Overborrowing, Financial Crises and 'Macro-prudential' Policy *

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Abstract

An equilibrium model of financial crises driven by Irving Fisher's financial amplification mechanism features a pecuniary externality, because private agents do not internalize how the price of assets used for collateral respond to collective borrowing decisions, particularly when binding collateral constraints cause asset fire-sales and lead to a financial crisis. As a result, agents in the competitive equilibrium borrow "too much" ex ante, compared with a financial regulator who internalizes the externality. Quantitative analysis calibrated to U.S. data shows that average debt and leverage are only slightly larger in the competitive equilibrium, but the incidence and magnitude of financial crises are much larger. Excess asset returns, Sharpe ratios and the price of risk are also much larger, and the distribution of returns displays endogenous fat tails. State-contingent taxes on debt and dividends of about 1 and -0.5 percent on average respectively support the regulator's allocations as a competitive equilibrium.

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1 Introduction

A common argument in narratives of the causes of the 2008 global financial crisis is that economic agents "borrowed too much." The notion of "overborrowing," however, is often vaguely defined or presented as a value judgment, in light of the obvious fact that a prolonged credit boom ended in collapse. This lack of clarity makes it difficult to answer two key questions: First, is overborrowing a significant macroeconomic problem, in terms of causing financial crises and driving macro dynamics during both ordinary business cycles and crises episodes? Second, are the so-called "macro-prudential" policies that are being widely adopted in response to the crisis effective to contain overborrowing and reduce financial fragility, and if so what ought to be their main quantitative characteristics?

In this paper, we propose answers to these questions based on the quantitative predictions of a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We adopt an explicit definition of overborrowing and use quantitative methods to determine how much overborrowing the model predicts and how it affects business cycles, financial crises, and social welfare. We also show that there are state-contingent schedules of taxes on debt and dividends that can eliminate the overborrowing problem, and provide an analysis of their quantitative features.

Our definition of overborrowing is in line with the one used elsewhere in the literature (e.g. Lorenzoni, 2008, Korinek, 2009, Bianchi, 2011): The difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner, or financial regulator, who faces similar frictions but internalizes the general-equilibrium effects of its borrowing decisions. In the model, the credit friction is in the form of a collateral constraint on debt and working capital financing that has two important features. First, it introduces an externality that drives a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by the regulator. Second, when the constraint binds, it triggers Irving Fisher's classic debt-deflation financial amplification mechanism, which causes a financial crisis via a nonlinear feedback loop between asset fire sales and borrowing ability.

The model's collateral constraint limits private agents not to borrow more than a fraction of the market value of assets they can offer as collateral, which take the form of an asset in fixed aggregate supply. Private agents take the price of this asset as given, and hence a "systemic credit externality" arises, because they do not internalize that, when the collateral constraint binds, fire sales of assets cause a Fisherian debt-deflation spiral that causes asset prices to decline and the economy's borrowing ability to shrink. Moreover, when the constraint binds, production plans are also affected, because working capital financing is needed in order to pay for a fraction of labor costs, and working capital loans are also subject to the collateral constraint. As a result, output falls when the credit constraint binds, because of a sudden increase in the effective cost of labor. This affects dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the credit externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions.

The model is similar to the DSGE models with occassionally binding collateral constraints examined by Mendoza and Smith (2006) and Mendoza (2010). These studies showed that the cyclical dynamics of a competitive equilibrium with those constraints lead to periods of expansion in which leverage ratios raise enough so that the constraint becomes binding in response to shocks of standard magnitudes, triggering a Fisherian deflation that causes sharp declines in credit, asset prices, and macroeconomic aggregates. In this paper, we conduct instead a normative study that focuses on comparing the competitive equilibrium with the allocations attained by a financial regulator subject to the same collateral constraint but internalizing the credit externality, and on how the regulator's optimal plans can be implemented as a competitive equilibrium using policy tools.

We conduct a quantitative analysis in a version of the model calibrated to U.S. data. The results show that financial crises in the competitive equilibrium are significantly more frequent and more severe than in the equilibrium attained by the regulator. The incidence of financial crises is about three times larger. Asset prices drop about 25 percent in a typical crisis in the decentralized equilibrium, versus 5 percent in the regulator's equilibrium. Output drops about 50 percent more, because the fall in asset prices reduces access to working capital financing. The more severe asset price collapses also generate an endogenous "fat tail" in the distribution of asset returns in the decentralized equilibrium, which causes the price of risk to rise 1.5 times and excess returns to rise by 5 times, in both tranquil times and crisis times. The regulator can replicate exactly its equilibrium allocations as a decentralized equilibrium, and thus neutralize the credit externality, by imposing state-contingent taxes on debt and dividends of about 1 and -0.5 percent on average respectively.

This paper contributes to the recent literature in the intersection of Macroeconomics and Finance by developing a quantitative framework suitable for the normative analysis of overborrowing and macro-prudential policy. This task is challenging, because an accurate characterization of the macro implications of financial frictions requires using non-linear global methods to evaluate correctly the short- and long-run effects of these frictions in models with incomplete asset markets and subject to aggregate shocks. Studying the short-run effects is important for determining whether the model provides a reasonable approximation to the non-linear macroeconomic features of actual financial crises, and thus whether it is a useful laboratory for policy analysis. Studying the long-run mechanisms is equally important, because the prudential aspect of macro-prudential policy works by introducing policy changes that seek to alter the incentives for precautionary behavior in "good times," when credit and leverage are building up.

The recent Macro/Finance literature, including this article, follows in the vein of the classic studies on fire sales and financial accelerators (e.g. Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Shleifer and Vishny (2011), Jermann and Quadrini (2012)). In particular, we study a pecuniary externality similar to those examined in the work of Caballero and Krishnamurthy (2001), Lorenzoni (2008), Stein (2012), and Korinek (2009), which arises because private agents do not internalize the amplification effects caused by financial constraints that depend on market prices.

This externality was introduced into quantitative studies of macro-prudential policy by Bianchi (2011) and Benigno, Chen, Otrok, Rebucci, and Young (2011) using a model of emerging markets crises proposed by Mendoza (2002). In this model, private agents do not internalize the effect of their individual debt plans on the market price of nontradable goods relative to tradables, which influences their ability to borrow because debt is limited to a fraction of income valued in units of tradables. Bianchi examined how this externality leads to excessive debt accumulation and showed that a debt tax can restore constrained efficiency and reduce the vulnerability to financial crises. Benigno et al. studied how the effects of the externality are reduced when the planner has access to instruments that can affect directly labor allocations during crises.[1]

Our analysis differs from the above studies in that we focus on asset prices as a key factor driving debt dynamics and the credit externality, instead of the relative price of nontradables. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Moreover, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent when using a credit constraint linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. In addition, our model differs because it introduces working capital financing subject to the collateral constraint, which allows the credit externality to distort production and dividend rates, and thus again asset prices.

More recently, the quantitative studies by Nikolov (2009) and Jeanne and Korinek (2010) examine models of macro-prudential policy in which assets serve as collateral.[2] Nikolov found that simple rules that impose tighter collateral requirements may not be welfare-improving in a setup in which consumption is a linear function unaffected by precautionary savings. In contrast, in our model precautionary savings are critical determinants of optimal consumption and borrowing decisions, because of the strong non-linear amplification effects caused by the Fisherian debt-deflation dynamics, and for the same reason we find that macro-prudential taxes are welfare improving. Jeanne and Korinek construct estimates of a Pigouvian debt tax in a model in which output follows an exogenous Markov-switching process and individual credit is limited to the sum of a fraction of aggregate, rather than individual, asset holdings plus a constant term. In their calibration, this second term dominates and the probability of crises matches the exogenous probability of a low-output regime, and as result the tax cannot alter the frequency of crises and has small effects on their magnitude.[3] In

[1]In a related paper Benigno et al. (2012) found that intervening during financial crisis by subsidizing nontradable goods leads to large welfare gains.

[2]Hanson, Kashyap, and Stein (2011) and Galati and Moessner (2010) review the growing literature on the macroprudential approach to financial regulation.

[3]They also examined the existence of deterministic cycles in a non-stochastic version of the model.

contrast, in our model the probability of crises and their output dynamics are endogenous, and macro-prudential policy reduces sharply the incidence and magnitude of crises.

Our analysis is also related to other recent studies exploring alternative theories of inefficient borrowing and their policy implications. For instance, Schmitt-Grohé and Uribe (2012) and Farhi and Werning (2012) examine the use of prudential capital controls as a tool for smoothing aggregate demand in the presence of nominal rigidities and a fixed exchange rate regime. In earlier work, Uribe (2006) examined an environment in which agents do not internalize an aggregate borrowing limit and yet borrowing decisions are the same as in an environment in which the borrowing limit is internalized.[4] Our analysis differs in that the regulator internalizes not only the borrowing limit but also the price effects that arise from borrowing decisions. Still, our results showing small differences in average debt ratios across competitive and regulated equilibria are in line with his findings.

The literature on participation constraints in credit markets initiated by Kehoe and Levine (1993) is also related to our work, because it examines the role of inefficiencies that result from endogenous borrowing limits. In particular, Jeske (2006) showed that if there is discrimination against foreign creditors, private agents have a stronger incentive to default than a planner who internalizes the effects of borrowing decisions on the domestic interest rate, which affects the tightness of the participation constraint. Wright (2006) then showed that as a consequence of this externality, subsidies on capital flows restore constrained efficiency.

The rest of the paper is organized as follows: Section 2 presents the analytical framework. Section 3 analyzes the financial regulator's problem. Section 4 presents the quantitative analysis. Section 5 provides conclusions.

2 Competitive Equilibrium

We follow Mendoza (2010) in specifying the economic environment in terms of firm-household units who make production and consumption decisions. Preferences are given

[4]He provided analytical results for a canonical endowment economy model with a credit constraint where there is an exact equivalence between the two sets of allocations. In addition, he examined a model in which this exact equivalence does not hold, but still overborrowing is negligible.

by:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t - G(n_t))\right] \tag{1}$$

In this expression, $E(\cdot)$ is the expectations operator, β is the subjective discount factor, n_t is labor supply and c_t is consumption. The period utility function $u(\cdot)$ is assumed to have the constant-relative-risk-aversion (CRRA) form. The argument of $u(\cdot)$ is the composite commodity $c_t - G(n_t)$ defined by Greenwood, Hercowitz, and Huffman (1988). $G(n)$ is a convex, strictly increasing and continuously differentiable function that measures the disutility of labor supply. This formulation of preferences removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only.

Each household can combine physical assets and labor services purchased from other households to produce final goods using a production technology such that $y = \varepsilon_t F(k_t, h_t)$, where F is a decreasing-returns-to-scale production function, k_t represents individual asset holdings, h_t represents labor demand and ε_t is a productivity shock, which has compact support and follows a finite-state, stationary Markov process. Individual profits from are therefore given by $\varepsilon_t F(k_t, h_t) - w_t h_t$.

The budget constraint faced by the representative firm-household is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + w_t n_t + [\varepsilon_t F(k_t, h_t) - w_t h_t] \tag{2}$$

where b_t denotes holdings of one-period, non-state-contingent discount bonds at the beginning of date t, q_t is the market price of capital, R_t is the real interest rate, and w_t is the wage rate.

The interest rate is assumed to be exogenous. This is equivalent to assuming that the economy is a price-taker in world credit markets, as in other studies of the U.S. financial crisis like those of Boz and Mendoza (2010), Corbae and Quintin (2009) and Howitt (2011), or alternatively it implies that the model can be interpreted as a partial-equilibrium model of the household sector. This assumption is adopted for simplicity, but is also in line with evidence indicating that the observed decline in the U.S. risk-free rate in the era of financial globalization has been driven largely by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. Warnock and Warnock (2009) provide econometric evidence of the significant downward pressure exerted by foreign capital inflows on U.S. T-bill rates since the mid 1980s. Mendoza and Quadrini (2010) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents. From this perspective, assuming a constant R is conservative, as in reality the pre-crisis boom years were characterized by a *falling* real interest rate, which would strengthen our results.[5] Still, we study later how our quantitative results vary if we relax this assumption and consider instead an exogenous inverse supply-of-funds curve, which allows the real interest rate to increase as debt rises.

Household-firms are subject to a working capital constraint. In particular, they are required to borrow a fraction θ of the wages bill $w_t h_t$ at the beginning of the period and repay it at the end of the period. In the conventional working capital setup, a cash-in-advance-like motive for holding funds to pay for inputs implies that the wages bill carries a financing cost determined by R. In contrast, here we simply assume that working capital funds are within-period loans. Hence, the interest rate on working capital is zero, as in some recent studies on the business cycle implications of working capital and credit frictions (e.g. Chen and Song (2009)). We follow this approach so as to show that the effects of working capital in our analysis hinge only on the need to provide collateral for working capital loans, as explained below, and not on the effect of interest rate fluctuations on effective labor costs as in typical business cycle models with working capital (e.g. Uribe and Yue, 2006).[6]

As in Mendoza (2010), agents face a collateral constraint that limits total debt, including both intertemporal debt and atemporal working capital loans, not to exceed a fraction κ of

[5]This suggests that a complete model of the pre-crisis U.S. credit boom is not a closed-economy model, but an open-economy model with mechanisms leading the rest of the world to run a large current account surplus vis-a-vis the United States (see, for example, Mendoza and Quadrini (2010)).

[6]We could also change to the standard setup, but in our calibration, $\theta = 0.14$ and $R = 1.028$, and hence working capital loans would add 0.4 percent to the cost of labor implying that our findings would remain largely unchanged.

the market value of asset holdings (i.e. κ imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta w_t h_t \leq \kappa q_t k_{t+1} \tag{3}$$

Following Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999), we interpret this constraint as resulting from an environment where limited enforcement prevents lenders to collect more than a fraction κ of the value of a defaulting debtor's assets, but we abstract from modeling the contractual relationship explicitly.

2.1 Private Optimality Conditions

In the competitive equilibrium, agents maximize (1) subject to (2) and (3) taking asset prices and wages as given. This maximization problem yields the following optimality conditions for each date t:

$$w_t = G'(n_t) \tag{4}$$

$$\varepsilon_t F_h(k_t, h_t) = w_t \left[1 + \theta \mu_t / u'(t)\right] \tag{5}$$

$$u'(t) = \beta R E_t \left[u'(t+1)\right] + \mu_t \tag{6}$$

$$q_t(u'(t) - \mu_t \kappa) = \beta E_t \left[u'(t+1)\left(\varepsilon_{t+1} F_k(k_{t+1}, h_{t+1}) + q_{t+1}\right)\right] \tag{7}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint.

Condition (4) is the individual's labor supply condition, which equates the marginal disutility of labor with the wage rate. Condition (5) is the labor demand condition, which equates the marginal productivity of labor with the effective marginal cost of hiring labor. The latter includes the extra financing cost $\theta \mu_t / u'(t)$ in the states of nature in which the collateral constraint on working capital binds. The last two conditions are the Euler equations for bonds and physical assets respectively. When the collateral constraint binds, condition (6) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (7) equates the marginal cost of an extra unit of investment with its marginal gain. The marginal cost nets out from the marginal utility of foregone current consumption a fraction κ of the shadow value of the credit constraint, because the additional unit of asset holdings contributes to relax the borrowing limit.

Condition (7) yields the following forward solution for asset prices:

$$q_t = E_t\left[\sum_{j=0}^{\infty}\left(\prod_{i=0}^{j} m_{t+1+i}\right)d_{t+j+1}\right], \qquad m_{t+1+i} \equiv \frac{\beta u'(t+1+i)}{u'(t+i)-\mu_{t+i}\kappa} \quad , \quad d_t \equiv \varepsilon_t F_k(k_t, h_t) \quad (8)$$

Thus, we obtain what seems a standard asset pricing condition stating that, at equilibrium, the date-t price is equal to the expected present value of the future stream of dividends discounted using the stochastic discount factors m_{t+1+i}, for $i = 0, ..., \infty$. The key difference with the standard asset pricing condition, however, is that the discount factors are adjusted to account for the shadow value of relaxing the credit constraint by purchasing an extra unit of assets whenever the collateral constraint binds (at any date $t+i$ for $i = 0, ..., \infty$).

Combining (6), (7) and the definition of asset returns ($R^q_{t+1} \equiv \frac{d_{t+1}+q_{t+1}}{q_t}$), it follows that the expected excess return assets relative to bonds (i.e. the equity premium), $R^{ep}_t \equiv E_t(R^q_{t+1} - R)$, satisfies the following condition:

$$R^{ep}_t = \frac{\mu_t(1-\kappa)}{(u'(t)-\mu_t\kappa)E_t\left[m_{t+1}\right]} - \frac{cov_t(m_{t+1}, R^q_{t+1})}{E_t\left[m_{t+1}\right]}, \qquad (9)$$

where $cov_t(m_{t+1}, R^q_{t+1})$ is the date-t conditional covariance between m_{t+1} and R^q_{t+1}.

