecuniary externality, which is given by the sign of the pricing derivatives determined above. Moreover, everything else the same, the tax is higher when (a) the constraint is more likely to bind at  $t + 1$  (i.e., the larger is the set of states at  $t + 1$  for which  $\mu_{t+1} > 0$ ), (b) prices fall more in response to the collapse in demand at  $t + 1$  (i.e., the larger are the price derivatives), and (c) the larger the value of pledgeable collateral at  $t + 1$ .

The above argument uses debt taxes to decentralize the optimal macroprudential policy because they are a natural way of doing so given that we are dealing with an externality. As documented in Section 1, however, standard tax instruments are

<sup>17</sup> Technically, we assume implicitly also that if the planner chooses under commitment, all previous collateral constraints are also not binding (see Bianchi and Mendoza, 2018 for details).

not a widely used macroprudential policy instrument, compared with rules for capital buffers with countercyclical elements, or liquidity requirements such as the liquidity coverage ratio, and more recently limits on loan-to-value and debt-to-income ratios on borrowers. Still, under some conditions, it is possible to implement the optimal macroprudential policy with these instruments (Bianchi, 2011), echoing standard results on the equivalence between price and quantity instruments. An alternative macroprudential instrument that does not rely on financial regulation is the accumulation of international reserves (Arce et al., 2019).

In line with the properties of the optimal debt tax, the other MPP instruments would be deployed whenever there is a positive one-step-ahead conditional probability of the constraint becoming binding. These instruments, however, may have different comparative statics or cyclical properties, as discussed in Bianchi (2011) and Stavrageva (2020). Moreover, as we show in Appendix B, in a model in which households pledge assets as collateral, debt taxes and regulatory LTVs are not equivalent, because they have different effects on asset prices.

There are three important additional considerations to the above discussion:

1. Depending on model structure, the social planner may have incentives to intervene not just with macroprudential or ex-ante policy when  $\mu_t = 0$  and  $\mathbb{E}_t[\mu_{t+1} > 0]$  but also with ex-post policy when  $\mu_t > 0$ . Benigno et al. (2013) study a DTI model in which tradables and non-tradables are produced with labor, and wage income enters in the credit constraint.<sup>18</sup> The planner intervenes ex-post to increase the relative price of non-tradables by reallocating labor toward tradables or by subsidizing non-tradable goods. Similarly, Benigno et al. (2016) show how a subsidy on non-tradables can prop up the real exchange rate and relax the constraint. In Schmitt-Grohé and Uribe (2018), there are multiple equilibria, and a subsidy on borrowing when the constraint binds can rule out the bad equilibrium. Interestingly, in these cases, while the planner ends up borrowing more than in the unregulated equilibrium because of the ability to mitigate the Fisherian deflation ex-post, the tax on debt remains positive and given by the expression (16). Hernandez and Mendoza (2017) study a DTI model with sectoral production using imported inputs. There is overborrowing only, but again the planner reallocates inputs from non-tradables to tradables production ex-post to prop up the value of collateral and weaken the constraint. In Bianchi (2016), firms face collateral and dividend constraints, and when the dividend constraint binds, a pecuniary externality operates via wages to reduce profits and investment. The planner uses bailouts to stabilize firms' net-worth ex-post and taxes debt to hamper overborrowing ex-ante.
2. When collateral values at date  $t$  are determined jointly by date- $t$  and date- $t+1$  equilibrium outcomes, the planner's optimal plans become time-inconsistent under commitment. Time-inconsistency is important because it undermines the credibility of macroprudential policy and raises the well-known problems of rules v. discretion, which are examined in the broader literature on optimal policy. In the MPP context, this issue was raised first by Bianchi and Mendoza (2018) in their analysis of a planner's optimal policy problem in an LTV model with and without commitment. They showed that the time-inconsistency of the optimal policy under commitment emerges because the planner internalizes the Euler equation driving asset prices, which makes  $q_t$  increasing in  $c_t$  and decreasing in  $c_{t+1}$ . Hence, the planner realizes that rising  $c_t$  weakens the credit constraint at  $t$  but tightens it at  $t - 1$ . Intuitively, when the LTV constraint binds at  $t$ , the planner acting under commitment promises lower future consumption so as to prop up asset prices and borrowing capacity at  $t$ , but ex post when  $t + 1$  arrives, it is suboptimal to keep this promise. Bianchi and Mendoza show also that debt taxes can be used to decentralize the optimal policy of the planner with or without commitment. The debt tax can be separated into two components. One is macroprudential and again tackles the pecuniary externality, and the other captures the effects of the incentives to intervene when the credit constraint binds. The macroprudential component is not, however, characterized by the same expressions with and without commitment. They are the same only the first time the constraint binds along a given equilibrium path (i.e., if the collateral constraint has never been binding up to some date  $t$  and is expected to bind with some probability at  $t + 1$ ). Otherwise, in future dates  $t + j$  in which the constraint does not bind but can bind at  $t + j + 1$ , the expressions determining optimal macroprudential debt taxes differ with and without commitment.
3. Quantitatively, optimal macroprudential policy is very effective at reducing the magnitude and frequency of financial crises, but it requires complex, non-linear, state-contingent schedules for the management of policy instruments. Moreover, simpler policy rules are less effective and can be welfare reducing.<sup>19</sup> Using the DTI model, Bianchi (2011) found that the "best" (i.e., welfare-maximizing) constant debt tax reduces the probability of Sudden Stops from 5.5. to about 2.2 percent (v. 0.4 percent with the optimal debt taxes). Moreover, the optimal fixed tax is 3.6 percent, which is about 70 percent of the average of the state-contingent tax and achieves 62 percent of the welfare gains from implementing the constrained-efficient allocations. Hernandez and Mendoza (2017) found that the best constant debt and production taxes reduce the probability of Sudden Stops from 3 to 1.1 percent v. 0 with the optimal taxes, and the welfare gain is about half of that under the optimal taxes. In addition, a constant debt tax set to the average of the optimal debt tax is welfare reducing. In an LTV model, Bianchi and Mendoza (2018) found that several simple rules, including constant

<sup>18</sup> This formulation has nontrivial limitations in terms of matching the stylized facts of Sudden Stops. Since agents can borrow more by working harder, the model underestimates the magnitude of Sudden Stops and predicts that tradables output rises during Sudden Stops.

<sup>19</sup> This is in sharp contrast with findings for monetary and fiscal policies. The Taylor rule is generally sub-optimal in the New Keynesian DSGE models used in central banking, but quantitatively it is very effective at smoothing fluctuations and targeting inflation, and it is welfare-improving. Re-arrangements of the tax structure, even of simple constant tax rates, can produce large efficiency and welfare gains.

taxes and constant-elasticity rules akin to a Taylor rule that adjust debt taxes to target credit or asset prices, are significantly inferior to the optimal policy and can easily lead to welfare losses. The difference relative to the results with DTI models is that when asset prices determine the value of collateral, by altering marginal utilities and asset prices, a tax that is optimal in some states can have negative welfare effects on other states. In this situation, the lack of a state-contingent policy makes the application of macroprudential policies more challenging.

#### 4. Macroprudential policy tradeoffs

The existing quantitative assessments of simple MPP rules documented in the previous Section abstract from potentially important limitations of those rules, because they were obtained using models in which the main tradeoff of sub-optimal credit regulation is hampering consumption smoothing and distorting precautionary savings. In practice, however, in both the long and short-run, regulation is likely to also affect capital accumulation and the level of output. In this section, we shed light on these issues by studying a setup with endogenous capital accumulation that combines elements of both LTV and DTI Fisherian models.

##### 4.1. A two-sector model with investment

We propose a two-sector model in which tradables and non-tradables are used both for consumption and as inputs for production of investment goods. Final output in each sector is produced with capital, and capital is homogeneous across sectors. Households face an LTV collateral constraint. The model is designed so that the relative goods price of non-tradables determines also the price of capital as collateral.

*Investment goods production* A representative firm produces investment goods  $i_t$  using as input a composite good  $x_t = x(x_t^T, x_t^N)$  formed by a constant elasticity-of-substitution (CES) aggregator of inputs from the tradables and nontradable sectors  $x_t^T$  and  $x_t^N$ :

$$x_t = \left[ \pi \left(x_t^T\right)^{-\theta} + (1 - \pi) \left(x_t^N\right)^{-\theta} \right]^{-\frac{1}{\theta}}, \theta > -1, \pi \in (0, 1), \tag{17}$$

where  $1/(1 + \theta)$  is the elasticity of substitution between  $x_t^T$  and  $x_t^N$ .

This firm’s optimization problem is solved using standard two-stage budgeting and duality results. In the first stage, the producer minimizes the cost of inputs required to obtain a target amount of  $x$  denoted  $\bar{x}$ :

$$C(p_t^N, \bar{x}) = \min_{x_t^T, x_t^N} \left[ x_t^T + p_t^N x_t^N \right] \tag{18}$$

s.t.

$$x(x_t^T, x_t^N) = \bar{x}. \tag{19}$$

As in the DTI models reviewed earlier, tradable goods are the numeraire and  $p_t^N$  denotes the relative price of non-tradables in units of tradables.

The first-order conditions of this problem imply that:

$$x_{x^N}(x_t^T, x_t^N) / x_{x^T}(x_t^T, x_t^N) = p_t^N \tag{20}$$

Using the linear homogeneity of  $x(x_t^T, x_t^N)$ , we obtain an homogeneous price index  $P(p_t^N)$  in units of tradables such that the cost function of the producer is  $C_t = P(p_t^N)x_t$ . Hence,  $P(p_t^N)$  is the minimum expenditure needed to produce one unit of investment goods for a given  $p_t^N$  (i.e.,  $C(p_t^N, 1) = P(p_t^N) = x_t^T + p_t^N x_t^N$ ). Given the CES structure of  $x_t$ , the price index of investment goods is:

$$P_t(p_t^N) = \left[ \pi^{\frac{1}{1+\theta}} + (1 - \pi)^{\frac{1}{1+\theta}} \left(p_t^N\right)^{\frac{\theta}{1+\theta}} \right]^{\frac{1+\theta}{\theta}}. \tag{21}$$

In the second stage, the firm chooses the optimal  $x_t$  so as to maximize profits of selling investment goods at a relative price  $q_t$  subject to the investment goods-production technology denoted  $f(x)$ . In order to obtain an equilibrium in which the price of investment goods  $q_t$  is determined by the cost price index  $P_t$  in “static” fashion (since  $P_t$  is determined by the relative price of nontradable goods), we assume a simple linear technology  $f(x_t) = z_t^x x_t$ , where  $z_t^x$  is a shock to the productivity of producing investment goods. The profit maximization problem is

$$\max_{i_t, x_t} [q_t i_t - P_t x_t] \tag{22}$$

s.t.

$$i_t = z_t^x x_t.$$

The first-order condition implies that  $q_t = P_t/z_t^\alpha$ . Hence, at equilibrium, profits are zero, and the price of investment goods is equal to the productivity-adjusted price index of the inputs used to produce them (which is a function of  $p_t^N$ ).

