Financial Integration and Monetary Policy Coordination*

Javier Bianchi
Federal Reserve Bank of Minneapolis

Louphou Coulibaly
University of Wisconsin-Madison and NBER

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Abstract

Financial integration generates macroeconomic spillovers that may require international monetary policy coordination. We show that individual central banks may set nominal interest rates too low or too high relative to the cooperative outcome. We identify three sufficient statistics that determine whether the non-cooperative equilibrium exhibits under-tightening or over-tightening: the output gap, sectoral differences in labor intensity, and the response of the trade balance to changes in nominal rates. In the case of higher labor intensity in the non-tradable sector, nominal interest rates are too low under two conditions: when the economy is in a recession and lower nominal rates increase the trade surplus, or when the economy is overheated and lower nominal rates reduce the trade surplus.

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Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is not so much that current and planned moves will fail eventually to quell inflation. It is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account.

Maurice Obstfeld "Uncoordinated monetary policies risk a historic global slowdown," blog post, Peterson Institute, 09/12/2022

1 Introduction

After a prolonged period of expansionary monetary policy, in 2022 central banks around the world shifted to a tightening cycle to quell rising inflation. The rapid pace and synchronous nature of the increase in interest rates, however, have raised concerns about the risk that the monetary tightening could lead to a significant economic downturn (Obstfeld, 2022).[1]

In light of these concerns, there has been a renewed discussion on the necessity of cooperation to avert a global recession and achieve a soft landing. Many important questions remain open: Does cooperative monetary policy prescribe lower interest rates compared to the non-cooperative scenario? Or is it possible that countries may be insufficiently tightening relative to the social optimum? In a broader sense, what are the benefits from international coordination of monetary policy and how do they depend on the degree of financial integration?

The study of international monetary policy cooperation has a long history in international macro literature, dating back to Hamada (1976), and Canzoneri and Henderson (1991). Early studies argued that countries have incentives to weaken their currencies to gain a trade advantage, leading to concerns about competitive devaluations and widespread inflation. On the contrary, modern international macro-models, as exemplified by Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2005) predict that countries have incentives to appreciate their currencies to improve their terms of trade and extract more rents from foreign countries. From this perspective, the motivations for strategic manipulation of terms of trade actually call for cooperation towards more expansionary monetary policies. Moreover, the gains from cooperation in this literature emerge purely from trade

[1] See the Peterson Institute blog post quoted at the beginning of the paper and Figure 1.

flows and are present even in the absence of financial flows.[2]

In this paper, we approach the questions on international monetary coordination from a different, intertemporal perspective. Central to our model is the notion that monetary policy has effects on an intertemporal price, namely, the world real rate, and through this channel, a central bank's policy affects the ability of other central banks to achieve their output and inflation stability objectives. Our analysis builds on the analysis of international spillovers in Bianchi and Coulibaly (2021) where we show how countries use monetary policy to raise their net foreign asset position and reduce their vulnerability to liquidity traps. When the world interest rate is lower, this results in larger incentives for households' borrowing, driving the central bank to deviate further from its efficient level of output in an attempt to raise its net foreign asset position. Given the intertemporal nature of this mechanism, distinct from the static terms of trade manipulation, we refer to it as the *financial channel of international spillovers*. Here, we compare the Nash equilibrium without monetary policy cooperation with the equilibrium under the optimal cooperative monetary policy and provide a general characterization of whether this financial channel requires coordinating on a more restrictive or expansionary monetary policy.[3]

Our main result is that the Nash equilibrium may feature nominal rates that are too low or too high relative to the cooperative outcome and to elucidate how the outcome depends on a small set of sufficient statistics, specifically, the output gap, the difference in labor intensity across sectors, and the response of the trade balance to changes in nominal rates. In particular, we establish that when the economy is undergoing a recession, the cooperative monetary policy prescribes lower interest rates relative to the Nash equilibrium (i.e., there is over-tightening) if the product of the difference between the labor intensity in the non-tradable sector and the tradable sector and the response of the trade balance to a domestic monetary policy expansion is positive. Conversely, if the sign of the product is the same, the cooperative monetary policy prescribes lower interest rates relative to the Nash equilibrium (i.e., there is under-tightening) when the economy is facing overheating.

The intuition for our results is as follows. Consider a global economy facing a recessionary shock. To the extent that wages are rigid and inflation is costly, central banks in individual countries expand monetary policy to help reduce the output gap and face an increase in inflation. In such a scenario, we argue that a reallocation of employment from

[2]From a quantitative standpoint, however, the consensus in the literature following Obstfeld and Rogoff (1995) is that the gains from cooperation due to this trade channel are negligible. Bodenstein, Corsetti and Guerrieri (2020) argue the gains can be somewhat more sizable away from a symmetric steady state.

[3]Fornaro and Romei (2022) tackles the problem of monetary policy coordination from a similar perspective. We discuss below how our framework and conclusions differ from theirs.

a low labor-intensity sector to a high labor-intensity sector helps mitigate inflation. This is because, in sectors with higher labor intensity, prices are less responsive to changes in production under wage stickiness. Consequently, to the extent that the non-tradable sector is more labor intensive than the tradable sector, a shift in employment towards non-tradables would lead to an overall reduction in inflation.

In turn, the allocation of employment across sectors depends crucially on financial flows. When households borrow more from abroad, they increase their demand for consumption of both tradable and non-tradable goods. But employment of tradables remains fixed for a given monetary policy. Therefore, higher capital inflows result in more employment in the non-tradable sector (to satisfy the increase in demand) and help reduce inflation.

Even though the central bank cannot control directly financial flows, it can use monetary policy to steer them to help control inflation. In the case where an increase in the interest rate decreases the trade surplus (i.e., Marshall Lerner holds), a central bank thus perceives that raising interest rates helps to contain inflation for given total employment, as the increase in capital inflows generates in equilibrium a reallocation of employment towards non-tradables. However, as all economies seek to run a trade deficit, this pushes up real interest rates keeping capital flows unchanged. The increase in the world real rate then feeds back into each central bank's problem. Notably, for an individual central bank facing a recession, the increase in the real interest rate induces households to reduce their demand, pushing the economy further down into the recession. Because individual countries do not internalize this pecuniary externality, they tend to over-tighten in this situation. Under cooperative monetary policy, it is optimal to lower nominal interest rates because this helps central banks improve their macroeconomic tradeoffs.

In a situation where the economy is overheating instead, the above conclusions reverse for the same sign of the difference in labor intensities and the same response of the trade balance to a monetary policy expansion. In this case, now, a lower world real rate worsens the macroeconomic tradeoffs faced by central banks, and therefore, central banks tighten too little monetary policy relative to the cooperative outcome (i.e., there is under-tightening).

When a monetary expansion in one country leads to a decrease in its trade surplus instead (i.e., Marshall-Lerner fails), we obtain that the Nash equilibrium displays under-tightening even if the global economy faces a recession, insofar as the non-tradable sector is more labor-intensive than the tradable sector. The wedge changes sign because now an individual central bank is hurt by a more expansionary monetary policy abroad, as this raises the world real rate and worsens its monetary tradeoffs.

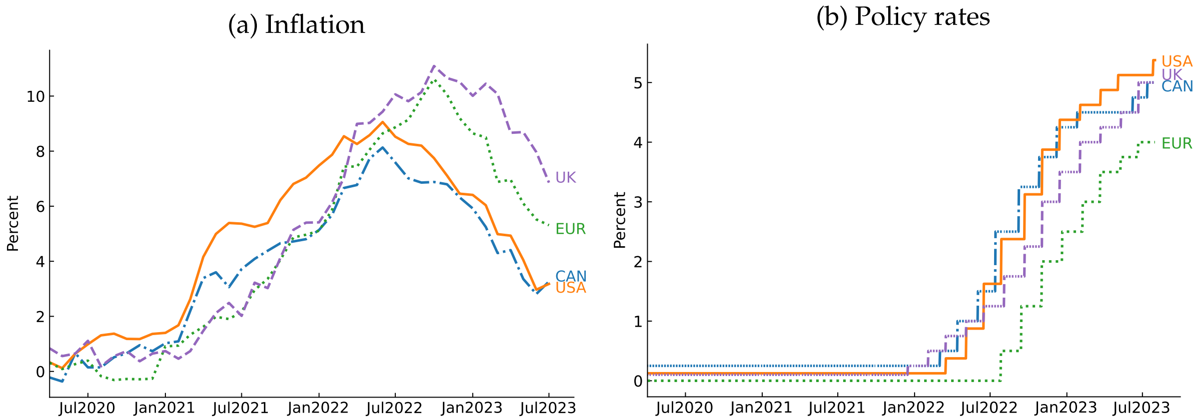

Figure 1: Synchronous Monetary Policy Tightening

When the non-tradable sector is more labor intensive than the tradable sector, we also obtain under-tightening if the economy is in a recession and the Marshall-Lerner condition holds. This is because central banks facing a recession now benefit from a higher world real rate.

To summarize, whether cooperation calls for lower or higher rates can be framed entirely in terms of the sign of the output gap and the sign of the product of the differences in labor intensity between the tradable sector and the non-tradable sector and the response of the trade balance to a monetary expansion. We show in the paper that these findings generalize when we allow for additional features. In one extension, we allow for the anticipation of future shocks. In this case, the comparison between cooperative and non-cooperative outcomes is quite stark. While the cooperative solution maintains zero inflation and zero output in response to the news shock, the Nash equilibrium exhibits either one of these scenarios overheating and inflation or a recession and deflation. Moreover, the sign of the output gap, the differences in labor intensity, and the response of the trade balance to a monetary expansion remain the three key sufficient statistics. Furthermore, we show that our results also hold when we allow for costly labor reallocation or oil price shocks.

Related literature. Our paper belongs to a vast literature on international monetary policy coordination. Early contributions in the context of static Mundell-Flemming models are Hamada (1976), Oudiz and Sachs (1984) and Canzoneri and Henderson (1991). Two pioneer papers adopting a microfounded approach to cooperative monetary policy are Obstfeld and Rogoff (1995, 2002) and Corsetti and Pesenti (2005) (see also, e.g., Tille, 2001; Clarida, Galı and Gertler, 2002; Devereux and Engel, 2003; Egorov and Mukhin, 2020; and

Bodenstein, Corsetti and Guerrieri, 2020). A key theme in this literature is that individual countries have incentives to reduce their own production to change terms of trade in their favor at the expense of other countries. According to this optimal tariff argument, central banks generally over-tighten monetary policy relative to the socially optimal level, a result that is independent of the degree of financial integration. Instead, we highlight a financial channel, involving an intertemporal price (i.e., the world real interest rate) and show that this generates the possibility of under-tightening.

Our paper is most closely related to Fornaro and Romei (2022) who also consider the scope for monetary policy cooperation in a two-sector New Keynesian model with tradables and non-tradables. They show how an increase in the preference for tradable goods leads to inflation and a negative output gap in equilibrium. Moreover, they find that cooperative monetary policy prescribes higher output levels relative to the Nash equilibrium. In terms of the model, we differ by considering a more general structure with elastic labor supply, diminishing returns in labor, and non-unitary elasticities of substitution.[4] Our analysis shows that the Nash equilibrium may also exhibit under-tightening and elucidates how this outcome depends on a set of sufficient statistics. Namely, we establish analytically that, independently of the shocks, whether cooperation calls for lower or higher rates depends on the degree of slack in the economy, the differences in labor intensities across sectors, and the response of the trade balance to a monetary expansion.[5]

Our paper is also related to the literature that examines the potential for international coordination in the context of various government policies. Chang (1990) and Kehoe (1987) study the coordination of fiscal policies when fiscal deficits in some countries make it more costly for others to finance their deficit (see also Azzimonti, De Francisco and Quadrini, 2014). In Halac and Yared (2018), governments exhibit present bias and fiscal rules are slacker under coordination. Obstfeld and Rogoff (1996) and Costinot, Lorenzoni and Werning (2014) consider the case for excluding capital controls when countries are

[4]In their setup, a fixed endowment of hours implies that overheating cannot occur, linear production for non-tradables rules out inflation in non-tradables and unitary elasticities imply that the trade balance always increases in response to a depreciation.

[5]As mentioned earlier, we also draw from our previous work on international spillovers (Bianchi and Coulibaly, 2021) which focuses on a prudential aspect of monetary policy. Another recent paper is Caldara, Ferrante, Iacoviello, Prestipino and Queralto (2023), which studies non-linear effects from monetary spillovers in a model with global banks. Previous work by Acharya and Bengui (2018), Eggertsson, Mehrotra, Singh and Summers (2016), Caballero, Farhi and Gourinchas (2021), and Fornaro and Romei (2019) study the propagation of liquidity traps across countries, but did not consider the scope for monetary policy cooperation. For the empirical literature on international monetary policy spillovers, see, for example, Rey (2013) and Kalemli-Ozcan (2019).

large and have market power over the world interest rate.[6] In our case, countries are infinitesimal, and the case for coordination is grounded in a pecuniary externality, where the world interest rate influences monetary policy tradeoffs.

The key mechanism at play in our model is also related to the literature on aggregate demand externalities. In Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), nominal rigidities and constraints on monetary policy create a rationale for capital controls. In our model, monetary policy faces no constraints, but inflation is costly, and divine coincidence fails, generating aggregate demand externalities. Crucially, the scope for monetary policy cooperation emerges because of the interaction between this aggregate demand externality and a pecuniary externality operating through the world real rate.

Finally, there has been an active recent literature on the rise of inflation following the Covid-19 pandemic and the connection with sectoral reallocation.[7] Besides our open economy focus, we also contribute to this literature by highlighting, for the first time, to the best of our knowledge, the importance of differences in labor intensity across sectors for the determination of inflation and output.

2 Model

Time is discrete and infinite. We model the world economy as a continuum of identical small open economies indexed by $k \in [0,1]$. There are two consumption goods, a tradable good and a non-tradable good, which are produced in each economy using labor in a competitive market with nominally rigid wages. For simplicity, we focus on a deterministic environment.

We first describe the problem faced by households and firms in each economy k, and then describe the competitive equilibrium. The notation does not index variables in each country by k to avoid clutter. We will use $\{x_t\}$ to refer to the sequence $\{x_{k,t}\}_{t=0}^{\infty}$ for some variable x and country k.

[6] Other recent examples are Clayton and Schaab (2022) on macroprudential policy with multinational banks and Chari, Nicolini and Teles (2023) on fiscal and trade policies in a multi-country business cycle model.

[7] See, for example, Rubbo (2020), Guerrieri, Lorenzoni, Straub and Werning (2021), di Giovanni, Kalemli-Özcan, Silva and Yildirim (2022, 2023), and Baqaee and Farhi (2022).