Following Mendoza (2010), we characterize the first term in the right-hand-side of (9) as the direct (first-order) effect of the collateral constraint on the equity premium, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell assets, depressing the current price.[7] There is also an indirect (second-order) effect given by the fact that $cov_t(m_{t+1}, R^q_{t+1})$ is likely to become more negative when there is a possibility of a binding credit constraint, because the collateral constraint makes it harder for agents to smooth consumption.

Given the definitions of the Sharpe ratio ($S_t \equiv \frac{R^{ep}_t}{\sigma_t(R^q_{t+1})}$) and the price of risk ($s_t \equiv$

[7]Notice that this effect vanishes when $\kappa = 1$, because when 100 percent of the value of asset holdings can be collateralized, the shadow value of relaxing the constraint by acquiring an extra unit of assets equals the shadow value of relaxing it by reducing the debt by one unit.

$\sigma_t(m_{t+1})/E_t m_{t+1})$, we can rewrite the expected excess return and the Sharpe ratio as:

$$R_t^{ep} = S_t \sigma_t(R_{t+1}^q), \qquad S_t = \frac{\mu_t(1-\kappa)}{(u'(t) - \mu_t \kappa) E_t\left[m_{t+1}\right] \sigma_t(R_{t+1}^q)} - \rho_t(R_{t+1}^q, m_{t+1}) s_t \tag{10}$$

where $\sigma_t(R_{t+1}^q)$ is the date-t conditional standard deviation of asset returns and $\rho_t(R_{t+1}^q, m_{t+1})$ is the conditional correlation between R_{t+1}^q and m_{t+1}. Thus, the collateral constraint has direct and indirect effects on the Sharpe ratio analogous to those it has on the equity premium. The indirect effect reduces to the usual expression in terms of the product of the price of risk and the correlation between asset returns and the stochastic discount factor. The direct effect is normalized by the variance of returns. These relationships will be useful later to study the quantitative effects of the credit externality on asset pricing.

Since $q_t E_t[R_{t+1}^q] \equiv E_t[d_{t+1} + q_{t+1}]$, we can rewrite the asset pricing condition in this way:

$$q_t = E_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} E_{t+i} R_{t+1+i}^q \right)^{-1} d_{t+j+1}, \tag{11}$$

Notice that (9) and (11) imply that a binding collateral constraint at date t implies an increase in expected excess returns and a drop in asset prices at t. Moreover, since expected returns exceed the risk free rate whenever the collateral constraint is expected to bind at any future date, asset prices at t are affected by collateral constraint not just when the constraints binds at t, but whenever it is expected to bind at any future date.

2.2 Recursive Competitive Equilibrium

The competitive equilibrium is defined by stochastic sequences of allocations $\{c_t, k_{t+1}, b_{t+1}, h_t, n_t\}_{t=0}^{\infty}$ and prices $\{q_t, w_t\}_{t=0}^{\infty}$ such that: (A) agents maximize utility (1) subject to the sequence of budget and credit constraints given by (2) and (3) for $t = 0, ..., \infty$, taking as given $\{q_t, w_t\}_{t=0}^{t=\infty}$; (B) the markets of goods, labor and assets clear at each date t. We assume that assets are in fixed supply $\bar{K}$, hence the market-clearing condition for assets is $k_t = \bar{K}$. [8] The market clearing condition in the goods and labor markets are $c_t + \frac{b_{t+1}}{R} = \varepsilon_t F(\bar{K}, n_t) + b_t$ and $h_t = n_t$ respectively.

We now characterize the competitive equilibrium in recursive form. The state variables for a particular individual's optimization problem at time t are the individual bond holdings (b), aggregate bond holdings (B), individual asset holdings (k), and the TFP realization (ε). Aggregate capital is not carried as a state variable because it is in fixed supply. Denoting by $\Gamma(B, \varepsilon)$ the agents' perceived law of motion of aggregate bonds and $q(B, \varepsilon)$ and $w(B, \varepsilon)$ the pricing functions for assets and labor respectively, the agents' recursive optimization problem is:

$$V(b, k, B, \varepsilon) = \max_{b', k', c, n, h} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} \left[V(b', k', B', \varepsilon') \right] \tag{12}$$

$$\begin{aligned} s.t. \quad q(B, \varepsilon)k' + c + \frac{b'}{R} &= q(B, \varepsilon)k + b + w(B, \varepsilon)n + [\varepsilon F(k, h) - w(B, \varepsilon)h] \\ B' &= \Gamma(B, \varepsilon) \\ -\frac{b'}{R} + \theta w(B, \varepsilon)h &\leq \kappa q(B, \varepsilon)k' \end{aligned}$$

The solution to this problem is characterized by the decision rules $\hat{b}'(b, k, B, \varepsilon)$, $\hat{k}'(b, k, B, \varepsilon)$, $\hat{c}(b, k, B, \varepsilon)$, $\hat{n}(b, k, B, \varepsilon)$ and $\hat{h}(b, k, B, \varepsilon)$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}'(B, \bar{K}, B, \varepsilon)$. In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

Definition 1 ***(Recursive Competitive Equilibrium)***
A recursive competitive equilibrium is defined by an asset pricing function $q(B, \varepsilon)$, a pricing function for labor $w(B, \varepsilon)$, a perceived law of motion for aggregate bond holdings $\Gamma(B, \varepsilon)$, and a set of decision rules $\left\{ \hat{b}'(b, k, B, \varepsilon), \hat{k}'(b, k, B, \varepsilon), \hat{c}(b, k, B, \varepsilon), \hat{n}(b, k, B, \varepsilon), \hat{h}(b, k, B, \varepsilon) \right\}$ with associated value function $V(b, k, B, \varepsilon)$ such that:

[8] This assumption is to preserve tractability in the quantitative comparison between the competitive equilibrium and the equilibrium with financial regulation. The credit externality would still be present if we allowed for capital accumulation, as long as the price of capital fluctuates. Mendoza (2010) conducts a positive analysis of the competitive equilibrium of a similar model with capital accumulation and capital adjustment costs.

1. $\left\{\hat{b}'(b,k,B,\varepsilon),\hat{k}'(b,k,B,\varepsilon),\hat{c}(b,k,B,\varepsilon),\hat{n}(b,k,B,\varepsilon),\hat{h}(b,k,B,\varepsilon)\right\}$ *and* $V(b,k,B,\varepsilon)$ *solve the agents' recursive optimization problem, taking as given* $q(B,\varepsilon)$, $w(B,\varepsilon)$ *and* $\Gamma(B,\varepsilon)$.

2. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion:* $\Gamma(B,\varepsilon)=\hat{b}'(B,\bar{K},B,\varepsilon)$.

3. *Wages satisfy* $w(B,\varepsilon) = G'(\hat{n}(B,\bar{K},B,\varepsilon))$ *and asset prices satisfy* $q(B,\varepsilon) = E_{\varepsilon'|\varepsilon}\left\{\frac{\beta u'\left(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right)\ \left[\varepsilon' F_k\left(\bar{K},\hat{n}(\Gamma(B,\varepsilon)\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right)+q(\Gamma(B,\varepsilon),\varepsilon')\right]}{u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right)-\kappa\max\left[0,u'\left(\hat{c}(B,\bar{K},B,\varepsilon)\right)-\beta R E_{\varepsilon'|\varepsilon}u'(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')\right]}\right\}$

4. *Goods, labor and asset markets clear:* $\frac{\hat{b}'(B,\bar{K},B,\varepsilon)}{R}+\hat{c}(B,\bar{K},B,\varepsilon)=\varepsilon F(\bar{K},\hat{n}(B,\bar{K},B,\varepsilon))+B$, $\hat{n}(B,\bar{K},B,\varepsilon)=\hat{h}(B,\bar{K},B,\varepsilon)$ *and* $\hat{k}(B,\bar{K},B,\varepsilon)=\bar{K}$

3 Financial Regulator's Equilibrium

3.1 Equilibrium without collateral constraint

We start the normative analysis by briefly comparing the competitive equilibrium with an efficient equilibrium in the absence of the collateral constraint (3). The allocations of this equilibrium can be represented as the solution to the following standard planning problem:

$$H(B,\varepsilon)=\max_{b',c,n} u(c-G(n))+\beta E_{\varepsilon'|\varepsilon}\left[H(B',\varepsilon')\right] \tag{13}$$
$$s.t. \qquad c+\frac{B'}{R}=\varepsilon F(\bar{K},n)+B$$

and subject also to either this problem's *natural debt limit*, which is defined by $B' > \frac{\varepsilon^{\min}F(\bar{K},n^*(\varepsilon^{\min}))}{R-1}$, where $\varepsilon^{\min}$ is the lowest possible realization of TFP and $n^*(\varepsilon^{\min})$ is the optimal labor allocation that solves $\varepsilon^{\min}F_n(\bar{K},n)=G'(n)$, or to a tighter ad-hoc time- and state-invariant borrowing limit.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (see, for example, Mendoza and Smith, 2006 and Mendoza, 2010) is to compare the allocations of the competitive equilibrium with the collateral constraint with those arising from the above benchmark case. The competitive equilibria with and without the collateral constraint differ in that in the former private agents borrow *less* (since the collateral constraint limits the amount they can borrow, and also because they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint), and there is financial amplification of the effects of the underlying exogenous shocks (since binding collateral constraints produce large recessions and drops in asset prices). Compared with the financial regulator's equilibrium we define next, however, we will show that the competitive equilibrium with collateral constraints displays overborrowing (i.e. agents borrow *more* than in the regulator's equilibrium).

3.2 Recursive Financial Regulator's Equilibrium

Consider a benevolent regulator who maximizes the agents' utility subject to the resource constraint, the collateral constraint and the same menu of assets and credit allocations that the competitive equilibrium supports. In particular, the regulator is constrained to face the same "borrowing ability" at every given state as agents in the decentralized equilibrium (i.e. the same set of values of $\kappa q(B,\varepsilon)\bar{K}$ determined by the competitive market price of collateral assets), but with the key difference that the regulator internalizes the effects of its borrowing decisions on the market prices of assets and labor.[9]

The assumption that the regulator values collateral using the pricing function of the competitive equilibrium is related to the concept of conditional or financial efficiency proposed by Kehoe and Levine (1993) and Lustig (2000). They define conditional efficiency in terms of Pareto efficient allocations that satisfy credit market participation constraints conditional on a given set of market prices of either Arrow-Debreu state-contingent claims (Kehoe and Levine, 1993) or Arrow securities traded sequentially (Lustig, 2000). In our setup, there are no allocations other than the regulator's which can both satisfy the collateral constraint, for a given competitive equilibrium pricing function, and produce higher welfare. In contrast with Kehoe and Levine (1993) and Lustig (2000), however, our regulator's allocations

[9] We could also allow the regulator to manipulate the borrowing ability state by state (i.e., by allowing it to alter $\kappa q(B,\varepsilon)\bar{K}$). Allowing for this possibility can potentially increase the welfare gains of macroprudential policy but the macroeconomic effects of the externality are similar. In addition, since asset prices are forward-looking, this would make the regulator's problem time-inconsistent, because allowing the regulator to commit to future actions would lead it to internalize not only how today's choice of debt affects tomorrow's asset prices but also how it affects asset prices and the tightness of collateral constraints in previous periods.

are not guaranteed to satisfy participation constraints, and hence these allocations are not conditionally efficient in the terms they defined.

Formulating the regulator's problem in this way has three advantages. First, despite the forward-looking nature of asset prices, it renders the regulator's optimization problem time-consistent, which guarantees that macro-prudential policy, if effective, improves welfare across all states and dates in a time-consistent fashion. Second, it allows for a simple decentralization of the regulator's allocations based on the use of Pigouvian taxes on debt and dividends, as we show later. Third, even though we do not solve for an optimal policy that implements conditionally-efficient allocations or a Ramsey plan in which the pricing function can be altered, our setup yields a policy that neutralizes fully the credit externality and leads to a sharp reduction in the probability and severity of financial crises. In fact, the regulator's allocations are close to those of an equilibrium in which assets are valued at a constant price for collateral, and thus there is no Fisherian deflation and no credit externality.

The recursive problem of the financial regulator is defined as follows:

$$W(B,\varepsilon) = \max_{B',c,n} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} W(B',\varepsilon') \tag{14}$$

$$s.t. \quad c + \frac{B'}{R} = \varepsilon F(\bar{K}, n) + B$$

$$-\frac{B'}{R} + \theta w(B,\varepsilon) n \leq \kappa q(B,\varepsilon)\bar{K}$$

where $q(B,\varepsilon)$ is the equilibrium pricing function obtained in the competitive equilibrium. Wages can be treated in a similar fashion, but it is easier to decentralize the regulator's allocations as competitive equilibrium if we assume that the it takes wages as given and wages need to satisfy $w(B,\varepsilon) = G'(n)$.[10] Under this assumption, we impose the optimality condition of labor supply as a condition that the regulator's equilibrium must satisfy, in addition to solving problem (14) for given wages.

Applying the envelope theorem to the first-order conditions of problem (14) and imposing

[10]This implies that the financial regulator does not internalize the direct effects of choosing the contemporaneous labor allocation on contemporaneous wages–only the effect of debt choices on future wages. We have also investigated the possibility of having the regulator internalize these effects but results are very similar. This occurs because our calibrated interest rate and working capital requirement are very small.

the labor supply optimality condition, we obtain the following optimality conditions for the regulator's problem:

$$u'(t) = \beta R E_t \left[u'(t+1) + \mu_{t+1}\psi_{t+1} \right] + \mu_t, \qquad \psi_{t+1} \equiv \kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}} \tag{15}$$

$$\varepsilon_t F_n(\bar{K}, n_t) = G'(n_t) \left[1 + \theta \mu_t / u'(t) \right] \tag{16}$$

The key difference between the competitive equilibrium and the regulator's equilibrium follows from examining the Euler equations for bond holdings in both problems. In particular, the term $\mu_{t+1}\psi_{t+1}$ in condition (15) represents the additional marginal benefit of savings considered by the regulator at date t, because it takes into account how an extra unit of bond holdings alters the tightness of the credit constraint through its effects on the prices of assets and labor at $t+1$. Note that, since $\frac{\partial q_{t+1}}{\partial b_{t+1}} > 0$ and $\frac{\partial w_{t+1}}{\partial b_{t+1}} \geq 0$, ψ_{t+1} is the difference of two opposing effects and hence its sign is in principle ambiguous. The term $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ is positive, because an increase in net worth increases demand for assets and assets are in fixed supply. The term $\frac{\partial w_{t+1}}{\partial b_{t+1}}$ is positive, because the effective cost of hiring labor increases when the collateral constraint binds, reducing labor demand and pushing wages down. We found, however, that the value of ψ_{t+1} is positive in all our quantitative experiments with baseline parameter values and variations around them, and this is because $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ is large and positive when the credit constraint binds due the effects of the Fisherian debt-deflation mechanism.

Definition 2 ***(Recursive Financial Regulator's Equilibrium)***
The recursive equilibrium of the financial regulator is given by a set of decision rules $\left\{ \hat{B}'(B,\varepsilon), \hat{c}(B,\varepsilon), \hat{n}(B,\varepsilon) \right\}$ *with associated value function* $W(B,\varepsilon)$*, and wages* $w(B,\varepsilon)$ *such that:*

1. $\left\{ \hat{B}'(B,\varepsilon), \hat{c}(B,\varepsilon), \hat{n}(B,\varepsilon) \right\}$ *and* $W(B,\varepsilon)$ *solve the regulator's recursive optimization problem, taking as given* $w(B,\varepsilon)$ *and the competitive equilibrium's asset pricing function* $q(B,\varepsilon)$*.*

2. *Wages satisfy* $w(B,\varepsilon) = G'(\hat{n}(B,\varepsilon))$*.*

3.3 Comparison of Equilibria & 'Macro-prudential' Policy

Using a simple variational argument, we can show that the allocations of the competitive equilibrium are suboptimal, in the sense that they violate the conditions that support the regulator's equilibrium. In particular, private agents undervalue net worth in periods during which the collateral constraint binds. To see this, consider first the marginal utility of an increase in individual bond holdings. By the envelope theorem, in the competitive equilibrium this can be written as $\frac{\partial V}{\partial b} = u'(t)$. For the regulator, however, the marginal benefit of an increase in bond holdings takes into account the fact that prices are affected by the increase in bond holdings, and is therefore given by $\frac{\partial W}{\partial b} = u'(t) + \psi_t \mu_t$. If the collateral constraint does not bind, $\mu_t = 0$ and the two expressions coincide. If the collateral constraint binds, the social benefits of a higher level of bonds assessed by the regulator include the extra term given by $\psi_t \mu_t$, because one more unit of aggregate bonds increases the inter-period ability to borrow by ψ_t which has a marginal value of μ_t.