**Final goods production** Households own the firms that produce final goods, and these firms operate a production technology with decreasing returns that uses capital  $k$  as the only input. Capital depreciates at a rate  $(1 - \delta)$ . The production function is  $z_t k_t^\alpha$ , where  $z_t$  is a productivity shock to final goods production, and  $0 < \alpha < 1$ . Each period, this technology produces a flow of tradable goods equal to a fraction  $a$  of  $z_t k_t^\alpha$  and a fraction  $((1 - a)$  of  $z_t k_t^\alpha$  of non-tradables.

$$y_t^T = a z_t k_t^\alpha \quad (23)$$

$$y_t^N = (1 - a) z_t k_t^\alpha. \quad (24)$$

**Households** A representative household makes optimal consumption, saving and investment decisions so as to maximize expected lifetime utility:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right], \quad (25)$$

where  $c_t$  is a CES aggregator of tradables and non-tradables consumption  $c_t(c_t^T, c_t^N)$ , which takes the same form as in Section 2. The household faces the following budget and credit constraints:

$$q_t [k_{t+1} - k_t(1 - \delta)] + p_t^N c_t^N + c_t^T + \frac{b_{t+1}}{R} = b_t + p_t^N (1 - a) z_t k_t^\alpha + a z_t k_t^\alpha \quad (26)$$

$$\frac{b_{t+1}}{R} \geq \bar{b}_t - \kappa q_t k_{t+1} \quad (27)$$

The collateral constraint is of the LTV type but with the variation that it includes the term  $\bar{b}_t < 0$  aimed at capturing fluctuations in credit regimes. There are two regimes,  $\bar{b}_t = \{\bar{b}^L, \bar{b}^H\}$ , where  $\bar{b}^L$  ( $\bar{b}^H$ ) is the tight (loose) regime with one-step transition probability  $P_{LL}$  ( $P_{HH}$ ). Hence, the mean durations of the  $L$  and  $H$  regimes are determined by  $1/P_{LH}$  and  $1/P_{HL}$  respectively.

In the left-hand side of the budget constraint, we used the law of motion of the capital stock to substitute for investment (i.e., purchases of investment goods are given by  $q_t i_t = q_t [k_{t+1} - k_t(1 - \delta)]$ ). As in the case of the producer of investment goods, the household's total consumption expenditures can be expressed as  $P_t^C c(c_t^T, c_t^N) = c_t^T + p_t^N c_t^N$ , where  $P_t^C$  is the CES price index for consumption expenditures (which is again a linear homogeneous function of  $p_t^N$ ):

$$P_t^C = \left[ \omega^{\frac{1}{1+\eta}} + (1 - \omega)^{\frac{1}{1+\eta}} \left( p_t^N \right)^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}}. \quad (28)$$

Notice that since  $q_t$  is both the market price of new capital at which collateral is valued and the market price of investment goods, and the latter is a function of  $p_t^N$  at equilibrium, the model features financial amplification and externality effects similar to those resulting from having the price of non-tradables in the credit constraint of the standard DTI models. In addition, since the collateral constraint is of the LTV form, the Fisherian mechanism affecting excess returns and capital accumulation is also at work.

Let  $\mu$  denote the Lagrange multiplier on the collateral constraint. The first-order conditions of the household's problem imply the following optimality conditions:

$$c_{c^N}(c_t^T, c_t^N)/c_{c^T}(c_t^T, c_t^N) = p_t^N \quad (29)$$

$$u_{c^T}(t) - \mu_t = \beta R \mathbb{E}_t \{ u_{c^T}(t + 1) \} \quad (30)$$

$$q_t [u_{c^T}(t) - \kappa \mu_t] = \beta \mathbb{E}_t \left\{ u_{c^T}(t + 1) \left[ (1 - \delta) q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha-1} ((1 - a) p_{t+1}^N + a) \right] \right\}. \quad (31)$$

The Euler equations for bonds and capital imply that the excess return on capital simplifies to

$$\mathbb{E}_t (R_{t+1}^q) - R = \frac{-\text{cov}(\beta u_{c^T}(t + 1), R_{t+1}^q) + (1 - \kappa) \mu_t}{\beta \mathbb{E}_t [u_{c^T}(t + 1)]}, \quad (32)$$

where  $R_{t+1}^q \equiv [q_{t+1} - \delta q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha-1} ((1 - a) p_{t+1}^N + a)]/q_t$  is the rate of return of capital. Hence, in the absence of uncertainty and if the credit constraint does not bind, the model yields the standard property of small open economy models equating the return on domestic capital to the world real interest rate. Uncertainty introduces an equity-risk-premium term, and the credit constraint adds an additional premium given by the fraction  $1 - \kappa$  of the shadow value of the constraint. This premium is earned because of the fraction of new capital that cannot be pledged as collateral.

**Market-clearing and unregulated equilibrium** The market-clearing conditions in the markets for nontradable goods and investment goods are:

$$x_t^N + c_t^N = y_t^N \tag{33}$$

$$k_{t+1} - k_t(1 - \delta) = z_t^x x_t^T, x_t^N. \tag{34}$$

Using the above conditions together with  $q_t = P_t/z_t^x$ ,  $i_t = z_t^x x_t$  and  $P_t x_t = p_t^N x_t^N + x_t^T$ , we obtain the following resource constraint for tradables<sup>20</sup>:

$$x_t^T + c_t^T = y_t^T + b_t - \frac{b_{t+1}}{R}. \tag{35}$$

The unregulated competitive equilibrium is defined by sequences of prices  $\{p_t^N, q_t, P_t\}_{t=0}^\infty$  and allocations  $\{c_t^T, c_t^N, b_{t+1}, k_{t+1}, x_t^T, x_t^N\}_{t=0}^\infty$  such that: (i)  $\{c_t^T, c_t^N, b_{t+1}, k_{t+1}\}_{t=0}^\infty$  solve the household’s optimization problem; (ii)  $\{x_t^T, x_t^N, i_t\}_{t=0}^\infty$  solve the optimization problem of investment goods producers; (iii) the market-clearing conditions for non-tradable goods and investment goods hold (i.e., equations (33) and (34) hold). The equilibrium of a variant of the model without credit frictions is defined in the same way but without  $\mu$  terms in the household’s optimality conditions.

**Equilibrium with regulation** The government implements simple macroprudential policy rules, which take the form of a tax on debt  $\tau_t$  or a regulatory LTV ratio  $\chi_t$  such that  $0 \leq \chi_t \leq 1$  (to reflect the assumption that regulation *reduces* borrowing capacity below what private markets can provide). The equilibrium conditions are the same as before except for these key changes<sup>21</sup>:

$$u_{c^T}(t) = \beta R(1 + \tau_t) \mathbb{E}_t \{u_{c^T}(t + 1)\} + \mu_t(1 + \tau_t) \tag{38}$$

$$q_t [u_{c^T}(t) - \mu_t \kappa(1 - \chi_t)] = \beta \mathbb{E}_t \left\{ u_{c^T}(t + 1) \left[ (1 - \delta)q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha-1} ((1 - a)p_{t+1}^N + a) \right] \right\} \tag{39}$$

$$\mathbb{E}_t (R_{t+1}^q) = R(1 + \tau_t) + \frac{(1 - \kappa(1 - \chi_t))\mu_t - cov(\beta u_{c^T}(t + 1), R_{t+1}^q)}{\beta \mathbb{E}_t [u_{c^T}(t + 1)]} \tag{40}$$

Consider first the role of the debt tax. Condition (38) implies that the debt tax increases the effective cost of borrowing and thus reduces incentives to borrow. Hence, the debt tax aims to hamper incentives to borrow in non-crisis times. The debt tax, however, distorts investment decisions by requiring a higher marginal return on capital as eq. (40) shows. Consequently, this reduces investment. In fact, the debt tax has an effect analogous to that of a capital income tax. This is clearer assuming perfect-foresight and no credit frictions: The debt tax rises the opportunity cost of accumulating capital and is equivalent to taxing capital returns at a rate equal to  $\tau_t/(1 + \tau_t)$ . This is the standard efficiency distortion of capital income taxation.<sup>22</sup> Under uncertainty, the debt tax has an additional effect operating in the opposite direction, because to the extent that it improves consumption smoothing, it reduces risk premia. Overall, this imply that the debt tax has potentially costly tradeoffs in terms of investment and output.

These debt-investment tradeoffs interact with the dynamics of debt and borrowing capacity. On one hand, the debt tax makes it less likely that the constraint binds and induces lower debt levels, so that when it binds its shadow value is smaller, which in turn contributes to reduce capital returns when the constraint binds. On the other hand, the debt tax imposed when the constraint was not binding can also result in the economy attaining higher leverage ratios because of

<sup>20</sup> This constraint is derived as follows:

$$\begin{aligned} q_t [k_{t+1} - k_t(1 - \delta)] + p_t^N c_t^N + c^T + \frac{b_{t+1}}{R} &= b_t + p_t^N(1 - a)k_t^\alpha + ak_t^\alpha \\ P_t x_t + p_t^N(y_t^N - x_t^N) + c^T + \frac{b_{t+1}}{R} &= b_t + p_t^N(1 - a)k_t^\alpha + ak_t^\alpha \\ (p_t^N x_t^N + x_t^T) + p_t^N(y_t^N - x_t^N) + c^T + \frac{b_{t+1}}{R} &= b_t + p_t^N(1 - a)k_t^\alpha + ak_t^\alpha. \end{aligned}$$

In the last expression, the terms with  $p_t^N x_t^N$  cancel out, and we arrive at the result in the text.

<sup>21</sup> The household’s budget and borrowing constraints are modified as follows:

$$q_t k_{t+1} + p_t^N c_t^N + c_t^T + \frac{b_{t+1}}{R(1 + \tau_t)} = b_t + \left[ p_t^N(1 - a)z_t k_t^\alpha + az_t k_t^\alpha + q_t k_t(1 - \delta) \right] + T_t \tag{36}$$

$$\frac{b_{t+1}}{R} \geq \bar{b}_t - \kappa(1 - \chi_t)q_t k_{t+1}. \tag{37}$$

The tax on debt is rebated as a lump-sum transfer, so that  $T_t = -\tau_t \frac{b_{t+1}}{R(1 + \tau_t)}$  at equilibrium.

