MONETARY INDEPENDENCE AND ROLLOVER CRISES*

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This article shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. We study a sovereign default model with self-fulfilling rollover crises, foreign currency debt, and nominal rigidities. When the government lacks monetary independence, lenders anticipate that the government would face a severe recession in the event of a liquidity crisis and are therefore more prone to run on government bonds. In a quantitative application to the Eurozone debt crisis, we find that the lack of monetary autonomy played a central role in making Spain vulnerable to a rollover crisis. Finally, we argue that a lender of last resort can go a long way toward reducing the costs of giving up monetary independence. *JEL Codes:* E4, E5, F34, G15.

I. Introduction

A prominent concern during the Eurozone crisis was the risk of a rollover crisis. Policy makers feared that an adverse shift in market expectations would restrict governments' ability to roll over their debt, creating liquidity problems that would feed back into investors' expectations and ultimately lead governments to default. At the same time, the premise was that the lack of monetary independence was aggravating sovereign debt problems in Southern Europe. In this context, the European Central Bank (ECB) took unprecedented policy measures aimed at stabilizing

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financial markets and reducing the risk of a breakup of the monetary union.[1]

The goal of this article is to investigate whether and how the lack of monetary independence affects the vulnerability to a rollover crisis. A central question we tackle is: does a country become more vulnerable after joining a monetary union?

We present a theory in which the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. The key insight is that lenders' pessimism can trigger a demand-driven recession, making the option to default more attractive for the government and, in turn, validating lenders' pessimism. With an independent monetary policy, a government can alleviate the recession that results from the fiscal contraction during a rollover crisis, making investors less prone to run in the first place. Quantitative simulations also show that while an economy that possesses monetary independence is almost immune to a rollover crisis, it can become significantly vulnerable once it joins a monetary union. Moreover, we argue that a lender of last resort can significantly mitigate the welfare costs from joining a monetary union and therefore enhance its stability.

The environment we consider is a version of the canonical model of sovereign default à la Eaton and Gersovitz (1981) that incorporates the possibility of rollover crises, as in Cole and Kehoe (2000). The government issues debt before deciding whether to repay or default. When lenders expect the government to default, the government is shut off from credit markets and forced to repay the maturing debt exclusively out of its tax revenues. When the maturing debt is large enough, repayment becomes too costly for the government, lenders' pessimistic expectations are validated, and a self-fulfilling rollover crisis arises. We depart from the standard endowment economy setup by considering nominal rigidities, which creates scope for a stabilization role for monetary policy. External debt is denominated in real terms, or equivalently in foreign currency, eliminating the possibility of inflating away the debt. The model features tradable and nontradable goods and

1. On September 6, 2012, Mario Draghi, the president of the European Central Bank, expressed that "the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium,' namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Preceding these remarks, Draghi famously pledged to do "whatever it takes to preserve the euro."

downward nominal wage rigidity, as in Schmitt-Grohé and Uribe (2016). In this environment, a shock leading to a contraction in aggregate demand reduces the price of nontradables in equilibrium, generating a decline in labor demand. When wages cannot fall sufficiently quickly to clear the labor market, involuntary unemployment arises, and the economy goes through a recession. Following the classic principles from Friedman (1953), a government with an independent monetary policy can use the nominal exchange rate as a shock absorber, altering real wages and reducing unemployment.

Our main theoretical result is that the lack of an independent monetary policy increases the vulnerability to a rollover crisis. To understand the mechanisms in the model, consider what happens when a government is trying to roll over its debt and investors suddenly panic and refuse to lend to it. As the government is shut off from credit markets, it needs to raise tax revenues and cut down on expenditures to service the maturing debt. In the presence of nominal rigidities and constraints on monetary policy, this situation has macroeconomic implications. The fiscal contraction generates a decline in aggregate demand, which leads to involuntary unemployment and makes repayment less attractive for the government. If the increase in unemployment is sufficiently large, the government finds it optimal to default, which in turn validates the initial panic by investors and generates a self-fulfilling rollover crisis. Interestingly, for this pessimistic equilibrium to emerge, unemployment does not have to be realized in equilibrium. In fact, it is the off-equilibrium outcome of a large recession that pushes the government to default and triggers the rollover crisis.

Under monetary independence, the government can offset the recessionary effects from the fiscal contraction that results from being shut off from credit markets. Because of this ability, the government's willingness to repay becomes relatively less affected by the lenders' pessimistic expectations. Compared with an economy that lacks monetary independence, a panic is therefore less likely to occur in the first place. We also show that these theoretical insights carry over to several variations of the baseline model. Among others, we show that the same results apply when the source of nominal rigidity is on prices rather than wages, when there is production in both sectors, and when there are benefits from following a fixed exchange rate regime.

We conduct a quantitative investigation. We start by considering a calibration of the model under monetary independence, specifically, a flexible exchange rate regime under which the government chooses the exchange rate optimally at each point in time. In this regime, the government finds it optimal to implement the full-employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. (Notice, however, that the government cannot alter the value of the debt, since it is denominated in foreign currency.) Our simulations show that rollover crises play a modest role under a flexible exchange rate regime: only 1 out of 100 default episodes are driven by rollover crises.

We examine the effects of giving up monetary independence. One can think of a small open economy that has a fixed exchange rate regime or, equivalently, a single small economy in a monetary union in which wages (and debt) are denominated in the currency of the union and the conduct of monetary policy is exogenous to the single small economy. Keeping the same parameter values for the calibration of the flexible exchange rate regime, we find that the economy faces a significantly larger fraction of defaults due to rollover crises, which can reach about 11% (compared with 1% in the flexible exchange rate regime). Our findings therefore suggest that joining a monetary union entails significant costs in terms of a higher exposure to rollover crises.

Using the calibrated model for the fixed exchange rate regime, we then simulate the path of the Spanish economy, starting at the time of its adoption of the euro. We find that the economy hits the “crisis zone” precisely around the time of turmoil in sovereign debt markets. As a counterfactual, we show that if Spain had exited the Eurozone, it would have remained immune to a rollover crisis. The goal of this exercise is not to argue that being part of a monetary union is undesirable but to point out that a cost of giving up monetary independence is higher vulnerability to rollover crises. An important welfare consequence that emerges from our analysis is that a lender of last resort can significantly reduce the costs of remaining in a monetary union. Consistent with our model, Mario Draghi pledged to do “whatever it takes,” and after the speech, spreads fell immediately and Spain ultimately did not default on the debt.

I.A. Related Literature

Our article contributes to a vast literature on monetary unions, pioneered by the seminal work of Mundell (1961). The traditional view is that the benefit of joining a monetary union is more international trade, fostered by lower transaction costs. A more modern view, stressed by Alesina and Barro (2002), has emphasized the benefits from reduced inflationary bias generated by the time inconsistency problem of monetary policy identified in the seminal work of Barro and Gordon (1983). The main theme in the literature is that these benefits have to be traded off against the losses from inefficient macroeconomic fluctuations due to nominal rigidities and the lack of monetary independence. A comprehensive discussion of these issues, which have taken center stage since the formation of the Eurozone, is provided in Alesina, Barro, and Tenreyro (2003), Santos Silva and Tenreyro (2010), and De Grauwe (2020). A related literature compares the performance of fixed versus flexible exchange rates. Several studies in particular study the role of exchange rate policies in the presence of firms' balance sheet constraints (e.g., Céspedes, Chang, and Velasco 2004; Gertler, Gilchrist, and Natalucci 2007).[2]

Our article adds a new dimension to the costs from giving up monetary independence, namely, a higher exposure to rollover crises. Our welfare analysis shows that the higher exposure to rollover crises can be substantial and suggests that these costs should be part of the overall evaluation of a cost-benefit analysis. In this respect, our results shed some light on the Outright Monetary Transactions facility established by the ECB, following Draghi's July 2012 speech. Indeed, the article shows that a lender of last resort can enhance a monetary union by substantially reducing the costs from the lack of monetary independence.

This article also belongs to the literature on rollover crises in sovereign debt markets, starting with Sachs (1984), Alesina, Prati, and Tabellini (1990), and Cole and Kehoe (2000). Our formulation follows Cole-Kehoe, which has become the workhorse model in the quantitative sovereign default literature in the tradition of Aguiar and Gopinath (2006) and Arellano (2008). Examples include Chatterjee and Eyigungor (2012), Aguiar et al. (2016), Conesa and Kehoe (2017), Roch and Uhlig (2018), and Bocola and Dovis (2019). Different from these contributions, we consider an economy with production and nominal rigidities and establish how the exchange rate regime is central to the risk of exposure to

2. Also related is an active closed economy literature on how the interaction between household deleveraging and a zero lower bound can amplify demand shocks (Eggertsson and Krugman 2012; Guerrieri and Lorenzoni 2017).

rollover crises. With a flexible exchange rate regime, we find the exposure to a rollover crisis to be minimal, which is in line with Chatterjee and Eyigungor (2012), who showed that in a canonical endowment economy model with long-term debt calibrated to the data, the presence of rollover crises has a negligible effect on debt and spreads. By contrast, we show that with a fixed exchange rate regime, the multiplicity region expands significantly, and the government is heavily exposed to a rollover crisis.[3]

The paper that is perhaps most closely related to ours is Aguiar et al. (2013), who address the question of whether the government's ability to inflate away its debt reduces its exposure to rollover crises, an argument notably raised by De Grauwe (2013) and Krugman (2011), who made the observation that Spain and Portugal had higher levels of sovereign spreads compared with those of the United Kingdom, despite having lower levels of debt. Aguiar et al. consider an endowment economy with domestic currency debt and show that when commitment to low inflation is weak, an independent monetary policy can actually increase the vulnerability to a rollover crisis, contrary to De Grauwe's and Krugman's argument. Our article also studies how monetary policy matters for the exposure to a rollover crisis but considers instead a model with nominal rigidities and foreign currency debt. Our results show that the lack of monetary autonomy does increase vulnerability to a rollover crisis and provide a new perspective that ascribes a role for monetary policy to deal with rollover crises, even when debt is denominated in foreign currency.[4]

3. With one-year maturity, as in Cole and Kehoe (1996, 2000), the exposure to a rollover crisis is typically large because the government has to roll over a large amount of debt relative to output every period. While they were motivated by the Mexican crisis in 1994 with maturity of less than a year, the typical maturity for sovereign bonds is much larger, averaging around six years for the Eurozone. With debt duration calibrated to the Eurozone, Conesa and Kehoe (2017) and Bocola and Dovis (2019) achieve a somewhat more significant role for rollover crises but rely implicitly on a minimum subsistence level for consumption, which they set to about 70% of income, and require debt levels of around 100% of GDP for a typical rollover crisis. Overall, the quantitative analysis in Bocola and Dovis still finds that nonfundamental risk played a limited role during the Italian debt crisis.

4. Aguiar et al. (2015) consider a setup similar to Aguiar et al. (2013), but with multiple countries and a union-wide monetary policy. They show that for a country with a high level of debt, it is preferable to join a monetary union with a mix of high- and low-debt countries as a way to balance the costs from inflationary bias and the reduction in the vulnerability to rollover crises by inflating away the debt ex post. Other papers addressing issues of debt crises with a focus on the

A related literature studies sovereign debt crises, but in the tradition of Calvo (1988), where the government lacks commitment to debt issuances. If investors expect high inflation, the government borrows at a high rate and finds it optimal to inflate ex post, validating the initial expectations. In this line of work, the fact that debt is denominated in domestic currency and that the government can inflate away the debt is at the core of the fragility problem.[5] We consider a baseline model with debt in real terms, which allows us to abstract from the use of inflation to reduce the real value of the debt (and the associated multiplicity issues) to highlight a new channel by which monetary policy can actually help reduce a fragility problem originating from rollover crises.

Our study is also related to an emerging literature that integrates nominal rigidities into the workhorse sovereign default model. Na et al. (2018) study a sovereign default model with downward nominal wage rigidity and show that it can account for the joint occurrence of large nominal devaluations and defaults, a phenomenon known as the "twin Ds." Bianchi, Ottonello, and Presno (2019) analyze the trade-off between the expansionary effects of government spending and the increase in sovereign risk and show how it can generate the observed fiscal procyclicality. Other recent papers include Arellano, Bai, and Mihalache (2019), who study the comovements of sovereign spreads with domestic nominal rates and inflation, and Bianchi and Sosa-Padilla (2020), who study the accumulation of international reserves as a macroeconomic stabilization tool. In contrast to this literature, we consider the possibility of rollover crises, which allows us to provide the first analysis of how nominal rigidities and monetary policy affect vulnerability to rollover crises.

Eurozone are Broner et al. (2014) and Gourinchas, Martin, and Messer (2017) (see also De Ferra and Romei 2020; Fornaro 2020).

5. A large literature on multiple equilibria follows this tradition, including Corsetti and Dedola (2016), Farhi and Maggiori (2018), Bacchetta, Perazzi, and Van Wincoop (2018), and Lorenzoni and Werning (2019). The role of inflation as a partial default also plays a key role in recent work by Araujo, Leon, and Santos (2013), Du and Schreger (2016), Bassetto and Galli (2019), Nuño and Thomas (2017), Camous and Cooper (2019), and Hur, Kondo, and Perri (2018).

II. Model

We study a small open economy (SOE) model with nominal rigidities, in which the government is unable to commit to repay the sovereign debt and is subject to rollover crises. We describe the decision problems of households, firms, lenders, and the government.

