

MACROPRUDENTIAL POLICY WITH LEAKAGES*

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August 2021

Abstract

We explore how imperfect regulation enforcement affects the design of optimal macroprudential policy. We study an open economy workhorse model of macroprudential regulation motivated by pecuniary externalities. Our analytical characterization shows that imperfect enforcement generates two opposing effects. While tighter regulation leads to higher borrowing by unregulated agents, a “leakage effect”, mitigating the increase in fragility calls for “squeezing” regulated agents further. Quantitative simulations show that, overall, a macroprudential policy that accounts for the leakages remains successful at mitigating the vulnerability to financial crises.

Keywords: Macroprudential policy, capital flow management, regulatory arbitrage, financial crises, limited regulation enforcement

JEL Classifications: F32, E58, G28

*For useful comments and suggestions, we thank Yan Bai, Frédéric Boissay, Roberto Chang, Marcel Fratzscher, Olivier Jeanne, Sebnem Kalemli-Ozcan, Michael Klein, Enrique Mendoza, Alessandro Rebucci, Paulina Restrepo-Echavarria and Alp Simsek, as well as conference/seminar participants at the IMF, Einaudi Institute, Laval, UQAM, NBER-CRBT Conference on Monetary Policy and Financial Stability in Emerging Markets, 2015 SED, Barcelona GSE Summer Workshop on International Capital Flows, the Concluding Conference of the ESCB’s Macro-prudential Research Network, the 2015 Canadian Macroeconomic Study Group Annual Meeting, the IMF 2015 Annual Research Conference and the CBC-IDB Conference on Financial Frictions, Macroeconomic Implications and Policy Options for Emerging Economies. An earlier version of this paper circulated under the title “Capital Flow Management When Capital Controls Leak.” We are grateful to the Social Sciences and Humanities Research Council of Canada for funding this research under grant 430-2016-00995. We thank Louphou Coulibaly, Wontae Han, and Yanis Sbih for excellent research assistance. The views expressed herein are those of the authors and not necessarily those of the Bank of Canada, the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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1 Introduction

Macroprudential regulation has emerged as a central element of the post-financial crisis dominant policy paradigm, in which the management of credit cycles has been elevated to the rank of first-order policy concerns. Central to this view is the notion that curbing credit booms reduces financial sector vulnerabilities to future reversals in financial conditions. The effectiveness of macroprudential regulation, however, is not being taken for granted and has been the subject of a growing empirical literature (see, e.g., [Cerutti, Claessens and Laeven, 2017](#)). Indeed, a common policy concern is that such new regulation could be bypassed and have unintended consequences.¹

How is the optimal design of macroprudential regulation altered by the presence of leakages? Is the scope for macroprudential policy significantly altered by the possibility of leakages? Does macroprudential policy remain desirable in the presence of leakages? The theoretical literature on macroprudential policy has so far abstracted from the imperfect regulation enforcement, and thus a proper framework to address these questions is lacking. Our goal in this paper is to fill this gap by providing a theoretical model suited to tackle these issues.

In line with the emerging policy paradigm and the theoretical literature, we adopt a framework in which inefficient private borrowing decisions generate excessive financial fragility (e.g., [Lorenzoni 2008](#); [Bianchi 2011](#)). In our theory, a pecuniary externality resulting from frictions in financial markets makes macroprudential policy desirable, yet the introduction of such policy endogenously leads to increased risk taking by a shadow sector endowed with the ability to bypass regulation. These unintended spillover effects in turn feed into the economy's exposure to financial crises, limiting the effectiveness of macroprudential policy and altering its optimal design.

We start our analysis with a tractable three-period model. In the model, agents initially make a borrowing decision and are then subject to income shocks in the intermediate period while facing a collateral constraint that limits their ability to smooth consumption. The presence of a market price in the collateral constraint implies that the higher the aggregate leverage chosen in the initial period, the larger the contraction in the borrowing capacity in the intermediate period for all households. This pecuniary externality and the associated financial amplification effects are not internalized by private households, providing a rationale for a macroprudential policy aimed at limiting private leverage.

Different from existing studies, macroprudential regulation is subject to imperfect enforcement. In particular, the macroprudential authority is able to curb risk taking within a narrow (regulated) sector, while internalizing the endogenous response of the shadow (unregulated) sector. A stricter macroprudential policy seeking to make regulated agents internalize the pecuniary externality creates a safer environment but has the unintended consequence of encouraging higher borrowing by unregulated agents. These spillovers undermine the effectiveness of macroprudential policy and in-

¹ Empirical studies analyzing such leakages in macroprudential policies include [Magud et al. \(2011\)](#), [Aiyar, Calomiris and Wieladek \(2014\)](#), [Klein \(2012\)](#), [Jiménez, Ongena, Peydró and Saurina \(2012\)](#), [Forbes, Fratzscher and Straub \(2015\)](#), [Dassatti and Peydró \(2013\)](#), and [Ahnert, Forbes and Reinhardt \(2018\)](#).

crease the economy's exposure to financial crises.

We show that despite destabilizing spillover effects on the unregulated sphere, a small macroprudential tax on regulated agents is always strictly welfare improving for all agents. Going beyond a perturbation argument, we derive an explicit formula for the optimal macroprudential tax. Relative to the standard Pigouvian tax expression familiar from the literature, accounting for leakages yields two new elements. One reflects the weaker influence of the financial regulator onto the price of collateral due to the unregulated sector partially offsetting the regulated sector's response to the regulation. The second one reflects the regulator's higher valuation of relaxing future collateral constraints resulting from the unregulated sector's higher exposure to financial crises. Taken together, these two effects imply an ambiguous effect of leakages on the size of optimal macroprudential taxes.

We then turn to an infinite horizon model to pursue a quantitative analysis. In the infinite horizon context, leakages further give rise to a time inconsistency problem. To see why, suppose agents expect loose regulation in the future. This expectation generates a perception of a riskier environment, which induces unregulated agents to take less risk today, narrowing the gap between the borrowing choices of regulated and unregulated agents. Interestingly, this channel works despite unregulated agents not being directly subject to regulation. Given our goal of evaluating the extent to which leakages undermine the effectiveness of regulation, we find it useful to focus on a discretionary regime in which a regulator has limited ability to exploit the disciplining effect just described. We thus consider a government that sets optimal regulation sequentially and without commitment, and we focus on Markov-perfect equilibria. We then calibrate our infinite horizon model to match the features of emerging market crises and experiment with degrees of leakages ranging from 0 to 50% of agents making up the unregulated sphere of the economy.

The main take away from our quantitative analysis is that even subject to significant leakages, macroprudential policy remains highly effective and desirable. To offset the increase in borrowing by the unregulated sector, the planner induces even tighter regulation on the regulated sector, and this results in aggregate levels of borrowing that are comparable to the constrained-efficient allocation. Average welfare gains are also surprisingly stable with respect to leakages. Nonetheless, the welfare gains are spread unevenly in the economy, with the lion's share going to unregulated agents. Intuitively, while the financial stability benefits of macroprudential policy are partially offset by the burden of regulation for regulated agents, they are not for unregulated agents, who free-ride on others' precautionary behavior.

Related Literature. This paper belongs to a growing literature providing foundations for macroprudential policies. A first strand of this literature, most closely related to our paper, examines pecuniary externalities and incomplete markets when financial constraints depend on market prices (Caballero and Krishnamurthy, 2001, Lorenzoni, 2008, Bianchi, 2011, Jeanne and Korinek, 2012, Bianchi and Mendoza, 2018, Korinek, 2018). Abstracting from imperfect regulation enforcement, this literature

has tackled several issues, including the interactions between macroprudential and stabilization policies (Benigno et al., 2013), monetary policy (Fornaro, 2015, Ottonello, 2015, Sergeyev, 2016, Coulibaly, 2018, Devereux, Young and Yu, 2018), foreign reserves (Arce, Bengui and Bianchi, 2019; Davis et al., 2020), international coordination (Bengui, 2013), policy cyclicalities (Schmitt-Grohé and Uribe, 2017), multiple equilibria (Schmitt-Grohé and Uribe, 2016), learning (Bianchi, Boz and Mendoza, 2012), news and regime switching shocks (Bianchi, Liu and Mendoza, 2016), uncertainty shocks (Reyes-Heroles and Tenorio, 2018), trend shocks and the implications for the cyclicalities of capital controls (Fleming, L’Huillier and Piguillem, 2019, Seoane and Yurdagul, 2017). A second strand of the literature on macroprudential policy examines aggregate demand externalities in the presence of nominal rigidities (Schmitt-Grohé and Uribe, 2013, Farhi and Werning, 2016). Recent research in this area has studied Mundell’s Trilemma (Farhi and Werning, 2012), liquidity traps (Korinek and Simsek, 2016, Acharya and Bengui, 2018, Fornaro and Romei, 2018, Bianchi and Coulibaly, 2021), and fiscal unions (Farhi and Werning, 2017). See Bianchi and Lorenzoni (2021) and Bianchi and Mendoza (2020) for reviews of the literature.

A common theme in this literature is the presence of a wedge between the private and social cost of borrowing, which generates scope for Pigouvian taxes. Our contribution is to explicitly model the imperfect enforcement of macroprudential regulation, pursuing both a qualitative and quantitative analysis of optimal policy. The paper also speaks to an empirical literature that examines the effectiveness of macroprudential regulation.² In particular, our quantitative predictions are consistent with the findings that tighter regulation leads to opposite risk-taking responses by different agents, although tighter regulation leads to an overall reduction in risk taking (Dassatti and Peydró, 2013, Aiyar, Calomiris and Wieladek, 2014).

Finally, our paper relates to a literature in financial intermediation studying the interaction between regulation and shadow banking activity. Examples include Huang (2014), Plantin (2015), Grochulski and Zhang (2015), Farhi and Tirole (2017), Ordonez (2018), Ordonez and Piguillem (2018), Begenau and Landvoigt (2018) and Bengui, Bianchi and Coulibaly (2019). Begenau and Landvoigt, for example, study a model in which traditional banks are covered by deposit insurance but shadow banks are not, and find that a tightening of capital requirements for traditional banks causes shadow banks to take over a larger fraction of the credit market. We do not model banks explicitly but instead consider a generic setting with regulated and unregulated agents, and point to a distinct channel involving leverage responses to endogenous changes in systemic risk generated by regulation. From a normative standpoint, the above mentioned literature adopts a primarily microprudential approach to financial regulation. Overall, our paper provides the first analysis of macroprudential regulation in an economy with an unregulated sector.

The paper is organized as follows. Section 2 presents a three-period model that shows analytical results for the main mechanisms in the paper. Section 3 presents the results from the infinite horizon

²See footnote 1 for a list of papers.

quantitative model. Section 4 concludes. Proofs are collected in the appendix.

2 A Three-Period Model

In this section, we present an analytically tractable three-period model of financial crises in which macroprudential policy cannot be enforced on a subset of agents that are outside the regulated sector. We adopt specific assumptions on preferences that deliver closed-form solutions for continuation equilibria and help us obtain an analytical characterization of the optimal macroprudential policy problem in the presence of leakages. Later, in Section 3, we study the same mechanisms we outline in this simple model in the context of a more general quantitative infinite horizon model.

2.1 Environment

We consider a small open economy that lasts for three periods $\ell = 0, 1, 2$. There are two types of goods, tradables and nontradables, and no production. The only source of uncertainty is over the endowment of tradable goods in period $\ell = 1$.

The economy is populated by a continuum of agents of one of two types $\theta = \ast, \prime$ that differ in their ability to bypass regulation: a fraction \mathbb{W} is unregulated (\ast) and the remaining $1 - \mathbb{W}$ is regulated (\prime). The distinction between unregulated and regulated agents is intended to capture in a broad sense the existence of a shadow banking sector, differences in access to sources of funding, or differences in the ability to circumvent regulation by different agents. Our focus will be on how these different types of agents respond differently to policy and its implications for optimal regulation. For the most part, we consider an environment where the size of the unregulated sector \mathbb{W} is exogenous, but will also provide a simple way to endogenize \mathbb{W} based on an idiosyncratic cost of bypassing regulation.

Both types of agents have identical preferences and endowments. Preferences are given by

$$u_{\theta} = z_{\theta 0}^{\gamma} E_0 [V \ln z_{\theta 1}^{1-\gamma} V^2 \ln z_{\theta 2}^{\gamma}] \quad (1)$$

with

$$z_{\theta \ell} = z_{\theta \ell}^{\gamma} z_{\theta \ell}^{\# 1-\gamma}.$$

where E is the expectation operator and V is a discount factor. Date 0 utility is linear in tradable consumption z^{γ} , while date 1 and date 2 utility is logarithmic in a consumption basket z , which is a Cobb-Douglas aggregator of tradable goods consumption z^{γ} and nontradable goods consumption $z^{\#}$. The parameter γ is the share of tradables in total consumption.

