Monetary Independence and Rollover Crises *

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Abstract

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. We study a sovereign default model with self-fulfilling rollover crises, foreign currency debt, and nominal rigidities. When the government lacks monetary independence, lenders anticipate that the government would face a severe recession in the event of a liquidity crisis, and are therefore more prone to run on government bonds. In a quantitative application to the Eurozone debt crisis, we find that the lack of monetary autonomy played a central role in making Spain vulnerable to a rollover crisis. Finally, we argue that a lender of last resort can go a long way towards reducing the costs of giving up monetary independence.

Keywords: Sovereign Debt Crises, Rollover Risk, Monetary unions
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1 Introduction

A prominent concern during the Eurozone crisis was the risk of a rollover crisis. Policymakers feared that an adverse shift in market expectations would restrict governments' ability to roll over their debt, creating liquidity problems that would feed back into investors' expectations and ultimately lead governments to default. At the same time, the premise was that the lack of monetary independence was aggravating sovereign debt problems in Southern Europe. In this context, the European Central Bank (ECB) took unprecedented policy measures aimed at stabilizing financial markets and reducing the risk of a breakup of the monetary union.[1]

The goal of this paper is to investigate whether and how the lack of monetary independence affects the vulnerability to a rollover crisis. A central question we tackle is: Does a country become more vulnerable after joining a monetary union?

We present a theory in which the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. The key insight is that lenders' pessimism can trigger a demand-driven recession, making the option to default more attractive for the government and, in turn, validating lenders' pessimism. With an independent monetary policy, a government can alleviate the recession that results from the fiscal contraction during a rollover crisis, making investors less prone to run in the first place. Quantitative simulations also show that while an economy that possesses monetary independence is almost immune to a rollover crisis, it can become significantly vulnerable once it joins a monetary union. Moreover, we argue that a lender of last resort can significantly mitigate the welfare costs from joining a monetary union and therefore enhance its stability.

The environment we consider is a version of the canonical model of sovereign default à la Eaton and Gersovitz (1981) that incorporates the possibility of rollover crises, as in Cole and Kehoe (2000). The government issues debt before deciding whether to repay or default. When lenders expect the government to default, the government is shut off from credit markets and forced to repay the maturing debt exclusively out of its tax revenues. When the maturing debt is large enough, repayment becomes too costly for the government, and lenders' pessimistic expectations are validated, a self-fulfilling rollover crisis arises. We depart from the standard endowment economy setup by considering nominal rigidities, which creates scope for a stabilization role for monetary policy. External debt is denominated in real terms, or equivalently in foreign currency, eliminating the possibility of inflating away the debt. The model features tradable and non-tradable goods and downward nominal wage rigidity, as in Schmitt-Grohé and Uribe (2016). In this environment, a shock leading to a contraction

[1] On September 6, 2012, Mario Draghi, the president of the European Central Bank, expressed that "the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium,' namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Preceding these remarks, Draghi famously pledged to do "whatever it takes to preserve the euro."

in aggregate demand reduces the price of non-tradables in equilibrium, generating a decline in labor demand. When wages cannot fall sufficiently quickly to clear the labor market, involuntary unemployment arises, and the economy goes through a recession. Following the classic principles from Friedman (1953), a government with an independent monetary policy can use the nominal exchange rate as a shock absorber, altering real wages and reducing unemployment.

Our main theoretical result is that the lack of an independent monetary policy increases the vulnerability to a rollover crisis. To understand the mechanisms in the model, consider what happens when a government is trying to roll-over its debt and investors suddenly panic and refuse to lend to it. As the government is shut off from credit markets, it needs to raise tax revenues and cut down on expenditures in order to service the maturing debt. In the presence of nominal rigidities and constraints on monetary policy, this situation has macroeconomic implications. The fiscal contraction generates a decline in aggregate demand, which leads to involuntary unemployment and makes repayment less attractive for the government. If the increase in unemployment is sufficiently large, the government finds it optimal to default, which in turn validates the initial panic by investors and generates a self-fulfilling rollover crisis. Interestingly, for this pessimistic equilibrium to emerge, unemployment does not have to be realized in equilibrium. In fact, it is the off-equilibrium outcome of a large recession that pushes the government to default and triggers the rollover crisis.

Under monetary independence, the government can offset the recessionary effects from the fiscal contraction that results from being shut off from credit markets. Because of this ability, the government's willingness to repay becomes relatively less affected by the lenders' pessimistic expectations. Compared with an economy that lacks monetary independence, a panic is therefore less likely to occur in the first place. We also show that these theoretical insights carry over to several variations of the baseline model. Among others, we show that the same results apply when the source of nominal rigidity is on prices rather than wages, when there is production in both sectors, and when there are benefits from following a fixed exchange rate regime.

We then proceed to conduct a quantitative investigation. We start by considering a calibration of the model under monetary independence. Specifically, a flexible exchange rate regime under which the government chooses the exchange rate optimally at each point in time. In this regime, the government finds it optimal to implement the full employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. (Notice, however, that the government cannot alter the value of the debt, since it is denominated in foreign currency.) Our simulations show that rollover crises play a modest role under a flexible exchange rate regime: only one out of 100 default episodes are driven by rollover crises.

We then examine the effects of giving up monetary independence. One can think of a small open economy that has a fixed exchange rate regime or, equivalently, a single small economy within a monetary union in which wages (and debt) are denominated in the currency of the union and the conduct of monetary policy is exogenous to the single small economy. Keeping the same parameter

values for the calibration of the flexible exchange rate regime, we find that the economy faces a significantly larger fraction of defaults due to rollover crises, which can reach about 11% (compared with 1% in the flexible exchange rate regime). Our findings therefore suggest that joining a monetary union entails significant costs in terms of a higher exposure to rollover crises.

Using the calibrated model for the fixed exchange rate regime, we then simulate the path of the Spanish economy, starting at the time of its adoption of the euro. We find that the economy hits the "crisis zone" precisely around the time of turmoil in sovereign debt markets. As a counterfactual, we show that if Spain had exited the Eurozone, it would have remained immune to a rollover crisis. The goal of this exercise, however, is not to argue that being part of a monetary union is undesirable but to point out that a cost of giving up monetary independence is higher vulnerability to rollover crises. An important welfare consequence that emerges from our analysis is that a lender of last resort can significantly reduce the costs of remaining in a monetary union. Consistent with our model, Mario Draghi pledged to do "whatever it takes," and after the celebrated speech, spreads fell immediately and Spain ultimately did not default on the debt.

Related literature. Our paper contributes to a vast literature on monetary unions, pioneered by the seminal work of Mundell (1961). The traditional view is that the benefit of joining a monetary union is larger international trade, fostered by lower transaction costs. A more modern view, stressed by Alesina and Barro (2002), has emphasized the benefits from a reduction in the inflationary bias generated by the time inconsistency problem of monetary policy identified in the seminal work of Barro and Gordon (1983). The main theme in the literature is that these benefits have to be traded off against the losses from inefficient macroeconomic fluctuations due to nominal rigidities and the lack of monetary independence. A comprehensive discussion of these issues, which have taken center stage since the formation of the Eurozone, is provided in Alesina, Barro, and Tenreyro (2003), Santos Silva and Tenreyro (2010), and De Grauwe (2020). A related literature compares the performance of fixed versus flexible exchange rates. Several studies in particular study the role of exchange rate policies in the presence of firms' balance sheet constraints (e.g., Céspedes, Chang, and Velasco, 2004; Gertler, Gilchrist, and Natalucci, 2007).[2]

Our paper adds a new dimension to the costs from giving up monetary independence, namely a higher exposure to rollover crises. Our welfare analysis shows that the higher exposure to rollover crises can be substantial and suggests that these costs should be part of the overall evaluation of a cost-benefit analysis. In this respect, our results shed some light on the Outright Monetary Transactions facility established by the ECB, following Mario Draghi's July 2012 speech pledging to do "whatever it takes to preserve the euro." Indeed, the paper shows that a lender of last resort can enhance a monetary union by substantially reducing the costs from the lack of monetary independence.

[2] Also related is an active closed economy literature on how the interaction between household deleveraging and a zero lower bound can amplify demand shocks (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017).

Our paper also belongs to the literature on rollover crises in sovereign debt markets, starting with Sachs (1984), Alesina, Prati, and Tabellini (1990), and Cole and Kehoe (2000). Our formulation follows Cole-Kehoe, which has become the workhorse model in the quantitative sovereign default literature in the tradition of Aguiar and Gopinath (2006) and Arellano (2008). Examples include Chatterjee and Eyigungor (2012), Aguiar, Chatterjee, Cole, and Stangebye (2016), Conesa and Kehoe (2017), Roch and Uhlig (2018), and Bocola and Dovis (2019). Different from these contributions, we consider an economy with production and nominal rigidities, and establish how the exchange rate regime is central to the risk of exposure to rollover crises. With a flexible exchange rate regime, we find the exposure to a rollover crisis to be minimal, which is in line with Chatterjee and Eyigungor (2012), who showed that in a canonical endowment economy model with long-term debt calibrated to the data, the presence of rollover crises has a negligible effect on debt and spreads. By contrast, we show that with a fixed exchange rate regime, the multiplicity region expands significantly, and the government is heavily exposed to a rollover crisis.[3]

The paper that is perhaps most closely related to ours is Aguiar, Amador, Farhi, and Gopinath (2013), who address the question of whether the government's ability to inflate away its debt reduces its exposure to rollover crises, an argument notably raised by De Grauwe (2013) and Krugman (2011), who made the observation that Spain and Portugal had higher levels of sovereign spreads compared with those of the UK, despite having lower levels of debt. Aguiar et al. consider an endowment economy with domestic currency debt and show that when commitment to low inflation is weak, an independent monetary policy can actually increase the vulnerability to a rollover crisis, contrary to De Grauwe and Krugman's argument. Our paper also studies how monetary policy matters for the exposure to a rollover crisis but considers instead a model with nominal rigidities and foreign currency debt. Our results show that the lack of monetary autonomy does increase vulnerability to a rollover crisis and provides a new perspective that ascribes a role for monetary policy to deal with rollover crises, even when debt is denominated in foreign currency.[4]

A related literature studies sovereign debt crises, but in the tradition of Calvo (1988), in which the government lacks commitment to debt issuances. If investors expect high inflation, the government

[3]With one-year maturity, as in Cole and Kehoe (1996, 2000), the exposure to a rollover crisis is typically large because the government has to roll over a large amount of debt relative to output every period. While they were motivated by the Mexican crisis in 1994 with maturity of less than a year, the typical maturity for sovereign bonds is much larger, averaging around six years for the Eurozone. With debt duration calibrated to the Eurozone, Conesa and Kehoe (2017) and Bocola and Dovis (2019) achieve a somewhat more significant role for rollover crises but rely implicitly on a minimum subsistence level for consumption, which they set to about 70% of income, and require debt levels of around 100% of GDP for a typical rollover crisis. Overall, the quantitative analysis in Bocola and Dovis still finds that non-fundamental risk played a limited role during the Italian debt crisis.

[4]Aguiar, Amador, Farhi, and Gopinath (2015) consider a setup similar to Aguiar, Amador, Farhi, and Gopinath (2013), but with multiple countries and a union-wide monetary policy. They show that for a country with a high level of debt, it is preferable to join a monetary union with a mix of high and low debt countries as a way to balance the costs from inflationary bias and the reduction in the vulnerability to rollover crises by inflating away the debt ex post. Other recent papers addressing issues of debt crises with a focus on the Eurozone are Broner, Erce, Martin, and Ventura (2014) and Gourinchas, Martin, and Messer (2017) (see also, De Ferra and Romei, 2020 and Fornaro, 2020).

borrows at a high rate and finds it optimal to inflate ex post, validating the initial expectations. In this line of work, the fact that debt is denominated in domestic currency and that the government can inflate away the debt is at the core of the fragility problem.[5] We consider a baseline model with debt in real terms, which allows us to abstract from the use of inflation to reduce the real value of the debt (and the associated multiplicity issues) to highlight a new channel by which monetary policy can actually help reduce a fragility problem originating from rollover crises.

Our paper is also related to an emerging literature that integrates nominal rigidities into the workhorse sovereign default model. Na, Schmitt-Grohé, Uribe, and Yue (2018) study a sovereign default model with downward nominal wage rigidity and show that it can account for the joint occurrence of large nominal devaluations and defaults, a phenomenon known as the "twin Ds." Bianchi, Ottonello, and Presno (2019) analyze the trade-off between the expansionary effects of government spending and the increase in sovereign risk and show how it can generate the observed fiscal procyclicality. Other recent papers include Arellano, Bai, and Mihalache (2019), who study the comovements of sovereign spreads with domestic nominal rates and inflation, and Bianchi and Sosa-Padilla (2019), who study the accumulation of international reserves as a macroeconomic stabilization tool. In contrast to this literature, we consider the possibility of rollover crises, which allows us to provide the first analysis of how nominal rigidities and monetary policy affect vulnerability to rollover crises.

2 Model

We study a small open economy (SOE) model with nominal rigidities, in which the government is unable to commit to repay the sovereign debt and is subject to rollover crises. We next describe the decision problems of households, firms, lenders, and the government.

2.1 Households

There is a unit measure of households with preferences over consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where c_t is a constant elasticity of substitution (CES) composite of tradable goods c_t^T and non-tradable goods c_t^N

$$c_t = [\omega (c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu}]^{-1/\mu}, \quad \omega \in (0,1),\ \mu > -1.$$

[5] A large literature on multiple equilibria follows this tradition, including Corsetti and Dedola (2016), Farhi and Maggiori (2018), Bacchetta, Perazzi, and Van Wincoop (2018), and Lorenzoni and Werning (2019). The role of inflation as a partial default also plays a key role in recent work by Araujo, Leon, and Santos (2013), Du and Schreger (2016), Bassetto and Galli (2019), Nuño and Thomas (2017), Camous and Cooper (2019), and Hur, Kondo, and Perri (2018).

Each period, households receive y_t^T units of tradable endowment, which is stochastic and follows a stationary first-order Markov process. We assume a constant unit price of tradable goods in terms of foreign currency and that the law of one price holds. The value of the tradable endowment in domestic currency is therefore given by $e_t y^T$, where e_t denotes the exchange rate measured as domestic currency per foreign currency (an increase in e_t denotes a depreciation of the domestic currency). Households also receive firms' profits, which we denote by ϕ_t^N, and labor income, $W_t h_t$, where W is the wage expressed in domestic currency and h is the amount of hours worked. Households inelastically supply $\bar{h}$ hours of work to the labor markets but will work a strictly lower amount of hours when the downward wage rigidity is binding.

As is standard in the sovereign debt literature, we assume that households do not have direct access to external credit markets, although the government can borrow abroad and distribute the net proceedings to the households using lump-sum taxes or transfers. The households' budget constraint is therefore given by

$$e_t c_t^T + P_t^N c_t^N = e_t y^T + \phi_t^N + W_t h_t - T_t, \tag{2}$$

where P_t^N denotes the price of non-tradables and T_t denotes lump-sum taxes, both in units of domestic currency.

The households' problem consists of choosing c_t^T and c_t^N to maximize (1), taking as given the sequence of prices for non-tradables, labor income, profits, and taxes $\left\{P_t^N, W_t h_t, \phi_t^N, T_t\right\}_{t=0}^{\infty}$. The static optimality condition equates the relative price of non-tradables to the marginal rate of substitution between tradables and non-tradables:

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{3}$$

Thanks to homotheticity, the relative demand of tradable and non-tradable consumption goods is a function only of the relative price.[6]

2.2 Firms

Firms operate a production function $y^N = F(h)$, where y_t^N denotes the output of non-tradable goods and h_t denotes employment, the sole input. The production function $F(\cdot)$ is a differentiable, increasing, and concave function. In particular, we will consider a homogeneous production function $F(h) = h^\alpha$, where $\alpha \in (0, 1]$. Firms operate in perfectly competitive markets, and in each period,

[6]Homotheticity, as implied by CES structure, is assumed to simplify the analysis. For our results, it suffices that non-tradable goods are normal.

they maximize profits that are given by

$$\phi_t^N = \max_{h_t} P_t^N F(h_t) - W_t h_t. \tag{4}$$

The optimal choice of labor employment h_t equates the value of the marginal product of labor to the nominal wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

Employment demand is decreasing in wages and increasing in the price of non-tradables.

2.3 Downward Nominal Wage Rigidity

Wages in domestic currency are downward rigid:

$$W_t \geq \overline{W}, \tag{6}$$

where the parameter $\overline{W}$ determines the severity of the rigidity.[7] If the nominal wage that clears the labor market is higher than $\overline{W}$, the economy is at full employment. If, however, the nominal wage that would clear the market is below $\overline{W}$, the wage rigidity binds and the economy experiences involuntary unemployment. In this case, the amount of employment in equilibrium is determined by firms' labor demand (5). Formally, wages and employment need to satisfy the following slackness condition:

$$\left(W_t - \overline{W}\right)\left(\bar{h} - h_t\right) = 0. \tag{7}$$

It will be convenient to define the corresponding market wage and wage lower bound in foreign currency as $w_t \equiv W_t/e_t$ and $\bar{w} \equiv \overline{W}/e_t$. An alternative representation of (6) is therefore $w \geq \bar{w}$.[8]

2.4 Government

The government issues a non-contingent, long-term bond with geometrically decaying coupons.[9] In particular, a bond issued in period t promises to pay $\delta(1-\delta)^{j-1}$ units of foreign currency in period $t+j$ for all $j \geq 1$. Debt dynamics can be represented by the following law of motion: $b_{t+1} = (1-\delta)b_t + i_t$,

[7] In Schmitt-Grohé and Uribe (2016), $\overline{W}$ depends on the previous period's wage and a parameter that controls the speed of wage adjustment. For numerical tractability, we take $\overline{W}$ as an exogenous (constant) value, as in Bianchi et al. (2019). A vast empirical literature documents the importance of downward wage rigidity. In particular, a recent literature has used micro-level data to highlight the important role that this friction played in the European crisis (e.g., Faia and Pezone, 2018; Ronchi and Di Mauro, 2017).

[8] For an economy within a currency union, wages are set in foreign currency (the currency of the union), and therefore the lower bound is also, in effect, in foreign currency.

[9] We take maturity as a primitive, following Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). There is an active literature studying maturity choices in sovereign default models (Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2019; Sanchez, Sapriza, and Yurdagul, 2018).

where b_t is the stock of debt owed at the beginning of period t, and i_t is the stock of new bonds issued in period t.

Debt contracts cannot be enforced. If the government chooses to default, it faces two punishments. First, the government remains in financial autarky for a stochastic number of periods. Second, there is a utility loss $\kappa(y^T)$, which we think of as capturing various default costs related to reputation, sanctions, or the misallocation of resources.[10]

The government's budget constraint in a period starting with good credit standing is

$$\delta e_t b_t(1-d_t) = T_t + e_t q_t i_t(1-d_t), \tag{8}$$

where q_t is the price of the bond in foreign currency and d_t is a default indicator that takes the value of 1 if the government repays and zero otherwise. The budget constraint indicates that repayment of outstanding debt obligations is made by collecting lump-sum taxes and issuing new debt.[11]

The timing within each period follows Cole and Kehoe (2000). At the beginning of each period, the government has outstanding debt liabilities b_t and could be in good or bad credit standing. If the government is in good credit standing, it chooses new debt issuances at the price schedule offered by investors. At the end of each period, the government decides whether to default or repay the initial debt outstanding. The difference with respect to Eaton and Gersovitz (1981) that will give rise to multiplicity is that here the government cannot commit to repaying within the period.[12] As we will see, negative beliefs about the decision of the government to repay can become self-fulfilling.

Monetary regimes. We consider two regimes: a flexible exchange rate and a fixed exchange rate. In the flexible exchange rate regime, the government chooses the optimal exchange rate at all dates without commitment. In the fixed exchange rate regime, we assume that the government sets the exchange rate to an exogenous fixed level at all times. Equivalently, one can interpret the fixed exchange rate regime as the policy of a single economy that enters a monetary union and gives up its

[10] Utility losses from default in sovereign debt models are also used by Bianchi, Hatchondo, and Martinez (2018) and Roch and Uhlig (2018), among others. An alternative often used is an output cost. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications. In any case, as will become clear below, what will be crucial for our mechanism is the gap between the value of repayment for the government when investors are willing to lend and when they refuse to lend, which is indepenent of the form of default costs.

[11] As is well understood, allowing for specific taxes on consumption or payroll subsidies can mimic a nominal depreciation (see e.g. Farhi, Gopinath, and Itskhoki 2013; Schmitt-Grohé and Uribe, 2016; Correia, Nicolini, and Teles, 2008). As long as there are some limitations on the use of these policies (either political or economic), there remains a role for explicit nominal depreciations. From a normative standpoint, the importance of the exchange rate regime that we will uncover applies therefore to the role of fiscal devaluation policies.

[12] A different source of multiplicity following Calvo (1988) arises if the government has to issue a fixed amount of debt revenues. In this case, the fact that bond prices decrease with debt generates a Laffer curve, which leads, directly through the budget constraint, to a high debt/high spreads equilibrium and a low debt/low spreads equilibrium. Lorenzoni and Werning (2019) and Ayres, Navarro, Nicolini, and Teles (2016) are recent explorations of this type of multiplicity in dynamic setups.

currency.[13] In the case of a monetary union, wages are directly set in the currency of the union. The important point, whether in a fixed exchange rate or in a monetary union, is that the government loses the ability to conduct its own monetary policy and use the exchange rate as a shock absorber.

2.5 International Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-free security denominated in units of foreign currency that pays a net interest rate r. By a no-arbitrage condition, equilibrium bond prices when the government repays are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{9}$$

2.6 Equilibrium

In equilibrium, the market for non-tradable goods must clear domestically:

$$c_t^N = F(h_t). \tag{10}$$

Combining the budget constraints of households (2) and the government (8) with firms' profits (4) and the market clearing condition (10), we obtain the resource constraint for tradable goods in the economy:

$$c_t^T = y^T + (1-d_t)[\delta b_t - q_t(b_{t+1} - (1-\delta)b_t)]. \tag{11}$$

Before proceeding to study a Markov equilibrium in which the government chooses policies optimally without commitment, let us examine the equilibrium for given government policies.

Definition 1 (Competitive Equilibrium)**.** Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that:

i) Households and firms solve their optimization problems.

ii) Government policies satisfy the government budget constraint (8).

iii) The bond pricing equation (9) holds.

iv) The market for non-tradable goods (10) clears

[13] One could also allow some degree of correlation between the small open economy and the monetary policy conducted at the union level by allowing P^* to follow a stochastic process correlated with the shocks to the small open economy. Theoretically, all our results would remain unchanged.

v) The labor market satisfies conditions (6), (7), and $h \leq \overline{h}$.

Employment, consumption, and wages. At the core of our framework is a relationship between between aggregate demand and employment. By combining the optimality for households and firms together with market clearing for non-tradable goods, we can obtain a useful (partial) characterization of equilibrium. As we show in Lemma A1, employment follows:

$$\mathcal{H}\left(c_t^T; \bar{w}\right) = \min\left\{\left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}, \bar{h}\right\}. \tag{H-demand}$$

Condition (H-demand) establishes that employment is increasing in tradable consumption (strictly so, if the level of employment is below $\bar{h}$). The core intuition is that if there are fewer resources available for tradable consumption, the demand for non-tradable goods must also fall (because both are normal goods). In turn, the fall in the households' demand for non-tradable goods leads in equilibrium to a lower employment demand by firms. When the downward wage rigidity is binding and monetary policy does not respond, the reduction in demand generates underutilization of labor.

Remark. The model features a Keynesian positive relationship between aggregate demand and employment. While we obtain this relationship under a specific structure with wage rigidity and a two-sector open economy model with external government borrowing, the positive relationship between aggregate demand and employment is a general feature of a large class of models with nominal rigidities. For example, in a SOE model with home and foreign goods and price rigidities à la Gali and Monacelli, a contraction in domestic resources available also generates a recession.

As we will see below, this Keynesian feature will crucially affect the incentives of the government to repay and its vulnerability to a rollover crisis.

2.7 Recursive Government Problem and Markov Equilibrium

We consider the optimal policy of a benevolent government that chooses without commitment. We focus on Markov equilibria. The payoff relevant states are $(b, \mathbf{s})$, where $\mathbf{s} = (y^T, \zeta)$ denotes the vector of exogenous states in every period. The variable ζ is a sunspot variable to index for the possibility of multiplicity of equilibria, which is assumed to be *i.i.d.* over time. We use $q(b', b, \mathbf{s})$ to denote the bond price schedule faced by the government. In contrast to the equilibrium according to the Eaton and Gersovitz (1981) timing, the possibility of a rollover crisis implies that the bond price is a function of the initial debt position and the sunspot, in addition to the debt choice and current income shock.

When the government has access to financial markets, it compares the values of repayment and

default, denoted respectively by $V_R(b, \mathbf{s})$ and $V_D(y^T)$.

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \{(1-d)V_R(b, \mathbf{s}) + dV_D(y^T)\}, \tag{12}$$

Fixed exchange rate regime. We start by focusing on a fixed exchange rate regime. The government chooses allocations, subject to the resource constraint and the implementability constraints associated with the labor market. Let us use $u\left(c_t^T, c_t^N\right)$ to denote the utility flow. The value function under repayment can be written as

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h} \left\{u(c^T, F(h)) + \beta \mathbb{E}\left[V(b', , \mathbf{s}')\right]\right\} \tag{13}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$h \leq \mathcal{H}\left(c^T, \bar{w}\right),$$

where $\mathcal{H}$ is defined in (H-demand). Meanwhile, the value of default is given by

$$V_D(y^T) = \max_h \left\{u\left(y^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})\right]\right., \tag{14}$$

subject to

$$h \leq \mathcal{H}\left(y^T, \bar{w}\right),$$

where $\psi \in [0, 1]$ denotes the probability of reentering financial markets after a default.

Definition 2 (Markov-perfect equilibrium)**.** A Markov-perfect equilibrium under a fixed exchange rate regime is defined by value functions $\{V(b, \mathbf{s}), V_R(b, \mathbf{s}), V_D(y^T)\}$, policy functions $\{\hat{d}(b, \mathbf{s}), \hat{c}^T(b, \mathbf{s}), \hat{b}(b, \mathbf{s}), \hat{h}(b, \mathbf{s})\}$, and a bond price schedule $q(b', b, \mathbf{s})$ such that

1. given the bond price schedule, the value functions and policy functions solve problems (12), (13), and (14);

2. the debt price schedule satisfies

$$q(b', b, \mathbf{s}) = \begin{cases} \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))] & \text{if } \hat{d}(b, \mathbf{s}) = 0, \\ 0 & \text{if } \hat{d}(b, \mathbf{s}) = 1, \end{cases}$$

where $b'' = \hat{b}\left(b', \mathbf{s}'\right)$ and $d' = d\left(b', \mathbf{s}'\right)$.