2.1 Households

Each economy is populated by a continuum of identical households of measure one. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \kappa_t n_t + \frac{\chi}{2} \left(\pi_t - \overline{\pi}_t\right)^2 \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and U is a strictly increasing and concave utility function over a consumption good c_t, which is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a Cobb-Douglass aggregator

$$c_t = \left(c_t^T\right)^{\phi^T} \left(c_t^N\right)^{\phi^N},$$

with $\phi^T \in (0,1)$ and $\phi^N = 1 - \phi^T$. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods and $\sigma_t \equiv \frac{-c_t U''(c_t)}{U'(c_t)}$ to denote the inverse of the intertemporal elasticity of substitution. Households face a linear disutility from working given by κ. Aggregate hours n_t is the sum of hours worked in the tradable sector n_t^T and in the non-tradable sector n_t^N, that is $n_t = n_t^T + n_t^N$. Implicit in the formulation is that labor is perfectly mobile across sectors, which, in turn, implies that in equilibrium the wage is equated in both sectors. In Section 5, we generalize preferences by allowing for imperfect labor mobility and a CES composite for consumption.

The last term in (1) represents a utility cost of inflation, which represents standard losses from price adjustments that emerge in models with costly price adjustments á la Rotemberg or staggered prices à la Calvo. This cost is assumed to be quadratic in the deviations of the inflation rate of the consumer price index $\pi_t \equiv P_t/P_{t-1} - 1$ from the central bank target $\bar{\pi}_t$. Given a unitary elasticity of substitution between tradables and non-tradables, the consumer price index P_t satisfies

$$P_t = \left(\frac{P_t^T}{\phi^T}\right)^{\phi^T} \left(\frac{P_t^N}{\phi^N}\right)^{\phi^N} \tag{2}$$

where P_t^N and P_t^T denote respectively the price of non-tradables and tradables in terms of the domestic currency.

We assume that the law of one price holds for the tradable good. Let us denote by P_{jt}^T the price of the tradable good in terms of the country j currency. Thus, it follows that $P_{kt}^T = P_{jt}^T e_{kt}^j$ for any pair of countries k and j, where e_{kt}^j is the nominal exchange rate defined as the price of the country j' currency in terms of the country k' currency.

In each period, households receive their labor income, $W_t(n_t^T + n_t^N)$. They also collect profits φ_t from domestic firms. Households have two assets available, a real international bond that pays R_t^* units of tradables and a nominal domestic bond that pays R_t in units of the domestic currency. These assets are referred to as b_t^* and b_t respectively. The budget constraint is, therefore, given by

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(n_t^T + n_t^N) + \varphi_t + b_t + P_t^T b_t^*. \tag{3}$$

We assume that wages are rigid in period 0 at a given value W. For $t = 0$, households are off their labor supply and hours worked are determined by firms' labor demand. For $t > 0$, we assume that wages are flexible.

The problem of the household consists of choosing a sequence of consumption $\left\{c_t^N, c_t^T\right\}_{t=0}^{\infty}$, asset positions $\left\{b_{t+1}, b_{t+1}^*\right\}_{t=0}^{\infty}$, and hours $\left\{n_t^T, n_t^N\right\}_{t=1}^{\infty}$, to maximize the expected present discounted value of utility (1), subject to (3) and taking as given profits $\{\varphi_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}_{t=0}^{\infty}$.

The optimality condition with respect to c_t^T and c_t^N equates the marginal rate of substitution between the two goods to the relative price. Given the Cobb-Douglas aggregator, households allocate a constant share of their expenditures to each good, implying that

$$\phi^T P_t^N c_t^N = \phi^N P_t^T c_t^T. \tag{4}$$

The linearity of the disutility from working implies that for $t > 0$ (when wages are flexible) the wage in both sectors must satisfy

$$\frac{W_t}{P_t^N} = \frac{\kappa_t}{u_N(c_t^T, c_t^N)}, \qquad \frac{W_t}{P_t^T} = \frac{\kappa_t}{u_T(c_t^T, c_t^N)}. \tag{5}$$

where we use u_T and u_N to denote the respective partial derivatives.

Finally, the optimality condition with respect to asset holdings yield

$$u_T\left(c_t^T, c_t^N\right) = \beta R_t^* u_T\left(c_{t+1}^T, c_{t+1}^N\right) \tag{6}$$

$$R_t^* = R_t \frac{P_t^T}{P_{t+1}^T}, \tag{7}$$

Condition (6) is the Euler equation for the real bond. Condition (7) is a no-arbitrage condition that equates the return on real international bonds and domestic currency bonds, both expressed in units of tradables.

2.2 Firms

There is a continuum of firms of measure one producing tradable goods and non-tradable goods. Output of the two goods $i = \{T, N\}$ is produced using labor with a production function F such that

$$y_t^i = F^i(h_t^i, A_t^i)$$

We assume an isoelastic production function such that $F^i(h_t^i, A_t^i) = A_t^i (h_t^i)^{\alpha^i}$. We refer to α^i as the *labor intensity* parameter.

Profits are given by $P_t^i F^T(h_t^i, A_t^i) - W_t h_t^i$. At the optimum, firms equate the marginal product of labor to the nominal wage in the two sectors.

$$P_t^T F_h^T(h_t^T, A_t^T) = W_t, \tag{8}$$

$$P_t^N F_h^N(h_t^N, A_t^N) = W_t. \tag{9}$$

Given competitive markets, the labor intensity equals the labor share for each sector in equilibrium. As we will see, differences in labor intensity across sectors, $\alpha^N - \alpha^T$, will play an important role in the analysis. We note that the fact that labor is the only factor of production or that the production function exhibits decreasing returns to scale is not restrictive.[8]

2.3 Monetary Policy

In each small open economy, there is a central bank that chooses nominal interest rates $\{R_t\}$. Because of the assumption that prices are flexible for $t > 0$, monetary policy is neutral starting from period 1. Therefore, we assume that monetary policy implements a strict inflation targeting regime such that $\pi_t = \bar{\pi}_t$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.

[8] Adding a factor in fixed unit of supply with a flexible factor price does not alter allocations. In Section 5, we incorporate oil as an additional factor of production.

2.4 Competitive equilibrium

In each country, the market for non-tradable goods must clear. That is,

$$c_t^N = F^N(h_t^N, A_t^N). \tag{10}$$

At $t = 0$, households in each country supply hours in the tradable and non-tradable sectors to meet the demand by firms. For $t > 0$, the labor clears the labor market. That is, $n_t^T = h_t^T$ and $n_t^N = h_t^N$.

We assume without loss of generality that the bond denominated in domestic currency is only traded domestically in each country. Market clearing therefore implies

$$b_{t+1} = 0. \tag{11}$$

Finally, at the world level, real bonds are in zero net supply. To account for market clearing at the world level, we now explicitly index the policies of each country by k. We have that

$$\int b_{k,t+1}^* dk = 0. \tag{12}$$

We now define a competitive equilibrium in the global economy.

Definition 1 (Competitive Equilibrium)**.** Given initial positions $b_{k,0}^*$, a sticky wage W, and a sequence of central bank policies $\{R_t\}$ in each country k, an equilibrium is a sequence of world real rates $\{R_t^*\}$, prices $\{P_t^T, P_t^N, W_t, e_{k,t}^j\}$ and allocations $\{c_t^T, c_t^N, h_t^T, h_t^N, b_{t+1}, b_{t+1}^*\}$ in each country k such that

(i) Households optimize, and hence the following conditions hold: (4), (6), (7) for all $t \geq 0$ and (5) holds for all $t \geq 1$;

(ii) Firms optimize, implying (8) and (9) hold for all $t \geq 0$;

(iii) The law of one price holds for tradables: $P_{k,t}^T = P_{j,t}^T e_{k,t}^j$ for any country-pair k and j,

(iv) The market for non-tradables (10) and domestic bonds (11) clears; moreover, the labor market clears for $t \geq 1$.

(v) Globally, the market for the real bond clears. That is, (12) holds.

If we combine the budget constraints of households and firms as well as market clearing conditions, we arrive at the country budget constraint for tradables, or the balance of payment condition:

$$c_t^T - F^T(h_t^T, A_t^T) = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{13}$$

which says that if a country runs a trade deficit, it accumulates net debt and if it runs a trade surplus, it accumulates net external assets.

We assume that all countries start at $t = 0$ with zero net foreign asset position. To the extent that all countries follow the same policies, we can therefore restrict the analysis to symmetric competitive equilibrium.

2.5 Efficient allocation, output gaps, and the natural wage

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations to maximize welfare subject to a resource constraint. The planner's problem can be written as

$$\max_{\{h_t^N, h_t^T\}} \sum_{t=0}^{\infty} \beta^t \left[u\left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) - \kappa_t \left(h_t^T + h_t^N \right) \right].$$

First-order conditions with respect to tradable and non-tradable employment yield

$$F_h^T(h_t^T, A_t^T)\; u_T\left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) = \kappa_t. \tag{14}$$

$$F_h^N(h_t^T, A_t^N)\; u_N\left(F^T(h_t^T, A_t^T), F^N(h_t^N, A_t^N) \right) = \kappa_t, \tag{15}$$

These conditions imply zero labor wedges for the tradable and non-tradable sectors. Let us denote by $\bar{h}_t^T$ and $\bar{h}_t^N$ the employment levels in the two sectors in the first-best allocation. We obtain the following lemma characterizing the ratio of employment levels solely as the product of the relative weights in preferences and the relative labor intensities:

Lemma 1 (First-best)**.** *The optimal ratio of hours in the first-best allocation is given by*

$$\frac{\bar{h}_t^N}{\bar{h}_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \tag{16}$$

Proof. In Appendix A.1 □

We highlight that the first-best allocations coincide with those in a competitive equilibrium in a flexible wage version of our model. This can be seen by noting that if the nominal wage were flexible, we would arrive at (14) and (15) by combining firms' demand for labor (8) and (9) with households' labor supply decisions (5). This result will provide a clear benchmark for the normative analysis.

Output gaps. To characterize the central banks' tradeoff and to highlight the differences between the competitive equilibrium and the first-best allocation, we define a measure of *output gaps* as the deviations of employment relative to the first-best levels

$$\widehat{h}_t^N \equiv \frac{h_t^N}{\bar{h}_t^N} - 1, \qquad \widehat{h}_t^T \equiv \frac{h_t^T}{\bar{h}_t^T} - 1$$

The assumption that good prices are flexible and that labor is perfectly mobile across sectors implies that output gaps are equated across sectors in any symmetric equilibrium.

Lemma 2. *In any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are equalized $\widehat{h}_t^T = \widehat{h}_t^N = \widehat{h}_t$. Furthermore, the employment ratio in the first-best allocations coincides with those in a competitive symmetric equilibrium for any monetary policy.*

Proof. In Appendix A.2 □

We note that this result applies to any symmetric competitive equilibrium given any monetary policy. Notice that even though the ratio of hours in the competitive equilibrium coincides with the ratio in the first-best, this does not mean that the level of hours will coincide. Depending on the parameters and the stance of monetary policy, output gaps can be negative (implying a recession) or positive (implying overheating).

The natural wage. We define the *natural wage* as the nominal wage that would prevail in equilibrium if wages were flexible and the central bank stabilized inflation at $\bar{\pi}_t$. Denoting variables without a subscript as $t = -1$ variables, we can write the natural wage at date $t = 0$ as described in the following lemma:

Lemma 3 (Natural Wage)**.** *The natural wage at date t is given by*

$$W_t^n = (1 + \overline{\pi}_t)\, P_{-1} \left[\prod_{i=T,N} \left(\alpha^i A_t^i\right)^{\phi^i} \left(\bar{h}_t^i\right)^{-(1-\alpha^i)\phi^i} \right] \tag{17}$$

Proof. In Appendix A.3 □

Equation (17) characterizes the natural wage as a function of parameters for productivity, preferences, and the inflation target. In particular, the natural wage falls in period 0 when there is a decline in productivity for tradables or non-tradables, when there is a positive labor supply shock, or when there is a negative shock to the inflation target. In what follows, we assume that in period -1 the nominal wage is at its natural level, but not in period 0 where the nominal wage is fixed at an arbitrary value W.

3 Monetary Policy in a Nash Equilibrium

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad.

3.1 Optimal Monetary Policy for a Single Country

We first study the individual problem of a central bank that takes as given $\{R_t^*\}$ and policies conducted in other countries.

Time $t > 0$ problem. Recall that because prices are flexible for $t \geq 1$, we can focus on a situation where the central bank implements the flexible price allocation with $\pi_t = \bar{\pi}$ for all $t \geq 1$. The lifetime welfare for a central bank with net foreign asset b_1^* in period 1 is given by

$$V_1(b_1^*) = \max_{\{c_t^T, c_t^N, h_t^T, h_t^N\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^T, c_t^N) - \kappa_t (h_t^T + h_t^N) \right] \tag{18}$$

subject to (10), (14),(15)

$$b_1^* = -\sum_{t=1}^{\infty} \frac{F^T(h_t^T, A_t^T) - c_t^T}{\prod_{j=1}^{t-1} R_j^*}$$

Given that wages are flexible for $t \geq 1$, hours worked in each sector imply a zero labor wedge. The last constraint is the intertemporal budget constraint that requires that future trade balances must be consistent with the initial level of debt.

Time 0 problem. Turning to the date-0 problem, the central bank's policy choice is the nominal interest rate. The central bank's objective is to choose R_0 that maximizes the welfare of the domestic household subject to domestic allocations and prices consistent with a competitive equilibrium given policies $\{R_{k,t}\}$ conducted in other countries.

Following a primal approach, we proceed to combine constraints to express the problem in terms of allocations. First, combining the optimality conditions of firms (8) and (9), with the optimality condition of households (4), we arrive at an equation that determines the relative demand for hours in the two sectors.

$$\frac{h_0^N}{h_0^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T}(1 - \hat{z}_0) \tag{19}$$

where $\hat{z}_0 \equiv (y_0^T - c_0^T)/F^T(h_0^T, A_0^T)$ represents the trade balance to output ratio. This condition will play a key role in the analysis. According to (19), if a country runs a larger trade balance surplus, it will face in equilibrium a lower amount of hours in the non-tradable sector relative to the tradable sector. The logic is as follows. Running a larger trade surplus requires an accumulation of net foreign assets and lower resources available for consumption. Because preferences are homothetic, this implies lower demand for both tradables and non-tradables. As non-tradable goods are produced domestically, the decline in the demand for non-tradables leads to a decrease in the hours worked in the non-tradable sector.

Second, the level of inflation can be expressed as

$$\frac{\hat{\pi}_0}{1 + \overline{\pi}_0} = \frac{W}{W_0^n} \left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T} \left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} - 1 \tag{20}$$

This condition is an open economy version of the Phillips curve that relates employment in both sectors to inflation (20).[9] For a given wage, higher employment in tradables or non-tradables requires higher prices in the respective sectors.

One observation, which to our knowledge has not received attention in the literature, is that the labor intensities of the sectors play a crucial role in determining the extent to which higher employment in each sector raises inflation. To see this more clearly, we can totally differentiate firms' first-order conditions and using that the wage is constant we obtain

$$d \log P_t^i = \frac{1 - \alpha^i}{\alpha^i} d \log y_t^i$$

The higher is the labor intensity in each sector, the lower is the rise in prices needed to achieve a certain increase in output. Crucial for this result is that wages are sticky. Thus, if a good is more labor intensive, this means that firms can scale up production without significant raises in prices. As the curvature in the production becomes lower, an increase in employment leads to a faster decline in the marginal product, thus necessitating a larger increase in prices to induce higher employment to be optimal for firms.

[9] To obtain (20) we use the definition of the price index (2) at dates $t = 0$ and $t = -1$ and combine with firms' optimality (8) and (9).