Agents receive endowments of tradable goods and nontradable goods of $\tilde{z}_1^{\gamma} B^0 \cdot \tilde{z}_1^{\#}$ at date 1 and date 2, but do not receive any endowment at date 0. The date 1 endowment of tradables $\tilde{z}_1^{\gamma} B^0$ is a random variable depending on the event $B \in \mathcal{B}$, which can be interpreted as the aggregate state of the economy. For simplicity, we assume that \tilde{z}_2 is deterministic and that the nontradable goods

endowment is constant, $\tilde{w}_1^\# = \tilde{w}_2^\# = \tilde{w}^\#$.

Agents have access to a single, one-period, non-state-contingent bond denominated in units of tradable goods that pays a fixed interest rate A , determined exogenously in the world market. Denoting the relative price of nontradables by $\tau^\#$, the agents' budget constraints in all three periods are given by

$$2_{\beta 0}^1 1_{\beta 0} + g_{\beta 0}^0 1_{\beta 0} = \gamma_{\beta 0} \quad (2)$$

$$2_{\beta 1}^1 B^0 + \tau_1^\# 1_{\beta 1} B^0 + 2_{\beta 1}^{\#} 1_{\beta 1} B^0 = 1_{\beta 1} + A^0 1_{\beta 1} + \tau_1^\# 1_{\beta 1} B^0 + \tau_1^\# 1_{\beta 1} B^0 \quad \text{SB} \quad (3)$$

$$2_{\beta 2}^1 B^0 + \tau_2^\# 1_{\beta 2} B^0 + 2_{\beta 2}^{\#} 1_{\beta 2} B^0 = 1_{\beta 2} + A^0 1_{\beta 2} + \tau_2^\# 1_{\beta 2} B^0 + \tau_2^\# 1_{\beta 2} B^0 \quad \text{SB} \quad (4)$$

where $1_{\beta, \ell}$ denotes the bond holdings an agent chooses at the beginning of period ℓ , g_{β} denotes the tax rate on date 0 borrowing, and γ_{β} is a lump-sum transfer. Crucially, we impose $g^* = 0$, that is, unregulated agents face no tax on borrowing.³ To abstract from the distributional side effects of macroprudential policy, we assume that the planner rebates the tax proceeds to the agents who pay the tax.

At date 1, agents are subject to a credit constraint preventing them from borrowing more than a fraction $\hat{\alpha}$ of their current income:

$$1_{\beta 2}^1 B^0 \leq \hat{\alpha} (\tau_1^\# 1_{\beta 1} B^0 + \tau_1^\# 1_{\beta 1} B^0) \quad \text{SB}'' \quad (5)$$

This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit market access. The form of this constraint follows [Mendoza \(2002\)](#) and is common in the literature on financial crises and macroprudential policy (e.g., [Bianchi, 2011](#) and [Korinek, 2018](#)).⁴

Agents choose consumption and savings to maximize their utility (1) subject to budget constraints (2), (3), (4), and the credit constraint (5), taking $\tau_1^\# 1_{\beta 1} B^0$ and $\tau_2^\# 1_{\beta 2} B^0$ as given. Necessary and sufficient

³We assume that unregulated agents are prevented from arbitraging via borrowing abroad and lending domestically to the regulated sphere (e.g., because of technological reasons). Equivalently, we could assume that the tax on borrowing on the regulated sphere applies to both domestic and foreign forms of borrowing.

⁴This collateral constraint can be derived endogenously from limited enforcement assuming that: (i) households can default at the end of the period (before observing next period income realization), (ii) upon default, foreign creditors can seize a fraction $\hat{\alpha}$ of the current income and households regain access immediately to credit markets. Non-tradable goods enter the collateral constraint because while foreign creditors do not value non-tradable goods, they can sell it in exchange for tradable goods after seizing these goods after default. The current and not the future price appears in the constraint because the opportunity to default is at the end of the current period, before the realization of future shocks (see [Bianchi and Mendoza, 2018](#)).

conditions for optimality are given by

$$\frac{u'_c(B^0)}{u'_c(B^1)} = \frac{1 + \beta \frac{u'_c(B^0)}{u'_c(B^1)}}{1 + \beta \frac{u'_c(B^0)}{u'_c(B^1)}} \quad (6)$$

$$1 = V_1 + A^0 + g^0 E_0 \frac{1}{2_{\beta_1} B^0} \quad (7)$$

$$\frac{1}{2_{\beta_1} B^0} = V_1 + A^0 \frac{1}{2_{\beta_2} B^0} \quad \text{with equality if } 1_{\beta_2} B^0 < \beta \frac{u'_c(B^0)}{u'_c(B^1)} \quad (8)$$

for all B . Condition (6) is a static optimality condition equating the marginal rate of substitution between tradable and nontradable goods to their relative price. Conditions (7) and (8) are the Euler equations for bonds at date 0 and date 1. The latter holds with strict inequality if the credit constraint binds, in which case current marginal utility exceeds the expected marginal utility costs from borrowing one unit and repaying next period.

Finally, the government budget constraint says that taxes on borrowing are rebated back to regulated agents:

$$\tau = \frac{g}{1 + g} \tau_1 \quad \text{and} \quad \tau^* = 0 \quad (9)$$

Definition of competitive equilibrium. Given a tax policy (g, τ) , a competitive *regulated* equilibrium of the model is a set of date 0 choices $(2_{\beta_0} B^0, 1_{\beta_1} B^0, g^0, \tau)$, date 1 choices $(2_{\beta_1} B^1, 2_{\beta_2} B^1, 1_{\beta_2} B^1, g^1, \tau_1)$, and prices $(\frac{u'_c(B^0)}{u'_c(B^1)}, \frac{u'_c(B^0)}{u'_c(B^1)})$ such that (1) given prices and taxes, agents' decisions are optimal, (2) the market for the nontradable good clears at all dates, and (3) the government budget constraint holds.

In what follows, we proceed by backward induction. We first analyze the date 1 continuation equilibrium for given date 0 bond choices and then turn to the determination of date 0 borrowing decisions.

2.2 Date 1 continuation equilibrium

To simplify the analysis, we make the following parametric assumptions.

Assumption 1. *The domestic agents' discount factor and the international interest rate satisfy $V_1 + A^0 = 1$.*

This assumption, common in small open economy models, implies that domestic agents are as patient as international investors. A result of this assumption is that there is no intrinsic motivation for consumption tilting in the domestic economy between date 1 and date 2, which simplifies the households' optimization problem.

Assumption 2. *The consumption shares and collateralizable fraction of income are such that $0 < \beta < \beta^*$.*

This assumption simplifies the analysis by guaranteeing that there is a unique (continuation) equilibrium. As will become clear below, when Assumption 2 holds, an increase in aggregate consumption by one unit does not relax agents' credit constraint by more than one unit in equilibrium.

For given date 0 savings choices, the agent's date 1 problem conveniently admits a closed-form solution. If an agent θ is unconstrained at date 1, his consumption plan for all B is given by

$$z_{\theta 1}^> B^0 = z_{\theta 2}^> B^0 = \frac{1}{1 + V} F4_{\theta 1}^> B^0 \quad \text{and} \quad z_{\theta c}^{\#} B^0 = \frac{1}{1 + V} \frac{F4_{\theta 1}^> B^0}{z_{\theta c}^{\#} B^0} \quad \text{for} \quad c = 1, 2 \quad (10)$$

where $F4_{\theta 1}^> B^0$ is the agent's date 1 lifetime wealth:

$$F4_{\theta 1}^> B^0 = (1 + A^0) z_{\theta 1}^> B^0 + z_{\theta 1}^{\#} B^0 + \frac{z_{\theta 2}^{\#} B^0}{1 + A}$$

To finance this consumption plan, the agent borrows the shortfall between his expenditures $F4_{\theta 1}^> B^0 \cdot (1 + V)$ and cash on hand $(1 + A^0) z_{\theta 1}^> B^0 + z_{\theta 1}^{\#} B^0$ at date 1:

$$z_{\theta 2}^> B^0 = z_{\theta 2}^{D=2} B^0 = \frac{V}{1 + V} \left[(1 + A^0) z_{\theta 1}^> B^0 + z_{\theta 1}^{\#} B^0 - \frac{z_{\theta 2}^{\#} B^0}{1 + A} \right] \quad (11)$$

The agent is constrained at date 1 if the bond position in (11) violates the credit constraint (5). In this case, he borrows the maximum amount:

$$z_{\theta 2}^> B^0 = z_{\theta 2}^{>=} B^0 = \frac{z_{\theta 1}^{\#} B^0}{1 + A} \quad (12)$$

and chooses a consumption plan given by

$$\begin{aligned} z_{\theta 1}^> B^0 &= \frac{1}{1 + V} F4_{\theta 1}^> B^0 & z_{\theta 2}^> B^0 &= \frac{1}{1 + V} \left[(1 + A^0) F4_{\theta 1}^> B^0 - F4_{\theta 1}^> B^0 \right] \\ z_{\theta 1}^{\#} B^0 &= \frac{1}{1 + V} \frac{F4_{\theta 1}^> B^0}{z_{\theta 1}^{\#} B^0} & z_{\theta 2}^{\#} B^0 &= \frac{1}{1 + V} \frac{1 + A^0}{z_{\theta 2}^{\#} B^0} F4_{\theta 1}^> B^0 \end{aligned} \quad (13)$$

where $F4_{\theta 1}^> B^0$ is the agent's date 1 constrained wealth

$$F4_{\theta 1}^> B^0 = (1 + A^0) z_{\theta 1}^> B^0 + z_{\theta 1}^{\#} B^0 + \frac{z_{\theta 2}^{\#} B^0}{1 + A}$$

which corresponds to the sum of actual date 1 wealth and the maximum amount that can be borrowed.

The nontradable goods market clearing condition is

$$W_{*c}^{\#} B^0 = 1 - W_{'c}^{\#} B^0 = \tilde{\#} \quad \text{for} \quad c = 1, 2 \quad (14)$$

where uppercase letters with $*$ or $'$ subscripts denote aggregates over an agent's type.

The aggregation of the two sets of agents' intertemporal budget constraints yields the economy's

intertemporal resource constraint:

$$\gamma_1^1 B^0 \leq \frac{\gamma_2^1 B^0}{1 + A} = \gamma_1^1 A^0 \gg W^* \gamma_1^1 B^0 \leq W^0 \gamma_1^1 B^0 \leq \frac{\gamma_2^1 B^0}{1 + A}. \quad (15)$$

where $\gamma_1^1 B^0 = W^* \gamma_1^1 B^0 \leq \gamma_1^1 B^0$ is aggregate tradable consumption.

Combining the nontradable market clearing condition (14) with the agents' static optimality condition (6) indicates that the equilibrium price of nontradables is proportional to the economy's absorption of tradables:

$$q_1^{\#} B^0 = \frac{1}{I} \frac{\gamma_1^1 B^0}{\gamma_1^{\#}}. \quad (16)$$

This condition establishes a positive equilibrium relationship between the price of nontradables and aggregate tradable consumption for a given level of nontradable output. Intuitively, when tradable consumption is high relative to nontradables consumption, the relative price of nontradables has to be high. Given homothetic preferences, a negative shock to the tradable goods endowment γ_1^1 generates a decline in the demand for both consumption goods. For a given level of nontradable output - and, by market clearing, nontradable consumption - the equilibrium relative price of nontradables $q_1^{\#}$ must fall to induce agents to substitute tradable with nontradable consumption. Crucial for our analysis is that a higher aggregate level of debt accumulated at date 0 will similarly imply a lower level of tradable consumption and thus a lower price of nontradables for any income shock at date 1. Through the credit constraint (5), this implies a lower borrowing capacity and a tighter borrowing constraint when this constraint is binding.⁵ Individual agents do not internalize these effects, which generates a scope for welfare-improving macroprudential regulation at date 0, as in Bianchi (2011) and others. In this context, the novelty of our analysis regards the planner's inability to tax or regulate the borrowing decision of a subset of agents.