The above argument explains why bond holdings are valued differently by the regulator and the private agents "ex post," when the collateral constraint binds. Since both the regulator and the agents are forward looking, however, it follows that those differences in valuation lead to differences in the private and social benefits of debt accumulation "ex ante," when the constraint is not binding. Consider the marginal cost of increasing the level of debt at date t evaluated at the competitive equilibrium in a state in which the constraint is not binding. This cost is given by the discounted expected marginal utility from the implied reduction in consumption next period $\beta R E\left[u'(t+1)\right]$. In contrast, the regulator internalizes the effect by which the larger debt reduces tomorrow's borrowing ability by ψ_{t+1}, and hence the marginal cost of borrowing at period t that is not internalized by private agents is given by $\beta R E_t \left[\mu_{t+1}\left(\kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}}\right)\right]$.

We now show that the regulator's equilibrium allocations can be implemented as a competitive equilibrium in the decentralized economy by introducing a macro-prudential policy that taxes debt and dividends (the latter can turn into a subsidy too, as we show in the next Section, but we refer to it generically as a tax).[11] In particular, the regulator can do this

[11] See Bianchi (2011) for other decentralizations using capital and liquidity requirements, or loan-to-value ratios.

by constructing state-contingent schedules of taxes on bond purchases (τ_t) and on dividends (δ_t). The tax on bonds ensures that the regulator's optimal plans for consumption and bond holdings are consistent with the Euler equation for bonds in the competitive equilibrium. This requires setting $\tau_t = E_t\mu_{t+1}\psi_{t+1}/E_t u'(t+1)$. The tax on dividends ensures that these optimal plans and the pricing function $q(B,\varepsilon)$ are consistent with the private agents' Euler equation for asset holdings.

With the taxes in place, the budget constraint of private agents becomes:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t(1+\tau_t)} = q_t k_t + b_t + w_t n_t + [\varepsilon_t F(k_t,h_t)(1-\delta_t) - w_t h_t] + T_t. \tag{17}$$

Here, T_t represents lump-sum transfers by which the government rebates all its tax revenue (or a lump-sum tax in case the tax rates are negative, which is not ruled out).

The Euler equations of the competitive equilibrium become:

$$u'(t) = \beta R(1+\tau_t)E_t\left[u'(t+1)\right] + \mu_t \tag{18}$$

$$q_t(u'(t) - \mu_t\kappa) = \beta E_t\left[u'(t+1)\left(\varepsilon_{t+1}F_k(k_{t+1},n_{t+1})(1+\delta_{t+1}) + q_{t+1}\right)\right] \tag{19}$$

By combining these two Euler equations we can derive the expected excess return on assets under the macro-prudential policy. In this case, after-tax returns on assets and bonds are defined as $\tilde{R}^q_{t+1} \equiv \frac{d_{t+1}(1+\delta_{t+1})+q_{t+1}}{q_t}$ and $\tilde{R}_{t+1} \equiv R(1+\tau_t)$ respectively, and the after-tax expected equity premium reduces to an expression analogous to that of the decentralized equilibrium:

$$\tilde{R}^{ep}_t = \frac{\mu_t(1-\kappa)}{E_t\left[(u'(t)-\mu_t\kappa)m_{t+1}\right]} - \frac{Cov_t(m_{t+1},\tilde{R}^q_{t+1})}{E_t\left[m_{t+1}\right]} \tag{20}$$

This excess return also has a corresponding interpretation in terms of the Sharpe ratio, the price of risk, and the correlation between asset returns and the pricing kernel as in the case of the competitive equilibrium without macro-prudential policy.

It follows from comparing the expressions for R^{ep}_t and $\tilde{R}^{ep}_t$ that differences in the after-tax expected equity premia of the competitive equilibria with and without macro-prudential policy are determined by differences in the direct and indirect effects of the credit constraint in the two environments. As shown in the next Section, these effects are stronger in the decentralized equilibrium without policy intervention, in which the inefficiencies of the credit externality are not addressed. Intuitively, higher leverage and debt in this environment imply that the constraint binds more often, which strengthens the direct effect. In addition, lower net worth implies that the stochastic discount factor covaries more strongly with the excess return on assets, which strengthens the indirect effect. Notice also that dividends in the regulator's allocations are discounted at a rate which depends positively on the tax on debt. This premium is required by the regulator so that the excess returns reflect the social costs of borrowing.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to annual frequency using data from the U.S. economy. The functional forms for preferences and technology are the following:

$$u(c - G(n)) = \frac{\left[c - \varkappa \frac{n^{1+\omega}}{1+\omega}\right]^{1-\sigma} - 1}{1-\sigma} \quad \omega > 0, \sigma > 1$$

$$F(k, h) = \varepsilon k^{\alpha_K} h^{\alpha_h}, \qquad \alpha_K, \alpha_h \geq 0 \quad \alpha_K + \alpha_h < 1$$

The real interest rate is set to $R - 1 = 0.028$ per year, which is the ex-post average real interest rate on U.S. three-month T-bills during the period 1980-2005. We set $\sigma = 2$, which is a standard value in quantitative DSGE models. The parameter $\varkappa$ is inessential and is set so that mean hours are equal to 1, which requires $\varkappa = 0.64$. Aggregate capital is normalized to $\bar{K} = 1$ without loss of generality and the share of labor in output α_h is equal to 0.64, the standard value. The Frisch elasticity of labor supply $(1/\omega)$ is set equal to 1, in line with evidence by Kimball and Shapiro (2008).

We follow Schmitt-Grohe and Uribe (2007) in taking M1 money balances in possession of firms as a proxy for working capital. Based on the observations that about two-thirds of M1 are held by firms (Mulligan, 1997) and that M1 was on average about 14 percent of annual GDP over the period 1980 to 2009, we calibrate the working capital-GDP ratio to

Table 1: Calibration

textitDescription	textitValue	*Source / target*
Interest rate	$R - 1 = 0.028$	U.S. data
Risk aversion	$\sigma = 2$	Standard DSGE value
Share of labor	$\alpha_n = 0.64$	U.S. data
Labor disutility coefficient	$\chi = 0.64$	Normalization
Frisch elasticity parameter	$\omega = 1$	Kimball and Shapiro (2008)
Supply of Assets	$\bar{K} = 1$	Normalization
Working capital coefficient	$\theta = 0.14$	Working Capital-GDP=9%
Discount factor	$\beta = 0.96$	Debt-GDP ratio= 38%
Collateral coefficient	$\kappa = 0.36$	Frequency of Crisis = 3%
Share of assets	$\alpha_K = 0.05$	$q\overline{K}/GDP = 1.35$
TFP process	$\sigma_\varepsilon = 0.014, \rho_\varepsilon = 0.53$	Std. dev. and autoc. of U.S. GDP

match $(2/3)0.14 = 0.093$. Given the 64 percent labor share in production, and assuming the collateral constraint does not bind, we obtain $\theta = 0.093/0.64 = 0.146$.

The value of β is set to 0.96, which is also a standard value, but in addition it supports an average household debt-income ratio in a range that is in line with U.S. data from the Federal Reserve's *Flow of Funds* database. Before the mid-1990s this ratio was stable at about 30 percent. Since then and until just before the 2008 crisis, it rose steadily to a peak of almost 70 percent. By comparison, the average debt-income ratio in the stochastic steady-state of the model with the baseline calibration is 38 percent. A mean debt ratio of 38 percent is sensible because 70 percent was an extreme at the peak of a credit boom and 30 percent is an average from a period before the substantial financial innovation of recent years.

TFP shocks follow a log-normal AR(1) process $log(\varepsilon_t) = \rho \log(\varepsilon_{t-1}) + \eta_t$. We construct a discrete approximation to this process using the quadrature procedure of Tauchen and Hussey (1991) using 15 nodes. The values of σ_ε and ρ are set so that the standard deviation and first-order autocorrelation of the output series produced by the model match the corresponding moments for the cyclical component of U.S. GDP in the sample period 1947-2007 (which are 2.1 percent and 0.5 respectively). This procedure yields $\sigma_\varepsilon = 0.014$ and $\rho = 0.53$.

Since capital in the model is in fixed supply, we do not set the capital share to the standard 1/3rd, which is based on capital income accrued to the entire capital stock. Instead, we set α_K so that the model matches an estimate of the ratio of capital in fixed supply to GDP based on the value of the housing stock. Using data from the *Flow of Funds* database, we estimate this ratio at about 1.35. The model matches it, given the other parameter values, if we set $\alpha_K = 0.05$.[12]

The parameter κ is difficult to calibrate with actual data because of the model's high level of aggregation and the wide dispersion in loan-to-value restrictions and ability to leverage across households and firms of various characteristics. Hence, following Mendoza (2010), we chose to set κ so as to match the frequency of financial crises in U.S. data. To be consistent with the empirical literature, we define a financial crisis as an event in which both the credit constraint binds and there is a decrease in credit of more than one standard deviation. Then, we set κ so that financial crises in the stochastic steady state of the baseline model simulation occur about 3 percent of the time, which is consistent with the fact that the U.S. has experienced three major financial crises in the last hundred years.[13] This procedure yields $\kappa = 0.36$. This value of κ is also consistent with measures of household and corporate leverage, which were respectively 0.2 and 0.45 at the onset of the 2007 financial crisis.[14]

We recognize that several of the parameter values are subject of debate (e.g. the Frisch elasticity of labor supply), or relate to variables that do not have a clear analog in the data (e.g. κ or θ). Hence, we will perform extensive sensitivity analysis to examine the effects of changes in the model's key parameters.

The model is solved using a global, nonlinear solution method that solves the recursive competitive equilibrium by iterating on the equilibrium asset pricing function and the optimality conditions. The algorithm is described in detail in the Appendix. Since mean output

[12]Estimates of the value of capital in fixed supply vary depending on whether they include land used for residential or commercial purposes, or owned by government at different levels. We used an estimate based on residential property because it is a closer match to the structure of the model (for example, the discount factor was set to match the household debt ratio). This is also a conservative assumption, because a lower α_K weakens the Fisherian deflation mechanism, and hence if anything our findings would be strengthened with a broader definition of capital in fixed supply, which would imply a higher α_K.

[13] The three crises correspond to the Great Depression, the Savings and Loans Crisis and the Great Recession (see Reinhart and Rogoff (2008)). While a century may be a short sample for estimating accurately the probability of a rare event in one country, Mendoza (2010) estimates a probability of about 3.6 percent for financial crises using a similar definition but applied to all emerging economies using data since 1980.

[14]These leverage ratios were computed using asset and liabilities data from the *Flow of Funds* (total assets and credit market debt outstanding of households and nonprofit organizations, and total assets and debt outstanding of the domestic nonfinancial business sector). The resulting leverage ratios are lower than maximum loan-to-value ratios in home mortgages, which peaked above 95 percent in the sub-prime market at the peak of the housing boom, but the lower ratio in the *Flow of Funds* data suggests that this is not a representative figure for the broader housing sector.

is normalized to 1, all quantities can be interpreted as fractions of mean output.

4.2 Borrowing decisions

We start the quantitative analysis by exploring the effects of the externality on optimal borrowing plans and the financial amplification mechanism. The two panels of Figure 1 show the bond decision rules (b') of private agents and the regulator as a function of b (left panel) as well as the asset pricing function (right panel), both for a negative two-standard-deviations TFP shock.

The first important result illustrated in these plots is that the Fisherian deflation mechanism generates V-shaped bond decision rules, instead of the typical monotonically increasing decision rules, and a concave pricing function with a very steep slope at high debt levels (i.e. low values of b). The point at which bond decision rules switch slope corresponds to the value of b at which the collateral constraint holds with equality but does not bind. To the right of this point, the collateral constraint does not bind and the bond decision rules are upward sloping. To the left of this point, the bond decision rules are *decreasing* in b, because a reduction in current bond holdings results in a sharp reduction in the price of assets, as can be seen in the right panel, and tightens the borrowing constraint, thus increasing b'.

As in Bianchi (2011), we can separate the bond decision rules in the left panel of Figure 1 into three regions: a "constrained region," a "high-externality region" and a "low-externality region." The "constrained region" is defined by the range of b in the horizontal axis with sufficiently high initial debt such that the collateral constraint binds for the regulator. This is the range with $b \leq -0.385$. In this region, the collateral constraint binds for both the regulator and private agents in the competitive equilibrium, because the credit externality implies that the constraint starts binding at higher values of b in the latter than in the former, as we show below.

By construction, *the total amount of debt* (i.e. the sum of bond holdings and working capital) in the constrained region is the same under the regulator's allocations and the competitive equilibrium. If working capital were not subject to the collateral constraint, the two *bond decision rules* would also be identical. But with working capital in the constraint the two can differ. This is because the effective cost of labor differs between the two equilibria,

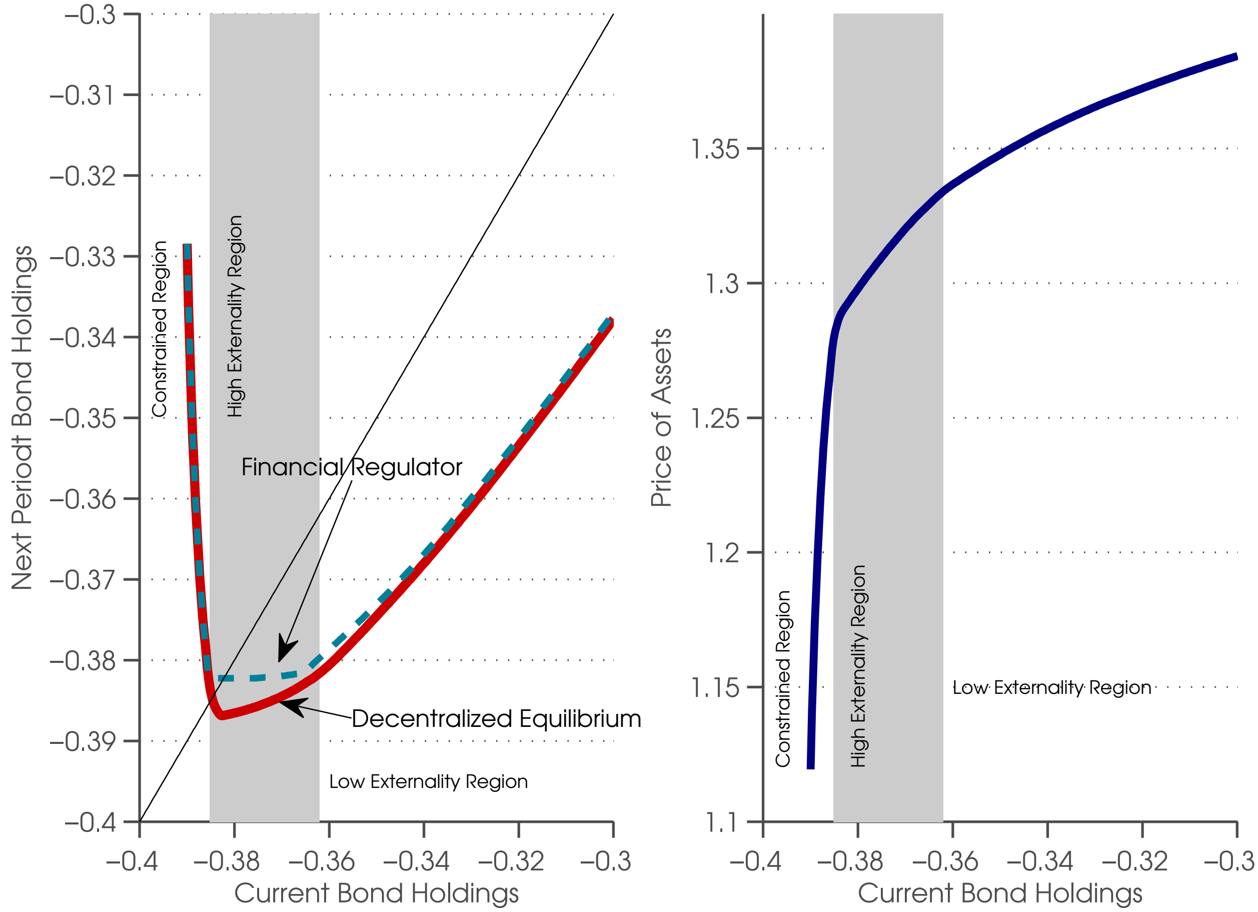

Figure 1: Bond Decision Rules (left panel) and Asset Pricing Function (right panel) for a Negative Two-standard-deviations TFP Shock

since the increase in the marginal financing cost of labor when the constraint binds, $\theta\mu_t/u'(t)$, is different. These differences, however, are very small in the numerical experiments, and thus the bond decision rules are approximately the same in the constrained region.[15]

The high-externality region is located to the right of the constrained region, and it includes the interval $-0.385 < b < -0.363$. Here, the regulator chooses uniformly higher bond positions (lower debt) than private agents, because of the effect of the externality on the regulator's decisions when the constrained region is near. In fact, private agents hit the credit constraint at $b = -0.383$, while at this initial b the regulator still retains some borrowing capacity. Moreover, this region is characterized by "financial instability," in the sense that

[15]The choice of b' becomes slightly higher for the regulator as b gets closer to the upper bound of the constrained region, because the deleveraging that occurs around this point is small enough for the probability of a binding credit constraint next period to be strictly positive. As a result, for given allocations, conditions (15) and (6) imply that μ is lower for the regulator.

the levels of debt chosen for $t+1$ are high enough so that a negative TFP shock of standard magnitude in that period can lead to a binding credit constraint that leads to large falls in consumption, output, asset prices and credit. We will show later that this is also the region of the state space in which the regulator uses actively its macro-prudential policy.