Finally, given the assumption that the initial net foreign asset position is zero, the country budget constraint for tradables (13) implies the following balance-of-payment condition

$$\frac{b_1^*}{R_0^*} = F^T(h_0^T, A_0^T)\,\widehat{z}_0, \tag{21}$$

We can then write the Lagrangian associated with the central bank's problem can be written as:

$$\begin{aligned}
&u\left((1-\widehat{z}_0)F^T(h_0^T,A_0^T), F^N(h_0^N,A_0^N)\right) - \kappa_0(h_0^T+h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2 + \beta V_1\left(R_0^* F^T(h_0^T,A_0^T)\widehat{z}_0\right)\\
&+\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} + 1\right] + \eta_0\left[(1-\widehat{z}_0)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N} - 1\right]\\
&+\mu_0\left[u_T\left((1-\widehat{z}_0)F^T(h_0^T,A_0^T), F^N(h_0^N,A_0^N)\right) - \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)\right]
\end{aligned} \tag{22}$$

where b_1^* is given by (21) and W_0^n is given by (17).

Two important observations from this problem are worth making. First, the only foreign variable that appears is the world real rate. Although foreign monetary policies can alter the exchange rate vis-à-vis the domestic country, the domestic central bank can alter these movements by varying the nominal rate. As we will see, this will imply that the international spillovers operate through the world real rate. Because this is an intertemporal channel, we refer to it as the "financial channel."

Second, the trade balance affects welfare not just through the utility of current consumption and the continuation value, but also through the last two implementability constraints. Taking first-order condition for the trade balance $\widehat{z}_0$ we obtain:

$$\eta_0 = \left[\delta_0 - \phi^T + \sigma_0\phi^T\right] u_T(c_0^T, c_0^N)\,\mu_0 \tag{23}$$

where δ_0 is given by (A.5) in Appendix A.4 and satisfies $\delta_0 > 1$. This condition shows that the Lagrange multipliers on households' Euler equation (6) and households' intratemporal allocation of hours worked (19) have the same sign. The intuition is as follows. Suppose the central bank would like to increase the ratio of non-tradable employment to tradable employment (that is, $\eta_0 > 0$). Notice that if households were to borrow more, the increase in consumption would lead to higher demand for tradables and non-tradables.

But for given monetary policy, employment of tradables remains fixed.[10] Therefore, a higher level of borrowing would result in more hours in the non-tradable sector relative to the tradable sector. From the small open economy central bank's perspective, therefore, a positive shadow value from higher non-tradable to tradable hours is associated with a positive shadow value from higher household borrowing.

Targeting rules. By optimality conditions of firms and households, summarized by the implementability constraint (19), the marginal value of hours worked $F^i(h_0^i, A_0^i)u_i(c_0^T, c_0^N)$ are equalized across sectors. We use this equilibrium condition and combine it with the optimality conditions for tradable employment and non-tradable employment to obtain the optimal monetary policy for a single country in target form which we present in the proposition below.

Proposition 1 (Targeting rule)**.** *For a given sequence of world real rate $\{R_t^*\}$ the optimal monetary policy of a central bank in a small open economy targets*

$$\sum_{i=T,N} \delta_0^i \alpha^i \phi^i \left[\tau\left(h_0^N, \widehat{z}_0\right) - \kappa_0\right] = \frac{\alpha^N \phi^N}{h_0^N}(1+\psi_z \widehat{z}_0) \cdot \sum_{i=T,N} \delta_0^i (1-\alpha^i)\phi^i \cdot \chi(1+\pi_0)\widehat{\pi}_0 \qquad (24)$$

where δ_0^T, δ_0^N are positive coefficients defined in (A.7), and where

$$\tau\left(h_0^N, \widehat{z}_0\right) \equiv F_h^N(h_0^N, A_0^N)\, u_N\left((1-\widehat{z}_0)F^T\left(\frac{\alpha^T\phi^T}{\alpha^N\phi^N}\frac{h_0^N}{1-\widehat{z}_0}, A_0^T\right), F^N\left(h_0^N, A_0^N\right)\right) \qquad (25)$$

Proof. In Appendix A.4 □

The targeting rule (24) describes what a central bank in a small open economy optimally targets. The left-hand side of the rule captures the net benefit of an increase in employment in both sectors induced by an expansionary monetary, which corresponds to a weighted average of the marginal utility benefits from raising employment in each sector i net of the disutility from working more hours sector i.[11] The right-hand side captures the average marginal cost of higher price of goods produced in both sectors.

The key implication of the targeting rule (24) is that in the face of inflationary pressures, the domestic central bank must respond by driving output in the non-tradable sector

[10] This is because tradable employment depends only on the wage in units of tradables, and the price of tradables is given by $P_t^{T*}e_t$.

[11] A monetary policy expansion directly raises the price of tradables leading to more output and employment in the tradable sector by (8). On the other hand, the increase in the price of tradables shifts expenditures toward non-tradables and raises prices and employment in the non-tradable sector.

below its efficient level $\widehat{h}_0^N < 0$. By (19), this can be achieved by either driving output in the tradable sector below its efficient level $\widehat{h}_0^T < 0$ or by running a trade surplus $\widehat{z}_0 > 0$.[12,13] The former is a usual consideration in the literature when a central bank tries to minimize the output gap and inflation gap. The trade balance consideration is novel to our open economy. The idea is that if the economy faces high inflation, the central bank finds it optimal to run a trade surplus in order to help cool down inflation (as larger trade surplus implies lower demand for domestic consumption). Next, we delve into the incentives for an individual central bank to manage the trade balance.

Trade-balance-management. When households borrow, they equate the marginal benefits of consuming today to the marginal costs of repaying tomorrow, as given by (6). However, a central bank also perceives that changes in international borrowing (and thus changes in the trade balance) affect the reallocation of hours worked across sectors, by (23), which in turn can help bring inflation closer to the target. In particular, the perceived social benefit of the reallocation of hours worked η_0 across sectors is, to a first order, given by

$$\eta_0 = \frac{\phi^N \phi^T}{\sum_i \delta_0^i \alpha^i \phi^i} (\alpha^N - \alpha^T) \chi \widehat{\pi}_0, \tag{26}$$

We can then infer from condition (26) that the sign of η_0 depends on the difference in labor intensity across sectors $\alpha^N - \alpha^T$ and the sign of the inflation gap. When the economy has high inflation, the central bank in the small open economy would like to redistribute labor toward the more labor-intensive sector (i.e., $\eta_0 > 0$). When a sector is more labor intensive, this means that prices respond less to a change in production in that sector. Therefore, starting from a situation with high inflation, the central bank can achieve a reduction in inflation by shifting employment towards the more labor-intensive sector, for a given monetary policy.

When the two sectors are equally labor intensive $\alpha^N = \alpha^T$, households' borrowing choices are optimal from the perspective of the central bank, as it does not perceive any social benefit from changing the composition of hours between the tradable sector and non-tradable sector.

[12]Notice that monetary policy directly controls P_0^T and therefore $\widehat{h}_0^T$.

[13]It is also possible to see this by combining the first-order approximation of (24) around the efficient allocation (where variables with *bar* are evaluated at the efficient allocation) with $\widehat{h}_0^N = \widehat{h}_0^T - \widehat{z}_0$ by (19) to get

$$-\Big[1+(\bar{\sigma}_0-1)\sum_{i=T,N}\alpha^i\phi^i\Big]\widehat{h}_0^T + \Big[1+(\bar{\sigma}_0-1)(\phi^T+\alpha^N\phi^N)\Big]\widehat{z}_0 = \frac{\sum_i \bar{\delta}_0^i (1-\alpha^i)\phi^i}{\sum_i \bar{\delta}_0^i \alpha^i \phi^i}\chi\widehat{\pi}_0$$

Consider the case in which the non-tradable sector is more labor intensive than the tradable sector (i.e., $\alpha^N > \alpha^T$). In this case, if the inflation gap is positive, the central bank internalizes that a reallocation of hours away from the less labor-intensive sector (tradables) toward the most labor-intensive sector (non-tradables) would help bring down inflation and improve social welfare. Individual households do not internalize this aggregate demand externality and under-borrow (relative to the socially optimal level of borrowing from the central bank's perspective). On the other hand, if the inflation gap is negative, the central bank internalizes that a reallocation of hours away from the more labor-intensive sector (non-tradables) toward the less labor-intensive sector (tradables) would help raise inflation towards the target and improve welfare.

The signs of η_0 and μ_0 are reverted when it is the tradable sector that is more labor intensive than the non-tradable sector. In that case, for example, if the inflation gap is positive, the economy features over-borrowing instead of under-borrowing.

The key question then is how monetary policy affects households' borrowing choices and thus the trade balance in the small open economy. There are three distinct forces. First, there is a direct effect by which a lower nominal rate increases the price of tradables by the interest parity condition and leads to higher output. Second, there is an intertemporal substitution effect by which given prices and income, households borrow more externally, tilting consumption towards the present. Finally, there is an income general equilibrium effect by which the resulting increase in current aggregate demand increases output and translates into higher external savings. Formally, the response of the trade balance satisfies[14]

$$\left[\delta_0 + (\sigma_0 - 1) \sum_{i=T,N} \alpha^i \phi^i \right] \left(\delta_0 - \alpha^T\right) d\widehat{z}_0 = -\left[\alpha^T + (\sigma_0 - 1) \sum_{i=T,N} \alpha^i \phi^i \right] \frac{dR_0}{R_0}. \tag{27}$$

In the lemma below, we summarize the effects of monetary policy on the trade balance.

Lemma 4 (Generalized Marshall-Lerner Condition)**.** *The response of the trade balance to a domestic monetary expansion satisfies*

$$\frac{d\widehat{z}_0}{dR_0} > 0 \iff \sigma_0 > \widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \alpha^N \phi^N}$$

Proof. In Appendix A.5 □

The lemma generalizes existing results in the literature to a situation with multi-sector

[14]The derivation of equation (27) uses (6), (7), (8), (19). See Appendix A.5 for more details.

production.[15] Whether an expansionary monetary policy expands the trade balance depends on the elasticities of substitution and labor intensities in the two sectors. If the tradable sector were an endowment, $\alpha^T = 0$, we would obtain the familiar result that the trade balance increases in response to a fall in the nominal rate (i.e., $d\widehat{z}_0/dR_0 < 0$) if and only if the intertemporal elasticity of substitution was lower than the intratemporal elasticity of substitution between tradables and non-tradables (which in this case is assumed to be one).[16] In our model with endogenous production in the tradable sector, the lower interest rate expands tradable output and thus is an additional force toward a trade surplus. Therefore, to obtain a decrease in net exports in response to a lower nominal interest rate, the intertemporal elasticity of substitution must be lower. In addition, it also follows that if $\alpha^T \geq \alpha^N$, a monetary expansion increases the trade surplus for *any* intertemporal elasticity of substitution. Intuitively, a higher α^T implies that tradable output responds more to an increase in the price of tradables (for a given wage), and through consumption smoothing, this means a higher trade surplus.

3.2 Nash Equilibrium

Having characterized individual central banks' best response functions, we can now define a Nash equilibrium.

Definition 2 (Nash Equilibrium)**.** Let $\mathcal{U}(R_0, R_0^*)$ denote the utility of the representative household in a competitive equilibrium where all countries set the nominal rate to R_0. The Nash equilibrium is the fixed point in the central banks' best response function $R_0(R_0^*)$ that maximizes $\mathcal{U}(R_0, R_0^*)$.

We saw that an individual central bank has incentives to alter the trade balance when the labor intensity differs across sectors and the first-best allocation is not achieved. However, by symmetry in any competitive equilibrium in the global economy where all central banks are optimizing, there are no capital flows and exchange rates are constant. From (24) with $\widehat{z}_0 = 0$, we arrive at

$$\tau\left(h_0^N, 0\right) - \tau\left(\bar{h}_0^N, 0\right) = \chi \psi_0^{NE} (1 + \pi_0) \widehat{\pi}_0 \tag{28}$$

with $\psi_0^{NE} = \frac{\alpha^N \phi^N}{h_0^N} \sum_i \delta_0^i (1 - \alpha^i) \phi^i / \sum_i \delta_0^i \alpha^i \phi^i > 0$ and where we used $\kappa_0 = \tau\left(\bar{h}_0^N, 0\right)$ by (14).

[15] The classic Marshall-Lerner condition depends on static elasticities of exports and imports, but as is well-understood, in dynamic general equilibrium model, the effects depend on intertemporal considerations (see e.g., Lane, 2001; Bianchi and Coulibaly, 2021).

[16] When the elasticities are equal, changes in the nominal rate do not affect capital flows. The Cole-Obstfeld parameterization, which is adopted in much of the literature, is a special case.

Given that the marginal value of hours worked is decreasing in hours, $\frac{d\tau}{dh^N} < 0$, condition (28) reveals that under optimal policy, only one of the two scenarios can emerge in the Nash equilibrium: either the world economy is overheating, $\widehat{h}_0^N > 0$, and inflation is below target or there is a recession $\widehat{h}_0^N < 0$ and inflation is above target. To understand the intuition, consider the possibility that in a coordinated equilibrium, there is a negative output gap in the tradable sector (and the non-tradable) and inflation is below the target. In that case, by lowering the nominal interest rate and allowing for higher prices, the central bank can narrow the output gap and the inflation gap. By the same token, if there is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. It is also clear from these conditions that if the inflation cost is zero $\chi = 0$, central banks can implement the first-best allocation for any shocks.

It is possible to uniquely determine the level of employment by combining the targeting rule (28) with (20) which by Lemma 2 can be rewritten as

$$\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} = (1+\widehat{w}_0)\left(1+\widehat{h}_0^N\right)^{\sum_i (1-\alpha^i)\phi^i} - 1, \tag{29}$$

where $\widehat{w}_0 \equiv \frac{W}{W_0^n} - 1$ is the wage gap. The proposition below relates the level of employment in each country in the Nash equilibrium to the wage gap.

Proposition 2. *In the Nash equilibrium, employment in each country is given by $\widehat{h}_0^T = \widehat{h}_0^N$ and*

$$\widehat{w}_0 \cdot \widehat{h}_0^N \leq 0, \quad \text{with} \quad \widehat{h}_0^N = 0 \text{ if and only if } \widehat{w}_0 = 0. \tag{30}$$

Proof. In Appendix A.6 □

Proposition 2 establishes that the sign of the output gap in the Nash equilibrium is the opposite of the wage gap. It follows that in the Nash equilibrium when the market wage is above the natural wage, countries face a recession whereas when the market wage is below the natural wage, countries face overheating. Importantly, this result is independent of what shocks drive the market wage above or below the natural wage.

4 Monetary Policy under Cooperation

We now turn to analyze the optimal monetary policy under cooperation. The key question we tackle is whether coordination calls for tighter or looser monetary policy relative to the

Nash equilibrium.

4.1 Cooperative Equilibrium

We define the optimal cooperative monetary policy as the outcome of a planner's problem that chooses the interest rates on behalf of all countries to maximize average welfare. Because all countries are identical, this means it maximizes the welfare of any given country.