At date 1, the economy's aggregate state variables are given by the tradable goods endowment $\gamma_1^1 B^0$ and by the respective aggregate bond positions of unregulated and regulated agents, γ_1^* and $\gamma_1^{\#}$. We denote the functions mapping these state variables into date ℓ aggregate tradable consumption, unregulated agents' tradable consumption, and regulated agents' tradable consumption by $\gamma_1^{\ell} \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$, $\gamma_1^{\ell} \gamma_1^* \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$, and $\gamma_1^{\ell} \gamma_1^{\#} \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$, respectively. Similarly, we respectively denote the functions mapping the state variables into date ℓ aggregate nontradable consumption, unregulated agents' nontradable consumption, and regulated agents' nontradable consumption by $\gamma_1^{\ell} \gamma_1^{\#} \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$, $\gamma_1^{\ell} \gamma_1^* \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$, and $\gamma_1^{\ell} \gamma_1^{\#} \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$. Finally, we denote the pricing function by $q_1^{\#} \gamma_1^{\#} \gamma_1^1 B^0 \cdot \gamma_1^* \gamma_1^0$. Depending on which set(s) of agents is (are) credit constrained, the economy can be in four regions at date 1: 22 where both types of agents are constrained, $2D$ where $*$ agents are constrained and $^{\#}$ agents are unconstrained, $D2$ where $*$ agents are unconstrained and $^{\#}$ agents are constrained, and DD where both types of agents are unconstrained. In each of these regions,

⁵From equation (16), an increase in aggregate consumption raises the price of nontradables by $\frac{1}{1 + I} \frac{1}{\gamma_1^{\#}}$ and hence raises overall borrowing capacity by $\frac{1}{1 + I}$, which by Assumption 2 is strictly lower than one.

tradable consumption is linear in all three state variables. In particular, date 1 aggregate tradable consumption is increasing in \tilde{y}_1^B , \tilde{y}_1^N , and \tilde{y}_1^R . As a result, the date 1 price of nontradables is increasing in \tilde{y}_1^B , \tilde{y}_1^N , and \tilde{y}_1^R . In addition, these two date 1 variables are more sensitive to tradable income when credit constraints are binding. Similarly, \tilde{y}_1^B and \tilde{y}_1^N are more sensitive to \tilde{y}_1^B and \tilde{y}_1^R in the regions where the credit constraints are binding. Appendix A formally establishes these results and related ones.

Figure 1 illustrates the sensitivity of the price of nontradables \tilde{p}_1^N with respect to the wealth positions \tilde{y}_1^B and \tilde{y}_1^R in the four regions, for a given realization of date 1 tradable income. The downward-sloping light lines represent iso-price curves. Lines farther to the northeast represent higher levels of \tilde{p}_1^N . Naturally, agents are constrained at lower wealth levels. The smaller distance between the iso-price curves in the constrained regions reflects a higher sensitivity of \tilde{p}_1^N in these regions. The intuition is as follows. When the credit constraint does not bind, consumption is increasing in wealth because of a standard permanent income effect. When it does bind, however, the sensitivity is higher because of an additional financial amplification effect working through the price of nontradables. The larger the mass of constrained agents, the stronger this financial amplification effect and thus the stronger the sensitivity of \tilde{p}_1^N to debt.

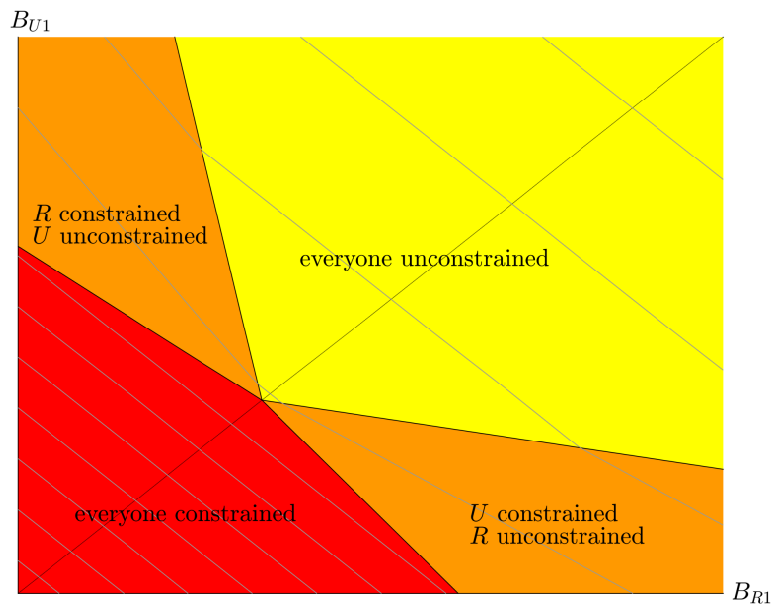


Figure 1: Date 1 price of nontradable good as a function of savings positions \tilde{y}_1^B and \tilde{y}_1^R for a given tradable endowment \tilde{y}_1^B .

The properties of the equilibrium nontradable price function discussed above have key implications for the spillover effects of macroprudential policy onto the unregulated sphere of the economy. In particular, the increasingness of \tilde{p}_1^N in \tilde{y}_1^B means that the date 1 total income at market prices and the date 1 borrowing capacity of unregulated agents are both increasing in the date 0 savings of the regulated agents. This in turn implies that for a given level of debt of unregulated agents, the

consumption of these agents is higher the lower the debt of regulated agents. This is illustrated in Figure 2, which represents unregulated agents' date 1 tradable consumption c_{U1}^T as a function of the realization of tradable income y_1^T , for two different regulated savings levels $\bar{y}_1^R \bar{y}_1^U$ and a given unregulated savings level $\bar{y}_1^U = \bar{y}_1^U$. The light curve represents the consumption function for a high level of regulated agents' debt (i.e., high \bar{y}_1^R), while the dark curve represents the function for a lower level of regulated agents' debt (i.e., a lower \bar{y}_1^R). The figure shows that a lower level of regulated agents' debt has several general equilibrium effects on unregulated agents' consumption. First, by increasing total income evaluated at market prices, it shifts consumption up when all agents are unconstrained. Second, by propping up collateral value, it shifts the region where unregulated agents become constrained to the left. Third, and maybe more subtly, it reduces the sensitivity of (unregulated agents') consumption to income for intermediate levels of income at which unregulated agents are constrained but regulated agents are not.

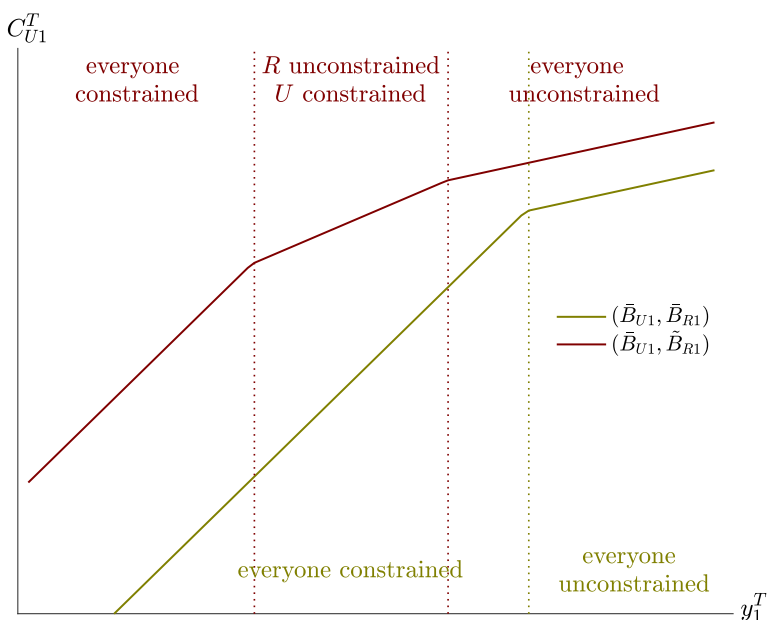


Figure 2: Unregulated agents' date 1 consumption as function of the tradable endowment y_1^T , for given savings pairs $\bar{y}_1^R \bar{y}_1^U$ and $\bar{y}_1^R \bar{y}_1^U$, with $\bar{y}_1^U = \bar{y}_1^U$.

The general equilibrium effects just described of regulated agents' borrowing on unregulated agents' date 1 consumption profile naturally translate into spillovers from regulated agents' borrowing into unregulated agents' borrowing at date 0. We elaborate on these in Section 2.4.

At this stage, we can define the date 1 value in equilibrium of an agent of type θ as a function of the aggregate state variables of the economy as

$$v_{\theta 1}^{\theta} = \sum_{\ell=1,2} \frac{\partial v_{\theta 1}^{\theta}}{\partial y_{\ell}^T} \ln \left(\frac{y_{\ell}^T}{y_{\ell}^T} \right) + \ln \left(\frac{y_{\ell}^T}{y_{\ell}^T} \right) \quad (17)$$

This value function will be useful for the normative analysis coming up in Section 2.5.

2.3 Date 0 unregulated equilibrium

We refer to the competitive equilibrium that prevails when the tax g^* is set to zero as the *unregulated* equilibrium. The unregulated equilibrium naturally displays symmetric borrowing choices ($b_1^* = b_1^0$) and is characterized by the following Euler equation:

$$1 = E_0 \frac{I}{\beta^0 \cdot \beta^0} \quad (18)$$

Lemma 1. *At date 0 competitive equilibrium exists and is unique.*

2.4 Equilibrium with exogenous tax

To lay the groundwork for our analysis of optimal macroprudential policy in the presence of leakages, we start by characterizing the private sector's response to an exogenous tax on borrowing. In a second step, we will then solve for the optimal tax chosen by a benevolent government.

A competitive regulated equilibrium can be conveniently characterized by a sole pair of Euler equations,

$$1 = E_0 \frac{I}{\beta^0 \cdot \beta^0} \quad (19)$$

$$\frac{1}{1 - g^*} = E_0 \frac{I}{\beta^0 \cdot \beta^0} \quad (20)$$

where (19) and (20) are the respective Euler equations for date 0 borrowing of unregulated and regulated agents.

Given a value for b_1^* , (19) implicitly defines the equilibrium borrowing choice of unregulated agents. Likewise, for a given value of b_1^* and a given tax g^* , (20) implicitly defines the equilibrium borrowing choice of regulated agents. We can thus formally define the following equilibrium responses of the two sectors.

Definition 1 (Equilibrium borrowing responses). $b_1^* = q^* \cdot b_1^0$ denotes the equilibrium borrowing responses of $*$ agents to the borrowing of $'$ agents, as implicitly defined by (19). Similarly, $b_1^* = q^* \cdot b_1^0 \cdot g^*$ denotes the equilibrium borrowing responses of $'$ agents to the borrowing of $*$ agents and a tax rate, as implicitly defined by (20).

To understand the effect of macroprudential policy with leakages, a key issue is the impact of a change in borrowing by one type of agent on the other agents' borrowing choice. Our next result characterizes such responses.

Proposition 1 (Substitutability in borrowing decisions). *For a given tax rate, the equilibrium borrowing of unregulated agents is decreasing in the amount of borrowing of $'$ agents and vice versa; that is, $\frac{\partial b_1^*}{\partial b_1^0} < 0$ and $\frac{\partial b_1^0}{\partial b_1^*} < 0$.*

Proposition 1 establishes that borrowing decisions by the two sets of agents are akin to strategic substitutes. The less regulated agents borrow, the more unregulated agents find it optimal to borrow (and vice versa). This result follows naturally from our discussion of how unregulated agents' date 1 consumption depended on regulated agents' borrowing in the context of Figure 2.⁶ Lower borrowing by regulated agents at date 0 shifts unregulated agents' consumption up for any realization of the date 1 endowment, notably because it supports higher nontradable goods prices and thus uniformly relaxes everyone's date 1 borrowing constraint. Higher date 1 (tradable) consumption, and thus lower date 1 marginal utility, for a given level of x_1 , in turn induces unregulated agents to increase borrowing. The exact same logic applies to regulated agents' response to unregulated agents' borrowing.

While the substitutability emphasized in Proposition 1 is key to grasping our leakage phenomenon, our ultimate interest lies in understanding how both sectors react to macroprudential policy. Our next proposition describes how date 0 borrowing by the two sets of agents and date 1 borrowing capacity respond to changes in the tax rate.

Proposition 2 (Positive effect of small tax). *Starting from the unregulated equilibrium, imposing a small tax leads to strictly less borrowing by regulated agents, strictly more borrowing by unregulated agents, and to an unambiguously larger borrowing capacity at date 1.*

Quite intuitively, Proposition 2 states that a tax on debt generates a decrease in regulated agents' date 0 borrowing. More importantly, and consistent with Proposition 2, it also says that a larger tax causes an increase in unregulated agents' date 0 borrowing.

Figure 3 illustrates these results by representing the equilibrium response functions of the two sets of agents in the (x_1, g_1) space. The solid line is the equilibrium response of unregulated agents, while the other two lines are the equilibrium responses of regulated agents associated with a zero tax (dash-dotted line) and a positive tax (dashed line). The intersection between the unregulated agents' equilibrium response and the regulated agents' equilibrium response associated with a zero tax coincides by definition with the unregulated equilibrium. A positive tax causes a shift of the regulated agents' equilibrium response to the right: for a given x_1 choice, regulated agents respond to the tax by borrowing less (it makes borrowing more costly). But unregulated agents respond to this lower borrowing by ' agents by borrowing more themselves. This extra borrowing by ' agents in turn induces ' agents to borrow even less. And ' agents again respond by borrowing even more. This "process" continues until equilibrium is reached at point (g_1^*, x_1^*) . The equilibrium response of unregulated agents can thus be thought of as the set of (x_1, g_1) combinations that a government could achieve by varying the tax g .