Flexible exchange rate regime. For the economy with a flexible exchange rate, the only difference in the government problem is that the optimization also includes the choice of the exchange rate. As should be clear from the nominal rigidity constraint $w \geq \overline{W}/e_t$, an exchange rate depreciation enables the government to offset the wage rigidity and achieve the flexible wage allocation. As shown in Proposition C1 in the Appendix, this is also the optimal time-consistent policy. Indeed, notice from problem (13) that a depreciation relaxes the implementability constraint $h \leq \mathcal{H}\left(c^T, \bar{w}\right)$. This result is of course in line with the traditional benefit of having a flexible exchange rate in the presence of nominal rigidities, going back to Friedman (1953) and Mundell (1961).

The stabilization of unemployment through the adjustment of real wages is indeed a central channel of monetary policy in open economies. Milton Friedman, for example, highlighted the dangers of Europe's eliminating the exchange rate adjustment precisely because of possible misalignments in real wages.[14] A subtle, yet important, difference in our theory is that a government depreciation may be a purely off-equilibrium policy.

2.8 Multiplicity of Equilibrium

We will look for two possible equilibria, one where investors lend and the government repays, and one in which investors refuse to lend and the government defaults.

Let us define the *fundamental price* as the price at which the government bond would trade if investors were willing to lend:

$$\tilde{q}(b', y^T) \equiv \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))]. \tag{15}$$

Denote by V_R^+ the value of repayment for the government when facing the fundamental price:

$$\begin{aligned} V_R^+(b, y^T) = \max_{b', c^T, h \leq \bar{h}} & \left\{u(c^T, F(h)) + \beta\mathbb{E}\left[V(b', \mathbf{s}')\right]\right\}, \\ \text{s.t.} \qquad & c^T = y^T - \delta b + \tilde{q}(b', \mathbf{s})[b' - (1-\delta)b], \\ & h \leq \mathcal{H}\left(c^T, \bar{w}\right). \end{aligned} \tag{16}$$

Consider now the situation in which investors are unwilling to lend to the government, a restriction that is relevant only when the government would want to issue debt when facing the fundamental

[14] As expressed by Milton Friedman in "Why Europe Can't Afford the Euro," *Times* (London), November 19, 1997, "If one country is affected by negative shocks that call for, say, lower wages relative to other countries, that can be achieved by a change in one price, the exchange rate, rather than by requiring changes in thousands on thousands of separate wage rates, or the emigration of labour. The hardships imposed on France by its 'franc fort' policy illustrate the cost of a politically inspired determination not to use the exchange rate to adjust to the impact of German unification. Britain's economic growth after it abandoned the exchange-rate mechanism a few years ago to refloat the pound illustrates the effectiveness of the exchange rate as an adjustment mechanism."

price. Denoting by $V_R^-(b)$ the value of repaying in this case, we have that

$$V_R^-(b, y^T) = \max_{c^T, h \leq \bar{h}} \left\{ u(y^T - \delta b, F(h)) + \beta \mathbb{E}\left[V((1-\delta)b, \mathbf{s}')\right] \right\}, \tag{17}$$
$$\text{s.t.} \qquad c^T = y^T - \delta b,$$
$$h \leq \mathcal{H}\left(c^T, \bar{w}\right),$$

Because the government can always choose not to borrow when lenders are willing to issue new debt, we have that $V_R^+ \geq V_R^-$.[15] Moreover, tradable consumption is necessarily lower when the government does not have access to borrowing, and hence employment will also be lower—recall that $\mathcal{H}(\cdot)$ is increasing in c^T. A key implication, which is at the heart of our model, is that the presence of wage rigidity will have a more substantial adverse effect on V_R^- than on V_R^+.

Three zones. Following Cole and Kehoe (2000), let us separate the state space into three zones: the safe zone, the default zone, and the crisis zone. In the *safe zone*, the government finds it optimal to repay its debt even if international lenders are unwilling to roll over the debt. In the *default zone*, the government finds it optimal to default regardless of whether international lenders are willing to lend. Finally, in the *crisis zone,* the government finds it optimal to repay if investors are willing to lend at the fundamental debt price schedule but finds it optimal to default if investors are unwilling to lend. These three zones can be characterized respectively as follows:

$$\mathcal{S} = \left\{ \left(b, y^T\right) : \; V_D\left(y^T\right) \leq V_R^-\left(b, y^T\right) \right\},$$
$$\mathcal{D} = \left\{ \left(b, y^T\right) : \; V_D\left(y^T\right) > V_R^+\left(b, y^T\right) \right\},$$
$$\mathcal{C} = \left\{ \left(b, y^T\right) : \; V_R^-\left(b, y^T\right) < V_D\left(y^T\right) \leq V_R^+\left(b, y^T\right) \right).$$

It is in the crisis zone that the equilibrium outcome is undetermined and depends on investors' beliefs. If investors believe the government will repay, the government will find it optimal to repay, whereas if they believe that the government will default, the government will find it optimal to default. To select an equilibrium, we will use a sunspot $\zeta \in \{0, 1\}$. If $\zeta = 0$, we will say there is a "good sunspot," in which case the equilibrium with repayment is selected. If $\zeta = 1$, we will say there is a "bad sunspot," in which case the equilibrium with default is selected. The probability of selecting the bad sunspot is denoted by π.

Using that the repayment value functions are strictly decreasing with respect to debt and that the value of default is independent of debt, we can show that for every y^T, there exists a pair of debt thresholds (b^-, b^+) that separates these three regions, as illustrated in Figure 1. In the next section,

[15] One element implicit in the budget constraint in problem (17) is that if the government were to repurchase debt when investors are unwilling to lend, the price of bonds would rise to the fundamental price, as reflected in (17). See Aguiar and Amador (2013) and Bocola and Dovis (2019) for an elaboration of this point.

we will study how the exchange rate regime affects these thresholds.

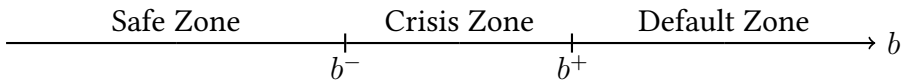


Figure 1: Debt thresholds for given y^T

3 Monetary Policy and Rollover Crises

In this section, we establish that the crisis zone is larger under a fixed exchange rate. We first present a graphical illustration of the mechanism and key insights and then provide a formal theoretical analysis.

3.1 Graphical Illustration

We consider a version of the model where wages are rigid in period t, and become fully flexible for $t+1, t+2, \ldots$. The goal is to analyze the vulnerability to a debt crisis at t depending on the ability to depreciate the exchange rate. Because wages become flexible in the future, the *fundamental* price schedule faced by the government is the same for a flexible or a fixed exchange rate regime, but as we will see, the exposure to rollover crises will be different in the initial period.[16]

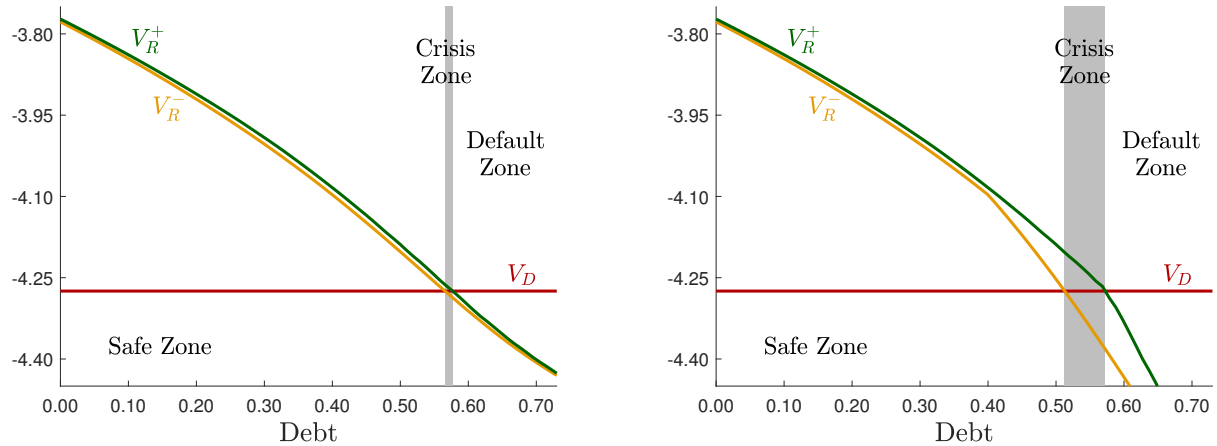
The crisis zone. To examine how we arrive at the crisis zone, Figure 2 presents the value functions $\{V_D, V_R^-, V_R^+\}$ for the government at time t as a function of the initial debt—the actual equilibrium value function V is given by the upper envelope of V_R^+ and V_D in case of the good sunspot and by the upper envelope of V_R^- and V_D in the case of the bad sunspot. The left panel is for flexible exchange rates and the right panel is for fixed exchange rates. We consider a mean value for y^T and the value of $\bar{w}$ is set to the highest rigidity such that the default zone remains unchanged by the exchange rate regime. The parameter values correspond to the calibrated economy, to be described in Section 4.

The value of default V_D is a constant because it does not depend on the amount of debt the government owes. The values of repayment V_R^+ and V_R^- are decreasing in debt because the resource constraint becomes tighter. At the intersection of V_R^+ and V_D, the government is indifferent between repaying when it has access to credit markets and defaulting. For debt positions higher than this level, the government defaults regardless of the lenders' beliefs. This is the default zone. At the intersection of V_R^- and V_D, the government is indifferent between repaying when unable to roll over the debt and

[16] Appendix C.1 provides formal details of this exercise. Appendix C.3 also considers a similar exercise in which the wage rigidity is permanent but there is a deterministic path for the tradable endowment that leads to a binding rigidity only in the initial period.

Figure 2: Value functions and crisis zone

(a) Flexible exchange rate (b) Fixed exchange rate



Note: We plot the value functions for mean tradable output. The value of $\bar{w}$ is set to the highest rigidity such that the default zone remain unchanged. We use the parameter values corresponding to the calibrated flexible exchange rate economy to be described in Section 4.1. The value functions under fixed corresponds to an economy in which wages are rigid today and flexible tomorrow. Debt is expressed as a fraction of the unconditional mean of total output.

defaulting. For debt positions lower than this level, the value of repayment is higher than the value of default, and the government repays its debt regardless of lenders' beliefs. This is the safe zone. Finally, in between, the government repays if international lenders are willing to roll over the debt and defaults otherwise. This is the crisis region, which appears shaded in the two panels.

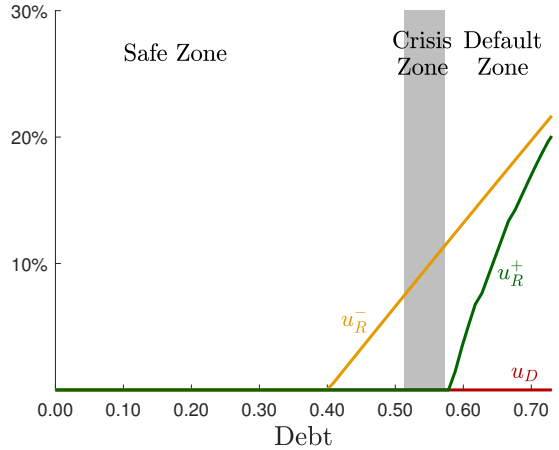
As Figure 2(a) shows, the size of the crisis zone under a flexible exchange rate is small: the government is vulnerable to a rollover crisis only when debt is between 57% and 58%. On the other hand, Figure 2(b) shows that the crisis zone becomes much larger under a fixed exchange rate regime, and and now ranges between 52% to 58%. Debt positions that were safe under a flexible exchange rate now leave the government vulnerable to a rollover crisis.

Inspecting the mechanism. To delve into the mechanism that gives rise to a larger crisis zone under a fixed exchange rate regime, we analyze the behavior of unemployment and how it varies with investors' beliefs and government policy. Figure 3 presents two panels: the left one presents the the unemployment levels under a fixed exchange rate, and the right one presents the value functions from repaying $\{V_R^-, V_R^+\}$ under both a flexible and a fixed exchange rate regime.

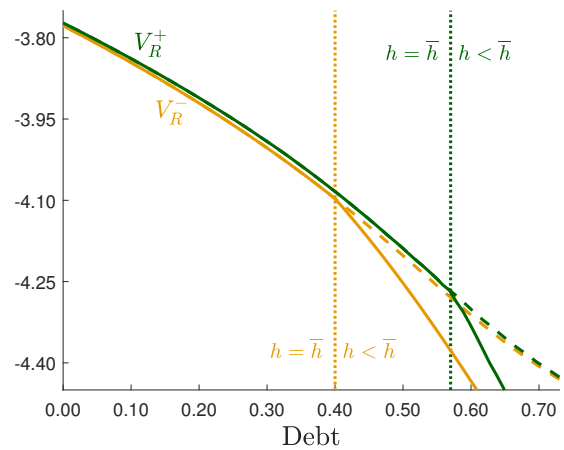
In Figure 3(a), there are three lines: u_D denotes unemployment if the government chooses to default, u_R^+ is unemployment if the government chooses to repay when investors are willing to roll over, and u_R^- is unemployment if the government chooses to repay when investors refuse to roll over. When the government repays, unemployment is weakly increasing in the initial amount of debt in the two cases. This is because a higher initial debt level reduces available resources and aggregate

Figure 3: Unemployment and repayment value functions

(a) Unemployment under fixed



(b) Repayment value functions (fixed vs. flex)



Note: In panel (b), dashed lines correspond to the flexible exchange rate regime and straight lines correspond to the fixed exchange rate regime. We use the same parameter values as in Figure 2..

demand, raising unemployment once the downward rigidity on wages becomes binding.

Comparing u_R^+ and u_R^- reveals that when investors are unwilling to roll over the debt, unemployment starts rising for strictly lower levels of debt and reaches higher values compared with the situation in which investors are willing to roll over the debt. The reason is that when the government is forced to raise tax revenues to repay the maturing debt, this generates a severe contraction in aggregate demand, leading to a surge in unemployment.

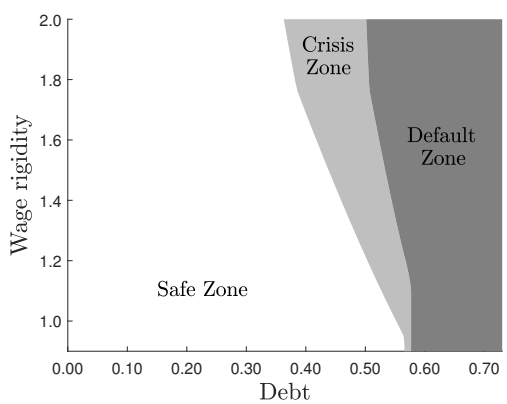
It is interesting to note that the on-path equilibrium value for unemployment turns out to be zero in Figure 3(a). Even though u_R^- can take large values, these high levels of unemployment are not realized in equilibrium. For debt positions such that the government is better-off repaying even if investors were to run, we have that in equilibrium, investors do not refuse to roll over, and hence the level of unemployment is u_R^+. For debt positions such that the government defaults in the event of a run, we have that investors do run and the government defaults on the equilibrium path, resulting in a level of unemployment of u_D. In both cases, unemployment is zero. The takeaway is that what leads the government to default (and investors to run) in a rollover crisis is not the realization of unemployment per se but the desire to avoid the large levels of unemployment that would emerge if the government were to repay while being unable to borrow.

These differences in unemployment translate into substantial effects on the value functions, as can be seen in Figure 3(b). Notice that these are the same value functions from both panels of Figure 2, which we now present in the same plot to highlight the key differences. Dashed (straight) lines denote the value functions under flexible (fixed) exchange rate regime. The two vertical lines indicate the debt thresholds when unemployment emerges under fixed exchange rate, depending on whether

investors are willing to lend. To the left of these thresholds, the value functions under fixed and flex coincide. To the right of these thresholds, the value functions of repayment under fixed exchange rate drop relative to the flexible case. Crucially, V_R^- is reduced by more than V_R^+, resulting in a much wider gap between the two value functions compared with the flexible exchange rate. This wider gap emerges from the large levels of unemployment that the economy suffers when investors refuse to lend and the government has to conduct a fiscal contraction to repay the debt. Moreover, the widening of the gap between V_R^+ and V_R^- occurs precisely at debt levels at which lenders' beliefs matter for the repayment decision. The outcome is a wide crisis zone.

Crisis regions for range of $\bar{w}$. In Figure 2, we compared the crisis region for fixed and flexible exchange rates for a value of $\bar{w}$ sufficiently low such that the default region is not affected.In Figure 4 we show how the regions change for a whole range of $\bar{w}$, keeping y^T at the average value.[17] Recall that a reduction in $\bar{w}$ is equivalent to a nominal exchange rate depreciation, a higher price of foreign tradables, or a lower $\overline{W}$. In particular, one can think about a lower $\bar{w}$ as a scenario in which the government is able to allow for certain depreciation of the currency. The figure shows that, given a normalization, as soon as $\bar{w}$ rises above one, the wage rigidity becomes binding and the safe region contracts. For low values of wage rigidity, b^+ remains unaffected, and hence the crisis region expands at the expense of the safe region without changes in the default region. Once $\bar{w}$ reaches around 1.2, the value function V_R^+, starts to fall, leading in turn to an expansion of the default region at the expense of the crisis region. However, we can see that the crisis region continues to expand significantly because the safe region contracts by an amount greater than the default region expansion.

Figure 4: Crisis region for different $\bar{w}$



Note: The figure shows the safe, crisis, and default zone change under different values for $\bar{w}$. The figure normalizes by the highest value of $\bar{w}$ that is consistent with a non-binding wage rigidity. A rigidity lower or equal than one therefore corresponds to the flexible exchange rate regime.

[17] Appendix C.2 presents the value functions for other values of $\bar{w}$ and the crisis region for other values of y^T.

3.2 Formalizing the Results

We now formally analyze how the exposure to debt crises varies with the exchange rate regime. We consider the baseline case when wage rigidities may bind for all t but focus the comparison on the case in which tradable output is constant and $\beta(1+r) = 1$. As shown by Cole and Kehoe (1996), under a constant output and $\beta(1+r) = 1$, the government seeks to exit the crisis zone by deleveraging until the economy reaches the safe zone. Here we study how the crisis zone differs under the two regimes.

We use b^-_{flex} and b^+_{flex} to denote the debt thresholds that separate the safe zone, crisis zone, and default zone under under a flexible exchange rate regime. Recall that as illustrated in Figure 1, the crisis zone corresponds to debt levels such that $b^-_{flex} \leq b \leq b^+_{flex}$. By the same token, let $b^-_{fix}(\bar{w})$ and $b^+_{fix}(\bar{w})$ be the thresholds under fixed exchange rate for a given $\bar{w}$. We can establish the following results.

Proposition 1 (Vulnerability and exchange rate regime)**.** *Assume that $\beta(1+r) = 1$, $y^T_t = y^T$ for all $t \geq 0$, and let $\left\{\bar{w}^D, \bar{w}^-, \bar{w}^+\right\}$ be wage rigidity thresholds defined in Appendix B. We have the following:*

(i) The safe zone is smaller under fixed exchange rates: The debt thresholds satisfy $b^-_{fix}(\bar{w}) \leq b^-_{flex}$ for any rigidity $\bar{w} \leq \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \bar{w} \leq \bar{w}^D$. Furthermore, if preferences are separable, we have $b^-_{fix}(\bar{w}) < b^-_{flex}$ for any $\bar{w} > \bar{w}^-$.

(ii) A devaluation expands the safe zone: Assume that preferences are separable. We have that for every $e' > e$ then $b^-_{fix}\left(\overline{W}/e'\right) \geq b^-_{fix}\left(\overline{W}/e\right)$ for any nominal rigidity and exchange rate such that $\overline{W}/e \leq \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \overline{W}/e \leq \bar{w}^D$.

(iii) Crisis and default zones: Assume $\pi = 0$. Then, we have $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{w})$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^+$. Moreover, if preferences are separable, $b^+_{fix}(\bar{w}) < b^+_{flex}$ for any $\bar{w} > \bar{w}^+$.

Item *(i)* of Proposition 1 establishes the key result: the safe zone is smaller when the government lacks monetary independence. As illustrated in Figure 2, a government that fixes the exchange rate is vulnerable to a rollover crisis with lower levels of debt. Item *(ii)* shows that a higher nominal exchange rate depreciation helps expand the safe zone and reduce the vulnerability.[18]

Finally, item *(iii)* establishes conditions under which the crisis region under flexible exchange rate is strictly contained in the crisis region under a fixed exchange rate. The previous statement already showed that the crisis zone expands to the left under fixed exchange rate. However, notice that unlike the example in Section 3.1, the government here faces wage rigidities in the future, which tend to increase default incentives in the future and lead to an increase in the default region today (while contracting the crisis region on the right). With $\pi = 0$, constant output and sufficiently low rigidity, this effect is muted and the default zone remains unchanged. In addition, the second part of the

[18]The same result follows if we consider a lower $\overline{W}$ or a rise in the foreign price of tradables instead of a nominal depreciation.

statement demonstrates that for sufficiently high $\bar{w}$, the default zone strictly expands under a fixed exchange rate relative to the flexible exchange rate.

3.3 Extensions and Generalizations

In this section, we discuss briefly how the main theoretical results can be extended and generalized. (Details can be found in the Online Supplemental Appendix.)

Tradable production. The model features an endowment of tradables, whereas non-tradable goods are produced with labor. In Appendix D.1, we allow for a symmetric production structure in which both goods are produced with labor and show that our results are preserved. To see why, consider a panic by foreign investors in this extended version of the model. As the government raises tax revenues to repay the debt, the demand for non-tradable goods falls, leading to a reallocation of labor from the non-tradable to the tradable sector (which faces a perfectly elastic demand from abroad). To the extent that labor has decreasing marginal returns, however, the reallocation is limited. In fact, once the wage rigidity becomes binding, the demand for tradable employment is entirely determined by the condition $F_T'\left(h^T\right) = \bar{w}$. Hence, further declines in aggregate demand do not lead to a reallocation of labor towards the tradable sector, and overall employment remains depressed.

Sticky prices. The same results can be obtained in a model in which the source of nominal rigidity is prices instead of wages. When prices are sticky, rationing takes place in the goods market rather than in the labor market. Either way, a panic generates a contraction in aggregate demand, which makes repayment more costly under a fixed exchange rate. Appendix D.2 shows how the theoretical results extend to the case of price stickiness.

Costs from nominal depreciations. In our model, a higher exchange rate unambiguously increases the utility flow at any particular state, given that it reduces unemployment and does not involve any cost. In practice, a depreciation of the exchange rate may also come with some costs, which could result, for example, from adverse redistributive effects or monetary distortions. To capture these costs, we consider in Appendix D.3., an additively utility cost from exchange rate fluctuations and show that our results continue to hold. In this extension of the model, the government faces a trade-off between the benefits from higher employment and the costs of exchange rate fluctuations. Regardless of how large the costs are, however, an economy under flexible exchange rate regime displays a smaller crisis zone. Intuitively, even though depreciating is costly, it is still the case that the exchange rate flexibility is especially valuable during a rollover crisis, which makes investors less prone to run in the first place.

Benefits from currency unions. We have not explicitly modeled the government's choice to fix the exchange rate or join a monetary union. We argue, however, that while considering these benefits may alter the welfare ranking between a fixed and a flexible exchange rate, our central result that a fixed exchange rate is more vulnerable to a rollover crisis continues to hold. There are potentially several ways to model benefits from being part of monetary union. (See Appendix D.4 for details of the analysis that follows.)

A first possibility is that being in a monetary union allows for the mitigation of inflationary bias, one of the key arguments for joining a monetary union (Barro and Gordon, 1983; Alesina and Barro, 2002). To allow for this possibility, we consider a variant of the model in which the costs from exchange rate fluctuations arise from *expected* depreciations. Lacking commitment to an exchange rate policy, the government always finds it optimal to depreciate the currency ex post to deliver full employment and generates excessive fluctuations ex ante. By entering a monetary union, an economy is able to avoid the resulting inflationary bias costs, and doing so can be desirable if these costs are sufficiently large. However, the result that the lack of exchange rate flexibility makes the economy more vulnerable to a rollover crisis remains.[19]

A second possibility to consider is that being in a monetary union raises the economy's tradable output because of enhanced trade linkages. This is indeed one of the traditional arguments for joining a monetary union Mundell, 1961. Moreover, in the context of the recent Spanish crisis, Almunia, Antràs, Lopez-Rodriguez, and Morales (2018) argues that amid the decline in domestic demand, a high level of integration with Europe made it possible to prevent a further drop in tradable output. In the appendix, we consider a version of the model in which an economy with a fixed exchange rate has a higher permanent level of tradable output. We argue that the higher vulnerability of a member of the monetary union extends to this case as the increase in tradable output raises both the value functions from both repayment and default—specifically, under the assumption of linear utility in tradable consumption, Proposition 1 remains intact. The logic is that what matters for the difference in crises exposure is that the panic generates an endogenous contraction in output under a fixed exchange rate.

A third possibility is that being integrated in a monetary union improves enforcement of external debt payments. Indeed, some observers have argued that defaulting while being in a monetary union might be more costly. Interestingly, an increase in the default cost has the direct implication of always reducing the fundamental default zone, but the crisis zone may expand. In fact, a simple inspection of Figure 2 underscores that upon a parallel shift in the value of default, the crisis region may increase or decrease depending on the slopes of the two values of repayment at the intersection point with V^D. Key for the results is that the crisis zone depends mainly on the gap between V_R^+ and V_R^-, and

[19] In fact, inflationary bias does not increase the gap between V_R^+ and V_R^-, which is the key determinant of the crisis zone. On the contrary, when the government cannot rollover the debt, the government starts the next period with lower debt, and therefore the expected depreciation is lower compared to the case in which the government can rollover.

this gap is larger when the government cannot use monetary policy to stabilize output when facing a liquidity problem.

These three extensions highlight that our result that a fixed exchange rate is more vulnerable to a rollover crisis does not hinge on the fact that we abstracted from modeling the reasons why the government implements a fixed exchange rate regime.

Nominal debt. In the baseline model, the only difference between a flexible exchange rate regime and a fixed exchange rate regime is that in the former, the government can use monetary policy to stabilize macroeconomic fluctuations. We made this assumption partly to better highlight the new channel regarding the role of monetary policy in reducing the vulnerability to rollover crises. In principle, however, an economy that is outside a monetary union can also issue debt in domestic currency, which opens the possibility to inflating away the debt, and introduces another difference between the two regimes. In Appendix D.5, we describe a version of the model in which a nominal depreciation allows for simultaneously affecting the real value of the debt as well as the level of employment. In this economy, depreciating the currency allows for an increase in the amount of tradable consumption by effectively diluting the real value of the debt. Importantly, this allows for an increase in aggregate demand and, through the mechanism highlighted above, also reduces unemployment and makes repayment less costly in the event of a run. We therefore argue that the main insight of the paper remains when we allow for domestic currency denominated debt.

