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Banks, Liquidity Management, and Monetary Policy*

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ABSTRACT

We develop a new framework to study the implementation of monetary policy through the banking system. Banks finance illiquid loans by issuing deposits. Deposit transfers across banks must be settled using central bank reserves. Transfers are random and therefore create liquidity risk, which in turn determines the supply of credit and the money multiplier. We study how different shocks to the banking system and monetary policy affect the economy by altering the trade-off between profiting from lending and incurring greater liquidity risk. We calibrate our model to study quantitatively why banks have recently increased their reserve holdings but have not expanded lending despite policy efforts. Our analysis underscores an important role of disruptions in interbank markets, followed by a persistent credit demand shock.

Keywords: Banks; Monetary policy; Liquidity; Capital requirements
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1 Introduction

The conduct of monetary policy around the world is changing. During the past five years, banking systems have experienced unprecedented financial losses and subsequent freezes in interbank markets. A major reduction in bank lending and a protracted recession followed. In response, central banks in developed economies have expanded their balance sheets in an open attempt to preserve financial stability and reinvigorate lending. However, in response to these unprecedented policy interventions, banks seem to have accumulated central bank reserves without renewing their lending activities as intended. Why? Can central banks do more to stimulate lending? These questions remain open. In the context of this policy debate, the role of banks in the transmission of monetary policy has taken center stage. However, few modern macroeconomic models take into account that monetary policy is implemented through the banking system, as it occurs in practice. Instead, most macroeconomic models assume that central banks directly control interest rates or monetary aggregates and abstract from how the transmission of monetary policy may depend on the conditions of banks. This paper presents a model that contributes towards filling this gap.

The Mechanism. The main building block of our model is a liquidity management problem. Liquidity management is recognized as one of the fundamental problems in banking and can be explained as follows. When a bank grants a loan, it simultaneously creates demand deposits—or credit lines. These deposits can be used by the borrower to perform transactions at any time. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, more lending relative to a given amount of central bank reserves increases a bank's liquidity risk. When deposits are transferred out of a bank, that bank must transfer reserves to other banks in order to settle transactions. Central bank reserves are critical to clear settlements because loans cannot be sold immediately. Thus, the lower a bank's reserve holdings of a bank, the more likely it is to be short of reserves in the future. This introduces liquidity risk because the bank must incur expensive borrowing from other banks—or the central bank's discount window—if it falls short of reserves. This friction—the liquidity mismatch—induces a trade-off between profiting from lending and incurring additional liquidity risks, which we call liquidity management trade-off. It is by having an impact on banks' liquidity management that monetary policy has real effects in the model.

Implementation of Monetary Policy. In the model, the central bank has access to various tools. A first set of instruments are reserve requirements, discount rates, and interest payments on reserves. This first set of instruments affects the demand for reserves by altering the relative return on reserves. A second set of instruments are open-market operations (OMO) and direct lending to banks. This second set of instruments alters the volume of reserves in the system. Both types of instruments carry real effects by tilting the liquidity management trade-off. Macroeconomic effects result from their indirect effect on aggregate lending and interest rates. However, as much as the central bank can influence bank decisions, shocks to the banking system may limit the ability of monetary policy to stabilize lending and output.

Model Features. We introduce this liquidity management problem into a dynamic stochastic general equilibrium model with rational, profit-maximizing banks. Banks are subject to random deposit transfers. Since loans are illiquid, banks use central bank reserves to settle deposit transfers. To accommodate their reserve surpluses or deficits, banks borrow or lend in an over-the-counter (OTC) interbank market. The central bank conducts OMO and sets corridor rates, which in turn affect liquidity risk management and the volume of lending.

Despite the richness of bank portfolio decisions, idiosyncratic withdrawal risk, and an OTC interbank market, we are able to reduce the state space into a single aggregate endogenous state: the aggregate value of bank equity. Moreover, the bank's problem satisfies portfolio separation. In turn, this allows us to analyze the liquidity management problem through a portfolio problem with non-linear returns that depend only on aggregate market conditions. These results make the analysis of the model transparent and amenable to various extensions and applications.[1]

Testable Implications. The model delivers a rich set of testable implications. For individual banks, it explains the behavior of their reserve, leverage, and dividend ratios. It also provides predictions for aggregate lending, interbank borrowing, equity, and excess reserves, as well as for the return on loans and the return on equity. The model also generates endogenous money multipliers and liquidity premia.

Quantitative Application. As an application of our model, we exploit the lessons derived from the theoretical framework to investigate qualitatively and quantitatively why banks are not lending despite all the policy efforts. Thanks to its testable implications, our model is able to contrast different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of the following:

Hypothesis 1 - Bank Equity Losses: Lack of lending responds to an optimal behavior by banks given the equity losses suffered in 2008.

Hypothesis 2 - Capital Requirements: The anticipation of higher capital requirements is leading banks to hold more reserves and simultaneously lend less.

Hypothesis 3 - Increased Precautionary Holdings of Reserves: Banks hold more reserves because they now face greater liquidity risk.

Hypothesis 4 - Interest on Excess Reserves: Interest payments on excess reserves has led banks to substitute reserves for loans.

Hypothesis 5 - Weak Demand: Banks face a weaker demand for loans. This hypothesis encompasses a direct shock to the demand for loans or a decline in the effective demand for loans that could follow from increases in credit risk.

To evaluate these hypotheses, we calibrate our model and simulate it with shocks associated with each hypothesis that we obtain from the data. We use the model's predictions to uncover

[1] Except for the bank's portfolio problem, the model can be solved analytically.

which shocks are quantitatively more relevant to explain why lending has declined while reserves have increased by several multiples. Our model suggests that a combination of two shocks best fits the data: In particular, the model points to an early disruption in the interbank market during the US financial crisis, followed by a substantial and persistent contraction in loan demand. The underlying intuition is simple. Shocks to loanable funds (Hypotheses 1–2) reduce the supply of loans, but through general equilibrium effects, loan rates rise and banks end up holding *less* liquid assets (i.e., reserves) as a fraction of total assets. Disruptions in interbank markets (Hypothesis 3) as well as interest payments on reserves (Hypothesis 4) increase the benefits of holding reserves relative to loans, and hence can explain the observed pattern in the data. Quantitatively, when we feed these shocks into the model, it successfully predicts the immediate effects that occurred during the crisis, but not the persistence. As Fed policies contributed to stabilize financial markets, the model suggests that it is credit demand shocks that account for the persistent decline in lending. We interpret these results as suggestive of a deeper economic phenomenon in which an initial contraction in the supply of loans eventually leads to a credit demand collapse.