Definition 3 (Optimal Cooperative Monetary Policy)**.** Let $\boldsymbol{R}_0$ denote the vector of nominal rate for all countries $k \in [0,1]$, $\mathcal{R}^*(\boldsymbol{R}_0)$ the implied real rate, and $\mathcal{U}(\boldsymbol{R}_0, \mathcal{R}^*(\boldsymbol{R}_0))$ the associated welfare for each country. The optimal cooperative monetary policy consists of choosing $\boldsymbol{R}_0$ to maximize welfare in the competitive equilibrium. That is,

$$\max_{\boldsymbol{R}_0} \mathcal{U}_0(\boldsymbol{R}_0, \mathcal{R}^*(\boldsymbol{R}_0))$$

Following this definition, the optimality condition for the nominal rate for the planner is as follows:

$$\frac{\partial \mathcal{U}_0(\boldsymbol{R}_0, \mathcal{R}_0^*)}{\partial \boldsymbol{R}_0} + \frac{d\mathcal{R}_0^*}{d\boldsymbol{R}_0} \frac{\partial \mathcal{U}_0}{\partial R_0^*} = 0$$

In a non-cooperative equilibrium, each country sets the nominal rate to maximize their welfare, which implies that $\frac{\partial \mathcal{U}_0}{\partial \boldsymbol{R}_0} = 0$. The social planner instead realizes that changing nominal rates alters the real rate. To understand how the planner would deviate from the non-cooperative equilibrium, there are two crucial considerations: how welfare changes with R_0^* and how R_0^* changes with R_0. Regarding the former, we have that the change in welfare evaluated at the Nash equilibrium is given by

$$\left.\frac{\partial \mathcal{U}_0}{\partial R_0^*}\right|_{R_0^* = R_0^{*,NE}} = -\frac{\Delta}{R_0^*}(\alpha^N - \alpha^T) \left[\tau(h_0^N, 0) - \tau(\bar{h}_0^N, 0)\right] \frac{h_0^N}{\alpha^N \phi^N} \tag{31}$$

with $\Delta \equiv \phi^T \phi^N [(\delta_0 - \phi^T + \sigma_0 \phi^T) \sum_i \delta_0^i (1-\alpha^i) \phi^i]^{-1} > 0$.

This expression is a result of an envelope condition, derived in Appendix A.7, which echoes the findings in Bianchi and Coulibaly (2021). Equation (31) shows that the first-order effects of changes in the world real rate on welfare are determined by the output gap and the differences in labor intensity.[17] In particular, welfare goes up when interest

[17] Notice that because countries are neither net borrowers nor net savers, a marginal change in the world real rate does not affect the country's resource constraint.

rates rise if the sign of the product of the output gap and the difference in labor intensity $\alpha^N - \alpha^T$ is positive.

When the output gap was zero, changes in the world real rate would have no effect on welfare. Notice that by the targeting rule, if the output gap is zero, the inflation is gap is also zero. Therefore, evaluated at the first-best allocations, changes in R^* cancel out on welfare, a result of the envelope theorem.

When labor intensities are equal $\alpha^N = \alpha^T$, we can see that a change in the world real rate has no first-order effects on welfare, regardless of the sign of the output gap. The intuition for this result is that when the two sectors have the same labor intensity, the social and private marginal benefits of borrowing are aligned. This can be seen by combining (23) and (26), which yields

$$u_T(c_0^T, c_0^N)\mu_0 = \Delta \frac{\alpha^N - \alpha^T}{\alpha^N} \left[\tau\left(h_0^N, 0\right) - \tau\left(\bar{h}_0^N, 0\right)\right] h_0^N \tag{32}$$

implying that $\mu_0 = 0$. Even though central banks may not be achieving the efficient allocation, changes in R_0^* are irrelevant for welfare.

Consider instead the case where $\alpha^N > \alpha^T$. In this case, we can see that starting from $\widehat{z}_0 = 0$, if the economy is in a recession $\widehat{h}_0^N < 0$, it also faces positive inflation. Moreover, the central bank from every small open economy would like to relocate employment towards non-tradables and induce more households' borrowing (i.e., $\eta_0 > 0$ and $\mu_0 > 0$ as explained above). A reduction in the real rate of the world, therefore, relaxes the implementability constraints (6) and (19) and improves social welfare.

If instead the economy faces overheating $\widehat{h}_0^N > 0$ and the inflation gap is negative, the reduction in the world real rate now reduces welfare. That is, households are borrowing too much in the competitive equilibrium, and a reduction in the world real rate exacerbates overheating.

The key takeaway is that when countries are not at the first-best allocation and labor intensities are different across sectors, changes in the world real rate have first-order effects on welfare. However, when individual countries set their monetary policy, they do not internalize the potential effects on the world real rate and how this affects welfare in other countries.

The second key consideration is how the real rate changes with the nominal rate. We have[18]

$$\sigma_0 \frac{d\mathcal{R}_0^*}{R_0^*} = (\sigma_0 - \widetilde{\sigma}) \frac{d\boldsymbol{R}_0}{\boldsymbol{R}_0}, \tag{33}$$

This result is in line with Lemma 4. When $\sigma_0 > \widetilde{\sigma}$, a monetary policy expansion in one country raises its trade balance. When all countries simultaneously expand their monetary policy, the real rate must fall to clear the asset market.

Putting together (31) and(33) implies that a monetary policy expansion will improve welfare when $\alpha^N - \alpha^T > 0$, and $\sigma_0 > \widetilde{\sigma}$ and $\widehat{h}_0^N < 0$ in the Nash equilibrium. In the next section, we build on these insights to solve for the planner's allocations and its targeting rules, contrasting how they differ with the Nash equilibrium.

4.2 Optimal Targeting Rule under Cooperation

Following a primal approach we can reduce the global planning problem to choosing $\{h_0^N, h_0^T, \widehat{\pi}_0\}$ and write the associated Lagrangian as follows:

$$\begin{aligned} & u\left(F^T(h_0^T, A_0^T), F^N(h_0^N, A_0^N)\right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2 \\ & + \vartheta_0 \left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} + 1\right] + \eta_0\left[\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N} - 1\right] \end{aligned} \tag{34}$$

The optimal monetary policy under cooperation is similar to the one in the Nash equilibrium (28), but now the planner internalizes that $c_0^T = F^T(h_0^T, A_0^T)$. Optimality implies that

$$\tau(h_0^N, 0) - \tau(\bar{h}_0^N, 0) = \chi\psi_0^{GP}(1+\pi_0)\widehat{\pi}_0, \quad \text{with } \psi_0^{GP} = \frac{\psi_0^{NE}}{1+(\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\Delta} \tag{35}$$

(see Appendix A.8 for details.) Equation (35) shows that, independently of the shocks, whether the planner puts more weight on inflation than individual central banks depends on the product of two sufficient statistics, the difference in labor intensities, $\alpha^N - \alpha^T$ and the response of the trade balance to an expansionary policy, i.e. the sign of $\sigma_0 - \widetilde{\sigma}$. To understand the intuition, consider the case in which the economy is in a recession. When the global planner increases output, it internalizes that this has effects on the world real rate and in turn, the world real rate alters the output-inflation tradeoff faced by individual countries. In particular, if $\sigma_0 > \widetilde{\sigma}$, an expansionary policy that raises output also raises demand for global real assets and lowers the world real rate. Moreover, if $\alpha^N > \alpha^T$, a lower

[18] Equation (33) uses (6), (7), (8), (19) as in (27), with market clearing for global assets $\widehat{z}_0 = 0$.

world real rate helps countries in a recession. Therefore, the planner puts more weight on output and less on inflation.

Together (29) and (35) uniquely determine the level of employment. As in the Nash equilibrium, the sign of the output gap in the cooperative equilibrium is the opposite of the sign of the wage gap. However, the levels of employment may differ as it can be inferred from the targeting rule (35). The next Proposition compares the levels of employment in the Nash equilibrium and in the cooperation equilibrium.

Proposition 3. *The output gaps in the Nash equilibrium* $\widehat{h}_0^{NE}$ *and in the cooperative equilibrium* $\widehat{h}_0^{GP}$ *have the same sign. Moreover, we have that*

$$\widehat{h}_0^{GP} < \widehat{h}_0^{NE} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{w}_0 > 0 \tag{36}$$

Proof. In Appendix A.9 □

Illustration. Figure 2 illustrates the comparison between the outcomes in the Nash equilibrium outcome and the optimal monetary policy under cooperation under the case of $\alpha^N > \alpha^T$ and $\sigma_0 > \widetilde{\sigma}$. The figure presents the inflation-output tradeoff, represented by (28) and (35), respectively for the Nash equilibrium and the optimal cooperative monetary policy, and the level of inflation as a function of output, as represented by (29) (which is the same for both economies). The x-axis denotes the output gap and the y-axis denotes the inflation gap. Notice that the slope of the inflation-output curve for the planner is steeper and intersects with the Nash at the ideal point (0,0).

The figure displays three panels depending on the sign of the wage gap: zero wage gap (panel [a]), positive wage gap (panel [b]), and negative wage gap (panel [c]).[19] Panel (a) shows that when the Nash equilibrium features a zero wage gap, the allocations under cooperative and non-cooperative monetary policy coincide and equal the first-best allocation. The equilibrium, which occurs at the intersection of the two curves, goes through the ideal point (0,0). When the wage gap is positive (panel [b]), both economies feature overheating. However, because the planner puts more weight on output and less weight on inflation, the planner reduces the degree of overheating relative to the Nash equilibrium (and lowers inflation). Finally, when the wage gap is negative (panel [c]), the planner now chooses a higher level of output (and higher inflation) compared to the Nash equilibrium.

[19] Recall that the wage gap depends on parameters as given by (17).

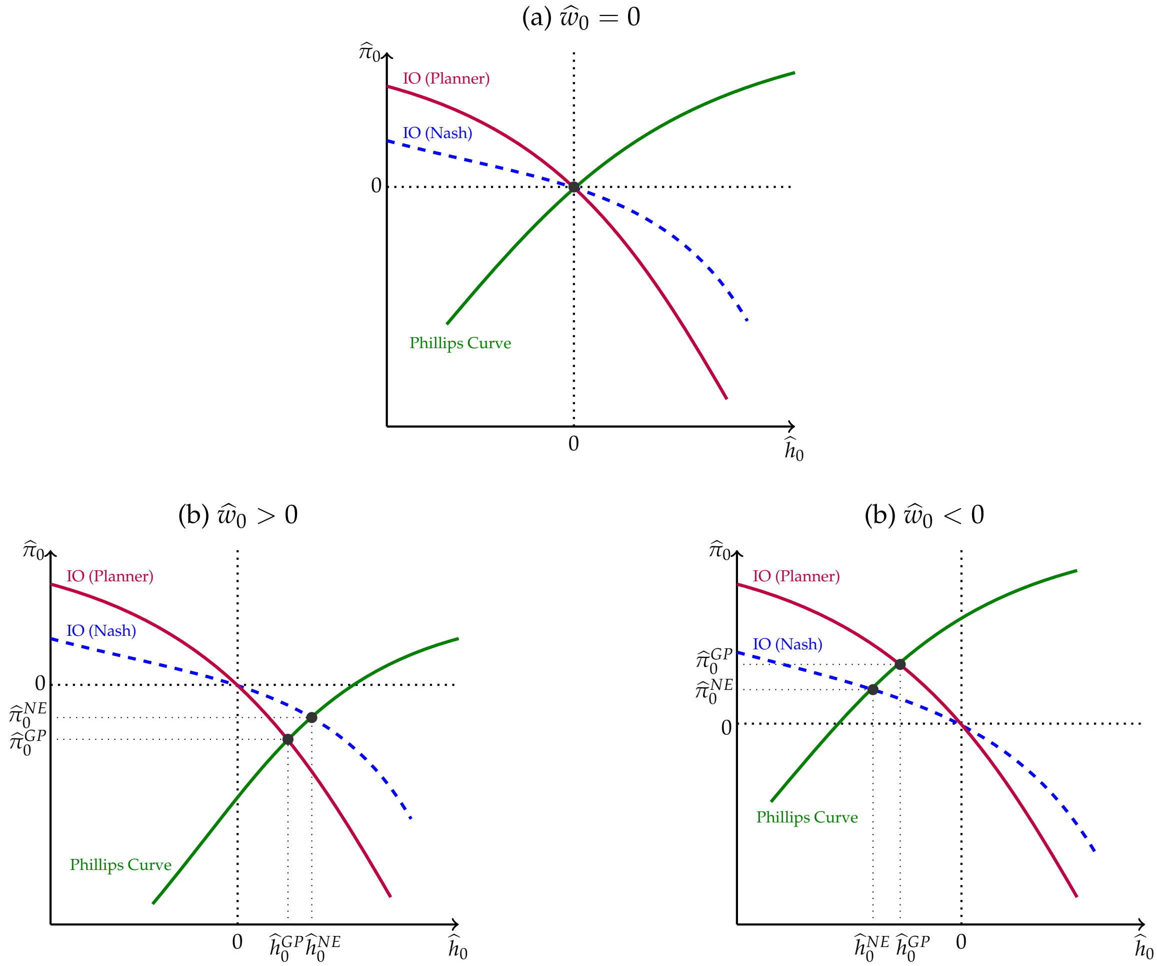

Figure 2: Nash equilibrium vs under cooperation for $\alpha^N > \alpha^T$ and $\sigma_0 > \widetilde{\sigma}$

Note: IO stands for Inflation-Output trade-off. IO (Nash) and IO (Planner) correspond respectively to (28) and (35). Phillips curve corresponds to (29).

In the next section, we examine the implication of these differences in the targeting rules for the policy rates.

4.3 Over-tightening or Under-tightening?

We now turn to comparing the choice of nominal interest rates in the Nash equilibrium and in the cooperative equilibrium. The following proposition summarizes our results

Proposition 4 (Sufficient statistics)**.** *Denote h_0^N the output gap in the Nash equilibrium. Then, the Nash equilibrium displays under-tightening $R_0^{NE} < R_0^{GP}$ if and only if $(\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^N > 0$.*

Proof. In Appendix A.10 □

The result of the proposition can be understood by tracing the difference in the targeting rules highlighted above. Namely, depending on the sign of the differences $\alpha^N - \alpha^T$, and $\sigma_0 - \widetilde{\sigma}$, the planner puts more or less weight on inflation relative to the Nash equilibrium. In the case where both of these differences are positive, the planner puts more weight on output. As Proposition 4 shows, this means that if the economy faces overheating $\widehat{h}_0^N > 0$, the planner keeps nominal rates higher compared to the Nash equilibrium. Conversely, if the economy faces a recession $\widehat{h}_0^N < 0$, the planner keeps nominal rates lower compared to the Nash equilibrium. That is, the extent of over-tightening or under-tightening is state-dependent.

In the case where $\alpha^N < \alpha^T$, in which case $\sigma_0 - \widetilde{\sigma} > 0$, the state dependence is reversed. That is, facing overheating, the planner reduces the interest rate relative to the Nash equilibrium whereas when facing a recession, the planner raises the interest rate relative to the Nash equilibrium.