<sup>22</sup> For the same reason, it follows that an additional policy instrument, in the form of a subsidy on capital returns, could be used to mitigate the adverse effect of MPP instruments on the effective opportunity cost of investing in physical capital.

lower capital values. Thus, the debt tax distorts equilibrium outcomes both when the constraint is not binding and when it binds.

Regulatory LTVs targeted to borrowers have similar implications. When the regulatory LTV binds, the shadow value of the binding credit constraint takes the place of the debt tax, increasing the effective marginal cost of borrowing in the right-hand-side of condition (38). In addition, the LTV regulation increases the marginal cost of accumulating capital in the left-hand-side of condition (39), which leads to higher marginal returns on capital and reduced investment in condition (40). There are, however, two important differences between the debt tax and the regulatory LTV. First, while the direct effects of the debt tax on the marginal cost of borrowing and capital returns are exogenous (since  $R(1 + \tau_t)$  is a product of exogenous variables), the effects of the LTV are partly endogenous, since they depend on the value of  $\mu_t$ . Hence, regulators have full control of the policy instrument direct effect on equity returns with the debt tax but not with the LTV regulation. Second, the regulatory LTV distorts the optimality conditions only when it binds, while a debt tax governed by a simple rule may remain active even when debt is too low for taxing debt to be optimal for macroprudential reasons (i.e., when the credit constraint is not binding at  $t$  and has zero probability of being binding at  $t + 1$ ). If in some of these low-debt states the regulatory LTV does not bind, the debt tax still has adverse distortionary effects on capital accumulation but the LTV does not, and it does not because of a self-adjusting mechanism that turns off the policy instrument when the LTV regulation is not binding. Moreover, in general it is not true that an equilibrium supported by a given debt-tax policy  $\tau_t$  can be supported by some LTV regulation  $\chi_t$ . As shown in Appendix B, LTV regulation aimed at matching the allocations of the debt-tax regime at a given date  $t$  implies a higher price of capital than under debt taxes and, because of the forward-looking nature of asset prices, different allocations in previous periods. Hence, qualitatively a debt tax and a regulatory LTV have similar effects in terms of distorting optimality conditions, but they are not equivalent instruments.

## 5. Quantitative analysis

This section of the paper examines the model's quantitative implications. We calibrate the model to emerging-markets data and study its implications for the properties of Sudden Stop events. Then, we use the model to examine the effects of constant debt taxes and the tradeoffs of these taxes in terms of capital accumulation and output.

Quantitative solutions of optimal macroprudential policy in Fisherian models are generally based on first-order condition methods that apply fixed-point iteration or endogenous-grids algorithms to the models' optimality conditions. These methods are standard, and since they are described in several published articles (with replication codes available online), we do not provide a detailed description here. For DTI models, see for example, the appendices to Bianchi (2011) and Bianchi et al. (2016). For LTV models, see for example, Sections B and E of the appendix to Bianchi and Mendoza (2018). For a fast fixed-point iteration method that is easy to implement in Sudden Stop models, see Mendoza and Villalvazo (2020). The solution method used to generate the solutions discussed in this section is described in the replication files provided online, which include also the computer codes.

### 5.1. Calibration

The calibrated parameter values are reported in Table 1. A period in the model represents a year, so the calibration uses annual data. The calibration strategy splits the parameters into three sets: (1) parameters set "directly" based on commonly used values ( $R, \delta, \alpha, \sigma$ ), (2) parameters calibrated to data estimates or to match data targets in the deterministic steady state of the model ( $\eta, \theta, \omega, \pi, \beta, P_{LL}, P_{HH}$ ), and (3) parameters determined so that the model's stochastic equilibrium matches statistics observed in the data ( $\sigma_\epsilon, \rho, \bar{b}^L, \bar{b}^H, \kappa$ ).

The values of the parameters set to commonly used values are  $R = 1.02$ ,  $\delta = 0.1$ ,  $\alpha = 0.35$ , and  $\sigma = 4$ . The real interest rate, depreciation rate, and capital share are standard for a calibration at annual frequency. The value of  $\sigma$  is consistent with empirical evidence in emerging markets (see Reinhart and Vegh (1995)) suggesting that the intertemporal elasticity of substitution is in the 0.2–0.5 range.

The second set of parameters is calibrated as follows. We assume, for simplicity, the same elasticity parameters in the two CES aggregators ( $\theta = \eta$ ), and set them to the same value used by Bianchi (2011), which is based on a range of cross-country estimates for Latin American economies. The values of  $P_{LL}, P_{HH}$  are set based on the estimated durations of the loose and tight phases of financial cycles reported by Borio (2014). In particular, we set  $P_{LL} = 0.77$ ,  $P_{HH} = 0.93$ , which imply durations of 4.5 and 16 years of the loose and tight financial regime.

The values of the weights of tradables in the CES investment and consumption aggregators ( $\pi$  and  $\omega$ ) and the discount factor ( $\beta$ ) are determined jointly with the model's deterministic steady state-equilibrium conditions, for a given steady state of NFA denoted  $\bar{b}$ . We assume  $\pi = \omega$ , based on the evidence from Lombardo and Ravenna (2012) showing that for several emerging economies (e.g., Mexico, Turkey and Poland) the share of tradable goods in consumption demand equals that in investment inputs. The values of  $\beta$  and  $\omega$  are then determined jointly with the deterministic steady-state values of  $x^T, x^N, k, p^N, q$  by solving the following system of deterministic steady-state conditions targeting a sectoral consumption ratio of  $c^T/p^N c^N = 0.5$ , a capital output ratio of  $qk/y = 2.5$ , and an NFA position of  $-0.40$  as a share of GDP (based on the

**Table 1**  
Calibration.

	Value	Source/Target
Interest rate	$R - 1 = 0.02$	Standard value
Depreciation rate	$\delta = 0.1$	Standard value
Share of capital	$\alpha = 0.35$	Standard value
Elasticity of substitution consumption	$\eta = 0.205$	Bianchi (2011)
Elasticity of substitution production	$\theta = 0.205$	Same as $\eta$
Preference weight $c^T$	$\omega = 0.314$	$y^T/p^N y^N = 0.5$
Production weight $x^T$	$\pi = 0.314$	Lombardo and Ravenna (2012)
Productivity parameter	$\bar{z} = 1$	Normalization
Risk aversion	$\sigma = 4$	Standard value
Discount factor	$\beta = 0.962$	$qk/(y^T + p^N y^N) = 2.5$
Duration tight financial regime	$P_{LL} = 0.77$	Borio (2014)
Duration loose financial regime	$P_{HH} = 0.93$	Borio (2014)
Parameters set by simulation	Value	Target
Std. of TFP	$\sigma_\epsilon = 0.025$	STD of Output = 0.029
Autoc. of TFP	$\rho = 0.3$	Autocorrelation of Output = 0.54%
Tight financial regime	$\bar{b}^L = -0.27$	Probability of sudden stop = 3%
Loose financial regime	$\bar{b}^H = -0.31$	Std of CAY = 1.1%
Collateral parameter	$\kappa = 0.09$	Mean NFA

data from Lane and Milesi-Ferreti, (2007) or  $-1.2$  as a share of  $y^T$  (using a ratio of tradables GDP in total GDP of one-third), which implies  $\tilde{b} = -1.2ak^\alpha$ <sup>23</sup>:

$$\frac{\omega}{1 - \omega} \left( \frac{(1 - a)k^\alpha - x^N}{ak^\alpha - x^T + \tilde{b}(1 - R^{-1})} \right)^\eta = 0.5$$

$$\frac{qk}{ak^\alpha + p^N(1 - a)k^\alpha} = 2.5$$

$$p^N = \frac{1 - \omega}{\omega} \left( \frac{ak^\alpha - x^T + \tilde{b}(1 - R^{-1})}{(1 - a)k^\alpha - x^N} \right)^{\eta+1}$$

$$q = \left[ \omega^{\frac{1}{1+\eta}} + (1 - \omega)^{\frac{1}{1+\eta}} (p^N)^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}}$$

$$1 = \beta[(1 - \delta) + \frac{\alpha}{q}k^{\alpha-1}((1 - a)p^N + a)]$$

$$x^N = \frac{\delta k}{[\pi(x^T)^{-\theta} + 1 - \pi]^{-1/\theta}}$$

$$x^T = x^N \frac{\pi}{1 - \pi} (p^N)^{1/(1+\theta)}.$$

Solving the above system yields  $\pi = \omega = 0.34$  and  $\beta = 0.962$ .

The third set of parameter values is determined by solving the stochastic model given the rest of the parameter values so as to match data targets as follows: for the stochastic process of TFP, we assume that TFP follows a first-order autoregressive process  $\ln z_t = \rho \ln z + \varepsilon_t$ . The values of the autocorrelation coefficient  $\rho$  and the standard deviation of the TFP innovations  $\sigma_\epsilon$  are set so as to match the standard deviation and autocorrelation of the cyclical component of annual HP-filtered GDP for Mexico for the 1950–2016 period. The values of  $\bar{b}^L$  and  $\bar{b}^H$  are set so as to match the long-run frequency of Sudden Stops (3 percent) and the standard deviation of the current account for the 1993-2016 period (1.1 percent), respectively. A Sudden Stop in the model is defined as an increase in excess of 4 percent in the current-account-GDP ratio, in line with the 3.7 percent magnitude of the median Sudden Stop event identified in the empirical analysis of Section 2.<sup>24</sup> The value of  $\kappa$  was set so as to match a mean NFA position of  $-0.4$  of GDP, which required  $\kappa = 0.09$ .

<sup>23</sup> With unitary elasticity and balanced trade, and since expenditure shares are constant, it follows that  $p^N y^N / y^T = [(1 - \omega)/\omega] y^N / y^T$ .

<sup>24</sup> 4 percent is about 2.3 times the standard deviation of the current account in the calibrated model, so this criterion also fits within the 95 percentile criterion used in Section 2 to define Sudden Stops. Conditioning on a binding borrowing constraint in addition to the current-account reversal does not change the probability of a sudden stop in the model.