Related Literature. A tradition in macroeconomics dating back to at least Bagehot (1873) stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework to study policy with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was abandoned from macroeconomics for many years. Until the Great Recession, the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.[2]

In the aftermath of the global financial crisis, numerous calls have been made for the development of macroeconomic models with an explicit role for banks.[3] Some early steps were taken by Gertler and Karadi (2011) and Curdia and Woodford (2009), who show how shocks that disrupt financial intermediation can have important effects on the real economy. Following these papers, a large literature has studied how various policies affect bank equity and macroeconomic outcomes. Our model also belongs to the banking channel view, but it emphasizes instead how monetary policy affects the tradeoff banks face in holding assets of different liquidity. In turn, this approach relates our model to classic models of bank liquidity management and monetary policy.[4] Our contribution to this literature is to bring the classic insights from the liquidity management literature into a modern, dynamic, general equilibrium model that can be used for policy analysis and the

[2] This was a natural simplification by the literature. In the United States, the behavior of banks did not seem to matter for monetary policy. In fact, the banking industry was among the most stable industries in terms of returns and the pass-through from policy tools to aggregate conditions had little variability.

[3] See, for example, Woodford (2010) and Mishkin (2011).

[4] Classic papers that study static liquidity management—also called reserve management—by individual banks are Poole (1968) and Frost (1971). Bernanke and Blinder (1988) present a reduced form model that blends reserve management with an IS-LM model. Many modern textbooks for practitioners that deal with liquidity management. For example, Saunders and Cornett (2010) and Duttweiler (2009) provide managerial and operations research perspectives. Many modern banking papers have focused on bank runs. See, for example, Diamond and Dybvig (1983), Allen and Gale (1998), Ennis and Keister (2009), or Holmstrom and Tirole (1998). Gertler and Kiyotaki (2013) is a recent paper that incorporates bank runs into a dynamic macroeconomic model.

study of banking crises.[5]

We share common elements with recent work by Brunnermeier and Sannikov (2012). Brunnermeier and Sannikov (2012) also introduce inside and outside money into a dynamic macro model. Their focus is on the real effects of monetary policy through the redistributive effects of inflation when there are nominal contracts. The use of reserves for precautionary motives also places our model close to Stein (2012) and Stein et al. (2013). Those papers study the effects of an increase in the supply of reserves given an exogenous demand for short-term liquid assets.

Our paper also builds on the search theoretic literature of monetary exchange (see the survey by Williamson and Wright, 2010). Williamson (2012) studies an environment in which assets of different maturity have different properties as mediums of exchange. Cavalcanti et al. (1999) provide a theoretical foundation to our setup because reserves there emerge as a disciplining device to sustain credit creation under moral hazard and to guarantee the circulation on deposits. Atkeson et al. (2012) present a model to study trading decisions of banks in an OTC market and draw implications for policy. Finally, Afonso and Lagos (2012) develop an OTC model of the federal funds market and use it to study the intraday evolution of the distribution of reserve balances and the dispersion in loan sizes and fed funds rates. Our market for reserves is a simplified version of that model.[6]

Organization. The paper is organized as follows. Section 2 presents the model and Section 3 provides theoretical results. Section 4 reports the calibration exercises. There, we study the steady state and policy functions under that calibration. In Section 5, we analyze the transitional dynamics generated after shocks associated with each hypothesis. In Section 6, we evaluate and discuss the plausibility of each hypothesis. Section 7 concludes. All proofs are in the appendix.

2 The Model

We present a dynamic equilibrium model of heterogenous banks that chooses loans, reserves and deposits. We start the description of the model by presenting a description of the dynamic decision of banks. The goal is to derive the supply of loans and the demand for reserves given an exogenous demand for loans, central bank policies and aggregate shocks. After this, we describe the central bank instruments, introduce a demand for loans and a supply of deposits that closes the model.

[5]Kashyap and Stein (2000) exploit cross-sectional variation in liquidity holdings by banks and find empirical evidence for the monetary policy transmission mechanism that we study here. Recently, Jimenez et al. (2012); Jiménez et al. (2014) exploit both, firm heterogeneity in loan demand and variation in bank liquidity ratios to identify the presence of the bank lending channel in Spain. Chodorow-Reich (2014) analyze the effects of credit contractions on employment outcomes.

[6]Ashcraft and Duffie (2007) and Afonso and Lagos (2014) provide empirical support for search frictions in the federal funds market and the presence of substantial liquidity costs.

2.1 Environment

Time is discrete, is indexed by t, and has an infinite horizon. Each period is divided into two stages: a lending stage (l) and a balancing stage (b). The economy is populated by a continuum of competitive banks whose identity is denoted by $z \in [0,1]$. Banks face a demand for loans and a vector of shocks that we describe later. An exogenous deterministic monetary policy is chosen by the monetary authority, which we refer to as the Fed. There are three types of assets: deposits, loans and central bank reserves. Deposits and loans are denominated in real terms. Reserves are denominated in nominal terms. Deposits play the role of a numeraire.

Banks. A bank's preferences over real dividend streams $\{DIV_t\}_{t\geq 0}$ are evaluated via an expected utility criterion:

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t U(DIV_t)$$

where $U(DIV) \equiv \frac{DIV^{1-\gamma}}{1-\gamma}$ and DIV_t is the banker's consumption at date t.[7] Banks hold a portfolio of loans, B_t, and central bank reserves, C_t, as part of their assets. Demand deposits, D_t, are their only form of liabilities. These holdings are the individual state variables of a bank.

Loans. Banks make loans during the lending stage. The flow of new loan issuances is I_t. These loans constitute a promise to repay the bank $I_t(1-\delta)\delta^n$ in period $t+1+n$ for all $n \geq 0$, in units of numeraire. Thus, loans promise a geometrically decaying stream of payments as in the Leland-Toft model—see Leland and Toft (1996). We denote by B_t the stock of loans held by a bank at time t. Given the structure of payments, the stock of loans has a recursive representation:

$$B_{t+1} = \delta B_t + I_t.$$

When banks grant a loan, they provide the borrower a demand deposit account that amounts to $q_t^l I_t$, where q_t is the price of the loan. Banks take q_t as given. Consequently, the bank's immediate accounting profits are $\left(1-q_t^l\right) I_t$. To focus on the liquidity management problem that will be explained below, we assume that there is no default risk.

A key feature of our model is that bank loans are illiquid—they cannot be traded—during the balancing stage.[8] The lack of a liquid market for loans in the balancing stage can be rationalized by several market frictions. For example, loans may be illiquid assets if banks specialize in particular

[7] Introducing curvature into the objective function is important in generating dividend smoothing and slow-moving bank equity, as observed empirically. One way to rationalize these preferences is through undiversified bank equity holders; similar preferences are often found in dynamic corporate finance models. Alternatively, a wedge between the marginal cost of equity and the marginal benefit of dividends would deliver curvature, which could arise due to agency frictions.