Finally, Proposition 2 indicates that the effect of a larger tax on the date 1 borrowing capacity is positive. This effect can be traced back to the net effect of an increase in regulated agents' wealth on

⁶Note that this discussion focused on the effect of regulated agents' borrowing on unregulated agents' consumption. But a similar logic applies for the effect of unregulated agents' borrowing on regulated agents' consumption.

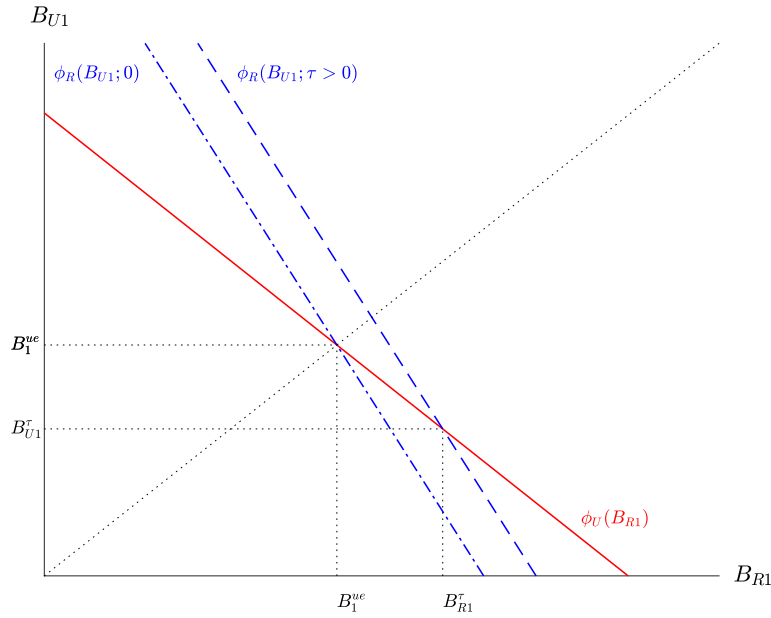


Figure 3: Equilibrium response functions of regulated and unregulated agents in equilibrium with exogenous tax ($0 < \tau < 1$).

the date 1 price of nontradable goods:

$$\frac{\partial \beta_1^{\#}}{\partial \tau} = \frac{\partial \beta_1^{\#}}{\partial \tau} + \underbrace{\frac{\partial \beta_1^{\#}}{\partial \tau} \frac{\partial \tau}{\partial \tau}}_{\text{Leakage effect}} \quad (21)$$

That is, the direct effect of the tax on the date 1 price of nontradables via regulated agents' borrowing (first term in (21)) necessarily dominates its indirect effect via unregulated agents' borrowing (second term in (21)), resulting in a positive net effect. This suggests that, at least locally, leakages may reduce the effectiveness of macroprudential policy by making future borrowing capacity less responsive to a tax on current borrowing, but are not powerful enough to overturn its effect.

2.5 Optimal macroprudential policy

The preceding section characterized the private sector's response to an exogenous tax. Building on this positive analysis, we now take a normative perspective and endogenize the level of the tax as the outcome of an optimal policy problem.

Before turning to a formal analysis of optimal policy, we find it useful to highlight the scope for welfare improvements of macroprudential policy with the following preliminary result.

Proposition 3 (Welfare effect of small tax). *If borrowing constraints bind with positive probability in the unregulated equilibrium, a small positive tax is welfare improving for all agents.*

This result, based on a perturbation argument, indicates that in spite of leakages, it is always

desirable for the planner to impose a small tax on regulated agents. According to Proposition 2, such a tax leads to less borrowing by regulated agents, more borrowing by unregulated agents, and a higher date 1 borrowing capacity. The only first-order effect on welfare (for both agents) arises from the relaxation of the borrowing constraint at date 1 in states of nature where this constraint binds and is positive. All other effects are of second order. This suggests that there exists potential welfare gains from macroprudential policy despite leakages.

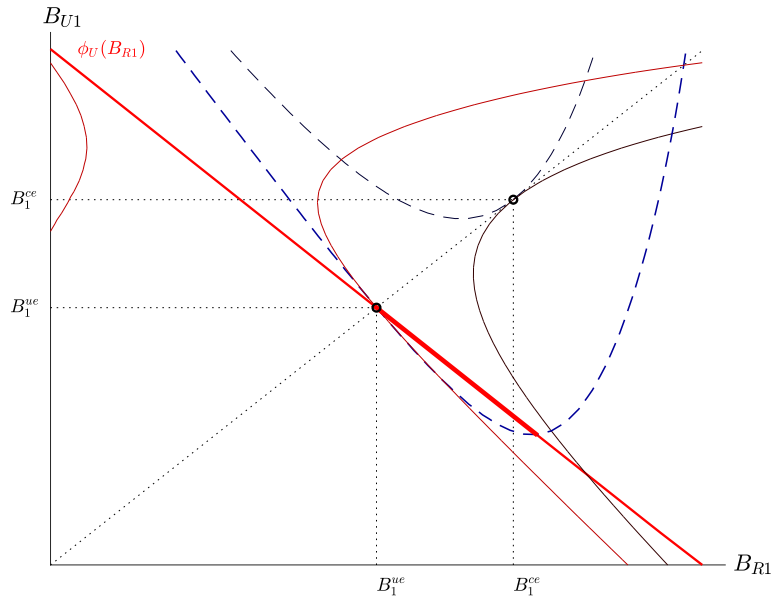


Figure 4: Contract curve and implementability constraint

Figure 4 plots the unregulated agents' equilibrium response function $q^{*1} \cdot 1^0$ together with both types of agents' iso-utility curves passing through the unregulated equilibrium $1 \cdot 1^0$. Proposition 3 implies that the segment of the unregulated agents' equilibrium response function situated immediately to the right of the unregulated equilibrium necessarily lies in the lens formed by the two types of agents' iso-utility curves passing through the unregulated equilibrium.

Turning to optimal policy, we consider a planner maximizing a weighted sum of the agents' utility subject to the date 1 and date 2 objects being the outcomes of a continuation equilibrium, and subject to the unregulated agents' private optimality condition for borrowing (19). The planner's problem in primal form is thus given by

$$\max_{\{B_{1^*}^h, B_{1^*}^i\}} W^h \cdot V_{E_0}^{h, B_{1^*}^h} \cdot \{B_{1^*}^h\} + W^i \cdot V_{E_0}^{i, B_{1^*}^i} \cdot \{B_{1^*}^i\} \quad (22)$$

subject to

$$1 = E_0 \left[\frac{1}{\{B_{1^*}^h\} \cdot \{B_{1^*}^i\}} \right]$$

where the functions $V_{E_0}^{h, B_{1^*}^h}$ and $V_{E_0}^{i, B_{1^*}^i}$ are defined in (17) and W is a relative Pareto weight on unregulated

agents. When $\chi = 1$, the planner's objective becomes utilitarian, with the weight on agents' welfare being equal to their share in the population. When, in contrast, $\chi \neq 1$, the planner puts a relatively larger weight on regulated agents.

Before formally characterizing the optimal policy, let us stress how the planner's problem differs from typical planning problems in environments with pecuniary externalities. As is standard, the planner effectively controls b_1 by means of the tax on borrowing. However, here the choice of b_1 is up to unregulated agents. As a result, the planner is constrained to choosing a pair (b_1, c_1) from within the equilibrium best response function of unregulated agents, as depicted by the solid line in Figure 4.⁷ Absent this constraint, the solution to the planning problem would be the constrained-efficient allocation, depicted as the point (b_1^*, c_1^*) in Figure 4.

The planner's optimal choice of b_1 is characterized by the following generalized Euler equation (GEE):⁸

$$1 = E_0 \frac{1}{\gamma_1} \left(V_1^{\chi} E_0 \left(\frac{\chi W}{1 - W} \right)^{\chi} \frac{3\lambda_1^{\chi}}{3 - \lambda_1} \sum_{c=1}^C \tilde{V}_c^{\chi} \left(\frac{1}{\gamma_c} \right)^{\chi} \frac{1}{\gamma_c} \frac{3\lambda_1^{\chi}}{3 - \lambda_1} \right) \quad (23)$$

where λ_1 and λ_2 are the shadow costs associated with the credit constraints at date 1, and $3\lambda_1^{\chi}$ is given by (21).

This GEE equates the marginal costs from reducing borrowing and consumption today with the marginal benefits of having lower debt tomorrow. It resembles the private Euler equation (7), but contains additional terms reflecting the planner's internalization of pecuniary externalities and spillovers to unregulated agents. The left-hand side is the same and corresponds to the marginal utility cost of decreasing consumption by one unit. Given linear utility, this term is a constant equal to one. The first term on the right-hand side of (23) corresponds to the private marginal utility benefit from borrowing one less unit and raising consumption tomorrow, also present in (7). The second term in (21) constitutes the pecuniary externality: the planner internalizes that a lower level of debt leads to an increase in the price of nontradable goods at date 1 and a relaxation of collateral constraints for both regulated and unregulated agents at that date, where the benefit of relaxing the collateral constraint is an average of the Lagrange multipliers, weighted by the relevant Pareto weights.

The key element in this GEE that differs from models with perfect financial regulation enforcement is that it embeds the unregulated agents' response to the borrowing choice of the planner for the regulated agents, through the negative partial derivative $\frac{\partial m}{\partial b_1}$ present in the price derivative term $3\lambda_1^{\chi}$.⁹ This leakage effect lowers the marginal value of saving on behalf of regulated agents for the planner and therefore reduces the wedge between the planner's and private agent's perceived

⁷Naturally, the unregulated equilibrium belongs to the set of feasible (b_1, c_1) pairs.

⁸We assume in what follows that the second-order conditions are satisfied. We verified numerically that this is the case.

⁹Absent leakages, the expression would be the one in Bianchi (2011). Similar expressions arise in models with aggregate demand externalities, in which savings affect the output gap.

benefit of having lower debt. The final term of the GEE reflects the planner's marginal value of the wealth redistribution induced by a higher price of nontradables.¹⁰

The discrepancy between the planner's GEE (23) and the regulated agent's private Euler equation for debt (7) calls for a Pigouvian tax on debt, as in the existing literature, with the tax given by

$$g^* = \frac{V_{E_0}^h \left(\frac{\partial W}{\partial \tau} \right)^* \left(\frac{d\tau^*}{d\tau} \right)^i}{E_0 \left(\frac{\partial W}{\partial \tau} \right)^i} \int_{c=1}^h V_c^h \left(X \frac{\partial V_c}{\partial \tau} - \frac{\partial V_c}{\partial \tau} \right)^* \left(\frac{d\tau_c^*}{d\tau} \right)^i \quad (24)$$

The optimal tax captures the uninternalized marginal costs of borrowing from the GEE (23). The expression suggests an ambiguous effect of leakages on the magnitude of the tax. Two key forces work in opposite directions. First, the leakage term in (21) reduces the magnitude of the price derivative term $\frac{\partial V_c^*}{\partial \tau} \cdot \frac{\partial V_c}{\partial \tau}$ calling for a lower tax—a pure *leakage effect*. Second, leakages lead to higher average shadow values of relaxing the collateral constraint, calling for a higher tax on the regulated sphere and reflecting an indirect *squeezing effect*. In other words, the macroprudential policy's reduced effectiveness speaks in favor of a lower tax, while larger benefits from financial stabilization speak in favor of a higher tax, resulting in an ambiguous total effect. Similarly, the relationship between the size of the unregulated sector W and the optimal tax is a priori ambiguous. Through numerical explorations, we have found that the leakage effect appears to dominate, resulting in an optimal tax that is decreasing in W .

Planner's objective and Pareto improvements While Proposition 3 indicates that sufficiently small (positive) taxes necessarily lead to Pareto improvements relative to the laissez-faire, it may be that for some values of X (possibly including $X = 1$), the optimal intervention characterized by (23)-(24) results in a welfare loss for a subset of agents. The shape of indifference curves in Figure 4 suggests that unregulated agents are always made better off by an intervention, but that may not be the case for regulated agents if the optimal tax exceeds a certain threshold (i.e., if the pair (τ^*, τ^0) picked by the planner is located to the right of the contract curve represented in bold). Setting $X = 0$ would guarantee that both agents are made better off by the intervention.¹¹

2.6 Endogenous fraction of unregulated agents

So far, our analysis of macroprudential policy with leakages has taken the size of the unregulated sector as exogenously given. In this section, we present a simple way to endogenize this variable with a model of free entry into the unregulated sphere and show that the resulting framework can be

¹⁰A higher price of nontradables redistributes wealth from the net buyer of nontradables to the net sellers of nontradables.