The low-externality region is the interval for which $b \geq -0.363$. In this region, the probability of a binding constraint next period is zero for both the regulator and the competitive equilibrium. The bond decision rules still differ, however, because expected marginal utilities differ for the two equilibria. But the regulator does not set a tax on debt, because negative shocks cannot lead to a binding credit constraint in the following period.

The long-run probabilities with which the regulator's (competitive) equilibrium visit the three regions of the bond decision rules are 2 (4) percent for the constrained region, 69 (70) percent for the high-externality region, and 29 (27) percent for the low-externality region. Both economies spend more than 2/3rds of the time in the high-externality region, but the prudential actions of the regulator reduce the probability of entering in the constrained region by a half. Later we will show that this is reflected also in financial crises that are much less frequent and less severe than in the competitive equilibrium.

The larger debt (i.e. lower b') choices of private agents relative to the regulator, particularly in the high-externality region, constitute our first measure of the overborrowing effect at work in the competitive equilibrium. The regulator accumulates extra precautionary savings above and beyond what private individuals consider optimal in order to self-insure against the risk of financial crises. This effect is quantitatively small in terms of the difference between the two decision rules, but this does not mean that its macroeconomic effects are negligible. Later in this Section we illustrate this point by comparing financial crises events in the two economies. In addition, we use Figure 2 to study further the dynamics implicit in the bond decision rules so as to show that small differences in borrowing decisions lead to major differences in financial amplification when a crisis hits.

Figure 2 shows bond decision rules for the regulator and the competitive equilibrium over the range (-0.39,-0.36) for two TFP scenarios: average TFP and TFP two-standard-deviations below the mean. The ray from the origin is the stationary choice line, where $b' = b$. We use a narrower range of bond values than in Figure 1 to "zoom in" and highlight

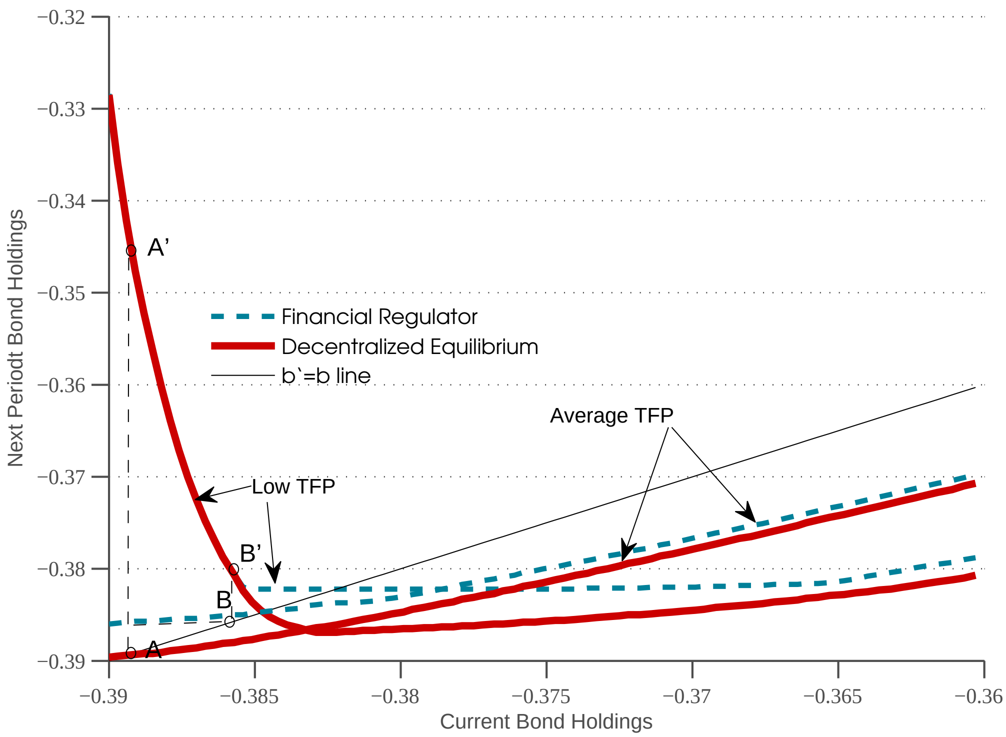

Figure 2: Comparison of Debt Dynamics

the differences in decision rules.

Assume both economies start at a value of b such that at average TFP the debt of agents in the competitive equilibrium remains unchanged. This is point A, where the agents' decision rule cuts the stationary choice line, so that $b' = b = -0.389$. If the TFP realization is indeed the average, private agents in the decentralized equilibrium keep that level of debt. On the other hand, starting from that same $b = -0.389$, the regulator builds precautionary savings and reduces its debt to point B with $b = -0.386$. Hence, the next period the two economies start at the debt levels in A and B respectively. Assume now that at this time TFP falls by two standard deviations. Now we can see the large dynamic implications of the small differences in the bond decision rules of the two economies: The competitive equilibrium suffers a major correction caused by the Fisherian deflation mechanism. The collateral constraint becomes binding and the economy is forced to a large deleveraging that results in a sharp reduction in debt (an increase in b to -0.347 at point A'). Consumption falls leading to a drop in the stochastic discount factor and a drop in asset prices. In contrast, the regulator, while also facing a binding credit constraint, adjusts its debt marginally to

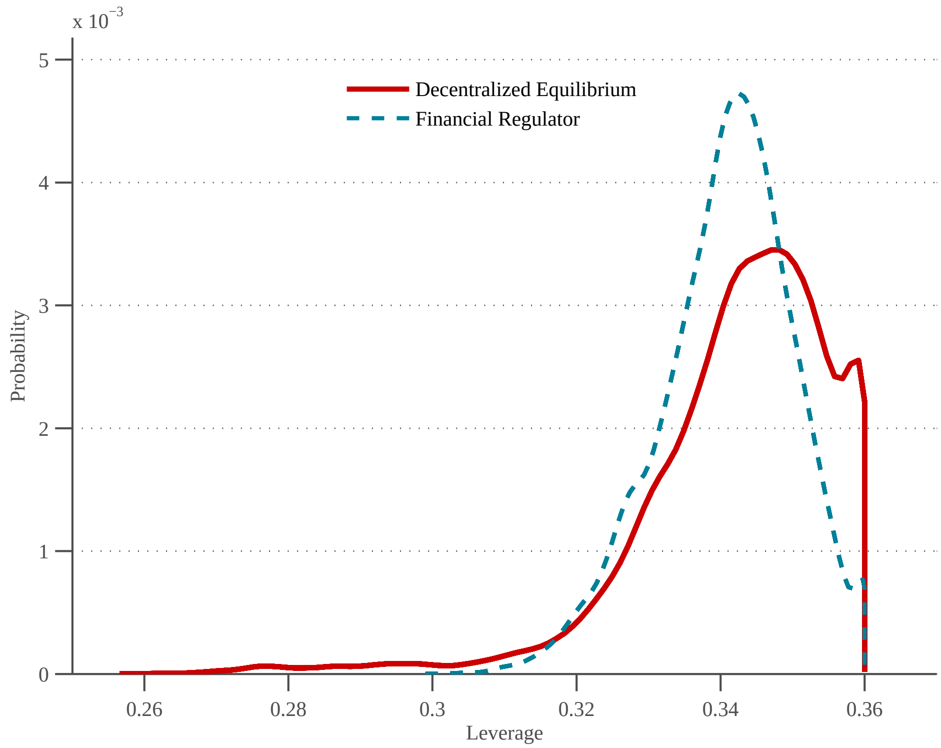

Figure 3: Ergodic Distribution of Leverage ($\frac{-b_{t+1}+\theta w_t h_t}{q_t K}$)

just about $b = -0.379$ at point B'. This was possible for the regulator because, taking into account the risk of a Fisherian deflation *and* internalizing its price dynamics, the regulator chose to borrow less than agents in the decentralized equilibrium a period earlier.

Overborrowing can also be assessed by comparing the long-run distributions of debt and leverage of private agents and the regulator. The fact that the regulator accumulates more precautionary savings implies that its ergodic distribution concentrates less probability at higher leverage ratios than in the competitive equilibrium. Figure 3 shows the ergodic distributions of leverage ratios (measured as $\frac{-b_{t+1}+\theta w_t n_t}{q_t K}$) in the two economies. The maximum leverage ratio in both economies is given by κ but notice that the decentralized equilibrium concentrates higher probabilities in higher levels of leverage. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than 1 percent. Hence, overborrowing seems a relatively minor problem if measured by comparing unconditional long-run averages of leverage ratios.[16]

[16]Measuring "ex ante" leverage as $\frac{-b_t+\theta w_t h_t}{q_t K}$, we find that leverage ratios in the competitive equilibrium can exceed the maximum of those for the regulator 3 percent of the time and by up to 12 percentage points.

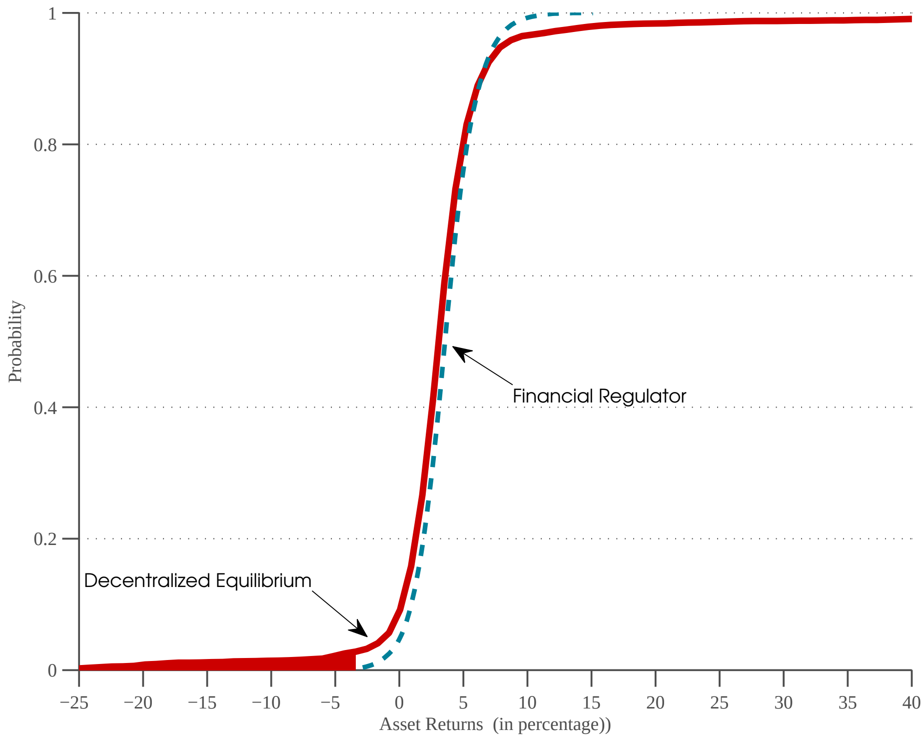

Figure 4: Ergodic Cumulative Distribution of Realized Asset Returns

4.3 Asset Pricing

Overborrowing has important quantitative implications for asset returns and their determinants. Figure 4 shows the long-run distributions of realized equity returns for the competitive equilibrium and the regulator.

A key result from our analysis is that the distribution of equity returns for the competitive equilibrium displays fatter tails. In fact, the 99th percentile of returns is about -17.5 percent, v. -1.6 percent for the regulator. The fatter left tail in the competitive equilibrium corresponds to states in which a negative TFP shock hits when agents have a relatively high level of debt. Intuitively, as a negative TFP shock hits, expected dividends decrease and this puts downward pressure on asset returns. In addition, if the collateral constraint becomes binding, asset fire-sales lead to a further drop in asset prices. Following a similar logic, the fatter right-tail in the distribution of returns of the competitive equilibrium corresponds to periods with positive TFP shocks, which were preceded by unusually low asset prices due to fire sales.

We show below that the fatter tails of the distribution of asset returns, and the associated time-varying risk of financial crises, have substantial effects on the risk premium. These features of our model are similar to those examined in the literature on asset pricing and "disasters" (see Barro, 2009). Note, however, that this literature generally treats financial disasters as resulting from exogenous stochastic processes with fat tails and time-varying volatility, whereas in our setup financial crises and their time-varying risk are both endogenous.[17] The underlying shocks driving the model are standard TFP shocks, even in periods of financial crises. In our model, as in Mendoza (2010), financial crises are endogenous outcomes that occur when shocks of standard magnitudes trigger financial amplification via a Fisherian deflation.

Table 2 reports statistics that characterize the main properties of asset returns for the regulator and the competitive equilibrium. We also report statistics for a competitive equilibrium in which assets in the collateral constraint are valued at a fixed price set equal to the average price across the ergodic distribution $\bar{q}$ (i.e. the credit constraint becomes $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} k_{t+1}$).[18] This fixed-valuation scenario allows us to compare the properties of asset returns in the competitive and regulator's equilibria with a setup in which a collateral constraint exists but the Fisherian deflation channel and the credit externality are removed.

Table 2 lists expected excess returns, the direct and indirect (or covariance) effects of the credit constraint on excess returns, the log standard deviation of returns, the price of risk, and the Sharpe ratio. These moments are reported for the unconditional long-run distributions of each model economy, as well as for distributions conditional on the collateral constraint being binding and not binding. In addition, Figure 5 plots Sharpe ratios and excess returns as a function of the bond position. These plots provide further evidence of important non-linearities present in the model, now for the compensation for risk taking and the excess returns, both of which rise very sharply as debt enters the region where the

[17] The literature on disasters typically uses Epstein-Zin preferences so as to be able to match the large observed equity premia (Gourio, 2011). Here we use standard CRRA preferences with a risk aversion coefficient of 2, and as we show later, we can obtain larger risk premia than in the typical CRRA setup without credit frictions. Moreover, we obtain realistically large risk premia when the credit constraint binds.

[18] Because the asset is in fixed supply, these allocations would be the same if we use instead an ad-hoc borrowing limit such that $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} \bar{K}$. The price of assets, however, would be lower since with the ad-hoc borrowing constraint assets do not have collateral value.

Table 2: Asset Pricing Moments

	Excess Return	Direct Effect	Covariance Effect	s_t	$\sigma_t(R^q_{t+1})$	S_t
			Decentralized Equilibrium			
Unconditional	1.09	0.87	0.22	5.22	3.05	0.36
Constrained	13.94	13.78	0.16	4.05	2.71	5.14
Unconstrained	0.23	0.00	0.23	5.31	3.08	0.07
			Financial Regulator			
Unconditional	0.17	0.11	0.06	2.88	1.85	0.09
Constrained	4.86	4.80	0.06	3.02	2.07	2.34
Unconstrained	0.06	0.00	0.06	2.86	1.84	0.03
			Fixed Valuation Equilibrium			
Unconditional	0.89	0.82	0.04	2.59	1.69	0.52
Constrained	1.29	1.23	0.05	2.81	1.84	0.74
Unconstrained	0.03	0.00	0.03	2.16	1.39	0.02

Note: The table reports averages of the conditional excess return after taxes, the direct effect, the covariance effect, the market price of risk s_t, the (log) volatility of asset returns denoted $\sigma_t(R^q_{t+1})$, and the Sharpe ratio (S_t) . The Sharpe ratio is computed as the unconditional average of the excess return divided by the standard deviation of excess returns. All numbers except the Sharpe ratios are in percentage.

collateral constraint binds.