II.A. Households

There is a unit measure of households with preferences over consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where c_t is a constant elasticity of substitution (CES) composite of tradable goods c_t^T and nontradable goods c_t^N

$$c_t = \left[\omega\left(c_t^T\right)^{-\mu} + (1-\omega)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1}{\mu}}, \quad \omega \in (0,1), \ \mu > -1.$$

Each period, households receive y_t^T units of tradable endowment, which is stochastic and follows a stationary first-order Markov process. We assume a constant unit price of tradable goods in terms of foreign currency and that the law of one price holds. The value of the tradable endowment in domestic currency is therefore given by $e_t y_t^T$, where e_t denotes the exchange rate measured as domestic currency per foreign currency (an increase in e_t denotes a depreciation of the domestic currency). Households also receive firms' profits, which we denote by ϕ_t^N, and labor income, $W_t h_t$, where W is the wage expressed in domestic currency and h is the amount of hours worked. Households inelastically supply $\bar{h}$ hours of work to the labor markets but will work a strictly lower amount of hours when the downward wage rigidity is binding.

As is standard in the sovereign debt literature, we assume that households do not have direct access to external credit markets, although the government can borrow abroad and distribute the net proceedings to the households using lump-sum taxes or transfers. The households' budget constraint is therefore given by

$$e_t c_t^T + P_t^N c_t^N = e_t y_t^T + \phi_t^N + W_t h_t - T_t, \tag{2}$$

where P_t^N denotes the price of nontradables and T_t denotes lump-sum taxes, both in units of domestic currency.

The households' problem consists of choosing c_t^T and c_t^N to maximize equation (1), taking as given the sequence of prices for nontradables, labor income, profits, and taxes $\{P_t^N, W_t h_t, \phi_t^N, T_t\}_{t=0}^{\infty}$. The static optimality condition equates the relative price of nontradables to the marginal rate of substitution between tradables and nontradables:

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{3}$$

Thanks to homotheticity, the relative demand of tradable and nontradable consumption goods is a function only of the relative price.[6]

II.B. Firms

Firms operate a production function $y^N = F(h)$, where y_t^N denotes the output of nontradable goods and h_t denotes employment, the sole input. The production function $F(\cdot)$ is a differentiable, increasing, and concave function. In particular, we consider a homogeneous production function $F(h) = h^\alpha$, where $\alpha \in (0, 1]$. Firms operate in perfectly competitive markets, and in each period, they maximize profits given by

$$\phi_t^N = \max_{h_t} P_t^N F(h_t) - W_t h_t. \tag{4}$$

The optimal choice of labor employment h_t equates the value of the marginal product of labor to the nominal wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

Employment demand is decreasing in wages and increasing in the price of nontradables.

II.C. Downward Nominal Wage Rigidity

Wages in domestic currency are downwardly rigid:

$$W_t \geqslant \bar{W}, \tag{6}$$

6. Homotheticity, as implied by the CES structure, is assumed to simplify the analysis. For our results, it suffices that nontradable goods are normal.

where the parameter $\bar{W}$ determines the severity of the rigidity.[7] If the nominal wage that clears the labor market is higher than $\bar{W}$, the economy is at full employment. If the nominal wage that would clear the market is below $\bar{W}$, the wage rigidity binds and the economy experiences involuntary unemployment. In this case, the amount of employment in equilibrium is determined by firms' labor demand equation (5). Formally, wages and employment need to satisfy the following slackness condition:

$$(W_t - \bar{W})(\bar{h} - h_t) = 0. \tag{7}$$

It will be convenient to define the corresponding market wage and wage lower bound in foreign currency as $w_t \equiv \frac{W_t}{e_t}$ and $\bar{w} \equiv \frac{\bar{W}}{e_t}$. An alternative representation of inequality (6) is therefore $w_t \geqslant \bar{w}$.[8]

II.D. Government

The government issues a noncontingent, long-term bond with geometrically decaying coupons.[9] In particular, a bond issued in period t promises to pay $\delta(1-\delta)^{j-1}$ units of foreign currency in period $t+j$ for all $j \geqslant 1$. Debt dynamics can be represented by the following law of motion: $b_{t+1} = (1-\delta)b_t + i_t$, where b_t is the stock of debt owed at the beginning of period t, and i_t is the stock of new bonds issued in period t.

Debt contracts cannot be enforced. If the government chooses to default, it faces two punishments. First, the government remains in financial autarky for a stochastic number of periods. Second, there is a utility loss $\kappa(y^T)$, which we think of as

7. In Schmitt-Grohé and Uribe (2016), $\bar{W}$ depends on the previous period's wage and a parameter that controls the speed of wage adjustment. For numerical tractability, we take $\bar{W}$ as an exogenous (constant) value, as in Bianchi, Ottonello, and Presno (2019). A vast empirical literature documents the importance of downward wage rigidity. In particular, a recent literature has used micro-level data to highlight the important role this friction played in the European crisis (e.g., Ronchi and Di Mauro 2017; Faia and Pezone 2018).

8. For an economy in a currency union, wages are set in foreign currency (the currency of the union), and therefore the lower bound is also, in effect, in foreign currency.

9. We take maturity as a primitive, following Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). There is an active literature studying maturity choices in sovereign default models (Arellano and Ramanarayanan 2012; Sanchez, Sapriza, and Yurdagul 2018; Bocola and Dovis 2019).

capturing various default costs related to reputation, sanctions, or the misallocation of resources.[10]

The government's budget constraint in a period starting with good credit standing is

$$\delta e_t b_t (1 - d_t) = T_t + e_t q_t i_t (1 - d_t), \tag{8}$$

where q_t is the price of the bond in foreign currency and d_t is a default indicator that takes the value of 1 if the government repays and 0 otherwise. The budget constraint indicates that outstanding debt obligations are repaid by collecting lump-sum taxes and issuing new debt.[11]

The timing in each period follows Cole and Kehoe (2000). At the beginning of each period, the government has outstanding debt liabilities b_t and could be in good or bad credit standing. If the government is in good credit standing, it chooses new debt issuances at the price schedule offered by investors. At the end of each period, the government decides whether to default or repay the initial debt outstanding. The difference with respect to Eaton and Gersovitz (1981) that will give rise to multiplicity is that here the government cannot commit to repaying within the period.[12] As we will see, negative beliefs about the decision of the government to repay can become self-fulfilling.

10. Utility losses from default in sovereign debt models are also used by Bianchi, Hatchondo, and Martinez (2018) and Roch and Uhlig (2018), among others. An alternative often used is an output cost. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications. In any case, as will become clear, what will be crucial for our mechanism is the gap between the value of repayment for the government when investors are willing to lend and when they refuse to lend, which is independent of the form of default costs.

11. As is well understood, allowing for specific taxes on consumption or payroll subsidies can mimic a nominal depreciation (see Correia, Nicolini, and Teles 2008; Farhi, Gopinath, and Itskhoki 2013; Schmitt-Grohé and Uribe 2016). As long as there are some limitations on the use of these policies (either political or economic), there remains a role for explicit nominal depreciations. From a normative standpoint, the importance of the exchange rate regime that we will uncover applies therefore to the role of fiscal devaluation policies.

12. A different source of multiplicity following Calvo (1988) arises if the government has to issue a fixed amount of debt revenues. In this case, the fact that bond prices decrease with debt generates a Laffer curve, which leads directly through the budget constraint to a high debt/high spreads equilibrium and a low debt/low spreads equilibrium. Lorenzoni and Werning (2019) and Ayres et al. (2016) explore this type of multiplicity in dynamic setups.

1. *Monetary Regimes.* We consider two regimes: a flexible exchange rate and a fixed exchange rate. In the flexible exchange rate regime, the government chooses the optimal exchange rate at all dates without commitment. In the fixed exchange rate regime, we assume that the government sets the exchange rate to an exogenous fixed level at all times. Equivalently, one can interpret the fixed exchange rate regime as the policy of a single economy that enters a monetary union and gives up its currency.[13] In the case of a monetary union, wages are directly set in the currency of the union. The important point, whether in a fixed exchange rate or in a monetary union, is that the government loses the ability to conduct its own monetary policy and use the exchange rate as a shock absorber.

II.E. International Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-free security denominated in units of foreign currency that pays a net interest rate r. By a no-arbitrage condition, equilibrium bond prices when the government repays are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{9}$$

II.F. Equilibrium

In equilibrium, the market for nontradable goods must clear domestically:

$$c_t^N = F(h_t). \tag{10}$$

Combining the household budget constraint (2) and the government budget constraint (8) with firms' profits equation (4) and the market-clearing condition (10), we obtain the resource constraint for tradable goods in the economy:

$$c_t^T = y_t^T + (1-d_t)[\delta b_t - q_t(b_{t+1} - (1-\delta)b_t)]. \tag{11}$$

13. One could also allow some degree of correlation between the small open economy and the monetary policy conducted at the union level by allowing P^* to follow a stochastic process correlated with the shocks to the small open economy. Theoretically, our results would remain unchanged.

Before proceeding to study a Markov equilibrium in which the government chooses policies optimally without commitment, we examine the equilibrium for given government policies.

DEFINITION 1. (Competitive equilibrium) Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a competitive equilibrium is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that:

i. Households and firms solve their optimization problems.
ii. Government policies satisfy the government budget constraint (8).
iii. The bond pricing equation (9) holds.
iv. The market for nontradable goods equation (10) clears
v. The labor market satisfies conditions (6), (7), and $h_t \leqslant \bar{h}$.

1. *Employment, Consumption, and Wages.* At the core of our framework is a relationship between aggregate demand and employment. By combining the optimality for households and firms together with market clearing for nontradable goods, we can obtain a useful (partial) characterization of equilibrium. As we show in Lemma A1, employment follows:

$$\text{(H-demand)} \qquad \mathcal{H}(c_t^T, \bar{w}) = \min\left\{\left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}, \bar{h}\right\}.$$

Condition (H-demand) establishes that employment is increasing in tradable consumption (strictly so, if the level of employment is below $\bar{h}$). The core intuition is that if there are fewer resources available for tradable consumption, the demand for nontradable goods must also fall (because both are normal goods). In turn, the fall in the households' demand for nontradable goods leads in equilibrium to lower employment demand by firms. When the downward wage rigidity is binding and monetary policy does not respond, the reduction in demand generates underutilization of labor.

2. *Remark.* The model features a Keynesian positive relationship between aggregate demand and employment. Although we obtain this relationship under a specific structure with wage rigidity and a two-sector open economy model with external government borrowing, the positive relationship between aggregate demand and employment is a general feature of a large class of models with nominal rigidities. For example, in a SOE model with home and foreign goods and price rigidities à la Gali and Monacelli (2005), a contraction in domestic resources available also generates a recession.

As we will see, this Keynesian feature will crucially affect the incentives of the government to repay and its vulnerability to a rollover crisis.

II.G. The Recursive Government Problem and Markov Equilibrium

We consider the optimal policy of a benevolent government that chooses without commitment. We focus on Markov equilibria. The payoff-relevant states are $(b, \mathbf{s})$, where $\mathbf{s} = (y^T, \zeta)$ denotes the vector of exogenous states in every period. The variable ζ is a sunspot variable to index for the possibility of multiplicity of equilibria, which is assumed to be i.i.d. over time. We use $q(b', b, \mathbf{s})$ to denote the bond price schedule faced by the government. In contrast to the equilibrium according to the Eaton and Gersovitz (1981) timing, the possibility of a rollover crisis implies that the bond price is a function of the initial debt position and the sunspot, in addition to the debt choice and current income shock.

When the government has access to financial markets, it compares the values of repayment and default, denoted respectively by $V_R(b, \mathbf{s})$ and $V_D(y^T)$:

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \{(1-d)V_R(b, \mathbf{s}) + dV_D(y^T)\}. \tag{12}$$

1. *Fixed Exchange Rate Regime.* We start by focusing on a fixed exchange rate regime. The government chooses allocations, subject to the resource constraint and the implementability constraints associated with the labor market. Let us use $u(c_t^T, c_t^N)$ to denote the utility flow. The value function under repayment

can be written as

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h} \{u(c^T, F(h)) + \beta \mathbb{E}[V(b', \mathbf{s}')]\} \tag{13}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$h \leqslant \mathcal{H}(c^T, \bar{w}),$$

where $\mathcal{H}$ is defined in (H-demand). Meanwhile, the value of default is given by

$$V_D(y^T) = \max_h \{u(y^T, F(h)) - \kappa(y^T) + \beta \mathbb{E}[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})]\}, \tag{14}$$

subject to

$$h \leqslant \mathcal{H}(y^T, \bar{w}),$$

where $\psi \in [0, 1]$ denotes the probability of reentering financial markets after a default.

DEFINITION 2. (Markov-perfect equilibrium) A Markov-perfect equilibrium under a fixed exchange rate regime is defined by value functions $\{V(b, \mathbf{s}), V_R(b, \mathbf{s}), V_D(y^T)\}$, policy functions $\{\hat{d}(b, \mathbf{s}), \hat{c}^T(b, \mathbf{s}), \hat{b}(b, \mathbf{s}), \hat{h}(b, \mathbf{s})\}$, and a bond price schedule $q(b', b, \mathbf{s})$ such that

i. given the bond price schedule, the value functions and policy functions solve problems (12), (13), and (14);
ii. the debt price schedule satisfies

$$q(b', b, \mathbf{s}) = \begin{cases} \frac{1}{1+r} \mathbb{E}[(1-d')(\delta + (1-\delta) q(b'', b', \mathbf{s}'))] & \text{if } \hat{d}(b, \mathbf{s}) = 0, \\ 0 & \text{if } \hat{d}(b, \mathbf{s}) = 1, \end{cases}$$

where $b'' = \hat{b}(b', \mathbf{s}')$ and $d' = d(b', \mathbf{s}')$.