[8] Loans can be sold during the *lending* stage. The asymmetry between the lending and balancing stages allows us to reduce the state space. In particular, it is not necessary to keep track of the composition, but only the size of bank balance sheets, thanks to this assumption. Dispensing this assumption, would require us to keep track of the cross-sectional joint distribution of liquidity and leverage ratios through time.

customers or if they face agency frictions.[9]

Demand Deposits. Deposits earn a real gross interest rate $R^D = \left(1 + r^d\right)$. Behind the scenes, banks enable transactions between third parties. When they obtain a loan, borrowers receive deposits. This means that banks make loans—a liability for the borrower—by issuing their own liabilities—an asset ultimately held by a third party. This swap of liabilities enables borrowers to purchase goods because deposits are effective mediums of exchange. After the transaction, the holder of those deposits may, in turn, transfer those funds again to the accounts of others, make payments, and so on.

A second key feature of the environment is that deposits are callable on demand. In the balancing stage, banks are subject to random deposit withdrawals $\omega_t D_t$, where $\omega_t \sim F_t(\cdot)$ with support in $(-\infty, 1]$. Here, F_t is the time-varying cumulative distribution for withdrawals. The operator $\mathbb{E}_\omega(\cdot)$ is the expectation under F_t. For simplicity, we assume F_t is common to all banks.[10] When ω_t is positive (negative), the bank loses (receives) deposits. The shock ω_t captures the idea above that deposits are constantly circulating when payments are executed or in response to a loss of confidence in a given bank. The complexity of these transfers is approximated by the random process of ω_t. [11]

For simplicity, we assume that no withdrawal of deposits are made outside of the banking system, which in turn implies that reserves do not leave the banking system:[12]

Assumption 1 (Deposit Conservation)**.** *Deposits remain within the banking system:* $\int_{-\infty}^{1} \omega_t dF_t(\omega) = 0, \forall t$.

When deposits are transferred across banks, the receptor bank absorbs a liability issued by another bank. Therefore, this transaction needs to be settled with the transfer of an asset. Since bank loans are illiquid, deposit transfers are settled with reserves. Thus, the illiquidity of loans induces a demand for reserves.

Reserves. Reserves are special assets issued by the Fed and used by banks to settle transactions. Banks can buy or sell reserves without frictions during the lending stage. However, during the balancing stage, they can only borrow or lend reserves in the interbank market we detail below. We denote by p_t the price of reserves in terms of deposits. This term is also the inverse of the price level because deposits are in real terms.

[9] Diamond (1984) and Williamson (1987) introduce specialized monitoring technologies. Holmstrom and Tirole (1997) build a model in which bankers must hold a stake in the loans because of moral hazard. Finally, Bolton and Freixas (2009) introduce a differentiated role for different bank liabilities following from asymmetric information.

[10] We could assume that F is a function of the bank's liquidity or leverage ratio. This would add complexity to the bank's decisions but would not break any aggregation result. This tractability is lost if F_t is a function of the bank's size.

[11] For simplicity, we do not allow banks to issue liabilities not subject to withdrawal shocks, e.g., time deposits. A bound on ω strictly lower than one, however, captures in that not all liabilities can be withdrawn immediately a simple way. Moreover, it would be relatively straight forward to include a second liability not subject to withdrawal shocks that bears an exogenous liquidity premium.

[12] This assumption can be relaxed to allow for a demand for currency or system-wide bank runs. Ennis (2014) is a recent paper that studies the endogenous decomposition of the monetary base in currency and reserves.

By law, banks must hold a minimum amount of reserves within the balancing stage. In particular, the law states that $p_t C_t \geq \rho D_t(1-\omega_t)/R^D$, where $\rho \in [0,1]$ is a reserve requirement chosen by the Fed.[13] The case $\rho = 0$ requires banks to finish with a positive balance of reserves; banks cannot issue these liabilities. Given the reserve requirement, if ω_t is large, reserves may be insufficient to settle the outflow of deposits. In turn, banks that receive a large unexpected inflow will hold reserves in excess of the requirement.

To meet reserve requirements or allocate reserves in excess, banks can lend and borrow from each other or from the Fed. These trades constitute the interbank market. As part of its toolbox, the Fed chooses two policy rates: a lending rate, r_t^{DW}, and a borrowing rate, r_t^{ER}. The lending rate—or discount window rate—is the rate at which the Fed lends reserves to banks in deficit. The borrowing rate—the interest on excess reserves—is the interest paid by the Fed to banks that deposit excess reserves at the Fed. These rates satisfy $r_t^{DW} \geq r_t^{ER}$ and are paid within the period with deposits. This determines what in practice is known as the corridor system. [14] Banks have the option to trade with the Fed or with other banks.

Interbank Market. We assume that the interbank market for reserves is a directed OTC market.[15] This interbank market works in the following way. After the realization of withdrawal shocks, banks end with either positive or negative balances relative to their reserve requirements. At that point, there are distributions for reserve deficits and surpluses. A bank that wishes to lend excess reserves can place lending orders. A bank that needs to borrow reserves to patch its deficit can place borrowing orders. Thus, the market is directed in the sense that orders are placed in either the borrowing or lending sides of the market —orders in the same side never meet.

In addition, banks can place multiple orders at a time. In fact, they place a continuum of orders; an important feature that greatly simplifies the problem is that orders are assumed to be of infinitesimal size, as in Atkeson et al. (2012). We develop this notion formally in Appendix E where we describe this environment when banks can place multiple orders but of a fixed size. There, we also study the environment as the order size tends to 0. Here, we present that limit case. Thus, from here on, we refer to an infinitesimal order simply as a dollar —that is, as a per-unit of account order.

After orders are directed to either market side, a dollar in excess is randomly matched with a dollar in deficit. Once a match is realized, the lending bank transfers a dollar of excess reserves

[13] Some operating frameworks compute reserve balances over a maintenance period. Bank choices in our model would correspond to averages over the maintenance period.

[14] We do not model here the reasons why the central bank chooses to have a corridor system, and simply take as given that this is a standard policy instrument to affect credit creation. This can actually be motivated in our setup by the existence of a fire sale externality that arises because of a marked-to-market capital requirement constraint (see e.g. Bianchi and Mendoza (2013); Stein (2012)). Another natural motivation for the corridor system, which is outside the model, is aggregate demand management in the presence of nominal rigidities. What is critical for our analysis is the presence of liquidity risk which arises in our model when $r_{DW} > 0$. In practice, other frictions in interbank markets make a shortfall of reserves costly such as the stigma from borrowing at the discount window (see, e.g., Armantier et al. (2011) and Ennis and Weinberg (2013)).

[15] The features of this interbank market are borrowed from work by Afonso and Lagos (2012).

overnight to the borrower. Orders use Nash bargaining to split the surplus of the dollar transfer. In the bargaining problem that emerges, the outside option for the lending bank is to deposit the dollar at the Fed earning r^{ER}. For the bank in deficit, the outside option is the discount window rate r^{DW}. Because the principal of the loan—the dollar itself—is returned within the period, without loss of generality, banks bargain only about the net interest rate of this transaction. We call this net rate the fed funds rate, r^{FF}.