¹¹In the quantitative analysis of our infinite horizon model in Section 3, all agents are made better off by the intervention, both in an unconditional sense and conditional on being in a Sudden Stop (see Figure 9), despite an assumed Pareto weight of $X = 1$.

used to map any \mathbb{W} to a distribution of abilities to circumvent regulation without getting caught in the population.

We extend our baseline model by adding a regulation circumvention decision to the private agents' date 0 problem. When avoiding regulation, agents run the risk of getting caught with an idiosyncratic probability i , which is randomly distributed over the population and realized before the decision to circumvent is made. When caught, agents must pay a non-pecuniary penalty associated with a utility cost of λ .

We assume that private agents and the planner move simultaneously. Based on the tax choice by the government, agents choose whether they want to join the unregulated sector and attempt to avoid taxes, or remain in the regulated sector. Likewise, the government chooses the optimal tax, taking as given the decisions of agents to bypass regulation or not.

The debt tax households anticipate is critical to their circumvention decision. Given a conjectured tax g^2 to be set by the government, an agent joins the unregulated sphere when

$$u^1(g^2) - i\lambda > u^0(g^2)$$

and remains in the regulated sphere otherwise.

The equilibrium outcome is a fixed point. For a tax conjecture, households choose whether they want to be unregulated. In turn, given the implied size of the unregulated sector, the government chooses the tax on debt. Formally, an equilibrium of this extended model is a set of consumption, savings and consumption decisions by private agents and a regulation decision by the planner such that (1) given prices and the tax, agents' decisions are optimal, (2) given the \mathbb{W} implied by private agents' circumvention's choices, the government chooses the tax optimally, and (3) all markets clear.

Since the debt tax is rebated lump-sum to regulated agents, the optimal consumption allocation of a regulated agent is always available to an unregulated agent. It follows that $u^1(g^2) > u^0(g^2)$, with i when $g^2 < 0$. Therefore, any value of \mathbb{W} can be rationalized in the endogenous \mathbb{W} model by a distribution function for the probability of getting caught featuring $\mathbb{1}^0 = \mathbb{W}$ for t satisfying

$$u^1(g^1, \mathbb{W}^0) - t\lambda = u^0(g^1, \mathbb{W}^0)$$

where g^1, \mathbb{W}^0 denotes the optimal tax in the exogenous \mathbb{W} model, as characterized in (24).¹²

For instance, in the case of an exponential distribution for the probability of getting caught $\mathbb{1}^0 =$

¹²An interesting aspect that we leave for future research is the role of the timing of the regulation decision. In the case of sequential decisions with the planner moving first, the planner would solve

$$\max_{\tau^1, \tau^0} \int_0^1 i^1 g^0 \left[u^1(\tau^1, X^1) - i^1 g^0 \right] h \left(\tau^1, \mathbb{E}_{0^+} \tau^1 \right) \mathbb{1}^0 \left(\tau^1, \tau^0 \right) d\mathbb{W}$$

1 4^{-j} , the rate parameter required to rationalize a given value of W would be given by

$$\lambda = \frac{-\ln(1 - W^0)}{g \cdot 1W^0} \quad 0''$$

This mapping from W to λ is illustrated numerically in Figure 5, where it is apparent that a larger size of the unregulated sphere can be supported by a smaller mean probability of getting caught when circumventing the regulation.¹³

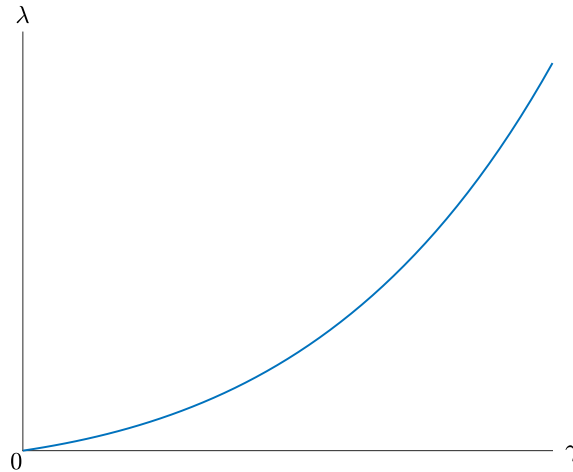


Figure 5: Rate parameter λ of exponential distribution for idiosyncratic probability of getting caught $i^0 = 1 - 4^{-j}$ necessary to rationalize given level of W when share of unregulated agents is endogenous.

2.7 Insights from three-period model

This section presented a highly stylized model of imperfectly enforced macroprudential policy, where the inherent motivation for a tax on borrowing derived from a pecuniary externality caused by financial constraints linked to a market price. The key prediction of the model is that in response to a tax on borrowing for the regulated sphere, borrowing by the unregulated sphere increases. The main normative insight is that this leakage phenomenon exerts two counteracting forces on the magnitude of the optimal macroprudential tax on borrowing. On the one hand, a leakage effect makes the macroprudential tax on the regulated sphere less effective because the reduction in the regulated sphere's indebtedness is partially offset by an increase in borrowing by the unregulated sphere. On the other hand, the leakages make the macroprudential tax introduce a new distortion that takes the form of an even more excessive indebtedness of the unregulated sphere. Correcting this distortion requires

where i^0 is the threshold satisfying $i^0 = g^2$ and $i^0 = g^2$ and $F(\cdot)$ denotes the cumulative density function of i . The optimal tax when the government moves first would feature an extra term related to the sensitivity of the cumulative density function $F(\cdot)$. The economy with an exogenous W would behave similarly to an economy with an endogenous W featuring a steep density function at $\lambda = W$.

¹³The mean probability of getting caught implied by the exponential distribution is equal to $1 - \lambda^{-1}$.

reducing the economy's indebtedness further and therefore calls, paradoxically, for even tighter borrowing restrictions on the regulated sphere, a *squeezing* effect.

3 Quantitative Model

In this section, we embed the leakage phenomenon into a canonical quantitative model of financial crisis. The goal is to assess the extent to which leakages limit the ability of macroprudential regulation to reduce the exposure to financial crisis and to study how leakages affect the optimal policy design. From a theoretical perspective, an additional element emerges in the infinite horizon model. Given the forward-looking nature of the unregulated agents' problem, these agents' borrowing decision depends not only on current regulation but also on their expectation of future regulation. As a result, a new time inconsistency problem emerges that would not be present with perfect enforcement of regulation.

3.1 Preferences and constraints in the infinite horizon model

As in the three-period model, there are two types of agents with identical preferences and endowments, who only differ on whether they are subject to borrowing taxes. Preferences are given by

$$E_0 \sum_{\ell=0}^{\infty} \beta^{\ell} D^{\ell} z_{\ell}^{\circ} \quad (25)$$

where D^{ℓ} is a standard concave, twice continuously differentiable function that satisfies the Inada condition. The consumption basket z is an Armington-type CES aggregator with elasticity of substitution $\sigma > 1$ between tradable goods z^T and nontradable goods $z^{\#}$, given by

$$z = \left[\lambda z^T + (1-\lambda) z^{\#} \right]^{\frac{1}{\sigma}}$$

In each period ℓ , agents receive endowments of tradable goods \tilde{z}_{ℓ}^T and nontradable goods $\tilde{z}_{\ell}^{\#}$ and choose a one-period non-state-contingent bond denominated in units of tradables. The vector of endowments $y_{\ell} = (\tilde{z}_{\ell}^T, \tilde{z}_{\ell}^{\#})'$ follows a first-order Markov process. The agents' budget constraints and credit constraints are given by

$$\frac{1}{1+r_{\ell}} + g_{\ell} = z_{\ell}^T + \tau_{\ell}^{\#} z_{\ell}^{\#} = 1 + \tau_{\ell}^{\#} \tilde{z}_{\ell}^{\#} \quad (26)$$

and

$$1 + r_{\ell} \leq \tau_{\ell}^{\#} \tilde{z}_{\ell}^{\#} \quad (27)$$

As in the three-period model, we assume that $g_{\ell} = 0$ for all $\ell \geq 0$. Given a tax policy $\tau_{\ell}^{\#}$ and initial levels of debt $1 + r_0$, a competitive equilibrium is defined as a stochastic sequence of prices $\tau_{\ell}^{\#}$ and households' policies $(z_{\ell}^T, z_{\ell}^{\#}, 1 + r_{\ell})$ such that (i) households

maximize (25) subject to sequences of budget constraints (26) and credit constraints (27), (ii) the market clears for nontradable goods ($W_2^* \cdot 1 - W_2^* = \tilde{c}^{\#}$), and (iii) the government budget constraint holds ($\frac{1}{1+g} = \frac{1}{1+r}$).¹⁴ As discussed in Section 2.6, the value for W could be endogenized by assuming a heterogeneous costs of avoiding regulation.¹⁵

3.2 Optimal time-consistent regulated equilibrium

As in Section 2.5, we consider the problem of a planner choosing the tax policy that delivers the highest welfare in the regulated equilibrium.

We assume that the planner makes decisions sequentially and without commitment, and we study Markov-perfect equilibria. Focusing on a discretionary regime is useful for our purpose given our goal of studying how leakages can undermine the effectiveness of regulation. We let $x_t = (b_t, c_t^{\#}, g_t)$ denote the aggregate state vector of the economy, B_t denote the policy rule for regulated bond holdings of future planners that the current planner takes as given, and $(C_t^{\#}, c_t^{\#}, p_t^{\#})$ denote the associated recursive functions returning unregulated agents' bond holdings, consumption allocations, and the price of nontradables under this policy rule.¹⁶

The forward-looking nature of unregulated agents' borrowing decisions introduces a time consistency problem. To see why, consider the Euler equation of unregulated agents when it holds with equality:

$$D_t \cdot 2_t^{\#} = V'(E D_t \cdot C_t^{\#}) \quad (28)$$

As was the case of constraint (19) in the three-period model, this constraint is a key implementability constraint for the government in the infinite horizon model. It captures the spillover effects from the planner's debt choice for $'$ agents onto $*$ agents' debt choice. However, in the infinite horizon context where the planner regulates the economy in every period, this implementability constraint depends on next period's regulatory policy. For example, a policy that induces low consumption of $*$ agents tomorrow (through a low tax on $'$ agents in ℓ_{t+1}) will indirectly push down these agents' consumption today and moderate the overborrowing externality in the present. Moreover, if unregulated agents expect loose regulation in the future, they have incentives to accumulate more precautionary savings today and borrow less. Tomorrow, however, the planner acting without commitment will not internalize the benefits of such a loose tax policy over previous periods. The time-inconsistency problem of macroprudential policy that we highlight is distinct from the one in Bianchi and Mendoza (2018). In that paper, a time-inconsistency problem arises because of the presence of asset prices, a forward-looking object, in collateral constraints. Here instead, it is due to imperfect regulation

¹⁴The definition of equilibrium in recursive form is given in Appendix C.

¹⁵If the decision to bypass regulation is made once and for all at $\ell = 0$, it is straightforward to compute the cost that would endogenize the value of W .

¹⁶Notice that by a form of block recursivity, once the policy for regulated bond holdings is chosen, the rest of the equilibrium objects can be obtained from the implementability constraints.

enforcement.¹⁷

Following again the primal approach adopted in the three-period model context (see Section 2.5), assuming a utilitarian objective, the optimal time-consistent (TC) planner's problem can be described by the following Bellman equation:

say here we do utilitarian...

$$V^{1-\theta} = \max_{\{c_t^*, \tau_t^*, \lambda_t^*\}} \sum_{t=0}^{\infty} \beta^t \lambda_t^* \left[\frac{1}{1-\theta} \log(c_t^*) - \lambda_t^* \right] \quad (\text{Optimal TC})$$

subject to

$$\lambda_t^* = \beta \lambda_{t+1}^* \quad \text{for } t \geq 0$$

$$\lambda_t^* = \beta \lambda_{t+1}^* \quad \text{for } t \geq 0$$

$$c_t^* = \frac{\lambda_{t+1}^*}{\lambda_t^*} \quad \text{for } t \geq 0$$

$$\lambda_t^* = \frac{1}{1-\theta} \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \right]^{1-\theta}$$

$$D_t \lambda_t^* = V'(E D_t) C_t^{1-\theta} \cdot C_t^{\theta-1}$$

$$\lambda_t^* = 0$$

A **Markov equilibrium** is defined by policy functions $\{B^{1-\theta}, B^*, C_t^{1-\theta}, C_t^*, C_t^{\theta-1}, C_t^{\theta-1}\}$, a value function $V^{1-\theta}$, and a pricing function $P^{1-\theta}$ such that the value function and policy functions solve (Optimal TC) given perceived policies $\{B_t^{1-\theta}, C_t^{1-\theta}, C_t^{\theta-1}\}$ for $t \geq 0$ and $P^{1-\theta}$.