2. *Flexible Exchange Rate Regime.* For the economy with a flexible exchange rate, the only difference in the government problem is that the optimization also includes the choice of the

exchange rate. As should be clear from the nominal rigidity constraint $w \geqslant \frac{\bar{W}}{e_t}$, an exchange rate depreciation enables the government to offset the wage rigidity and achieve the flexible wage allocation. As shown in Proposition C1 in the Appendix, this is also the optimal time-consistent policy. Indeed, notice from problem (13) that a depreciation relaxes the implementability constraint $h \leqslant \mathcal{H}(c^T, \bar{w})$. This result is in line with the traditional benefit of having a flexible exchange rate in the presence of nominal rigidities, going back to Friedman (1953) and Mundell (1961).

The stabilization of unemployment through the adjustment of real wages is indeed a central channel of monetary policy in open economies. Milton Friedman, for example, highlighted the dangers of Europe's eliminating the exchange rate adjustment precisely because of possible misalignments in real wages.[14] A subtle yet important difference in our theory is that a government depreciation may be a purely off-equilibrium policy.

II.H. Multiplicity of Equilibrium

We will look for two possible equilibria, one where investors lend and the government repays, and one in which investors refuse to lend and the government defaults.

Let us define the fundamental price as the price at which the government bond would trade if investors were willing to lend:

$$\tilde{q}(b', y^T) \equiv \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))]. \tag{15}$$

14. As expressed by Milton Friedman in "Why Europe Can't Afford the Euro," *Times* (London), November 19, 1997, "If one country is affected by negative shocks that call for, say, lower wages relative to other countries, that can be achieved by a change in one price, the exchange rate, rather than by requiring changes in thousands on thousands of separate wage rates, or the emigration of labour. The hardships imposed on France by its 'franc fort' policy illustrate the cost of a politically inspired determination not to use the exchange rate to adjust to the impact of German unification. Britain's economic growth after it abandoned the exchange-rate mechanism a few years ago to refloat the pound illustrates the effectiveness of the exchange rate as an adjustment mechanism."

Denote by V_R^+ the value of repayment for the government when facing the fundamental price:

$$(16)\qquad V_R^+(b, y^T) = \max_{b', c^T, h \leqslant \bar{h}} \{u(c^T, F(h)) + \beta \mathbb{E}[V(b', \mathbf{s}')]\},$$

$$\text{s.t.} \qquad c^T = y^T - \delta b + \tilde{q}(b', \mathbf{s})[b' - (1-\delta)b],$$

$$h \leqslant \mathcal{H}(c^T, \bar{w}).$$

Consider now the situation in which investors are unwilling to lend to the government, a restriction that is relevant only when the government would want to issue debt when facing the fundamental price. Denoting by V_R^- the value of repaying in this case, we have that

$$(17)\quad V_R^-(b, y^T) = \max_{c^T, h \leqslant \bar{h}} \{u(y^T - \delta b, F(h)) + \beta \mathbb{E}[V((1-\delta)b, \mathbf{s}')]\},$$

$$\text{s.t.} \qquad c^T = y^T - \delta b,$$

$$h \leqslant \mathcal{H}(c^T, \bar{w}),$$

Because the government can always choose not to borrow when lenders are willing to issue new debt, we have that $V_R^+ \geqslant V_R^-$.[15] Moreover, tradable consumption is necessarily lower when the government does not have access to borrowing, and hence employment will also be lower—recall that $\mathcal{H}(\cdot)$ is increasing in c^T. A key implication, which is at the heart of our model, is that the presence of wage rigidity will have a more substantial adverse effect on V_R^- than on V_R^+.

1. *Three Zones.* Following Cole and Kehoe (2000), let us separate the state space into three zones: the safe zone, the default zone, and the crisis zone. In the safe zone, the government finds it optimal to repay its debt even if international lenders are unwilling to roll over the debt. In the default zone, the government finds it optimal to default regardless of whether international lenders are willing to lend. Finally, in the crisis

15. One element implicit in the budget constraint in problem (17) is that if the government were to repurchase debt when investors are unwilling to lend, the price of bonds would rise to the fundamental price, as reflected in problem (17). See Aguiar and Amador (2013) and Bocola and Dovis (2019) for an elaboration of this point.

FIGURE I

Debt Thresholds for Given y^T

zone, the government finds it optimal to repay if investors are willing to lend at the fundamental debt price schedule but finds it optimal to default if investors are unwilling to lend. These three zones can be characterized respectively as follows:

$$\mathcal{S} = \{(b, y^T):\ V_D(y^T) \leqslant V_R^-(b, y^T)\},$$
$$\mathcal{D} = \{(b, y^T):\ V_D(y^T) > V_R^+(b, y^T)\},$$
$$\mathcal{C} = \{(b, y^T):\ V_R^-(b, y^T) < V_D(y^T) \leqslant V_R^+(b, y^T)\}.$$

It is in the crisis zone that the equilibrium outcome is undetermined and depends on investors' beliefs. If investors believe the government will repay, the government will find it optimal to repay, whereas if they believe that the government will default, the government will find it optimal to default. To select an equilibrium, we use a sunspot $\zeta \in \{0, 1\}$. If $\zeta = 0$, we will say there is a "good sunspot," in which case the equilibrium with repayment is selected. If $\zeta = 1$, we will say there is a "bad sunspot," in which case the equilibrium with default is selected. The probability of selecting the bad sunspot is denoted by π.

Using that the repayment value functions are strictly decreasing with respect to debt and that the value of default is independent of debt, we can show that for every y^T, there exists a pair of debt thresholds $\{b^-, b^+\}$ that separates these three regions, as illustrated in Figure I. In the next section, we study how the exchange rate regime affects these thresholds.

III. MONETARY POLICY AND ROLLOVER CRISES

In this section, we establish that the crisis zone is larger under a fixed exchange rate. We first present a graphical illustration of the mechanism and key insights and then provide a formal theoretical analysis.

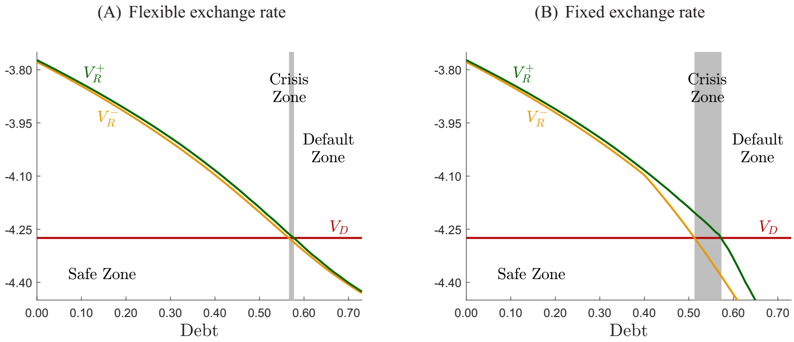

FIGURE II

Value Functions and Crisis Zone

We plot the value functions for mean tradable output. The value of $\bar{w}$ is set to the highest rigidity such that the default zone remain unchanged. We use the parameter values corresponding to the calibrated flexible exchange rate economy to be described in Section IV.A. The value functions under the fixed exchange rate correspond to an economy in which wages are rigid today and flexible tomorrow. Debt is expressed as a fraction of the unconditional mean of total output.

III.A. Graphical Illustration

We consider a version of the model where wages are rigid in period t, and become fully flexible for $t+1, t+2, \ldots$. The goal is to analyze the vulnerability to a debt crisis at t depending on the ability to depreciate the exchange rate. Because wages become flexible in the future, the fundamental price schedule faced by the government is the same for a flexible or a fixed exchange rate regime, but as we will see, the exposure to rollover crises will be different in the initial period.[16]

1. *The Crisis Zone.* To examine how we arrive at the crisis zone, Figure II presents the value functions $\{V_D, V_R^-, V_R^+\}$ for the government at time t as a function of the initial debt—the actual equilibrium value function V is given by the upper envelope of V_R^+ and V_D in the case of the good sunspot and by the upper envelope of V_R^- and V_D in the case of the bad sunspot. The left panel is for flexible exchange rates, and the right panel is for fixed exchange

16. Online Appendix D.1 provides formal details of this exercise. Online Appendix D.3 considers a similar exercise in which the wage rigidity is permanent but there is a deterministic path for the tradable endowment that leads to a binding rigidity only in the initial period.

rates. We consider a mean value for y^T and the value of $\bar{w}$ is set to the highest rigidity such that the default zone remains unchanged by the exchange rate regime. The parameter values correspond to the calibrated economy, to be described in Section IV.

The value of default V_D is a constant because it does not depend on the amount of debt the government owes. The values of repayment V_R^+ and V_R^- are decreasing in debt because the resource constraint becomes tighter. At the intersection of V_R^+ and V_D, the government is indifferent between repaying when it has access to credit markets and defaulting. For debt positions higher than this level, the government defaults regardless of the lenders' beliefs. This is the default zone. At the intersection of V_R^- and V_D, the government is indifferent between repaying when unable to roll over the debt and defaulting. For debt positions lower than this level, the value of repayment is higher than the value of default, and the government repays its debt regardless of lenders' beliefs. This is the safe zone. Finally, in between, the government repays if international lenders are willing to roll over the debt and defaults otherwise. This is the crisis region, which appears shaded in the two panels.

As Figure II, Panel A shows, the size of the crisis zone under a flexible exchange rate is small: the government is vulnerable to a rollover crisis only when debt is between 57% and 58%. On the other hand, Figure II, Panel B shows that the crisis zone becomes much larger under a fixed exchange rate regime, and now ranges between 52% to 58%. Debt positions that were safe under a flexible exchange rate now leave the government vulnerable to a rollover crisis.

2. *Inspecting the Mechanism.* To delve into the mechanism that gives rise to a larger crisis zone under a fixed exchange rate regime, we analyze the behavior of unemployment and how it varies with investors' beliefs and government policy. Figure III presents two panels: the left one presents the unemployment levels under a fixed exchange rate, and the right one presents the value functions from repaying $\{V_R^-, V_R^+\}$ under both a flexible and a fixed exchange rate regime.

In Figure III, Panel A, there are three lines: u_D denotes unemployment if the government chooses to default, u_R^+ is unemployment if the government chooses to repay when investors are willing to roll over, and u_R^- is unemployment if the government chooses

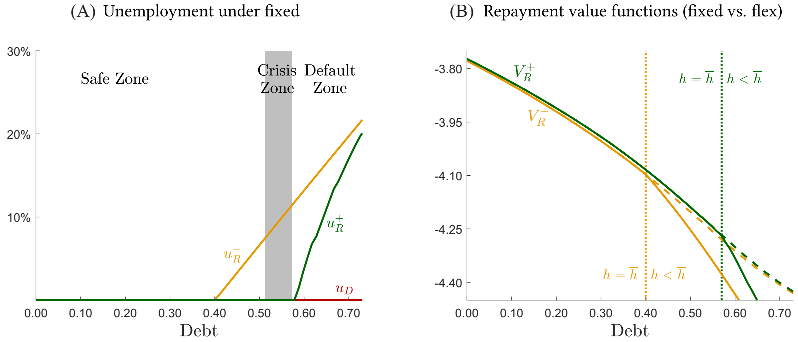

FIGURE III

Unemployment and Repayment Value Functions

In Panel B, dashed lines correspond to the flexible exchange rate regime and solid lines correspond to the fixed exchange rate regime. We use the same parameter values as in Figure II.

to repay when investors refuse to roll over. When the government repays, unemployment is weakly increasing in the initial amount of debt in the two cases. This is because a higher initial debt level reduces available resources and aggregate demand, raising unemployment once the downward rigidity on wages becomes binding.

Comparing u_R^+ and u_R^- reveals that when investors are unwilling to roll over the debt, unemployment starts rising for strictly lower levels of debt and reaches higher values compared with the situation in which investors are willing to roll over the debt. The reason is that when the government is forced to raise tax revenues to repay the maturing debt, this generates a severe contraction in aggregate demand, leading to a surge in unemployment.

It is interesting to note that the on-the-equilibrium-path value for unemployment turns out to be zero in Figure III, Panel A. Even though u_R^- can take large values, these high levels of unemployment are not realized in equilibrium. For debt positions such that the government is better off repaying even if investors were to run, we have that in equilibrium, investors do not refuse to roll over, and hence the level of unemployment is u_R^+. For debt positions such that the government defaults in the event of a run, we have that investors do run and the government defaults on the equilibrium path, resulting in a level of unemployment of u_D. In both cases, unemployment is zero. The takeaway is that what leads the government to default (and investors to run) in a rollover crisis is not the realization of unemployment but the desire to avoid the high levels of unemployment that would emerge if the government were to repay while being unable to borrow.

These differences in unemployment translate into substantial effects on the value functions, as can be seen in Figure III, Panel B. Notice that these are the same value functions from both panels of Figure II, which we now present in the same plot to highlight the key differences. Dashed (solid) lines denote the value functions under a flexible (fixed) exchange rate regime. The two vertical lines indicate the debt thresholds when unemployment emerges under a fixed exchange rate, depending on whether investors are willing to lend. To the left of these thresholds, the value functions under fixed and flex coincide. To the right of these thresholds, the value functions of repayment under the fixed exchange rate drop relative to the flexible case. Crucially, V_R^- is reduced by more than V_R^+, resulting in a much wider gap between the two value functions compared with the flexible exchange rate. This wider gap emerges from the large levels of unemployment that the economy suffers when investors refuse to lend and the government has to conduct a fiscal contraction to repay the debt. Moreover, the widening of the gap between V_R^+ and V_R^- occurs precisely at debt levels at which lenders' beliefs matter for the repayment decision. The outcome is a wide crisis zone.