Inflation targeting. In our baseline model, the key constraint on monetary policy is a fixed exchange rate regime. An alternative constraint is a strict inflation targeting regime, in which the government keeps constant the price of the composite consumption good in domestic currency. Under an inflation targeting regime, the government has the ability to depreciate the currency in response to a rollover crisis, but the target for inflation may prevent the government from allowing a sufficiently large depreciation that achieves full stabilization. As a result, a strict inflation targeting regime still leaves the government more vulnerable to a rollover crisis (see Appendix D.6).

Domestic debt. The results can also be applied to rollover crises with domestic borrowing in open or closed economies. In fact, a panic by domestic investors of government bonds can trigger a reduction in government spending or a redistribution away from households with a high marginal propensity to consume. If the government cannot offset the recessionary effects on economic activity by using monetary policy (e.g., because of a fixed exchange rate or a zero lower bound), it will become more costly for the government to repay. As in our model, these Keynesian features would make investors more prone to run.

Key takeaway. Beyond these specific extensions, our main result is quite general in the sense that it hinges on only two key robust elements: (i) a sudden panic by investors triggers capital outflows, if the government chooses to repay; (ii) the costs of sudden capital outflows are more severe under monetary policy constraints because the government is unable to mitigate the contraction in aggregate demand. The combination of these two elements implies that the government is more tempted to default during a panic under a fixed exchange rate regime, and hence investors are more prone to run.

4 Quantitative Analysis

This section presents the quantitative analysis of the stochastic version of the model. (Appendix G presents all the details of the computational approach.) We conduct three policy experiments with the model. First, we perform simulations to assess how often an economy is exposed to rollover crises and examine how this exposure depends on the exchange rate regime. Second, we assess the welfare costs from monetary independence are and the potential gains from a lender of last resort. Third, we perform a counterfactual experiment applied to the recent sovereign debt crisis in Spain to shed light on whether the crisis was triggered by fundamentals or self-fulfilling beliefs.

4.1 Calibration

We calibrate the model at an annual frequency, using Spain as a case study. Table 1 shows all the baseline calibration values for the parameters of the model.

We parameterize the default utility cost as $\kappa\left(y^T\right) = \max\left\{0, \kappa_0 + \kappa_1 \ln\left(y^T\right)\right\}$. As shown in Arellano (2008) and Chatterjee and Eyigungor (2012), a non-linear specification of the cost of default is important to allow the model to match the levels of debt and spreads in the data.

Tradable output follows a log normal AR(1) process $\ln\left(y_{t+1}^T\right) = \rho \ln\left(y_t^T\right) + \sigma_y \varepsilon_t$, where $|\rho| < 1$ and $\varepsilon_t \sim N(0,1)$. To estimate this process, we use the European Classification of Economic Activities (NACE-2) from the Eurostat database to compute the value added of the tradable sectors in Spain between 1995 and 2018. We define an activity as tradable when the Tradability Index defined as (Exports + Imports)/Value Added) is on average above 10% from 2010 to 2015, following De Gregorio, Giovannini, and Wolf (1994).[20] Using this classification, we obtain that the share of tradables' value added relative to GDP is on average 32%. We estimate the log quadratically detrended tradable output by OLS and obtain $\rho = 0.826$ and $\sigma_y = 2.7\%$.

[20]Under this criteria, we label as tradable activities "Agriculture, Forestry, and Fishering" (A), "Mining and Quarrying" (B), "Manufacturing" (C), "Electricity Gas, Steam, and Air Conditioning Supplies" (D), and "Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles" (G).

Table 1: Parameter values

Parameter	Value		Description
$\bar{h}$	1.000		Normalization
σ	5.000		Standard risk aversion
ω	0.298		Share of tradables
μ	1.000		Elasticity of substitution between T-NT =1/2
ρ	0.826		Tradable output persistence
σ_y	0.027		Standard deviation of tradable output shock
α	0.750		Labor share in non-tradable sector
r	0.020		German six-year government bond yield
δ	0.141		Spanish bond maturity six years
ψ	0.240		Reentry to financial markets probability
π	0.050		Sunspot probability
Calibration	Flexible	Fixed	Target
β	0.935	0.853	Average external debt-GDP ratio 29.05%
κ_0	0.140	0.131	Average spread 2.01%
κ_1	1.116	0.395	Standard deviation of spread 1.42%
$\bar{w}$	-	1.442	Δ unemployment rate 2.00%

Parameters set externally. A first subset of parameters $\{\sigma, \mu, \omega, \bar{h}, \alpha, r, \delta, \psi, \pi\}$ is specified directly. The parameters governing the preferences and the technology of the model take standard values found in the literature. The coefficient of risk aversion is set to $\sigma = 5$, and the elasticity of substitution between tradable and non-tradable goods is set to $1/(1+\mu) = 0.5$, both standard values in the literature. In addition, the share of tradable goods in the consumption aggregator is set to $\omega = 0.298$, so it matches the share of tradable output, which we estimated at 32.[21] We set the technology parameter $\alpha = 0.75$, an estimate from Uribe (1997). Lastly, we normalize the inelastic labor supply of households to $\bar{h} = 1$.

The parameters from financial markets are set as follows. We set the international risk-free interest rate to $r = 2\%$, which is the average annual gross yield on German six-year government bonds over the period 2000 to 2015. We use a maturity parameter of $\delta = 0.141$ to reproduce an average bond duration of six years, in line with Spanish data.[22] We set the reentry to financial markets probability after default to $\psi = 0.24$ to capture an average autarky spell of four years, in line with Gelos, Sahay, and Sandleris (2011). Finally, we need to set the sunspot probability, which is a more difficult parameter to calibrate. Our baseline value is $\pi = 5\%$, but we examine a wide range as well.

[21] In a non-stochastic version of the model with a value of debt $\bar{b}$ interest rate $\bar{r}$, and average employment $\bar{h}$, the value of ω can be pinned down from $y^T/[y^T + \frac{1-\omega}{\omega}\left(\frac{y^T+\bar{r}/(1+r)\bar{b}}{F(\bar{h})}\right)^{1+\mu} F(\bar{h})] = 31.72\%$.

[22] The Macaulay duration of a bond with price q and our coupon structure is given by $D = \sum_{t=1}^{\infty} t\frac{\delta}{q}\left(\frac{1-\delta}{1+i_b}\right)^t = \frac{1+i_b}{\delta+i_b}$, where the constant per-period yield i_b is determined by $q = \sum_{t=1}^{\infty} \delta\left(\frac{1-\delta}{1+i_b}\right)^t$.

Parameters set by simulation. A second subset of parameters $\{\beta, \kappa_0, \kappa_1, \bar{w}\}$ is set so that the moments in the model match the counterparts in the data. Since we have two different exchange rate regimes, we have two sets of parameters. While $\bar{w}$ is irrelevant under flexible exchange rate, we need to calibrate this parameter for the fixed exchange rate regime. In particular, we calibrate $\bar{w}$ in the fixed exchange rate regime to be consistent with the increase in unemployment during episodes of high sovereign spreads. As a reference, we use the increase in unemployment relative to the HP-filtered trend in 2011, the year before to the EU and ECB's intervention, which was close to 2%.[23]

For both regimes, we calibrate the parameters β, κ_0, and κ_1 to match three moments from the data, and we follow Hatchondo, Martinez, and Sosa-Padilla (2016) in considering the moments in the years following 2008 to concentrate on the period around the crisis. The three moments targeted are the average debt-GDP ratio, and the average and standard deviation of spreads. For the average debt-GDP ratio, we target an average external debt of 29%. For the average and the standard deviation of spreads, we target 2.0 and 1.4, respectively.[24] The resulting values for these parameters appear in the bottom section of Table 1.

4.2 Simulation Results: Exposure to Rollover Crises

We conduct simulations to investigate how the exchange rate regime determines which type of default—fundamental or rollover crisis—is more likely.

Degree of wage rigidity. We start from the flexible exchange rate economy. In this economy, only 1 out of 100 default episodes are due to a rollover crisis. Moreover, on average, the economy is in the crisis zone and therefore vulnerable to a rollover crisis only 0.53% of the time. To examine how the degree of wage rigidity and the exchange rate regime matters for the exposure to a rollover crisis, we vary $\bar{w}$ while keeping all parameters from the calibrated flexible exchange rate economy. In Figure 5(a), we can see here that the tighter wage rigidity is, the larger the fraction of defaults that are explained by non-fundamentals. In fact, the fraction of defaults due to rollover crisis can reach about 11%, compared with 1% for the case under flexible exchange rates. In line with this result, Figure 5(b) shows how time spent in the crisis zone increases with the degree of rigidity.

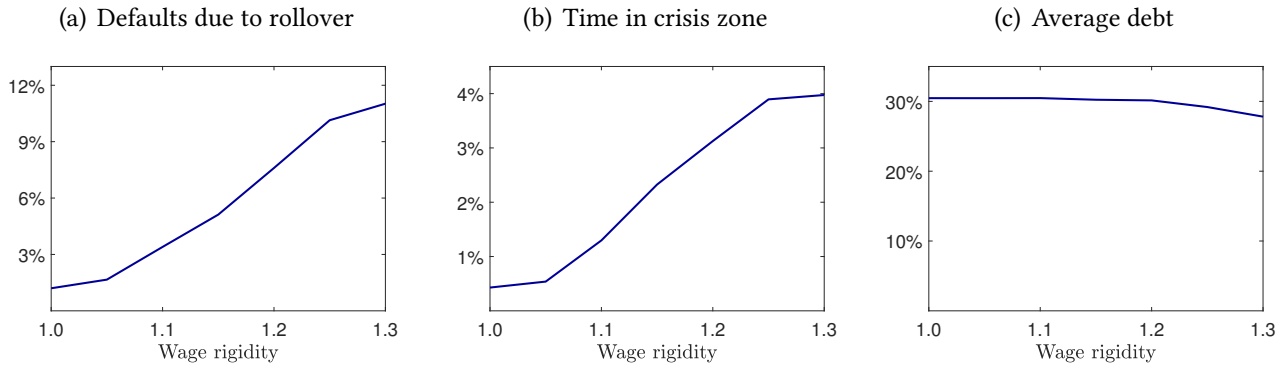
Figure 5(c) also shows that the average debt-to-GDP ratio falls with the degree of wage rigidity. Two reasons explain this. First, a higher $\bar{w}$ implies that the government faces borrowing terms that are more adverse, given that incentives to default in the future are higher. Second, a higher $\bar{w}$ implies that the government faces a larger crisis zone, and the increased vulnerability prompts the government to

[23] As we mentioned in footnote 11, governments have available to them fiscal instruments to stimulate employment, such as payroll subsidies. In terms of our model, this would imply that the wage rigidity would be governed by $\bar{w}$ net of these subsidies. Our approach to calibrating $\bar{w}$ therefore implicitly incorporates these effects.

[24] The debt level in the model is computed as the present value of future payment obligations discounted at the risk-free rate r. Given the maturity structure, the debt level is given by $\frac{\delta}{1-(1-\delta)/(1+r)} b_t$.

reduce the debt level in the long run.[25]

Figure 5: Simulation results under different rigidities



Note: We use the benchmark calibration of the flexible exchange rate regime. The wage rigidity grid is normalized with the highes wage rigidity that shares the same policy functions and debt pricing solution for the flexible exchange rate regime. We collect 5,000 observations series of 50 periods before a default experience. The moments are computed for the last 35 periods before the default experience with simple averages.

Fixed vs. flexible (recalibrated). In the previous experiment, we kept constant all parameters except for $\bar{w}$. Because the long-run moments that we target in the calibration change with $\bar{w}$, it is useful to complement the results by recalibrating the parameters for the discount factor and the default cost to hit the same baseline targets. In Table 2, we consider the simulation statistics for the two economies calibrated to match the same targets for different values of π. Let us start by considering the intermediate columns, which correspond to the baseline values for $\pi = 5\%$. The first three rows correspond to the targeted moments calibrated for both economies and therefore have about the same values for the first two columns.[26] The key result appears in the last two rows. The calibrated fixed exchange rate economy experiences close to 10 defaults due to rollover crises for every 100 default episodes, whereas, as mentioned, the flexible exchange rate experiences only one default due to rollover crises for every 100 default episodes. Similarly, the share of time spent in the crisis zone increases by an order of magnitude in the calibrated fixed exchange rate economy.

Sunspot probability. The fraction of defaults that are the outcome of a rollover crisis depends on two factors. One factor is the probability of a bad sunspot (i.e., the probability of selecting the bad equilibrium whenever the economy is in the crisis zone). The second factor is the probability of ending up in the crisis zone in the first place, which is an endogenous outcome that depends critically on borrowing decisions and on the monetary policy regime. Next, we analyze the sensitivity of our

[25] Notice that despite this reduction in the average debt level, the fact that the crisis zone expands significantly implies that the government still ends up being more heavily exposed to a rollover crisis. This result, however, does not apply in the entire state space. Once debt is substantially reduced, rollover crises become less likely.

[26] The modest differences in the targets for mean debt, mean spreads, and volatility of spreads are because of the nonlinear nature of the model that makes difficult an exact calibration (see Aguiar, Chatterjee, Cole, and Stangebye, 2016).

results by considering different values of π while keeping the rest of the parameter values at their respective baseline values with the exception of $\{\beta, \kappa_{0,}\kappa_1, \bar{w}\}$, which are recalibrated to match the same baseline targets in the two economies.

Table 2 presents the results for three values of π. The table shows how a higher likelihood of a bad sunspot increases the fraction of defaults due to a rollover crisis for the two economies, and particularly for the economy under a fixed exchange rate regime. When the probability of a bad sunspot is 10%, about one-fifth of all defaults in the fixed exchange rate economy are for non-fundamental reasons. Moreover, one can see that the fraction of time spent in the crisis region decreases as the government finds optimal to reduce its exposure, but this duration is not enough to offset the effects of a higher likelihood of a bad sunspot.

Table 2: Simulation Statistics

Sunspot probability	Spain	$\pi = 1\%$		$\pi = 5\%$		$\pi = 10\%$	
(percentage %)	Data	Flexible	Fixed	Flexible	Fixed	Flexible	Fixed
Average spread	2.01	2.01	1.94	1.95	2.00	1.96	1.90
Average debt-income	29.05	30.64	31.88	30.45	29.96	28.55	31.84
Spread volatility	1.42	1.62	1.62	1.59	1.71	1.61	1.72
Unemployment increase	2.00	0.00	1.78	0.00	1.71	0.00	1.87
Time in crisis zone		0.59	6.03	0.53	4.30	0.49	3.59
Defaults due to rollover crisis		0.53	3.87	1.20	10.12	2.22	17.88

Note: All parameter values correspond to the benchmark calibration for flexible and fixed exchange rate regimes with exception of $\{\beta, \kappa_0, \kappa_1, \bar{w}\}$. The benchmark calibration uses $\pi = 5\%$. All values are in percent.

4.3 Welfare Consequences

We now tackle two central normative considerations: (i) what is the welfare cost of the lack of monetary independence? (ii) what are the welfare gains from a lender of last resort (LOLR)?

Our first result is that the possibility of a rollover crisis substantially increases the welfare costs of giving up monetary independence. We examine, for all initial states, how much household are willing to give up of the composite consumption good to move from the fixed exchange rate economy to a flexible exchange rate for one period (see Appendix E for the technical details).

Figure 6 shows these calculation, denoted by $\theta_0^{flex}(b, \mathrm{s})$, for a range of debt levels, for the good sunspot $\zeta = 0$ and the bad sunspot $\zeta = 1$, and for a given endowment shock. For reference, the three zones are displayed for the economy under a fixed exchange rate. Starting from the left, we see that if debt is very low, there is no unemployment and no cost from having a fixed exchange rate. As debt approaches 0.25, unemployment emerges on equilibrium, and there is a positive welfare cost.

Under the good sunspot, the welfare cost increases continuously until debt reaches about 0.36, at which point the government chooses to default under a fixed exchange rate. This helps mitigate the effects from the wage rigidities. Here, the welfare costs from a fixed exchange rate become decreasing in the level of debt, because the value function is independent of debt under a fixed exchange rate—since the government defaults—but decreasing under a flexible exchange rate. Importantly, while the economy under a fixed exchange rate features no unemployment, there is still a welfare cost from a fixed exchange rate because it is precisely the lack of flexibility that triggers the government default, and the economy suffers from the default costs. For debt levels higher than 0.41, the government under a flexible exchange rate also chooses to default and there are no costs from rigidity. Under the bad sunspot, the welfare costs increase discretely once the debt enters the crisis zone. This occurs because the lack of exchange rate flexibility prompts the government to default if investors refuse to roll over the government bonds.

The next welfare consideration that we tackle is the welfare gains from having a LOLR. As is well understood, a third party with deep pockets can eliminate the coordination problem behind a rollover crisis. The basic argument is that by purchasing a sufficiently large amount of government bonds in either the primary or the secondary market, this can induce the government to repay and therefore make investors willing to lend to the government.[27]

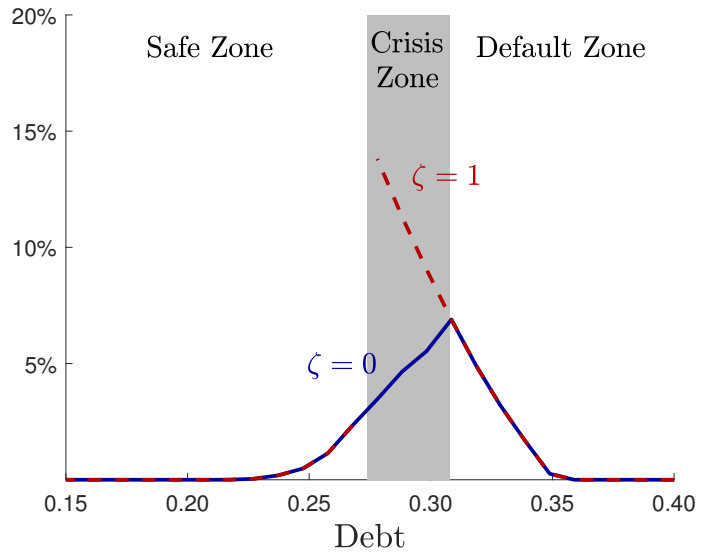
We ask how much households would be willing to pay in terms of consumption to have access to a LOLR (or equivalently to permanently eliminate the possibility of a rollover crisis). To compute these welfare costs, we take the fixed and flexible exchange rate economies with their respective calibrations and solve for the Markov equilibrium after setting the sunspot probability to zero. For each exchange rate regime, we compute the welfare gains in terms of the composite consumption in every state. Under a fixed exchange rate regime, the gains from having a LOLR can reach about 0.43% of permanent consumption and average 0.14% over the long-run simulations. Having access to a LOLR allows for both an improvement in the borrowing terms and a reduction in default costs. For the flexible exchange rate, however, the unconditional welfare gains from having a LOLR are negligible, in line with the minimal exposure to rollover crises.

It is worth highlighting that a successful implementation of a LOLR hinges on the ability to correctly identify whether a default is being driven by fundamentals or by self-fulfilling beliefs. Moral hazard concerns would naturally emerge when the government and investors expect interventions in defaults driven by fundamentals. Therefore, in a scenario in which the LOLR does not observe the source of the default, a trade-off is likely to emerge between the benefits from offsetting the coordination problem and the moral hazard effects.[28] Our analysis shows that while economies that lack monetary independence are likely to strongly benefit from a LOLR, this is less valuable for a flexible

[27] See Roch and Uhlig (2018) and Bocola and Dovis (2019) for an analysis of a LOLR in the context of the Outright Monetary Transactions (OMT) program by the ECB.

[28] See Bianchi (2016) for a quantitative analysis of the trade-off between the moral hazard effects from bailouts and the stabilization benefits in the context of firms' borrowing.

Figure 6: Welfare gains from exiting a currency union



Note: The figure presents the value of exiting a currency union for one period when the current output level is 7% below the mean, which corresponds to the drop in y^T in Spain in 2012 (see Figure 7 below). We use the parameters calibrated for the benchmark fixed exchange rate regime. The solid line represents the economy when there is no sunspot. The dashed line represents the economy when the sunspot is activated. The y-axis represents the welfare gain of being in a flexible exchange rate regime in terms of the composite consumption good.

exchange rate regime, since defaults are driven almost exclusively by fundamental reasons.

Overall, this welfare analysis provides two important policy lessons. First, the lack of monetary independence can become very costly in the presence of rollover crises. Second, a LOLR can help ease the costs for an economy of giving up monetary independence.

4.4 The Path to Spain's Rollover Crisis

In this section, we use the model to shed light on the path to Spanish debt crisis. We start the simulations in 1999, when Spain gave up the peseta and adopted the euro. Given Spain's external debt-GDP ratio in 1999, we feed the sequence of shocks to tradable output and simulate the model under a fixed exchange rate regime.

From 1999 until 2011, we find that the economy remains in the safe zone (and hence the sunspot realization is irrelevant). As it turns out, the model predicts that the economy is in the crisis zone in 2012–2013, and a negative sunspot would trigger a rollover crisis and a default. Even though Spain did not ultimately default on the debt, it received a €100 billion assistance package from the European Union, channeled through the European Financial Stability Fund and the European Stability Mechanism. Even more important for our analysis was the announcement of the ECB's OMT bond purchasing program following the "whatever it takes" speech, which dissipated concerns over the emergence of a rollover crisis. Indeed, after the speech, there was a substantial reduction in sovereign

spreads.[29] Consistent with this, our model predicts that Spain would not default in the presence of a lender of last resort.[30]

Figure 7 summarizes the results of the exercise. Panel (a) displays the tradable output we feed into the model. Panels (b) and (c) show the dynamics of debt and spreads in these simulations. In early 2000, given the low initial debt and the relatively good income shocks, the government increases its debt thanks to the favorable borrowing terms.[31] These dynamics are fairly similar to those in the data, except that the model overpredicts the initial increase. One can also see that the model is able to replicate the low and stable spreads before 2008 in the data. Finally, the evolution of the probability of being in the crisis zone in panel (d) reveals interesting dynamics. [32] After the debt accumulation that occurs initially and the negative income shocks that pile up after 2008, the economy's probability of a rollover crisis becomes more significant.

The final block of the exercise is a series of policy counterfactuals presented in Figure 7(e). We first ask what the welfare gains are from recovering monetary independence, as examined in Section 4.3. As the blue line shows, the gains are close to zero until 2011 but reach about 10 in 2012 and 2013. We then ask how these welfare gains would be modified in the presence of a LOLR. We can see that while the gains from a LOLR are close to nil under a flexible exchange rate regime, they are significant for a fixed exchange rate, representing 60% of the gains from regaining monetary independence in 2012 and 33% in 2013. In other words, the attractiveness of exiting a monetary union can be substantially reduced by providing a LOLR.

According to this experiment, if Spain had exited the monetary union, it would not have been subject to a rollover crisis.[33] Two remarks about this counterfactual experiment are in order. First, we are keeping everything else constant when we analyze the implications of exiting the Eurozone. We

[29]This was the case not only for Spain but for other Eurozone countries that had previously experienced a substantial rise in spreads, like Greece, Portugal and Italy.

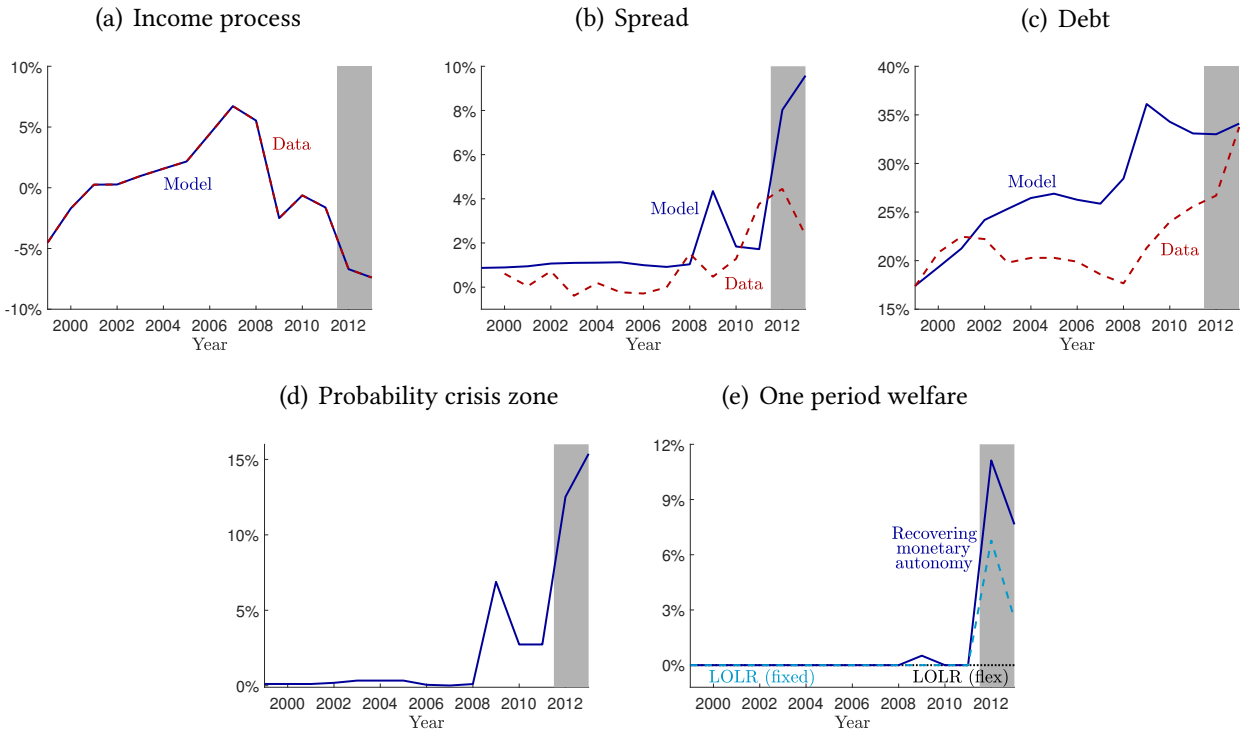
[30]We note here that by starting in 1999, we abstract from many other factors at play in the transition to the Euro. One relevant observation is the reduction in sovereign yields as Spain approached the date for joining the Euro. While this reduction may seem to be at odds with the model's predictions, the inference of default risk is complicated by the change in the currency denomination of the Spanish bonds. Codogno, Favero, and Missale (2003, see Figure 4) construct synthetic Deutsche Mark bonds by swapping the flows of the Spanish bond into German Deutsche Marks, using outright forward contracts, and find that default risk did not actually fall.