3.3 Calibration

The calibration follows Bianchi (2011). The time period is one year. A first subset of parameters is set independently using standard values from the literature: $\beta = 0.94$, $\theta = 1.1$, $\lambda_0 = 0.83$, and the endowment process is estimated based on the HP-filtered component of tradable and nontradable GDP for Argentina. Assuming a first-order bivariate autoregressive process: $\ln y_t = d \ln y_{t-1} + \gamma_t$, where $\gamma_t = \gamma_c \cdot \gamma_c^0 + \gamma_n$, we obtain the estimates

$$d = \begin{pmatrix} 0.901 & 0.453 \\ 0.495 & 0.225 \end{pmatrix} \quad \gamma = \begin{pmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{pmatrix}$$

The second subset of parameters $\{V, I, \lambda\}$ is set to match Argentina's average net foreign asset position, the share of nontradable output in Argentina, and the average frequency of financial crises for

¹⁷Other contributions studying optimal policy problems with Markov perfect equilibria include Klein, Quadrini and Rios-Rull (2005), Klein, Krusell and Rios-Rull (2008), and Debortoli, Nunes and Yared (2017).

emerging markets.¹⁸ This yields $V = 0.91 \cdot l = 0.31 \cdot \lambda = 0.32$.

Finally, we solve our regulated equilibrium for different values of \mathbb{W} . For the most part, we focus on values of \mathbb{W} ranging from 0 to 0.5. A value of 0.5 entails a substantial amount of leakages by which 50% of the economy can evade regulation. Moreover, a value of $\mathbb{W} = 0.5$ is the value at which the losses from a dispersion in consumption across agents are given the highest weight by the planner.

Table 1: Calibration

	Value	Source/Target
Interest rate	$A = 0.04$	Standard value
Risk aversion	$\mathcal{F} = 2$	Standard value
Elasticity of substitution	$1 + \frac{1}{\sigma} = 0.83$	Conservative value
Weight on tradables in CES	$l = 0.31$	Share of tradable output=32%
Discount factor	$V = 0.91$	Average NFA-GDP ratio = 29%
Credit coefficient	$\lambda = 0.32$	Frequency of crises = 5.5%
Size of unregulated sector	$\mathbb{W} = 0.05$	Baseline range

3.4 Numerical solution

The computation of the optimal regulated equilibrium follows a nested fixed point algorithm, common to those used in studies of Markov perfect equilibria (e.g. Bianchi and Mendoza, 2018). For a given conjectured policy followed by governments in the future, we solve for the current optimal policy using value function iteration. Using this solution, we update our conjectured policy. We iterate until the optimal policy coincides with the conjectured policy to obtain a Markov-perfect equilibrium. Details are provided in Appendix D.

3.5 Overborrowing and leakages

We start our quantitative analysis by looking at how the distribution of debt of regulated and unregulated agents differs across regimes. To show this, we conduct a 10,000-period simulation for the unregulated equilibrium, the constrained-efficient allocation and the regulated equilibrium for $\mathbb{W} = 0.5$ and provide a scatterplot of the bond positions.

Panels (a) and (b) of Figure 6 correspond to the scatterplots of bond positions in the unregulated equilibrium and constrained-efficient allocation. Vertical and horizontal lines show sample averages. Because there is no distinction between $*$ and $'$ agents in these economies, all the points line up on the 45-degree line. One can see, as expected, that the unregulated equilibrium displays simulations with higher levels of debt, which in turn are associated with a larger frequency and severity of financial crises.

¹⁸In both the model and the data, financial crises are defined as episodes in which the current account increases by more than one standard deviation.

(a) Unregulated equilibrium

(b) Constrained efficient

(c) Macroprudential policy with leakages ($\lambda = 0.5$)

Figure 6: Scatterplots of debt positions in unregulated equilibrium, constrained-efficient allocation, and macroprudential policy with leakages.

Panel (c) of Figure 6 displays the case of the regulated equilibrium with $\lambda = 0.5$. In this case, most pairs of bond positions are located below the 45-degree line, indicating higher levels of debt for unregulated agents than for regulated agents. Interestingly, many simulation periods display levels of debt for unregulated agents that are much higher than the maximum values of debt observed in the unregulated equilibrium. This is the leakage effect at play: regulation worsens the overborrowing problem for unregulated agents. Conversely, regulated agents' borrowing in the regulated equilibrium with leakages is lower on average than in the constrained-efficient equilibrium. This suggests that the planner (at least partially) compensates for the unregulated agents' extra borrowing by commanding less borrowing for regulated agents.

3.6 Frequency and severity of crises

Next, we study the extent to which leakages undermine the effectiveness of regulation at reducing the vulnerability to financial crises. We define financial crises as episodes in which the current account increases by more than one standard deviation (or, equivalently, credit falls by more than one standard deviation). Based on this definition, we study how the probability of crises varies in regulated equilibria associated with a range of values for λ and compare the severity of crises in a regulated equilibrium where $\lambda = 0.5$ to the severity in the constrained-efficient allocation and unregulated equilibrium.

Figure 7 shows how the frequency of financial crises changes with the size of the unregulated sphere. In the absence of leakages (i.e., with $\lambda = 0$), the frequency of crises is about 0.5%, which is about 1/10th of the frequency of crises in the unregulated equilibrium. As expected, the frequency of crises increases with λ . Quite strikingly, however, the increase is very modest even for values of λ as large as 0.5. This suggests that as λ increases, the planner adjusts its desired borrowing for regulated agents to offset the effect of leakages and achieve a given level of financial stability.

To study how leakages alter the government's ability to reduce the severity of financial crises using macroprudential regulation, we construct a comparable event analysis following a procedure similar

Figure 7: Long-run frequency of financial crises as a function of

to Bianchi and Mendoza (2018). First, we simulate the decentralized equilibrium for a large number of periods, identify all the financial crisis episodes, and construct 11-year window events centered on the financial crisis episodes. Second, we take the average of key variables across the window period for the decentralized equilibrium. Third, we feed in the initial state and shock sequence that characterizes all financial crises in the unregulated equilibrium to the policy functions of the regulated equilibrium. We do this for two degrees of leakages $\lambda = 0$, which corresponds to the constrained-efficient allocation, and $\lambda = 0.5$.¹⁹ Finally, we average the key variables across the window period for the regulated equilibria. This experiment allows us to do a counterfactual analysis that highlights how leakages lead to different financial crises dynamics, controlling for the same sequence of shocks and the same initial states.

Figure 8 shows the results of these simulations. In the top panels, we plot the income shocks, the current account to GDP ratio, and the real exchange rate. In the bottom panels, we show the debt of regulated agents, the debt of unregulated agents, and the optimal tax. All the plotted paths correspond to averages across all the simulation samples from the event analysis. The unregulated equilibrium (solid line) clearly displays a larger decline in credit and a larger current account reversal, as well as a larger collapse in the real exchange rate (defined as the inverse of the price of the composite good). The crises are preceded by increases in the amount of credit and negative income shocks, and are triggered on impact by income shocks that are about 1.5 standard deviations on average. In contrast, the constrained-efficient equilibrium (dashed line) displays a much smaller decline in credit (1% percent versus 10% for the unregulated equilibrium) and a much smaller decline in the real exchange rate. Note that these differences in the event dynamics emerge despite the two economies having the same initial conditions and being subject to the same shock sequence.

¹⁹In some sense, the economy with optimal macroprudential policy and $\lambda = 0$ is still constrained-efficient, but we reserve this term for the economy without leakages.

(a) Income

(b) Current account to GDP (%)

(c) Real exchange rate

(d) Debt of agents

(e) Debt of agents

(f) Optimal tax (%)

Figure 8: Event analysis.

In terms of aggregate variables, one can see that the regulated equilibrium (macroprudential policy with leakages, dash-dotted line) is much closer to the constrained-efficient equilibrium than to the unregulated equilibrium. This is consistent with the message from Figure 7 that the frequency of crises increases only modestly with leakages, that is, overall, neither the frequency nor the severity of financial crises increases substantially with leakages, even when as much as 50% of the economy is 100% unregulated. However, these aggregate results hide important disparities between the debt dynamics of the regulated and unregulated spheres across the event window. Both debt positions start at exactly the same level (by construction), but unregulated agents start accumulating debt very rapidly (panel e), while regulated agents reduce their indebtedness at a significantly faster pace than in the constrained-efficient equilibrium. Higher taxes than in the constrained-efficient case (panel f), together with stronger precautionary motives due to the spillback effects from unregulated agents' overborrowing, generates this sharp deleveraging by regulated agents in the run-up to the crisis event.

3.7 Welfare effects

Finally, we study the welfare implications of macroprudential policy with leakages, focusing on two key questions: By how much does average welfare fall because of the presence of leakages? How are the welfare benefits of macroprudential policy distributed across regulated and unregulated agents?

Figure 9 displays measures of the welfare effects of macroprudential policy. In the 100% panel, we

Figure 9: Welfare gains from macroprudential policy in the presence of leakages.

Note: Welfare gains are computed in consumption equivalence terms and expressed in percentages.

report the unconditional welfare gains of moving from the unregulated equilibrium to the regulated equilibrium for different degrees of leakages. We report welfare gains by agent type, as well as average welfare gains.²⁰ For comparison, we also show the welfare gains of moving from the unregulated equilibrium to the constrained-efficient allocation, which are of course the same for regulated and unregulated agents. A first observation is that the average welfare gains of being in the regulated equilibrium decrease with the size of the unregulated sector. This finding is natural, since with larger λ the planner directly controls a smaller share of the economy and thus becomes less effective at correcting the overborrowing externality. However, it is also apparent that the decline in the average welfare gains associated with leakages is modest. This suggests that macroprudential policy remains not only effective but also desirable, even with significant leakages.

This figure also reveals interesting insights about the distribution of these welfare effects across the two spheres. First, welfare gains are higher for unregulated agents than for regulated agents. For small values of λ the welfare gains of macroprudential policy are about twice as large for unregulated agents. The intuition is straightforward: unregulated agents enjoy the same financial stability benefits of macroprudential regulation as regulated agents, but unlike the latter, they do not bear the costs that arise from lower consumption ahead of (potential) future crises. Second, welfare gains for regulated agents fall sharply as λ gets larger. Intuitively, larger leakages imply that more unregulated agents overborrow and therefore impose a larger externality on regulated agents, who bear a more concentrated cost regulation.

The right panel of Figure 9 complements this analysis by representing the welfare gains of macro-

²⁰To compute unconditional gains, we first compute, for every possible state $(\tilde{c}, \tilde{b}, \tilde{w})$, the proportional increase in consumption across all possible future histories that would make households indifferent between remaining in the unregulated equilibrium and switching to the regulated equilibrium with leakages. Then, we compute the mean of this variable in the simulations.

prudential policy with leakages (for $\alpha = 0.5$), but through the event windows of Section 3.6 rather than unconditionally. The results are broadly consistent with the unconditional analysis, with unregulated agents capturing the lion's share of the gains. In addition, it is apparent that the increase in the welfare gains of macroprudential policy in the run-up to a crisis event falls disproportionately on unregulated agents. This strengthens the conclusion that these agents become the main beneficiaries of macroprudential policy when leakages are large.

4 Conclusion

This paper conducted an investigation of macroprudential policies with limited regulation enforcement. We characterize the optimal policy for different degrees of enforcement and examine the extent to which leakages undermine the effectiveness of capital flow management. Our results show that the presence of leakages does not necessarily call for weaker intervention, as commonly argued in policy discussions. Instead, our framework suggests that a stronger intervention may well be needed. Quantitative results show that, thanks to a larger intervention, macroprudential policy remains highly effective at reducing the vulnerability to financial crises.

Even though we conduct our analysis in a small open economy model featuring excessive external borrowing, we think that our insights regarding the two-way interaction between the regulated and unregulated spheres of the economy and the trade-offs that emerge for optimal regulation are likely to apply to a general class of models in which either financial or nominal frictions generate excessive risk taking from a social point of view. Studying the interaction of these frictions and exploring alternative policy responses is an interesting agenda for future research.

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A Formal characterization of continuation equilibria

Lemma 2. For $G \in \{D_1, D_2, D_3, D_4\}$ aggregate date 1 consumption in region G is given by $\{c_1^G = U_{c_1}^G, c_2^G, U_{c_2}^G\}$.

Proof. See Appendix B.2.