A few conventions are implicit: First, if an order does not find a match, the bank does not lose the opportunity to lend/borrow to/from the Fed. Second, a bank cannot place orders beyond its reserve needs or excess holdings. Without this restriction, a bank could place a higher number of orders than needed to increase its probability of allocating or borrowing funds. Third, matched orders take as given the outcome of other orders in the same bank —orders do not bargain collectively. Finally, interests are paid with deposits; this is innocuous, since all assets are liquid during the lending stage.

The probability that a lending or borrowing order finds a match depends on the relative mass on each side of the market. We denote by M^+ the mass of dollars that are lending orders and by M^- the mass in the borrowing side. The probability that a borrowing order finds a lending order is given by $\gamma^- = \min(1, M^+/M^-)$. Conversely, the probability that a lending order finds a borrowing order is $\gamma^+ = \min(1, M^-/M^+)$. These probabilities will affect the average cost of being short or long in reserves, which will in turn affect banks' portfolio decisions and aggregate liquidity. In the quantitative analysis, we will study shocks that reduce the probability of matching for given $\{M^+, M^-\}$ to capture disturbances in interbank markets.

In this environment, as we show in Appendix E, the result of the bargaining problem as the order size becomes infinitesimal is reduced to:

Problem 1 (Interbank Market Bargaining Problem) *The rate at which bank orders trade in the interbank solves*

$$\max_{r^{FF}} \left(m_b r_t^{DW} - m_b r^{FF}\right)^{\xi} \left(m_l r^{FF} - m_l r_t^{ER}\right)^{1-\xi}.$$

In the objective function, m_l is the marginal utility of the bank lending reserves and m_b is the corresponding term for the bank borrowing reserves.[16] These constants depend on the identities of the banks in the match, are part of the value of the match, but do not appear in the first-order condition of this problem:

$$\frac{r^{FF} - r_t^{ER}}{(1 + r_t^{DW}) - (1 + r^{FF})} = \frac{(1-\xi)}{\xi}.$$

This condition yields an implicit solution for r^{FF}. Since $(1-\xi)/(\xi)$ is positive, it is clear that r^{FF} will fall within the Fed's corridor of interest rates, $\left[r_t^{ER}, r_t^{DW}\right]$. For simplicity, we set $\xi = 0.5$

[16] We note that if the bargaining took place between banks, rather than dollars, the value functions would appear in the problem, making the model significantly less tractable. One possible interpretation of bargaining on a per-order basis is that each bank places orders through a large number of traders that take as given the trades conducted by the other traders.

so that the Fed funds rate is at the middle of the bounds.[17]

Note that unlike in Afonso and Lagos (2014), r^{FF} is constant across matches and, thus, the model does not feature a distribution of fed funds rates. Afonso and Lagos (2014) allow for multiple rounds of bargaining which deliver a distribution of rates that depend on the time. Also, because bargaining is between banks and not dollars, there is also a cross-sectional distribution of fed fund rates at every round of trade. In Atkeson et al. (2012), trades are infinitesimal and happen only once, as occurs here. However, that model also delivers a cross-sectional distribution of prices. The reason for this is that trade in Atkeson et al. (2012) occurs before the realization of shocks. The infinitesimal nature of trade implies that bargaining also depends only on marginal utilities. However, in that case, these marginal utilities cannot be factored out of the bargaining problem due to the timing of uncertainty.

2.2 Timing, Laws of Motion and Bank Problems

This section describes the model recursively: we drop time subscripts from now on. We adopt the following notation: let Z be a variable at the beginning of the period, $\tilde{Z}$ is its value by the end of the lending stage and the beginning of the balancing stage. Similarly, Z' denotes its value by the end of the balancing stage and the beginning of the following period. The aggregate state, summarized in the vector X, includes all policy decisions by the Fed, the distribution of withdrawal shocks, F, and a shock to the demand for loans—to be specified below.

Lending Stage. Banks enter the lending stage with reserves, C, loans, B, and deposits, D. The bank chooses dividends, DIV, loan issuances, I, and purchases of reserves, φ.[18] The evolution of deposits is as follows:

$$\frac{\tilde{D}}{R^D} = D + qI + DIV + \varphi p - B(1-\delta).$$

Several actions affect this evolution. First, deposits increase when the bank credits qI deposits in the accounts of borrowers—or whomever they trade with. Second, banks pay dividends to shareholders with deposits. Third, the bank issues $p\varphi$ deposits to buy φ reserves. Finally, deposits fall by $B(1-\delta)$ because loans are amortized with deposits.

At the end of the lending stage, reserves are the sum of the previous stock plus purchases of reserves, $\tilde{C} = C + \varphi$. Loans evolve according to $\tilde{B} = \delta B + I$. Banks choose $\{I, DIV, \varphi\}$ subject to these laws of motion and a capital requirement constraint. The capital requirement constraint imposes an upper bound, κ, on the stock of deposits relative to equity—marked-to-market.[19]

[17]In a Walrasian setting, the interbank rate would equal the discount rate or the excess reserve rates depending on whether there are enough reserves in the system to satisfy the reserve requirements of all banks.

[18]The purchase of reserves φ occurs during the lending stage. Thus, this flow differs from the flow that follows from loans in the interbank market, which occurs during the balancing stage.

[19]Observe that if a bank arrives at a node with negative equity, the problem is not well defined. However, when choosing its policies, the bank will make decisions that guarantee that it does not run out of equity.

Denoting by V^l and V^b the bank's value function during the lending and balancing stages, we have the following recursive problem in the lending stage:

Problem 2 *In the lending stage, banks solve the following:*

$$\begin{aligned} V^l(C,B,D;X) &= \max_{I,DIV,\varphi} U(DIV) + \mathbb{E}\left[V^b(\tilde{C},\tilde{B},\tilde{D};\tilde{X})\right] \\ \frac{\tilde{D}}{R^D} &= D + qI + DIV + p\varphi - B(1-\delta) \\ \tilde{C} &= C + \varphi \\ \tilde{B} &= \delta B + I \\ \frac{\tilde{D}}{R^D} &\leq \kappa\left(q\tilde{B} + p\tilde{C} - \frac{\tilde{D}}{R^D}\right); \tilde{B},\tilde{C},\tilde{D} \geq 0. \end{aligned}$$

Balancing Stage. During the balancing stage, withdrawal shocks shift deposits and reserves across the banking system, leading to a distribution of reserve deficits and surpluses. Let x be the reserve deficit for an individual bank. Given that withdrawals are settled with reserves, this deficit is

$$x = \rho\underbrace{\left(\frac{\tilde{D} - \omega\tilde{D}}{R^D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Deposits}}} - \underbrace{\left(\tilde{C}p - \frac{\omega\tilde{D}}{R^D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Reserves}}}.$$