The result in Proposition 4 generalizes and clarifies results in the literature. In particular, Fornaro and Romei (2022) considers $\alpha^N = 1, \sigma_0 = 1$ and $\widehat{h}_0^N \leq 0$. Thus countries benefit from lower world real rates and monetary policy expansions lower world real rates. This implies over-tightening. In Bianchi and Coulibaly (2021) $\alpha^T = 0$, $\chi = 0$ and countries benefit from higher world real rates as this reduces their vulnerability to a liquidity trap. If the intertemporal elasticity of substitution is lower than the intratemporal elasticity, then lower nominal rates lower real rates and this reduces welfare.

4.4 The Need for Cooperation

In this section, we delve into the reasons why cooperation is necessary by inspecting how individual countries would unilaterally deviate from the coordinated solution.

After linearizing the equilibrium conditions for a single small open economy, we obtain the following system:[20]

$$\widehat{z} = a_1 \left[-(\sigma - \widetilde{\sigma})\, \widehat{R} + \sigma\, \widehat{R}^* \right] \tag{MP}$$

$$\widehat{\pi} = \widehat{w} + \sum_{i=T,N} (1-\alpha^i)\phi^i\, \widehat{h}^N + \widehat{z}\left(\widehat{h}^N, \widehat{R}^*\right) \tag{AS}$$

[20](MP) combines linearized (7) and (6), and (AS) combines linearized (29) and (19). Moreover, $a_1 \equiv \sum_{i=T,N} \alpha^i \phi^i \left[(\delta - \alpha^T)\left(\delta + (\sigma-1)\sum_{i=T,N} \alpha^i \phi^i\right)\right]^{-1} > 0$ and $a_2 \equiv \left[\delta + (\sigma-1)\left(\phi^T + \alpha^N \phi^N\right)\right]^{-1} > 0$.

where $\widehat{z}(\widehat{h}^N, \widehat{R}^*)$, which follows from the linearization of the Euler equation (6), satisfies

$$\widehat{z}\left(\widehat{h}^N, \widehat{R}^*\right) = a_2\left[(\sigma - \widetilde{\sigma}) \sum_{i=T,N} \alpha^i \phi^i \, \widehat{h}^N + \widehat{R}^*\right] \tag{37}$$

In the cooperation solution, the global planner internalizes the general equilibrium effect of individual central banks' decision, that is the equilibrium conditions under cooperation are described by (MP) and (AS) with $\widehat{z} = 0$. By (37), and (AS), the aggregate supply faced by central banks can be steeper or flatter than the aggregate supply (AS) curve faced by the global planner depending on the sign of $\sigma - \widetilde{\sigma}$. Moreover, the world real rate plays the role of a shifter of the (AS) curve faced by central banks.

The indifference curve represents the combinations of $\widehat{h}^N$ and $\widehat{\pi}$ that will leave the small open economy with the same welfare loss

$$\mathbb{L} \equiv \frac{1}{2}\left\{ \frac{\kappa \bar{h}^N}{\alpha^N \phi^N}\left[1 + (\sigma - 1) \sum_{i=T,N} \alpha^i \phi^i\right] \sum_{i=T,N} \alpha^i \phi^i \left(\widehat{h}^N\right)^2 + \chi \left(\widehat{\pi}\right)^2 + \left(\delta - \phi^T + \sigma \phi^T\right) \phi^T \left(\widehat{z}\right)^2 \right\} \tag{38}$$

where the welfare-based loss function (38) corresponds to the second-order approximation of the difference between households' welfare in the efficient allocation and households' welfare given current allocation.

Figure 3 presents a graphical illustration, focusing on the case where $\sigma < \widetilde{\sigma}$, $\alpha^N > \alpha^T$ and a W such that $\widehat{h}^N < 0$ and $\widehat{\pi} > 0$. The blue-dashed lines and the red lines describe respectively the equilibrium relationships under cooperation and the Nash equilibrium. In addition, the outcomes are represented by points G and E. Starting from the cooperative equilibrium, the green-solid lines describe the benefit from a unilateral deviation by an individual central bank. The figure illustrates a situation where in the cooperative equilibrium, the planner picks R_{GP}, the economy is in a recession and inflation is above target, as illustrated in point G.

From an individual central bank's perspective, given $R^*_{0,GP}$, a decrease in the nominal lowers the trade surplus, as illustrated in panel (a). In addition, the decrease in the interest rate stimulates demand and raises output, as illustrated in the shift downward in the EE curve in panel (c). Furthermore, the trade deficit implies that now less employment is allocated in the tradable sector, as can be seen from the shift downward in the (TN) curve in panel (d). Because tradables are less labor intensive, this means that for given output, inflation decreases. As a result, the individual central bank would deviate towards a point E' where it faces a lower output gap compared to the cooperative outcome and lower

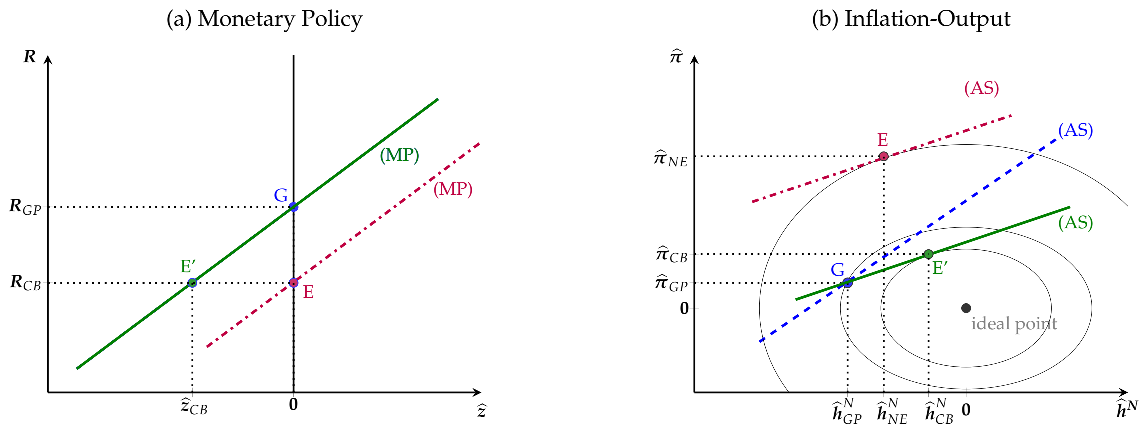

Figure 3: Under-tightening for $\sigma < \widetilde{\sigma}$ and $\alpha^N > \alpha^T$

inflation.

In general equilibrium, however, the trade balance must be zero. That is, the economy must be on a vertical line in panels (a) and (c) that go through zero. As countries raise their nominal rate and countries seek to run trade deficits, the world real rate increases in general equilibrium. This means that now there would be a reduction in demand that would push output away from the efficient level. This is represented in point E, which drifts away from the ideal point (0,0).

4.5 Anticipated Shocks: A Case of Prudential Undertightening

Until now, we considered an economy that faces a sudden shock that creates an output-inflation tradeoff at $t = 0$. In this environment, under optimal monetary policy, the economy features either a positive output gap and low inflation relative to the target or a negative output gap and high inflation relative to the target. However, the economy cannot feature both inflation and output above or below target under the optimal policy.[21]

We now extend the environment to allow for anticipated shock. We show how this can give rise to a situation where a positive output gap coexists with high inflation relative to the target in the Nash equilibrium and show that there are analogous implications for monetary policy cooperation.

We assume that the economy is initially at period $t = -1$. The wage is rigid at a value W such that $W = W^n_{-1}$. We assume that agents suddenly anticipate a shock to the economy at period $t = 0$.

[21]This is a feature that is common with standard New Keynesian models.

Let us start with the analysis of the non-cooperative solution. The problem the central bank faces at $t = -1$ is analogous to the one described in (22) with the difference that now the continuation value is not the one associated with the flexible wage allocation. The individual central bank at period can still achieve the efficient allocation at $t = -1$, given that the shock will hit at $t = 0$. However, the central bank perceives that by changing its net foreign asset position, it will improve the output-inflation tradeoff at $t = 0$ when the shock hits. In particular, the central bank wants to run a trade deficit at $t = 0$ when a trade surplus at $t = 0$ would help improve domestic policy tradeoff, which by Lemma 4, may require a monetary expansion or contraction depending on the sign of $\sigma_{-1} - \widetilde{\sigma}$. More formally, the output gap in the Nash equilibrium satisfies

$$-\left[\tau\left(h^N_{-1}, 0\right) - \tau\left(\bar{h}^N_{-1}, 0\right)\right] + \chi\psi^{NE}(1 + \pi_{-1})\widehat{\pi}_{-1} = \beta\Theta\left(\sigma_{-1} - \widetilde{\sigma}\right) u_T(c^T_0, c^N_0)\mu_0, \tag{39}$$

with $\Theta > 0$, and where μ_0 satisfies (32) and the inflation rate satisfies

$$\frac{\widehat{\pi}_{-1}}{1 + \overline{\pi}_{-1}} = \left(1 + \widehat{h}^N_{-1}\right)^{\sum_i (1-\alpha^i)\phi^i} - 1. \tag{40}$$

Under the assumption that $\alpha^N > \alpha^T$, the central bank would try to boost its net foreign asset position if the shock tomorrow leads to a recession (and reduce its net foreign asset position if the shock tomorrow leads to overheating). In turn, to the extent that $\sigma_{-1} > \widetilde{\sigma}$, the central bank would cut the nominal rate if the shock tomorrow will lead to a recession (and increase the nominal interest rate if the shock tomorrow will lead to overheating).

On the other hand, the anticipation of the shock has no effect on the optimal monetary policy under cooperation at period $t = -1$. That is, the planner sets the nominal rate to achieve the efficient allocation. Intuitively, the desire to accumulate net foreign asset position for individual countries is a zero-sum game. When central banks depart from the efficient allocation at $t = -1$, they end up worsening the allocation without any future gains. These findings are summarized in the next proposition.

Proposition 5. *Consider $\widehat{w}_{-1} = 0$. Then,*

1. *the optimal monetary policy under cooperation features $\widehat{h}^N_{-1} = \widehat{\pi}_{-1} = 0$*
2. *the Nash equilibrium features*

   i) $\widehat{h}^N_{-1} > 0$ and $\widehat{\pi}_{-1} > 0$ if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}^N_0 > 0$

   ii) $\widehat{h}^N_{-1} < 0$ and $\widehat{\pi}_{-1} < 0$ if $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}^N_0 < 0$

Proof. In Appendix A.11 □

A feature of our environment with anticipated shocks is that countries can experience both overheating labor markets and high inflation. This is an interesting feature because this is hard to rationalize within most New Keynesian models where the central bank faces unemployment and high inflation or overheating and low inflation. Another implication of this Proposition is presented in the Corollary below.

Corollary 1 (Prudential under-tightening)**.** *Suppose countries anticipate a recession. Then, the Nash equilibrium displays under-tightening if and only if* $(\alpha^N - \alpha^T)(\sigma_{-1} - \widetilde{\sigma})\widehat{h}^N_{-1} > 0$*.*

Proof. In Appendix A.12 □

Or sufficient statics therefore remain valid in the presence of anticipated shocks. That is, the extent to which there is over or under-tightening depends on the product of the difference in labor intensity $\alpha^N - \alpha^T$ and the difference between the intertemporal elasticity of substitution, and the sign of the output gap.

The inefficiency of the non-cooperative outcome can be referred to as a problem of "prudential under-tightening". That is, by attempting to increase the future net foreign asset position, with a prudential goal, central banks will conduct a monetary policy that inefficiently boosts output when there is an expected recession (and inefficiently depresses output when there is an expectation of overheating).

4.6 Quantitative Gains from Monetary Policy Coordination

We evaluate in this section the quantitative gains from monetary policy coordination. We calibrate the economy using standard parameters from the literature. Households' utility function has the constant relative risk-aversion form, $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$.

The time period is a year. Following Schmitt-Grohé and Uribe (2016), we set the inverse of inter-temporal elasticity of consumption to $\sigma = 5$, the labor intensity in the non-tradable sector to $\alpha^N = 0.75$, and the weight on tradable consumption in the CES function $\phi^T = 0.26$. The labor intensity in the tradable sector is set to ensure an aggregate labor share of 2/3, which implies $\alpha^T = 0.43$.[22] The discount factor β is set to 0.96 which ensures a steady state value of 4 percent for the risk-free world real interest rate. We set the parameter governing the inflation cost χ to 10. With this value, we obtain using a second-order approximation

[22]The aggregate labor share is given by $\frac{W_t h_t^T + W_t h_t^N}{P_t^T y_t^T + P_t^N y_t^N} = \alpha^T \phi^T + \alpha^N \phi^N$.

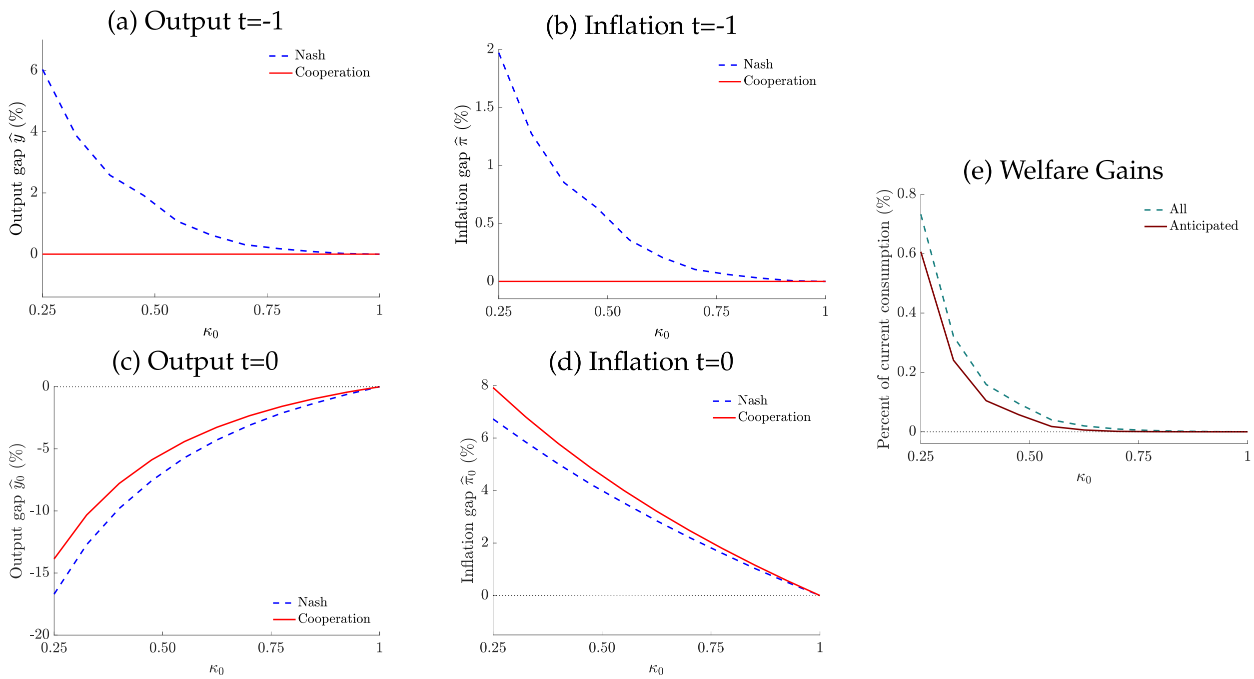

Figure 4: Cooperation versus Nash Equilibrium

Note: The parameter values are $\sigma = 5$, $\alpha^N = 0.75$, $\alpha^T = 0.43$, $\phi^T = 0.26$, $\beta = 0.96$, $\chi = 10$.

of the objective function, an effective weight on the output and inflation of 20% and 80%, which are in the range of those used in standard New Keynesian models.[23]

The experiment we consider is a labor disutility shock κ_0, which is anticipated at $t = -1$. Figure 4 plots the output gap and the inflation rate under cooperation and in the Nash equilibrium in periods $t = -1$ and $t = 0$, as well as the welfare gains.