Lemma 2 says that, within each region, date 1 aggregate consumption is linear in each of the aggregate state variables $\{c_1^G, c_2^G, U_{c_2}^G\}$. Date 2 aggregate consumption follows from the economy's intertemporal resource constraint (15) and is therefore also linear in $\{c_1^G, c_2^G, U_{c_2}^G\}$. Finally, according to (16), the equilibrium prices $\{p_1^G, p_2^G\}$ are linear in $\{c_1^G, c_2^G, U_{c_2}^G\}$ (respectively) and therefore also linear in $\{c_1^G, c_2^G, U_{c_2}^G\}$. The next lemma characterizes the aggregate consumption solution in more detail.

Lemma 3. The coefficients of the decision rule for c_1^G are such that:

1. $U_{c_1}^G \geq 0$ and $U_{c_1}^G \leq U_{c_2}^G = 0$ with $U_{c_1}^G = 0$ (resp. $U_{c_2}^G = 0$) if $W = 0$ (resp. $W = 1$), and $U_{c_2}^G = 0$ if $G = D_2$
2. if $0 < W < 0.5$ (resp. $0.5 < W < 1$), then $U_{c_1}^{D_1} > U_{c_1}^{D_2} > U_{c_1}^{D_3} > U_{c_1}^{D_4}$, (resp. $U_{c_1}^{D_1} > U_{c_1}^{D_2} > U_{c_1}^{D_3} > U_{c_1}^{D_4}$), with strict inequalities if $0 < W < 0.5$ (resp. $0.5 < W < 1$).
3. $U_{c_2}^{D_1} > U_{c_2}^{D_2} > U_{c_2}^{D_3}$ and $U_{c_2}^{D_1} > U_{c_2}^{D_2} > U_{c_2}^{D_4}$, with strict inequalities if $W_i < 0$.
4. $U_{c_2}^{D_1} > U_{c_2}^{D_2} > U_{c_2}^{D_3}$ and $U_{c_2}^{D_1} > U_{c_2}^{D_2} > U_{c_2}^{D_4}$, with strict inequalities if $W_i > 1$.

Proof. See Appendix B.3.

Part 1. of Lemma 3 establishes that aggregate consumption is increasing in each of the three aggregate state variables $\{c_1^G, c_2^G, U_{c_2}^G\}$, always strictly for c_1^G , and strictly for c_2^G unless $W = 0$ and for $U_{c_2}^G$ unless $W = 1$. Part 2. of the lemma says that aggregate tradable consumption is more sensitive to tradable income in the regions where the credit constraints are binding. Similarly, parts 3. and 4. establish that aggregate tradable consumption is more sensitive to the two sets of agents' wealth positions in the regions where the credit constraints are binding.

We define an individual's credit constraint set as the set of tradable endowment realizations such that her credit constraint is binding:

$$Q_1^G = \{c_1^G, c_2^G, U_{c_2}^G\} \in R^3 : \int_{G^D} p_1^G c_1^G + p_2^G c_2^G = \int_{G^D} p_1^G \tilde{c}_1^G + p_2^G \tilde{c}_2^G$$

where $G \in \{D_1, D_2, D_3, D_4\}$ notes the region in which the economy is and determines the mapping between $\{c_1^G, c_2^G, U_{c_2}^G\}$ and $\{\tilde{c}_1^G, \tilde{c}_2^G, U_{c_2}^G\}$ relevant to compute $\int_{G^D} p_1^G c_1^G + p_2^G c_2^G$ and $\int_{G^D} p_1^G \tilde{c}_1^G + p_2^G \tilde{c}_2^G$. The four regions can

hence be represented by the following sets:

$$X^{22i} = \{ \omega \in \Omega \mid \omega \in Q^1 \text{ and } \omega \in 22 \setminus Q^1 \}; \quad (A.1)$$

$$X^{D2i} = \{ \omega \in \Omega \mid \omega \in Q^{21} \text{ and } \omega \in D2 \setminus Q^1 \}; \quad (A.2)$$

$$X^{2Di} = \{ \omega \in \Omega \mid \omega \in Q^1 \text{ and } \omega \in 2D \setminus Q^{21} \}; \quad (A.3)$$

$$X^{DDi} = \{ \omega \in \Omega \mid \omega \in Q^{21} \text{ and } \omega \in DD \setminus Q^{21} \}; \quad (A.4)$$

Further, we define unions of some of these sets as $X^{22} = X^{22i}, X^{2\phi} = X^{22} \cup X^{D2}, X^{D\phi} = X^{D2} \cup X^{D2i}, X^{DD} = X^{DDi} \cup X^{D\phi}$ and $X^{\phi D} = X^{DDi} \cup X^{2D}$. These sets have some intuitive properties, summarized in the following lemmas.

Lemma 4. There are thresholds 0_G and 1_G satisfying $0 < 0_G < 1_G$ (with $0_G = 1_G$ if $\alpha_1 = \beta_1$) such that $\omega \in X^{22i}, \omega \in X^{D2i}, \omega \in X^{DDi}, \omega \in X^{2Di}, \omega \in X^{D\phi}$ and $\omega \in X^{\phi D}$ if and only if $\omega \in X^{22i}, \omega \in X^{D2i}, \omega \in X^{DDi}, \omega \in X^{2Di}, \omega \in X^{D\phi}$ and $\omega \in X^{\phi D}$.

Proof. See Appendix B.4.

Lemma 4 says that for a given pair (α_1, β_1) , the regions are ordered along the real line, that the poorest type of agents is never unconstrained when the other type is constrained, and that when both types of agents have the same wealth only the symmetric regions X^{22} and X^{DD} can arise. It notably implies that X^{22}, X^{D2}, X^{2D} and X^{DD} are disjoint, and that their union is \mathbb{R} , meaning that for any triplet $(\alpha_1, \beta_1, \omega)$ the economy is always in one and only one region. Finally, the next lemma offers comparative statics results.

Lemma 5. For a given α_1 (resp. β_1) and any two $\beta_1 \succ \beta_1'$ (resp. $\alpha_1 \succ \alpha_1'$) such that $\beta_1 \succ \beta_1'$ (resp. $\alpha_1 \succ \alpha_1'$):

1. for $X = X^{22} \cup X^{2\phi} \cup X^{\phi 2}$, if $\omega \in X^1$ for (α_1, β_1) (resp. $\omega \in X^1$ for (α_1', β_1')), then $\omega \in X^1$ for (α_1, β_1') (resp. $\omega \in X^1$ for (α_1', β_1)).
2. for $X = X^{DD} \cup X^{D\phi} \cup X^{\phi D}$, if $\omega \in X^1$ for (α_1, β_1) , then $\omega \in X^1$ for (α_1, β_1') (resp. $\omega \in X^1$ for (α_1', β_1)).

Proof. See Appendix B.5.

Part 1. of Lemma 5 says that the region X^{22} where both types of agents are credit constrained, and the regions $X^{2\phi}$ and $X^{\phi 2}$ where at least one type of agents is constrained are all shrinking if $\beta_1 \succ \beta_1'$. Part 2. says that the region X^{DD} where both types of agents are unconstrained, and the regions $X^{D\phi}$ and $X^{\phi D}$ where at least one type of agents is unconstrained are all expanding if $\beta_1 \succ \beta_1'$.

A.1 Price and consumption functions

The price functions are related to the aggregate tradable consumption function, given in Lemma 2, by

$$p_1^{\#}(\omega) = \frac{1}{\#} \frac{c_1^{\#}(\omega)}{c_1^{\#}(\omega)}$$

and

$$?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 = \frac{1 \mid 0^1 \mathbf{1}, A^0 \mathbf{1}, A^0 \mathbf{W} * \mathbf{1}, \mathbf{1} \mathbf{1} \mathbf{W} \cdot \mathbf{1} \frac{1}{4}, \sim_1^1 \mathbf{B}^{\bullet}, \frac{1}{1, A}}{\sim^{\#}} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0,$$

'e remaining consumption functions are related to these price functions but the relationships depend on which agents are constrained and unconstrained.

When * agents are unconstrained (i.e., in regions D1 and D2), their consumption functions are given by

$$\begin{aligned} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \sim_2^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 = \\ &= \frac{1}{1, V} \mathbf{1}, A^0 * \mathbf{1}, \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}, \frac{\sim_2^1, ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}}{1, A} \cdot \end{aligned}$$

and

$$\begin{aligned} ?_C^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \frac{1 \mid}{1, V ?_C^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0} \\ &= \mathbf{1}, A^0 * \mathbf{1}, \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}, \frac{\sim_2^1, ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}}{1, A} \cdot \end{aligned}$$

for C= 1•2. In contrast, when these agents are constrained (i.e., in regions D1 and D2), their consumption functions are given by

$$\begin{aligned} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \mathbf{1} \mid \mathbf{1}, A^0 * \mathbf{1}, \mathbf{1}, A^0 \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#} \cdot \\ \sim_2^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \mathbf{1} \mid \sim_2^1, ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#} \wedge \mathbf{1}, A^0 \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#} \cdot \\ ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \mathbf{1} \mid 0 \frac{\mathbf{1}, A^0 * \mathbf{1}, \mathbf{1}, A^0 \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}}{?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0} \cdot \\ ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \mathbf{1} \mid 0 \frac{\sim_2^1, ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#} \wedge \mathbf{1}, A^0 \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}}{?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0} \cdot \end{aligned}$$

Similarly, when' agents are unconstrained (i.e., in regions D1 and D2), their consumption functions are given by

$$\begin{aligned} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 &= \sim_2^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 = \\ &= \frac{1}{1, V} \mathbf{1}, A^0 \cdot \mathbf{1}, \sim_1^1 \mathbf{B}^{\bullet}, ?_1^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}, \frac{\sim_2^1, ?_2^{\#} \sim_1^1 \mathbf{B}^{\bullet} * \mathbf{1}^{\bullet} \cdot \mathbf{1}^0 \sim^{\#}}{1, A} \cdot \end{aligned}$$

and

$$c_1^{\#} = \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} = \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} \cdot \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}$$

for $C=1 \cdot 2$. In contrast, when these agents are constrained (i.e., in region 2), their consumption functions are given by

$$\begin{aligned} c_1^{\#} &= \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} \\ c_2^{\#} &= \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} \\ c_1^{\#} &= \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} \\ c_2^{\#} &= \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} \end{aligned}$$

B Proofs

B.1 Proof of Lemma 1

Note that the private Euler equation (18) is given by $g_1^{D4} = 0$, where

$$g_1^{D4} = \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}} - \frac{1}{1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}}$$

g_1^{D4} is continuous, satisfies $g_1^{D4} > 0$ for some ϵ , $\lim_{\epsilon \rightarrow 0} g_1^{D4} = 1$, and $g_1^{D4} < 0$ for the 1 range over which consumption is positive. It follows that there exists a single ϵ for which $g_1^{D4} = 0$.

B.2 Proof of Lemma 2

We consider each region in turn.

22 In this case equilibrium is given by the system (12), (13), (14), (15) and (16). This system is block recursive in a linear system in $c_1^{\#}$ and $c_2^{\#}$. Solving this linear system yields the following coefficients for $c_1^{\#}$: $U_1^2 = 1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}$, $U_1^2 = W(1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}})$, $U_1^2 = 1 + \beta \frac{1}{1 + \beta \frac{1}{1 + \beta \dots}}$ and $U_2^2 = 0$.

It can be easily verified that

1. $1_{\beta_2}^{D=21} \beta_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot 22^0 \dot{Y} 1_{\beta_2}^{2>=1} \beta_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot 22^0$ for $\beta = \ast \cdot \prime$ is equivalent to $\sim_1^1 \beta^0 \dot{Y} O_G$,
2. $1_{\beta_2}^{D=21} \beta_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot DD^0 \quad 1_{\beta_2}^{2>=1} \beta_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot DD^0$ for $\beta = \ast \cdot \prime$ is equivalent to $\sim_1^1 \beta^0 1_G$,
3. $1_{\ast_2}^{D=21} \ast_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot 2D^0 \dot{Y} 1_{\ast_2}^{2>=1} \ast_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot 2D^0$ and $1_{\prime_2}^{D=21} \prime_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot 2D^0$ is equivalent to $O_G \quad \sim_1^1 \beta^0 \dot{Y} 1_G$ iff $\ast_1 \dot{Y} \prime_1$, and
4. $1_{\ast_2}^{D=21} \ast_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot D2^0 \quad 1_{\ast_2}^{2>=1} \ast_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot D2^0$ and $1_{\prime_2}^{D=21} \prime_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot D2^0 \dot{Y} 1_{\prime_2}^{2>=1} \prime_1; \sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1 \cdot D2^0$ is equivalent to $O_G \quad \sim_1^1 \beta^0 \dot{Y} 1_G$ iff $\ast_1 j \prime_1$.