Given the structure of the OTC market described above, a bank with a reserve surplus obtains a return of r^{FF} if it lends a unit of reserves in the interbank market and r^{ER} if it lends to the Fed. Notice that for any Nash bargaining parameter $r^{FF} > r^{ER}$, banks always attempt to lend first in the interbank market. Thus, they place lending orders for every dollar in excess. In equilibrium, only a fraction γ^+ of those orders are matched and earn a return of r^{FF}. The rest earns the Fed's borrowing rate r^{ER}. Thus, the average return on excess reserves is

$$\chi_l = \gamma^+ r^{FF} + \left(1 - \gamma^+\right) r_t^{ER}.$$

Analogously, a bank with a reserve deficit borrows from the interbank market before borrowing from the Fed because $r^{FF} < r_t^{DW}$. The cost of reserve deficits is

$$\chi_b = \gamma^- r^{FF} + \left(1 - \gamma^-\right) r_t^{DW}.$$

The difference between χ_l and χ_b is an endogenous wedge between the marginal value of excess reserves and the cost of reserve deficits. The simple rule that characterizes orders in the interbank market problem yields a value function for the bank during the balancing stage:

Problem 3 *The value of the bank's problem during the balancing stage is as follows:*

$$\begin{aligned}
V^b(\tilde{C},\tilde{B},\tilde{D};\tilde{X}) &= \beta\mathbb{E}\left[V^l(C',B',D';X')|\tilde{X}\right]\\
D' &= \tilde{D}(1-\omega)+\chi(x)\\
B' &= \tilde{B}\\
x &= \rho\left(\frac{\tilde{D}-\omega\tilde{D}}{R^D}\right)-\left(\tilde{C}p-\frac{\omega\tilde{D}}{R^D}\right)\\
C' &= \tilde{C}-\frac{\omega\tilde{D}}{p}.
\end{aligned}$$

Here χ represents the illiquidity cost, the return/cost of excess/deficit of reserves:

$$\chi(x)=\begin{cases}\chi_l x \;\; if \;\; x\leq 0\\ \chi_b x \;\; if \;\; x>0\end{cases}$$

We can collapse the problem of a bank for the entire period through a single Bellman equation by substituting V^b into V^l:

Problem 4 *The bank's problem during the lending stage is as follows:*

$$\begin{aligned}
V^l(C,B,D,X) &= \max_{\{I,DIV,\varphi\}} U\left(DIV\right)... \qquad (1)\\
&+\beta\mathbb{E}\left[V^l\left(\tilde{C}-\frac{\omega'\tilde{D}}{p},\tilde{B},\tilde{D}(1-\omega')+\chi\left(\frac{(\rho+\omega'\left(1-\rho\right))\tilde{D}}{R^D}-\tilde{C}p\right),X'|X\right)\right]\\
\frac{\tilde{D}}{R^D} &= D+qI+DIV_t+p\varphi-B(1-\delta)\\
\tilde{B} &= \delta B+I\\
\tilde{C} &= \varphi+C\\
\frac{\tilde{D}}{R^D} &\leq \kappa\left(\tilde{B}q+\tilde{C}p-\frac{\tilde{D}}{R^D}\right).
\end{aligned}$$

The following section provides a characterization of this problem.

2.3 Characterization of the Bank Problem

The recursive problem of banks can be characterized through a single state variable, the banks' equity value after loan amortizations, $E\equiv pC+(\delta q+1-\delta)B-D$. Substituting the laws of motion for reserves and loans $\tilde{C}=\varphi+C$ and $\tilde{B}=\delta B+I$ into the law of motion for deposits 2.2, we have that the evolution of deposits takes the form of a budget constraint:

$$E=q\tilde{B}+\tilde{C}p+DIV-\tilde{D}/R^D.$$

In this budget constraint, E is the value of the bank's available resources, which is predetermined for an individual bank. We use an updating rule for E that depends on the bank's current decisions to express the bank's value function through a single-state variable:

Proposition 1 (Single-State Representation)

$$\begin{aligned}
V(E) &= \max_{\tilde{C},\tilde{B},\tilde{D},DIV} U(DIV) + \beta\mathbb{E}\left[V(E')|X\right] \qquad (2)\\
E &= q\tilde{B} + p\tilde{C} + DIV - \frac{\tilde{D}}{R^D}\\
E' &= \left(q'\delta + 1 - \delta\right)\tilde{B} + p'\tilde{C} - \tilde{D} - \chi\left(\frac{(\rho + \omega'(1-\rho))\tilde{D}}{R^D} - \tilde{C}p\right)\\
\frac{\tilde{D}}{R^D} &\leq \kappa\left(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D}\right).
\end{aligned}$$

This problem resembles a standard consumption-savings problem subject to a leverage constraint. Dividends play the role of consumption; the bank's savings are allocated into loans, $\tilde{B}$, and reserves, $\tilde{C}$, and it can leverage its position by issuing deposits $\tilde{D}$.[20] Its choice is subject to a capital requirement constraint—the leverage constraint. The budget constraint is linear in E, and the objective is homothetic. Thus, by the results in Alvarez and Stokey (1998), the solution to this problem exists and is unique, and policy functions are linear in equity. Formally,

Proposition 2 (Homogeneity—γ) *The value function $V(E;X)$ satisfies*

$$V(E;X) = v(X)\,E^{1-\gamma},$$

where $v(\cdot)$ satisfies

$$v(X) = \max_{\tilde{c},\tilde{b},\tilde{d},div} U(div) + \beta\mathbb{E}\left[v(X')\,|X\right]\mathbb{E}_{\omega'}(e')^{1-\gamma} \qquad (3)$$

subject to

$$\begin{aligned}
1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D}\\
e' &= (q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi\left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - p\tilde{c}\right)\\
\frac{\tilde{d}}{R^D} &\leq \kappa\left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D}\right).
\end{aligned}$$

Moreover, the policy functions in (2) satisfy $\left[\tilde{C}\ \tilde{B}\ \tilde{D}\right] = \left[\tilde{c}\ \tilde{b}\ \tilde{d}\right]\cdot E$.

[20] From here on, we use the terms cash and reserves interchangeably. These terms are not to be confused with cash holdings by firms which may refer to deposits.

According to this proposition, the policy functions in (2) can be recovered from (3) by scaling them by equity, i.e., if c^* is the solution to (3), we have that $C = Ec^*$, and the same applies for the rest of the policy functions. An important implication is that two banks with different equity are scaled versions of a bank with one unit of equity.[21] This also implies that the distribution of equity is not a state variable but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity—the variance of distribution grows over time—the model yields predictions about the cross-sectional dispersion of equity growth.

An additional useful property of the bank's problem is that it satisfies portfolio separation. In particular, the choice of dividends can be analyzed independently—through a consumption savings problem with a single asset—from the portfolio choices between deposits, reserves and loans. We use the principle of optimality to break the Bellman equation (3) into two components.