The mean unconditional excess return is 1.09 percent in the competitive equilibrium v. only 0.17 percent for the regulator and 0.86 percent in the fixed-valuation economy.[19] The risk premium in the competitive equilibrium is large, about half as large as the risk-free rate. The fact that the other two economies produce lower premia indicates that the high premium of the competitive equilibrium is the combined result of the Fisherian deflation mechanism and the credit externality. Note also that the high premium produced by our model contrasts sharply with the findings of Heaton and Lucas (1996) and Gomes, Yaron, and Zhang (2003), who found that credit frictions without the Fisherian deflation mechanism do not produce

[19] Note that the asset in our model is a claim to a unit of output payable at $t+1$, which is akin to an unlevered claim on equity. We can also compute the returns of what would be a levered claim on equity, by calculating the returns of an asset that promises to pay c_{t+1} units of consumption. The returns on the levered and unlevered claims are highly correlated, but the excess returns of the former are larger (1.7 v. 1.09 percent in terms of unconditional averages in the competitive equilibrium). Computing the returns of a levered claim with the standard formula, assuming a leverage ratio of 2/3rds, yields a premium of 4 percent, but this estimate is problematic because, since the Modigliani-Miller theorem does not apply, a claim with this return is actually not traded.

large premia.[20]

The excess returns for the regulated and fixed-valuation economies in the region where the collateral constraint does not bind are in line with those obtained in classic asset pricing models that display the "equity premium puzzle." The equity premia we obtained in these two scenarios are driven only by the covariance effect, as in the classic models, and they are negligible: 0.03 percent in the fixed-valuation economy and 0.06 percent for the regulator. This is natural because, without the constraint binding and with the effects of the credit externality and the Fisherian deflation removed or weakened, the model is in the same class as those that display the equity premium puzzle. In contrast, our baseline competitive economy yields a 0.23 percent premium conditional on the constraint not binding, which is small relative to data estimates that range from 6 to 18 percent, but 4 to 8 times larger than in the other two economies.

Conditional on the collateral constraint being binding, mean excess returns are quite large: Nearly 14 percent in the competitive equilibrium, 4.86 percent for the regulator, and 1.29 percent in the fixed-valuation economy. Interestingly, the lowest unconditional premium is the one for the regulated economy (0.17 percent), but conditional on the constraint binding, the lowest premium is the one for the fixed-valuation economy (1.29 percent).This is because on one hand the Fisherian deflation effect is still at work when the collateral constraint binds for the regulated, but not in the fixed-valuation economy, while on the other hand the regulator has a lower probability of hitting the collateral constraint (so that the higher premium when the constraint binds does not weigh heavily when computing the unconditional average). In turn, the probability of hitting the collateral constraint is higher for the fixed-valuation economy, because the incentive to build precautionary savings is weaker when there is no Fisherian amplification.

The unconditional direct and covariance effects of the collateral constraint on excess returns are significantly stronger in the competitive equilibrium than for the regulated and fixed-valuation economies, and even more so if we compare them conditional on the con-

[20]The unconditional premium in the fixed valuation economy, at 0.86 percent, is not trivial, but note that it results from the fact that the constraint binds with very high probability, given the smaller incentives to accumulate precautionary savings. The risk premium in the unconstrained region of the fixed-valuation model is only 0.03 percent, v. 0.23 in our baseline model.

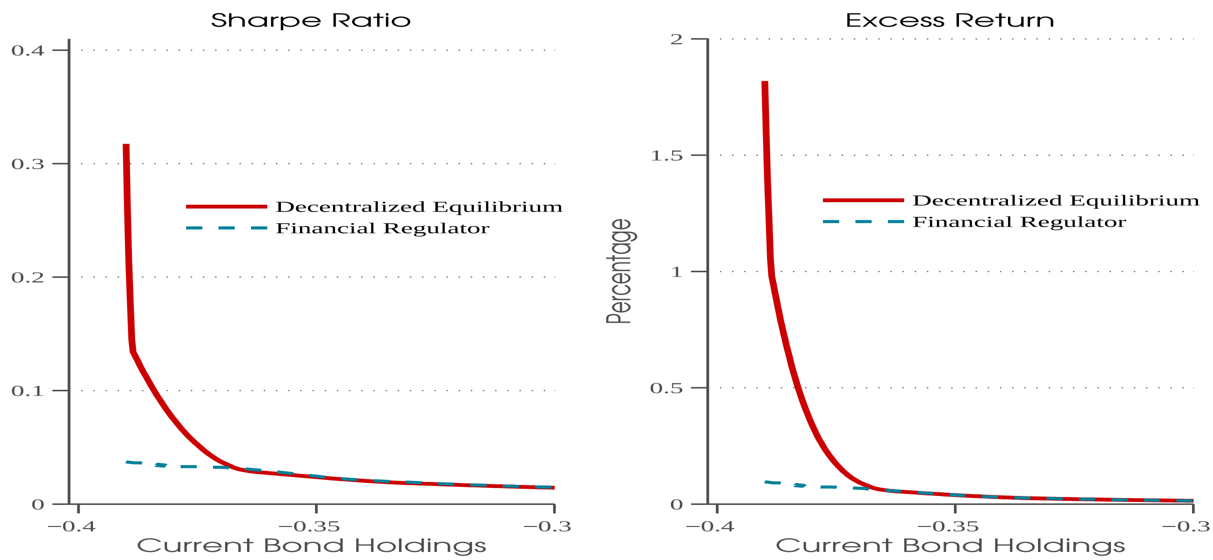

Figure 5: Sharpe Ratio and Excess Return for negative one-standard-deviation TFP Shock

straint being binding. Again, the direct and covariance effects are larger in the competitive equilibrium because of the effects of the overborrowing externality and the Fisherian deflation mechanism.

In terms of the decomposition of excess returns based on condition (10), Table 2 shows that the unconditional average of the price of risk is about twice as large in the decentralized equilibrium than for the regulated and fixed-valuation economies. This reflects the fact that consumption, and therefore the pricing kernel, fluctuate significantly more in the decentralized equilibrium. The Sharpe ratio and the variability of asset returns are also much larger in the competitive equilibrium across the two regions. The increase in the former indicates, however, that the mean excess return rises significantly more than the variability of returns, which indicates that risk-taking is "overcompensated" in the competitive equilibrium (relative to the compensation it receives when the regulator internalizes the credit externality). Interestingly, the unconditional Sharpe ratio in the competitive equilibrium is similar to U.S. data estimates of around 0.3 (Campbell (2003)). Note also that the correlations between asset returns and the stochastic discount factor, not shown in the Table, are very similar under the three equilibria and very close to 1. This is important because it implies that the differences in excess returns and Sharpe ratios cannot be attributed to differences in this correlation.

4.4 Incidence and Magnitude of Financial Crises

We show now that overborrowing in the competitive equilibrium increases the incidence and severity of financial crises. To demonstrate this result we construct an event analysis of financial crises with simulated data obtained by performing long (100,000-period) stochastic time-series simulations of the competitive equilibrium, the regulator's equilibrium and the fixed-valuation economy, removing the first 1,000 periods. A financial crisis episode is defined as a situation in which the credit constraint binds and this causes a decrease in credit larger than a crisis threshold, which is set at one standard deviation of the first difference of credit in the ergodic distribution corresponding to each economy.

The event analysis is important not only for studying the effects of overborrowing on the incidence and magnitude of financial crises, but also to shed light on whether the model can produce financial crises with realistic features. This is an important first step in making the case for treating the normative implications of the model as relevant. In this regard, the results show that, while we did build a rich equilibrium business cycle model so we could keep the analysis of the externality tractable, and hence our match to the data is not perfect, the model does produce financial crises with realistic features in terms of abrupt, large declines in allocations, credit, and asset prices, and it supports non-crisis output fluctuations roughly in line with observed U.S. business cycles. Other studies have also shown that the Fisherian deflation mechanism can do well at explaining observed financial crisis dynamics nested within realistic long-run business cycle co-movements in full blown business cycle models (see Mendoza (2010)).

The first important result of the event analysis is that the incidence of financial crises is significantly higher in the competitive equilibrium. As explained earlier, we calibrated κ so that the competitive economy experiences financial crises with a long-run probability of 3.0 percent. With this same value of κ, the regulator experiences financial crises only with 0.9 percent probability in the long run. Thus, the credit externality increases the frequency of financial crises by a factor of 3.33.[21]

[21] If we define financial crises for the regulator by using the crisis threshold of the competitive equilibrium, instead of the threshold based on the regulator's own credit fluctuations, the probability of crises for the regulator is even lower. In fact, credit declines equal to at least one standard deviation of the first-difference of credit in the competitive equilibrium are zero-probability events for the financial regulator.

The second important result is that financial crises are more severe in the competitive equilibrium. This is illustrated in the event windows shown in Figure 6. The event windows are for total credit (bonds plus working capital), consumption, labor, output,TFP and asset prices, all expressed as deviations from long-run averages.

We construct comparable event windows for the competitive equilibrium, the regulator and the fixed-valuation economy by following this procedure: First we identify financial crisis events in the competitive equilibrium, and isolate five-year event windows centered in the period in which the crisis takes place. That is, each event window includes five years, the two years before the crisis, the year of the crisis, and the two years after. Second, we calculate the median TFP shock across all of these event windows in each year $t-2$ to $t+2$, and the median initial debt at $t-2$. This determines an initial value for bonds and a five-year sequence of TFP realizations. Third, we feed this sequence of shocks and initial value of bonds to the decisions rules of each model economy and compute the corresponding endogenous variables plotted in Figure 6. By proceeding in this way, the event dynamics for the three equilibria are simulated using the same initial state and the same sequence of shocks.[22]

The features of financial crises at date t in the competitive economy are in line with the results in Mendoza (2010): The debt-deflation mechanism produces financial crises characterized by sharp declines in credit, consumption, asset prices and output.

The five macro variables illustrated in the event windows show similar dynamics across the three economies in the two years before the financial crisis. When the crisis hits, however, the collapses observed in the competitive equilibrium are much larger. Credit falls about 20 percentage points more, and two years after the crisis the credit stock of the competitive equilibrium remains 10 percentage points below that of the regulated economy.[23] Consumption, asset prices, and output also fall much more sharply in the competitive equilibrium than in the regulated economy. The declines in consumption and asset prices are particu-

[22]The sequence of TFP shocks is $0.9960, 0.9881, 0.9724, 0.9841, 0.9920$ and the initial level of debt is 1.6 percent above the average.

[23]The model overestimates the drop in credit relative to what we have observed so far in the U.S. crisis (which as of the third quarter of 2010 reached about 7 percent of GDP) . One reason for this is that in the model, credit is in the form of one-period bonds, whereas in the data, loans have on average a much larger maturity. In addition, our model does not take into account the strong policy intervention that took place with the aim to prevent what would have been a larger credit crunch.

larly larger (-16 percent v. -5 percent for consumption and -24 percent v. -7 percent for asset prices). The asset price collapse also plays an important role in explaining the more pronounced decline in credit in the competitive equilibrium, because it reflects the outcome of the Fisherian deflation mechanism.

Output falls by 2 percentage points more and labor almost 3 percentage points more in the competitive equilibrium than in the regulated economy, because of the higher shadow cost of hiring labor due to the effect of the tighter binding credit constraint on access to working capital. Moreover, by comparing the output and labor drops at date t in the competitive equilibrium with the drops experienced in the fixed-valuation economy, and since both economies experience identical TFP shocks, we can infer that the model's Fisherian deflation mechanism amplifies the decline in output by 50 percent (6 percent v. 4 percent drop) and in labor by 300 percent (6 percent drop v. 2 percent drop).

The event analysis results can also be used to illustrate the relative significance of the wage and asset price components in the externality term $\psi_t \equiv \kappa \bar{K} \frac{\partial q_t}{\partial b_t} - \theta n_t \frac{\partial w_t}{\partial b_t}$ identified in condition (15). Given the unitary Frisch elasticity of labor supply, wages decrease one-to-one with labor (and hence the event plot for wages would be identical to the one shown for employment in Figure 6). As a result, the extent to which the drop in wages can help relax the collateral constraint is very limited. Wages and employment fall about 6 percent at date t , and with a working capital coefficient of $\theta = 0.14$, this means that the effect of the drop in wages on the borrowing capacity is $0.14(1-0.06)0.06 = 0.79$ percent. On the other hand, given that $\bar{K} = 1$ and that asset prices fall about 25 percent below trend at date t, and since $\kappa = 0.36$, the effect of asset fire sales on the collateral constraint is $0.36(0.25) = 9$ percent. Thus, the asset price effect of the externality is about 10 times bigger than the wage effect. This finding will play an important role in our quantitative analysis of macro-prudential policy later in this section.

The fixed-valuation economy displays very little amplification given that the economy is free from the Fisherian deflation mechanism. Credit increases slightly at date t in order to smooth consumption and remains steady in the following periods. The fact that assets are valued at the average price, and not the market price, contributes to mitigate the drop in the asset price, since it remains relatively more attractive as a source of collateral.

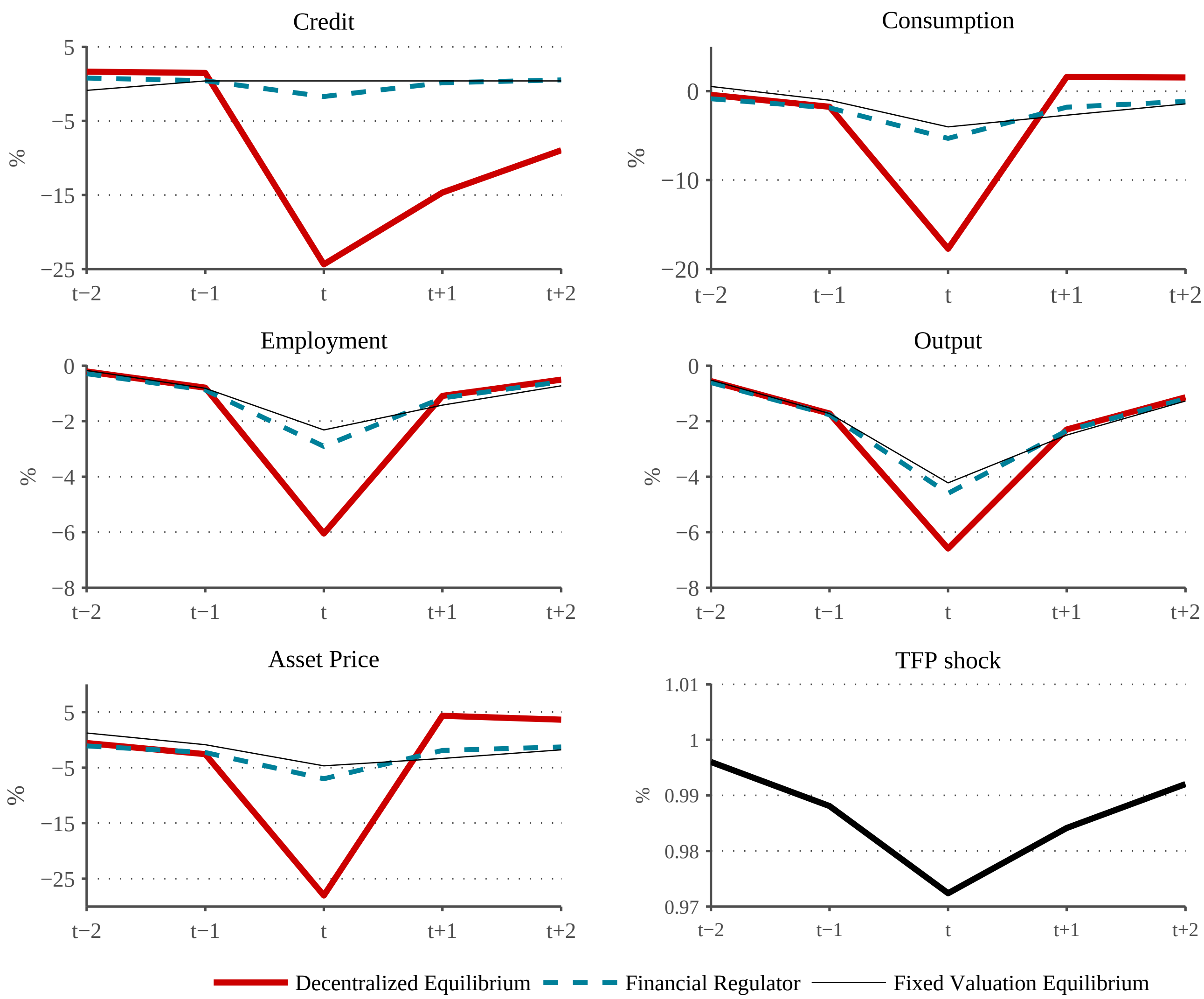

Figure 6: Event Analysis: percentage differences relative to unconditional averages