[31]The prediction that borrowing tends to be increasing in income shocks is relatively standard in the literature (although it is less strong under a fixed exchange rate because deleveraging is more costly). Notice that the figure plots beginning of period debt b_t. After the negative shock to output in 2009, the deleveraging translates into lower debt levels in 2010. We also note that we conducted the same experiment with a target for average debt 5 percentage points lower and find that just like in the baseline calibration, the government enters the crisis zone in 2012-2013 (see Figure F.3)..

[32]This crisis probability is computed as the probability of receiving in the following period an income shock that pushes the economy into the crisis zone, given the end-of-period level of debt level.

[33] While the welfare results of Figure 7 correspond to a situation in which Spain regains monetary autonomy for one period, the same result would hold if there were a permanent exit from the Eurozone. In both cases, we continue to assume that debt remains denominated in a foreign currency. A natural assumption, since a currency redenomination would be akin to a default. While it is quite likely that Spain would start issuing debt in its domestic currency after exiting, this would apply only to new issuances of debt, not the existing stock, which is to a large extent the most relevant in understanding the incentives to default and how they change if the government remains in or exits the monetary union.

Figure 7: Path to Spain's rollover crisis



Note: Welfare gains in panel (e) correspond to policies that are in place for one period, reverting to the baseline Markov equilibrium. Crisis probability denotes the probability that the economy would be in the crisis zone the following period. The tradable endowment shock was obtained by applying a log-quadratic filter to the Spanish tradable output from 1999 to 2018. Debt levels in the data correspond to Spain's external debt-GDP ratio. The shaded interval denotes that the economy is in the crisis zone.

are therefore abstracting from any possible structural changes that Spain could experience upon exiting a monetary union. Nevertheless, to the extent that these structural changes would symmetrically affect V_R^+ and V_R^-, we expect that the large gap between these two values that arise because of the inability to depreciate the currency would remain intact, and hence these structural changes should not significantly alter the crisis region. Second, we do not suggest that Spain would have been better off by exiting the monetary union. Being in a monetary union indeed has many benefits that we are not modeling. Our goal is to point out an additional cost of remaining in a monetary union, which arises from higher exposure to rollover crises.[34]

[34]Interestingly, the ECB's policy measures since the COVID crisis appear to recognize the importance of a more permanent scheme of liquidity assistance because of the lack of exchange rate flexibility of the members (see a keynote speech by the ECB's Chief Economist Lane, 2019). Nonetheless, the measures remain controversial, as evidenced by the ruling of the Germany's Federal Constitutional Court in May 2020 questioning the proportionality of the ECB's policy measures. See also Lane (2021).

4.5 Empirical Exploration

Besides its application to the Eurozone crisis, to Spain more specifically, our model suggests more generally that, everything else constant, a fixed exchange rate economy is more vulnerable to a rollover crisis.

An ideal test of our model would be to compare in the data the probability of rollover crises in countries with fixed exchange rates against that in countries with flexible exchange rates, with both countries borrowing in foreign currency and the choice of the exchange rate regime being exogenous. Conducting this test is difficult for at least two reasons. First, one needs to have a plausible categorization of when a default is due to a rollover crisis and have a sufficiently large amount of observations to achieve statistical significance. Second, the choice of the exchange rate regime is endogenous, and according to the theory, it should depend on the likelihood of experiencing a rollover crisis. While resolving these challenges is outside the scope of this paper, we present a simple empirical exercise that confirms the theoretical predictions.

A prediction of the model is that sovereign spreads are less connected to fundamentals in a fixed exchange rate. To verify this prediction, we run a standard sovereign spread regression in the data. We consider countries that belong to the EMBI and sort countries according to the degree of exchange rate flexibility following the classification of Ilzetzki, Reinhart, and Rogoff (2019)—see details in Appendix H.[35] We then estimate for each group the following regression:

$$\text{spread}_{it} = \alpha_i + \beta X_{it} + \varepsilon_{it}, \tag{18}$$

where spread_{it} is the spread for a country i in period t, α_i is a country fixed effect, and X_{it} is a vector of controls. We find an R^2 of $0.45, 0.64, 0.95$, respectively, for countries with low, intermediate and high degree of exchange rate flexibility. When we estimate (18) in the model, we also find that fundamentals also better predict spreads for the flexible exchange rate economy—the R^2 is 0.82 for the fixed exchange rate and 0.96 for the flexible exchange rate.

We therefore find that both in the model and in the data, fundamentals have more explanatory power under flexible exchange rates. These results are only suggestive, and we leave for future research a more definite test.[36]

[35] To isolate our mechanism, we restrict to countries with with 60% or more share of foreign currency-denominated debt (an 80% threshold does not change the results that follow).

[36] Also consistent with the predictions of the model is a result recently documented in Born, Müller, Pfeifer, and Wellmann (2020) that post 2008, countries with a fixed exchange rate have experienced much larger variation in spreads.

5 Conclusion

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis and points to a new cost from joining a monetary union. When a government lacks monetary autonomy, a run on government bonds can lead to a large recession in the presence of nominal rigidities. In turn, anticipating that the government will find it more costly to repay, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that an economy with a flexible exchange rate is relatively immune to a rollover crisis. On the other hand, a substantial fraction of defaults under a fixed exchange rate regime are driven by rollover crises.

Our analysis provides a new perspective on discussions about whether the lack of monetary autonomy in Southern European countries made them more vulnerable to a rollover crisis. The popular narrative is that the inability to resort to the printing press contributed to raising their vulnerability. We argue instead that monetary policy, by enhancing macroeconomic stabilization, has a role in preventing rollover crises that goes beyond the ability to inflate away the debt. Our analysis also suggests that a lender of last resort contributes to easing the costs from giving up monetary independence and could be highly beneficial for the stability of a monetary union.

Several avenues remain for future work. In terms of debt management, our model suggests that economies with more rigid labor markets or a less flexible monetary policy should seek longer debt maturities. Another interesting avenue is to provide a more explicit modeling of the benefits from joining a monetary union and quantify the relevant trade-offs involved. Finally, the key mechanism that we highlight is not specific to an open economy setting. In particular, one could extend the analysis to consider rollover crises in closed economies that face other types of constraints on monetary policy.

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A Employment characterization

Lemma A1. *In any equilibrium, employment is given by the following function of tradable consumption:*

$$\mathcal{H}\left(c_t^T;\bar{w}\right) \equiv \min\left[\left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}},\bar{h}\right], \tag{H-demand}$$

or equivalently,

$$\mathcal{H}\left(c_t^T;\bar{w}\right) = \begin{cases} \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \text{if } c^T < \bar{c}_{\bar{w}}^T \\ \bar{h} & \text{if } c^T > \bar{c}_{\bar{w}}^T \end{cases}, \tag{A.1}$$

where $\bar{c}_{\bar{w}}^T \equiv \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\bar{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}}\bar{h}^{\frac{1+\alpha\mu}{1+\mu}}$. *In addition, we have that employment is increasing in* c^T *and decreasing in* $\bar{w}$ *(strictly so, if* $c^T < \bar{c}_{\bar{w}}^T$*).*

Proof. Using (3),(5), and (10), we obtain the following value for employment:

$$h_t = \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{w_t}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}. \tag{A.2}$$

Using labor market conditions (6), (7), and $h \leq \bar{h}$, we arrive at (H-demand).

In addition, we have that $\mathcal{H}\left(\bar{c}_{\bar{w}}^T;\bar{w}\right) = \bar{h}$. The first argument of $\mathcal{H}\left(c_t^T;\bar{w}\right)$ is increasing in c^T and decreasing in $\bar{w}$ given that $\mu > -1$, $\alpha \in (0,1]$, and $\omega \in (0,1)$. We can therefore write (H-demand) as (A.1). □

B Proof of Proposition 1

Preliminaries

The existence of debt thresholds $\{b^-, b^+\}$ are guaranteed by the following lemma.

Lemma B2. *(Existence of Thresholds) For every level of tradable endowment* y^T*, there exists a debt threshold* b^+ *such that* $V_D\left(y^T\right) \geq V_R^+\left(b, y^T\right)$ *if and only if* $b \geq b^+$*. Likewise, there exists a debt threshold* b^- *such that* $V_D\left(y^T\right) \geq V_R^-\left(b, y^T\right)$ *if and only if* $b \geq b^-$*. In addition, we have that* $b^+ \geq b^-$*.*

Proof. First, realize that $V_D\left(y^T\right) < V_R^-\left(0, y^T\right)$. This follows the fact that a government in repayment can choose the same amount of consumption as a defaulting government and avoid the strictly positive costs of defaulting. By the Inada condition on the utility function, we can find a sufficiently high value for debt b^+ such that $V_D\left(y^T\right) \geq V_R^+\left(b^+, y^T\right)$. Using that V_R^+ is strictly decreasing in b, we get that $V_D\left(y^T\right) \geq V_R^+\left(b, y^T\right)$ if and only if $b \geq b^+$. The proof for b^- is identical and we omit it here. Finally,

$b^+ \geq b^-$ is evident from the fact that the problem in (17) is the same as in (16), but with an additional constraint. □

Let us define

$$\mathcal{Y}^N\left(c^T, \bar{w}\right) \equiv F\left(\mathcal{H}\left(c^T, \bar{w}\right)\right).$$

Lemma B3. *(Value functions in the safe zone) Assuming $\beta(1+r)=1$ and a constant tradable endowment y^T, we have*

$$V^+_{R,fix}\left(b;\bar{w}\right) = \frac{1}{1-\beta}\left[u\left(y^T - \frac{\delta r}{r+\delta}b, \mathcal{Y}^N\left(y^T - \frac{\delta r}{r+\delta}b, \bar{w}\right)\right)\right], \quad \forall\, b \leq b^+_{fix}\left(\bar{w}\right); \tag{B.1}$$

$$\begin{aligned} V^-_{R,fix}\left(b;\bar{w}\right) &= \frac{1}{1-\beta}u\left(y^T - \delta b, \mathcal{Y}^N\left(y^T - \delta b, \bar{w}\right)\right) \\ &+ \frac{\beta}{1-\beta}u\left(y^T - \left(\frac{\delta r}{r+\delta}\right)(1-\delta)b, \mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)b, \bar{w}\right)\right), \quad \forall\, b \leq b^-_{fix}\left(\bar{w}\right); \end{aligned} \tag{B.2}$$

$$V_{D,fix}(\bar{w}) = \frac{1}{1-\beta}u\left(y^T, \mathcal{Y}^N\left(y^T, \bar{w}\right)\right) - \frac{\kappa}{1-\beta(1-\psi)}. \tag{B.3}$$

Proof. Following the proof in Cole and Kehoe (2000), under $\beta(1+r)=1$ and constant tradable endowment y^T, the economy becomes stationary once debt is in the safe zone. This implies that for any $b \leq b^-$, the value function is given by the present value of the utility of a constant consumption stream. Without default risk, arbitrage requires $q = \delta/(r+\delta)$. Using this bond price and $b' = b$, the tradable resource constraint (11) implies a constant consumption given by $c^T = y^T - \frac{\delta r}{r+\delta}b$. This gives (B.1). When the government cannot roll over the debt, we have that $b_{t+1} = (1-\delta)b_t$. Following the same logic as above, the economy is in the safe zone in the next period and consumes $c^T = y^T - \frac{\delta r}{r+\delta}(1-\delta)b$ in all future periods. This gives (B.2). Finally, the value of default (B.3) follows simply by using $c^T_t = y^T$ for all t and the fact that the expected discounted default cost is $\frac{\kappa}{1-\beta(1-\psi)}$. □

Now, to ease notation, let us use that under a flexible exchange rate regime, the values of repayment and default can be expressed as

$$V_{R,flex}(b) \equiv V_{R,fix}\left(b;0\right) \quad .$$

$$V_{D,flex} \equiv V_{D,fix}(0).$$

In addition, let us define

$$\bar{w}^D \equiv \alpha\frac{1-\omega}{\omega}\left(y^T\right)^{1+\mu}\left(\bar{h}\right)^{-(1+\alpha\mu)}$$

$$\bar{w}^{-} \equiv \alpha \frac{1-\omega}{\omega} \left(y^{T} - \delta b_{flex}^{-}\right)^{1+\mu} \left(\bar{h}\right)^{-(1+\alpha\mu)}$$
$$\bar{w}^{+} \equiv \alpha \frac{1-\omega}{\omega} \left(y^{T} - \left(\frac{r}{r+\delta}\right) \delta b_{flex}^{+}\right)^{1+\mu} \left(\bar{h}\right)^{-(1+\alpha\mu)}.$$

These values represent wage thresholds. The value of $\bar{w}^{D}$ represents the value of $\bar{w}$ at the point at which the rigidity becomes binding under default; the value of $\bar{w}^{-}$ represents the value of $\bar{w}$ at the point at which the wage rigidity becomes binding under fixed exchange rate when the initial debt is b_{flex}^{-} and the government cannot borrow; and $\bar{w}^{+}$ represents the value of $\bar{w}$ at the point at which the wage rigidity becomes binding under fixed exchange rate when the initial debt is b_{flex}^{+} and the government keeps the debt constant. It is easy to see that $\bar{w}^{+} > \bar{w}^{D} > \bar{w}^{-}$. We summarize these results in the following lemma.

Lemma B4. *(Wage thresholds) Consider levels of tradable consumption given by y^{T}, $y^{T} - \delta b_{flex}^{-}$, and $y^{T} - \left(\frac{r}{r+\delta}\right) \delta b_{flex}^{+}$. In this case, the economy experiences unemployment if and only if the wage rigidity is given respectively by $\bar{w} > \bar{w}^{D}$, $\bar{w} > \bar{w}^{-}$, and $\bar{w} > \bar{w}^{+}$.*

Proof. Replacing $\bar{w} = \bar{w}^{D}$ and $c^{T} = y^{T}$ in (H-demand), we obtain $\mathcal{H}\left(y^{T}, \bar{w}^{D}\right) = \bar{h}$. Since $\mathcal{H}$ is strictly decreasing in $\bar{w}$,the first result follows. The second part is analogous. Once we replace $\bar{w} = \bar{w}^{-}$ and $c^{T} = y^{T} - \delta b_{flex}^{-}$ in (H-demand), we obtain $\mathcal{H}\left(y^{T} - \delta b_{flex}^{-}, \bar{w}^{-}\right) = \bar{h}$. □

We proceed now with the proof of the two items in the proposition.

Proof of item (i), part 1

Proof. We want to first prove that the debt thresholds satisfy $b_{fix}^{-}\left(\bar{w}\right) \leq b_{flex}^{-}$ for any rigidity $\bar{w} \leq \bar{w}^{D}$. By definition of $\bar{w}^{D}$ and the fact that $\mathcal{H}$ is decreasing in wages, we have that for all $\bar{w} \leq \bar{w}_{D}$, $\mathcal{Y}^{N}\left(c^{T}, \bar{w}\right) = F\left(\bar{h}\right)$, and

$$V_{D,fix}\left(\bar{w}\right) = V_{D,flex}. \tag{B.4}$$

It is immediate that for any $\bar{w}$,

$$V_{R,fix}^{-}\left(b_{fix}^{-}\left(\bar{w}\right); \bar{w}\right) \leq V_{R,fix}^{-}\left(b_{fix}^{-}\left(\bar{w}\right); 0\right). \tag{B.5}$$

For all $\bar{w} \leq \bar{w}_{D}$, we can obtain

$$V_{R,flex}^{-}\left(b_{flex}^{-}\right) = V_{D,flex} = V_{D}\left(\bar{w}\right) = V_{R,fix}^{-}\left(b_{fix}^{-}\left(\bar{w}\right); \bar{w}\right) \leq V_{R,flex}^{-}\left(b_{fix}^{-}\left(\bar{w}\right)\right). \tag{B.6}$$

The first equality in (B.6) follows from the definition of b_{flex}^{-}. The second equality follows from (B.4). The third equality follows from the definition of $b_{fix}^{-}\left(\bar{w}\right)$, and the inequality follows from (B.5) and

the definition of $V_{R,flex}(b)$. Given that $V^-_{R,flex}\left(b^-_{flex}\right) \leq V^-_{R,flex}\left(b^-_{fix}(\bar{w})\right)$ and $V^-_{R,flex}$ is decreasing in debt, we have demonstrated that $b^-_{flex} \geq b^-_{fix}(\bar{w})$.

For the strict part, we have that $V^-_{R,fix}\left(b^-_{fix}(\bar{w});\bar{w}\right) < V^-_{R,flex}\left(b^-_{fix}(\bar{w})\right)$ for all $\bar{w} > \bar{w}^-$. Proceeding analogously as in (B.6), we obtain

$$V^-_{R,flex}\left(b^-_{flex}\right) = V_{D,flex} = V_D(\bar{w}) = V^-_{R,fix}\left(b^-_{fix}(\bar{w});\bar{w}\right) < V^-_{R,flex}\left(b^-_{fix}(\bar{w})\right).$$

It therefore follows that $b^-_{flex} > b^-_{fix}(\bar{w})$. This completes the proof of the first statement of (i). □

Proof of item (i), part 2

Proof. Let us prove now that if preferences are separable, we have $b^-_{fix}(\bar{w}) < b^-_{flex}$ for any $\bar{w} > \bar{w}^-$. Define

$$u\left(c^T, c^N\right) = U^T\left(c^T\right) + U^N\left(c^N\right), \tag{B.7}$$

where both $U^T$and U^N are strictly increasing and strictly concave functions. A particular case to obtain (B.7) would be to set the intertemporal elasticity of substitution equal to the intratemporal elasticity of substitution in the baseline utility function (i.e., $1/(1+\mu) = 1/\sigma$).

Combining (3),(5), and (10), we now arrive at $w = F'(h)U^{N\prime}(F(h))/U^{T\prime}\left(c^T\right)$. Given the standard conditions over preferences, we have that there exists a function $\mathcal{H}$ increasing in tradable consumption c^T and decreasing in w such that $h = \mathcal{H}(c^T, w)$, analogous to (H-demand).

Using (B.1) and (B.3) and replacing the utility function (B.7), we have that $b^-_{fix}(\bar{w})$ is implicitly given by $V^-_{R,fix}\left(b^-_{fix}\right) = V_{D,fix}$,

Equating $V^-_{R,flex}\left(b^-_{flex}\right) = V_{D,flex}$, we can obtain an implicit function for b^-_{flex} . Using Lemma B3, replacing (B.8), and noting that $U^N\left(c^N\right)$ cancel out, we arrive at

$$\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} = U^T\left(y^T\right) - (1-\beta)U^T\left(y^T - \delta b^-_{flex}\right) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{flex}\right). \tag{B.8}$$

Similarly, equating $V^-_{R,fix}\left(b^-_{fix};\bar{w}\right) = V_{R,fix}(\bar{w})$ and rearranging, we obtain

$$\begin{aligned} U^T\left(y^T\right) - (1-\beta)U^T\left(y^T - \delta b^-_{fix}(\bar{w})\right) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{fix}(\bar{w})\right) - \frac{(1-\beta)\kappa}{1-\beta(1-\psi)} \\ = \beta\left[U^N\left(\mathcal{Y}^N\left(y^T - \left(\frac{r(1-\delta)}{r+\delta}\right)\delta b^-_{fix}(\bar{w}), \bar{w}\right)\right) - U^N\left(\mathcal{Y}^N\left(y^T, \bar{w}\right)\right)\right] \\ + (1-\beta)\left[U^N\left(\mathcal{Y}^N\left(y^T - \delta b^-_{fix}(\bar{w}), \bar{w}\right)\right) - U^N\left(\mathcal{Y}^N\left(y^T, \bar{w}\right)\right)\right] < 0, \end{aligned} \tag{B.9}$$

where the inequality follows from the fact that $\mathcal{Y}^N$ is increasing in tradable consumption for $\bar{w} > \bar{w}^-$.

$$\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} > U^T\left(y^T\right) - (1-\beta)U^T\left(y^T - \delta b^-_{fix}\left(\bar{w}\right)\right) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{fix}\left(\bar{w}\right)\right). \tag{B.10}$$

Combining (B.8) and (B.10) we arrive at

$$\begin{aligned} &U^T\left(y^T\right) - (1-\beta)U^T\left(y^T - \delta b^-_{flex}\right) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{flex}\right) \\ &\quad > U^T\left(y^T\right) - (1-\beta)U^T\left(y^T - \delta b^-_{fix}\left(\bar{w}\right)\right) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{fix}\left(\bar{w}\right)\right). \end{aligned}$$

This can be rewritten to the following expression:

$$\begin{aligned} &\beta\left[U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{flex}\right) - U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{fix}\left(\bar{w}\right)\right)\right] \\ &+ (1-\beta)\left[U^T\left(y^T - \delta b^-_{flex}\right) - U^T\left(y^T - \delta b^-_{fix}\left(\bar{w}\right)\right)\right] < 0. \end{aligned} \tag{B.11}$$

Since U^T is strictly increasing and $\frac{(1-\delta)r}{r+\delta} < 1$, we can conclude $b^-_{flex} > b^-_{fix}\left(\bar{w}\right)$ for any rigidity $\bar{w} > \bar{w}^-$. □

Proof of item (ii)

We want to show that a devaluation expands the safe zone. More precisely, when preferences are separable, we have that for every $e' > e$, then $b^-_{fix}\left(\overline{W}/e'\right) > b^-_{fix}\left(\overline{W}/e\right)$ for any nominal rigidity and exchange rate such that $\overline{W}/e \leq \bar{w}^D$.

Proof. First, note that $V_{D,fix}\left(\overline{W}/e\right) = V_{D,fix}\left(\overline{W}/e'\right) = V_{D,flex}$. Using this result and the definition of $b^-_{fix}\left(\overline{W}/e\right)$ and $b^-_{fix}\left(\overline{W}/e'\right)$, we can equate $V^-_{R,fix}\left(b^-_{fix}\left(\overline{W}/e'\right);\overline{W}/e'\right)$ to $V^-_{R,fix}\left(b^-_{fix}\left(\overline{W}/e\right);\overline{W}/e\right)$. Using the functional form in (B.7) and applying a Lemma B3, we obtain

$$\begin{aligned} &U^T\left(y^T - \delta b^-_{fix}\left(\overline{W}/e\right)\right) - U^T\left(y^T - \delta b^-_{fix}\left(\overline{W}/e'\right)\right) + \\ &\frac{\beta}{1-\beta}\left[U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\overline{W}/e\right) - U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\overline{W}/e'\right)\right)\right) = \end{aligned}$$

$$U^N\left(\mathcal{Y}^N\left(y^T-\delta P b^-_{fix}\left(\overline{W}/e'\right),\overline{W}/e'\right)\right)-U^N\left(\mathcal{Y}^N\left(y^T-\delta b^-_{fix}\left(\overline{W}/e\right),\overline{W}/e\right)\right)+$$
$$\frac{\beta}{1-\beta}\left[U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\overline{W}/e'\right),\overline{W}/e'\right)\right)\right.$$
$$\left.-U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\overline{W}/e\right),\overline{W}/e\right)\right)\right].$$

By way of contradiction, suppose that $b^-_{fix}\left(\overline{W}/e\right) > b^-_{fix}\left(\overline{W}/e'\right)$. Then, we have that the left hand side is negative while the right hand side is positive. The former follows from U^T being strictly increasing in tradable consumption, while the latter follows from the fact that $\mathcal{Y}^N$ is increasing in c^T and decreasing in $\overline{W}$. It thus follows that $b^-_{fix}\left(\overline{W}/e\right) \leq b^-_{fix}\left(\overline{W}/e'\right)$.

By way of contradiction, suppose that $b^-_{fix}\left(\overline{W}/e\right) > b^-_{fix}\left(\overline{W}/e'\right)$. Then, we have that the left hand side is negative while the right hand side is positive. The former follows from U^T being strictly increasing in tradable consumption, while the latter follows from the fact that $\mathcal{Y}^N$ is increasing in c^T and decreasing in $\overline{W}$. It thus follows that $b^-_{fix}\left(\overline{W}/e\right) \leq b^-_{fix}\left(\overline{W}/e'\right)$. □

Proof of item (iii), part 1

Proof. We want to show that $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}\left(\bar{w}\right)$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^+$. Recall that by definition, the crisis zone is given by

$$\mathcal{C} = \left\{b : b^- < b \leq b^+\right\}.$$

We already showed that $b^-_{fix}(\bar{w}) < b^-_{flex}$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^D$. To deliver the desired result, it suffices to show that $b^+_{fix}\left(\bar{w}\right) = b^+_{flex}$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^+$. Recall that $\bar{w}^D > \bar{w}^+ > \bar{w}^-$. We can see that $\mathcal{H}\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}, \bar{w}^-\right) < \bar{h}$.