**Table 2**  
Long-Run and Sudden Stop Moments.

	Unregulated Equilibrium	Constant Tax
Long-run Moments	(1)	(2)
Average $B/Y$ %	-39.5	-39.0
Average $K/Y$	2.53	2.51
Welfare Gain <sup>1</sup> %	n/a	0.0179
Prob. of Sudden Stops <sup>2</sup> %	3.15	1.06
Prob( $\mu_t > 0$ ) %	18.1	8.7
Debt Tax Rate $\tau^B$ %	n/a	1.31
Average change in Sudden Stops (%)		
$c$	-0.5	-0.2
$I$	-22.1	-16.0
$p^N$	-17.1	-11.6
$q^K, p^C$	-12.1	-8.2
$CA/Y$	5.2	3.4

<sup>1</sup> Welfare gains are computed as compensating variations in the argument of period utility constant across dates and states that equate welfare in the economy with regulation with that in the unregulated decentralized equilibrium. The welfare gain  $W$  at state  $(b, k, s)$  is given by  $(1 + W(b, k, s))^{1-\sigma} V^{DE}(b, k, s) = V^i(b, k, s)$ . The unconditional (long-run) average is computed using the ergodic distribution of the unregulated economy.

## 5.2. Results

### 5.2.1. Financial crisis events

We first examine the characteristics of financial crises in the unregulated competitive equilibrium (DE). Column (1) of Table 2 shows key statistical moments. The top panel shows the long-run averages of the NFA-GDP and capital-GDP ratios, the probability of Sudden Stops and the probability of a binding credit constraint. The bottom panel shows the average changes in macro variables displayed in Sudden Stop events, identified using a long time-series simulation of the model of 100,000 periods. As mentioned earlier, a Sudden Stop in the model occurs when the current-account-GDP ratio rises by more than 4 percent (with 4 percent corresponding to 2.3 times the standard deviation of the current account). Fig. 5 shows seven-year even windows of all the main macroeconomic aggregates centered on the date Sudden Stops hit. For each period, we report the mean of each variable across all the events. The straight blue line corresponds to the unregulated equilibrium. The figure also includes the simulations for the economy with an optimal fixed tax, depicted by a broken red line to be described below.

The probability of Sudden Stops is 3.15 percent, close to the 3 percent calibration target of based on the actual frequency of Sudden Stops in emerging economies reported in Section 2. The event windows show that the model qualitatively matches several of the documented features of Sudden Stops, indicating strong financial amplification effects via Fisherian deflation during Sudden Stop events. Quantitatively, the model predicts that when a Sudden Stop hits (at  $t = 0$ ) the reversal in the current account and the drops in investment, asset prices and the real exchange rate are large and in line with the data, but the drops in aggregate consumption and output are small (see column [1] of Table 2 and Fig. 5).

Output falls little because on the date a Sudden Stop hits it can respond only to TFP shocks and the capital stock, which is predetermined, is on average above the mean.<sup>25</sup> TFP shocks are only about  $-1$  percent on average, roughly 0.38 of the standard deviation of TFP. Hence, the large fluctuations observed in the model's Sudden Stops are driven largely by the effects of the credit constraint, which operate because of the fluctuations in  $\bar{b}_t$  and the Fisherian amplification affecting  $q_t$  and  $k_{t+1}$ .

The small fall in aggregate consumption masks the fact that sectoral consumption allocations do move significantly, albeit in opposite directions:  $c^T$  falls more than 10 percent while  $c^N$  rises 5 percent. The sharp decline in  $c^T$  is standard in Sudden Stops models. It reflects the sudden loss of access to credit markets that forces a sharp reversal of both the current account and net exports. In the standard model, however, non-tradables consumption is often modeled as an exogenous endowment. In contrast, in this model, households cut investment expenditures seeking to reallocate their resources towards tradables consumption, and the fall in demand for investment goods lowers their price, leading producers to demand less of both  $x^T$  and  $x^N$  (see Fig. 5). The supply of non-tradables ( $y^N = (1-a)zk^\alpha$ ) slightly falls because of the TFP shock, but the fall in  $x^N$  is larger, and therefore market clearing requires  $c^N$  to rise. Since we are abstracting from productivity shocks in the production of investment goods,  $q = P$ . Hence, the fall in  $q$  implies a fall in  $P$  and also in  $p^N$ , since  $P$  is a monotonic function of  $p^N$ . But since  $p^N$  is the same price for households buying consumer goods or firms buying inputs, and since the calibration features  $\omega = \pi$  and  $\eta = \theta$ , the ratios  $x^T/x^N$  and  $c^T/c^N$  are always equal to each other. When the Sudden Stop

<sup>25</sup> Producing large drops in output requires mechanisms that link Fisherian amplification with the supply side of the model—for instance if working-capital financing is needed to pay for variable inputs and is subject to the credit constraint (e.g., Mendoza, 2010, Bianchi and Mendoza, 2018).

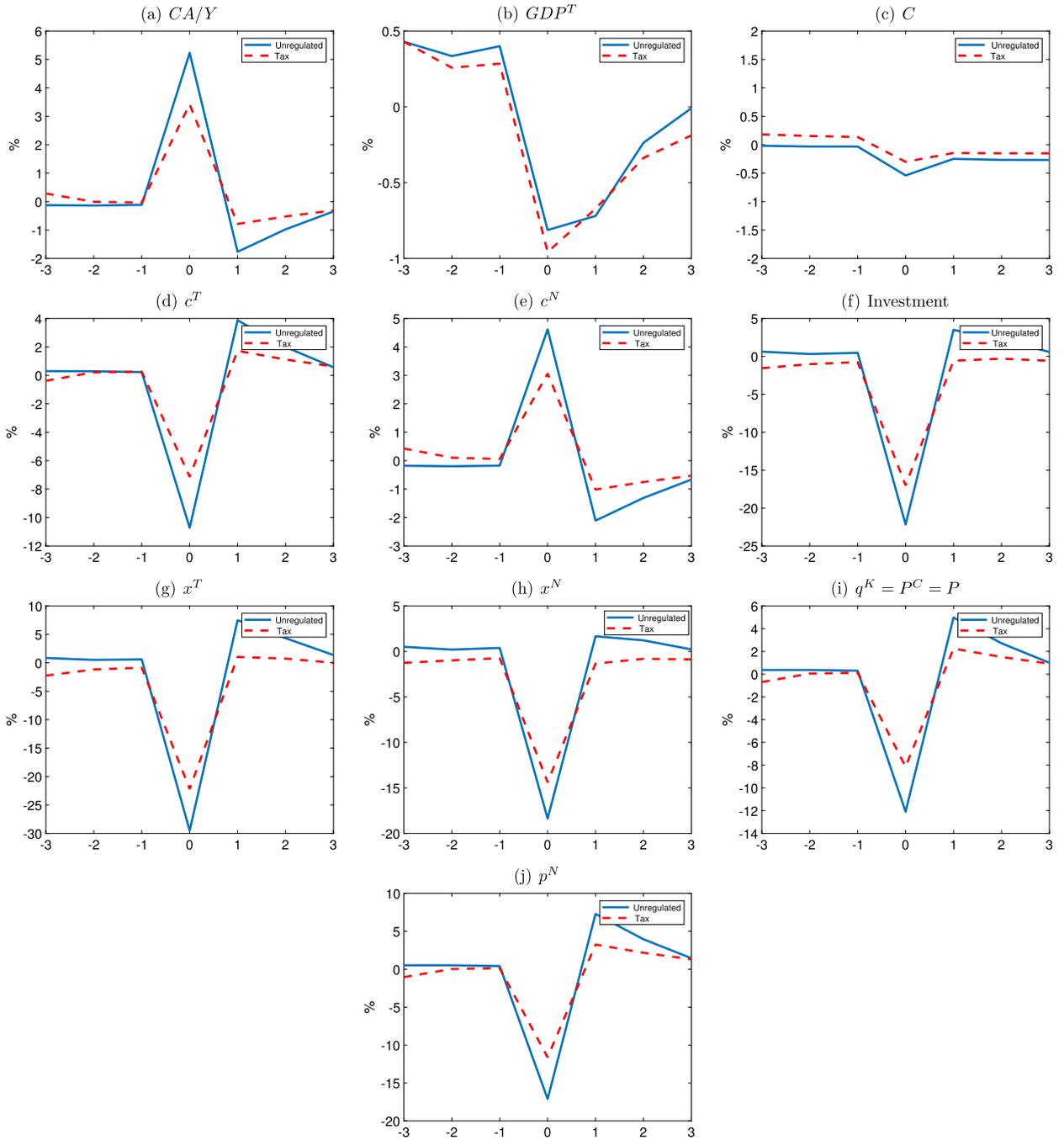


Fig. 5. Sudden Stop events with and without debt taxes.

occurs,  $x^T$  falls by more than  $x^N$  and  $c^T$  falls more than  $c^N$  rises so that the two sectoral ratios are equal and yield the same  $p^N$ .

Output remains low and, in fact, drops slightly at  $t + 1$ . This occurs because the sharp investment drop at  $t$  reduces capital used in production at  $t + 1$ , and the recovery of TFP is gradual. Investment demand and tradables consumption rise sharply, as the credit constraint is relaxed. This sets in motion the opposite dynamics from those observed at  $t$ . Now, the price of investment goods rises with investment demand, which causes  $x^T$ ,  $x^N$ ,  $P$  and  $p^N$  to increase. Non-tradables consumption falls, because the supply of non-tradables is still low but demand of  $x^N$  surged, so market clearing requires lower consumption of non-tradables.

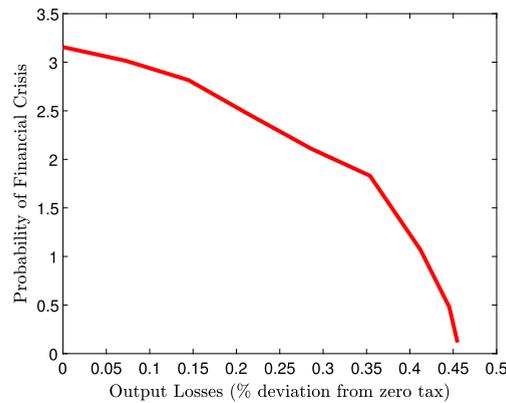


Fig. 6. Long-run financial stability-growth frontier.

### 5.3. Investment tradeoffs of macroprudential policy

Next we conduct a quantitative analysis of the tradeoffs of MPP induced by the reduction in output and the capital stock. The results show that there is a potentially significant tradeoff between losses in output and investment, on one hand, and financial stability on the other. We examine features of this tradeoff in both the long-run (using averages from stochastic steady states) as well as in the short run (conditional on temporary policies implemented in the initial period).

*Long-run frontier* We first study a long-run crisis probability–output frontier, defined as the locus formed by the values of the probability of Sudden Stops and the long-run average of GDP that the solution of the model yields for different values of the time-invariant debt tax. Hence, this frontier shows the sacrifice in terms of long-run output that must be made in order to achieve higher levels of financial stability using constant debt taxes as a simple MPP rule. Fig. 6 shows this frontier for debt taxes ( $\tau$ ) that vary from zero to 1.8 percent. This tax is constant across all states in which the collateral constraint does not bind and zero otherwise. The vertical axis shows the unconditional (long-run) probability of Sudden Stops, and the horizontal axis shows the associated long-run output loss in percent of the long-run average of output in the unregulated economy.<sup>26</sup> The probability of a crisis corresponds to the probability of observing a reversal in the current account-GDP ratio of at least 4 percentage points, which is the same threshold used to define Sudden Stops in the economy without regulation.