To gain more intuition on why asset prices drop more because of the credit externality, we plot in Figure 7 the projected conditional sequences of future dividends and asset returns up to 30 periods ahead of a financial crisis that occurs at date $t = 0$ (conditional on information available on that date). These are the sequences of dividends and returns used to compute the present values of dividends that determine the equilibrium asset price at t in the event analysis of Figure 6. The expected asset returns start very high when the crisis hits in both the competitive equilibrium and the regulator's equilibrium, but significantly more for the former (at about 40 percent) than the latter (at 10 percent). On the other hand, expected dividends do not differ significantly, and therefore we conclude that the sharp change in the

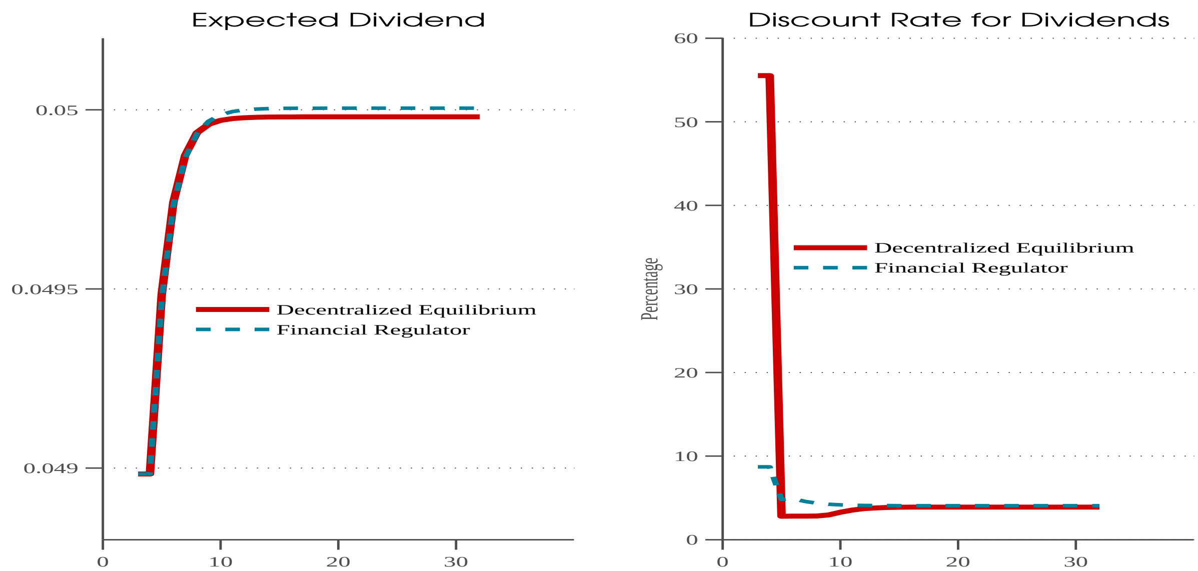

Figure 7: Forecast of expected dividends and asset returns.

pricing kernel reflected in the surge in projected asset returns when the crisis hits is what drives the large differences in the drop of asset prices.

The large deleveraging that takes places when a financial crisis occurs in the competitive equilibrium implies that projected asset returns for the immediate future (i.e. the first 6 periods after the crisis) drop significantly. Returns are also projected to fall for the regulator, but at a lower pace, so that in fact the regulator projects higher asset returns than agents in the competitive equilibrium for a few periods. Projected dividends for the same immediate future after the crisis are slightly smaller than the long-run average of 0.05 in both economies because of the persistence of the TFP shock. In the long-run, expected dividends are slightly higher for the regulator, because the marginal productivity of capital drops less during the financial crisis as a result of the lower amount of debt. Notice also that the regulator projects to discount dividends with slightly higher asset returns in the long run, because the tax on debt more than offsets the fact that the risk premium of the regulator is lower (recall that we are comparing after-tax returns as defined in Section 2). This arises because the tax on debt makes bonds relatively more attractive and this leads in equilibrium to a higher required return on assets.

4.5 Long-Run Business Cycles

Table 3 reports the long-run business cycle moments of the competitive equilibrium, the regulator's equilibrium, and the fixed-valuation economy, which are computed using each economy's ergodic distribution. The business cycle moments are roughly consistent with U.S. data. One aspect to notice is that consumption variability is slightly higher than output variability in the competitive equilibrium, which is not the case in U.S. data. However, if we exclude the crisis periods, the ratio of the variability of consumption to that of GDP would be 0.87 (compared with 0.88 in annual U.S. data from 1960 to 2007).[24]

The strong financial amplification mechanism at work in the competitive equilibrium produces higher business cycle variability in output and labor, and especially in consumption, compared with the regulated and fixed-valuation economies. The high variability of consumption and credit are consistent with the results in Bianchi (2011), but we find in addition that the credit externality produces a moderate increase in the variability of labor and a substantial increase in the variability of asset prices and leverage. It may seem puzzling that we can obtain non-trivial differences in long-run business cycle moments even though financial crises are low probability events. To explain this result, it is useful to go back to Figure 1. This plot shows that even during normal business cycles the optimal plans of the competitive equilibrium and the regulator differ, particularly in the high-externality region. Because the economy spends about 70 percent of the time in this region, where private agents borrow more and are more exposed to the risk of financial crises, long-run business cycle moments differ. In addition, the larger effects that occur during crises have a non-trivial effect on long-run moments. This is particularly noticeable in the case of consumption where the variability drops from 2.7 to 1.7 in the decentralized equilibrium when we exclude the crises episodes.

The business cycle moments of consumption, output and labor for the regulated economy are about the same as those of the fixed-valuation economy. This occurs even though the regulator is subject to the Fisherian deflation mechanism and the fixed-valuation economy

[24]In fact, the variability of consumption over output fluctuates significantly depending on the sample period, reaching 1.0 in the period 1945-2010, using annual data and detrending the data with a standard HP filter and detrending parameters equal to 100.

is not. The reason for this is because the regulator accumulates extra precautionary savings, which compensate for the sudden change in the borrowing ability when the credit constraint binds. The constraint binds less often and when it does it has weak effects on macro variables. On the other hand, the regulator does display lower variability in leverage and asset prices than the fixed-valuation economy, and this occurs because the regulator internalizes how a drop in the price tightens the collateral constraint.

The correlations of output with leverage, credit, and asset prices also differ significantly across the model economies. The GDP correlations of leverage and credit are significantly higher in the competitive equilibrium, while the correlation between asset prices and GDP is lower. The model without credit frictions would have a natural tendency to produce countercyclical credit because consumption-smoothing agents want to save in good times and borrow in bad times. This effect still dominates for the regulated and fixed-valuation economies, but in the competitive equilibrium the collateral constraint and the Fisherian deflation hamper consumption smoothing enough to produce procyclical credit and a higher GDP-leverage correlation. Similarly, the GDP-asset price correlation is nearly perfect when the Fisherian deflation mechanism is weakened (regulator's case) or removed (fixed-valuation case), but falls to about 0.8 in the competitive equilibrium. Because of the strong procyclicality of asset prices, leverage is countercyclical with a GDP correlation of -0.57. This is in line with the observed countercyclicality of household and corporate leverage in U.S. data, although the correlation is lower than in the data. Using data asset and liabilities data from the *Flow of Funds* between 1950-2010, we find that the correlation between the ratio of net household (corporate) leverage and GDP is -0.25 (-0.34) at the business cycle frequency. [25]

In terms of the first-order autocorrelations, the competitive equilibrium displays lower autocorrelations in all its variables compared to the other two economies. This occurs because crises in the competitive equilibrium are characterized by deep but not very prolonged recessions.

[25] Two important observations on this point. First, at lower frequencies the correlation is positive. As Boz and Mendoza (2010) report, the household leverage ratio rose together with GDP, land prices and debt between 1997 and 2007. Second, the countercyclicality of leverage for the household and non-financial corporate sectors differs sharply from the strong procyclicality of leverage in the financial sector (see Adrian and Shin (2010)).

Table 3: Long Run Moments

	Standard Deviation			Correlation with GDP			Autocorrelation		
	DE	SP	FV	DE	SP	FV	DE	SP	FV
Output	2.10	1.98	1.97	1.00	1.00	1.00	0.50	0.51	0.51
Consumption	2.71	1.87	1.85	0.86	0.99	0.99	0.23	0.56	0.57
Employment	1.25	1.02	0.98	0.97	1.00	1.00	0.42	0.50	0.51
Leverage	3.92	2.72	3.80	-0.57	-0.93	-0.95	0.59	0.69	0.71
Total Credit	3.55	0.95	0.76	0.27	-0.35	-0.42	0.58	0.77	0.81
Asset Price	3.95	2.24	3.48	0.79	0.97	0.97	0.16	0.56	0.60
Working capital	2.48	2.04	1.97	0.97	1.00	1.00	0.42	0.50	0.51

Note: 'DE' represents the decentralized equilibrium,'SP' represents the social planner/financial regulator, 'FV' represents the fixed valuation equilibrium, i.e. an economy with collateral valued at a fixed price equal to the average of the price of assets in the competitive equilibrium.

4.6 Properties of Macro-prudential Policies

Table 4 shows the statistical moments that characterize the macro-prudential taxes on debt and dividends. To make the two comparable, we express the dividend tax as a percent of the price of assets.

The unconditional average of the debt tax is 1.07 percent, v. 0.09 when the constraint binds and 1.09 when it does not. The tax remains positive, albeit small, on average when the collateral constraint binds, because in some of these states the regulator wants to allocate borrowing ability across bonds and working capital in a way that differs from the competitive equilibrium. If there is a positive probability that the credit constraint will bind again next period, the regulator allocates less debt capacity to bonds and more to working capital. As a result, a tax on debt remains necessary in a subset of the constrained region. Note, however, that these states are not associated with financial crisis events in our simulations. They correspond to events in which the collateral constraint binds but the deleveraging that occurs is not strong enough for a crisis to occur.

The debt tax fluctuates about 2/3rds as much as GDP and is positively correlated with leverage, i.e. $\frac{-b_{t+1}+\theta w_t n_t}{q_t K}$. This is consistent with the macro-prudential rationale behind the tax: The tax is high when leverage is building up and low when the economy is deleveraging.

Table 4: Long Run Moments of Macro-prudential Policies

	Average		Standard Deviation		Correlation with Leverage	
	Debt Tax	Dividend Tax	Debt Tax	Dividend Tax	Debt Tax	Dividend Tax
Unconditional	1.07	-0.46	1.41	0.62	0.73	-0.64
Constrained	0.09	0.52	0.41	0.04	0.0	0.0
Unconstrained	1.09	-0.49	1.40	0.61	0.81	-0.79

Note, however, that since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation. When the constraint binds, the correlation between the tax and leverage is zero by construction, because leverage remains constant at the value of κ.

The unconditional average of the dividend tax is negative (i.e. it is a subsidy), and it is small at about -0.46 percent. When the constraint binds, dividends are taxed at an average rate of about 0.52 percent, compared with an average subsidy of 0.49 percent when the constraint does not bind. The fact that on average the regulator requires a subsidy on dividends may seem puzzling, given that assets are less risky in the regulated economy. There is another effect at work, however, because the debt tax exerts downward pressure on assets prices by making bonds relatively more attractive, and this effect turns out to be quantitatively larger. Thus, since by definition the regulator's allocations are required to support the same asset pricing function of the competitive economy without policy intervention, the regulator calls for a dividend subsidy on average in order to offset the effect of the debt tax on asset prices. The variability of the tax on dividends is 0.62 percentage points, less than 1/3rds the variability of GDP. The correlation between this tax on dividends and leverage is negative in the unconstrained region, reflecting the negative correlation between the tax on debt and the tax on dividends explained above.

The dynamics of the debt and dividend taxes around crisis events are shown in Figure 8. The debt tax is high relative to its average, at about 2.7 percent, at $t-2$ and $t-1$, and this again reflects the macro-prudential nature of these taxes: Their goal is to reduce borrowing so as to mitigate the magnitude of the financial crisis if bad shocks occur. At date t the debt tax falls to zero, and it rises again at $t+1$ and $t+2$ to about 2 percent. The latter occurs because this close to the crisis the economy still remains financially fragile (i.e. there is still a non-zero probability of agents becoming credit constrained next period). The tax on dividends follows a similar pattern. Dividends are subsidized at a similar rate before and after financial crises events, but they are actually taxed when crises occur. The reason is again that the regulator needs to support the same pricing function of the competitive equilibrium that would arise without policy intervention. Hence, with the tax on debt falling to almost zero, there is pressure for asset prices to be higher than what that pricing function calls for, and hence dividends need to be taxed to offset this effect.

The macro-prudential behavior of the debt tax is intuitive and follows easily from the precautionary behavior of the regulator we described. To complement this result, we analyze the relationship between the conditional probability of a financial crisis and the optimal tax on debt in our time-series simulation. Figure 9 presents a scatter diagram of the one-step ahead probability of a crisis at time $t+1$ conditional on time t against the debt tax at t across the simulated time series. This Figure shows that there is a clear positive relationship between the tax on debt and the one-step-ahead probability of a financial crisis. That is, periods in which leverage is building up are associated with increasing probability of a financial crisis, which causes the financial regulator to react introducing higher taxes on debt. Notice also that financial crises are forecastable in the model, in the sense that the conditional probability assigned to a financial crisis occurring within a year is 18 percent. In the long run, however, the unconditional probability of a crisis is about 1 percent for the financial regulator.

On the other hand, the tax on dividends and its dynamic behavior seem less intuitive and harder to justify as a policy instrument (i.e. proposing a tax on dividends at the through of a financial crisis is bound to be unpopular). The two policy instruments are required, however, in order to implement exactly the financial regulator's allocations as a decentralized competitive equilibrium. Moreover, the regulator's allocations are guaranteed to attain a level of welfare at least as high as that of the competitive equilibrium without macro-prudential policy, since this equilibrium remains feasible to the regulator. If one takes the debt tax and not the tax on dividends, this Pareto improvement cannot be guaranteed.

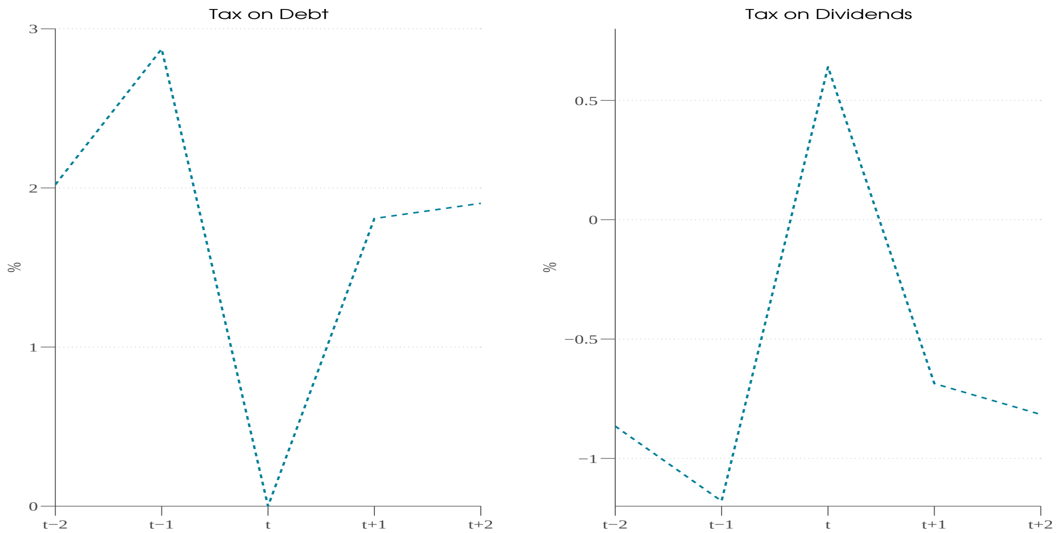

Figure 8: Event Analysis: Macroprudential Policies

Indeed, we solved a variant of the model in which we introduced the optimal debt taxes but left the taxes on dividends out, and found that average welfare is actually lower than without policy intervention by -0.02 percent. This occurs because welfare in the states of nature in which the constraint is already binding is lower than without policy intervention.[26]

The state-contingent nature of the macro-prudential taxes raises a familiar criticism posed in the context of Ramsey optimal taxation analysis: State-contingent policy schedules are impractical because of the limited flexibility of policy-making institutions to adhere to complex, pre-determined, time-varying rules for adjusting policy instruments.[27] For this reason, we studied the performance of an alternative regulated decentralized equilibrium in which the policy rules are simple time and state-invariant taxes on debt and dividends, with tax rates set equal to their long-run averages under the optimal macro-prudential policy.