With $\pi = 0$, we have that if the government is not in the default zone today, it keeps the debt constant. For $\bar{w} \leq \bar{w}^+$, we have that wage rigidities are not binding as long as $b \leq b^+_{flex}$. Thus, the value of repaying for the government is the same under fixed and flexible exchange rates:

$$V^+_{R,fix}\left(b;\bar{w}\right) = V^+_{R,flex}\left(b\right) \text{ for all } \quad \bar{w} \leq \bar{w}^+,\ b \leq b^+_{flex} \tag{B.12}$$

In addition, given that $\bar{w}^+ < \bar{w}^D$,we have that for any $\bar{w} \leq \bar{w}^+$,

$$V_{D,flex} = V_{D,fix}\left(\bar{w}\right). \tag{B.13}$$

Using (B.12) and (B.13), we have that $b^+_{flex} = b^+_{fix}\left(\bar{w}\right)$ for any $\bar{w} \leq \bar{w}^+$. This completes the proof of the first part. □

Proof of item (iii), part 2

Proof. This part requires first showing that if preferences are separable, $b^+_{flex} \geq b^+_{fix}(\bar{w})$ for any $\bar{w}$. Given $\pi = 0$ and replacing the utility function (B.7), we have

$$V^+_{flex}(b) = \frac{U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b\right) + U^N\left(F(\bar{h})\right)}{1-\beta} \qquad \forall\, b \leq b^+_{flex}. \tag{B.14}$$

By definition, $V^+_{flex}\left(b^+_{flex}\right) = V^D_{flex}$. Replacing (B.3) and (B.14) and using the functional form (B.7), we obtain

$$\begin{aligned}\frac{1}{1-\beta}\left(U^T\left(y^T\right) + U^N\left(F(\bar{h})\right)\right) - \frac{\kappa}{1-\beta(1-\psi)} \\ = \frac{1}{1-\beta}\left(U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right) + U^N\left(F\left(\bar{h}\right)\right)\right),\end{aligned}$$

which, simplifying, yields

$$\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} = U^T\left(y^T\right) - U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^-_{flex}\right). \tag{B.15}$$

We now proceed to obtain analogous expressions under a fixed exchange rate regime. We have

$$V_{D,fix}(\bar{w}) = \frac{U^T\left(y^T\right) + U^N\left(\mathcal{Y}^N\left(y^T, \bar{w}\right)\right)}{1-\beta} - \frac{\kappa}{1-\beta(1-\psi)}, \tag{B.16}$$

and

$$V^+_{R,fix}\left(b^+_{fix}(\bar{w}), \bar{w}\right) = \frac{U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}(\bar{w})\right) + U^N\left(\mathcal{Y}\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}(\bar{w}), \bar{w}\right)\right)}{1-\beta} \quad \forall\, b \leq b^+_{fix}. \tag{B.17}$$

Equating (B.16) and (B.17), by construction we can determine b^+_{fix} as

$$\begin{aligned}\frac{1}{1-\beta}\left[U^T\left(y^T\right) + U^N\left(\mathcal{Y}^N\left(y^T, \bar{w}\right)\right)\right] - \frac{\kappa}{1-\beta(1-\psi)} \\ = \frac{1}{1-\beta}\left(U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}(\bar{w})\right) + U^N\left(\mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}(\bar{w}), \bar{w}\right)\right)\right).\end{aligned}$$

Manipulating the expression, we arrive at

$$U^T\left(y^T\right) - U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}\left(\bar{w}\right)\right) - \frac{(1-\beta)\kappa}{1-\beta(1-\psi)}$$
$$= U^N\left(\mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}\left(\bar{w}\right), \bar{w}\right)\right) - U^N\left(\mathcal{Y}^N\left(y^T, \bar{w}\right)\right) \geq 0, \tag{B.18}$$

where the inequality follows from $\mathcal{Y}^N\left(y^T, \bar{w}\right) \geq \mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}\left(\bar{w}\right), \bar{w}\right)$. Hence, we can rewrite (B.18) as

$$\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} \geq U^T\left(y^T\right) - U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}\left(\bar{w}\right)\right). \tag{B.19}$$

Combining expressions (B.15) and (B.19) and simplifying, we arrive at

$$U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right) \leq U^T\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}\left(\bar{w}\right)\right).$$

Hence, we can conclude $b^+_{flex} \geq b^+_{fix}\left(\bar{w}\right)$ for any $\bar{w}$. Notice that if we take $\bar{w} > \bar{w}^+$, we have $\mathcal{Y}^N\left(y^T, \bar{w}\right) > \mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{fix}, \bar{w}\right)$. Hence, the inequality becomes strict if we take any rigidity $\bar{w} > \bar{w}^+$. □

C Optimal exchange rate policy

Proposition C1 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government always chooses an exchange rate that achieves full employment.*

Proof. The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{e, b', c^T, h \leq \bar{h}} \left\{u\left(c^T, F(h)\right) + \beta\mathbb{E}\left[V(b', \mathbf{s})\right]\right\} \tag{C.1}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$
$$h \leq \mathcal{H}\left(c^T, \overline{W}/e\right).$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the

following Bellman equation:

$$V_D(y^T) = \max_{e, c^T, h \leq \bar{h}} \left\{ u\left(c^T, F(h)\right) - \kappa\left(y^T\right) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D\left(y^{T\prime}\right)\right] \right\} \tag{C.2}$$

$$\begin{aligned} &\text{subject to} \\ &c^T = y^T \\ &h \leq \mathcal{H}\left(c^T, \overline{W}/e\right). \end{aligned}$$

It is immediate from (C.1) and (C.2) that for any level of tradable consumption, an increase in e increases the employment demand without tightening any other constraint (see Lemma A1). When employment demand falls short of full employment, the government depreciates the exchange rate to strictly increase employment until the point at which $\bar{h} = \mathcal{H}\left(c^T, \overline{W}/e\right)$. □

Online Appendix to "Monetary Independence and Rollover Crises"

By Javier Bianchi and Jorge Mondragon

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C Appendix to Graphical Analysis

C.1 Details on the construction

Let $\left\{V^{flex}(b,\mathbf{s}), V_R^{flex}(b,\mathbf{s}), V_D^{flex}(y^T)\right\}$ denote the continuation values and $\tilde{q}^{flex}(b,y^T)$ the price schedule in this environment in which the future Markov equilibrium has a flexible exchange rate (or flexible wages). With some abuse of notation, let us denote the current value functions with a one-period wage rigidity $\bar{w}$ as $V_D(y^T;\bar{w}), V_R^+(b,y^T;\bar{w}), V_R^-(b,y^T;\bar{w})$. These values are given by

$$V_D(y^T;\bar{w}) = \max_{c^T, F(h)\leq \bar{h}} u\left(c^T,h\right) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V^{flex}(0,\mathbf{s}') + (1-\psi)V_D^{flex}(y^{T\prime})\right], \tag{C.1}$$

$$\begin{aligned} &\text{subject to} \\ &c^T = y^T, \\ &h \leq \mathcal{H}\left(y^T,\bar{w}\right); \end{aligned}$$

$$V_R^+(b,y^T;\bar{w}) = \max_{b',c^T,h\leq\bar{h}} u(c^T,F(h)) + \beta\mathbb{E}V^{flex}(b',\mathbf{s}'), \tag{C.2}$$

$$\begin{aligned} &\text{subject to} \\ &c^T = y^T - \delta b + \tilde{q}^{flex}(b',y^T)(b' - (1-\delta)b), \\ &h \leq \mathcal{H}\left(y^T,\bar{w}\right); \end{aligned}$$

and

$$V_R^-(b,y^T;\bar{w}) = \max_{c^T,F(h)\leq\bar{h}} \left\{u(c^T,h) + \beta\mathbb{E}V^{flex}((1-\delta)b,\mathbf{s}')\right\}, \tag{C.3}$$

$$\begin{aligned} &\text{subject to} \\ &c^T = y^T - \delta b, \\ &h \leq \mathcal{H}\left(y^T,\bar{w}\right). \end{aligned}$$

Figures 2 in the main text shows these value functions for a given $\bar{w}$ and y^T and shows they give rise to a larger crisis zone relative to the one in the economy with flexible exchange rate.

C.2 Crisis zone for alternative values

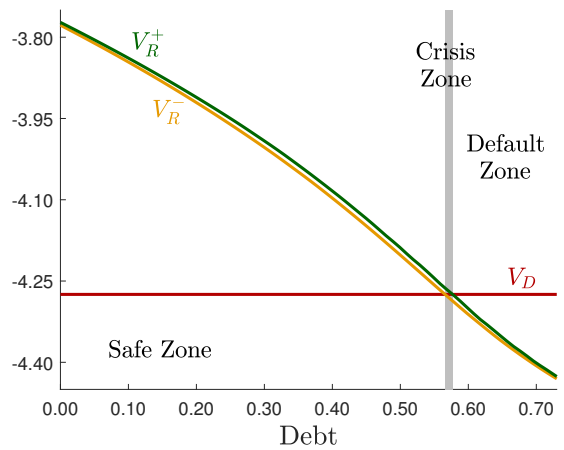
In Figure 2, we compared the crisis region for a flexible exchange rate with the crisis region for a fixed exchange rate for the largest value of $\bar{w}$ such that the default region is not affected. In Figure C.1, we now show the value function for a higher $\bar{w}$ that expands the default region. Figure C.2 also presents the levels of unemployment for the different values of $\bar{w}$.

These regions were constructed for a given level of the tradable endowment. To have a more complete picture, we show in Figure C.3 the three zones in the $\left(b,y^T\right)$ state space. For any given level of debt, the economy is in the default zone for a sufficiently low level of tradable endowment. As we increase the tradable endowment, the economy arrives in the crisis zone at some point. Finally, increasing it even further makes the economy reach the safe zone. Again, we can clearly see how vulnerability to a rollover crisis is lower in a flexible exchange rate regime compared with a fixed

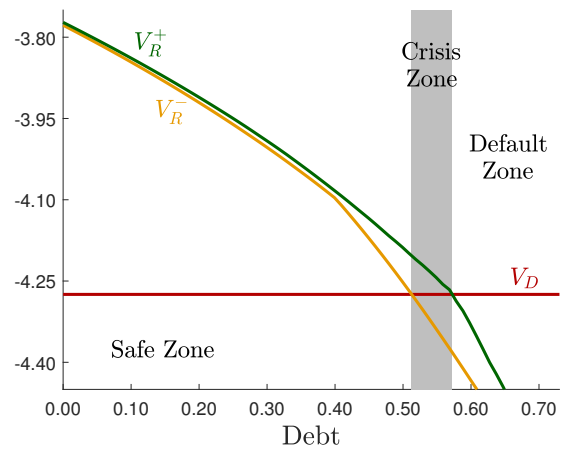
exchange rate regime, and this occurs for all income levels.

Figure C.1: Value functions for different rigidities

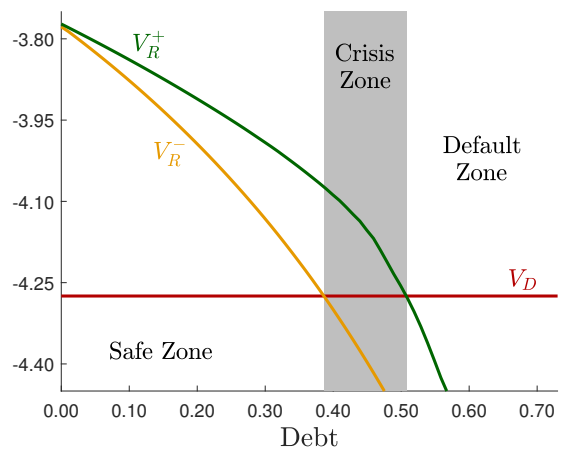
(a) Flexible exchange rate



(b) Low Rigidity

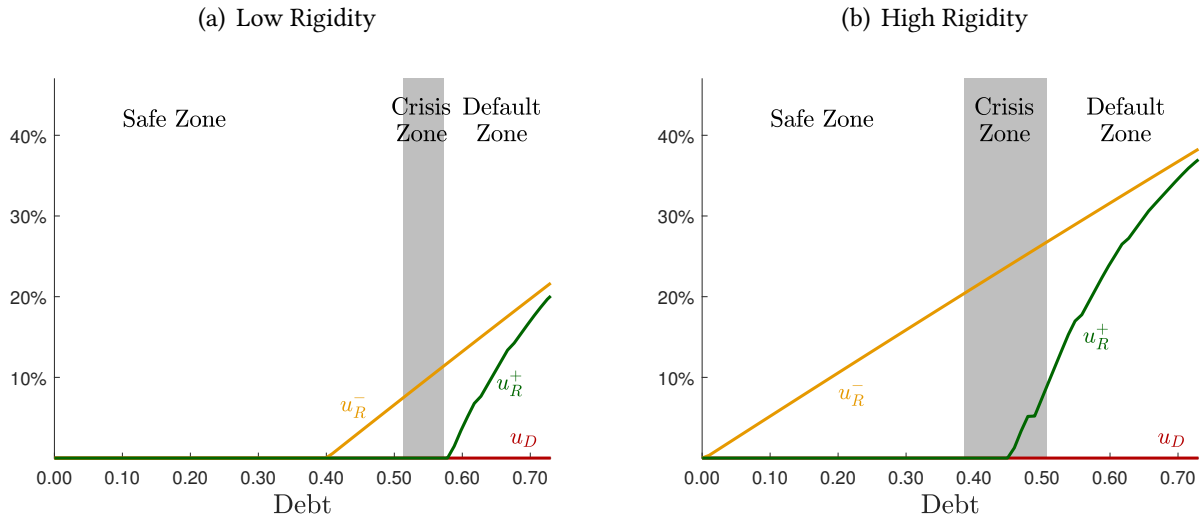


(c) High Rigidity



Note: Low rigidity corresponds to the value of $\bar{w}$ in Figure 2, which is set highest rigidity such that the default zone remain unchanged. High rigidity corresponds to the largest value such that there is full employment in default.

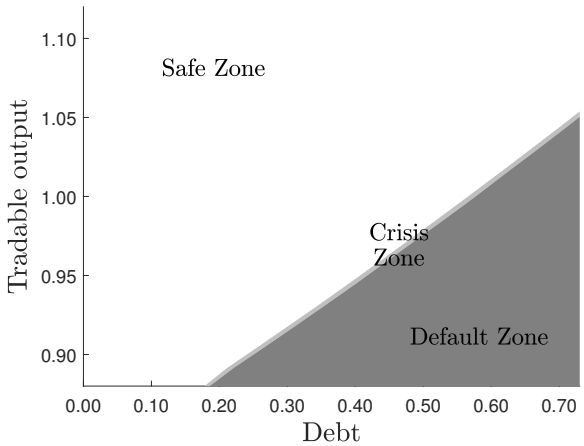
Figure C.2: Unemployment



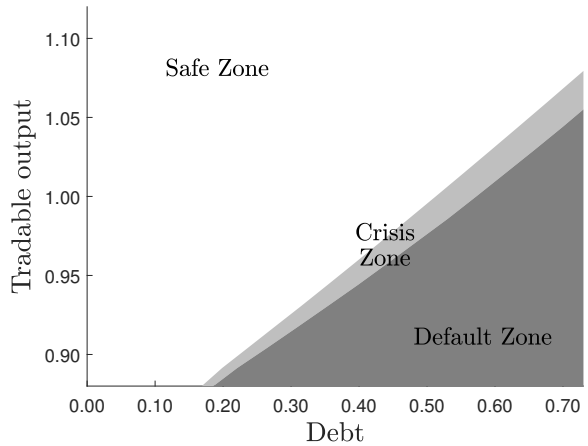
Note: Low rigidity corresponds to the value of $\bar{w}$ in Figure 2, which is set highest rigidity such that the default zone remains unchanged. High rigidity corresponds to the largest value such that there is full employment in default.

Figure C.3: Value functions and crisis zone for different y^T

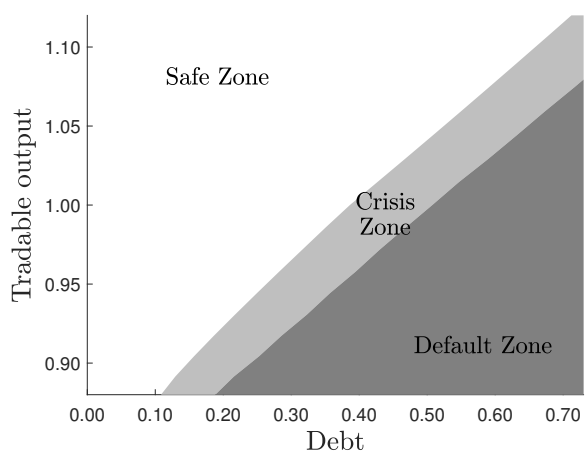
(a) Flexible exchange rate



(b) Fixed exchange rate (low rigidity)



(c) Fixed exchange rate (high rigidity)



Note: Same parameter values as Figure C.2.

C.3 Devaluation in response to output contraction

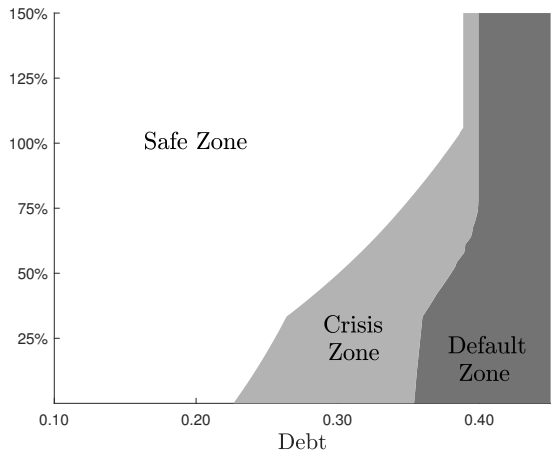
In Section 3.1, we considered a full stochastic environment in which the wage rigidities apply only in the first period. Here, we examine an exercise in which $\bar{w}$ is fixed over time but consider a deterministic path for tradable endowment such that the wage rigidity binds only in the first period, as it does in Section 3.1.

We consider a deterministic economy with $\beta R = 1$ and start from a constant level of tradable output such that the economy is initially in the safe zone and features full employment under fixed exchange rate. We then consider a negative shock to the initial tradable endowment y_0^T that can potentially generate unemployment, and we show how a depreciation of the exchange rate helps make the economy less vulnerable to a rollover crises.

In Figure C.4, we show how the three regions vary with the extent to which the government can depreciate the exchange rate for a drop in y^T of 7%. When the depreciation exceeds 100%, full employ-

ment is guaranteed in and off equilibrium. Effectively, the allocations are the same as in the flexible exchange rate regime. As the depreciation rate falls below 100%, wage rigidity becomes binding when investors panic and the safe zone contracts. Once the depreciation falls below around 75%, the default zone also expands. However, we can see that the crisis zone continues to expand as the depreciation rate cap lowers, because the safe zone contracts in a greater magnitude than the default zone expansion. Finally, once the depreciation rate falls below 35%, unemployment emerges under default and lowers the value of default. Nevertheless, the crisis zone and the default zone continue to expand because the values of repayment fall more than the value of default.

Figure C.4: Depreciation and Crisis Zones



Note: The figure shows the safe, crisis, and default zone change under different nominal exchange rate depreciations under $\beta R = 1$ and a deterministic path of output such that in the first period, $y_t^T = 0.93 \times y^T$ (value that matches the tradable output contraction in Spain in 2012), and $y_t^T = y^T$ for rest of the periods.

D Extensions of the Basic Model

In this section, we show that our main theoretical results hold in various extensions of our baseline model. In Section D.1, we consider a version of the model in which tradable goods are produced endogenously using labor as its sole input. In Section D.2, we consider sticky prices. In Section D.3, we consider a version of the model in which there are costs from exchange rate fluctuations. In Section D.4, we incorporate benefits from belonging to a currency union or adopting a fixed exchange rate regime. In Section D.5, we consider a version of the model in which debt is denominated in domestic currency. In Section D.6, we consider an inflation targeting regime. In Section D.7, we consider an elastic labor supply.

D.1 Tradable Labor Production

In this section, we expand the baseline model to incorporate production in the tradable sector. We will show that our main proposition remains unchanged. In addition, while the quantitative effects are mitigated, the differences in the exposure to rollover crises remain substantial.

Define the production functions for tradables and non-tradables respectively as $F_T(h)$ and $F_N(h)$, where both production functions are strictly increasing and concave. In particular, define $F_T(h) = zh^{\alpha_T}$ and $F_N(h) = h^{\alpha_N}$, where z is a productivity shock to the tradable production function and $\alpha_T < 1, \alpha_N < 1$.

Every period, firms solve the following maximization problem:

$$\max_{h^T} \left\{ ezF_T\left(h^T\right) - Wh^T \right\} \qquad \text{and} \qquad \max_{h^N} \left\{ P^N F_N\left(h^N\right) - Wh^N \right\}.$$

First-order conditions are

$$\begin{aligned} ezF_T'\left(h^T\right) &= w_t \\ P_t^N F_N'\left(h_t^N\right) &= W_t. \end{aligned}$$

Combining these first-order conditions and using that $p_t^N = P_t^N / e_t$, we obtain

$$zF_T'\left(h_t^T\right) = p_t^N F_N'\left(h_t^N\right). \tag{D.1}$$

This equation implies that if there is a reallocation of labor from non-tradables to tradables, we must have in equilibrium an increase in the price of non-tradables.

From the households' first-order condition, recall that

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{D.2}$$

Combining (D.2) and (D.1) implies that the marginal rate of substitution must be equal to the marginal rate of transformation:

$$\frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu} = \frac{F_N'\left(h_t^N\right)}{zF_T'\left(h_t^T\right)}. \tag{D.3}$$

Let us define $h_t = h_t^T + h_t^N$. As before, we have that the following slackness condition must be

satisfied:

$$\left(W_t - \overline{W}\right)\left(\bar{h} - h_t\right) = 0. \tag{D.4}$$

Notice that when the wage rigidity constraint binds under a fixed exchange rate, the demand for labor is determined entirely by $(z\alpha_T/\bar{w})^{\frac{1}{1-\alpha_T}}$. On the other hand, the demand for non-tradable labor depends on the domestically available resources as this pins down the relative price of non-tradables. As we have seen, this element is crucial to making a fixed exchange rate more vulnerable to a rollover crisis.

A competitive equilibrium, given government policies in our economy, is defined as before with the modification that the demand for labor now includes firms in the tradable sector.

Definition 1. *Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous stochastic process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a competitive equilibrium is a sequence of allocations $\{c_t^T, c_t^N, h_t, h^T, h^N, h\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that (i) households and firms optimize; (ii) the government budget constraint holds; (iii) the bond price satisfies investors' optimality; (iv) the market for non-tradable goods clears; and (v) the labor market satisfies conditions (D.4), $W_t \geq \overline{W}$, and $h \leq \bar{h}$.*

The values of repayment and default for the government are given by

$$\begin{aligned} V_R(b, \mathbf{s}) = \max_{b', c^T, h^T} & \left\{u\left(c^T, F_N(h^N)\right) + \beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t.} \quad & c^T - q\left(b', b, \mathbf{s}\right)\left(b' - (1-\delta)b\right) = zF_T\left(h^T\right) - \delta b \\ & zF_T'\left(h_t^T\right) = \frac{1-\omega}{\omega}\left(\frac{c^T}{F_N\left(h - h^T\right)}\right)^{1+\mu} F_N'\left(h_t^N\right) \\ & \frac{1-\omega}{\omega}\left(\frac{c^T}{F_N\left(h - h^T\right)}\right)^{1+\mu} F_N'\left(h_t^N\right) \geq \bar{w} \\ & h_t^T + h_t^N \leq \bar{h}; \end{aligned} \tag{D.5}$$

$$\begin{aligned} V_D(z) = \max_{c^T, h^T} & \left\{u\left(zF_T'\left(h_t^T\right), F_N(h^N)\right) - \kappa(z) + \beta \mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right) + (1-\psi)V_D\left(z'\right)\right]\right\} \\ \text{s.t.} \quad & zF_T'\left(h_t^T\right) = \frac{1-\omega}{\omega}\left(\frac{c^T}{F_N\left(h - h^T\right)}\right)^{1+\mu} F_N'\left(h_t^N\right) \\ & \frac{1-\omega}{\omega}\left(\frac{c^T}{F_N\left(h - h^T\right)}\right)^{1+\mu} F_N'\left(h_t^N\right) \geq \bar{w} \\ & h_t^T + h_t^N \leq \bar{h}. \end{aligned} \tag{D.6}$$

Similar to the results of the benchmark model, these show that full employment is always achieved under a flexible exchange rate regime.

Proposition D2. *Under a flexible exchange rate regime, the government always chooses an exchange rate that replicates the flexible wage allocation.*

Proof. As in the benchmark case, the proof follows directly from the fact that an increase in the exchange rate relaxes the wage rigidity constraint without tightening any other constraint. □

Let us construct the rigidity thresholds :

$$\bar{w}^D \equiv \frac{1-\omega}{\omega}\left(\frac{y^T}{F_N\left(\bar{h}-h^T_{flex}\right)}\right)^{1+\mu} F'_N\left(\bar{h}-h^T_{flex}\right) \tag{D.7}$$

$$\bar{w}^- \equiv \frac{1-\omega}{\omega}\left(\frac{z\left(h^T_{flex}(b^-_{flex})\right)^{\alpha_T}-\delta b^-_{flex}}{F_N\left(\bar{h}-h^T_{flex}(b^-_{flex})\right)}\right)^{1+\mu} F'_N\left(\bar{h}-h^T_{flex}(b^-_{flex})\right), \tag{D.8}$$

$$\bar{w}^+ \equiv \alpha\left((1-\omega)/\omega\right)\left(z\left(h^T_{flex}(b^+_{flex})\right)^{\alpha_T}-\left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right)^{1+\mu} F'_N\left(\bar{h}-h^T_{flex}(b^+_{flex})\right), \tag{D.9}$$

where $h^T_{flex}(b)$ denotes the optimal labor under flexible exchange rates. We can then obtain the same proposition as in the baseline model without demand for tradable labor.

Proposition D3. *Assume that $\beta(1+r)=1$ and $z_t = z$.*

i) Smaller safe zone under fixed: The debt thresholds satisfy $b^-_{fix}(\bar{w}) \leq b^-_{flex}$ for any rigidity $\bar{w} \leq \bar{w}^D$. Moreover, there exists $\bar{w}^-$ such that $b^-_{fix}(\bar{w}) < b_{flex}$ for every $\bar{w}^- < \bar{w} \leq \bar{w}^D$. If preferences are separable between tradables and non-tradables, we have $b^-_{fix}(\bar{w}) < b_{flex}$ for every $\bar{w}^- < \bar{w}$.

ii) Crisis and default zones: Assume $\pi = 0$. Then, there exists $\{\bar{w}^-, \bar{w}^+\}$ such that $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{w})$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^+$. Moreover, if preferences are separable between tradables and non-tradables, $b^+_{flex} > b^+_{fix}(\bar{w})$ for any rigidity $\bar{w} > \bar{w}^+$.

The proof is analogous to that of Proposition 1 , once we use the wage thresholds (D.7)-(D.9).

Quantitative results. We show in a numerical analysis the exposure to a rollover crisis in this extension of the benchmark model. We keep most of the shared parameters with the benchmark model the same. We modify $\omega = 0.355$ to target the share of tradable GDP to 31.72%, as in the baseline calibration. For the labor share in tradable production, we set $\alpha_T = 0.50$, an estimate from Uribe (1997) for the tradable sector. The stochastic process of productivity in the tradable production follows an AR(1) process:

$$\ln z' = (1-\rho)\ln\bar{z} + \rho \ln z + \sigma_\varepsilon \varepsilon, \qquad \text{where } \varepsilon \sim N(0,1).$$

We keep the same persistence parameter as in the baseline stochastic process, $\rho = 0.826$. We normalize $\ln\bar{z} = 0.752$ to match the long-run mean of the log tradable output AR(1) process from the baseline stochastic process $\mu\left(\ln y^T\right) = 0$. The parameters $\{\beta, \kappa_0, \kappa_1, \bar{w}\}$ are calibrated specific to the two different types of exchange rate regimes in the same way as in the baseline calibration. The values of the parameters are $\beta = 0.900, \kappa_0 = -0.085$ and $\kappa_1 = 0.335$ for the flexible exchange rate regime and $\beta = 0.895, \kappa_0 = 0.290$ and $\kappa_1 = 0.075$ for the fixed exchange rate regime. Finally, the standard deviation σ_ε is set to match the standard deviation of tradable output, which yields $\sigma_\varepsilon = 0.053$ for the flexible exchange rate regime and $\sigma_\varepsilon = 0.062$ for the fixed exchange rate regime.