The figure shows that there is an important long-run tradeoff between output and financial stability. To reduce the probability of crisis down to roughly half of what it is without regulation (1.5 v. 3.2 percent), the economy gives up roughly 0.4 percent of the long-run average of output. This outcome is attained with a debt tax of just 1.3 percent. Reducing the crisis probability to zero requires a sacrifice of 0.45 of a percent of long-run output. These declines are due to the distortionary effect of the debt tax on the no-arbitrage condition between capital and bonds discussed earlier, which implies that the debt tax is akin to a capital tax.

*Short-run tradeoffs of temporary debt taxes* The above analysis considers the long-run effects of permanent taxes. It is also useful to examine the immediate effects of a temporary tax on debt, because these effects also shed light on the incentives that a financial regulator has for adjusting MPP in the model.

Consider a policy scenario in which a debt tax is set at date zero,  $\tau_0 > 0$ , and from the following period onward, there are no MPP instruments in place ( $\tau_t = 0$  for  $t \geq 1$ ). The goal is to show how  $\tau_0$  affects current allocations—in particular, investment—and how this affects continuation values through the two endogenous state variables, capital and bonds. Starting at  $t = 1$ , the economy becomes the unregulated equilibrium, but the debt tax present at  $t = 0$  critically affects how the economy reacts to adverse shocks at  $t = 1$ .<sup>27</sup> This experiment is also equivalent to calculating the effects of an unanticipated debt tax introduced for only one period.

Fig. 7 shows the effects of varying  $\tau_0$  on allocations and prices (in the vertical axes) for values of  $\tau_0$  ranging from 0 to 7 percent (in the horizontal axes). We assume that at date 0, the economy starts with  $b_0, k_0$  around their long-run averages in the unregulated equilibrium, and there are positive shocks to TFP and  $\bar{b}$ .<sup>28</sup> For these initial states, the collateral constraint is

<sup>26</sup> To make real output comparable across economies with different taxes, we evaluate GDP at constant prices. Measuring GDP using the prices in each economy gives a similar picture, but with somewhat lower GDP losses. This is because the economy with higher taxes, features a higher NFA position, which implies a more appreciated real exchange rate.

<sup>27</sup> Given the debt levels and capital accumulation policies, it is possible to compute an LTV policy  $\chi$  that would deliver the same level of debt by solving for  $\chi$  such that  $b_{t+1} = -\kappa q_t k_{t+1} (1 - \chi_t) + \bar{b}$ . As we argue in Appendix B, however, asset prices would differ under both policies.

<sup>28</sup> Results are qualitatively similar for other initial conditions in which the collateral constraint is not binding.

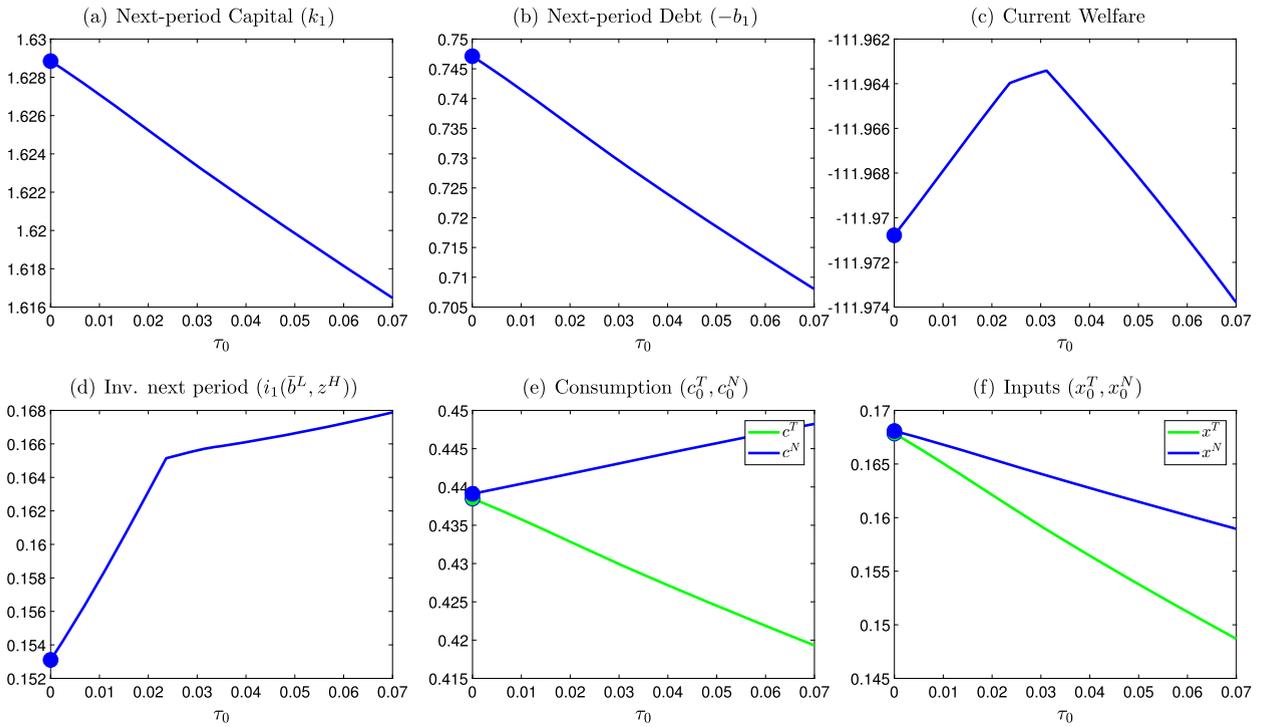


Fig. 7. Short-run effects of a temporary debt tax.

not binding in the first period. Absent regulation, the constraint binds in the next period if a negative shock to  $\bar{b}$  is realized. By construction, the allocations and prices for  $\tau_0 = 0$  (illustrated with solid dots) coincide with the unregulated equilibrium.

As Fig. 7 shows, a higher  $\tau_0$  leads to a reduction in debt accumulation but at the expense of reducing capital accumulation (i.e.,  $-b_1$  falls but  $k_1$  also falls, see panels [a] and [b]). Moreover, panel (f) shows that both tradable and nontradable inputs fall, while panel (e) shows that tradable consumption falls but nontradable consumption rises. This implies that  $p_0^N$  falls and hence  $q_0$  falls and the real exchange rate depreciates. This occurs because of the same mechanism described earlier to explain the Sudden Stop effects: the tax on debt makes tradable goods more expensive, reducing both investment and consumption. As investment drops, so do the prices of capital and investment goods and the relative price of non-tradables. Producers of investment goods reduce  $x_0^T, x_0^N$ , but on the households' side, market clearing requires  $c_0^N$  to rise, because the supply of non-tradables is the same for all tax rates (since  $k_0$  and the initial TFP shock are the same) and demand for non-tradables by producers fell.

The dynamic benefits of the debt tax can be seen in panel (d), which shows investment chosen at  $t = 1$  for a low  $\bar{b}$  realization and a positive TFP shock that period. A higher  $\tau_0$  induces higher investment at  $t = 1$ , partly because of a direct effect of lower investment at  $t = 0$ , but it is also important to note that the lower accumulation of debt leaves more resources to invest in the future, especially when the constraint binds and higher investment contributes to relax it. This accounts for the kink observed in panel (d). Investment at  $t = 1$  increases more with  $\tau_0$  in the region in which the constraint becomes binding. For high enough  $\tau_0$ , the reduction in debt at  $t = 0$  is sufficiently large so that the constraint does not bind at  $t = 1$ , and in that region,  $i_1$  is less responsive to tax hikes. Moreover, there is also an amplification effect: lower aggregate debt level implies that demand for consumption and investment is relatively higher tomorrow, which in turn leads to a higher market price for assets tomorrow and mitigates the Fisherian deflation that arises when a negative shock triggers a binding collateral constraint.

Panel (c) plots lifetime utility evaluated as of  $t = 0$ . This graph shows an important result: the welfare-maximizing debt tax is strictly positive (date-0 welfare peaks with  $\tau_0$  around 3.1 percent). By taxing debt, the government induces less borrowing, which in turn mitigates the general equilibrium effects that tighten the credit constraint via the pecuniary externality. The tax on debt weakens this externality and prevents over-consumption and overborrowing (Bianchi, 2011, Bianchi and Mendoza, 2018). In addition, the debt tax induces less investment at  $t = 0$  but more investment at  $t = 1$ . Note that the kinks in the welfare plot are related to the kinks in the plot for  $i_1$  in panel (d). The first kink in both plots occurs at the same tax rate slightly above 2 percent, and the second, less noticeable one in panel (d) occurs with a tax a notch above 3 percent. This second kink occurs precisely at the tax rate at which the credit constraint ceases to bind at  $t = 1$ . For taxes higher than that value, welfare sharply falls as the tax rises. The economy is better off increasing the debt tax if there is no debt tax, but after a debt tax of slightly more than 3 percent, welfare declines with the tax, as the efficiency losses of the higher  $\tau_0$  captured in the fall in  $k_1$  dominate the financial-stability benefits.

It is also worth noting that the short-run effects on capital and output at  $t=1$  can be potentially larger than the long-run effects on steady state capital and output with a permanent tax. This is because the NFA position increases in the long run with the latter, since the permanent tax incentivizes additional precautionary savings and reduces equity risk premia. Therefore, for a given cost of borrowing given by the risk free rate plus the tax, once the economy adjusts to a higher NFA, it invests more in capital.

The previous experiment shows how a temporary debt tax reduces investment, borrowing, and consumption in the short run, relative the unregulated economy, and can also improve welfare. To evaluate the efficiency of investment decisions, we consider now a related scenario in which a social planner compares different levels of debt and investment at date 0, taking as given a level of  $c_0^T$  and, from date 1 forward, a given continuation unregulated competitive equilibrium. The difference in this experiment is that we keep constant the initial level of consumption and consider alternative  $(k', b')$  that would deliver this level of consumption.