Proposition 3 (Separation) *The value function $v(\cdot)$ defined in (3) solves*

$$v(X) = \max_{div \in \mathbb{R}_+} U(div) + \beta \mathbb{E}\left[v(X')\,|X\right] \Omega(X)^{1-\gamma} (1-div)^{1-\gamma}. \quad (4)$$

Here, $\Omega(X)$ is the value of the certainty-equivalent portfolio value of the bank, which is the outcome of the following liquidity management portfolio problem:

$$\begin{aligned} \Omega(X) &\equiv \max_{\{w_b, w_c, w_d\} \in \mathbb{R}^3_+} \left\{ \mathbb{E}_{\omega'} \left[R^B_X w_b + R^C_X w_c - \ w_d R^D_X - R^\chi_X(w_d, w_c) \ \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \quad (5) \\ w_b + w_c - w_d &= 1 \\ w_d &\leq \kappa (w_b + w_c - w_d) \quad (6) \end{aligned}$$

with $R^B_X \equiv \frac{q'\delta + (1-\delta)}{q}$, $R^C_X \equiv \frac{p'}{p}$, $R^\chi_X \equiv \chi((\rho + \omega'(1-\rho)) w_d - w_c)$.

Once we solve the policy functions of this portfolio problem, we can reverse the solution for $\tilde{c}, \tilde{b}, \tilde{d}$ that solves (3) via the following formulas: $\tilde{b} = (1-div) w_b/q$, $\tilde{c} = (1-div) w_c/p$, and $\tilde{d} = (1-div) w_d R^D$.

The maximization problem that determines $\Omega(X)$ consists of choosing portfolio shares among assets of different risk, liquidity and return. This problem is a liquidity management portfolio problem with the objective of maximizing the certainty equivalent return on equity, where the return on equity is given by:

$$R^E(\omega'; w_b, w_d, w_c) \equiv R^B w_b + R^C w_c - R^D w_d - R^\chi(w_d, w_c, \omega').$$

This portfolio problem is not a standard portfolio problem because it features non-linear returns. The return on loans is linear and equals the sum of the coupon payment plus the resale price

[21] Studying differences between large and small banks is beyond the scope of this paper. See Corbae and D'Erasmo (2013) and Corbae and D'Erasmo (2014) for recent contributions on this dimension. For our purpose, an important fact is that reserves are widely distributed across the banking sector, as documented by Wolman and Ennis (2011).

of loans: $R^B \equiv (\delta q + (1-\delta))/q$. The return on reserves and deposits can be separated into independent—intrinsic—return components and a joint return component. The intrinsic return on reserves is the deflation rate $R^C \equiv p'/p$. The independent return on deposits is the interest on deposits, R^D. The joint return component, which depends on ω', captures the cost—or benefit—of running out of reserves. This illiquidity cost depends on the conditions of the interbank market and is given by:

$$R^{\chi}(w_d, w_c, \omega') \equiv \chi\left(\left(\rho + (1-\rho)\,\omega'\right) w_d - w_c\right). \tag{7}$$

The risk and return of each asset varies with the aggregate state, making the solution to the liquidity management portfolio problem time varying. In addition, the solution for the dividend rate and marginal values of bank equity satisfy a system of equations described below.

Proposition 4 (Solution for Dividends and Bank Value) *Given the solution to the portfolio problem (5), the dividend ratio and the value of bank equity are given by*

$$div(X) = \frac{1}{1 + \left[\beta(1-\gamma)\mathbb{E}\left[v(X')\,|X\right]\Omega^*(X)^{1-\gamma}\right]^{1/\gamma}}$$

and

$$v(X) = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega^*(X)^{1-\gamma}\,\mathbb{E}\left[v(X')\,|X\right]\right)^{\frac{1}{\gamma}}\right]^{\gamma}.$$

The policy functions of banks determine the loan supply and demand for reserves. This concludes the partial equilibrium analysis of the bank's portfolio decisions. We now describe the demand for loans and the actions of the Fed.

2.4 Loan Demand

We consider a downward-sloping demand for loans with respect to the loan rate, i.e., the demand for loans is increasing on the price. In particular, we consider a constant elasticity demand function:

$$q_t = \Theta_t \left(I_t^D\right)^{\epsilon}, \epsilon > 0, \Theta_t > 0, \tag{8}$$

where ϵ is the inverse of the semi-elasticity of credit demand with respect to the price, which could capture the extent to which non-financial firms can substitute bank loans for other forms of liabilities. The term Θ_t captures possible credit demand shifts that could occur.

We do not take a particular stance about the causes of the shifts in demand. Shocks to demand for credit could occur because of weak investment opportunities, increases in uncertainty at the firm or household level, or a reduction in the pool of borrowers that meet minimum credit standards. Notice also that from the perspective of an individual bank, a drop in Θ_t would be isomorphic to introducing an exogenous probability of default on banks' loans portfolio. Below, we analyze a specific microfoundation for this loan demand function based on a working capital constraint for firms where the decline in demand can follow from exogenous reductions in total factor productivity or increases in the labor wedge.

2.5 The Fed's Balance Sheet and Its Operations

This section describes the Fed's balance sheet and how the Fed implements monetary policy. The Fed's balance sheet is analogous to that of commercial banks with an important exception: the Fed does not issue demand deposits as liabilities, it issues reserves instead. As part of its assets, the Fed holds commercial bank deposits, D_t^{Fed}, and private sector loans, B_t^{Fed}. As liabilities, the Fed issues $M0_t$ reserves—high power money. The Fed's assets and liabilities satisfy the following laws of motion:

$$\begin{aligned} M_{t+1}^0 &= M0_t + \varphi_t^{Fed} \\ \frac{D_{t+1}^{Fed}}{R^D} &= D_t^{Fed} + p_t\varphi_t^{Fed} + (1-\delta)\,B_t^{Fed} - q_t I_t^{Fed} + \chi_t^{Fed} - T_t \\ B_{t+1}^{Fed} &= \delta B_t^{Fed} + I_t^{Fed}. \end{aligned}$$

The laws of motion for these state variables are very similar to the laws of motion for banks. Here, φ_t^{Fed} represents the Fed's purchase of deposits by issuing reserves to commercial banks. Its deposits are affected by the purchase or sale of loans, I_t^{Fed}, and the coupon payments of previous loans, $(1-\delta)\,B_t^{Fed}$. In addition, the Fed's deposits vary with, T_t, the transfers to or from the fiscal authority—the analogue of dividends. Finally, χ_t^{Fed} represents the Fed's income revenue that stems from its participation in the fed funds market:

$$\chi_t^{Fed} = \underbrace{r_t^{DW}\left(1-\gamma^-\right)M^-}_{\substack{\text{Earnings from}\\ \text{Discount Loans}}} - \underbrace{r_t^{ER}\left(1-\gamma^+\right)M^+}_{\substack{\text{Losses from}\\ \text{Interest Payments}\\ \text{on Excess Reserves}}}.$$

The Fed's balance sheet constraint is obtained by combining the laws of motion for reserves, loans and deposits:

$$p_t\left(M_{t+1}^0 - M0_t\right) + (1-\delta)\,B_t^{Fed} + \chi_t^{Fed} = D_{t+1}^{Fed}/R^D - D_t^{Fed} + q_t\left(B_{t+1}^{Fed} - \delta B_t^{Fed}\right) + T_t. \quad (9)$$

The Fed has a monopoly over the supply of reserves, $M0_t$, and alters this quantity through several operations.