B.5 Proof of Lemma 5

The proof simply follows from the fact that O_G and 1_G are non-increasing in \ast_1 and \prime_1 (see expressions in proof of Lemma 4).

B.6 Proof of Proposition 1

We start by defining

$$\ast_1 \ast_1 \cdot \prime_1^0 = 1 \quad \# \quad \frac{1}{\sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1^0} \cdot \quad \prime_1 \ast_1 \cdot \prime_1; g^0 = \frac{1}{1 \cdot g} \quad \# \quad \frac{1}{\sim_1^1 \beta^0 \cdot \ast_1 \cdot \prime_1^0} \quad \# \quad (B.1)$$

and note that the unregulated agents' Euler equation (19) can be written as $\ast_1 \ast_1 \cdot \prime_1^0 = 0$, while the regulated agents' Euler equation (20) can be written as $\prime_1 \ast_1 \cdot \prime_1; g^0 = 0$.

Let us consider the equilibrium response of \ast agents. We note that we can write \ast as

$$\ast_1 \ast_1 \cdot \prime_1^0 = 1 \quad \underset{\sim}{=} \quad \frac{1}{\sim_1^1} \frac{O_G}{\ast_1} \frac{1}{3} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{O_G}{=} \quad \frac{1}{\sim_1^1} \frac{1_G}{\ast_1} \frac{1}{3} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{1_G}{=} \quad \frac{1}{\sim_1^1} \frac{1}{\ast_1} \frac{1}{3} \frac{1}{\sim_1^1} \beta^{00}$$

where the arguments of \sim_1^1 , O_G and 1_G are omitted in the interest of space. According to the implicit function theorem we have $3 \ast_1 \cdot 3 \prime_1 = \frac{m \ast \cdot m \prime_1}{m \ast \cdot m \ast_1}$, with

$$\frac{m \ast}{m \prime_1} = \underset{\sim}{=} \frac{1}{\sim_1^1} \frac{O_G}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{O_G}{=} \frac{1}{\sim_1^1} \frac{1_G}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{1_G}{=} \frac{1}{\sim_1^1} \frac{1}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00}$$

$$\frac{m \ast}{m \ast_1} = \underset{\sim}{=} \frac{1}{\sim_1^1} \frac{O_G}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{O_G}{=} \frac{1}{\sim_1^1} \frac{1_G}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00} \quad \underset{1_G}{=} \frac{1}{\sim_1^1} \frac{1}{\ast_1} \frac{1}{m} \frac{1}{\sim_1^1} \beta^{00}$$

and

$$j \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1} = \frac{[22] \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1}}{1}}{[22] \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1}}{1}} \quad (\text{B.3})$$

For any value of W , the numerator in (B.2) is smaller than the one in (B.3), and the denominator in (B.2) is larger than the one in (B.3). It follows that $j \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1} > j \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$ and therefore $q_{*1}^{D^4 \cdot 0^0} > j \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$.

Furthermore, we have

$$\frac{m q \cdot 1 \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{m q \cdot 1} = \frac{m \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{m \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}} = \frac{1 \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{1 \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}} > 0$$

since $m \cdot \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1} > 0$.

In the $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$ space, the curve $q_{*1}^{D^4 \cdot 0^0}$ crosses the curve $q_{*1}^{D^4 \cdot 0^0}$ from above at $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1} = \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$, and it shifts to the right as g' rises. Hence, in the neighborhood of $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$, a rise in g' results in a downward movement of $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$ along the downward sloping $q_{*1}^{D^4 \cdot 0^0}$ curve. It follows that for a small g' , $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$ is decreasing in g' and $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$ is increasing in g' .

The result on debt capacity follows from the sign of the derivative $\frac{\partial \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{\partial g'}$. Evaluated at $\frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1} = \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}$, this derivative is given by

$$\begin{aligned} \frac{\partial \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{\partial g'} &= \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} q_{*1}^{D^4 \cdot 0^0} \\ &= \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{[22] \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V}}{[22] \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1}} > 0 \end{aligned}$$

in region 22, and by

$$\begin{aligned} \frac{\partial \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{\partial g'} &= \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} q_{*1}^{D^4 \cdot 0^0} \\ &= \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} A^0}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{[22] \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V}}{[22] \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1 \wedge \frac{1}{1} \frac{1}{1}} \frac{1}{1} [DD] \frac{1}{1} \frac{1}{V} \frac{1}{1} \frac{1}{1} \frac{W^0 \frac{1}{1} \frac{1}{1} \frac{1}{1}}{1}} > 0 \end{aligned}$$

in region DD. Since 22 and DD are the only two relevant regions when debt choices are symmetric, it follows that $\frac{\partial \frac{D^4 \cdot 0^0 \cdot m \cdot \frac{1}{4} \cdot 1}{m q \cdot 1}}{\partial g'} > 0$.

B.8 Proof of Proposition 3

The proof relies on the result that starting from the unregulated equilibrium, a movement along the best response of unregulated agents associated with a small tax g' leads to welfare gains for agents of

both types. To establish this result, we start by observing that

$$u_{\ell}^1 = \max_{\{c_{\ell}^1\}_{c=1}^{\infty}} \sum_{c=1}^{\infty} \beta^c \ln \left(\frac{1}{1 + A^c} \right)$$

subject to

$$\sum_{c=1}^{\infty} \frac{1}{1 + A^c} = 1$$

and

$$u_{\ell}^0 = \sum_{c=1}^{\infty} \beta^c \ln \left(\frac{1}{1 + A^c} \right)$$

Therefore, using the Envelope theorem, we have

$$\frac{\partial u_{\ell}^1}{\partial m} = \sum_{c=1}^{\infty} \beta^c \frac{1}{1 + A^c} \frac{\partial A^c}{\partial m} \quad (B.4)$$

and

$$\frac{\partial u_{\ell}^0}{\partial m} = \sum_{c=1}^{\infty} \beta^c \frac{1}{1 + A^c} \frac{\partial A^c}{\partial m} \quad (B.5)$$

for $\ell < \ell$.

Next, the date 0 welfare of an agent of type ℓ associated with aggregate debt choices $\{c_{\ell}^0\}_{\ell=1}^{\infty}$ can be written as

$$u_{\ell}^0 = \sum_{c=1}^{\infty} \beta^c \ln \left(\frac{1}{1 + A^c} \right)$$

Thus, the variation in welfare of an agent of type ℓ caused by imposing a small tax on regulated agents is proportional to

$$\frac{\partial u_{\ell}^0}{\partial m} = \sum_{c=1}^{\infty} \beta^c \frac{1}{1 + A^c} \frac{\partial A^c}{\partial m}$$

For unregulated agents, this variation is given by

$$\frac{\partial u_{\ell}^0}{\partial m} = \sum_{c=1}^{\infty} \beta^c \frac{1}{1 + A^c} \frac{\partial A^c}{\partial m}$$

Using (B.4) and (B.5) and evaluating the expression at the unregulated equilibrium, the variation reduces to

$$\frac{\partial u_{\ell}^0}{\partial m} = \sum_{c=1}^{\infty} \beta^c \frac{1}{1 + A^c} \frac{\partial A^c}{\partial m}$$

Under the premise that borrowing constraints bind with positive probability in the unregulated equilibrium, the term in round brackets is positive in some states of nature (and zero in the others). Furthermore, Proposition 2 established that $\frac{\partial u_{\ell}^0}{\partial m} > 0$. It follows that $\frac{\partial u_{\ell}^0}{\partial m} > 0$.

Definition of unregulated Equilibrium. A recursive unregulated equilibrium is defined by a pricing function $\hat{p}^{\#1-\circ}$, perceived laws of motions $\hat{g}^{1-\circ}$ for $\theta \in \{f^*, g\}$, and decision rules $\hat{v}^1 1_{\theta^{\circ-\circ}}$, $\hat{v}^2 1_{\theta^{\circ-\circ}}$, $\hat{v}^{\#1} 1_{\theta^{\circ-\circ}}$ with associated value function $v^1 1_{\theta^{\circ-\circ}}$ such that:

1. Agents' optimization: $\hat{v}^1 1_{\theta^{\circ-\circ}}$, $\hat{v}^2 1_{\theta^{\circ-\circ}}$, $\hat{v}^{\#1} 1_{\theta^{\circ-\circ}}$ and $v^1 1_{\theta^{\circ-\circ}}$ solve the agent's θ recursive optimization problem for $\theta \in \{f^*, g\}$, taking as given $\hat{p}^{\#1-\circ}$ and $\hat{g}^{1-\circ}$ for $\theta = f^*, g$.
2. Consistency: the perceived laws of motion for aggregate bonds are consistent with the actual laws of motion: $\hat{g}^{1-\circ} = \hat{g}^1 \hat{g}^{\circ-\circ}$ for $\theta = f^*, g$.
3. Market clearing:

$$W^{\#1} \hat{p}^{\#1-\circ} \hat{g}^{1-\circ} = W^{\circ} \hat{p}^{\#1-\circ} \hat{g}^{\circ-\circ}$$

and

$$W^{\#1} \hat{p}^{\#1-\circ} \hat{g}^{1-\circ} \hat{g}^{\circ-\circ} = W^{\circ} \hat{p}^{\#1-\circ} \hat{g}^{\circ-\circ} \hat{g}^{\circ-\circ}$$

D Numerical solution

To solve for the regulated equilibrium when $W \neq 0$, we adopt a nested fixed point algorithm similar to those used in studies of Markov perfect equilibria (e.g., [Bianchi and Mendoza, 2018](#), [Klein et al., 2005](#), [Klein et al., 2008](#)). Given future policies, we solve for policy functions and value functions using value function iteration in an inner loop. In the outer loop, we update future policies with the solution of the Bellman equation from the inner loop. The algorithm follows these steps:

1. Generate a discrete grid for the bond position of regulated agents $\hat{b} = \{b_1, b_2, \dots, b_n\}$, the bond position of unregulated agents $\hat{b}^* = \{b_1^*, b_2^*, \dots, b_{n^*}^*\}$ and the shocks $\hat{\epsilon} = \{\epsilon_1, \epsilon_2, \dots, \epsilon_{\#}\}$ and choose an interpolation scheme for evaluating the functions outside the bond grids. We use 100 points for each bond grid and interpolate using piecewise linear approximation.
2. Guess policy functions U^0, P at step $t = 1, 2, \dots, T$ and $t = 1, 2, \dots, T^*$. We use as initial policies the policies of the unregulated equilibrium.
3. For given U^0, P , solve for the value function and policy functions associated with the following Bellman equation:

$$v^1 1_{\theta^{\circ-\circ}} = \max_{\hat{g}^1, \hat{g}^{\circ-\circ}} \left[W^{\#1} \hat{p}^{\#1-\circ} \hat{g}^{1-\circ} \hat{g}^{\circ-\circ} + V E v^1 1_{\theta^{\circ-\circ}} \right] \quad (\text{B.8})$$

subject to

$$1^0 = 1 \quad A^0 1 \quad 2^0 \quad ?^{\#} 1 \quad 2^{\#0} \quad (B.9)$$

$$1^0_* = 1 \quad A^0 1^*_* \quad 2^0_* \quad ?^{\#} 1 \quad 2^{\#0}_* \quad (B.10)$$

$$?^{\#} = \frac{1}{I} \frac{W 2^0_* \quad 1 \quad W^0 2^0_*}{\sim^{\#}} \quad (B.11)$$

$$2^{\#} = \frac{2^0_*}{W 2^0_* \quad 1 \quad W^0 2^0_*} \sim^{\#} \quad (B.12)$$

$$2^{\#}_* = \frac{2^0_*}{W 2^0_* \quad 1 \quad W^0 2^0_*} \sim^{\#} \quad (B.13)$$

$$1^0 \quad \wedge 1 P 1 \quad \sim^{\#} \quad \sim^0 \quad (B.14)$$

$$1^0_* \quad \wedge 1 P 1 \quad \sim^{\#} \quad \sim^0 \quad (B.15)$$

$$\cdot = D \quad 1 2^0_* \cdot 2^{\#0} \quad V 1 \quad A^0 E U^0_* 1 \quad \sim^0 \quad (B.16)$$

$$\cdot \gg 1^0_* \quad \wedge 1 P 1 \quad \sim^{\#} \quad \sim^0 \quad = 0 \quad (B.17)$$

$$\cdot \quad 0'' \quad (B.18)$$

The Bellman equation is solved using value function iteration. In each state, the maximization is performed using discrete search for 1^0 on the \cdot grid. For a given 1^0 , the value of all remaining variables (including 1^0_*), and therefore of the objective, follow from (B.9)-(B.18).

- Denote by f^U and f^P the policy functions that solve the recursive problem in step 3. Compute the sup distance between U^* and f^U , as well as between P and f^P . If the sup distance is higher than $1.0e-5$, update U^* and P and go back to step 2.