Start from an initial state  $(b, k, z, \bar{b})$  at date 0 and recall the resource constraint for tradables

$$c^T + x^T + qb' = ak^\alpha + b. \tag{41}$$

We want to examine how different combinations of  $(b', k')$  chosen at date 0 translate at date 1 into different investment choices, and levels of welfare and crisis exposure. To this end, we solve for all possible combinations of  $b', k'$  at date 0 that are consistent with the above resource constraint, a given level of tradables consumption—in particular the optimal one in the unregulated equilibrium  $c^T(b, k, z, \bar{b})$ —, optimal sectoral allocations of consumption and inputs, and market clearing in the non-tradables market. The solutions are determined as follows: given values of  $c^T$  and  $b'$ , the resource constraint yields a value of  $x^T$ . Then,  $x^N$  is determined as follows, using the households' and firms' first-order conditions for sectoral-demand allocations together with non-tradables market clearing:

$$\frac{1 - \omega}{\omega} \left( \frac{ak^\alpha + b - x^T - qb'}{(1 - a)k^\alpha - x^N} \right)^{\eta+1} = \frac{1 - \pi}{\pi} \left( \frac{x^T}{x^N} \right)^{\theta+1}.$$

Given the solutions for  $x^T$  and  $x^N$ , the value of  $k'$  follows from the CES aggregator of investment goods and the definition of investment,  $k' = x(x^T, x^N) + k(1 - \delta)$ , and  $c^N$  follows from the non-tradables market-clearing condition,  $c^N = (1 - a)zk^\alpha - x^N$ .

Fig. 8 presents the results of this experiment. We set  $b_0$  and  $k_0$  close to their long-run averages,  $z_0$  equal to a plus-one standard-deviation TFP shock, and  $\bar{b}$  at the level of the loose borrowing limit. In all panels, we show the debt choice of date  $t$  (i.e.,  $-b_1$ ) in the horizontal axis. In the vertical axis of each plot, we report the outcomes for  $k_1$  (panel (a)), welfare or lifetime utility (panel [b]) evaluated as of date 0, and the choice of investment in the following period  $i_1$  (panel [c]) following adverse shocks to TFP and the borrowing limit in that period. The solid dots identify the unregulated competitive equilibrium.

A key point that follows from panel (b) of this Figure, which echoes the one made in Fig. 7, is that the amount of debt that maximizes current welfare, keeping  $c_0^T$  constant, is lower than the unregulated equilibrium amount (i.e., there is overborrowing). Recall that because the amount of tradable consumption is kept constant, this implies that the capital stock chosen at date 0 ( $k_1$ ) is lower than in the unregulated equilibrium, as shown in panel (a). On the other hand, panel (c) shows that reducing the debt and investment chosen at date 0 leads to a higher investment at date 1.<sup>29</sup> At low debt levels chosen for date 0, welfare rises fast with debt and investment at date 1 falls slowly. At debt levels around 0.70, investment at date 1 begins to decline sharply with more debt, indicating that choosing debt in that range exposes the economy to a Sudden Stop at date 1. After a debt level of about 0.74, welfare becomes decreasing in debt and the unregulated equilibrium ends up with too much debt.

Throughout these experiments, allocations with lower borrowing and investment at date 0 deliver higher welfare. While this result suggests that there is not only overborrowing but also overinvestment, determining the socially optimal amount of investment is more subtle and a thorough analysis of this issue is left for future research. A social planner able to decide investment on behalf of firms would internalize how higher investment today provides more tradable resources tomorrow, which would raise asset prices and help relax the collateral constraint when it binds. At the same time, higher investment implies more non-tradable resources which would have the opposite effects. On the other hand, a government that only has a tax on debt available to tackle the pecuniary externality can affect investment only indirectly. In particular, as we discussed above, a tax on debt depresses investment, and therefore, a large tax on debt may go too far in terms of reducing investment and output.<sup>30</sup>

<sup>29</sup> It is possible to solve for a debt tax ( $\tau_0$ ) and a tax/subsidy on capital returns ( $s_0^k$ ) that implement this level of debt and capital as

$$1 + \tau_0 = \frac{u_T(0)}{\beta RE_t u_T(1)}, \quad 1 + s_0^k = \frac{q_0 u_{c^T}(0)}{\beta E_t \left\{ u_{c^T}(1) \left[ (1 - \delta)q_{t+1} + \alpha z_{t+1} k_1^{\alpha-1} ((1 - a)p_1^N + a) \right] \right\}}.$$

<sup>30</sup> In terms of the analysis contained in Fig. 7, we found that for low debt taxes on debt, incorporating a tax on investment is desirable. However, as we consider higher taxes on debt, including the welfare-maximizing one in Fig. 7, subsidizing investment becomes desirable. Overall, choosing jointly a welfare-maximizing pair of taxes on debt and investment yields a small negative tax on investment. The subsidy is  $-0.2$  percent for the same initial states

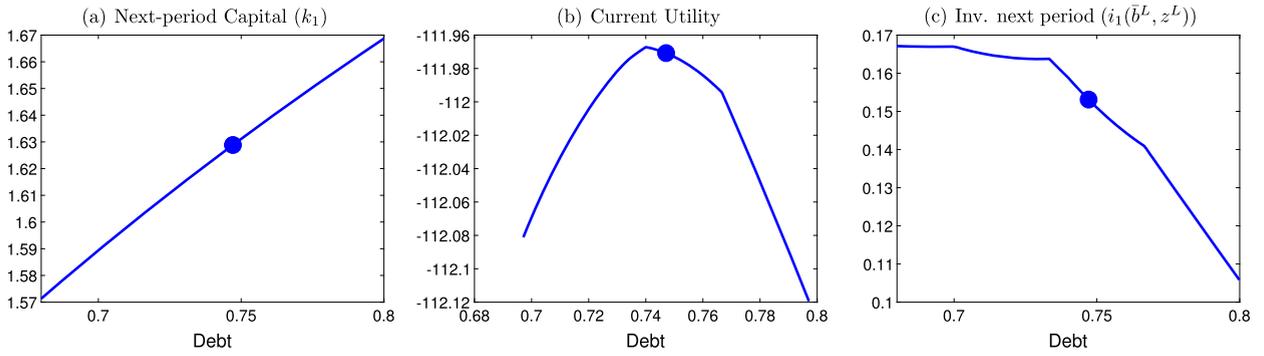


Fig. 8. Effects of choosing debt in the first period keeping  $c_0^T$  constant.

5.4. Optimal time-invariant debt tax

We now study the welfare implications of the constant debt-tax rule we examined in the context of the long-run frontier. For every initial state  $(b, k, z, \bar{b})$ , we compute the compensating variation in aggregate consumption that would make households indifferent between remaining in the unregulated economy v. switching to the economy with the debt tax. Let  $\gamma(b, k, z, \bar{b})$  denote the welfare gain for an initial value of  $(b, k, z, \bar{b})$  at  $t = 0$ .

$$\sum_{t=0}^{\infty} \beta^t u(c_t^T (1 + \gamma), c_t^N (1 + \gamma)) = \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t^T, \tilde{c}_t^N)$$

where “tilde” variables denote the economy with the constant tax and all consumption allocations are evaluated at the corresponding equilibrium values. Using the value functions and the homotheticity of the utility function, the welfare gain can be expressed as

$$\gamma(b, k, z, \bar{b}) = \left[ \frac{\tilde{V}(b, k, z, \bar{b})}{V(b, k, z, \bar{b})} \right]^{1/(1-\sigma)} - 1, \tag{42}$$

where

$$V(b, k, z, \bar{b}) = \sum_{t=0}^{\infty} \beta^t u(c_t^T, c_t^N), \quad \tilde{V}(b, k, z, \bar{b}) = \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t^T, \tilde{c}_t^N).$$

Note that each  $\gamma(b, k, z, \bar{b})$  includes long-run effects as well as the effects of the transitional dynamics of consumption across the unregulated and regulated economies. We average the welfare gains using the ergodic distribution of the unregulated equilibrium, denoted  $\Pi(b, k, z, \bar{b})$ . By averaging the gains in this way, we take into account the probability of introducing the regulation at each possible  $(b, k, z, \bar{b})$  in the unregulated economy. The average welfare gain is therefore given by

$$\Gamma = \int \gamma(b, k, z, \bar{b}) d\Pi(b, k, z, \bar{b}). \tag{43}$$

Panel (a) of Fig. 9 shows the welfare effects of varying  $\tau$  between 0 and 2 percent. This figure shows that unconditional welfare  $\Gamma$  is maximized with a debt tax of about 1.3 percent (denoted the “best fixed tax”). The welfare gains are less than 0.02 percentage points of consumption and fall sharply and turn into markedly bigger losses for taxes higher than 1.6 percent. The welfare losses that result when the tax is very high is accounted for by the fact that taxes depress current asset prices and further tighten the collateral constraint when it is binding (see Bianchi and Mendoza, 2018).

Panel (b) of Fig. 9 shows how the welfare gains and costs vary with debt for two different values of  $k$  and positive shocks to TFP and the borrowing limit, keeping the debt tax at its “best fixed” rate. The two values of capital are the mean and 15 percent below the mean. For the two  $k$  values, there is an interval of relatively high debt at which the best fixed tax yields welfare gains. Conversely, for low enough debt values, the tax on debt always yields welfare losses and these are much larger when  $k$  is high. These results follow from the fact that the debt tax is beneficial when it can tackle overborrowing, and this is the area with debt high enough so that the constraint does not bind at  $t$  but the economy is vulnerable to Sudden Stops at  $t + 1$ . As debt falls, the set of current debt and capital states for which this is possible shrinks, and it

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considered in Fig. 7 and the 3.1 percent welfare-maximizing debt tax. The effective wedge on the capital arbitrage condition is therefore  $1.031/1.002 = 1.029$ , which implies an effective tax of 2.9 percent.

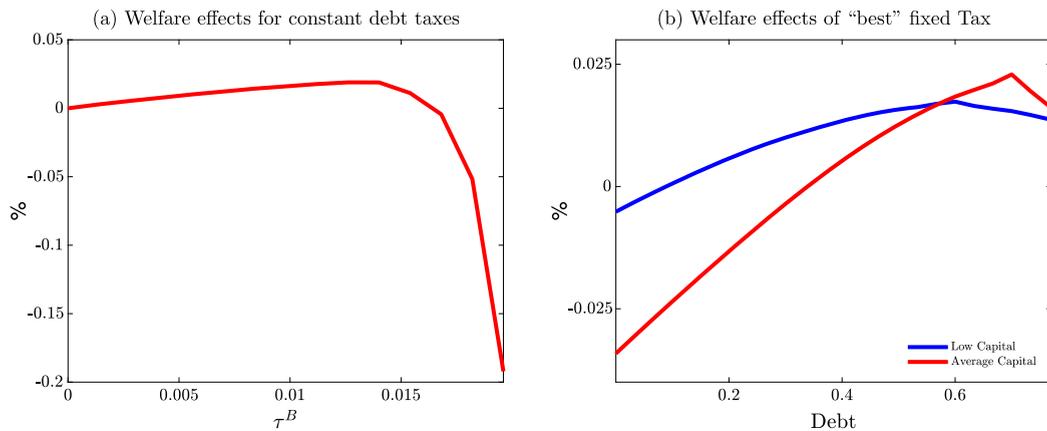


Fig. 9. Welfare effects of constant debt taxes.

shrinks faster for larger capital stocks. When the set is small enough, the distorting effects on investment dominate and result in efficiency losses that reduce welfare. There are also distortions on consumption smoothing, since debt is being taxed in states in which households could have to borrow more to smooth. On the other hand, when debt is sufficiently high and the constraint becomes binding, the welfare gains eventually fall.