At date $t = 0$, when the shock hits, the prevailing sticky wage is higher than the natural wage, and output is inefficiently low.[24] Central banks respond by loosening monetary policy in order to bring output closer to its efficient level. Owing, however, to the perceived private benefit from running a trade deficit, individual central banks over-tighten relative to the cooperative solution as shown in panels (c) and (d) of Figure 4. As a result, countries face a deeper recession in the Nash equilibrium (-8% versus -6% under cooperation for $\kappa_0 = 0.5 < \bar{\kappa}$) and a lower inflation rate (3.5% versus 4.5% under cooperation) as shown in panels (a) and (b) of Figure 4.

[23]The second-order approximation of the households' welfare in a symmetric equilibrium expressed in deviation from their welfare in the efficient allocation is given by

[24]This is because, under flexible wage allocation, the decrease in households' preference toward leisure leads to an increase labor supply and put upward pressure on the real wage. Under a policy that stabilizes prices, the nominal wage rises.

In an effort to mitigate the severity of the anticipated recession, central banks cut the nominal rate at $t = -1$ and overheat the economy with an output gap of about 1.8% for $\kappa_0 = 0.5 < \bar{\kappa}$ (see panels [a] and [b]), and the inflation rate rises to reach 0.5%. A global planner internalizes the effect of a prudential monetary policy expansion on the world real rate and focuses squarely on closing the output gap and the inflation gap. As a result, the Nash equilibrium features under-tightening (see panels [c] and [d]).

Finally, we assess the welfare implications of the lack of cooperation in panel (e). The welfare gain from cooperation is calculated as the compensating consumption variation at date $t = -1$ that equalizes the welfare of a household in the Nash equilibrium and the utility of a household under the cooperative monetary policy. This panel shows a significant welfare gain from cooperation.

5 Extensions

In this section, we extend our baseline model and show how our results can be generalized.

5.1 Consumption Bundle

In our baseline analysis, we consider a unitary elasticity of substitution between tradables and non-tradables. We now generalize the composite consumption to allow for a CES aggregator with elasticity $1/\gamma$. The elasticity of substitution between goods plays a crucial role in determining the response of the trade balance to a monetary expansion. In our case with multi-sector production, the Marshall-Lerner condition can be further generalized such that the trade balance increases in response to a monetary expansion if and only if $\sigma_0 > \gamma\widetilde{\sigma}$. That is, the lower is the elasticity of substitution across goods, the lower is the elasticity of intertemporal substitution that delivers an increase in the trade balance in response to a monetary expansion. Intuitively, as the intra-temporal elasticity of substitution increases, a depreciation leads to larger expenditure switching from tradables towards non-tradables. Consuming less tradables therefore implies that more tradable output can be exported and the trade balance increases.

5.2 Imperfect Labor Mobility

We first relax the assumption of perfect labor mobility. We assume that aggregate hours worked is a composite of hours worked in the tradable sector and in the non-tradable sector according to the following CES aggregator:

$$n_t = \left[\frac{1}{2}\left(n_t^T\right)^{1+\frac{1}{\eta}} + \frac{1}{2}\left(n_t^N\right)^{1+\frac{1}{\eta}}\right]^{\frac{\eta}{\eta+1}}, \tag{41}$$

where $\eta \geq 0$ measures the degree of labor mobility, that is how easy it is for a household to substitute hours worked in the tradable sector for hours worked in the non-tradable sector. When $\eta \to \infty$, there is perfect labor mobility and the aggregate hours worked reduce to $2n_t = n_t^T + n_t^N$ as in Section 2. For $\eta = 0$ labor is perfectly immobile across sectors.

Given that hours worked in the tradable sector and in the non-tradable sector are not perfect substitutes, the nominal need not be equal across the two sectors. We denote by W^N and W^T the prevailing wage at date $t = 0$ in the tradable and the non-tradable sector, respectively. The ratio of hours in a small open economy is given by

$$\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N}\frac{\alpha^N \phi^N}{\alpha^T \phi^T}\left(1 - \widehat{z}_0\right),$$

from which, it follows that, in any symmetric competitive equilibrium, the output gaps in the two sectors are proportional. For simplicity, we assume that parameters are time-invariant for $t \geq 1$. The optimal targeting rule under cooperation continues to be given by (28) and (35), but now the relative weights on the output gap satisfy

$$\frac{\psi_0^{NE}}{\psi_0^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\,\Delta(\eta), \quad \text{with} \quad \Delta(\eta) > 0 \text{ and } \Delta'(\eta) > 0. \tag{42}$$

Equation (42) shows that relative to individual central banks in the Nash equilibrium, the global planner put less weight puts less weight on closing the inflation gap (and therefore more weight on closing output gaps) if and only if the product of two sufficient statistics $\alpha^N - \alpha^T$ and $\sigma_0 - \widetilde{\sigma}$ is positive. In other words, Proposition 4 continues to hold.

Moreover, when labor becomes less mobile across sectors (η falls), central banks in a Nash equilibrium perceive changes in their net asset position to improve current policy tradeoffs would also induce costly reallocation of hours at dates $t \geq 1$ when wages are flexible as captured by $\Delta'(\eta) > 0$.

5.3 Oil shocks

Our baseline model assumes that labor is the sole factor of production. In this section, we incorporate oil as intermediate input and show how our results extend to this case. We assume that households in each country are endowed with M_t units of oil which are used as intermediate inputs for production and can be exchanged with the rest of the world. The endowment M_t is potentially time-varying and thus can give rise to "oil shocks."[25] The production functions, now given by $F^i(h_t^i, m_t^i, A_t^i)$, are differentiable, strictly increasing, concave, isoelastic with *intensity* parameters

$$\alpha^i \equiv \frac{d \log F^i(h_t^i, m_t^i, A_t^i)}{d \log h_t^i} \quad \text{and} \quad \alpha_m^i \equiv \frac{d \log F^i(h_t^i, m_t^i, A_t^i)}{d \log m_t^i}$$

The aggregate demand for oil in the domestic country is $m_t = m_t^T + m_t^N$. The law of one price is assumed to hold in the market for oil, that is $P_{mt} = e_t P_{mt}^*$ where P_{mt} and P_{mt}^* are the domestic and the world price of oil, respectively. The optimal policy problem of individual central banks in the Nash equilibrium and the problem under cooperation are presented in Appendix B.2. The optimal targeting rules are still given by (28) and (35) but now the relative weights on inflation are given by

$$\frac{\psi_0^{NE}}{\psi_0^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\, \Delta_m, \quad \text{with} \quad \Delta_m > 0.$$

The difference in labor intensity across sectors remains as in the baseline a key sufficient statistic. On the other hand, the difference in the intensity of oil in production across sectors is irrelevant to whether central banks over- or under-tighten in the Nash equilibrium. The takeaway is that the relevant factor intensity is the one corresponding to the sticky price factor.

6 Conclusion

We presented a simple general theory of monetary policy coordination under financial integration. Instead of terms of trade externalities as in the classic approach, we emphasize a pecuniary externality operating through the global capital market. Individual countries do not internalize how their monetary policy decisions affect the world real interest rate and alter the ability of foreign central banks to stabilize output and inflation.

We identify three sufficient statistics that determine whether the Nash equilibrium exhibits over or under-tightening. In particular, under the assumption that non-tradables

[25] Recent work by Auclert, Monnery, Rognlie and Straub (2023) focus on an energy price shock and show that from the perspective of the oil importer countries, coordinating on a tighter monetary policy is desirable to reduce import prices. These terms of trade manipulation motives are absent in our setup.

are more labor intensive, we find that when the economy faces a recession, the Nash equilibrium displays over-tightening (under-tightening) relative to the cooperative outcome if a monetary expansion generates a positive (negative) response of the trade balance. Conversely, when the economy faces overheating, the Nash equilibrium displays over-tightening (under-tightening) relative to the cooperative outcome if a monetary expansion generates a negative (positive) response of the trade balance. The characterization is independent of the specific shocks driving the economy and provides general guidelines for concrete policy discussions on monetary policy coordination.

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APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (14) and (15) and the specification of the utility function. □

A.2 Proof of Lemma 2

The mix of hours coincides with the competitive equilibrium. This follows from combining the optimality conditions of firms (8) and (9), with the optimality condition of households (4) to obtain

$$\frac{h_t^N}{h_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T}(1 - \widehat{z}_t), \tag{A.1}$$

and from the fact that $\widehat{z}_t = 0$ in a symmetric competitive equilibrium. Combining (A.1) with $\widehat{z}_t = 0$ and (16), we obtain

$$\frac{h_t^N}{\bar{h}_t^N} = \frac{h_t^T}{\bar{h}_t^T}$$

Rearranging this equation and using $1 + \widehat{h}_t^i = \frac{h_t^i}{\bar{h}_t^i}$, we arrive at $\widehat{h}_t^N = \widehat{h}_t^T$. □

A.3 Proof of Lemma 3

We combine (2) with (9) and (8) to get

$$P_t = W_t \left[\prod_{i=T,N} \left(F_h \left(h_0^i, A_0^i \right) \right)^{-\phi^i} \right]$$

Assume flexible wage $h_t^i = \bar{h}_t^i$ and zero inflation gap $\frac{P_t}{P_{t-1}} - 1 = \bar{\pi}_t$, then we get

$$(1 + \bar{\pi}_t) P_{-1} = W_t^n \left[\prod_{i=T,N} \left(F_h \left(\bar{h}_0^i, A_0^i \right) \right)^{-\phi^i} \right] \tag{A.2}$$

Rearranging (A.2) we obtain (17). □

A.4 Proof of Proposition 1

The first-order conditions of the central bank's problem (22) with respect to h_0^N and h_0^T are respectively given by

$$F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N) - \kappa_0 = \frac{\phi^N}{h_0^N}(1-\alpha^N)\chi(1+\widehat{\pi}_0)\widehat{\pi}_0 + \left[1-\alpha^N+\alpha^N\frac{(\delta_0-1)+\sigma_0}{\delta_0-\phi^T+\sigma_0\phi^T}\right]\frac{\eta_0}{h_0^N} \quad \text{(A.3)}$$

$$F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N) - \kappa_0 = \frac{\phi^T}{h_0^T}(1-\alpha^T)\chi(1+\widehat{\pi}_0)\widehat{\pi}_0 - \left[1-\alpha^T+\alpha^T\frac{(\delta_0-1)(1-\widehat{z}_0)^{-1}}{\delta_0-\phi^T+\sigma_0\phi^T}\right]\frac{\eta_0}{h_0^T} \quad \text{(A.4)}$$

where $\delta_0 > 1$ is given by

$$\delta_0 \equiv 1 + R_0^* c_0^T \left[u_{T,1}\frac{-du_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)}{db_1^*}\right] \quad \text{(A.5)}$$

Next, note by (19) that $F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N) = F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N)$. We then combine both first-order conditions to obtain

$$\left[\sum_{i\in T,N}\delta_0^i\alpha^i\phi^i - (1-\alpha^T)\alpha^N\phi^N\widehat{z}_0\right]\left[F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N) - \kappa_0\right]$$
$$= \left[\sum_{i=T,N}\delta_0^i(1-\alpha^i)\phi^i + \frac{\alpha^T(1-\alpha^N)(\delta_0-1)\phi^N}{\delta_0-\phi^T+\sigma_0\phi^T}\frac{\widehat{z}_0}{1-\widehat{z}_0}\right]\frac{\alpha^N\phi^N}{h_0^N}\chi(1+\pi_0)\widehat{\pi}_0 \quad \text{(A.6)}$$

where

$$\delta_0^T \equiv 1 + \frac{(\sigma_0-1)\alpha^N\phi^N}{\delta_0+(\sigma_0-1)\phi^T} \quad \text{(A.7)}$$

$$\delta_0^N \equiv 1 - \alpha^T\frac{1+(\sigma_0-1)\phi^T}{\delta_0+(\sigma_0-1)\phi^T} \quad \text{(A.8)}$$

$\delta_0^T > 0$ and $\delta_0^N > 0$ follows directly from $\delta_0 > 1$. Equation (A.6) can be rewritten as

$$\sum_{i=T,N}\delta_0^i\alpha^i\phi^i\left[F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N) - \kappa_0\right] = \frac{\alpha^N\phi^N}{h_0^N}(1+\psi_z\widehat{z}_0)\sum_{i=T,N}\delta_0^i(1-\alpha^i)\phi^i\chi(1+\pi_0)\widehat{\pi}_0$$

which corresponds to (24) and where

$$\psi_z \equiv \frac{\psi_1+\psi_2}{1-\psi_2\widehat{z}_0} \quad \text{with} \quad \psi_1 \equiv \frac{(1-\alpha^T)\alpha^N\phi^N}{\sum_i\delta_0^i\alpha^i\phi^i} \quad \text{and} \quad \psi_2 \equiv \frac{\alpha^T(1-\alpha^N)\phi^N}{\delta_0-\phi^T+\sigma_0\phi^T}\cdot\frac{\delta_0(1-\widehat{z}_0)^{-1}}{\sum_i\delta_0^i(1-\alpha^i)\phi^i} \qquad \square$$

A.5 Proof of Lemma 4

The proof follows directly from (27).