3. *Crisis Regions for Range of $\bar{w}$.* In Figure II, we compared the crisis region for fixed and flexible exchange rates for a value of $\bar{w}$ sufficiently low such that the default region is not affected. In Figure IV we show how the regions change for a whole range of $\bar{w}$, keeping y^T at the average value.[17] Recall that a reduction in $\bar{w}$ is equivalent to a nominal exchange rate depreciation, a higher price of foreign tradables, or a lower $\bar{W}$. In particular, one can think about a lower $\bar{w}$ as a scenario in which the government is able to allow for certain depreciation of the currency. The figure shows that, given a normalization, as soon as $\bar{w}$ rises above 1, the wage rigidity becomes binding and the safe region contracts. For low values of wage rigidity, b^+ remains unaffected, and hence the crisis region expands at the expense of the safe region without changes in the default region. Once $\bar{w}$ reaches around 1.2, the value function V_R^+, starts to fall, leading to an expansion of the default region at the

17. Online Appendix D.2 presents the value functions for other values of $\bar{w}$ and the crisis region for other values of y^T.

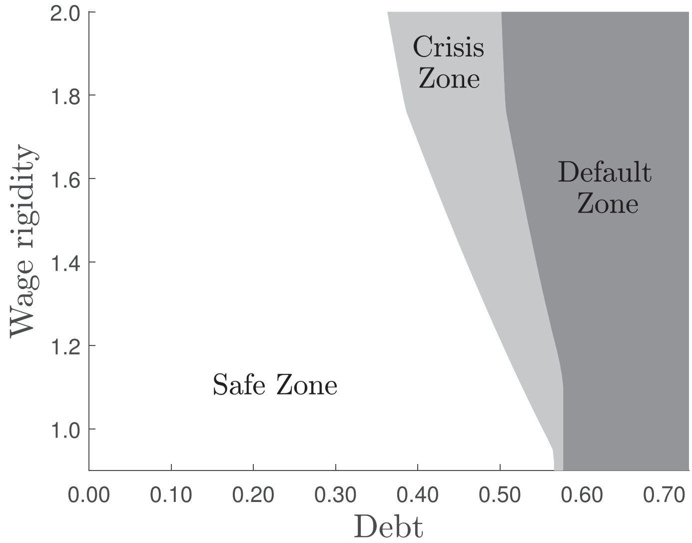

FIGURE IV

Crisis Region for Different $\bar{w}$

The figure shows the safe, crisis, and default zone change under different values for $\bar{w}$. The figure normalizes by the highest value of $\bar{w}$ that is consistent with a nonbinding wage rigidity. A rigidity lower than or equal to 1 therefore corresponds to the flexible exchange rate regime.

expense of the crisis region. However, we can see that the crisis region continues to expand significantly because the safe region contracts by an amount greater than the default region expansion.

III.B. Formalizing the Results

We formally analyze how the exposure to debt crises varies with the exchange rate regime. We consider the baseline case when wage rigidities may bind for all t but focus the comparison on the case in which tradable output is constant and $\beta(1 + r) = 1$. As shown by Cole and Kehoe (1996), under a constant output and $\beta(1 + r) = 1$, the government seeks to exit the crisis zone by deleveraging until the economy reaches the safe zone. Here we study how the crisis zone differs under the two regimes.

We use b^-_{flex} and b^+_{flex} to denote the debt thresholds that separate the safe zone, crisis zone, and default zone under a flexible exchange rate regime. Recall that as illustrated in Figure I, the crisis zone corresponds to debt levels such that

$b^-_{flex} < b \leqslant b^+_{flex}$. By the same token, let $b^-_{fix}(\bar{w})$ and $b^+_{fix}(\bar{w})$ be the thresholds under a fixed exchange rate for a given $\bar{w}$. We can establish the following results.

PROPOSITION 1. (Vulnerability and exchange rate regime) Assume that $\beta(1 + r) = 1$, $y^T_t = y^T$ for all $t \geqslant 0$, and let $\{\bar{w}^D, \bar{w}^-, \bar{w}^+\}$ be wage rigidity thresholds defined in Appendix B. We have the following:

i. The safe zone is smaller under fixed exchange rates: the debt thresholds satisfy $b^-_{fix}(\bar{w}) \leqslant b^-_{flex}$ for any rigidity $\bar{w} \leqslant \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \bar{w} \leqslant \bar{w}^D$. Furthermore, if preferences are separable, we have $b^-_{fix}(\bar{w}) < b^-_{flex}$ for any $\bar{w} > \bar{w}^-$.
ii. A devaluation expands the safe zone: assume that preferences are separable. We have that for every $e' > e$ then $b^-_{fix}\left(\frac{\bar{W}}{e'}\right) \geqslant b^-_{fix}\left(\frac{\bar{W}}{e}\right)$ for any nominal rigidity and exchange rate such that $\frac{\bar{W}}{e} \leqslant \bar{w}^D$. Moreover, the relationship is strict if $\bar{w}^- < \frac{\bar{W}}{e} \leqslant \bar{w}^D$.
iii. Crisis and default zones: assume $\pi = 0$. Then we have $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{w})$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leqslant \bar{w}^+$. Moreover, if preferences are separable, $b^+_{fix}(\bar{w}) < b^+_{flex}$ for any $\bar{w} > \bar{w}^+$.

Item i of Proposition 1 establishes the key result: the safe zone is smaller when the government lacks monetary independence. As illustrated in Figure II, a government that fixes the exchange rate is vulnerable to a rollover crisis with lower levels of debt. Item ii shows that a higher nominal exchange rate depreciation helps expand the safe zone and reduce the vulnerability.[18]

Finally, item iii establishes conditions under which the crisis region under a flexible exchange rate is strictly contained in the crisis region under a fixed exchange rate. The previous statement already showed that the crisis zone expands to the left under a fixed exchange rate. However, notice that unlike the example in Section III.A, the government here faces wage rigidities in the future, which tend to increase default incentives in the future and lead to an increase in the default region today (while contracting the crisis region on the right). With $\pi = 0$, constant output,

18. The same result follows if we consider a lower $\bar{W}$ or a rise in the foreign price of tradables instead of a nominal depreciation.

and sufficiently low rigidity, this effect is muted and the default zone remains unchanged. In addition, the second part of the statement demonstrates that for sufficiently high $\bar{w}$, the default zone strictly expands under a fixed exchange rate relative to the flexible exchange rate.

III.C. Extensions and Generalizations

In this section, we discuss briefly how the main theoretical results can be extended and generalized. (Details can be found in the Online Appendix.)

1. *Tradable Production.* The model features an endowment of tradables, whereas nontradable goods are produced with labor. In Online Appendix E.1, we allow for a symmetric production structure in which both goods are produced with labor and show that our results are preserved. To see why, consider a panic by foreign investors in this extended version of the model. As the government raises tax revenues to repay the debt, the demand for nontradable goods falls, leading to a reallocation of labor from the nontradable to the tradable sector (which faces a perfectly elastic demand from abroad). To the extent that labor has decreasing marginal returns, however, the reallocation is limited. In fact, once the wage rigidity becomes binding, the demand for tradable employment is entirely determined by the condition $F_T'(h^T) = \bar{w}$. Hence, further declines in aggregate demand do not lead to a reallocation of labor toward the tradable sector, and overall employment remains depressed.

2. *Sticky Prices.* The same results can be obtained in a model in which the source of nominal rigidity is prices instead of wages. When prices are sticky, rationing takes place in the goods market rather than in the labor market. Either way, a panic generates a contraction in aggregate demand, which makes repayment more costly under a fixed exchange rate. Online Appendix E.2 shows how the theoretical results extend to the case of price stickiness.

3. *Costs from Nominal Depreciations.* In our model, a higher exchange rate unambiguously increases the utility flow at any particular state, given that it reduces unemployment and does not involve any cost. In practice, a depreciation of the exchange rate may also come with some costs, which could result, for example,

from adverse redistributive effects or monetary distortions. To capture these costs, we consider (in Online Appendix E.3) an additively separable utility cost from exchange rate fluctuations and show that our results continue to hold. In this extension of the model, the government faces a trade-off between the benefits from higher employment and the costs of exchange rate fluctuations. Regardless of how large the costs are, however, an economy under a flexible exchange rate regime displays a smaller crisis zone. Intuitively, even though depreciating is costly, it is still the case that the exchange rate flexibility is especially valuable during a rollover crisis, which makes investors less prone to run in the first place.

4. *Benefits from Currency Unions.* We have not explicitly modeled the government's choice to fix the exchange rate or join a monetary union. We argue, however, that while considering these benefits may alter the welfare ranking between a fixed and a flexible exchange rate, our central result that a fixed exchange rate is more vulnerable to a rollover crisis continues to hold. There are potentially several ways to model benefits from being part of a monetary union. (See Online Appendix E.4 for details of the analysis that follows.)

A first possibility is that being in a monetary union allows for mitigating inflationary bias, one of the key arguments for joining a monetary union (Barro and Gordon 1983; Alesina and Barro 2002). To allow for this possibility, we consider a variant of the model in which the costs from exchange rate fluctuations arise from expected depreciations. Lacking commitment to an exchange rate policy, the government always finds it optimal to depreciate the currency ex post to deliver full employment and generates excessive fluctuations ex ante. By entering a monetary union, an economy is able to avoid the costs of the resulting inflationary bias, and doing so can be desirable if these costs are sufficiently large. However, the result that the lack of exchange rate flexibility makes the economy more vulnerable to a rollover crisis remains.[19]

A second possibility to consider is that being in a monetary union raises the economy's tradable output because of enhanced

19. In fact, inflationary bias does not increase the gap between V_R^+ and V_R^-, which is the key determinant of the crisis zone. On the contrary, when the government cannot roll over the debt, the government starts the next period with lower debt, and therefore the expected depreciation is lower compared to the case in which the government can roll over.

trade linkages. This is indeed one of the traditional arguments for joining a monetary union (Mundell 1961). Moreover, in the context of the recent Spanish crisis, Almunia et al. (2018) argue that amid the decline in domestic demand, a high level of integration with Europe made it possible to prevent a further drop in tradable output. In the Online Appendix, we consider a version of the model in which an economy with a fixed exchange rate has a higher permanent level of tradable output. We argue that the higher vulnerability of a member of the monetary union extends to this case as the increase in tradable output raises the value functions from both repayment and default—specifically, under the assumption of linear utility in tradable consumption, Proposition 1 remains intact. The logic is that what matters for the difference in crises' exposure is that the panic generates an endogenous contraction in output under a fixed exchange rate.

A third possibility is that being integrated in a monetary union improves enforcement of external debt payments. Indeed, some observers have argued that defaulting while being in a monetary union might be more costly. Interestingly, an increase in the default cost has the direct implication of always reducing the fundamental default zone, but the crisis zone may expand. In fact, a simple inspection of Figure II underscores that on a parallel shift in the value of default, the crisis region may increase or decrease depending on the slopes of the two values of repayment at the intersection point with V^D. Key for the results is that the crisis zone depends mainly on the gap between V_R^+ and V_R^-, and this gap is larger when the government cannot use monetary policy to stabilize output when facing a liquidity problem.

These three extensions highlight that our result that a fixed exchange rate is more vulnerable to a rollover crisis does not hinge on the fact that we abstracted from modeling the reasons the government implements a fixed exchange rate regime.

5. *Nominal Debt.* In the baseline model, the only difference between a flexible exchange rate regime and a fixed exchange rate regime is that in the former the government can use monetary policy to stabilize macroeconomic fluctuations. We made this assumption partly to better highlight the new channel regarding the role of monetary policy in reducing the vulnerability to rollover crises. In principle, an economy that is outside a monetary union can also issue debt in domestic currency, which opens the possibility of inflating away the debt and introduces another difference

between the two regimes. In Online Appendix E.5, we describe a version of the model in which a nominal depreciation allows for simultaneously affecting the real value of the debt and the level of employment. In this economy, depreciating the currency allows for an increase in the amount of tradable consumption by effectively diluting the real value of the debt. Importantly, this allows for an increase in aggregate demand and, through the mechanism highlighted above, reduces unemployment and makes repayment less costly in the event of a run. We therefore argue that the main insight of the article remains when we allow for debt denominated in domestic currency.

6. *Inflation Targeting.* In our baseline model, the key constraint on monetary policy is a fixed exchange rate regime. An alternative constraint is a strict inflation targeting regime, in which the government keeps constant the price of the composite consumption good in domestic currency. Under an inflation targeting regime, the government has the ability to depreciate the currency in response to a rollover crisis, but the target for inflation may prevent the government from allowing a sufficiently large depreciation that achieves full stabilization. As a result, a strict inflation targeting regime still leaves the government more vulnerable to a rollover crisis (see Online Appendix E.6).

7. *Domestic Debt.* The results can also be applied to rollover crises with domestic borrowing in open or closed economies. In fact, a panic by domestic investors in government bonds can trigger a reduction in government spending or a redistribution away from households with a high marginal propensity to consume. If the government cannot offset the recessionary effects on economic activity by using monetary policy (e.g., because of a fixed exchange rate or a zero lower bound), it will become more costly for the government to repay. As in our model, these Keynesian features would make investors more prone to run.