Table D.1 shows how even when tradable output can be adjusted by a reallocation of labor, the economy becomes more vulnerable to rollover crises under a fixed exchange rate regime, as in the baseline model. The second and third column show the moments of the model under a flexible and a fixed exchange rate regime, each calibrated to match the main moments in the data. (The calibration

Table D.1: Tradable labor model extension statistics

Exchange rate regime	Data	Flexible	Fixed	Fixed (u)
Average spread	2.01	2.29	2.69	3.55
Average debt-income	29.05	29.40	30.74	13.64
Spread volatility	1.42	1.37	3.71	4.91
Unemployment increase	2.00	0.00	2.80	2.40
Tradable output volatility	4.79	4.85	6.19	5.45
Tradable/Total GDP share	31.72	31.52	31.59	30.99
Time share in crisis zone	-	0.78	3.33	2.41
Defaults share due to rollover crisis	-	1.50	5.87	3.96

Note: Same simulations as in baseline. The second and third column are calibrated to match the targeted moments in the data. The fourth column uses the calibration for the flexible exchange rate regime economy, with the exception of the wage rigidity, which is set to approximately match the increase of unemployment in the data.

under fixed exchange rate has relatively more important misses in terms of the calibration targets; in particular, for the volatility of spreads.) Because of this, we add another column in which we show the moments in the model under the flexible exchange rate regime with a wage rigidity high enough to match the unemployment increase in the data. The main result still holds: under a fixed exchange rate regime the economy becomes more vulnerable to rollover crisis. The share of defaults due to rollover crises under a flexible exchange rate regime is 1.5% and under a fixed exchange rate regime is about 6% (4% with no recalibration). The difference in vulnerability is lower than that in the benchmark model but still substantial.

D.2 Sticky prices

In this section, we explore an alternative form of nominal rigidity with price rigidity instead of wage rigidity. We assume that wages and the price of tradables are flexible and that the nominal non-tradable price in the economy cannot fall lower than a threshold $\bar{P} > 0$.[37] Using the disequilibrium formulation of Barro and Grossman (1971), we have that whenever market clearing requires a price lower than $\bar{P}$, firms' amount of production is determined by household demand for non-tradable goods, and employment is therefore given by $h = F^{-1}(c^N)$. The slackness condition is then given by

$$(P_t^N - \bar{P})(P_t^N F'(h_t) - W_t) = 0 \tag{D.10}$$

Under a fixed exchange rate regime, the rigidity can be described as $p^N \geq \bar{p}$ where $\bar{p} = \bar{P}/e$.

Using the optimality condition of the household (3) and that the firms produce to satisfy demand.

[37] We could also assume that price rigidity goes in both directions, but we model the asymmetry to have a more direct comparison with the model with downward wage rigidity. The approach of downward price rigidity is commonly attributed to "social norms" and is followed, for example, by Caballero and Farhi (2018).

That is, $h^{\alpha} = c^N$. We can construct an employment demand function analogous to H-demand:

$$\tilde{\mathcal{H}}\left(c^T, p^N\right) \equiv \left(\left(\frac{1-\omega}{\omega}\frac{1}{p^N}\right)^{\frac{1}{1+\mu}} c^T\right)^{\frac{1}{\alpha}}. \tag{D.11}$$

We have the following definition of competitive equilibrium.

Definition D1. *Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous stochastic process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a competitive equilibrium is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that (i) households optimize c^T, c^N and are on their labor supply; (ii) the government budget constraint holds; (iii) the bond price satisfies investors' optimality; (iv) the labor market clears; and (v) the goods market satisfies conditions $h^{\alpha} = c^N$, (D.10), and $P_t^N \geq \bar{P}$.*

The government problem is equivalent to the baseline model after replacing (D.11) for H-demand. Thus, we have that under fixed exchange rate, the government problem under repayment is

$$\begin{aligned} V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \bar{h}} & \left\{u\left(c^T, F(h)\right) + \beta \mathbb{E}\left[V(b', \mathbf{s})\right]\right\} \\ \text{subject to} \quad & c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b) \\ & h \leq \tilde{\mathcal{H}}\left(c^T, \bar{P}/e\right), \end{aligned} \tag{D.12}$$

and we have an analogous problem under default.

We first prove that under linear production, for any equilibrium of the sticky wage economy, there exists a level of price rigidity such that the equilibrium under the sticky price is equivalent.

Proposition D1. *Assume a linear production $F(h) = h$ and rigidities $\bar{P} = \overline{W}$. The value functions $\{V, V_D, V_R\}$, policy rules $\left\{\hat{d}, \hat{c}^T, \hat{b}, \hat{h}\right\}$, and bond price schedule $\{q\}$ are the solution to the Markov recursive equilibrium with sticky wages if and only if they are also the solution to the Markov equilibrium for sticky prices .*

Proof. Using that $\alpha = 1$, we can write

$$\mathcal{H}\left(c^T, \bar{w}\right) = \left(\frac{1-\omega}{\omega}\frac{1}{\bar{w}}\right)^{\frac{1}{1+\mu}} c^T = \left(\frac{1-\omega}{\omega}\frac{1}{\bar{p}}\right)^{\frac{1}{1+\mu}} c^T = \tilde{\mathcal{H}}\left(c^T, \bar{p}\right).$$

Given that employment demand functions coincide under sticky prices and sticky wages, the government faces identical problems. As a result, we arrive to the same Markov equilibrium.

□

Away from a linear production, the Markov equilibria do not coincide. However, it is straightforward to obtain analogous propositions to the baseline model with nominal rigidities.

Let us define

$$\begin{aligned}
\bar{p}^D &\equiv \alpha \frac{1-\omega}{\omega} \left(y^T/\bar{h}^\alpha\right)^{1+\mu}, \\
\bar{p}^- &\equiv \alpha \frac{1-\omega}{\omega} \left(\left(y^T - \delta b^-_{flex}\right)/\bar{h}^\alpha\right)^{1+\mu}, \\
\bar{p}^+ &\equiv \alpha \frac{1-\omega}{\omega} \left(\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right)/\bar{h}^\alpha\right)^{1+\mu}.
\end{aligned}$$

Proposition D2. *Assume that $\beta(1+r) = 1$ and $y^T_t = y^T$ for all $t \geq 0$, and let $\left\{\bar{p}^D, \bar{p}^-, \bar{p}^+\right\}$ be price rigidity thresholds. We have the following:*

(i) *The safe zone is smaller under fixed exchange rates: The debt thresholds satisfy $b^-_{fix}(\bar{p}) \leq b^-_{flex}$ for any rigidity $\bar{p} \leq \bar{p}^D$. Moreover, the relationship is strict if $\bar{p}^- < \bar{p} \leq \bar{p}^D$. If preferences are separable, we have $b^-_{fix}(\bar{p}) < b_{flex}$ for every $\bar{p} > \bar{p}^-$.*

(ii) *A devaluation expands the safe zone: Assume that preferences are separable. We have that for every $e' > e$ then $b^-_{fix}\left(\overline{P}/e'\right) \geq b^-_{fix}\left(\overline{P}/e\right)$ for any nominal rigidity and exchange rate such that $\overline{P}/e \leq \bar{p}^D$. Moreover, the relationship is strict if $\bar{p}^- < \overline{P}/e \leq \bar{p}^D$.*

(iii) *Crisis and default zones: Assume $\pi = 0$. Then, there exists $\{\bar{p}^-, \bar{p}^+\}$ such that $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{p})$ for all $\bar{p}$ such that $\bar{p}^- < \bar{p} \leq \bar{p}^+$. Moreover, if preferences are separable, $b^+_{flex} > b^+_{fix}(\bar{p})$ for any rigidity $\bar{p} > \bar{p}^+$.*

Proof. Noting that $\tilde{\mathcal{H}}\left(c^T, \bar{p}\right)$ satisfies the same properties as $\mathcal{H}\left(c^T, \bar{w}\right)$, the proof becomes immediate based on the proof of Proposition 1. □

For the optimal exchange rate policy, we have the following

Proposition D3 (Optimal Exchange Rate Policy under sticky prices)**.** *Under a flexible exchange rate regime, the government always chooses an exchange rate that achieves full employment.*

Proof. The proof is immediate, based on the proof of Proposition C1. □

D.3 Devaluation costs

In this section, we explore a version of the model in which the government can choose the exchange rate every period, but a cost is associated with exchange rate fluctuations. We assume that exchange rate devaluations above a "natural" level $\bar{e} > 0$ incur a penalty $\Phi(e - \bar{e}) \geq 0$ that satisfies $\Phi(x) = 0$ for all $x \leq 0$, $\Phi'(0) = 0$, and $\Phi'(x) > 0$ for all $x > 0$.

The value of default for the government is given by

$$V_D\left(y^T\right) = \max_{e, c^T, h \leq \bar{h}} \left\{u\left(c^T, F(h)\right) - \kappa\left(y^T\right) - \Phi(e - \bar{e}) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D\left(y^{T\prime}\right)\right]\right\} \tag{D.1}$$

$$\begin{aligned}
c^T &= y^T \\
h &\leq \mathcal{H}\left(y^T, \overline{W}/e\right),
\end{aligned}$$

while the value of repayment is given by

$$V_R(b,\mathbf{s}) = \max_{e,b',c^T,h\leq\bar{h}} \left\{u\left(c^T, F(h)\right) - \Phi(e-\bar{e}) + \beta\mathbb{E}\left[V(b',\mathbf{s}')\right]\right\} \tag{D.2}$$
$$\text{s.t. } c^T = y^T - \delta b + q(b',b,\mathbf{s})(b' - (1-\delta)b).$$
$$h \leq \mathcal{H}\left(y^T, \overline{W}/e\right).$$

We can prove the following lemmas that characterize the optimal depreciation of the exchange rate in a flexible exchange rate regime:

Lemma D5 (Optimal Exchange Rate Policy)**.** *For a given c^T, the optimal exchange rate is given by*

$$\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right) = \underset{e}{\operatorname{argmax}} \left\{U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}/e\right)\right) - \Phi(e-\bar{e})\right\}. \tag{D.3}$$

Proof. The proof follows that for a given c^T, the maximizing value for e in (D.2) and (D.1) becomes static and satisfies (D.3). □

Lemma D6 (Optimal Exchange Rate Policy)**.** *For a given c^T, employment under flexible exchange is at least as high as under fixed exchange rate regime. In addition, $\mathcal{E}\left(c^T, \overline{W}\right) \geq \bar{e}$.*

Proof. By Lemma D5, the optimal exchange rate satisfies

$$U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}/\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)\right)\right) - \Phi\left(\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right) - \bar{e}\right) \geq U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}\right)\right).$$

We thus have

$$\begin{aligned} U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}/\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)\right)\right) &\geq U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}/\bar{e}\right)\right) + \Phi\left(\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right) - \bar{e}\right) \\ &\geq U\left(c^T, \mathcal{Y}^N\left(c^T, \overline{W}/\bar{e}\right)\right), \end{aligned}$$

where the second inequality follows from $\Phi \geq 0$. Because U is increasing in c^N, we arrive at

$$\mathcal{Y}^N\left(c^T, \overline{W}/\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)\right) \geq \mathcal{Y}^N\left(c^T, \overline{W}/\bar{e}\right),$$

concluding the proof of the first part. In addition, since $\mathcal{Y}^N$ is decreasing in $\overline{W}$, this means we must have $\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right) \geq \bar{e}$. □

We now characterize the employment function under a flexible exchange rate regime:

$$\tilde{\mathcal{H}}^{flex}\left(c^T, \overline{W}, \bar{e}\right) = \mathcal{H}\left(c^T, \overline{W}/\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)\right).$$

Using $\bar{w} \equiv \overline{W}/\bar{e}$ and H-demand, we thus have

$$\tilde{\mathcal{H}}^{flex}\left(c^T, \overline{W}, \bar{e}\right) = \mathcal{H}\left(c^T, \overline{W}/\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)\right) = \left(\frac{\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right)}{\bar{e}}\right)^{\frac{1}{1+\alpha\mu}} \mathcal{H}\left(c^T, \overline{W}/\bar{e}\right) \geq \mathcal{H}\left(c^T, \overline{W}/\bar{e}\right), \tag{D.4}$$

where the inequality follows $\mathcal{E}\left(c^T, \overline{W}, \bar{e}\right) \geq \bar{e}$. Let $b^-_{flex}\left(\overline{W}\right)$ and $b^+_{flex}\left(\overline{W}\right)$ be the thresholds under the flexible exchange rate regime for a given $\overline{W}$. Let us define wage thresholds:

$$
\begin{aligned}
\overline{W}^D &\equiv \bar{e}\alpha\frac{1-\omega}{\omega}\left(y^T\right)^{1+\mu}\left(\bar{h}\right)^{-(1+\alpha\mu)} \\
\overline{W}^- &\equiv \bar{e}\alpha\frac{1-\omega}{\omega}\left(y^T-\delta b^-_{flex}\right)^{1+\mu}\left(\bar{h}\right)^{-(1+\alpha\mu)} \\
\overline{W}^+ &\equiv \bar{e}\alpha\frac{1-\omega}{\omega}\left(y^T-\delta b^+_{flex}+\tilde{q}\left(\hat{b}^+_R-(1-\delta)b^+_{flex}\right)\right)^{1+\mu}\left(\bar{h}\right)^{-(1+\alpha\mu)}.
\end{aligned}
$$

We then have an analogous proposition to 1 for the comparison between fixed and flexible exchange rates.

Proposition D4. *Assume that $\beta(1+r)=1$ and $y^T_t = y^T$.*

i) Smaller safe zone under fixed: The debt thresholds satisfy $b^-_{fix}\left(\overline{W}\right) \leq b^-_{flex}\left(\overline{W}\right)$ for any rigidity $\overline{W} \leq \overline{W}^D$. Moreover, the relationship is strict if $\overline{W}^- < \overline{W} \leq \overline{W}^D$. Furthermore, if preferences are separable, we have $b^-_{fix}\left(\overline{W}\right) < b^-_{flex}\left(\overline{W}\right)$ for any $\bar{W} > \bar{W}^-$.

ii) Crisis and default zones: Assume $\pi = 0$. Then, there exists $\left\{\overline{W}^-, \overline{W}^+\right\}$ such that $\mathcal{C}_{flex}\left(\overline{W}\right) \subset \mathcal{C}_{fix}\left(\overline{W}\right)$ for all $\bar{w}$ such that $\overline{W}^- < \overline{W} \leq \overline{W}^+$. Moreover, if preferences are separable, $b^+_{flex}\left(\overline{W}\right) > b^+_{fix}\left(\overline{W}\right)$ for any $\overline{W} > \overline{W}^+$.

Proof. The proof follows the same steps as the proof of Proposition 1. □

D.4 Gains from a monetary union

In this section, we show our theoretical results are robust to incorporating gains from belonging to a currency union. We explore three different approaches to incorporating benefits from entering a monetary union: Section D.4.1 considers a version of the model in which monetary policy faces a time inconsistency problem; Section D.4.2 considers a version of the model in which entering a monetary union generates gains from trade; Section D.4.3 considers a version of the model in which entering a monetary union improves debt enforcement.

D.4.1 Inflationary bias

The first approach we consider is a simple time inconsistency problem in the flexible exchange rate regime, in the spirit of Barro and Gordon (1983). Assume that there are costs from the expectation of *future* depreciations (rather than from the current one as in Section D.3). In this context, the government would find it optimal to reduce future depreciations, but in the absence of commitment, it will find it optimal to deliver a high depreciation ex post, which results in an inflationary bias. Therefore, a government lacking commitment will experience a benefit from entering a monetary union (Alesina and Barro, 2002).

We assume that every period, there is a cost incurred today whenever there is positive expected depreciation. Following Section D.3, we assume an exogenous long-run level of exchange rate $\overline{e} > 0$

and a penalty $\mathbb{E}_t\left[\Phi\left(e_{t+1}-\bar{e}\right)\right] \geq 0$, now associated with expected depreciations from the target. We assume that the penalty satisfies $\Phi(x)=0$ for all $x \leq 0$, $\Phi'(0)=0$, and $\Phi'(x)>0$ for all $x>0$.

To define the government problem and the Markov equilibrium, it is important to consider the exchange rate that the government expects to be be followed in the next period. Given that the exchange rate is a function of $(b, \mathbf{s})$, we can write the expected devaluation cost as $\hat{\Phi}\left(b', \mathbf{s}\right)$. The value functions under fixed exchange rate are of course the same, while under a flexible exchange rate, the value of default reads as follows:

$$V_D\left(y^T\right)=\max_{c^T, e}\left\{u\left(c^T, F(h)\right)-\kappa\left(y^T\right)-\Phi\left(e-\bar{e}\right)+\beta \mathbb{E}\left[\psi V\left(0, \mathbf{s}'\right)+(1-\psi) V_D\left(y^{T\prime}\right)-\hat{\Phi}\left(0, \mathbf{s}'\right)\right]\right\} \tag{D.5}$$

$$\begin{aligned} \text{s.t. } c^T &= y^T. \\ h &\leq \mathcal{H}\left(y^T, \overline{W}/e\right). \end{aligned}$$

The values of repayment reads as follows:

$$\begin{aligned} V_R^+(b, \mathbf{s}) = \max_{b', c^T, e} & \left\{u\left(c^T, F(h)\right)-\Phi\left(e-\bar{e}\right)+\beta \mathbb{E}\left[V\left(b', \mathbf{s}'\right)-\hat{\Phi}\left(b', \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } c^T &= y^T-\delta b+q\left(b', \mathbf{s}\right)\left(b'-(1-\delta) b\right). \\ h &\leq \mathcal{H}\left(c^T, \overline{W}/e\right), \end{aligned} \tag{D.6}$$

$$\begin{aligned} V_R^-(b, \mathbf{s}) = \max_{b', c^T, e} & \left\{u\left(c^T, F(h)\right)-\Phi\left(e-\bar{e}\right)+\beta \mathbb{E}\left[V\left((1-\delta) b, \mathbf{s}'\right)-\hat{\Phi}\left((1-\delta) b, \mathbf{s}'\right)\right]\right\} \\ \text{s.t. } c^T &= y^T-\delta b. \\ h &\leq \mathcal{H}\left(c^T, \overline{W}/e\right). \end{aligned} \tag{D.7}$$

Comparing the gap between $V_R^+(b, \mathbf{s})$ and $V_R^-(b, \mathbf{s})$ reveals an important observation. Relative to the baseline version of the model without the inflationary bias costs, the gap between these two values is reduced by $\hat{\Phi}\left((b', \mathbf{s}')-\hat{\Phi}\left((1-\delta) b, \mathbf{s}'\right)\right.$. This term tends to be positive given that the exchange rate depreciation in the next period is increasing in the debt accumulated, conditional on repayment. As a result, inflationary bias costs per se do not necessarily increase the exposure to rollover crises. As we will see, this implies that a monetary union will remain much more exposed to rollover crises than a flexible exchange rate will be, even if we take into account problems of lack of commitment in monetary policy.

It is clear from (D.5) and (D.6) that the government always chooses ex post to depreciate sufficiently the constraint so that it achieves full employment.[38] That is, for every c^T, it chooses an exchange rate e^* such that $\bar{h}=\mathcal{H}\left(c^T, \bar{W}/e^*\right)$, just as characterized in Proposition 1

However, the presence of expected devaluation costs has two effects. First, the government will take into consideration how its current choices affect the future devaluation costs. In particular, if higher levels of debt are associated with higher costs from exchange rate depreciations, it may choose to borrow less than it would in the absence of these costs. Second, these welfare costs alter the overall welfare from a flexible exchange rate, even for the same allocations.

[38] We assume the government chooses the minimum depreciation that ensures full employment.

To examine the robustness of our finding to different inflationary bias costs, we solve the model for different values for the Φ function and re-run our main simulations. As Table D.1 shows, even with the time inconsistency problem in the exchange rate policy, the fraction of defaults due to rollover crises remains very small. In this regard, sacrificing monetary autonomy still leaves a government much more exposed to a rollover crisis.

Table D.1: Sensitivity to expected devaluation costs

	Fixed	Flex (ϕ =0)	ϕ =0.10	ϕ =0.20	ϕ =0.30
Average spread	2.00	1.96	1.67	1.15	1.05
Average debt-income	29.81	30.45	28.93	25.25	23.46
Spread volatility	1.72	1.60	1.39	1.52	1.02
Unemployment increase	1.71	0.00	0.00	0.00	0.00
Time in crisis zone	4.21	0.56	0.43	0.26	0.32
Defaults due to rollover crisis	10.48	1.04	1.44	0.92	1.64

Note: Parameter values correspond to the benchmark calibration for flexible exchange rate regimes. The devaluation cost is set to a quadratic penalty function as $\Phi(x) = \phi x^2$. The benchmark calibration uses $\phi = 0.0$; that is, there are no penalties from expected depreciations.

D.4.2 Gains from trade

The second approach we explore is that there are trade gains from being in a monetary union. In particular, we consider the possibility that under a monetary union, tradable endowment will be weakly higher than in a flexible exchange rate regime. In general, it is not possible to provide an analytical characterization of how a higher tradable endowment under a fixed exchange rate alters the exposure to rollover crises. There is, however, one case that is especially tractable. In fact, when preferences are separable, we can show that the debt-to-output thresholds that define the crisis zone are invariant to y^T.

Proposition D5. *Consider a constant level of output y^T , $\delta = 1$ and $\beta(1+r) = 1$. Assume that preferences are separable. Then, the debt thresholds are $b^-\left(y^T\right)/y^T$ and $b^+\left(y^T\right)/y^T$ are independent of y^T under a flexible exchange rate.*

Proof. We want to argue that $b^-(y^T(1+\Delta))/((1+\Delta)y^T)$ is independent of Δ. By definition, $b^-(y^T)$ satisfies

$$\ln\left(y^T - b^-(y^T)\right) + \frac{\beta}{1-\beta}\ln(y^T) = \frac{1}{1-\beta}\left[\ln(y^T) - \kappa\right]. \tag{D.8}$$

We can verify that at if $b = b^-(y^T)(1+\Delta)$, the government is also indifferent between repaying and defaulting when output is $y^T(1+\Delta)$. To see this, note that

$$\ln\left(y^T(1+\Delta) - b^-(y^T)(1+\Delta)\right) + \ln(y^T(1+\Delta)) = \tilde{\kappa}, \tag{D.9}$$

$$\ln(1+\Delta) + \log\left(y^T - b^-(y^T)\right) + \ln(1+\Delta) + \log(y^T) = \tilde{\kappa}.$$

□

This proposition implies that under a flexible exchange rate, a lower output relative to that of a monetary union does not increase the crisis zone in terms of debt-to-tradable output. The following proposition further shows that if the utility is linear in c^T,an economy within a monetary union is more vulnerable for a lower absolute value of debt even if tradable endowment is larger.

Proposition D6. *Assume that $\beta(1+r)=1$, $y_t^T = y_{flex}^T$ for flex and $y_t^T = y_{fix}^T$ for fixed for all $t \geq 0$, with $y_{fix}^T > y_{flex}^T$. In addition, assume the utility function is $u\left(c^T, c^N\right) = c^T + U^N\left(c^N\right)$. Then, the safe zone is smaller under fixed exchange rates. That is, the debt thresholds satisfy $b_{fix}^-(\bar{w}) < b_{flex}^-$ for any $\bar{w} < \bar{w}^D$, and there exists $\bar{w}^*$ such that the relationship is strict for any $\bar{w} > \bar{w}^*$.*

Proof. Under the utility function considered, we have for flexible exchange rates

$$V_{flex}^D = \frac{1}{1-\beta}(y_{flex}^T + U^N\left(F(\bar{h})\right) - \tilde{\kappa}$$

$$V_{flex}^-(b) = \frac{1}{1-\beta}(y_{flex}^T - \frac{\delta r}{r+\delta}b + U^N\left(F(\bar{h})\right).$$

Combining these two equations, we obtain an explicit value for b_{flex}^- as

$$\frac{1}{1-\beta}\frac{\delta r}{r+\delta}b_{flex}^- = \tilde{\kappa}. \tag{D.10}$$

In addition, under fixed exchange rates, we have that for any $\bar{w} < \bar{w}^D$,

$$V_{fix}^D(\bar{w}) = \frac{1}{1-\beta}(y_{fix}^T + U^N\left(F(\bar{h})\right) - \tilde{\kappa},$$

and

$$\begin{aligned} V_{R,fix}^-\left(b;\bar{w}\right) &= y_{fix}^T - \frac{\delta r}{r+\delta}b + \mathcal{Y}^N\left(y_{fix}^T - \frac{\delta r}{r+\delta}b, \bar{w}\right) \\ &+ \frac{\beta}{1-\beta}u\left(y_{fix}^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)b + \mathcal{Y}^{\mathcal{N}}\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)b, \bar{w}\right)\right). \end{aligned} \tag{D.11}$$

By definition, $b_{fix}^-(\bar{w})$ satisfies

$$\begin{aligned} \frac{1}{1-\beta}\frac{\delta r}{r+\delta}b_{fix}^-(\bar{w}) &= \tilde{\kappa} + \mathcal{Y}^N\left(y_{fix}^T - \frac{\delta r}{r+\delta}b, \bar{w}\right) - U^N(F(\bar{h})) \\ &\quad + \frac{\beta}{1-\beta}\left[\mathcal{Y}^{\mathcal{N}}\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)b, \bar{w}\right) - U^N\left(F(\bar{h})\right)\right] \\ &\leq \tilde{\kappa}, \end{aligned} \tag{D.12}$$

where the inequality follows from $\mathcal{Y}^N\left(y_{fix}^T - \frac{\delta r}{r+\delta}b, \bar{w}\right) \leq F(\bar{h})$. Combining (D.12) and (D.10), we therefore arrive at $b_{fix}^-(\bar{w}) \leq b_{flex}^-$. The strict part is straightforward to verify by taking $\bar{w} > \bar{w}^-$ □

The lesson is that the trade benefits from being in a monetary union do not overturn the result that a fixed exchange rate economy is more vulnerable. The logic is that what matters for the difference

in exposure to crises is how fewer available resources trigger an endogenous contraction in output under a fixed exchange rate. When output is different in an exogenous fashion, this effect does not necessarily lead to larger exposure.

D.4.3 Higher enforcement

The third approach we explore is that there are larger penalties from defaulting when a country is in a monetary union. The idea is to capture that within a monetary union, there may be a better enforcement of sovereign debt contracts.

We argue that even though a higher value of the default penalty κ naturally leads to a lower default zone, the crisis region may become larger. This result can be inspected from Figure 2. Upon a parallel shift in the value of default, the crisis region may increase or decrease, depending on the slopes of V_R^- and V_R^+ at the intersection point with V^D. This result is formalized in the proposition below.