This simple macro-prudential framework with fixed taxes achieves smaller welfare gains that are about 1/3rd of the welfare gains attained by the regulator's problem (as discussed below). But since the welfare gains of the regulator itself are small, it is more interesting to examine the implications of this policy on economic performance. In this regard the results are quite positive: The probability and severity of financial crises still fall sharply relative to

[26]If we reduce the debt tax we can obtain again average welfare gains, which again illustrates the interdependence of macro-prudential policies.

[27]A related criticism is that Ramsey optimal policies are time-inconsistent, but as we explained earlier, the financial regulator problem here is time-consistent by construction.

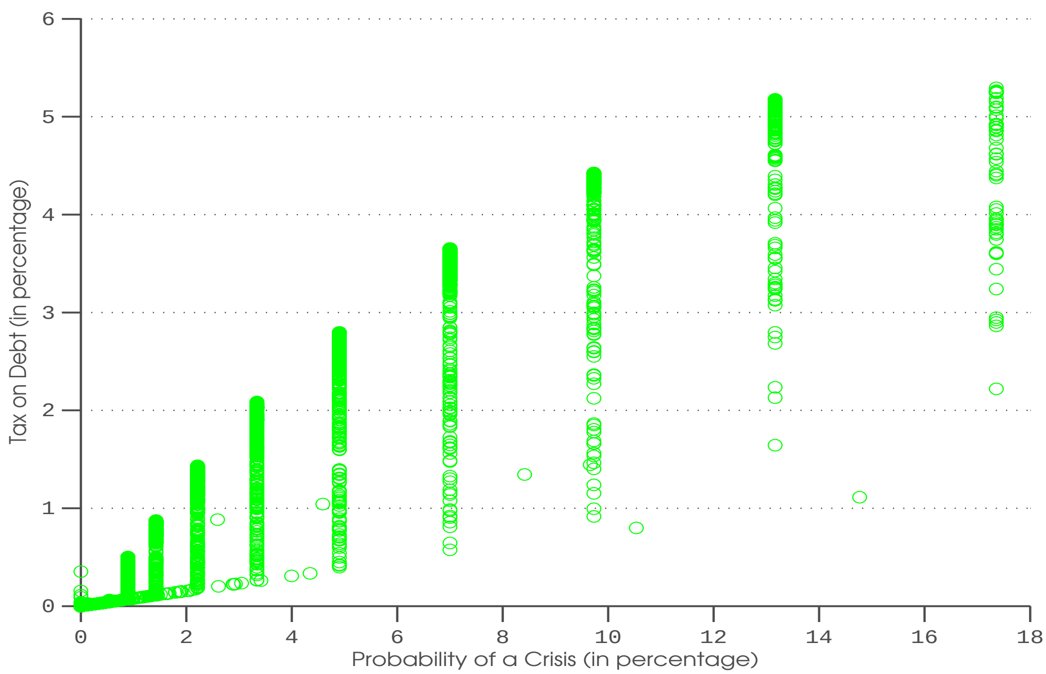

Figure 9: Tax on Debt is increasing in Crisis Probability

the competitive equilibrium, although relatively less than with the fully optimal policy (e.g. asset prices fall 7 percent for the optimal regulator v. 12 percent with fixed taxes, and 24 percent in the decentralized equilibrium). Partial use of this simple policy, by implementing only the fixed tax on debt, is again welfare reducing, and again the intuition is due to the fact that debt taxes have a depressing effect on asset prices, which tightens the collateral constraint in states where the collateral constraint already binds.

The results for the regulator's optimal policy and the fixed-taxes policy show that, while our results may provide a justification for the use of macro-prudential policies, they also provide a warning: Selective use of macro-prudential policies (i.e. partial implementation of the policy instruments) can reduce welfare in some states of nature. In this experiment, this happens because the selective use of the debt tax without the tax on dividends lowers asset prices in some states of nature, and reduces welfare in those states by reducing the value of collateral.

Jeanne and Korinek (2010) also computed a schedule of macro-prudential taxes on debt to correct a similar externality that arises because of a collateral constraint that depends on asset prices. Their findings, however, are quite different, because their results show that macro-prudential taxes lessen the effects of financial crises much less than in our setup and have no effect on the probability of crises. These different results are due to differences in the structure of the borrowing constraints, the behavior of output, and the design of the quantitative experiments.

Their credit constraint is determined by the aggregate level of assets $\bar{K}$ and by a linear state- and time-invariant term ψ (i.e. their constraint is defined as $\frac{b_{t+1}}{R} \geq -\kappa q_t \bar{K} - \psi$). The fact that this constraint depends on aggregate rather than individual asset holdings, as in our model, matters because it implies that agents do not value additional asset holdings as a mechanism to manage their borrowing ability.[28] But more importantly, in their quantitative analysis they set parameter values to $\kappa = 0.046$, $\psi = 3.07$ and $q_t \bar{K} = 4.8$, which imply that the effects of the credit constraint are driven mainly by ψ, and only 7 percent of the borrowing ability depends on the value of assets (0.07=0.046*4.8/(0.046*4.8+3.07)). As a result, the Fisherian deflation effect and the credit externality are weak, and thus macro-prudential policy is not very effective at containing financial crises. The asset price drop is reduced from 12.3 to 10.3 percent, and the consumption drop is reduced from 6.2 to 5.2 percent (compared with declines from 24 to 7 and 16 to 5 percent respectively in our model). Moreover, since they model output (or dividends) as an exogenous, regime-switching Markov process such that the probability of a crisis (i.e. binding credit constraint) coincides with the probability of a bad output realization, macro-prudential policy cannot affect the probability of crises.

4.7 Welfare Effects

We move next to explore the welfare implications of the credit externality. To this end, we calculate welfare costs as compensating consumption variations for each state of nature that make agents indifferent between the allocations of the competitive equilibrium and those attained by the financial regulator. Formally, for a given initial state (B, ε) at date 0, the

[28] If we adopt the same assumption in our model, and maintain the same baseline calibration, the fact that individual agents do not value assets as collateral causes even larger asset price drops during financial crises, and this leads agents to accumulate more precautionary savings, which results in crises having zero probability in the long-run for both the competitive equilibrium and the financial regulator.

welfare cost is computed as the value of γ that satisfies this condition:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1+\gamma_0) - G(n_t^{DE})) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(n_t^{SP})) \tag{21}$$

where the superscript DE denotes allocations in the decentralized competitive equilibrium and the superscript SP denotes the regulator's allocations. Note that these welfare costs measure also the (negative of) the welfare gains that would be obtained by introducing the regulator's optimal debt and dividend tax policies.

The welfare losses of the DE arise from two sources. The first source is the higher variability of consumption, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs due to the effect of the credit friction on working capital. Without the working capital constraint, the marginal disutility of labor equals the marginal product of labor. With the working capital constraint, however, the shadow cost of employing labor rises when the constraint binds, and this drives a wedge between the marginal product of labor and its marginal disutility. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss.

Figure 6 plots the welfare costs of the credit externality as a function of b for a negative, two-standard-deviations TFP shock. These welfare costs are sharply increasing in b at the high debt levels of the constrained region and some of the high externality region, and after that become decreasing in b and with a much flatter slope. This pattern is due to the differences in the optimal plans of the regulator vis-a-vis private agents in the decentralized equilibrium. Recall than in the constrained region, the current allocations of the DE essentially coincide with those of the SP, as described in Figure 1. Therefore, in this region the welfare gains from implementing the regulator's allocations only arise from how future allocations will differ. On the other hand, in the high-externality region, the regulator's allocations differ sharply from those of the DE, and this generally enlarges the welfare losses caused by the credit externality. Notice that, since the regulator's allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the regulator, but these are far outweighed by less vulnerability to sharp decreases in future

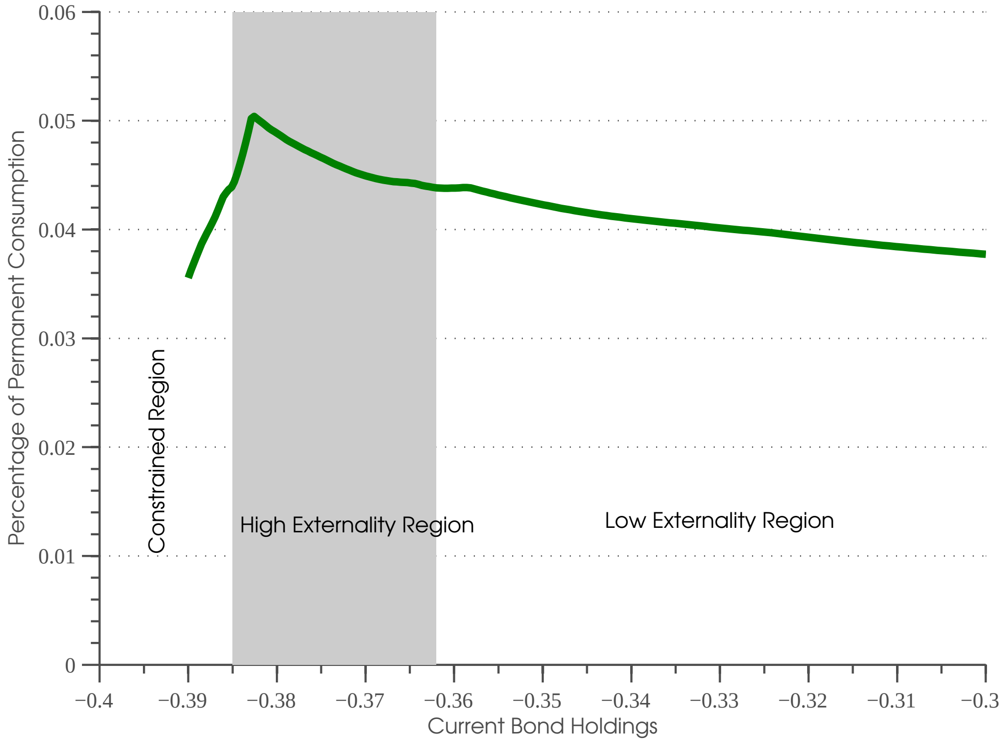

Figure 10: Welfare Costs of the Credit Externality for a two-standard-deviations TFP Shock

consumption during financial crises. Finally, as the level of debt is decreased further and the economy enters the low-externality region, financial crises are unlikely and the welfare costs of the inefficiency decrease.

The unconditional average welfare cost computed using the DE's ergodic distribution of bonds and TFP is 0.046 percentage points of permanent consumption. This cost is only about 1/3rd the cost found by Bianchi (2011). Note, however, that our results are in line with his if we express the welfare costs as a fraction of the variability of consumption. Consumption was more volatile in his setup because he examined a calibration to data for emerging economies, which are more volatile than the United States. Still, the welfare costs of the externality are small in both our analysis and Bianchi's.

The fact that welfare losses from the externality are small although the differences in consumption variability are large is related to the well-known Lucas result that models with CRRA utility, trend-stationary income, and no idiosyncratic uncertainty produce low welfare costs from consumption fluctuations. Moreover, the efficiency loss in the supply-side when the constraint binds produces low welfare costs on average because those losses have a low probability in the ergodic distribution.

4.8 Sensitivity Analysis

We examine now how sensitive are the effects of the credit externality to changes in the values of the model's key parameters. Table 5 shows the main model statistics for different values of σ, κ, ω, θ, σ_ε and ρ_ε, as well as experiments in which we make κ or R stochastic and an experiment in which we allow R to increase with aggregate bond holdings B' according to an exogenous function. The Table shows the unconditional averages of the tax on debt and the welfare loss, the covariance effect on excess returns, the probability of financial crises, and the impact effects of a financial crisis on key macroeconomic variables for both the DE and SP. In all of these experiments, only the parameter listed in the first column changes and the rest of the parameters remain at their baseline calibration values. Keep in mind that in reading this Table, differences between the DE and SP columns reflect how the parameter changes affect the effects of the externality, while differences across rows, for a given DE or SP column, reflect how the parameter changes affect each equilibria.

The results of the sensitivity analysis reported in Table 5 can be understood more easily by referring to the externality term derived in Section 2: The wedge between the social and private marginal costs of debt that separate the optimal plans of the competitive equilibrium and the regulator, $\beta R E_t\left[\mu_{t+1}\left(\kappa\bar{K}\frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1}\frac{\partial w_{t+1}}{\partial b_{t+1}}\right)\right]$. For given β and R, the magnitude of the externality is given by the expected product of two terms: the shadow value of relaxing the credit constraint, μ_{t+1}, and the associated price effects $\kappa\bar{K}\frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1}\frac{\partial w_{t+1}}{\partial b_{t+1}}$, which determine the effects of the externality on the ability to borrow when the constraint binds. As explained earlier, the price effects are driven mostly by $\frac{\partial q_{t+1}}{\partial b_{t+1}}$, because of the documented large asset price declines when the collateral constraint binds (see Figure 1). It follows, therefore, that the quantitative implications of the credit externality depend mainly on the parameters that affect $\mu_{t+1}$and $\frac{\partial q_{t+1}}{\partial b_{t+1}}$, as well as those that affect the probability of hitting the constraint.

Changes in Structural Parameters—The coefficient of relative risk aversion σ plays a key role because it affects both $\mu_{t+1}$and $\frac{\partial q_{t+1}}{\partial b_{t+1}}$. A high σ implies a low intertemporal elasticity of substitution in consumption, and therefore a high value from relaxing the constraint since a binding constraint hinders the ability to smooth consumption across time. A high σ also makes the stochastic discount factors more sensitive to changes in consumption, and therefore makes asset prices react more to changes in bond holdings. Accordingly, rising σ from 2 to 2.5 rises the welfare cost of the credit externality by a factor of 5, and widens the differences in the covariance effects across the DE and SP. In fact, the covariance effect in the decentralized equilibrium increases from 0.22 to 0.37 whereas for the SP the increase is from 0.06 to 0.08. Stronger precautionary savings reduce the probability of crises in the DE, and financial crises become a zero-probability event in the SP. Conversely, reducing σ to 1 makes the externality small, which results in much smaller gaps in the crisis probability and crisis impact effects across DE and SP, and negligible welfare cost of the externality.[29]

The collateral coefficient κ also plays an important role because it alters the effect of asset price changes on the borrowing ability. A higher κ implies that, for a given price response, the change in the collateral value becomes larger. Thus, this effect makes the externality stronger. On the other hand, a higher κ has two additional effects that go in the opposite direction. First, a higher κ implies that the direct effect of the collateral constraint on the asset price is weaker (recall eq. (9)), leading to a lower fall in the price of assets during crises. Second, a higher κ makes the constraint less likely to bind, reducing the externality. The effects of changes in κ are clearly non-monotonic. If κ is equal to zero, there is no effect of prices on the borrowing ability. At the same time, for high enough values of κ, the constraint never binds. In both cases, the externality does not play any role. Quantitatively, Table 5 shows that small changes in κ around the baseline value are positively associated with the size of the externality. In particular, an increase in κ from the baseline value of 0.36 to 0.40 increases the welfare cost of the externality by a factor of 6 and financial crises again become a zero-probability event for the regulator.

The above results have interesting policy implications. In particular, they suggest that while increasing credit access by rising κ may increase welfare relative to a more financially constrained environment, rising κ can also strengthen the effects of credit externalities and hence make macro-prudential policies more desirable (since the welfare cost of the externality

[29]Notice that the probability of a crisis in the DE increases to almost 10 percent, more than three times larger than the U.S. data target employed in the baseline calibration, because of the reduction in precautionary savings (albeit at the same time the impact effects of crises are significantly more tepid).

also rises).

Consider next the effects of changing the Frisch elasticity of labor supply. An elasticity higher than the baseline (1.2 v. 1) implies that output drops more when a negative TFP shock hits. If the credit constraint binds, this implies that consumption falls more, which increases the marginal utility of consumption and raises the return rate at which future dividends are discounted.[30] Moreover, everything else constant, a higher labor elasticity makes the externality term higher by weakening the effects of wages on the borrowing capacity. Hence, a higher elasticity of labor supply is associated with higher effects from the credit externality, captured especially by larger differences in the severity of financial crises, a higher probability of crises, and a larger welfare cost of the credit externality.

The fraction of wages that have to be paid in advance θ plays a subtle role. On one hand, a larger θ increases the shadow value of relaxing the credit constraint, since this implies a larger rise in the effective cost of hiring labor when the constraint binds. On the other hand, a larger θ implies, ceteris paribus, a weaker effect on borrowing ability, since the reduction of wages that occurs when the collateral constraint binds has a positive effect on the ability to borrow. Quantitatively, increasing (decreasing) θ by 5 percent increases (decreases) slightly the effects that reflect the size of the externality.