Unconventional Open-Market Operations. Since there are no government bonds, only unconventional open-market operations are available.[22] An unconventional OMO involves the purchase of loans and the issuance of reserves. This operation does not affect the stock of commercial bank deposits held by the Fed. To keep the amount of deposits constant, the Fed exchanges reserves for deposits with banks, and then sells those deposits to purchase loans.

Open-Market Liquidity Facilities. Liquidity facilities are deposits of reserves by the Fed at commercial banks.

Fed Profits and Transfers. In equilibrium, the Fed can return surpluses or losses. These operational results follow from the return on the Fed's loans and its profits/losses in the interbank market χ_t^{Fed}. We assume that the Fed transfers losses or profits immediately.

2.6 Market Clearing, Evolution of Bank Equity, and Equilibrium

Bank Equity Evolution. Define $\overline{E}_t \equiv \int_0^1 E_t(z)\,dz$ as the aggregate value of equity in the banking sector. The equity of an individual bank evolves according to $E_{t+1}(z) = e_t(\omega) E_t(z)$. Here, $e_t(\omega)$ is the growth rate of bank equity of a bank with withdrawal shock ω. The measure of equity holdings at each bank is denoted by Γ_t. Since the model is scale invariant, we only need to keep track of the evolution of average equity, $\overline{E}_t$ which by independence grows at the rate $\mathbb{E}_\omega[e_t(\omega)]$.[23]

Loans Market. Market clearing in the loans market requires us to equate the loan demand I_t^D to the supply of new loans made by banks and the Fed. Hence, equilibrium must satisfy

$$I_t^D \equiv (q_t/\Theta_t)^{\frac{1}{\epsilon}} = B_{t+1} - \delta B_t + B_{t+1}^{Fed} - \delta B_t^{Fed}. \tag{10}$$

Money Market. Reserves are not lent outside the banking system; there is no use for currency. This implies that the aggregate holdings of reserves during the lending stage must equal the supply of reserves issued by the Fed:

$$\int_0^1 \tilde{c}_t(z) E_t(z)\,dz = M0_t \longrightarrow \tilde{c}_t \overline{E}_t = M0_t.$$

Interbank Market. The equilibrium conditions for the interbank market depend on γ^+ and γ^-, the probability of matches in the reserve market. These probabilities, in turn, depend on M^- and M^+, the mass of reserves in deficit and surplus. During the lending stage, banks are identical replicas of each other scaled by equity. Thus, for every value of $E_t(z)$, there is

[22] Incorporating Treasury Bills (T-bills) and conventional open-market operations into our model is relatively straightforward. If T-bills are illiquid in the balancing stage, T-bills and loans become perfect substitutes from a bank's perspective and the model becomes equivalent to our baseline model—with an additional market-clearing condition for T-bills. If T-bills are perfectly liquid, we can show that banks that have a deficit in reserves first sell their holdings of T-bills before accessing the interbank market. In the intermediate case, T-bills are imperfect substitutes, the price of T-bills would depend on the distribution of assets in the economy.

[23] A limiting distribution for Γ_t is not well defined unless one adapts the process for equity growth.

an identical distribution of banks short and long of reserves. The shock that leads to $x = 0$ is $\omega^* = \left(\tilde{C}/p - \rho\tilde{D}\right)/(1-\rho)$. This implies that the mass of reserves in deficit is given by

$$M^- = \mathbb{E}\left[x(\omega)\,|\omega > \omega^*\right]\left(1 - F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1-\rho)}\right)\right)\overline{E}_t$$

and the mass of surplus reserves is,

$$M^+ = \mathbb{E}\left[x(\omega)\,|\omega < \omega^*\right] F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1-\rho)}\right)\overline{E}_t.$$

Money Aggregate. Deposits constitute the monetary creation by banks, $M_t^1 \equiv \int_0^1 \tilde{d}_t(z)\, E_t(z)\, dz$. The endogenous money multiplier is $\mu_t = \frac{M_t^1}{M0_t}$.

Equilibrium. The definition of equilibrium is as follows.

Definition. *Given M_0, D_0, B_0, a competitive equilibrium is a sequence of bank policy rules $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}_{t\geq 0}$, bank values $\{v_t\}_{t\geq 0}$, government policies $\left\{\rho_t, D_{t+1}^{Fed}, B_{t+1}^{Fed}, M0_t, T_t, \kappa_t, r_t^{ER}, r_t^{DW}\right\}_{t\geq 0}$, aggregate shocks $\{\Theta_t, F_t\}_{t\geq 0}$, measures of equity distributions $\{\Gamma_t\}_{t\geq 0}$, measures of reserve surpluses and deficits $\{M^+, M^-\}_{t\geq 0}$ and prices $\left\{q_t, p_t, r_t^{FF}\right\}_{t\geq 0}$, such that: (1) Given price sequences $\left\{q_t, p_t, r_t^{FF}\right\}_{t\geq 0}$ and policies $\left\{\rho_t, D_t^{Fed}, B_{t+1}^{Fed}, M0_t, \kappa_t, r_t^{ER}, r_t^{DW}\right\}_{t\geq 0}$, the policy functions $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}_{t\geq 0}$ are solutions to Problem 4. Moreover, v_t is the value in Proposition 3. (2) The money market clears: $\tilde{c}_t\overline{E}_t = M0_t$. (3) The loan market clears: $I_t^D = \Theta_t^{-1} q_t^{\frac{1}{\epsilon}}$, (4) Γ_t evolves consistently with $e_t(\omega)$, (5) the masses $\{M^+, M^-\}_{t\geq 0}$ are also consistent with policy functions and the sequence of distributions F_t. All the policy functions of Problem 4 satisfy $\left[\tilde{C}\ \tilde{B}\ \tilde{D}\right] = \left[\tilde{c}\ \tilde{b}\ \tilde{d}\right]\cdot E$.*

Before proceeding to the analysis of particular parameterizations of the model, we discuss a possible microfoundation for the demand for loans and the supply of deposits.

2.7 Non Banking Sector

The competitive equilibrium defined above assumes an exogenous demand for loans, given by (8), and an exogenous supply of deposits; the banking system faces a perfectly elastic supply of deposits at rate R^D. In Appendix D we provide a simple microfoundation for the demand for loans and the supply of deposits. This microfoundation has the following features.