We close this Section with an analysis of the macroeconomic implications of implementing the best fixed debt tax. Column 2 of Table 2 shows the key statistics for the economy regulated with this simple tax rule, which implies a debt tax of roughly 1.3 percent. This policy reduces the probability of Sudden Stops to 1.1 percent and yields a small welfare gain of 0.02 percent. The average reversals in investment, the price of non-tradables, and the price of capital when Sudden Stops occur are roughly 30 percent smaller than they are in the absence of regulation.

To show how the best fixed debt tax affects the dynamics of Sudden Stops, we use the counterfactual event analysis methodology proposed in Bianchi and Mendoza (2018). Based on the Sudden Stop events identified in the unregulated equilibrium (i.e., the periods in which the current account exceeds 4 percent), we feed the same shocks and initial conditions to the policy functions of the regulated economy. For each one of the sudden stops in the simulated sample, we consider the initial conditions 3 years before the sudden stops for the unregulated economy, and simulate the model using the policy functions for the best-fixed tax economy and the same shocks that hit the unregulated economy. The resulting simulations are shown in Fig. 5. The plots indicate that, by reducing investment and external borrowing before Sudden Stops, the debt tax mitigates the current-account reversal by about 2 percentage points and also dampens the fluctuations in the other macroeconomic aggregates.

We should emphasize that our analysis of optimal macroprudential taxes on debt is restricted to the search of a simple time-invariant tax. Allowing for optimal state contingent macroprudential policy leads to much larger gains, as studied in Bianchi (2011) and Bianchi and Mendoza (2018). However, a more complete study of macroprudential policy in the context of a model of capital accumulation, like the one presented here, remains an avenue for future research.

## 6. Conclusions

This paper reviews the positive and normative contributions of the research program on Fisherian models of Sudden Stops to the analysis of financial crises. From a positive perspective, the literature has shown that Fisherian deflation is a powerful financial amplification mechanism capable of producing model-generated Sudden Stops with realistic features, particularly their low long-run frequency and their large current-account reversals and collapses of real exchange rates, asset prices, output, consumption and investment. From a normative perspective, the literature has found that the large corrections in collateral prices induce large pecuniary externalities that distort borrowing choices and reduce social welfare. Optimal macroprudential policy is very effective at reducing the magnitude and frequency of financial crises, but requires complex state-contingent policies. Simpler policy rules closer to the ones used in practice (e.g., the countercyclical capital buffer and regulatory LTV and DTI ratios with limited flexibility) are less effective and need to be carefully evaluated because they can easily produce welfare-reducing outcomes. To date, finding an effective but simple MPP rule remains elusive.

The paper also adds to the literature by conducting a new analysis of MPP tradeoffs using a model that combines features of the DTI and LTV classes of Fisherian models and allows for capital accumulation and sectoral allocation of inputs. Macroprudential debt taxes and regulatory LTVs reduce overborrowing but also distort capital accumulation with effects analogous to those of capital income taxes. Starting from the unregulated economy, using debt taxes or LTVs is beneficial because they reduce overborrowing, but simple policy rules focusing purely on debt may result in output and welfare losses.

Despite the fact that both debt taxes and regulatory LTVs distort investment in the same direction in the model, lender-based MPP akin to debt taxes and borrower-based LTV regulation are not equivalent. If used to replicate the consumption

and debt allocations obtained with debt taxes, regulatory LTVs would yield higher asset prices. Since asset prices are forward-looking, this would also imply different allocations before that date.

Quantitative analysis of the new model we proposed shows that, in the absence of policy intervention, the model generates Sudden Stop dynamics in line with previous findings. The mechanism for production of investment goods using non-tradables and tradables generates sectoral reallocations that imply that when a Sudden Stop occurs consumption of tradables falls but that of non-tradables rises. Simple MPP rules in the form of constant debt taxes (but with the flexibility to be turned off when credit is constrained) have significant tradeoffs because of their distortionary effects on investment. These tradeoffs are illustrated by a long-run crisis probability–output loss frontier: higher debt taxes reduce the frequency of crises but also shave off up to 0.45 percent of GDP at the stochastic steady state. The welfare-maximizing constant debt tax of about 1.3 percent is somewhat effective at reducing the frequency and magnitude of crises and yields a modest welfare gain, but slightly higher tax rates above 1.5 percent produce increasingly large welfare losses.

Normative and positive quantitative analysis of financial crises based on Fisherian models is a fertile area for further research. Based on the material covered in this paper and our review of the recent literature, the following topics are particularly worth considering: i) enriching financial intermediation by introducing securitization and intermediaries with frictions (e.g., liquidity risk) and portfolio considerations (e.g., liquidity and maturity choices), ii) studying the interaction of monetary and macroprudential policies, iii) introducing nominal rigidities and interactions with aggregate demand externalities, iv) examining the interaction between ex-ante macroprudential policies and ex-post policies like FX intervention or bailouts, v) studying Fisherian amplification in environments with agent heterogeneity, vi) exploring the normative implications for the design of international financial regulation, vii) contrasting policies under commitment and under discretion and quantifying the value of commitment, and viii) studying the design of effective simple MPP rules.

### Appendix A. Optimal macroprudential policy

We describe below a constrained-efficient social planner’s problem that is standard in the literature in models of optimal macroprudential policy. We base the analysis on the workhorse DTI model of Bianchi (2011).<sup>31</sup> The planner chooses directly the holdings of non-state contingent bonds denominated in units of tradables subject to the collateral constraint, transfers the net proceeds from borrowing to the households and lets the goods market clear competitively. Households choose optimally the split between tradable and non-tradables, implying that the price of nontradables must equal the corresponding marginal rate of substitution in sectoral consumption. As a result, the planner internalizes how choosing debt affects consumption, which in turn affects the equilibrium price of collateral and thus borrowing capacity.

The planner’s optimal plans solve the following dynamic programming problem:

$$V(b, y^T, y^N) = \max_{p^N, c^T, c^N, b'} \left[ u \left( \left[ \omega (c^T)^{-\eta} + (1 - \omega) (c^N)^{-\eta} \right]^{-\frac{1}{\eta}} \right) + \beta \mathbb{E} V(b', z') \right] \tag{1}$$

subject to (2)

$$c^T + qb' = b + y^T \tag{3}$$

$$c^N = y^N \tag{4}$$

$$qb' \geq -\kappa(y^T + p^N y^N) \tag{5}$$

$$p^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c^T}{c^N} \right)^{\eta+1} \tag{6}$$

The endogenous state variable is the current bond position,  $b$ , and the exogenous states are the endowment realizations  $(y^T, y^N)$ . The planner chooses  $(p^N, c^T, c^N, b')$  subject to four constraints: the resource constraint for tradables (3), the market-clearing condition for nontradables (4), the credit constraint (5), and the implementability constraint requiring that the price of nontradables be consistent with a competitive equilibrium (6). Note that the price can be removed from the problem by substituting (6) into (5), to make it clear that what the planner is picking are the optimal allocations (i.e. optimal plans for bonds and consumption) but being mindful that these must be implementable as a competitive equilibrium.

The first-order conditions of the planner’s problem in sequential form are:

$$\lambda_t^* = u_T(t) + \mu_t^* \psi_t, \quad \psi_t \equiv \kappa \left[ \frac{1 - \omega}{\omega} (1 + \eta) \frac{c^T}{y^N} \right], \tag{7}$$

$$\lambda_t^* = \beta R \mathbb{E}_t [\lambda_{t+1}] + \mu_t^*. \tag{8}$$

where  $\lambda_t^*$  and  $\mu_t^*$  denote the planner’s Lagrange multipliers on the resource constraint and credit constraint, respectively. As explained in Section 3, the term  $\psi_t$  captures the effect of an additional unit of tradables consumption on borrowing capacity

<sup>31</sup> The macroprudential policy in this economy does not have any time inconsistency issues. We will return to this below.

via equilibrium effects on the price of collateral (i.e. the price of nontradables).<sup>32</sup> Hence,  $\mu_t^* \psi_t$  reflects the fact that, when the credit constraint binds, the social marginal benefit from  $c^T$  includes the gains resulting from how changes in  $c^T$  can help relax the credit constraint, in addition to the marginal utility of tradables consumption.

The above two conditions yield the following Euler equation:

$$u_T(t) = \beta R \mathbb{E}_t [u_T(t+1) + \mu_{t+1}^* \psi_{t+1}] + \mu_t^* (1 - \psi_t). \quad (9)$$

Consider a state of nature in which the constraint is not binding ( $\mu_t^* = 0$ ). Comparing this condition with the household's Euler equation for bonds (eq. (4)) shows that there is a wedge between the private and social marginal cost of borrowing, given by the term  $\mu_{t+1}^* \psi_{t+1}$ . In particular, when the credit constraint is expected to bind, the planner faces a strictly higher marginal cost of borrowing than the representative agent. This is a pecuniary externality, because it results from the fact that the planner makes borrowing choices at  $t$  taking into account that the credit constraint could bind at  $t+1$ , and if it does the Fisherian debt-deflation mechanism will cause a collapse of the relative price of nontradables that will shrink borrowing capacity. The representative agent takes prices as given, and thus does not internalize these effects.