Changes in the volatility and autocorrelation of TFP do not have significant effects. Increasing the variability of TFP implies that financial crises are more likely to be triggered by a large shock. This results in larger amplification and a higher benefit from internalizing price effects. In general equilibrium, however, precautionary savings increase too, resulting in a lower probability of financial crises for both DE and SP. Therefore, the overall effects on the externality of a change in the variability of TFP depend on the relative change in the probability of financial crisis in both equilibria and the change in the severity of these episodes. An increase in the autocorrelation of TFP leads to more frequent financial crises for given bond decision rules. Again, in general equilibrium, precautionary savings increase making ambiguous the effect on the externality.

[30]The increase in leisure mitigates the decrease in the stochastic discount factor but does not compensate for the fall in consumption

Table 5: Sensitivity Analysis

	Average	Welfare	Covariance Effect		Crisis Probab.		Average Effects on Impact during Financial Crises: Consumption		Credit		Asset Price		GDP	
	Tax	Loss	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP
(1) benchmark	1.1	0.05	0.22	0.06	3.0	0.9	-15.7	-5.3	-20.5	-1.7	-24.2	-7.0	-6.3	-4.6
(2) $\sigma = 1$	0.6	0.001	0.03	0.03	9.3	7.8	-3.8	-3.3	-2.2	-1.4	-2.8	-2.4	-2.6	-2.6
(3) $\sigma = 1.5$	1.0	0.01	0.08	0.04	5.7	2.5	-6.5	-3.1	-4.0	-0.2	-6.1	-2.5	-3.7	-2.9
(4) $\sigma = 2.5$	1.2	0.24	0.37	0.08	2.0	0.0	-15.9	-4.2	-19.7	0.64	-30.3	-7.0	-5.8	-4.0
(e)$\kappa = 0.32$	1.0	0.02	0.14	0.06	4.6	2.1	-8.8	-3.7	-9.4	0.4	-13.2	-5.1	-4.7	-3.6
(5) $\kappa = 0.4$	1.2	0.29	0.34	0.06	2.2	0.0	-17.2	-4.2	-21.6	1.7	-27.9	-5.7	-5.7	-4.0
(6) $1/\omega = 0.83$	1.0	0.03	0.16	0.05	3.7	1.1	-8.6	-3.4	-8.5	0.8	-12.3	-4.3	-4.4	-3.3
(7) $1/\omega = 1.2$	1.1	0.12	0.27	0.06	3.9	2.3	-18.5	-4.9	-25.9	-1.0	-29.8	-6.3	-6.7	-4.6
(8) $\theta = 0.13$	1.1	0.03	0.18	0.06	1.7	1.6	-15.3	-5.2	-20.0	-1.6	-23.7	-6.9	-6.2	-4.5
(9) $\theta = 0.15$	1.1	0.05	0.21	0.06	2.8	1.2	-17.5	-5.4	-24.0	-2.0	-27.5	-7.1	-6.6	-4.6
(10) $\sigma_\varepsilon = 0.010$	1.1	0.06	0.19	0.04	3.2	0.0	-13.9	-4.3	-19.3	-2.5	-21.9	-5.9	-4.9	-3.4
(11) $\sigma_\varepsilon = 0.018$	1.0	0.05	0.26	0.08	2.5	0.2	-17.3	-6.9	-21.4	-1.9	-26.7	-9.0	-7.5	-5.9
(12)$\rho_\varepsilon = 0.43$	1.1	0.05	0.22	0.05	2.8	0.00	-15.6	-4.9	-20.1	-0.9	-24.1	-6.4	-6.3	-4.5
(13) $\rho_\varepsilon = 0.63$	1.1	0.05	0.23	0.06	2.7	1.2	-16.1	-5.2	-21.7	-2.0	-25.1	-6.9	-6.3	-4.5
(14) Stochastic κ	1.1	0.04	0.25	0.06	3.0	1.2	-12.0	-3.8	-15.1	0.01	-18.3	-5.2	-5.3	-3.6
(15) Stochastic r	1.1	0.07	0.41	0.10	2.9	1.0	-28.6	-4.2	-45.3	-1.3	-48.8	-4.6	-7.2	-3.5
(16) Endogenous r	1.1	0.05	0.26	0.11	3.0	1.2	-17.8	-5.3	-24.5	-1.7	-28.1	-7.0	-6.6	-4.6

Note: 'DE' represents the decentralized equilibrium,'SP' represents the social planner/financial regulator. The average tax on debt corresponds to the average value of the state contingent tax on debt to decentralize the financial regulator allocations. The covariance effect represents the unconditional average of the covariance effect. Consumption, credit, asset prices and output are responses of these variables on impact during a financial crisis (see section 4.4 for a definition of the event analysis). The baseline parameter values are: $R-1=0.028, \beta=0.96, \sigma=2, \alpha_h=0.64, \chi=0.64, \omega=1, \bar{K}=1, \theta=0.14, \kappa=0.36, \alpha_K=0.05, \sigma_\varepsilon=0.014, \rho_\varepsilon=0.53$.

Collateral Shocks and Interest Rate Shocks — We consider now additional sources of uncertainty. First, we consider shocks that affect directly the collateral constraint by introducing a stochastic process for κ that follows a symmetric two-state Markov chain independent from TFP shocks. In line with evidence from Mendoza and Terrones (2008) on the mean duration of credit booms in industrial countries, we calibrate the probabilities of the Markov chain so that the average duration of each state is 7 years. We keep the average value of κ as in our benchmark model and consider fluctuations of κ of 10 percent. Under these assumptions, the collateral constraint only binds when κ takes the smallest value in the vector of Markov realizations. As shown in Table 5, the effects of the externality do not change significantly. Intuitively, this occurs again because shocks to κ strengthen incentives for precautionary savings for both DE and SP, and thus the effects of the externality are negligible.

Next we consider shocks to the real interest rate. We calibrate interest rate shocks using the ex-post real return of 3-month U.S. Treasury Bills , as a proxy for risk-free assets. This yields an interest rate process at the business cycle frequency with a standard deviation of 2 percent and an insignificant correlation with GDP. Thus, we assume the interest rate process is independent of TFP (in the model the correlation between output and R is -0.03 v. 0.005 in the data). As row (15) of Table 5 shows, this stochastic interest rate increases significantly the effects of the externality. Intuitively, periods of low interest rates, such as those observed in the United States before the 2008 crash, encourage a credit boom and a larger build-up of leverage in good times, which result in more severe effects on the economy when it enters a financial crises.

Endogenous Interest Rate — In this last sensitivity experiment, we consider a real interest rate that varies with aggregate bond holdings following an exogenous function. In our baseline model, there is a perfectly elastic supply of funds at a constant rate R. Consider instead a scenario in which households collectively affect the world interest rate, so that R depends on the aggregate bond holdings B'. In particular, we consider an interest rate such that $r(B') = r + \varrho(e^{(\overline{B}-B')} - 1)$. [31] With $\varrho > 0$, the interest rate increases as B' increases. We set the value of $\overline{B}$ equal to the mean value of bonds in the decentralized equilibrium with constant interest rate, and set ϱ to match the standard deviation of the interest rate in the data. In principle, this could work to attenuate the Fisherian deflation and the fire-sale externality, because of the endogenous self-correcting mechanism increasing the cost of borrowing as debt increases, but we found that the quantitative effects produced by our baseline remain largely unchanged. As Table 5 shows, except for a modest decline in the gap in the probability of a financial crisis and risk premium across DE and SP, the effects of the externality remain about the same.

Summary —Overall, the results of this sensitivity analysis show that parameter changes that weaken the model's financial amplification mechanism also weaken the magnitude of the externality. This results in smaller average taxes, smaller welfare costs and smaller differences in the incidence and severity of financial crises. The coefficient of risk aversion is particularly important also because it influences directly the price elasticity of asset demand, and hence it determines how much asset prices can be affected by the credit externality. This parameter plays a role akin to that of to the elasticity of substitution in consumption of tradables and non-tradables in Bianchi (2011), because in his model this elasticity drives the response of the price at which the collateral is valued. Accordingly, he found that the credit externality has significant effects only if the elasticity is sufficiently low.

The results of the sensitivity analysis also produce an important additional finding: The average debt tax of about 1.1 percent is largely robust to the parameter variations we considered. Except for the scenario that approximates logarithmic utility ($\sigma = 1$), in all other scenarios included in Table 5 the mean tax ranges between 1.01 and 1.2 percent.

[31] Schmitt-Grohe and Uribe (2003) make this assumption to avoid the problem of the unit root that arises when small open economy models are linearized around the steady state. Its purpose here is to allow us to approximate what would happen if we were to allow for the interest rate to respond to debt choices in a richer general equilibrium model. To make the analysis more comparable with our baseline, and in order to focus on the fire-sale externality, we do not allow the regulator to internalize its effects on the interest rate. Doing so, would only strengthen our results about the role of macro-prudential policy.

5 Conclusion

This paper examined the positive and normative effects of a credit externality in a dynamic stochastic general equilibrium model in which a collateral constraint limits access to debt and working capital to a fraction of the market value of an asset in fixed supply. We compared the allocations and welfare attained by private agents in a competitive equilibrium, in which agents face this constraint taking prices as given, with those attained by a financial regulator who faces the same borrowing limits but takes into account how current borrowing choices affect future asset prices and wages. This regulator internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs.

Our analysis quantifies the effects of the credit externality in a setup in which the credit friction has effects on both aggregate demand and supply. On the demand side, consumption drops as access to debt becomes constrained, and this induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and labor demand are affected by the collateral constraint because firms buy labor using working capital loans that are limited by the collateral constraint, and hence when the constraint binds the effective cost of labor rises, so the demand for labor and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the Macro/Finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing regulated v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to U.S. data. This analysis showed that, even though the credit externality results in only slightly larger average ratios of debt and leverage to output compared with the regulator's allocations, the credit externality does produce financial crises that are significantly more severe and more frequent than in the regulated equilibrium, and produces higher long-run business cycle variability. There are also important asset pricing implications. In particular, the credit externality and its associated higher macroeconomic volatility in the competitive equilibrium produce equity premia, Sharpe ratios, and market price of risk that are much larger than in the regulated economy. We also found that the degree of risk aversion plays a key role in our results, because it is a key determinant of the response of asset prices to volatility in dividends and stochastic discount factors. For the credit externality to be important, these price responses need to be nontrivial, and we found that they are nontrivial already at commonly used risk aversion parameters, and larger at larger risk aversion coefficients that are still in the range of existing estimates.

This analysis has important policy implications. In particular, the financial regulator can decentralize its optimal allocations as a competitive equilibrium by introducing state-contingent schedules of taxes on debt and dividends. By doing so, it can neutralize the credit externality and produce an increase in social welfare. In our calibrated model, the tax on debt necessary to attain this outcome is about 1 percent on average. The tax is higher when the economy is building up leverage and becoming vulnerable to a financial crisis, but before a crisis actually occurs, so as to induce private agents to value more the accumulation of precautionary savings than they do in the competitive equilibrium without taxes.

These findings are relevant for the ongoing debate on the design of new financial regulation to prevent financial crises, which emphasizes the need for "macro-prudential" regulation. Our results lend support to this approach by showing how to construct policy rules that can tackle credit externalities associated with fire-sales of assets with large adverse macroeconomic effects. At the same time, however, we acknowledge that actual implementation of macro-prudential policies remains a challenging task. In particular, the optimal design of these policies requires detailed information on a variety of credit constraints that private agents and the financial sector face, real-time data on their leverage positions, and access to a rich set of state-contingent policy instruments. Moreover, as we showed in this paper, implementing only a subset of the optimal policies because of these limitations (or limitations of the political process) can reduce welfare in some states. On the other hand, we did find that rules simpler than the optimal state-contingent taxes, such as time-invariant taxes on debt and dividends, can still reduce noticeable the incidence and magnitude of financial crises.

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Appendix: Numerical Solution Method

The computation of the competitive equilibrium requires solving for functions $\mathcal{B}(b,\varepsilon), q(b,\varepsilon), \mathcal{C}(b,\varepsilon), \mathcal{N}(b,\varepsilon), \mu(b,\varepsilon)$ such that:

$$\mathcal{C}(b,\varepsilon) + \frac{\mathcal{B}(b,\varepsilon)}{R} = \varepsilon F(\overline{K}, \mathcal{N}(b,\varepsilon)) + b \tag{22}$$

$$-\frac{\mathcal{B}(b,\varepsilon)}{R} + \theta G'(\mathcal{N}(b,\varepsilon))\mathcal{N}(b,\varepsilon) \leq \kappa q(b,\varepsilon)\overline{K} \tag{23}$$

$$u'(t) = \beta R E_{\varepsilon'/\varepsilon}\left[u'(\mathcal{C}(\mathcal{B}(b,\varepsilon),\varepsilon^{'}))\right] + \mu(b,\varepsilon) \tag{24}$$

$$\varepsilon F_n(\overline{K}, \mathcal{N}(b,\varepsilon)) = G'(\mathcal{N}(b,\varepsilon))\mathcal{N}(b,\varepsilon)(1 + \theta\mu(b,\varepsilon)/u'(\mathcal{C}(b,\varepsilon)) \tag{25}$$

$$q(b,\varepsilon) = \frac{\beta E_{\varepsilon'/\varepsilon}\left[u'(c(\mathcal{B}(b,\varepsilon),\varepsilon^{'}))\varepsilon' F_k(\overline{K}, \mathcal{N}(\mathcal{B}(b,\varepsilon),\varepsilon')) + q(\mathcal{B}(b,\varepsilon),\varepsilon')\right]}{(u'(\mathcal{C}(b,\varepsilon)\ ) - \mu(b,\varepsilon)\kappa)} \tag{26}$$

We solve the model using a time iteration algorithm developed by Coleman (1990) modified to address the occasionally binding endogenous constraint. The algorithm follows these steps:[32]

1. Generate a discrete grid for the economy's bond position $G_b = \{b_{1,}b_2, ...b_M\}$ and the shock state space $G_\varepsilon = \{\varepsilon_{1,}\varepsilon_2, ...\varepsilon_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture $\mathcal{B}_K(b,\varepsilon), q_K(b,\varepsilon), \mathcal{C}_K(b,\varepsilon), \mathcal{N}_K(b,\varepsilon), \mu_K(b,\varepsilon)$ at time K $\forall\ b \in G_b$ and $\forall\ \varepsilon \in G_\varepsilon$.

3. Set $j = 1$

4. Solve for the values of $\mathcal{B}_{K-j}(b,\varepsilon), q_{K-j}(b,\varepsilon), \mathcal{C}_{K-j}(b,\varepsilon), \mathcal{N}_{K-j}(b,\varepsilon), \mu_{K-j}(b,\varepsilon)$ at time $K - j$ using (22),(23),(24),(25), (26) and $\mathcal{B}_{K-j+1}(b,\varepsilon), q_{K-j+1}(b,\varepsilon), \mathcal{C}_{K-j+1}(b,\varepsilon)$, $\mathcal{N}_{K-j+1}(b,\varepsilon), \mu_{K-j+1}(b,\varepsilon) \forall\ b \in G_b$ and $\forall\ Y \in G_Y$:

[32] For the financial regulator's allocations, we use the same algorithm operating on the regulator's optimality conditions.

(a) Assume collateral constraint (23) is not binding. Set $\mu_{K-j}(b, \varepsilon) = 0$ and solve for $\mathcal{N}_{K-j}(b, \varepsilon)$ using (25). Solve for $\mathcal{B}_{K-j}(b, \varepsilon)$ and $\mathcal{C}_{K-j}(b, \varepsilon)$ using (22) and(24) and a root finding algorithm.

(b) Check whether $-\frac{\mathcal{B}_{K-j}(b,\varepsilon)}{R} + \theta G'(\mathcal{N}(b, \varepsilon))\mathcal{N}_{K-j}(b, \varepsilon) \leq \kappa q_{K-J+1}(b, \varepsilon)\overline{K}$ holds.

(c) If constraint is satisfied, move to next grid point.

(d) Otherwise, solve for $\mu(b, \varepsilon), \mathcal{N}_{K-J}(b, \varepsilon), \mathcal{B}_{K-j}(b, \varepsilon)$ using (23), (24) and(25) with equality.

(e) Solve for $q^N_{K-j}(b, \varepsilon)$ using (26)

5. Evaluate convergence. If $sup_{B,\varepsilon}\|x_{K-j}(B, \varepsilon) - x_{K-j+1}(B, \varepsilon)\| < \epsilon$ for $x = \mathcal{B}, \mathcal{C}, q, \mu, \mathcal{N}$ we have found the competitive equilibrium. Otherwise, set $x_{K-j}(B, \varepsilon) = x_{K-j+1}(B, \varepsilon)$ and $j \rightsquigarrow j + 1$ and go to step 4.