Derivation of (27). The change in $\widehat{z}_0$ induced by the change in R_0 can be decomposed as

$$\frac{d\widehat{z}_0}{dR_0} = \frac{d\widehat{z}_0}{dh_0^T}\frac{dh_0^T}{dR_0} \tag{A.9}$$

First, differentiating (6) which can be rewritten using market clearing conditions as

$$u_T\left((1-\widehat{z}_0)F^T\left(h_0^T, A_0^T\right), F^N\left(\frac{\alpha^N\phi^N}{\alpha^T\phi^T}(1-\widehat{z}_0)h_0^T, A_0^N\right)\right) = \beta R_0^* \frac{\kappa_1}{F_h\left(h_1^T(b_1^*), A_1^T\right)} \tag{A.10}$$

where it should be noted that b_1^* satisfies (21), we get

$$[\delta_0+(\sigma_0-1)(\phi^T+\alpha^N\phi^N)]d\widehat{z}_0 = [\alpha^T+(\sigma_0-1)(\alpha^T\phi^T+\alpha^N\phi^N)]dh_0^T \tag{A.11}$$

Next, we use (8) to express (7) as $R_0 = R_0^* \frac{W_1^n}{W}\frac{(h_0^T)^{1-\alpha^T}}{(h_1^T)^{1-\alpha^T}}$ and differentiate it to get

$$\frac{dR_0}{R_0} = (1-\alpha^T)\frac{dh_0^T}{h_0^T} + (\delta_0 - 1)d\widehat{z}_0 \tag{A.12}$$

Finally, we substitute (A.11) and (A.12) into (A.9) to obtain

$$\left(\delta_0 - \alpha^T\right)d\widehat{z}_0 = \frac{\alpha^T + (\sigma_0-1)\sum_{i\in\mathcal{S}}\alpha^i\phi^i}{\delta_0 + (\sigma_0-1)(\phi^T+\alpha^N\phi^N)}\frac{dR_0}{R_0}$$

which corresponds to (27). □

A.6 Proof of Proposition 2

We combine (28) and (29) to obtain that the level of employment in the Nash equilibrium, which we denote for convenience $h_{0,NE}^N$ satisfies $\mathcal{T}(h_{0,NE}^N;\widehat{w}_0) = 0$ with

$$\mathcal{T}(h_0^N;\widehat{w}_0) \equiv \left[\tau(h_0^N,0)-\tau(\bar{h}_0^N,0)\right] - \chi\psi_0^{NE}(1+\overline{\pi}_0)^2(1+\widehat{w}_0)\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{\sum_i(1-\alpha^i)\phi^i}\left[(1+\widehat{w}_0)\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{\sum_i(1-\alpha^i)\phi^i} - 1\right]$$

We have that

$$\frac{d\mathcal{T}(h_{0,NE}^N;\widehat{w}_0)}{dh_0^N} < 0 \tag{A.13}$$

and $\mathcal{T}(\bar{h}_0^N;\widehat{w}_0) = -\chi\psi_0^{NE}(1+\overline{\pi}_0)^2(1+\widehat{w}_0)\widehat{w}_0$. Therefore $h_{0,NE}^N = \bar{h}_0^N$, that is there is no output gap in the Nash equilibrium, if and only if $\widehat{w}_0 = 0$. Furthermore, if $\widehat{w}_0 < 0$ then

$\mathcal{T}(\bar{h}_0^N;\widehat{w}_0) > 0$ and by (A.13) $h_{0,NE}^N > \bar{h}_0^N$, that is $\widehat{h}_{0,NE}^N > 0$. By the same token, if $\widehat{w}_0 > 0$ then $\mathcal{T}(\bar{h}_0^N;\widehat{w}_0) < 0$ and by (A.13) $h_{0,NE}^N < \bar{h}_0^N$, that is $\widehat{h}_{0,NE}^N < 0$. The sign of the output gap is therefore the opposite of the sign of the wage gap in the Nash equilibrium. □

A.7 Derivation of (31) and (32)

The Lagrangian associated with the central bank's problem is as follows:

$$\begin{aligned} V_0 &= u\left((1-\widehat{z}_0)F^T(h_0^T,A_0^T),A_0^N F^N(h_0^N)\right)-\kappa_0(h_0^T+h_0^N)-\frac{\chi}{2}(\widehat{\pi}_0)^2+\beta V_1\left(R_0^* A_t^T F^T(h_0^T)\widehat{z}_0\right) \\ &+\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0}-\frac{W}{W_0^n}\left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T}\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N}+1\right]+\eta_0\left[(1-\widehat{z}_0)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N}-1\right] \\ &+\mu_0\left[u_T\left((1-\widehat{z}_0)F^T(h_0^T,A_0^T),A_0^N F^N(h_0^N)\right)-\beta R_0^* u_T\left(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*)\right)\right] \end{aligned}$$

Recalling that in the Nash equilibrium $\widehat{z}_0 = 0$, by the envelope theorem we have

$$\begin{aligned} \left.\frac{dV_0}{dR_0^*}\right|_{R_0^*=R_0^{*NE}} &= \beta\widehat{z}_0 F^T(h_0^T)\frac{dV_1(b_1^*)}{db_1^*}-\left[\frac{u_T(c_0^T,c_0^N)}{R_0^*}+\beta R_0^*\cdot\widehat{z}_0 F^T(h_0^T)\frac{du_T\left(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*)\right)}{db_1^*}\right]\mu_0 \\ &= -\frac{1}{R_0^*}u_T(c_0^T,c_0^N)\mu_0 \end{aligned} \tag{A.14}$$

Combining (A.3) and (A.4), we get

$$\eta_0 = \frac{\phi^T\phi^N}{\sum_i \delta_0^i\alpha^i\phi^i}(\alpha^N-\alpha^T)\chi(1+\pi_0)\widehat{\pi}_0 \tag{A.15}$$

Using the targeting rule (28) to express η_0 in (A.15) in terms of $\widehat{h}_0^N$ and substituting into (23) we arrive at (32). Finally, (31) is obtained by substituting (32) into (A.14). □

A.8 Derivation of (35)

Combining the first-order conditions of the global planning problem (34) with respect to h_0^N and h_0^T along with $F^T(h_0^T,A_0^T)u_T(c_0^T,c_0^N) = F^N(h_0^N,A_0^N)u_N(c_0^T,c_0^N)$ by (19) we arrive at

$$F_h^N(h_0^N,A_0^N)u_N(c_0^T,c_0^N)-\kappa_0 = \psi_0^{GP}\chi(1+\pi_0)\widehat{\pi}_0, \quad \text{with } \psi_0^{GP} \equiv \frac{\alpha^N\phi^N}{\kappa_0 h_0^N}\frac{\sum_i(1-\alpha^i)\phi^i}{\sum_i\alpha^i\phi^i} \tag{A.16}$$

We then take the ratio ψ_0^{NE}/ψ_0^{GP} where ψ_0^{NE} is defined in (28) to obtain the expression of ψ_0^{GP} in (35). □

A.9 Proof of Proposition 3

Proceeding similarly as in Appendix (A.6), we have the sign of $\widehat{h}_0^N$ in the cooperative solution is the opposite of $\widehat{w}_0$. By Proposition 2, the sign of the output gap is the same in the Nash and under cooperation. Let $h_{0,NE}^N$ and $h_{0,GP}^N$ denote the level of non-tradable employment in the Nash equilibrium and in the cooperative equilibrium, and let us define

$$\mathcal{T}(h_0^N;\psi_0) \equiv \left[\tau(h_0^N,0)-\tau(\bar{h}_0^N,0)\right]$$
$$-\chi\psi_0(1+\overline{\pi}_0)^2(1+\widehat{w}_0)\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{\sum_i(1-\alpha^i)\phi^i}\left[(1+\widehat{w}_0)\left(\frac{h_0^N}{\bar{h}_0^N}\right)^{\sum_i(1-\alpha^i)\phi^i}-1\right]$$

Notice that $\mathcal{T}(h_{0,NE}^N;\psi_0^{NE})=0$ and $\mathcal{T}(h_{0,GP}^N;\psi_0^{GP})=0$. Moreover, we have that

$$\mathcal{T}\left(h_{0,NE}^N,\psi_0^{GP}\right)=\underbrace{\mathcal{T}(h_{0,NE}^N,\psi_0^{NE})}_{=0}+\Delta(\sigma_0-\widetilde{\sigma})(\alpha^N-\alpha^T)\left[\tau(h_0^N,0)-\tau(\bar{h}_0^N,0)\right]\frac{\psi_0^{GP}}{\psi_0^{NE}}$$

which implies that $\mathcal{T}(h_{0,NE}^N,\psi_0^{GP})<0 \Leftrightarrow (\sigma_0-\widetilde{\sigma})(\alpha^N-\alpha^T)\widehat{h}_0^N>0$. In addition, we have $\frac{d\mathcal{T}(h,\psi)}{dh}<0$ by $\tau'(h,0)<0$. Therefore, $\mathcal{T}(h_{0,NE}^N,\psi_0^{GP})<0 \Longleftrightarrow h_{0,GP}^N<h_{0,NE}^N$ and thus

$$h_{0,GP}^N<h_{0,NE}^N \Longleftrightarrow (\alpha^N-\alpha^T)(\sigma_0-\widetilde{\sigma})\widehat{h}_{0,NE}^N>0. \tag{A.17}$$

A.10 Proof of Proposition 4

Using (2) and (4) and market clearing (10) and (13), we get in any symmetric equilibrium

$$\frac{P_1}{P_0}=\left(\frac{F^T(h_0^T,A_0^T)}{F^N(h_0^N,A_0^N)}\right)^{-\phi^N}\left(\frac{F^T(\bar{h}_1^T,A_1^T)}{F^N(\bar{h}_1^N,A_1^N)}\right)^{\phi^N}\frac{P_1^T}{P_0^T} \tag{A.18}$$

We next use $P_1=(1+\overline{\pi}_1)P_0$ since monetary policy stabilizes prices for $t\geq 1$ and then substitute (A.18), (A.10) and (19) with $\widehat{z}_0=0$ into (7) to arrive at

$$R_0=\frac{1+\overline{\pi}_1}{\beta\kappa_0}\left(\frac{A_0^T}{A_0^N}\frac{A_1^N}{A_1^T}\right)^{\phi^N}\left(\frac{h_0^N}{\bar{h}_1^N}\right)^{(\alpha^T-\alpha^N)\phi^N}F_h\left(\bar{h}_1^T,A_1^T\right)u_T\left(F^T\left(\frac{\alpha^T\phi^T}{\alpha^N\phi^N}h_0^N\right),F^N\left(h_0^N\right)\right)$$

Totally differentiating this equation we obtain

$$\frac{dR_0}{dh_0^N}=-\sigma_0\left[\alpha^T\phi^T+\alpha^N\phi^N\right]\frac{R_0}{h_0^N}<0 \tag{A.19}$$

from which it follows that $R_0^{GP}>R_0^{NE} \Longleftrightarrow h_0^{GP}<h_0^{NE}$. Combined with (A.17) we get $R_0^{GP}>R_0^{NE} \Longleftrightarrow (\alpha^N-\alpha^T)(\sigma_0-\widetilde{\sigma})\widehat{h}_0^{NE}>0$. □

A.11 Proof of Proposition 5

Given that the global planning problem is static, the solution to the problem date $t = -1$ (the targeting rule) is given by (A.16) where variables at $t=0$ are replaced with variables at $t=-1$. Combining this rule with the aggregate supply curve (40) we arrive at

$$\left[\tau\left(h_{-1}^N,0\right)-\tau\left(\bar{h}_{-1}^N,0\right)\right]-\chi\psi^{GP}(1+\overline{\pi}_0)^2\left(\frac{h_{-1}^N}{\bar{h}_{-1}^N}\right)^{\sum_i(1-\alpha^i)\phi^i}\left[\left(\frac{h_{-1}^N}{\bar{h}_{-1}^N}\right)^{\sum_i(1-\alpha^i)\phi^i}-1\right]=0$$

which implies that $h_{-1}^N = \bar{h}_{-1}^N$. Next, we turn to the solution under Nash. Combine (39) and (40), and use (32) to obtain

$$\mathcal{T}(h_{-1}^N) = -\beta\Theta\frac{\Delta}{\alpha^N}(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\left[\tau\left(h_0^N,0\right)-\tau\left(\bar{h}_0^N,0\right)\right]h_0^N \tag{A.20}$$

where

$$\mathcal{T}(h_{-1}^N) \equiv \left[\tau\left(h_{-1}^N,0\right)-\tau\left(\bar{h}_{-1}^N,0\right)\right]-\chi\psi^{GP}(1+\overline{\pi}_0)^2\left(\frac{h_{-1}^N}{\bar{h}_{-1}^N}\right)^{\sum_i(1-\alpha^i)\phi^i}\left[\left(\frac{h_{-1}^N}{\bar{h}_{-1}^N}\right)^{\sum_i(1-\alpha^i)\phi^i}-1\right]$$

The left-hand side of (A.20), that is $\mathcal{T}(h_{-1}^N)$, is decreasing in h_{-1}^N with $\mathcal{T}(\bar{h}_{-1}^N) = 0$. Therefore, $h_{-1}^N > \bar{h}_{-1}^N$ if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\widehat{h}_0^N > 0$. Moreover, for $h_{-1}^N > \bar{h}_{-1}^N$, we have by (40) that $\widehat{\pi}_{-1} > 0$. Conversely when $h_{-1}^N < \bar{h}_{-1}^N$ we have that $\widehat{\pi}_{-1} < 0$. □

A.12 Proof of Corollary 1

Suppose $\widehat{h}_0^N < 0$. Note that in cooperation solution features $\widehat{h}_{-1}^N = 0$. By (A.20) the Nash equilibrium coincides with the cooperation solution if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T) = 0$. Furthermore, the Nash equilibrium features under-tightening $\widehat{h}_{-1}^N > 0$ if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T) > 0$ or equivalently if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\widehat{h}_{-1}^N > 0$. □

B Proofs of Extensions

B.1 Consumption Bundle

This section extends the baseline model with CES aggregators. Households preferences are still described by (1) where the consumption good c_t is now a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a CES aggregator

$$c_t = \left[\sum_{i \in \mathcal{S}} \phi^i \left(c_t^i\right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

The budget constraint of households is identical to the one in the baseline model. The household's optimality condition with respect to c_t^T and c_t^N (4) is now given by

$$\frac{P_t^N}{P_t^T} = \frac{\phi^N}{\phi^T} \left(\frac{c_t^T}{c_t^N} \right)^{\gamma} \tag{A.21}$$

Using (A.21), we can express the share of expenditures in tradables $\tilde{\phi}_t^T \equiv P_t^T c_t^T / (P_t c_t)$ as $\tilde{\phi}_t^T = \phi^T (c_t^T / c_t)^{1-\gamma}$. and the share of expenditures in non-tradables is $\tilde{\phi}_t^N = 1 - \tilde{\phi}_t^T$. The remaining optimality conditions of the household's problem are given by (6), (7) (and (5) for $t > 0$) while for firms, (8), (9) continue to hold. Combining (A.21) with (8) and (9) condition we arrive at

$$\frac{h_t^N}{h_t^T} = \frac{\alpha^N \tilde{\phi}_t^N}{\alpha^T \tilde{\phi}_t^T} (1 - \widehat{z}_t) \tag{A.22}$$

While using (14) and (15), the optimal ratio of hours in the first-best allocation becomes

$$\frac{\bar{h}_t^T}{\bar{h}_t^N} = \frac{\alpha^T \tilde{\phi}_t^T}{\alpha^N \tilde{\phi}_t^N}$$

which corresponds to the employment ratio in a competitive symmetric equilibrium for any monetary policy. Therefore, in any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are proportional, and to a first-order

$$\widehat{h}_t^N = \Gamma \, \widehat{h}_t^T, \quad \text{where } \Gamma \equiv \frac{1 - \alpha^T + \alpha^T \gamma}{1 - \alpha^N + \alpha^N \gamma} > 0. \tag{A.23}$$

In the lemma below, we summarize the effects of monetary policy on the trade balance.