Proposition D7. *Consider an economy with an exogenous value of default V_D, discount factor $\beta(1+r)=1$, and constant level of tradable endowment y^T. The effect of a marginal change in the value of default over the length of the crisis zone depends on the slopes of V_R^- and V_R^+ and is given by*

$$\frac{\partial\left(b^+-b^-\right)}{\partial V_D}=\frac{1}{\frac{\partial V_R^+(b^+)}{\partial b}}-\frac{1}{\frac{\partial V_R^-(b^-)}{\partial b}}. \tag{D.13}$$

Because the derivatives of these value functions are negative, it follows that if the absolute value of the derivative of V_R^- is higher than the one of V_R^+ evaluated respectively at b^- and b^+, the distance between b^- and b^+ goes up. In other words, if V_R^- varies relatively more than V_R^+ with the level of debt, the crisis zone expands when default costs increase.

Proof. We have by definition

$$V_R^+(b^+)=V^D,$$
$$V_R^-(b^-)=V^D.$$

By the implicit function theorem, we have that

$$\partial b^+/\partial V^D=-\frac{1}{\frac{\partial V_R^+}{\partial b}} \tag{D.14}$$

$$\partial b^-/\partial V^D=-\frac{1}{\frac{\partial V_R^-}{\partial b}} \tag{D.15}$$

Combining (D.14) and (D.15), we arrive at (D.13). □

D.5 Nominal debt

In this section, we study a version of the model in which debt is denominated in domestic currency. This feature is of course relevant only for the flexible exchange rate economy. To allow for an equilibrium with positive debt under flexible exchange rate, we consider a cost from depreciating the currency as in section D.3. For simplicity, we restrict ourselves to one-period debt.

The resource constraint for tradables is now given by

$$c^T = y^T - \frac{b}{e} + q\frac{b'}{e}. \tag{D.16}$$

Notice in this equation how an increase in e reduces real payments to foreigners and increases tradable consumption.

Denote the optimal exchange rate as a function of the level of debt $e = \mathcal{E}(b)$. From the investors' side, arbitrage implies that the fundamental bond price must satisfy

$$q(b')(1+r) = \frac{e}{\mathcal{E}(b')}.$$

If investors are pessimistic, the value of repayment for the government is

$$V_R^-(b) = \max_{e,b'} u\left(y^T - \frac{b}{e}, \mathcal{H}\left(y^T - \frac{b}{e}, \frac{W}{e}\right)\right) - \psi(e/\bar{e}) + \beta V_R^+(0). \tag{D.17}$$

The value of default is

$$V^D = \max_e \frac{u(y^T, \mathcal{H}\left(y^T, \frac{W}{e}\right)) - \kappa - \psi(e/\bar{e})}{1-\beta}. \tag{D.18}$$

Inspecting (D.17), one can observe how the increase in e not only raises employment through the reduction in the real wage rigidity constraint but also through the increase in tradable consumption (by effectively reducing debt repayment to foreigners).

D.6 Inflation targeting

In this section, we present a version of the model in which the government adopts an inflation-targeting regime. The goal is to consider a regime with a monetary policy constraint other than a fixed exchange rate. In this regime, the exchange rate adjusts, acting as a shock absorber, but the inflation targeting imposes a limit on the size of the depreciation that can be tolerated.

In line with the inflation targeting regime, we assume there is a long-run consumer price index for the composite good $\bar{P} > 0$. Given the definition of the ideal consumer price index, we have that

$$P\left(P^T, P^N\right) \equiv \left(\omega^{\frac{1}{1+\mu}}\left(P^T\right)^{\frac{\mu}{1+\mu}} + (1-\omega)^{\frac{1}{1+\mu}}\left(P^N\right)^{\frac{\mu}{1+\mu}}\right)^{\frac{1+\mu}{\mu}}.$$

The constraint imposed by an inflation-targeting regime can be expressed as $P\left(P^T, P^N\right) = \bar{P}$, and the government must satisfy $W \geq \bar{W}$. Using conditions (3) and (5), we reach the following constraint:

$$(1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h) \geq \tilde{w}, \tag{D.19}$$

where $\tilde{w} = \frac{W}{\bar{P}}$. Now, define the function $\mathcal{F}\left(c^T, h, \tilde{w}\right)$ as

$$\mathcal{F}\left(c^T, h, \tilde{w}\right) \equiv (1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h) - \tilde{w},$$

and notice that the implementability constraint (D.19) can therefore be written as $\mathcal{F}\left(c^T, h, \tilde{w}\right) \geq 0$.

Lemma D7. *The function $\mathcal{F}$ is increasing in c^T, and decreasing in h and $\tilde{w}$.*

Proof. Taking the partial derivatives, we obtain

$$\begin{aligned}
\tfrac{\partial \mathcal{F}}{\partial \tilde{w}} &= -1 \\
\tfrac{\partial \mathcal{F}}{\partial c^T} &= \omega(1-\omega)(1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu}\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\left(\frac{F'(h)}{c^T}\right) > 0 \qquad \text{and} \\
\tfrac{\partial \mathcal{F}}{\partial h} &= \omega(1-\omega)\left(\frac{c\left(c^T, F(h)\right)}{F(h)}\right)^{1+\mu} F'(h)\left[\frac{F''(h)}{F'(h)} - (1+\mu)\left(\frac{c\left(c^T, F(h)\right)}{c^T}\right)^{\mu}\frac{F'(h)}{F(h)}\right] < 0,
\end{aligned}$$

because $F(\cdot)$ is concave, and thus $F''(\cdot) < 0$. Hence, the expression $(1-\omega)\left(c\left(c^T, F(h)\right)/F(h)\right)^{1+\mu} F'(h)$ is increasing in tradable consumption and decreasing in labor. □

Lemma D8. *Consider the inflation-targeting constraint binds, $\mathcal{F}\left(c^T, h, \tilde{w}\right) = 0$. There exists a function $\tilde{\mathcal{H}}(c^T, \tilde{w})$ such that $\mathcal{F}\left(c^T, \tilde{\mathcal{H}}(c^T, \tilde{w}), \tilde{w}\right) = 0$. Moreover,*

$$\frac{\partial h}{\partial c^T} > 0 \qquad \textit{and} \qquad \frac{\partial h}{\partial \tilde{w}} < 0.$$

Proof. Implicitly deriving $\mathcal{F}\left(c^T, h, \tilde{w}\right) = 0$,

$$\frac{\partial h}{\partial c^T} = \frac{-\partial \mathcal{F}/\partial c^T}{\partial \mathcal{F}/\partial h} > 0 \qquad \text{and} \qquad \frac{\partial h}{\partial \tilde{w}} = -\frac{\partial \mathcal{F}/\partial \tilde{w}}{\partial F/\partial h} < 0,$$

using the properties of $\mathcal{F}$ in Lemma D7. □

Using this implementability constraint, we can express the government problems as

$$\begin{aligned}
V_D\left(y^T\right) = \max_{b', c^T, h \leq \bar{h}} &\left\{u(c^T, F(h)) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi)V_D\left(y^{T\prime}\right)\right]\right\}, \qquad \text{(D.20)} \\
&\text{subject to} \\
c^T &= y^T \\
\mathcal{F}\left(c^T, h, \tilde{w}\right) &\geq 0;
\end{aligned}$$

$$\begin{aligned}
V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \bar{h}} &\left\{u(c^T, F(h)) + \beta\mathbb{E}\left[V(b', \mathbf{s}')\right]\right\}, \qquad \text{(D.21)} \\
&\text{subject to} \\
c^T &= y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b) \\
\mathcal{F}\left(c^T, h, \tilde{w}\right) &\geq 0.
\end{aligned}$$

Let $b^-_{inf}(\tilde{w})$ and $b^+_{inf}(\tilde{w})$ be the thresholds under the inflation targeting regime for a given $\tilde{w}$.

Proposition D8. *Assume that $\beta(1+r)=1$ and $y_t^T = y^T$.*

(i) *Smaller safe zone under fixed: The debt thresholds satisfy $b_{inf}^{-}(\tilde{w}) \leq b_{flex}^{-}$ for any rigidity $\tilde{w} \leq \tilde{w}^D \equiv \alpha(1-\omega)c\left(y^T, \bar{h}^{\alpha}\right)^{1+\mu}\bar{h}^{-(1+\alpha\mu)}$. Moreover, there exists $\tilde{w}^-$ such that $b_{inf}^{-}(\tilde{w}) < b_{flex}$ for every $\tilde{w}^- < \tilde{w} \leq \tilde{w}^D$. If preferences are separable, we have $b_{inf}^{-}(\tilde{w}) < b_{flex}$ for every $\tilde{w}^- < \tilde{w}$.*

(ii) *Crisis and default zones: Assume $\pi = 0$. Then, there exists $\{\tilde{w}^-, \tilde{w}^+\}$ such that $\mathcal{C}_{flex} \subset \mathcal{C}_{inf}(\tilde{w})$ for all $\tilde{w}$ such that $\tilde{w}^- < \tilde{w} \leq \tilde{w}^+$. Moreover, if preferences are separable between tradables and non-tradables, $b_{flex}^{+} > b_{inf}^{+}(\tilde{w})$ for any rigidity $\tilde{w} > \tilde{w}^+$.*

Proof. The proof follows the same steps as the proof of Proposition 1 by defining the rigidity thresholds $\tilde{w}^- \equiv \alpha(1-\omega)c\left(y^T - \delta b_{flex}^{-}, \bar{h}^{\alpha}\right)^{1+\mu}\bar{h}^{-(1+\alpha\mu)}$ and $\tilde{w}^+ \equiv \alpha(1-\omega)c\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b_{flex}^{+}, \bar{h}^{\alpha}\right)^{1+\mu}\bar{h}^{-(1+\alpha\mu)}$. In addition, even though there is not an explicit employment function, employment implicitly satisfies the same properties as $\mathcal{H}\left(c^T, \bar{w}\right)$ when the implementability constraint binds, as shown in Lemma D8. □

D.7 Endogenous Labor Supply

In this section, we expand the baseline model to allow for an elastic supply of labor by incorporating disutility from labor. In particular, we consider a utility function over consumption and employment defined as $u\left(C\left(c^T, c^N\right)\right) - g\left(h^s\right)$, where $g' > 0$, $g'' > 0$.

As in the baseline model, we assume that if the wage necessary to clear the market is above $\bar{W}$, then equilbrium employment is given by labor demand. Otherwise, employment must satisfy

$$\frac{g'\left(h_t\right)}{u_N\left(c_t^T, F(h_t)\right)} = F'\left(h_t\right). \tag{D.22}$$

Let us characterize the unconstrained supply of labor hours that satisfies (D.22) in equilibrium as $\mathcal{H}^S\left(c^T\right)$.

We have the following proposition.

Proposition D9 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government always chooses an exchange rate that replicates the flexible wage allocation and delivers* (D.22)*.*

Proof. The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$\begin{aligned} V_R(b,\mathbf{s}) = & \max_{b', c^T, h \leq \mathcal{H}^S(c^T)} \left\{u\left(c^T, F(h), h\right) + \beta\mathbb{E}\left[V(b', \mathbf{s}')\right]\right\}, \\ \text{subject to} \quad & c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b) \\ & h \leq \mathcal{H}\left(c^T, \bar{W}/e\right). \end{aligned} \tag{D.23}$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the

following Bellman equation:

$$V_D(y^T) = \max_{c^T, h \leq \mathcal{H}^S(c^T)} \left\{ u\left(c^T, F(h), h\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right], \right. \tag{D.24}$$

$$\text{subject to} \quad c^T = y^T$$
$$h \leq \mathcal{H}\left(c^T, \bar{W}/e\right).$$

It is immediate from (D.23) and (D.24) that an increase in e increases the employment demand without tightening any other constraint. The first-order condition then yields (D.22). □

To characterize how the crisis zone changes with the exchange rate regime, we now adadpt the wage thresholds to the fact that equilibrium employment could be different from $\bar{h}$:

$$\bar{w}^D \equiv \frac{\alpha(1-\omega)}{\omega} \left(y^T\right)^{1+\mu} \left(\mathcal{H}^S\left(y^T\right)\right)^{-(1+\alpha\mu)}$$
$$\bar{w}^- \equiv \frac{\alpha(1-\omega)}{\omega} \left(y^T - \delta b^-_{flex}\right)^{1+\mu} \left(\mathcal{H}^S\left(y^T - \delta b^-_{flex}\right)\right)^{-(1+\alpha\mu)}$$
$$\bar{w}^+ \equiv \frac{\alpha(1-\omega)}{\omega} \left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right)^{1+\mu} \mathcal{H}^S\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}\right)^{-(1+\alpha\mu)}.$$

We therefore have the same proposition as in the baseline model without utility costs from working.

Proposition D10. *Assume that $\beta(1+r) = 1$ and $y^T_t = y^T$ for all $t \geq 0$, and let $\left\{\bar{w}^D, \bar{w}^-, \bar{w}^+\right\}$, be wage rigidity thresholds defined in the appendix . We have the following:*

(i) The safe zone is smaller under fixed exchange rates: The debt thresholds satisfy $b^-_{fix}(\bar{w}) \leq b^-_{flex}$ for any rigidity $\bar{w} \leq \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \bar{w} \leq \bar{w}^D$. Furthermore, if preferences are separable, we have $b^-_{fix}(\bar{w}) < b^-_{flex}$ for any $\bar{w} > \bar{w}^D$.

(ii) A devaluation expands the safe zone: Assume that preferences are separable. We have that for every $e_2 > e_1$, $b^-_{fix}(\overline{W}/e_2) \geq b^-_{fix}(\overline{W}/e_1) >$ for any rigidity $\overline{W}/e_2 \leq \bar{w}^D$ and $\overline{W}/e_1 \leq \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \overline{W}/e_1 \leq \bar{w}^D$.

(iii) Crisis and default zones: Assume $\pi = 0$. Then, we have $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{w})$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leq \bar{w}^+$. Moreover, if preferences are separable, $b^+_{flex} > b^+_{fix}(\bar{w})$ for any $\bar{w} > \bar{w}^+$.

Proof. The proof is identical to Proposition 1. □

E Details on welfare analysis

Welfare costs of giving up monetary independence. To compute the welfare costs from the lack of monetary independence, the first step is to construct the value functions for the government under the assumption that it is able to exit a fixed exchange rate for the current period. Replacing $\bar{h} = 1$,we obtain values given by

$$V_{0,flex}(b,\mathbf{s}) = \max\left\{V^R_{0,flex}(b,\mathbf{s}), V^D_{0,flex}(\mathbf{s})\right\}, \tag{E.1a}$$

$$V^R_{0,flex}(b,\mathbf{s}) = \max_{b'}\left\{u\left(y^T - \delta b + q(b',b,\mathbf{s})(b' - (1-\delta)b)\right), F\left(\bar{h}\right)\right) + \beta\mathbb{E}\left[V(b',\mathbf{s}')\right]\right\}, \tag{E.1b}$$

$$V^D_{0,flex}(\mathbf{s}) = u\left(y^T, F\left(\bar{h}\right)\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V(0,\mathbf{s}') + (1-\psi)V^D\left(y^{T\prime}\right)\right], \tag{E.1c}$$

where the continuation values correspond to the values under a monetary union.

The second step is to compute the percentage increase in the composite consumption that households would need under a monetary union rate to remain indifferent between staying and exiting. Denote this value by $\theta_0^{flex}(b,\mathbf{s})$. Using the value from exiting in the equation above, the value by $\theta_0^{flex}(b,\mathbf{s})$ can be obtained numerically from the followign equation:

$$V_{0,flex}(b,\mathbf{s}) = \left(1 - \hat{d}(b,\mathbf{s})\right)\left[\left(1+\theta_0^{flex}(b,\mathbf{s})\right)^{1-\sigma} u\left(\hat{c}^T(b,\mathbf{s}), \hat{c}^N(b,\mathbf{s})\right) + \beta\mathbb{E}\left[V\left(\hat{b}(b,\mathbf{s}),\mathbf{s}'\right)\right]\right] +$$

$$\hat{d}(b,\mathbf{s})\left[\left(1+\theta_0^{flex}(b,\mathbf{s})\right)^{1-\sigma} u\left(\hat{c}^T(b,\mathbf{s}), \hat{c}^N(b,\mathbf{s})\right) - \kappa\left(y^T\right) + \beta\mathbb{E}\left[\psi V(0,\mathbf{s}') + (1-\psi)V^D\left(y^{T\prime}\right)\right]\right],$$

where $\left\{\hat{d}(b,\mathbf{s}), \hat{c}^T(b,\mathbf{s}), \hat{c}^N(b,\mathbf{s}), \hat{b}(b,\mathbf{s})\right\}$ correspond to the optimal policies that solve (12)-(14) from the Markov equilibrium under a fixed exchange rate.

Figure 6 presents the value of $\theta_0^{flex}(b,\mathbf{s})$ for a range of debt levels, for the good sunspot $\zeta = 0$ and the bad sunspot $\zeta = 1$, and for a given endowment shock.

Welfare gains of LOLR. Access to a lender of last resort is equivalent to setting $\pi = 0$. To compute the permanent welfare gains for a lender of last resort, we therefore compute the value functions when $\pi = 0$. Let $V^{LOLR}(b,s)$ denote the value functions associated with $\pi = 0$. The welfare gain in consumption equivalence in terms of moving from an economy without a lender of last resort to an economy with lender of last resort can be computed as

$$\theta^{LOLR}(b,\mathbf{s}) = \left(V^{LOLR}(b,\mathbf{s})/V(b,\mathbf{s})\right)^{1/(1-\sigma)} - 1. \tag{E.2}$$

This construction exploits homotheticity of the utility function and transforms default costs into consumption equivalence.

F Sensitivity Analysis

F.1 Risk Aversion

In this section, we study how the main results of the benchmark model vary with different degrees of risk aversion. We conduct two exercises: one in which we change the risk aversion and recalibrate all parameters to match the same moments as in the baseline calibration, and another one in which we change the risk aversion and keep all parameters constant.

Table F.1 presents the first approach. We consider three values for risk aversion: $\sigma = 2$, $\sigma = 5$ (our benchmark value), and $\sigma = 10$. We recalibrate the default costs $\{\kappa_0, \kappa_1\}$, the discount factor β, and the wage rigidity $\bar{w}$ to match the same targets as in the baseline calibration. All other parameters are as in the baseline calibration for $\sigma = 5$. Overall, we can see that a higher risk aversion tends to increase the importance of rollover crises, but in all cases, the fixed exchange rate is substantially more vulnerable to a rollover crises. In particular, when risk aversion is doubled to 10, the share of defaults due to rollover crises is 2% while under fixed exchange rate, it is 15%.

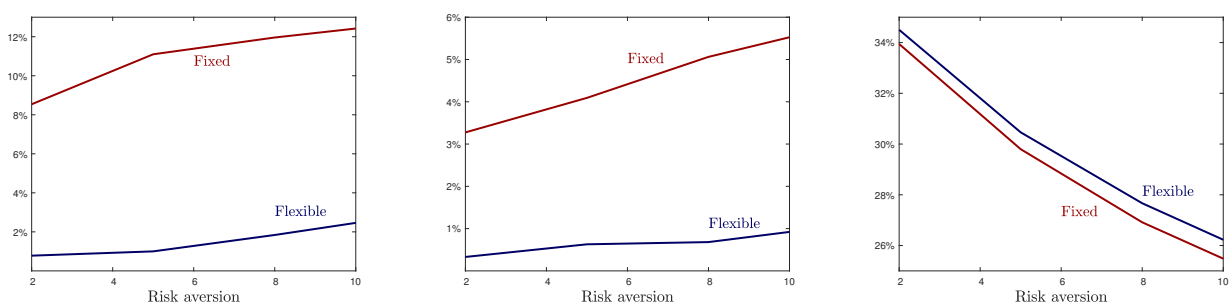
Table F.1: Statistics under different risk aversion

Risk aversion	Spain	$\sigma = 2$		$\sigma = 5$		$\sigma = 10$	
(percentage %)	Data	Flexible	Fixed	Flexible	Fixed	Flexible	Fixed
Average spread	2.01	1.97	1.92	1.96	2.00	1.90	2.32
Average debt-income	29.05	29.76	30.40	30.45	29.81	27.68	28.16
Spread volatility	1.42	1.46	1.73	1.60	1.72	1.67	2.07
Unemployment increase	2.00	0.00	1.69	0.00	1.71	0.00	1.57
Time in crisis zone	-	0.27	2.05	0.56	4.21	1.02	6.51
Defaults due to rollover crisis	-	0.56	5.02	1.04	10.48	2.00	15.38

Note: The parameter values for the default cost, β and $\bar{w}$, are recalibrated to match the same targets as in the baseline calibration. All other parameters are as in the baseline calibration for $\sigma = 5$.

Figure F.1: Risk-Aversion Sensitivity

(a) Defaults due to rollover (b) Time in crisis zone (c) Average debt



Note: All parameters except σ are for the baseline versions of the model for fixed and flexible exchange rates.

Figure F.1 presents the second approach. Here, we keep all parameters constant at the baseline values and vary risk aversion. The simulations show a similar pattern: a large robust difference in

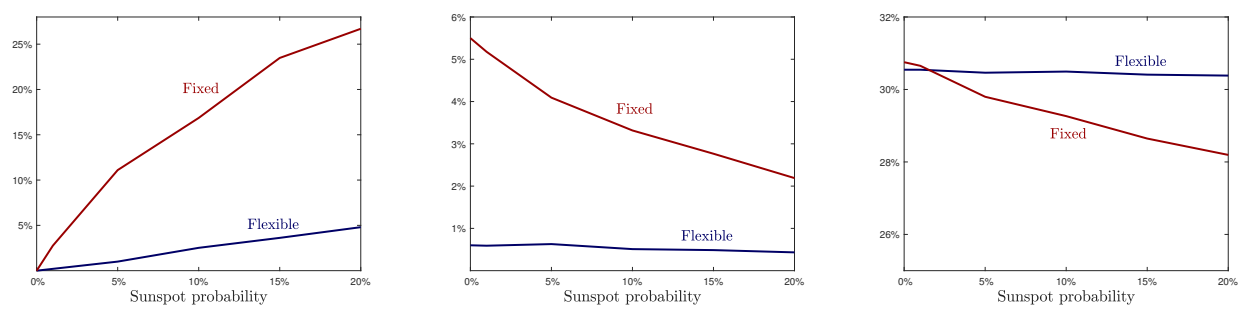
the exposure to rollover crises between fixed and flexible exchange rate. In addition, Figure 1(c) also shows that the fall of the debt-GDP ratio decreases for both regimes because of standard precautionary savings motives. However, because a higher risk aversion increases the crisis zone, the fraction of time spent in the crisis region still goes up under both regimes, but more so under fixed.

F.2 Sunspot Probability

In this section, we study how the main results of the benchmark model still hold under different sunspot probabilities. We complement Table 2 in the body of the paper by varying the sunspot probability without recalibrating any parameter from the model. Figure F.2 shows the the results. The first result we highlight is the share of defaults due to rollover crises increases for both regimes as the sunspot likelihood increases. However, the increase in the fixed exchange rate regime dwarfs the increase in the flexible exchange rate regime. Because the size of the crisis region is very small under a flexible exchange rate, varying the sunspot probability has modest effects. Similarly, the time spent in the crisis zone and the average debt change very little under flexible exchange rates. On the other hand, notice that when the sunspot probability reaches 20%, about a quarter of defaults under fixed exchange rates are due to rollover crises.

Figure F.2: Sunspot Sensitivity

(a) Share of rollover defaults (b) Crisis zone likelihood (c) Debt-GDP ratio

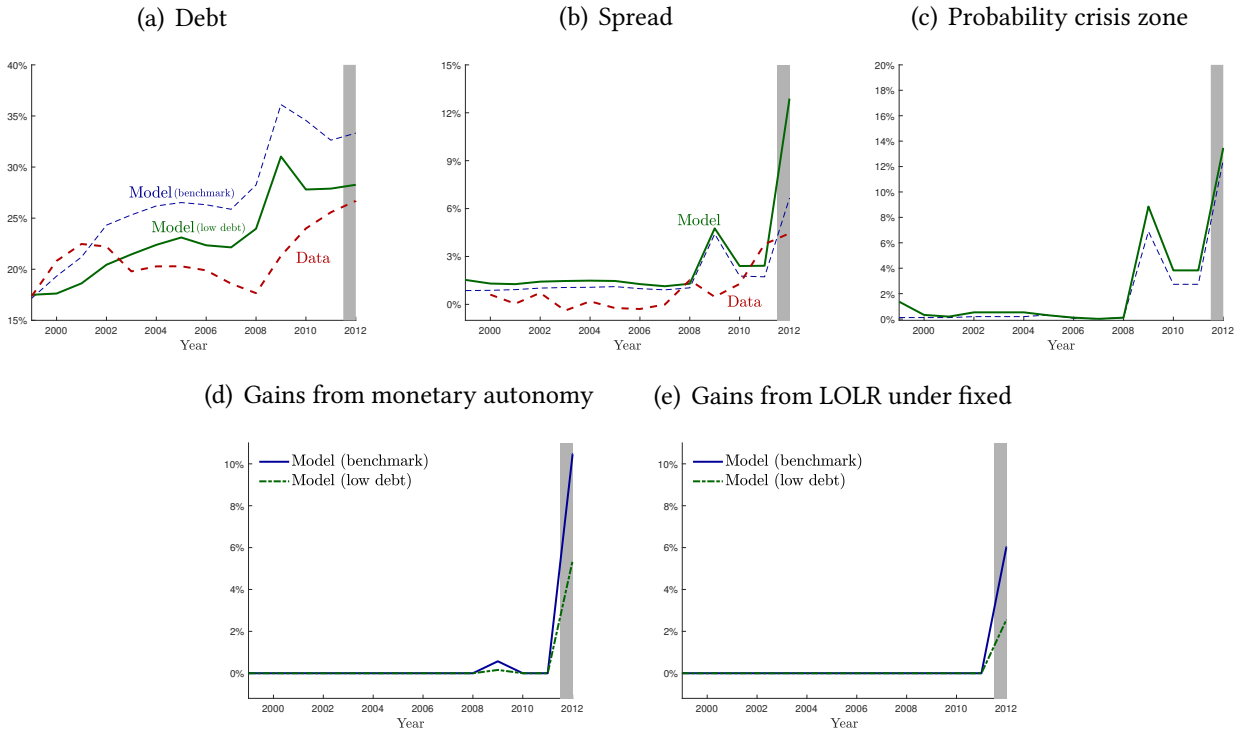


Note: All parameters except σ are for the baseline versions of the model for fixed and flexible exchange rates.

F.3 Discount Factor

In this section, we conduct a sensitivity analysis of the experiment in Section 4.4, in which we show that the model is able to generate a rollover crisis in Spain in 2012. Our benchmark calibration has a target debt level of 30%. The initial debt level in the data at the beginning of our simulation sample is lower by around 12 percentage poins. In this section, we recalibrate the discount factor so that the average debt level is 25%, which is the average debt-GDP ratio between 1999 and 2015. Figure F.3 shows the path to Spain's rollover crisis exercise under the benchmark and alternative calibrations. As one would expect, given the lower target for the debt level, the rise in debt is less strong than in the benchmark. Importantly, however, the alternative parameterization predicts that Spain reaches the crisis zone in 2012.