8. *Key Takeaway.* Beyond these specific extensions, our main result is quite general in the sense that it hinges on only two key robust elements: (i) a sudden panic by investors triggers capital outflows, if the government chooses to repay; and (ii) the costs of sudden capital outflows are more severe under monetary policy constraints because the government is unable to mitigate the contraction in aggregate demand. The combination of these

TABLE I
PARAMETER VALUES

Parameter	Value		Description
$\bar{h}$	1.000		Normalization
σ	5.000		Standard risk aversion
ω	0.298		Share of tradables
μ	1.000		Elasticity of substitution between T-NT = $\frac{1}{2}$
ρ	0.826		Tradable output persistence
σ_y	0.027		Standard deviation of tradable output shock
α	0.750		Labor share in nontradable sector
r	0.020		German six-year government bond yield
δ	0.141		Spanish bond maturity six years
ψ	0.240		Reentry to financial markets probability
π	0.050		Sunspot probability
Calibration	Flexible	Fixed	Target
β	0.935	0.853	Average external debt-GDP ratio 29.05%
κ_0	0.140	0.131	Average spread 2.01%
κ_1	1.116	0.395	Standard deviation of spread 1.42%
$\bar{w}$	–	1.442	Δ unemployment rate 2.00%

elements implies that the government is more tempted to default during a panic under a fixed exchange rate regime, and hence investors are more prone to run.

IV. Quantitative Analysis

This section presents the quantitative analysis of the stochastic version of the model. (Online Appendix H presents all the details of the computational approach.) We conduct three policy experiments with the model. First, we perform simulations to assess how often an economy is exposed to rollover crises and examine how this exposure depends on the exchange rate regime. Second, we assess the welfare costs from monetary independence and the potential gains from a lender of last resort. Third, we perform a counterfactual experiment applied to the recent sovereign debt crisis in Spain to shed light on whether the crisis was triggered by fundamentals or self-fulfilling beliefs.

IV.A. Calibration

We calibrate the model at an annual frequency, using Spain as a case study. Table I shows all the baseline calibration values for the parameters of the model.

We parameterize the default utility cost as $\kappa(y^T) = \max\{0, \kappa_0 + \kappa_1 \ln(y^T)\}$. As shown in Arellano (2008) and Chatterjee and Eyigungor (2012), a nonlinear specification of the cost of default is important to allow the model to match the levels of debt and spreads in the data.

Tradable output follows a log normal AR(1) process $\ln(y^T_{t+1}) = \rho \ln(y^T_t) + \sigma_y \varepsilon_t$, where $|\rho| < 1$ and $\varepsilon_t \sim N(0, 1)$. To estimate this process, we use the European Classification of Economic Activities (NACE-2) from the Eurostat database to compute the value added of the tradable sectors in Spain between 1995 and 2018. We define an activity as tradable when the Tradability Index defined as $\frac{\text{Exports+Imports}}{\text{Value Added}}$ is on average above 10% from 2010 to 2015, following De Gregorio, Giovannini, and Wolf (1994).[20] Using this classification, we obtain that the share of tradables' value added relative to GDP is on average 32%. We estimate the log quadratically detrended tradable output by OLS and obtain $\rho = 0.826$ and $\sigma_y = 2.7\%$.

1. *Parameters Set Externally.* A first subset of parameters $\{\sigma, \mu, \omega, \bar{h}, \alpha, r, \delta, \psi, \pi\}$ is specified directly. The parameters governing the preferences and the technology of the model take standard values found in the literature. The coefficient of risk aversion is set to $\sigma = 5$, and the elasticity of substitution between tradable and nontradable goods is set to $\frac{1}{1+\mu} = 0.5$, both standard values in the literature. In addition, the share of tradable goods in the consumption aggregator is set to $\omega = 0.298$, so it matches the share of tradable output, which we estimated at 32.[21] We set the technology parameter $\alpha = 0.75$, an estimate from Uribe (1997). Last, we normalize the inelastic labor supply of households to $\bar{h} = 1$.

The parameters from financial markets are set as follows. We set the international risk-free interest rate to $r = 2\%$, which is the average annual gross yield on German six-year government

20. Under this criteria, we label as tradable activities "Agriculture, Forestry, and Fishing" (A), "Mining and Quarrying" (B), "Manufacturing" (C), "Electricity Gas, Steam, and Air Conditioning Supplies" (D), and "Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles" (G).

21. In a nonstochastic version of the model with a value of debt $\bar{b}$ interest rate $\bar{r}$, and average employment $\bar{h}$, the value of ω can be pinned down from $\frac{y^T}{\left[y^T + \frac{1-\omega}{\omega}\left(\frac{y^T + \frac{\bar{r}}{1+\bar{r}}\bar{b}}{F(\bar{h})}\right)^{1+\mu} F(\bar{h})\right]} = 31.72\%$.

bonds from 2000 to 2015. We use a maturity parameter of $\delta = 0.141$ to reproduce an average bond duration of six years, in line with Spanish data.[22] We set the reentry to financial markets probability after default to $\psi = 0.24$ to capture an average autarky spell of four years, in line with Gelos, Sahay, and Sandleris (2011). Finally, we need to set the sunspot probability, which is a more difficult parameter to calibrate. Our baseline value is $\pi = 5\%$, but we examine a wide range as well.

2. *Parameters Set by Simulation.* A second subset of parameters $\{\beta, \kappa_0, \kappa_1, \bar{w}\}$ is set so that the moments in the model match the counterparts in the data. Because we have two different exchange rate regimes, we have two sets of parameters. Although $\bar{w}$ is irrelevant under a flexible exchange rate, we need to calibrate this parameter for the fixed exchange rate regime. In particular, we calibrate $\bar{w}$ in the fixed exchange rate regime to be consistent with the increase in unemployment during episodes of high sovereign spreads. As a reference, we use the increase in unemployment relative to the HP-filtered trend in 2011, the year before the EU and ECB's intervention, which was close to 2%.[23]

For both regimes, we calibrate the parameters β, κ_0, and κ_1 to match three moments from the data, and we follow Hatchondo, Martinez, and Sosa-Padilla (2016) in considering the moments in the years after 2008 to concentrate on the period around the crisis. The three moments targeted are the average debt-GDP ratio, and the average and standard deviation of spreads. For the average debt-GDP ratio, we target an average external debt of 29%. For the average and the standard deviation of spreads, we target 2.0 and 1.4, respectively.[24] The resulting values for these parameters appear in the bottom part of Table I.

22. The Macaulay duration of a bond with price q and our coupon structure is given by $D = \sum_{t=1}^{\infty} t \frac{\delta}{q} (\frac{1-\delta}{1+i_b})^t = \frac{1+i_b}{\delta+i_b}$, where the constant per period yield i_b is determined by $q = \sum_{t=1}^{\infty} \delta (\frac{1-\delta}{1+i_b})^t$.

23. As we mentioned in note 11, governments have available fiscal instruments to stimulate employment, such as payroll subsidies. In terms of our model, this would imply that the wage rigidity would be governed by $\bar{w}$ net of these subsidies. Our approach to calibrating $\bar{w}$ therefore implicitly incorporates these effects.

24. The debt level in the model is computed as the present value of future payment obligations discounted at the risk-free rate r. Given the maturity structure, the debt level is given by $\frac{\delta}{1-\frac{1-\delta}{1+r}} b_t$.

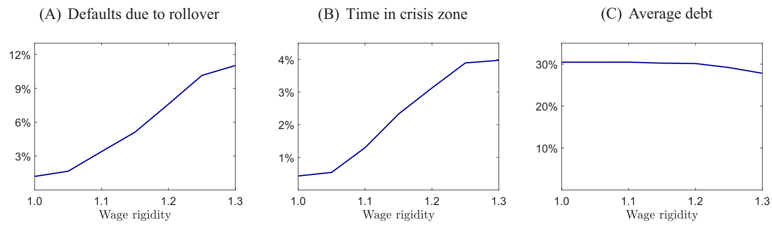

FIGURE V

Simulation Results under Different Rigidities

We use the benchmark calibration of the flexible exchange rate regime. The wage rigidity grid is normalized with the highest wage rigidity that shares the same policy functions and debt pricing solution for the flexible exchange rate regime. We collect 5,000 observations of a series of 50 periods before a default experience. The moments are computed for the last 35 periods before the default experience with simple averages.

IV.B. Simulation Results: Exposure to Rollover Crises

We conduct simulations to investigate how the exchange rate regime determines which type of default—fundamental or rollover crisis—is more likely.

1. *Degree of Wage Rigidity.* We start from the flexible exchange rate economy. In this economy, only 1 out of 100 default episodes are due to a rollover crisis. Moreover, on average, the economy is in the crisis zone and therefore vulnerable to a rollover crisis only 0.53% of the time. To examine how the degree of wage rigidity and the exchange rate regime matter for the exposure to a rollover crisis, we vary $\bar{w}$ while keeping all parameters from the calibrated flexible exchange rate economy. In Figure V, Panel A, we can see that the tighter wage rigidity is, the larger the fraction of defaults that are explained by nonfundamentals. In fact, the fraction of defaults due to rollover crisis can reach about 11%, compared with 1% for the case under flexible exchange rates. In line with this result, Figure V, Panel B shows how time spent in the crisis zone increases with the degree of rigidity.

Figure V, Panel C also shows that the average debt-to-GDP ratio falls with the degree of wage rigidity. Two reasons explain this. First, a higher $\bar{w}$ implies that the government faces borrowing terms that are more adverse, given that incentives to default in the future are higher. Second, a higher $\bar{w}$ implies that the government faces a larger crisis zone, and the increased vulnerability prompts the government to reduce the debt level in the long run.[25]

2. *Fixed versus Flexible (Recalibrated).* In the previous experiment, we kept constant all parameters except for $\bar{w}$. Because the long-run moments that we target in the calibration change with $\bar{w}$, it is useful to complement the results by recalibrating the parameters for the discount factor and the default cost to hit the same baseline targets. In Table II, we consider the simulation statistics for the two economies calibrated to match the same targets for different values of π. Let us start by considering the intermediate columns, which correspond to the baseline values for $\pi = 5\%$. The first three rows correspond to the targeted moments calibrated for both economies and therefore have about the same values for the first two columns.[26] The key result appears in the last two rows. The calibrated fixed exchange rate economy experiences close to 10 defaults due to rollover crises for every 100 default episodes, whereas the flexible exchange rate experiences only 1 default due to rollover crises for every 100 default episodes. Similarly, the share of time spent in the crisis zone increases by almost an order of magnitude in the calibrated fixed exchange rate economy.

3. *Sunspot Probability.* The fraction of defaults that are the outcome of a rollover crisis depends on two factors. One factor is the probability of a bad sunspot (i.e., the probability of selecting the bad equilibrium whenever the economy is in the crisis zone). The second factor is the probability of ending up in the crisis zone in the first place, which is an endogenous outcome that depends critically on borrowing decisions and the monetary policy regime. Next, we analyze the sensitivity of our results by considering different values of π while keeping the rest of the parameter

25. Notice that despite this reduction in the average debt level, the fact that the crisis zone expands significantly implies that the government still ends up being more heavily exposed to a rollover crisis. This result, however, does not apply in the entire state space. Once debt is substantially reduced, rollover crises become less likely.

26. The modest differences in the targets for mean debt, mean spreads, and volatility of spreads are because of the nonlinear nature of the model which makes an exact calibration difficult (see Aguiar et al. 2016).

TABLE II
SIMULATION STATISTICS

		$\pi = 1\%$		$\pi = 5\%$		$\pi = 10\%$	
Sunspot probability (%)	Spain data	Flexible	Fixed	Flexible	Fixed	Flexible	Fixed
Average spread	2.01	2.01	1.94	1.94	1.99	1.96	1.90
Average debt-income	29.05	30.64	31.88	30.16	29.95	28.55	31.84
Spread volatility	1.42	1.62	1.62	1.58	1.70	1.61	1.72
Unemployment increase	2.00	0.00	1.78	0.00	1.69	0.00	1.87
Time in crisis zone		0.59	6.03	0.51	4.28	0.49	3.59
Defaults due to rollover crisis		0.53	3.87	1.28	10.42	2.22	17.88

Note: All parameter values correspond to the benchmark calibration for flexible and fixed exchange rate regimes with the exception of $\{\beta, \kappa_0, \kappa_1, \bar{w}\}$. The benchmark calibration uses $\pi = 5\%$. All values are in percent.

values at their respective baseline values with the exception of $\{\beta, \kappa_{0,} \kappa_1, \bar{w}\}$, which are recalibrated to match the same baseline targets in the two economies.

Table II presents the results for three values of π. The table shows how a higher likelihood of a bad sunspot increases the fraction of defaults due to a rollover crisis for the two economies, and particularly for the economy under a fixed exchange rate regime. When the probability of a bad sunspot is 10%, about one-fifth of all defaults in the fixed exchange rate economy are for nonfundamental reasons. Moreover, one can see that the fraction of time spent in the crisis region decreases as the government finds it optimal to reduce its exposure, but this duration is not enough to offset the effects of a higher likelihood of a bad sunspot.

IV.C. Welfare Consequences

We now tackle two central normative considerations: (i) what is the welfare cost of the lack of monetary independence? and (ii) what are the welfare gains from a lender of last resort (LOLR)?

Our first result is that the possibility of a rollover crisis substantially increases the welfare costs of giving up monetary independence. We examine, for all initial states, how much households are willing to give up of the composite consumption good to move from the fixed exchange rate economy to a flexible exchange rate for one period (see Online Appendix F for the technical details).