The literature has shown that the planner's allocations can be decentralized as a regulated competitive equilibrium using various policy instruments, including taxes on debt, loan-to-value ratios, capital requirements or reserve requirements (see Bianchi, 2011). Most studies focus on a debt tax that taxes the cost of borrowing at a rate  $\tau_t$  with the revenue of the tax rebated to the household as a lump-sum transfer. With this policy in place in the regulated economy, the cost of purchasing bonds in the budget constraint becomes  $[1/R(1 + \tau_t)]b_{t+1}$  and the Euler equation becomes  $u_T(t) = \beta R(1 + \tau_t) \mathbb{E}_t [u_T(t+1)] + \mu_t$ .<sup>33</sup>

The optimal macroprudential debt tax is then defined as the value of  $\tau_t$  that equates the Euler equations of bonds of the social planner and the regulated economy with the tax (note that this tax varies across time and states of nature). Hence, the tax induces private agents to face the social marginal cost of borrowing when it differs from the private cost. Solving for  $\tau_t$  yields:

$$\tau_t = \frac{\mathbb{E}_t [\mu_{t+1}^* \psi_{t+1}]}{\mathbb{E}_t [u_T(t+1)]} \quad (10)$$

The tax rate equals the expected value of the pecuniary externality, which captures also the possibility of a financial crisis at  $t+1$ , normalized by the (higher) marginal utility of tradables consumption if a crisis does occur.

For states in which  $\mu_t > 0$ , there is a range of taxes that implements the constrained efficient allocations, each of which is associated with a different Lagrange multiplier on the borrowing constraint for households, while delivering the same allocations. The logic is that if a given tax implements the constrained-efficient allocation when the constraint binds, a slightly lower tax increases the shadow value of relaxing the constraint, but it does not modify the feasible level of debt. A standard case is such that when evaluated at the constrained-efficient allocation, we have  $u'_T(c_t) > \beta R E u'_T(c_{t+1})$ , in which case a zero tax would implement the constrained-efficient allocation. When two or more allocations are consistent with conditions (3)–(6) and (5) holding with equality (i.e. when the multiplicity conditions examined in Section 3.1 of the paper are satisfied), a subsidy may be necessary to induce a higher aggregate level of borrowing that can support a higher price of collateral, so that the allocations that the planner finds optimal can be implemented.

A final remark worth highlighting is that there are no credibility problems associated with the above optimal macroprudential policy, as implied by the fact that the planner's problem can be expressed as a standard Bellman equation. A feature of the planner's problem that is key for this result is that the price of collateral depends only on allocations determined at date  $t$ . When the price is forward looking, optimal macroprudential policy becomes time-inconsistent under commitment, as Bianchi and Mendoza (2018) showed for the case of a Fisherian model with the LTV constraint. In this case, the planner's implementability constraint is the Euler equation for capital:  $q_t u'(t) = \beta E_t [u'(t+1)(d_{t+1} + q_{t+1})]$ , where  $d_{t+1}$  represents dividends. Intuitively, the planner internalizes that it can affect  $q_t$  by altering  $c_{t+1}$ . When the constraint binds at  $t$ , it would like to pledge lower  $c_{t+1}$  to prop-up  $q_t$  and thus improve borrowing capacity at  $t$ , but delivering on that pledge at  $t+1$  is suboptimal.

The literature has followed two approaches to deal with the time-inconsistency problem: First, Bianchi and Mendoza (2018) solved for time-consistent policy by modeling the regulator of date  $t$  as taking as given the decision rules of future regulators, and then solving for a Markov stationary equilibrium in which these decision rules and the ones that are optimal for the current regulator are consistent. Second, an earlier version of Bianchi and Mendoza (2018) and studies like Mendoza and Rojas (2019) impose a treatment of the pricing function of collateral in the planner's problem akin to the notion of conditional efficiency: The planner takes as given the recursive equilibrium pricing function of the unregulated competitive equilibrium. It still internalizes how collateral prices respond to debt decisions and hence the pecuniary externality is still present, but it does not aim to alter the set of feasible borrowing positions allowed by the pricing function of the

<sup>32</sup> The product  $\left[ \frac{1-\omega}{\omega} (1+\eta) \frac{c^T}{y^T} \right]$  measures how total income in units of tradables changes with the choice of  $b_{t+1}$ , because this choice alters consumption of tradables and hence the relative price of nontradables.

<sup>33</sup> There are also results in the literature showing cases in which the problem of a Ramsey planner choosing debt tax rates yields the same allocations and taxes as the constrained efficient planner (see Section A.3 of the Appendix to Bianchi and Mendoza (2018) for a proof in the case of an LTV model).

unregulated equilibrium. When the planner is free to alter the pricing function, it will respond to the incentive to prop up  $q_t$  in the time-inconsistent problem under commitment or to the incentive to influence the actions of future regulators in the time-consistent, Markov-stationary problem.

**Appendix B. Debt taxes & LTV regulation are not equivalent**

This part of the Appendix shows that debt taxes and LTV regulation are not equivalent, even though both reduce credit and distort investment. Consider for simplicity a one-sector variant of the model with an asset in fixed supply of one unit. The budget and collateral constraints with debt taxes and LTV regulation are:

$$\frac{b_{t+1}}{R(1 + \tau_t)} + z_t F(k_t) \geq c_t - b_t + q_t(k_t - k_{t+1}) + T_t$$

$$\frac{b_{t+1}}{R} \geq -\kappa(1 - \chi_t)q_t k_{t+1}$$

and the first-order conditions of the household's problem (evaluated at  $k_{t+1} = 1$ ) are:

$$u'(c_t) = \beta R(1 + \tau_t)Eu'(c_{t+1}) + \mu_t$$

$$[u'(c_t) - \mu_t \kappa(1 - \chi_t)]q_t = \beta Eu'(c_{t+1})[q_{t+1} + z_{t+1}]$$

where we simplified by assuming  $F'(1) = 1$  without loss of generality.

**Proposition.** *Suppose there is a set of equilibrium allocations  $\{b_{t+1}^\tau, c_t^\tau\}$  and prices  $\{q_t^\tau\}$  associated with a debt-tax policy  $\{\tau_t\}$ . Then,  $\{b_{t+1}^\tau, c_t^\tau, q_t^\tau\}$  cannot be implemented with a regulatory LTV ratio  $\{\chi_t\}$ , and in particular implementing  $\{b_{t+1}^\tau, c_t^\tau\}$  yields higher asset prices under LTV regulation ( $q_t^\chi > q_t^\tau$ ).*

**Proof.** We show that implementing  $\{b_{t+1}^\tau, c_t^\tau\}$  with regulatory LTVs given by  $\{\chi_t\}$  yields higher equilibrium asset prices (i.e.  $q_t^\chi > q_t^\tau$ ), and hence the two regimes yield different equilibria. Consider first the equilibrium in which only taxes are used (i.e.  $\chi_t = 0$ ) and a state of nature in which the constraint is not binding at date  $t$  (which is when a positive debt tax would be used). The Euler equations of bonds and capital are:

$$u'(c_t^\tau) = \beta R(1 + \tau_t)Eu'(c_{t+1}^\tau)$$

$$u'(c_t^\tau)q_t^\tau = \beta Eu'(c_{t+1}^\tau)[q_{t+1}^\tau + z_{t+1}] \tag{11}$$

To implement the same level of debt with LTV regulation requires that the credit constraint binds ( $\mu_t^\chi > 0$ ) at the amount of debt of the debt-tax regime:

$$(1 - \chi_t) = \frac{-b_{t+1}^\tau}{R\kappa q_t^\tau} \tag{12}$$

In order to sustain also the same equilibrium asset prices, the Euler equations of the LTV regime must hold with the same allocations and prices using regulatory LTVs instead of taxes (i.e.  $\tau_t = 0$ ). Combining the two Euler equations under the LTV regime yields the following:

$$\{u'(c_t^\tau) - \kappa(1 - \chi_t)[u'(c_t^\tau) - \beta R Eu'(c_{t+1}^\tau)]\}q_t^\chi = \beta Eu'(c_{t+1}^\tau)(q_{t+1}^\tau + z_{t+1})$$

$$\{u'(c_t^\tau) - \kappa[u'(c_t^\tau) - \beta R Eu'(c_{t+1}^\tau)] + \kappa \chi_t [u'(c_t^\tau) - \beta R Eu'(c_{t+1}^\tau)]\}q_t^\chi = \beta Eu'(c_{t+1}^\tau)(q_{t+1}^\tau + z_{t+1}) \tag{13}$$

Subtracting (13) from (11) and simplifying we obtain:

$$u'(c_t^\tau)(q_t^\tau - q_t^\chi) = -\kappa(1 - \chi_t)\mu_t^\chi q_t^\chi \tag{14}$$

Since in order to tighten credit without taxes the regulatory LTV must bind ( $\mu_t^\chi > 0$ ) and since the LTV regulation requires  $0 < \chi_t < 1$ , the right-hand-side of this expression is negative and hence satisfying this condition requires  $q_t^\chi > q_t^\tau$ , which implies that asset prices would be higher under the LTV regime than with the debt tax. This also implies that allocations would be different in previous periods, because of the forward looking nature of asset prices. Moreover, this result suggests that it may be the case that LTVs dominate other macroprudential policy instruments, particularly debt-tax-like instruments (e.g. lender targeted instruments like capital controls and capital requirements), because of their ability to boost asset prices. □

## Appendix C. Sudden Stops event analysis

We constructed the Sudden Stops event analysis using the same methodology as in Korinek and Mendoza (2014), extending the dataset to cover the 1979–2016 period. The full details of their methodology are described in the data appendix of their paper (which is available for download at <https://www.annualreviews.org/doi/suppl/10.1146/annurev-economics-080213-041005>). Their methodology is in turn based on the one developed by Calvo et al. (2006), including Sudden Stops with both large and mild output collapses. Sudden Stops are identified by applying two filters: the capital flow reversal filter and the systemic filter. The capital flow filter flags years with a large fall in capital flows, measured as an increase in the current account-GDP ratio larger than two standard deviations. The systemic filter identifies years in which there were either aggregate EMBI spread spikes for emerging economies or aggregate VIX spikes for advanced economies.

A Sudden Stop event is identified for a particular country in a particular year when the two filter conditions are satisfied. Then we construct five-year event windows using macro data based on medians across all Sudden Stop events for emerging and advanced economies. For Sudden Stop events before 2004, we use the list of events identified by Calvo et al. (2006), except for the events they identified for Morocco in 1981 and 1995, which were too close to Sudden Stops they also identified for 1983 and 1997. We consider the 1981 and 1983 (1995 and 1997) events as part of a single event dated in 1983 (1997), which is the year with the largest current account reversal.

We used for the most part the same data sources as in Korinek and Mendoza (2014), but using additional data for the real exchange rate and real equity prices. For countries and/or periods where the Korinek-Mendoza data sources do not show data, we construct bilateral real exchange rates using domestic CPI, US CPI, and the nominal exchange rate of the domestic currency v. the US dollar for December of each year. For stock prices, missing country series were completed (when possible) using stock price index data from Bloomberg (December averages). Lastly, when possible, the stock price data were extended by extrapolating using the annual percentage change from S&P Global Equity Indices, obtained from the World Development Indicators.

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