Figure F.3: Spain's crisis simulation with higher β



Note: The figure presents the experiment conducted in Section 4.4 for two differents discount factors, the baseline value and a higher one calibrated to a mean value of debt of 25%.

G Computational Appendix

In this appendix, we describe the numerical algorithm to solve the benchmark model and provide a robustness analysis of the results.

G.1 Numerical algorithm

The model is solved using value function iteration. We approximate the value function using a discrete grid for the initial bond positions and for the tradable endowment. For the optimization of the government problem, we first find a candidate for the global optimum using a grid search and then optimize locally around that point using a Newton method. In addition, we use quadrature to approximate the integrals for the expected value functions.

The complete list of steps is as follows:

1. Set an equidistant grid space for debt $\mathcal{B} \equiv \{b_i\}_{i=1}^{N_b}$. We set $N_b = 75$ and the extreme values of the grid $b_1 = 0$ and $b_{N_b} = 1$.

2. Set a grid for tradable output $\mathcal{Y} \equiv \{y_j\}_{j=1}^{N_y}$. We set $N_y = 21$ and use Tauchen's discretization method with parameter $\lambda = 3$ to set the grid elements $\{y_j\}_{j=1}^{N_y}$.

3. Set a grid space for future tradable output $\mathcal{Y}^j \equiv \left\{y_k^j\right\}_{k=1}^{N_k}$ and a grid space for probabilities $\mathcal{P}^j \equiv \left\{p_k^j\right\}_{k=1}^{N_k}$ for each tradable output $y_j \in \mathcal{Y}$. The extreme values of future tradable output are computed as $y_1^j = e^{\rho \ln y_j - 4\sigma_y}$ and $y_{N_k}^j = e^{\rho \ln y_j + 4\sigma_y}$. The rest of the elements of the future tradable output grid $\left\{y_{j,k}^p\right\}_{k=2}^{N_k-1}$ are set equidistantly. Meanwhile, the probability of each element of the grid $\{p_{j,k}\}_{k=1}^{N_k}$ is set with its associated mean and standard deviation of a lognormal distribution. Finally, normalize each probability distribution for future tradable output using a Newton-Cotes numerical integration method with a Trapezoidal rule. We set $N_k = 51$.

4. Set the initial guess value functions $V_D(y_j) = V_R^+(b_i, y_j) = V_R^-(b_i, y_j) = 0$ and bond price schedule $\tilde{q}(b_i, y_j) = \frac{1}{1+r}$ for every $i = 1, 2, \ldots, N_b$ and $j = 1, 2, \ldots, N_y$

5. Compute the new value of default $W_D(y_j)$ for every $y_j \in \mathcal{Y}$ as

$$W_D(y_j) = u(y_j, F(\hat{h}_D)) - \kappa(y^T) + \beta \left[\frac{1}{2} \sum_{k=1}^{N_k-1} \left(y_{k+1}^j - y_k^j\right) \left(p_{k+1}^j \Psi(y_{k+1}^j) + p_k^j \Psi(y_k^j)\right) \right],$$

   where $\hat{h}_D = \min\left\{\mathcal{H}(y_j, \overline{w}), \bar{h}\right\}$ and $y_k^p \in \mathcal{Y}^j$. The function Ψ is defined as

$$\Psi(y_k^j) \equiv \psi V_R^+(0, y_k^j) + (1 - \psi) V_D(y_k^j),$$

   where every $y_k^j \in \mathcal{Y}^j$. If $y_k^j \notin \mathcal{Y}$, then $V_D(y_k^j)$ and $V_R^+(0, y_k^j)$ are interpolated/extrapolated linearly.

6. Compute all the possible repayment with rollover values $\tilde{W}_R^+(x_i, b_i, y_j)$ for every $x_i, b_i \in \mathcal{B}$ and

$y_j \in \mathcal{Y}$ as

$$\tilde{W}_R^+(x_i, b_i, y_j) = u(y_j - \delta b_i + \tilde{q}(x_i, y_j)\,(x_i - (1-\delta)b_i)\,, F(\hat{h}_R^+(x_i, b_i, y_j))) \\ + \beta\left[\frac{1}{2}\sum_{k=1}^{N_k-1}\left(y_{k+1}^j - y_k^j\right)\left(p_{k+1}^j V(x_i, y_{k+1}^j) + p_k^j V(x_i, y_k^j)\right)\right],$$

where $\tilde{h}_R^+(x_i, b_i, y_j) = \min\left\{\mathcal{H}(y_j - \delta b_i + \tilde{q}(x_i, y_j)\,(x_i - (1-\delta)b_i)\,, \overline{w}), \bar{h}\right\}$ and the function $V(\cdot)$ satisfies

$$V(x_i, y_k^j) = \begin{cases} V_R^+(x_i, y_k^j) & \text{if } V_R^-(x_i, y_k^j) \geq V_D(y_k^j) \\ V_D(y_k^j) & \text{if } V_R^+(x_i, y_k^j) < V_D(y_k^j) \\ \pi V_D(y_k^j) + (1-\pi)V_R^+(x_i, y_k^j) & \text{if } V_R^+(x_i, y_k^j) \geq V_D(y_k^j)\ \&V_R^-(x_i, y_k^j) < V_D(y_k^j) \end{cases},$$

where every $y_k^j \in \mathcal{Y}^j$ and if $y_k^j \notin \mathcal{Y}$, then $V_D(y_k^j)$ and $V_R^+(x_i, y_k^j)$ are interpolated/extrapolated linearly.

7. Compute the new value of repayment with rollover as $W_R^+(b) = \max_x \tilde{W}_R^+(b, x)$ and the optimal level in the grid $\hat{b}_R^+(b) = \operatorname{argmax}_x \tilde{W}_R^+(b, x)$ for every $b \in \mathcal{B}$.

8. For every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$, find the maximum element in the grid $x_i \in \mathcal{B}$ such that $x_i = \operatorname{argmax}_{x\in\mathcal{B}}\left\{\tilde{W}\left(x, b_i, y_j\right)\right\}$, and define the reduced debt interval $\mathcal{X} \equiv [x_0, x_1]$ where $x_0 = b_{\max\{1,i-1\}}$ and $x_1 = b_{\min\{N_b,i+1\}}$.

9. For every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$, apply the *Golden Search* method with a tolerance criteria of 1.0×10^{-12} over the reduced debt interval $\mathcal{X}$, and if in each iteration $x \notin \mathcal{B}$, use linear interpolation. Call $W_R^+(b_i, y_j) = \max_{x\in\mathcal{X}}\left\{\tilde{W}(x, b_i, y_j)\right\}$ and $\hat{b}_R^+(b_i, y_j) = \operatorname{argmax}_{x\in\mathcal{X}}\left\{\tilde{W}(x, b_i, y_j)\right\}$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$.

10. If $\hat{b}_R^+(b_i, y_j) \leq (1-\delta)b_i$, compute the new value of repayment without rollover as $W_R^-(b_i, y_j) = W_R^+(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$.

11. If $\hat{b}_R^+(b_i, y_j) > (1-\delta)b$, compute the new value of repayment without rollover $W_R^-(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$ as

$$\tilde{W}_R^-(b_i, y_j) = u(y_j - \delta b_i, F(\hat{h}_R^-(b_i, y_j)))+ \\ \beta\left[\frac{1}{2}\sum_{k=1}^{N_k-1}(y_{k+1}^j - y_k^j)(p_{k+1}^j V((1-\delta)b_i, y_{k+1}^j) - p_k^j V((1-\delta)b_i, y_k^j))\right],$$

where $\tilde{h}_R^-(b_i, y_j) = \min\left\{\mathcal{H}(y_j - \delta b_i, \overline{w}), \bar{h}\right\}$ and the function $V(\cdot)$ satisfies

$$V((1-\delta)b_i, y_k^j) = \begin{cases} V_R^+((1-\delta)b_i, y_k^j) & \text{if } V_R^-((1-\delta)b_i, y_k^j) \geq V_D(y_k^j) \\ V_D(y_k^j) & \text{if } V_R^+((1-\delta)b_i, y_k^j) < V_D(y_k^j) \\ \pi V_D(y_k^j) + (1-\pi)V_R^+((1-\delta)b_i, y_k^j) & \text{if } V_R^+((1-\delta)b_i, y_k^j) \geq V_D(y_k^j) \\ & \&V_R^-((1-\delta)b_i, y_k^j) < V_D(y_k^j) \end{cases},$$

where every $y_k^j \in \mathcal{Y}^j$ and, if $y_k^j \notin \mathcal{Y}$, then $V_D(y_k^j)$ and $V_R^+((1-\delta)b_i, y_k^j)$ are interpolated/extrapolated linearly.

12. Set an auxiliary price schedule $\tilde{r}(b_i, y_j) = \tilde{q}(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$.

13. Compute the arbitrage condition price schedule $\tilde{s}(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$

$$\tilde{s}(b_i, y_j) = \frac{1}{1+r}\left[\frac{1}{2}\sum_{k=1}^{N_k}\left(y_{k+1}^j - y_k^j\right)\left(p_{k+1}^j Q(b_i, y_{k+1}^j) + p_k^j Q(b_i, y_k^j)\right)\right],$$

where the function $Q(\cdot)$ satisfies

$$Q(b_i, y_k^j) = \begin{cases} \delta + (1-\delta)\tilde{r}(\hat{b}_R^+(b_i, y_k^j), y_k^j) & \text{if } W_R^-(b_i, y_k^j) \geq W_D(y_k^j) \\ 0 & \text{if } W_R^+(b_i, y_k^j) < W_D(y_k^j) \\ (1-\pi)\left(\delta + (1-\delta)\tilde{r}(\hat{b}_R^+(b_i, y_k^j), y_k^j)\right) & \text{if } W_R^+(b_i, y_k^j) \geq W_D(y_k^j) \\ & \quad \& W_R^-(b_i, y_k^j) < W_D(y_k^j) \end{cases},$$

where every $y_k^j \in \mathcal{Y}^j$ and if $y_k^j \notin \mathcal{Y}$, then $W_D(y_k^j)$, $W_R^+(b_i, y_k^j)$, and $W_R^-(b_i, y_k^j)$ are interpolated/extrapolated linearly.

14. Compute the error update for the arbitrage condition as $z_R^+ \equiv \max_{(b,y)\in\mathcal{B}\times\mathcal{Y}}\{|\tilde{r}(b,y) - \tilde{s}(b,y)|\}$.

15. Update the debt price schedule $\tilde{r}(b_i, y_j) = \tilde{s}(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$.

16. If $z_r > 1.0 \times 10^{-10}$, return to step 13.

17. Compute the error updates of the value functions and price schedule as $z_D \equiv \max_{y\in\mathcal{Y}}\{|V_D(y) - W_D(y)|\}$, $z_R^+ \equiv \max_{(b,y)\in\mathcal{B}\times\mathcal{Y}}\left\{\left|V_R^+(b,y) - W_R^+(b,y)\right|\right\}$, $z_R^- \equiv \max_{(b,y)\in\mathcal{B}\times\mathcal{Y}}\left\{\left|V_R^-(b,y) - W_R^-(b,y)\right|\right\}$, and $z_q \equiv \max_{(b,y)\in\mathcal{B}\times\mathcal{Y}}\{|\tilde{q}(b,y) - \tilde{s}(b,y)|\}$.

18. Update the policy functions $V_D(y_j) = \lambda_v W_D(y_j) + (1-\lambda_v)V_D(y_j)$, $V_R^+(b_i, y_j) = \lambda_v W_R^+(b_i, y_j) + (1-\lambda_v)V_R^+(b_i, y_j)$ and $V_R^-(b_i, y_j) = \lambda_v W_R^-(b_i, y_j) + (1-\lambda_v)V_R^-(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$. We use $\lambda_v = 1.0$ for the flexible exchange rate regime and $\lambda_v = 0.5$ for the fixed exchange rate regime.

19. Update the debt price schedule using a smoothing step parameter λ_q as $\tilde{q}(b_i, y_j) = \lambda_q \tilde{s}(b_i, y_j) + (1-\lambda_q)\tilde{q}(b_i, y_j)$ for every $b_i \in \mathcal{B}$ and $y_j \in \mathcal{Y}$. We use $\lambda_q = 0.2$ for the flexible exchange rate regime and $\lambda_q = 0.3$ for the fixed exchange rate regime.

20. If $\max\{z_D, z_R^+, z_R^-, z\} > 1.0 \times 10^{-5}$ or $z_q > 1.0 \times 10^{-3}$, return to step 5.

G.2 Simulations

We collect 5,000 observations series of 50 periods before a default episode. The moments are computed using a simple average of the last 35 periods before each default epsisode. Each simulation sample is constructed as follows:

1. Set the period counter $t = 1$, and set initial states of debt $b_t = 0$, the initial tradable output $y_t^T = \mathbb{E}\left[y^T\right]$, and the sunspot realization $\zeta_t = 0$.

2. Solve the maximization problems $V_D\left(y_t^T\right)$, $V_R^-\left(b_t, y_t^T\right)$, and $V_R^+\left(b_t, y_t^T\right)$.

3. If $\left\{V_D\left(y_t^T\right) > V_R^+\left(b_t, y_t^T\right)\right\}$ or $\left\{V_D\left(y_t^T\right) > V_R^-\left(b_t, y_t^T\right) \quad \& \quad \zeta = 1\right\}$, move to step 6.

4. Set the following period states as $b_{t+1} = \hat{b}_R^+(b_t, y_t^T)$ and tradable output $y_{t+1}^T = e^{\rho \ln(y_t^T) + \sigma \varepsilon_t}$, where ε_t is picked with a pseudo-random realization from a standard Normal distribution, and ζ_{t+1} picked with a pseudo-random realization from a Bernoulli distribution.

5. Update the period counter $t = t + 1$ and return to step 3.

6. If $t < 50$, return to step 1.

7. If $V_D\left(y_t^T\right) \leq V_R^+\left(b_t, y_t^T\right)$, update rollover default counter $d_r = d_r + 1$.

8. Compute the moments using the last 35 periods.

G.3 Robustness

As described above, our baseline algorithm uses $N_b = 75$ points for the grid of debt, $N_y = 21$ for the tradable output shock, and $N_k = 51$ to approximate the expectation of the tradable endowment shock. To analyze the robustness of the results, we solve the model using more points in the approximation. As Table G.1 shows, increasing the number of approximating points does not have any significant effects on the key results. Across all values of the knots, the time spent in the crisis zone is close 0.5% under flex and around 4% under fixed.

Table G.1: Simulations' vulnerability to rollover crisis under different grid nodes

Grid nodes	Time in crisis zone		Rollover defaults share	
(Percentage %)	Flexible	Fixed	Flexible	Fixed
Benchmark	0.56	4.21	1.04	10.48
$N_b = 125$	0.54	4.40	1.14	9.96
$N_b = 251$	0.56	4.31	1.38	10.80
$N_y = 51$	0.55	4.37	1.24	10.16
$N_y = 101$	0.52	4.31	1.30	10.58
$N_k = 151$	0.50	3.94	1.48	9.82
$N_k = 251$	0.47	4.31	1.22	10.58

Note: The benchmark model uses the nodes $N_b = 75, N_y = 21$, and $N_k = 51$. We keep the sensitivity analysis changing only one of the grid nodes, described in the first column of the table. We compute 5,000 default episodes for every simulation. The time in the crisis zone is computed as a standard average of the share of periods in the crisis zone. The rollover defaults share is the share of defaults that were the sunspot is active while being in the crisis zone.

Finally, the convergence criteria we use is 1.0×10^{-5} for the value functions and 1.0×10^{-3} for the debt price schedule. This convergence criteria is in line with what is standard in the literature, as in Bocola and Dovis (2019). Tables G.2 and G.3 also show the convergence criteria when we increase the number of grid points under the two regimes.

Table G.2: Flexible exchange rate regime policy functions error updates

Norm $\|\| \cdot \|\|_\infty$ Grid nodes	Flexible exchnage rate regime V_D	V_R^+	V_R^-	$\tilde{q}$	V	q
Benchmark	6.0×10^{-6}	6.0×10^{-6}	7.0×10^{-6}	2.5×10^{-4}	6.0×10^{-6}	2.3×10^{-5}
$N_b = 125$	8.0×10^{-6}	8.1×10^{-5}	2.3×10^{-5}	6.4×10^{-3}	5.3×10^{-5}	3.4×10^{-3}
$N_b = 251$	3.0×10^{-6}	1.2×10^{-4}	9.6×10^{-5}	1.6×10^{-2}	1.2×10^{-4}	6.9×10^{-3}
$N_y = 51$	9.0×10^{-6}	9.0×10^{-6}	1.0×10^{-5}	2.6×10^{-4}	9.0×10^{-6}	6.4×10^{-4}
$N_y = 101$	1.0×10^{-6}	3.6×10^{-4}	5.6×10^{-5}	1.7×10^{-2}	2.7×10^{-4}	1.7×10^{-2}
$N_k = 151$	2.0×10^{-6}	7.0×10^{-6}	4.0×10^{-6}	8.1×10^{-4}	4.0×10^{-6}	5.1×10^{-5}
$N_k = 251$	1.0×10^{-6}	7.0×10^{-6}	3.0×10^{-6}	6.8×10^{-4}	3.0×10^{-6}	8.0×10^{-4}

Note: The benchmark model uses the nodes $N_b = 75$, $N_y = 21$, and $N_k = 51$. We keep the sensitivity analysis, changing only one of the grid nodes, described in the first column of the table. The norm used is the maximum error between policies and their updates. We fix the step parameter for the price schedule update to $\lambda = 0.30$. If after 1,000 iterations, the convergence criterium is not satisfied, the algorithm is forced to exit.

Table G.3: Fixed exchange rate regime policy functions error updates

(Norm $\|\| \cdot \|\|_\infty$) Grid nodes	Fixed exchange rate regime V_D	V_R^+	V_R^-	$\tilde{q}$	V	q
Benchmark	1.0×10^{-7}	2.9×10^{-6}	9.0×10^{-6}	9.9×10^{-4}	2.4×10^{-6}	3.9×10^{-5}
$N_b = 125$	1.9×10^{-5}	7.2×10^{-4}	2.2×10^{-4}	1.2×10^{-2}	7.2×10^{-4}	1.1×10^{-2}
$N_b = 251$	1.5×10^{-5}	1.4×10^{-3}	3.5×10^{-4}	3.1×10^{-2}	1.3×10^{-3}	3.0×10^{-2}
$N_y = 51$	1.0×10^{-7}	6.5×10^{-4}	1.3×10^{-5}	8.2×10^{-3}	1.7×10^{-4}	1.3×10^{-3}
$N_y = 101$	8.0×10^{-6}	3.2×10^{-4}	8.4×10^{-5}	4.0×10^{-3}	3.2×10^{-4}	1.8×10^{-3}
$N_k = 151$	4.0×10^{-6}	3.2×10^{-4}	6.5×10^{-5}	6.3×10^{-3}	1.9×10^{-4}	1.0×10^{-3}
$N_k = 251$	1.0×10^{-7}	8.0×10^{-6}	2.0×10^{-6}	1.3×10^{-4}	8.0×10^{-6}	2.6×10^{-5}

Note: The benchmark model uses the nodes $N_b = 75$, $N_y = 21$, and $N_k = 51$. We keep the sensitivity analysis, changing only one of the grid nodes, described in the first column of the table. The norm used is the maximum error between policies and their updates. We fix the step parameter for the price schedule update to $\lambda = 0.15$. If after 1,000 iterations, the convergence criterium is not satisfied, the algorithm is forced to exit.

H Empirical Appendix

H.1 Data for Exchange Rate Classification

We use the exchange rate classification database provided in Ilzetzki, Reinhart, and Rogoff (2019). Their database constructs a monthly measure of exchange rate flexibility for 194 countries and territories from 1946 to 2016. Table H.1 shows the Ilzetzki, Reinhart, and Rogoff classification based on their coarse index.

For our analysis, "fixed regime" refers to observations in which the exchange rate index is equal to 1. Next, "intermediate regime" refers to observations where the exchange rate index is equal to 2 or 3. Finally, "flexible regime" refers to observations where the exchange rate index is equal to 4 or 5. In the database, there are no observations with an exchange rate index equal to 6. Since Ilzetzki, Reinhart, and Rogoff observations are monthly, we use the mode to construct a quartertly series.

Table H.1: Exchange rate coarse index of Ilzetzki, Reinhart, and Rogoff (2019)

Index	Exchange rate regime
1	No separate legal tender
1	Pre announced peg or currency board agreement
1	Pre announced horizontal band that is narrower than or equal to +/-2%
1	De facto peg
2	Pre announced crawling peg
2	Pre announced crawling band that is narrower than or equal to +/-2%
2	De facto crawling peg
2	De facto crawling band that is narrower than or equal to +/-2%
3	Pre announced crawling band that is wider than or equal to +/-2%
3	De facto crawling band that is narrower than or equal to +/-5%
3	Moving band that is narrower than or equal to +/-2%
3	Managed floating
4	Freely floating
5	Freely falling
6	Dual market in which parallel market data is missing

H.2 List of Countries

Our starting point in the list of EMBI countries. After we restrict ourselves to countries with higher than 60% of foreign currency denominated debt, we arrive at Table H.2.

Table H.2: Data country list and exchange rate classification

Country	Starting Quarter	Ending Quarter	Exchange Rate Type	Country	Starting Quarter	Ending Quarter	Exchange Rate Type
Bolivia	2012-Q4	2015-Q2	1	Brazil	2008-Q4	2015-Q2	3
Bulgaria	2005-Q3	2015-Q2	1	Chile	2003-Q1	2015-Q2	3
Croatia	2000-Q1	2015-Q2	1	Colombia	2013-Q2	2015-Q2	3
Ecuador	2003-Q1	2007-Q3	1	Hungary	2002-Q1	2009-Q1	3
Ecuador	2008-Q1	2015-Q2	1	India	2012-Q4	2012-Q4	3
Jordan	2011-Q1	2014-Q4	1	Indonesia	2005-Q1	2007-Q2	3
Lithuania	2009-Q4	2015-Q2	1	Indonesia	2014-Q2	2015-Q2	3
Romania	2013-Q1	2015-Q2	1	Korea	1999-Q1	2004-Q2	3
Slovakia	2013-Q3	2015-Q2	1	Malaysia	2012-Q1	2015-Q2	3
Ukraine	2003-Q4	2013-Q4	1	Mexico	2002-Q1	2015-Q2	3
Argentina	2003-Q3	2015-Q2	2	Paraguay	2013-Q1	2015-Q2	3
China	2015-Q1	2015-Q2	2	Peru	2003-Q2	2012-Q2	3
Costa Rica	2012-Q3	2015-Q2	2	Philippines	2010-Q3	2015-Q2	3
Egypt	2006-Q3	2015-Q2	2	Poland	1999-Q1	2015-Q2	3
Georgia	2008-Q2	2015-Q2	2	Russia	2008-Q4	2014-Q3	3
Guatemala	2012-Q3	2015-Q2	2	Thailand	2003-Q1	2006-Q1	3
Hungary	2009-Q2	2015-Q2	2	Turkey	2008-Q4	2015-Q2	3
India	2013-Q1	2015Q2	2	Uruguay	2003-Q4	2015-Q2	3
Indonesia	2007-Q3	2014-Q1	2	Brazil	2003-Q3	2008-Q3	4
Kazakhstan	2007-Q2	2015-Q2	2	South Africa	2003-Q4	2015-Q2	4
Kenya	2015-Q1	2015-Q2	2	Turkey	2003-Q2	2008-Q3	4
Mongolia	2013-Q3	2014-Q2	2	Russia	2014-Q4	2015-Q2	5
Mongolia	2015-Q1	2015-Q2	2	Turkey	2002-Q1	2003-Q1	5
Nigeria	2011-Q2	2013-Q1	2	Ukraine	2014-Q1	2015-Q2	5
Peru	2012-Q3	2015-Q2	2				
Romania	2012-Q1	2012-Q4	2				
Russia	2003-Q1	2008-Q3	2				
Sri Lanka	2012-Q4	2015-Q2	2				
Tunisia	2002-Q4	2015-Q2	2				

H.3 Data sources

Table H.3: Data series sources and details

Series	Source	Detail
Spreads	World Bank (GEM) Global Economic Monitor	J.P. Morgan Emerging Markets Bond Index (EMBI+)
Nominal GDP	World Bank (GEM) Global Economic Monitor	Quarterly, current US$, millions, seas. adj.,
Nominal Debt Government	World Bank (QEDS/SDDS) Quarterly External Debt Statistics	Gross Ext. Debt Pos., General Government, All maturities, All instruments, USD
Nominal Debt Foreign Currency	World Bank (QEDS/SDDS) Quarterly External Debt Statistics	Gross Ext. Debt Pos., All Sectors, All maturities, All instruments, Foreign currency, USD
Nominal Debt All Currencies	World Bank (QEDS/SDDS) Quarterly External Debt Statistics	Gross Ext. Debt Pos., All Sectors, All maturities, All instruments, All currencies, USD
Exchange rate type	Ilzetzki et al. (2019)	Exchange rate regime classification (coarse), monthly

H.4 Regressions

We run the following regression in the data:

$$\text{spread}_{it} = \beta_i^0 + \beta_b^1 \left(\frac{\text{Debt}}{\text{GDP}}\right)_{it} + \beta_y^1 (\text{GDP})_{it} + \beta_b^2 \left(\frac{\text{Debt}}{\text{GDP}}\right)_{it}^2 + \beta_y^1 (\text{GDP})_{it}^2 + \varepsilon_{it}, \tag{H.1}$$

where GDP is HP-filtered detrended. Section H.3 details the data sources for spreads, debt, and GDP.

Table H.4: Regression Analysis

Exchange Rate Regime	N	β_b^1	β_y^1	β_b^2	β_y^2	R^2
Data						
Fixed	260	- 5.595	-34.162***	3.454	482.110***	0.450
Intermediate	855	- 4.211***	-24.598***	3.771***	88.160	0.639
Flexible	108	- 6.605***	-10.841	12.512***	414.321***	0.949
Model						
Fixed	5000	-0.952***	-15.767***	1.773***	2.583***	0.866
Flexible	5000	-8.702***	-1.182***	13.589***	0.112***	0.977

Note: *** p<0.01, ** p<0.05, * p<0.1

We estimate equation (H.1) by ordinary least squares (OLS). Table H.4 shows the results of the fixed effects analysis for each different exchange rate regime using the data and model simulations. For the model, we present the average of the regression statistics that we obtain across all samples for the two economies. The tables show that the coefficients have the same signs in the model and in the

data. Remarkably, the R^2 coefficient is higher under flexible exchange rate, both in the model and in the data. Thus, fundamentals do not account as much for variation of spreads under a fixed exchange rate.