Figure VI shows these calculations, denoted by $\theta_0^{flex}(b, \mathbf{s})$, for a range of debt levels, for the good sunspot $\zeta = 0$ and the bad sunspot $\zeta = 1$, and for a given endowment shock. For reference, the three zones are displayed for the economy under a fixed exchange rate. Starting from the left, we see that if debt is very low, there is no unemployment and no cost from having a fixed exchange rate. As debt approaches 0.25, unemployment emerges in equilibrium, and there is a positive welfare cost. Under the good sunspot, the welfare cost increases continuously until debt reaches about 0.36, at which point the government chooses to default under a fixed exchange rate. This helps mitigate the effects from the wage rigidities. The welfare costs from a fixed exchange rate decrease in the level of debt, because the value function is independent of debt under a fixed exchange rate—since the government defaults—but decreasing under a flexible exchange rate. Importantly, although the economy under a fixed exchange rate features no

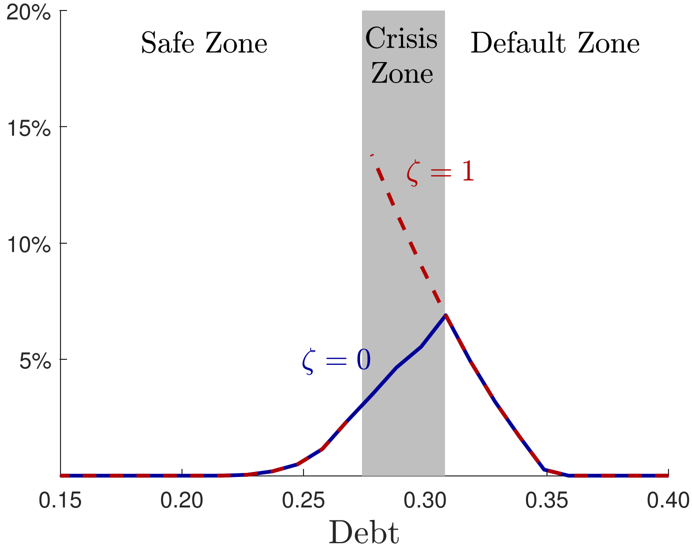

FIGURE VI

Welfare Gains from Exiting a Currency Union

The figure presents the value of exiting a currency union for one period when the current output level is 7% below the mean, which corresponds to the drop in y^T in Spain in 2012 (see Figure VII). We use the parameters calibrated for the benchmark fixed exchange rate regime. The solid line represents the economy when there is no sunspot. The dashed line represents the economy when the sunspot is activated. The y-axis represents the welfare gain of being in a flexible exchange rate regime in terms of the composite consumption good.

unemployment, there is still a welfare cost from a fixed exchange rate because it is precisely the lack of flexibility that triggers the government default, and the economy suffers from the default costs. For debt levels higher than 0.41, the government under a flexible exchange rate also chooses to default, and there are no costs from rigidity. Under the bad sunspot, the welfare costs increase discretely once the debt enters the crisis zone. This occurs because the lack of exchange rate flexibility prompts the government to default if investors refuse to roll over the government bonds.

The next welfare consideration that we tackle is the welfare gains from having a LOLR. As is well understood, a third party with deep pockets can eliminate the coordination problem behind a rollover crisis. The basic argument is that by purchasing a sufficiently large amount of government bonds in either the primary or the secondary market, this can induce the government to repay and therefore make investors willing to lend to the government.[27]

We ask how much households would be willing to pay in terms of consumption to have access to a LOLR (or equivalently, to permanently eliminate the possibility of a rollover crisis). To compute these welfare costs, we take the fixed and flexible exchange rate economies with their respective calibrations and solve for the Markov equilibrium after setting the sunspot probability to zero. For each exchange rate regime, we compute the welfare gains in terms of the composite consumption in every state. Under a fixed exchange rate regime, the gains from having a LOLR can reach about 0.43% of permanent consumption and average 0.14% over the long-run simulations. Having access to a LOLR allows for an improvement in the borrowing terms and a reduction in default costs. For the flexible exchange rate, however, the unconditional welfare gains from having a LOLR are negligible, in line with the minimal exposure to rollover crises.

It is worth highlighting that a successful implementation of a LOLR hinges on the ability to correctly identify whether a default is being driven by fundamentals or by self-fulfilling beliefs. Moral hazard concerns would naturally emerge when the government and investors expect interventions in defaults driven by fundamentals. Therefore, in a scenario where the LOLR does not observe the source of the default, a trade-off is likely to emerge between the benefits from offsetting the coordination problem and the moral hazard effects.[28] Our analysis shows that while economies that lack monetary independence are likely to strongly benefit from a LOLR, this is less valuable for a flexible exchange rate regime, since defaults are driven almost exclusively by fundamental reasons.

Overall, this welfare analysis provides two important policy lessons. First, the lack of monetary independence can become very costly in the presence of rollover crises. Second, a LOLR can help ease the costs for an economy of giving up monetary independence.

27. See Roch and Uhlig (2018) and Bocola and Dovis (2019) for an analysis of a LOLR in the context of the Outright Monetary Transactions (OMT) program by the ECB.

28. See Bianchi (2016) for a quantitative analysis of the trade-off between the moral hazard effects from bailouts and the stabilization benefits in the context of firms' borrowing.

IV.D. The Path to Spain's Rollover Crisis

In this section, we use the model to shed light on the path to the Spanish debt crisis. We start the simulations in 1999, when Spain gave up the peseta and adopted the euro. Given Spain's external debt-GDP ratio in 1999, we feed the sequence of shocks to tradable output and simulate the model under a fixed exchange rate regime.

From 1999 until 2011, we find that the economy remains in the safe zone (and hence the sunspot realization is irrelevant). As it turns out, the model predicts that the economy is in the crisis zone in 2012–2013, and a negative sunspot would trigger a rollover crisis and a default. Even though Spain did not ultimately default on the debt, it received a €100 billion assistance package from the European Union, channeled through the European Financial Stability Fund and the European Stability Mechanism. Even more important for our analysis was the announcement of the ECB's OMT bond purchasing program following the "whatever it takes" speech, which dissipated concerns over the emergence of a rollover crisis. Indeed, after the speech, there was a substantial reduction in sovereign spreads.[29] Consistent with this, our model predicts that Spain would not default in the presence of a LOLR.[30]

Figure VII summarizes the results of the exercise. Panel A displays the tradable output we feed into the model. Panels B and C show the dynamics of debt and spreads in these simulations. In early 2000, given the low initial debt and the relatively good income shocks, the government increases its debt thanks to the favorable borrowing terms.[31] These dynamics are fairly similar to

29. This was the case for Spain and other Eurozone countries that had previously experienced a substantial rise in spreads, like Greece, Portugal, and Italy.

30. We note here that by starting in 1999, we abstract from many other factors at play in the transition to the euro. One relevant observation is the reduction in sovereign yields as Spain approached the date for joining the euro. Although this reduction may seem to be at odds with the model's predictions, the inference of default risk is complicated by the change in the currency denomination of the Spanish bonds. Codogno, Favero, and Missale (2003, figure 4) construct synthetic Deutsche Mark bonds by swapping the flows of the Spanish bond into German Deutsche Marks, using outright forward contracts, and find that default risk did not actually fall.

31. The prediction that borrowing tends to be increasing in income shocks is relatively standard in the literature (although it is less strong under a fixed exchange rate because deleveraging is more costly). Notice that the figure plots beginning of period debt b_t. After the negative shock to output in 2009, the deleveraging translates into lower debt levels in 2010. We also note that we conducted the

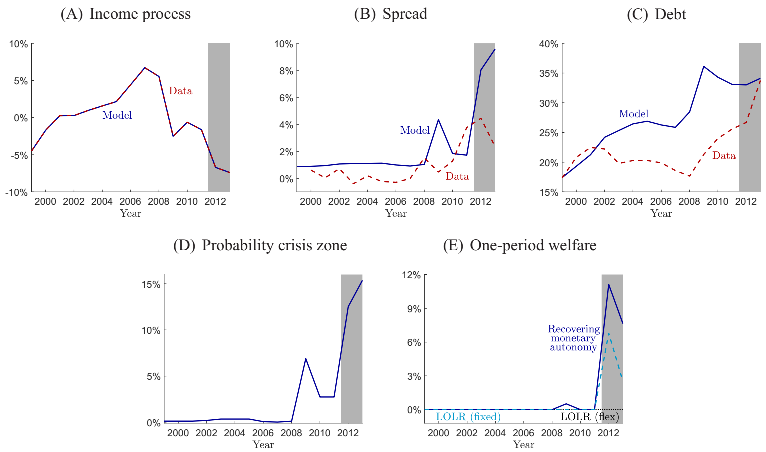

FIGURE VII

Path to Spain's Rollover Crisis

Welfare gains in Panel E correspond to policies that are in place for one period, reverting to the baseline Markov equilibrium. Crisis probability denotes the probability that the economy would be in the crisis zone the following period. The tradable endowment shock was obtained by applying a log-quadratic filter to Spanish tradable output from 1999 to 2018. Debt levels in the data correspond to Spain's external debt-GDP ratio. The shaded interval denotes that the economy is in the crisis zone.

those in the data, except that the model overpredicts the initial increase. One can also see that the model can replicate the low and stable spreads before 2008 in the data. Finally, the evolution of the probability of being in the crisis zone in Panel D reveals interesting dynamics.[32] After the debt accumulation that occurs initially and the negative income shocks that pile up after 2008, the economy's probability of a rollover crisis becomes more significant.

The final block of the exercise is a series of policy counterfactuals presented in Figure VII, Panel E. We first ask what the welfare gains are from recovering monetary independence, as examined in Section IV.C. As the solid blue line shows, the gains

same experiment with a target for average debt 5 percentage points lower and find that just like in the baseline calibration, the government enters the crisis zone in 2012–2013 (see Online Appendix Figure G.3).

32. This crisis probability is computed as the probability of receiving in the following period an income shock that pushes the economy into the crisis zone, given the end-of-period level of debt.

are close to zero until 2011 but reach around 10% in 2012 and 2013. We then ask how these welfare gains would be modified in the presence of a LOLR. We can see that while the gains from a LOLR are close to nil under a flexible exchange rate regime, they are significant for a fixed exchange rate, representing 60% of the gains from regaining monetary independence in 2012 and 33% in 2013. In other words, the attractiveness of exiting a monetary union can be substantially reduced by providing a LOLR.

According to this experiment, if Spain had exited the monetary union, it would not have been subject to a rollover crisis.[33] Two remarks about this counterfactual experiment are in order. First, we are keeping everything else constant when we analyze the implications of exiting the Eurozone. We are therefore abstracting from any possible structural changes that Spain could experience on exiting a monetary union. Nevertheless, to the extent that these structural changes would symmetrically affect V_R^+ and V_R^-, we expect that the large gap between these values arises because the inability to depreciate the currency would remain intact, and hence these structural changes should not significantly alter the crisis region. Second, we do not suggest that Spain would have been better off by exiting the monetary union. Being in a monetary union indeed has many benefits that we are not modeling. Our goal is to point out an additional cost of remaining in a monetary union, which arises from higher exposure to rollover crises.[34]

33. Although the welfare results of Figure VII correspond to a situation in which Spain regains monetary autonomy for one period, the same result would hold if there were a permanent exit from the Eurozone. In both cases, we continue to assume that debt remains denominated in a foreign currency, which is a natural assumption, since a currency redenomination would be akin to a default. While it is quite likely that Spain would start issuing debt in its domestic currency after exiting, this would apply only to new issuances of debt, not the existing stock, which is largely the most relevant in understanding the incentives to default and how they change if the government remains in or exits the monetary union.

34. Interestingly, the ECB's policy measures since the COVID crisis appear to recognize the importance of a more permanent scheme of liquidity assistance because of the lack of exchange rate flexibility of the members (see a keynote speech by the ECB's Chief Economist (Lane 2019)). Nonetheless, the measures remain controversial, as evidenced by the ruling of the Germany's Federal Constitutional Court in May 2020 questioning the proportionality of the ECB's policy measures. See also Lane (2021).

IV.E. Empirical Exploration

Besides its application to the Eurozone crisis, to Spain more specifically, our model suggests more generally that, everything else constant, a fixed exchange rate economy is more vulnerable to a rollover crisis.

An ideal test of our model would be to compare in the data the probability of rollover crises in countries with fixed exchange rates against that in countries with flexible exchange rates, with both countries borrowing in foreign currency and the choice of the exchange rate regime being exogenous. Conducting this test is difficult for at least two reasons. First, one needs to have a plausible categorization of when a default is due to a rollover crisis and have a sufficiently large amount of observations to achieve statistical significance. Second, the choice of the exchange rate regime is endogenous, and according to the theory, it should depend on the likelihood of experiencing a rollover crisis. While resolving these challenges is outside the scope of this article, we present a simple empirical exercise that confirms the theoretical predictions.

A prediction of the model is that sovereign spreads are less connected to fundamentals in a fixed exchange rate. To verify this prediction, we run a standard sovereign spread regression in the data. We consider countries that belong to the EMBI and sort countries according to the degree of exchange rate flexibility following the classification of Ilzetzki, Reinhart, and Rogoff (2019)—see the details in Online Appendix I.[35] We estimate for each group the following regression:

$$\text{spread}_{it} = \alpha_i + \beta X_{it} + \varepsilon_{it}, \tag{18}$$

where spread_{it} is the spread for a country i in period t, α_i is a country fixed effect, and X_{it} is a vector of controls. We find an R^2 of 0.45, 0.64, and 0.95, respectively, for countries with low, intermediate, and high degrees of exchange rate flexibility. When we estimate equation (18) in the model, we find that fundamentals also better predict spreads for the flexible exchange rate economy—the R^2 is 0.82 for the fixed exchange rate and 0.96 for the flexible exchange rate.

35. To isolate our mechanism, we restrict to countries with 60% or more share of foreign currency–denominated debt (an 80% threshold does not change the results that follow).