Lemma B.1 (Generalized Marshall-Lerner Condition)**.** *The response of the trade balance to a domestic monetary expansion satisfies* $\frac{d\widehat{z}_0}{dR_0} > 0 \iff \sigma_0 > \gamma \widetilde{\sigma}$ *where* $\widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T \phi^T + \Gamma \alpha^N \phi^N}$

Proof. Proceeding similarly as in Appendix A.5 by combining (6), (7), (8), (19) we arrive at

$$\left[\delta_0+(\sigma_0-1)(\alpha^T\phi^T+\Gamma\alpha^N\phi^N)\right](\delta_0-\alpha^T)d\widehat{z}_0 = -\left[\alpha^T+(\sigma_0-\gamma)(\alpha^T\phi^T+\Gamma\alpha^N\phi^N)\right]dR_0 \quad \text{(A.24)}$$

Thus $\frac{d\widehat{z}_0}{dR_0} > 0$ if and only if $\alpha^T+(\sigma_0-\gamma)(\alpha^T\phi^T+\Gamma\alpha^N\phi^N)>0$. Defining $\widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T\phi^T+\Gamma\alpha^N\phi^N}$, we obtain that $d\widehat{z}_0/dR_0 > 0$ if and only if $\sigma_0 > \gamma\widetilde{\sigma}$. □

Note that, given preferences, the consumer price index P_t now satisfies

$$P_t = \left[\sum_{i\in\mathcal{S}} (\phi^i)^{\frac{1}{\gamma}} (P_t^i)^{1-\frac{1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$$

Thus, using the definition of the natural wage we can express the inflation gap as

$$\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} = \frac{W}{W_0^n}\left[\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(h_t^i,A_t^i)\right)^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(\bar{h}_t^i,A_t^i)\right)^{\frac{\gamma}{1-\gamma}}}\right]^{\frac{\gamma}{\gamma-1}} - 1 \quad \text{(A.25)}$$

The Lagrangian associated with the central bank's problem can be written as follows

$$\begin{aligned} & u\left((1-\widehat{z}_0)A_0^TF^T(h_0^T), A_0^NF^N(h_0^N)\right) - \kappa_0(h_0^T+h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2 + \beta V_1\left(R_0^*A_t^TF^T(h_0^T)\widehat{z}_0\right) \\ & +\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(h_t^i,A_t^i)\right)^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(\bar{h}_t^i,A_t^i)\right)^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right] + \eta_0\left[(1-\widehat{z}_0)\frac{\alpha^N\tilde{\phi}_0^N}{\alpha^T\tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right] \\ & +\mu_0\left[u_T\left((1-\widehat{z}_0)A_0^TF^T(h_0^T), A_0^NF^N(h_0^N)\right) - \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)\right] \end{aligned} \quad \text{(A.26)}$$

The optimality condition for $\widehat{z}_0$ yields $\eta_0 = \left[\delta_0 + \left(\sigma_0\gamma^{-1}-1\right)\tilde{\phi}_0^T\right] u_T\left(c_0^T, c_0^N\right)\mu_0$ where δ_0 is given by (A.5). Using this equation and combining the first-order conditions for h_0^T and h_0^N, we obtain the following targeting rule in the Nash equilibrium (where $\widehat{z}_0 = 0$):

$$\tau(h_0^N,0) - \tau(\bar{h}_0^N,0) = \chi\psi_0^{NE}(1+\pi_0)\widehat{\pi}_0 \quad \text{with} \quad \psi_0^{NE} = \frac{\alpha^N\phi^N}{h_0^N}\frac{\sum_{i=T,N}\delta_0^i(1-\alpha^i)\tilde{\phi}_0^i}{\sum_{i=T,N}\delta_0^i\alpha^i\tilde{\phi}_0^i} \quad \text{(A.27)}$$

where as in (25) we have

$$\tau(h_0^N,0) \equiv F_h^N(h_0^N,A_0^N)\, u_N\left(F^T\left(\frac{\alpha^T\tilde{\phi}_0^T}{\alpha^N\tilde{\phi}_0^N}h_0^N, A_0^T\right), F^N\left(h_0^N, A_0^N\right)\right) \quad \text{(A.28)}$$

with $\tau'(h_0^N,0) < 0$ and $\tau(\bar{h}_0^N,0) = \kappa_0$, and where the parameters $\delta_0^T > 0$ and $\delta_0^N > 0$ are given by

$$\delta_0^T \equiv 1 + \alpha^N(\gamma - 1) + \frac{(\sigma_0 - \gamma)\alpha^N \tilde{\phi}_0^N}{\bar{\delta}_0 + (\sigma_0\gamma^{-1} - 1)\tilde{\phi}_0^T}$$
$$\delta_0^N \equiv 1 + \alpha^T(\gamma - 1) - \alpha^T \frac{\gamma + (\sigma_0 - \gamma)\tilde{\phi}_0^T}{\bar{\delta}_0 + (\sigma_0\gamma^{-1} - 1)\tilde{\phi}_0^T}$$

To see why $\delta_0^T > 0$ and $\delta_0^N > 0$, notice that for $\sigma > \gamma$ this is trivial. For $\sigma < \gamma$, it can be shown that δ_0^T and δ_0^N are increasing in γ, and we have $\lim_{\gamma \to 0} \delta_0^T > 0$ and $\lim_{\gamma \to 0} \delta_0^N > 0$.

Under cooperation, the Lagrangian associated with the global planning problem can be written as follows

$$u\left((1-\widehat{z}_0)A_0^T F^T(h_0^T), A_0^N F^N(h_0^N)\right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2$$
$$+\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{\sum_{i=T,N}\left(\phi^i\right)^{\frac{1}{\gamma}}\left(F_h(h_t^i, A_t^i)\right)^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}\left(\phi^i\right)^{\frac{1}{\gamma}}\left(F_h(\bar{h}_t^i, A_t^i)\right)^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right] + \eta_0\left[\frac{\alpha^N \tilde{\phi}_0^N}{\alpha^T \tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right]$$

The targeting rule, which combined the first-order condition, for h_0^T and h_0^N is given by

$$\tau(h_0^N, 0) - \tau(\bar{h}_0^N, 0) = \chi\psi_0^{GP}(1+\pi_0)\widehat{\pi}_0 \quad \text{with} \quad \psi_0^{GP} = \frac{\alpha^N \phi^N}{h_0^N}\frac{\sum_{i=T,N}\delta_x^i(1-\alpha^i)\tilde{\phi}_0^i}{\sum_{i=T,N}\delta_0^i \alpha^i \tilde{\phi}_0^i} \tag{A.29}$$

with $\delta_x^T = 1 + \alpha^N(\gamma - 1)$ and $\delta_x^N = 1 + \alpha^T(\gamma - 1)$. Taking the ratio between the relative weights in the targeting rules (A.27) and (A.29), we arrive at

$$\frac{\psi_0^{NE}}{\psi_0^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\,\Delta_m, \quad \text{with } \Delta_m \equiv \frac{\tilde{\phi}_0^T \tilde{\phi}_0^N}{\left(\delta_0 - \tilde{\phi}_0^T + \sigma_0\gamma^{-1}\tilde{\phi}_0^T\right)\sum_i \delta_0^i(1-\alpha^i)\tilde{\phi}_0^i} > 0$$

B.2 Oil Shock

This section extends the model to incorporate oil as an intermediate input. We assume that households receive an endowment of oil which is used by firms as inputs for production and can be exchanged with the rest of the world. The law of one price is assumed to hold in the market for oil, that is $P_{mt} = e_t P_{mt}^*$ where P_{mt} and P_{mt}^* are the domestic and the world price of oil, respectively. Combined with the law of one price for tradables, this implies that $\frac{P_{mt}}{P_t^T} = \frac{P_{mt}^*}{P_t^{T*}}$. We can thus express the budget constraint of the household as

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(n_t^T + n_t^N) + \varphi_t + P_{mt} m_t^S + P_{mt}(M_t - m_t^S) + b_t + P_t^T b_t^*$$

where m_t^S is the domestic supply of oil and $M_t - m_t^S$ is the net export of oil. The production functions are given by $F^i(h_t^i, m_t^i, A_t^i)$ with α^i, and α_m^i denoting the intensity of labor and oil respectively. At the optimum, the demand for labor is analogous to (8)-(9) and given by $P_t^i F_h(h_t^i, m_t^i, A_t^i) = W_t$ for all $i \in \mathcal{S}$; while their demand for oil is given by

$$m_t^T = \frac{\alpha_m^T}{\alpha^T}\frac{W_t}{P_{mt}}h_t^T, \quad m_t^N = \frac{\alpha_m^N}{\alpha^N}\frac{W_t}{P_{mt}}h_t^N \tag{A.30}$$

The Lemma below describes the allocation of oil in any symmetric competitive equilibrium.

Lemma B.2. *In any symmetric competitive equilibrium, the allocation of intermediate oil inputs is efficient and given by*

$$m_t^N = \frac{\alpha_m^N \phi^N}{\sum_{i=T,N}\alpha_m^i \phi^i}M_t, \quad m_t^T = \frac{\alpha_m^T \phi^T}{\sum_{i=T,N}\alpha_m^i \phi^i}M_t \tag{A.31}$$

Proof. The proof combines the ratio of the two equations in (A.30) with (19), together with $\widehat{z}_0$ and market clearing for oil $m_t^T + m_t^N = M_t$. □

Denoting by $\overline{m}_0^T$ and $\overline{m}_0^N$ the allocation in (A.31), the Lagrangian associated with the global planning problem can be expressed as

$$u\left(F^T(h_0^T, \overline{m}_0^T, A_0^T), F^N(h_0^N, \overline{m}_0^N, A_0^N)\right) - \kappa_0(h_0^T + h_0^N) - \frac{\chi}{2}(\widehat{\pi}_0)^2$$
$$+\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0} - \frac{W}{W_0^n}\left(\frac{F_h\left(\bar{h}_0^T, \overline{m}_0^T, A_0^T\right)}{F_h\left(h_0^T, \overline{m}_0^T, A_0^T\right)}\right)^{\phi^T}\left(\frac{F_h\left(\bar{h}_0^N, \overline{m}_0^N, A_0^N\right)}{F_h\left(h_0^N, \overline{m}_0^N, A_0^N\right)}\right)^{\phi^N} + 1\right] + \eta_0\left[\frac{\alpha^N \phi^N}{\alpha^T \phi^T}\frac{h_0^T}{h_0^N} - 1\right]$$

Notice that the allocation of oil is independent of policy. The targeting rule under cooperation, which combines the optimality condition for h_0^T and h_0^N, is therefore identical to (35) and given by

$$\tau\left(h_0^N, \overline{m}_0^N, 0\right) - \tau\left(\bar{h}_0^N, \overline{m}_0^N, 0\right) = \chi\psi_0^{GP}(1+\pi_0)\widehat{\pi}_0 \quad \text{with} \quad \psi_0^{GP} = \frac{\alpha^N\phi^N}{h_0^N}\frac{\sum_i(1-\alpha^i)\phi^i}{\sum_i \alpha^i \phi^i} \tag{A.32}$$

where

$$\tau\left(h_0^N, \overline{m}_0^N, 0\right) \equiv F_h^N(h_0^N, \overline{m}_0^N, A_0^N)\, u_N\left(F^T\left(\frac{\alpha^T\phi^T}{\alpha^N\phi^N}h_0^N, M_0 - \overline{m}_0^N, A_0^T\right), F^N\left(h_0^N, \overline{m}_0^N, A_0^N\right)\right)$$

We now turn to deriving the targeting rule in the Nash equilibrium. Combining (A.30) with (19), the allocation of oil input across sectors in a small open economy is given by

$$m_t^T(\widehat{z}_t) = \frac{\alpha_m^T\phi^T\left(1-\widehat{z}_t\right)}{\alpha_m^N\phi^N + \alpha_m^T\phi^T\left(1-\widehat{z}_t\right)}m_t^S \text{ and } m_t^N(\widehat{z}_t) = \frac{\alpha_m^N\phi^N}{\alpha_m^N\phi^N + \alpha_m^T\phi^T\left(1-\widehat{z}_t\right)}m_t^S \tag{A.33}$$

which can be used to express the Lagrangian associated with the central bank's problem as

$$
\begin{aligned}
& u\left((1-\widehat{z}_0)F^T(h_0^T, m^T(\widehat{z}_0), A_0^T), F^N(h_0^N, m^N(\widehat{z}_0), A_0^N)\right) \\
& -\kappa_0(h_0^T+h_0^N)-\frac{\chi}{2}(\widehat{\pi}_0)^2+\beta V_1\left(R_0^* F^T(h_0^T)\widehat{z}_0\right)+\eta_0\left[(1-\widehat{z}_0)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N}-1\right] \\
& +\vartheta_0\left[\frac{\widehat{\pi}_0}{1+\overline{\pi}_0}-\frac{W}{W_0^n}\left(\frac{F_h\left(\bar{h}_0^T, \overline{m}_0^T, A_0^T\right)}{F_h\left(h_0^T, m^T(\widehat{z}_0), A_0^T\right)}\right)^{\phi^T}\left(\frac{F_h\left(\bar{h}_0^N, \overline{m}_0^N, A_0^N\right)}{F_h\left(h_0^N, m^N(\widehat{z}_0), A_0^N\right)}\right)^{\phi^N}+1\right] \\
& +\mu_0\left[u_T\left((1-\widehat{z}_0)F^T(h_0^T, m^T(\widehat{z}_0), A_0^T), F^N(h_0^N, m^N(\widehat{z}_0), A_0^N)\right)-\beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)\right]
\end{aligned}
$$

The optimality condition with respect to $\widehat{z}_0$ is given by

$$\eta_0=\left[\delta_0^m+(\sigma_0-1)\phi^T\right]u_T(c_0^T, c_0^N)\mu_0 \tag{A.34}$$

where on the competitive equilibrium path, δ_0^m is given by

$$\delta_0^m=\delta_0+\frac{\alpha_m^T\phi^T\cdot\alpha_m^N\phi^N}{\alpha_m^T\phi^T+\alpha_m^N\phi^N}+\chi\left(\phi^T\frac{F_{hm}^T}{F_h^T}+\phi^N\frac{F_{hm}^N}{F_h^N}\right)(1+\pi_0)\widehat{\pi}_0 \tag{A.35}$$

Notice by (A.33) and (A.31) that in the Nash equilibrium where $\widehat{z}_0=0$, the allocation of oil is optimal. Moreover, the optimality condition for h_0^N and h_0^T are akin to (A.3) and (A.4) where δ_0 is replaced with δ_0^m. As a result, the targeting rule in the Nash equilibrium is

$$\tau(h_0^N, \overline{m}_0^N, 0)-\tau(\bar{h}_0^N, \overline{m}_0^N, 0)=\chi\psi_0^{GP}(1+\pi_0)\widehat{\pi}_0 \quad \text{with } \psi_0^{NE}=\frac{\alpha^N\phi^N}{h_0^N}\frac{\sum_i\delta_0^i(1-\alpha^i)\phi^i}{\sum_i\delta_0^i\alpha^i\phi^i} \tag{A.36}$$

where δ_0^T and δ_0^N satisfy (A.7) and (A.8) where δ_0 is replaced with δ_0^m. Taking the ratio of the relative weights on inflation in (A.32) and (A.36), we arrive at

$$\frac{\psi_0^{NE}}{\psi_0^{GP}}=1+(\alpha^N-\alpha^T)(\sigma_0-\widetilde{\sigma})\,\Delta_m, \quad \text{with} \quad \Delta_m\equiv\frac{\phi^T\phi^N}{(\delta_0^m-\phi^T+\sigma_0\phi^T)\sum_i\delta_0^i(1-\alpha^i)\phi^i}>0.$$