We therefore find that both in the model and in the data, fundamentals have more explanatory power under flexible exchange rates. These results are only suggestive, and we leave a more definite test for future research.[36]

V. Conclusion

This article shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis and points to a new cost from joining a monetary union. When a government lacks monetary autonomy, a run on government bonds can lead to a large recession in the presence of nominal rigidities. In turn, anticipating that the government will find it more costly to repay, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that an economy with a flexible exchange rate is relatively immune to a rollover crisis. On the other hand, a substantial fraction of defaults under a fixed exchange rate regime are driven by rollover crises.

Our analysis provides a new perspective on discussions about whether the lack of monetary autonomy in Southern European countries made them more vulnerable to a rollover crisis. The popular narrative is that the inability to resort to the printing press contributed to raising their vulnerability. We argue instead that monetary policy, by enhancing macroeconomic stabilization, has a role in preventing rollover crises that goes beyond the ability to inflate away the debt. Our analysis also suggests that a lender of last resort contributes to easing the costs from giving up monetary independence and could be highly beneficial for the stability of a monetary union.

Several avenues remain for future work. In terms of debt management, our model suggests that economies with more rigid labor markets or a less flexible monetary policy should seek longer debt maturities. Another interesting avenue is to provide a more explicit modeling of the benefits from joining a monetary union and quantify the relevant trade-offs involved. Finally, the key mechanism that we highlight is not specific to an open-economy setting. In particular, one could extend the

36. Also consistent with the predictions of the model is a result recently documented in Born et al. (2020) that after 2008 countries with a fixed exchange rate have experienced much larger variation in spreads.

analysis to consider rollover crises in closed economies that face other types of constraints on monetary policy.

Appendix A: Employment Characterization

Lemma A1. In any equilibrium, employment is given by the following function of tradable consumption:

(H-demand)
$$\mathcal{H}(c_t^T, \bar{w}) \equiv \min\left\{\left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}, \bar{h}\right\},$$

or equivalently,

$$(19)\qquad \mathcal{H}(c_t^T, \bar{w}) = \begin{cases} \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\bar{w}}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \text{if } c^T < \bar{c}_{\bar{w}}^T \\ \bar{h} & \text{if } c^T \geqslant \bar{c}_{\bar{w}}^T \end{cases},$$

where $\bar{c}_{\bar{w}}^T \equiv \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\bar{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \bar{h}^{\frac{1+\alpha\mu}{1+\mu}}$. In addition, we have that employment is increasing in c^T and decreasing in $\bar{w}$ (strictly so, if $c^T < \bar{c}_{\bar{w}}^T$).

Proof. Using equations (3), (5), and (10), we obtain the following value for employment:

$$(20)\qquad h_t = \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{w_t}\right)\right]^{\frac{1}{1+\alpha\mu}}\left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}}.$$

Using labor market conditions (6), (7), and $h \leqslant \bar{h}$, we arrive at (H-demand).

In addition, we have that $\mathcal{H}(\bar{c}_{\bar{w}}^T, \bar{w}) = \bar{h}$. The first argument of $\mathcal{H}(c_t^T, \bar{w})$ is increasing in c^T and decreasing in $\bar{w}$ given that $\mu > -1$, $\alpha \in (0, 1]$, and $\omega \in (0, 1)$. We can therefore write (H-demand) as (19). □

Appendix B: Proof of Proposition 1

Preliminaries

The existence of debt thresholds $\{b^-, b^+\}$ are guaranteed by the following lemma.

Lemma B1. (Existence of thresholds) For every level of tradable endowment y^T, there exists a debt threshold b^+ such that $V_D(y^T) \geqslant V_R^+(b, y^T)$ if and only if $b \geqslant b^+$. Likewise, there exists a debt threshold b^- such that $V_D(y^T) \geqslant V_R^-(b, y^T)$ if and only if $b \geqslant b^-$. In addition, we have that $b^+ \geqslant b^-$.

Proof. First, realize that $V_D(y^T) < V_R^-(0, y^T)$. This follows from the fact that a government in repayment can choose the same amount of consumption as a defaulting government and avoid the strictly positive costs of defaulting. By the Inada condition on the utility function, we can find a sufficiently high value for debt b^+ such that $V_D(y^T) \geqslant V_R^+(b^+, y^T)$. Using that V_R^+ is strictly decreasing in b, we get that $V_D(y^T) \geqslant V_R^+(b, y^T)$ if and only if $b \geqslant b^+$. The proof for b^- is identical and we omit it here. Finally, $b^+ \geqslant b^-$ is evident from the fact that problem (17) is the same as in equation (16), but with an additional constraint. □

Let us define

$$\mathcal{Y}^N(c^T, \bar{w}) \equiv F(\mathcal{H}(c^T, \bar{w})).$$

LEMMA B2. (Value functions in the safe zone) Assuming $\beta(1+r) = 1$ and a constant tradable endowment y^T, we have

$$V_{R,fix}^+(b; \bar{w}) = \frac{1}{1-\beta}\left[u\left(y^T - \frac{\delta r}{r+\delta}b, \mathcal{Y}^N\left(y^T - \frac{\delta r}{r+\delta}b, \bar{w}\right)\right)\right],$$

$$\forall\, b \leqslant b_{fix}^+(\bar{w}); \tag{21}$$

$$V_{R,fix}^-(b; \bar{w}) = \frac{1}{1-\beta}u(y^T - \delta b, \mathcal{Y}^N(y^T - \delta b, \bar{w})) + \frac{\beta}{1-\beta}u\left(y^T - \left(\frac{\delta r}{r+\delta}\right)(1-\delta)b, \mathcal{Y}^N\left(y^T - \left(\frac{r}{r+\delta}\right)\delta(1-\delta)b, \bar{w}\right)\right),$$

$$\forall\, b \leqslant b_{fix}^-(\bar{w}); \tag{22}$$

$$V_{D,fix}(\bar{w}) = \frac{1}{1-\beta}u(y^T, \mathcal{Y}^N(y^T, \bar{w})) - \frac{\kappa}{1-\beta(1-\psi)}. \tag{23}$$

Proof. Following the proof in Cole and Kehoe (2000), under $\beta(1+r) = 1$ and constant tradable endowment y^T, the economy becomes stationary once debt is in the safe zone. This implies that for any $b \leqslant b^-$, the value function is given by the present value of the utility of a constant consumption stream. Without default risk, arbitrage requires $q = \frac{\delta}{r+\delta}$. Using this bond price and $b' = b$, the tradable resource constraint (11) implies a constant consumption given by $c^T = y^T - \frac{\delta r}{r+\delta} b$. This gives equation (21). When the government cannot roll over the debt, we have that $b_{t+1} = (1 - \delta) b_t$. Following the same logic as above, the economy is in the safe zone in the next period and consumes $c^T = y^T - \frac{\delta r}{r+\delta}(1-\delta)b$ in all future periods. This gives equation (22). Finally, the value of default in equation (23) follows simply by using $c_t^T = y^T$ for all t and the fact that the expected discounted default cost is $\frac{\kappa}{1-\beta(1-\psi)}$. □

To ease notation, let us use that under a flexible exchange rate regime, the values of repayment and default can be expressed as

$$V_{R,flex}(b) \equiv V_{R,fix}(b;0).$$

$$V_{D,flex} \equiv V_{D,fix}(0).$$

In addition, let us define

$$\bar{w}^D \equiv \alpha \frac{1-\omega}{\omega} (y^T)^{1+\mu} \bar{h}^{-(1+\alpha\mu)}$$

$$\bar{w}^- \equiv \alpha \frac{1-\omega}{\omega} \left(y^T - \delta b^-_{flex}\right)^{1+\mu} \bar{h}^{-(1+\alpha\mu)}$$

$$\bar{w}^+ \equiv \alpha \frac{1-\omega}{\omega} \left(y^T - \left(\frac{r}{r+\delta}\right) \delta b^+_{flex}\right)^{1+\mu} \bar{h}^{-(1+\alpha\mu)}.$$

These values represent wage thresholds. The value of $\bar{w}^D$ represents the value of $\bar{w}$ at the point at which the rigidity becomes binding under default; the value of $\bar{w}^-$ represents the value of $\bar{w}$ at the point at which the wage rigidity becomes binding under a fixed exchange rate when the initial debt is b^-_{flex} and the government cannot borrow; and $\bar{w}^+$ represents the value of $\bar{w}$ at the point at which the wage rigidity becomes binding under a fixed exchange rate when the initial debt is b^+_{flex} and the government keeps the debt constant. It is easy to see that $\bar{w}^D > \bar{w}^+ > \bar{w}^-$. We summarize these results in the following lemma.

LEMMA B3. (Wage thresholds) Consider levels of tradable consumption given by y^T, $y^T - \delta b^-_{flex}$, and $y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}$. In this case, the economy experiences unemployment if and only if the wage rigidity is given respectively by $\bar{w} > \bar{w}^D$, $\bar{w} > \bar{w}^-$, and $\bar{w} > \bar{w}^+$.

Proof. Replacing $\bar{w} = \bar{w}^D$ and $c^T = y^T$ in (H-demand), we obtain $\mathcal{H}(y^T, \bar{w}^D) = \bar{h}$. Since $\mathcal{H}$ is strictly decreasing in $\bar{w}$,the first result follows. The second part is analogous. Once we replace $\bar{w} = \bar{w}^-$ and $c^T = y^T - \delta b^-_{flex}$ in (H-demand), we obtain $\mathcal{H}(y^T - \delta b^-_{flex}, \bar{w}^-) = \bar{h}$. □

We proceed now with the proof of the two items in the proposition.

Proof of Item i, First Part

Proof. We want to first prove that the debt thresholds satisfy $b^-_{fix}(\bar{w}) \leqslant b^-_{flex}$ for any rigidity $\bar{w} \leqslant \bar{w}^D$. By definition of $\bar{w}^D$ and the fact that $\mathcal{H}$ is decreasing in wages, we have that for all $\bar{w} \leqslant \bar{w}_D$, $\mathcal{Y}^N(c^T, \bar{w}) = F(\bar{h})$, and

$$V_{D,fix}(\bar{w}) = V_{D,flex}. \tag{24}$$

It is immediate that for any $\bar{w}$,

$$V^-_{R,fix}(b^-_{fix}(\bar{w}); \bar{w}) \leqslant V^-_{R,fix}(b^-_{fix}(\bar{w}); 0). \tag{25}$$

For all $\bar{w} \leqslant \bar{w_D}$, we can obtain

$$\begin{aligned} V^-_{R,flex}(b^-_{flex}) = V_{D,flex} = V_D(\bar{w}) = V^-_{R,fix}(b^-_{fix}(\bar{w}); \bar{w}) \\ \leqslant V^-_{R,flex}(b^-_{fix}(\bar{w})). \end{aligned} \tag{26}$$

The first equality in equation (26) follows from the definition of b^-_{flex}. The second equality follows from equation (24). The third equality follows from the definition of $b^-_{fix}(\bar{w})$, and the inequality follows from equation (25) and the definition of $V_{R,flex}(b)$. Given that $V^-_{R,flex}(b^-_{flex}) \leqslant V^-_{R,flex}(b^-_{fix}(\bar{w}))$ and $V^-_{R,flex}$ is decreasing in debt, we have demonstrated that $b^-_{flex} \geqslant b^-_{fix}(\bar{w})$.

For the strict part, we have that $V^-_{R,fix}(b^-_{fix}(\bar{w}); \bar{w}) < V^-_{R,flex}(b^-_{fix}(\bar{w}))$ for all $\bar{w} > \bar{w}^-$. Proceeding analogously as in equation (26), we obtain

$$V^{-}_{R,flex}(b^{-}_{flex}) = V_{D,flex} = V_D(\bar{w}) = V^{-}_{R,fix}(b^{-}_{fix}(\bar{w});\bar{w}) < V^{-}_{R,flex}(b^{-}_{fix}(\bar{w})).$$

It therefore follows that $b^{-}_{flex} > b^{-}_{fix}(\bar{w})$. This completes the proof of the first statement of i. □

Proof of Item i, Second Part

Proof. Let us prove now that if preferences are separable, we have $b^{-}_{fix}(\bar{w}) < b^{-}_{flex}$ for any $\bar{w} > \bar{w}^{-}$. Define

$$u(c^T, c^N) = U^T(c^T) + U^N(c^N), \tag{27}$$

where both U^T and U^N are strictly increasing and strictly concave functions. A particular case to obtain equation (27) would be to set the intertemporal elasticity of substitution equal to the intratemporal elasticity of substitution in the baseline utility function (i.e., $\frac{1}{1+\mu} = \frac{1}{\sigma}$).

Combining equations (3), (5), and (10), we now arrive at $w = \frac{F'(h)U^{N\prime}(F(h))}{U^{T\prime}(c^T)}$. Given the standard conditions over preferences, we have that there exists a function $\mathcal{H}$ increasing in tradable consumption c^T and decreasing in w such that $h = \mathcal{H}(c^T, w)$, analogous to (H-demand).

Using equations (21) and (23) and replacing the utility function (27), we have that $b^{-}_{fix}(\bar{w})$ is implicitly given by $V^{-}_{R,fix}(b^{-}_{fix}) = V_{D,fix}$.

Equating $V^{-}_{R,flex}(b^{-}_{flex}) = V_{D,flex}$, we can obtain an implicit function for b^{-}_{flex}. Using Lemma B2, replacing equation (28), and noting that $U^N(c^N)$ cancel out, we arrive at

$$\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} = U^T(y^T) - (1-\beta)U^T(y^T - \delta b^{-}_{flex}) - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^{-}_{flex}\right). \tag{28}$$

Similarly, equating $V^{-}_{R,fix}(b^{-}_{fix}; \bar{w}) = V_{R,fix}(\bar{w})$ and rearranging, we obtain

$$
\begin{aligned}
&U^T(y^T) - (1-\beta)U^T(y^T - \delta b^{-}_{fix}(\bar{w})) \\
&\quad - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^{-}_{fix}(\bar{w})\right) - \frac{(1-\beta)\kappa}{1-\beta(1-\psi)} \\
&= \beta\Bigg[U^N\left(\mathcal{Y}^N\left(y^T - \left(\frac{r(1-\delta)}{r+\delta}\right)\delta b^{-}_{fix}(\bar{w}), \bar{w}\right)\right) \\
&\quad - U^N(\mathcal{Y}^N(y^T, \bar{w}))\Bigg] \\
&\quad + (1-\beta)\Bigg[U^N(\mathcal{Y}^N(y^T - \delta b^{-}_{fix}(\bar{w}), \bar{w})) \\
&\quad - U^N(\mathcal{Y}^N(y^T, \bar{w}))\Bigg] < 0,
\end{aligned}
\tag{29}
$$

where the inequality follows from the fact that $\mathcal{Y}^N$ is increasing in tradable consumption for $\bar{w} > \bar{w}^{-}$.

$$
\begin{aligned}
\frac{(1-\beta)\kappa}{1-\beta(1-\psi)} &> U^T(y^T) - (1-\beta)U^T(y^T - \delta b^{-}_{fix}(\bar{w})) \\
&\quad - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^{-}_{fix}(\bar{w})\right).
\end{aligned}
\tag{30}
$$

Combining expressions (28) and (30) we arrive at

$$
\begin{aligned}
&U^T(y^T) - (1-\beta)U^T(y^T - \delta b^{-}_{flex}) \\
&\quad - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^{-}_{flex}\right) \\
&> U^T(y^T) - (1-\beta)U^T(y^T - \delta b^{-}_{fix}(\bar{w})) \\
&\quad - \beta U^T\left(y^T - \left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^{-}_{fix}(\bar{w})\right).
\end{aligned}
$$

This can be rewritten as the following expression:

$$\beta\left[U^T\left(y^T-\left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{flex}\right)-U^T\left(y^T-\left(\frac{(1-\delta)r}{r+\delta}\right)\delta b^-_{fix}(\bar{w})\right)\right]$$
$$+(1-\beta)\left[U^T(y^T-\delta b^-_{flex})-U^T(y^T-\delta b^-_{fix}(\bar{w}))\right]<0. \tag{31}$$

Because U^T is strictly increasing and $\frac{(1-\delta)r}{r+\delta}<1$, we can conclude $b^-_{flex}>b^-_{fix}(\bar{w})$ for any rigidity $\bar{w}>\bar{w}^-$. □

Proof of Item ii

We want to show that a devaluation expands the safe zone. More precisely, when preferences are separable, we have that for every $e'>e$, then $b^-_{fix}\left(\frac{\bar{W}}{e'}\right)>b^-_{fix}\left(\frac{\bar{W}}{e}\right)$ for any nominal rigidity and exchange rate such that $\frac{\bar{W}}{e}\leqslant\bar{w}^D$.

Proof. First, note that $V_{D,fix}\left(\frac{\bar{W}}{e}\right)=V_{D,fix}\left(\frac{\bar{W}}{e'}\right)=V_{D,flex}$. Using this result and the definition of $b^-_{fix}\left(\frac{\bar{W}}{e}\right)$ and $b^-_{fix}\left(\frac{\bar{W}}{e'}\right)$, we can equate $V^-_{R,fix}\left(b^-_{fix}\left(\frac{\bar{W}}{e'}\right);\frac{\bar{W}}{e'}\right)$ to $V^-_{R,fix}\left(b^-_{fix}\left(\frac{\bar{W}}{e}\right);\frac{\bar{W}}{e}\right)$. Using the functional form in equation (27) and applying Lemma B, we obtain

$$U^T\left(y^T-\delta b^-_{fix}\left(\frac{\bar{W}}{e}\right)\right)-U^T\left(y^T-\delta b^-_{fix}\left(\frac{\bar{W}}{e'}\right)\right)$$
$$+\frac{\beta}{1-\beta}\left[U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\frac{\bar{W}}{e}\right)\right)\right.$$
$$\left.-\;U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\frac{\bar{W}}{e'}\right)\right)\right]$$
$$=U^N\left(\mathcal{Y}^N\left(y^T-\delta P b^-_{fix}\left(\frac{\bar{W}}{e'}\right),\frac{\bar{W}}{e'}\right)\right)$$
$$-\,U^N\left(\mathcal{Y}^N\left(y^T-\delta b^-_{fix}\left(\frac{\bar{W}}{e}\right),\frac{\bar{W}}{e}\right)\right)$$

$$+\frac{\beta}{1-\beta}\left[U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\frac{\bar{W}}{e'}\right),\frac{\bar{W}}{e'}\right)\right)\right.$$
$$\left.-U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta(1-\delta)\delta b^-_{fix}\left(\frac{\bar{W}}{e}\right),\frac{\bar{W}}{e}\right)\right)\right].$$

By way of contradiction, suppose that $b^-_{fix}\left(\frac{\bar{W}}{e}\right) > b^-_{fix}\left(\frac{\bar{W}}{e'}\right)$. Then, we have that the left side is negative while the right side is positive. The former follows from U^T being strictly increasing in tradable consumption, whereas the latter follows from the fact that $\mathcal{Y}^N$ is increasing in c^T and decreasing in $\bar{W}$. It thus follows that $b^-_{fix}\left(\frac{\bar{W}}{e}\right) \leqslant b^-_{fix}\left(\frac{\bar{W}}{e'}\right)$.

By way of contradiction, suppose that $b^-_{fix}\left(\frac{\bar{W}}{e}\right) > b^-_{fix}\left(\frac{\bar{W}}{e'}\right)$. Then, we have that the left side is negative while the right side is positive. The former follows from U^T being strictly increasing in tradable consumption, and the latter follows from the fact that $\mathcal{Y}^N$ is increasing in c^T and decreasing in $\bar{W}$. It thus follows that $b^-_{fix}\left(\frac{\bar{W}}{e}\right) \leqslant b^-_{fix}\left(\frac{\bar{W}}{e'}\right)$. □

Proof of Item iii, First Part

Proof. We want to show that $\mathcal{C}_{flex} \subset \mathcal{C}_{fix}(\bar{w})$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leqslant \bar{w}^+$. Recall that by definition, the crisis zone is given by

$$\mathcal{C} = \{b : b^- < b \leqslant b^+\}.$$

We already showed that $b^-_{fix}(\bar{w}) < b^-_{flex}$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leqslant \bar{w}^D$. To deliver the desired result, it suffices to show that $b^+_{fix}(\bar{w}) = b^+_{flex}$ for all $\bar{w}$ such that $\bar{w}^- < \bar{w} \leqslant \bar{w}^+$. Recall that $\bar{w}^D > \bar{w}^+ > \bar{w}^-$. We can see that $\mathcal{H}\left(y^T - \left(\frac{r}{r+\delta}\right)\delta b^+_{flex}, \bar{w}^-\right) < \bar{h}$.

With $\pi = 0$, we have that if the government is not in the default zone today, it keeps the debt constant. For $\bar{w} \leqslant \bar{w}^+$, we have that wage rigidities are not binding as long as $b \leqslant b^+_{flex}$. Thus, the value of repaying for the government is the same under fixed and flexible exchange rates:

$$V^+_{R,fix}(b;\bar{w}) = V^+_{R,flex}(b) \text{ for all } \quad \bar{w} \leqslant \bar{w}^+,\ b \leqslant b^+_{flex}. \tag{32}$$

In addition, given that $\bar{w}^+ < \bar{w}^D$,we have that for any $\bar{w} \leqslant \bar{w}^+$,

$$V_{D,flex} = V_{D,fix}(\bar{w}). \tag{33}$$

Using equations (32) and (33), we have that $b^{+}_{flex} = b^{+}_{fix}(\bar{w})$ for any $\bar{w} \leqslant \bar{w}^{+}$. This completes the proof of the first part. □

Proof of Item iii, Second Part

Proof. This part requires first showing that if preferences are separable, $b^{+}_{flex} \geqslant b^{+}_{fix}(\bar{w})$ for any $\bar{w}$. Given $\pi = 0$ and replacing the utility function (27), we have

$$(34) \quad V^{+}_{flex}(b) = \frac{U^{T}\left(y^{T} - \left(\frac{r}{r+\delta}\right)\delta b\right) + U^{N}(F(\bar{h}))}{1-\beta} \qquad \forall\, b \leqslant b^{+}_{flex}.$$

By definition, $V^{+}_{flex}(b^{+}_{flex}) = V^{D}_{flex}$. Replacing equations (23) and (34) and using the functional form (27), we obtain

$$\frac{1}{1-\beta}(U^{T}(y^{T}) + U^{N}(F(\bar{h}))) - \frac{\kappa}{1-\beta(1-\psi)}$$
$$= \frac{1}{1-\beta}\left(U^{T}\left(y^{T} - \left(\frac{r}{r+\delta}\right)\delta b^{+}_{flex}\right) + U^{N}(F(\bar{h}))\right),$$

which, simplifying, yields

$$(35) \qquad \frac{(1-\beta)\kappa}{1-\beta(1-\psi)} = U^{T}(y^{T}) - U^{T}\left(y^{T} - \left(\frac{r}{r+\delta}\right)\delta b^{-}_{flex}\right).$$

We proceed to obtain analogous expressions under a fixed exchange rate regime. We have

$$(36) \qquad V_{D,fix}(\bar{w}) = \frac{U^{T}(y^{T}) + U^{N}(\mathcal{Y}^{N}(y^{T}, \bar{w}))}{1-\beta} - \frac{\kappa}{1-\beta(1-\psi)},$$

and

$$V^{+}_{R,fix}(b^{+}_{fix}(\bar{w}), \bar{w})$$
$$= \frac{U^{T}\left(y^{T} - \left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w})\right) + U^{N}\left(\mathcal{Y}\left(y^{T} - \left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w}), \bar{w}\right)\right)}{1-\beta}$$
$$\forall\, b \leqslant b^{+}_{fix}.$$

(37)

Equating (36) and (37), by construction we can determine b^{+}_{fix} as

$$\frac{1}{1-\beta}[U^T(y^T)+U^N(\mathcal{Y}^N(y^T,\bar{w}))]-\frac{\kappa}{1-\beta(1-\psi)}$$
$$=\frac{1}{1-\beta}\left(U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w})\right)\right.$$
$$\left.+U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w}),\bar{w}\right)\right)\right).$$

Manipulating the expression, we arrive at

$$U^T(y^T)-U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w})\right)-\frac{(1-\beta)\kappa}{1-\beta(1-\psi)}$$
$$=U^N\left(\mathcal{Y}^N\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w}),\bar{w}\right)\right)-U^N(\mathcal{Y}^N(y^T,\bar{w}))$$
$$\geqslant 0,$$

(38)

where the inequality follows from $\mathcal{Y}^N(y^T,\bar{w})\geqslant\mathcal{Y}^N(y^T-(\frac{r}{r+\delta})\delta b^{+}_{fix}(\bar{w}),\bar{w})$. Hence, we can rewrite equation (38) as

$$(39)\qquad \frac{(1-\beta)\kappa}{1-\beta(1-\psi)}\geqslant U^T(y^T)-U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w})\right).$$

Combining expressions (35) and (39) and simplifying, we arrive at

$$U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{flex}\right)\leqslant U^T\left(y^T-\left(\frac{r}{r+\delta}\right)\delta b^{+}_{fix}(\bar{w})\right).$$

Hence, we can conclude $b^{+}_{flex}\geqslant b^{+}_{fix}(\bar{w})$ for any $\bar{w}$. Notice that if we take $\bar{w}>\bar{w}^{+}$, we have $\mathcal{Y}^N(y^T,\bar{w})>\mathcal{Y}^N(y^T-(\frac{r}{r+\delta})\delta b^{+}_{fix},\bar{w})$. The inequality becomes strict if we take any rigidity $\bar{w}>\bar{w}^{+}$. □

Appendix C: Optimal exchange rate policy

Proposition C1. (Optimal exchange rate policy) Under a flexible exchange rate regime, the government always chooses an exchange rate that achieves full employment.

Proof. The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{e,b',c^T,h\leqslant\bar{h}} \{u(c^T, F(h)) + \beta\mathbb{E}[V(b', \mathbf{s})]\} \tag{40}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$h \leqslant \mathcal{H}\left(c^T, \frac{\bar{W}}{e}\right).$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the following Bellman equation:

$$V_D(y^T) = \max_{e,c^T,h\leqslant\bar{h}} \{u(c^T, F(h)) - \kappa(y^T) + \beta\mathbb{E}[\psi V(0, \mathbf{s}') + (1-\psi)V_D(y^{T\prime})]\}$$

subject to

$$c^T = y^T$$

$$h \leqslant \mathcal{H}\left(c^T, \frac{\bar{W}}{e}\right). \tag{41}$$

It is immediate from equations (40) and (41) that for any level of tradable consumption, an increase in e increases the employment demand without tightening any other constraint (see Lemma A1). When employment demand falls short of full employment, the government depreciates the exchange rate to strictly increase employment until the point at which $\bar{h} = \mathcal{H}(c^T, \frac{\bar{W}}{e})$. □

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Supplementary Material

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online.

Data Availability

Data and code replicating the tables and figures in this article can be found in Bianchi and Mondragon (2021) in the Harvard Dataverse, https://doi.org/10.7910/DVN/SIPPDD.

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