Deposit Supply. We introduce a continuum of households with quasi-linear utility. Deposits are their only savings instruments. They face convex disutility from labor and linear utility from consumption. The linearity in consumption leads to a perfectly elastic supply of savings, where R^D equals the inverse of the discount factor of households, $1/\beta^D$. The lump-sum tax T_t on the Fed's budget constraint is levied from these households. This assumption guarantees that taxes do not affect the supply of deposits or the demand for loans.

Derivation of Loan Demand. The demand for loans (8) emerges from the decisions of firms that need to borrow working capital to hire workers. Hiring decisions are made once, but production is realized slowly, in a way that delivers the maturity structure of debt that we described above.

3 Theoretical Analysis

3.1 Liquidity Premia and Liquidity Management

This section provides more insights about the implementation of monetary policy in the model. First, we derive an expression for a liquidity premium of reserves relative to loans. This liquidity premium has two components: the direct marginal benefit of avoiding borrowing in the interbank market and a risk premium. We then consider the case of risk-neutral banks. That exercise illustrates that monetary policy has real effects as long there is a kink in $\chi(\cdot)$. We then analyze the model when there are no withdrawals. In this case, excess reserves are zero, and hence, monetary policy has limited effects. Finally, we analyze equilibria when $r^{DW} = r^{ER} = 0$, a version of the zero lower bound (ZLB). For that case, lending is determined by the banking system's equity, the capital requirements, and demand shocks, but not by withdrawal risks.

Bank Portfolio Problem. Fix a state X. To spare notation, we suppress the X argument from prices and policy functions and leave this reference as implicit. We rewrite Problem 5 by inserting the budget constraint into the objective:

$$\Omega = \max_{\substack{w_d \in [0,\kappa] \\ w_c \in [0, 1+w_d]}} \left(\mathbb{E}_{\omega'} \left[\left(\underbrace{R^B}_{\text{Return on Loans}} \underbrace{- \left(R^B - R^C\right) w_c}_{\text{Opportunity Cost}} + \underbrace{\left(R^B - R^D\right)}_{\text{Arbitrage}} w_d - \underbrace{R^{\chi}\left(w_d, w_c, \omega'\right)}_{\text{Liquidity Cost}} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.$$

This objective can be read as follows. If banks hold neither reserves nor issue deposits, they obtain a return on equity of R^B. Issuing additional deposits provides a direct arbitrage of $R^B - R^D$ but also exposes the bank to greater liquidity costs $R^{\chi}(w_d, w_c, \omega')$. In turn, banks can reduce these liquidity costs by holding more reserves, although they must forgo an opportunity cost, the spread between loans and reserves, $R^B - R^C$.

Liquidity Premium. Assuming that reserves are strictly positive, first-order conditions with respect to reserves and deposits yield

$$w_C :: \qquad R^B - R^C = -\frac{\mathbb{E}_{\omega'}\left[\left(R^E_{\omega'}\right)^{-\gamma} R^{\chi}_c\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}} \tag{11}$$

and

$$w_D :: \quad R^B - R^D = \frac{\mathbb{E}_{\omega'}\left[\left(R^E_{\omega'}\right)^{-\gamma}\left(R^\chi_d\left(w_d, w_c, \omega'\right)\right)\right] + \mu}{\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}}, \tag{12}$$

where μ is the multiplier associated with the capital requirement constraint.[24] We rearrange (11) and define the stochastic discount factor $m' \equiv \text{div}\left(X'\right)\frac{\mathbb{E}_{\omega'}\left[\left(R^E_{\omega'}\right)^{-\gamma}\right]\mathbb{E}[1-\text{div}(X)]}{\mathbb{E}[\text{div}(X)]}$ to obtain:

$$\begin{aligned}
\underbrace{R^B - R^C}_{\text{Opportunity Cost}} &= -\frac{\mathbb{E}_{\omega'}\left[m' \cdot R^\chi_c\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left[m'\right]} \\
&= -\underbrace{\mathbb{E}_{\omega'}\left[R^\chi_c\left(w_d, w_c, \omega'\right)\right]}_{\text{Direct Liquidity Effect}} + \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m', R^\chi_c\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left[m'\right]}}_{\text{Liquidity-Risk Premium}}.
\end{aligned}$$

The left-hand side of this expression is the liquidity premium, i.e., the difference between the return on loans and reserves. This liquidity premium equals the direct benefit of holding additional reserves, $-\mathbb{E}_{\omega'}\left[R^\chi_c\left(w_d, w_c, \omega'\right)\right]$, adjusted by a liquidity risk premium. The direct benefit, $-\mathbb{E}_{\omega'}\left[R^\chi_c\left(w_d, w_c, \omega'\right)\right]$, is the expected marginal reduction in expected interest payments in the interbank market by holding additional reserves. The liquidity risk premium emerges because the stochastic discount factor varies with ω'.

We obtain a similar expression for the spread between loans and deposits:

$$\underbrace{R^B - R^D}_{\text{Arbitrage}} \geq \underbrace{\mathbb{E}_{\omega'}\left[R^\chi_d\left(w_d, w_c, \omega'\right)\right]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m', R^\chi_d\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left[m'\right]}}_{\text{Liquidity-Risk Premium}},$$

which holds at equality if $w_d < \kappa$.

This expression states that the direct arbitrage obtained by lending, $R^B - R^D$, equals the expected marginal increase in liquidity costs of additional deposits, $\mathbb{E}_{\omega'}\left[R^\chi_d\left(w_d, w_c, \omega'\right)\right]$, plus a liquidity risk premium. In addition, when the capital requirement constraint is binding, this excess return is larger.[25]

Define a bank's reserve rate as $L \equiv \left(w_c/w_d\right)$. The following lemma states that liquidity costs are linear in $\{w_d, w_c\}$:

Lemma 1 (Linear Liquidity Risk) *$\mathbb{E}_{\omega'}\left[R^\chi\left(w_d, w_c, \omega'\right)\right]$ is homogeneous of degree $\{w_d, w_c\}$.*

Moreover, we have an exact expression for the expected marginal benefit of additional reserves:

[24] We ignore the non-negativity constraints on deposits and loans because they are not binding in equilibrium. I

[25] These expressions are similar to other standard asset-pricing equations with portfolio constraints except for the liquidity adjustment. This expression may be useful for empirical investigations. For example, during the financial crises of 2008-2009, interest rate spreads widened. This increase has been attributed to greater credit risks and tighter capital requirements. The formulae above suggest that liquidity risks could also explain part of these spreads, and the expression may be useful in distilling these effects.

Lemma 2 (Marginal Liquidity Cost) *The marginal value of liquidity is*

$$-\mathbb{E}_{\omega'}\left[R_c^{\chi}(1,L,\omega')\right]=\chi_b \Pr\left[\omega'\geq\frac{L-\rho}{(1-\rho)}\right]+\chi_l \Pr\left[\omega'\leq\frac{L-\rho}{(1-\rho)}\right].$$

This lemma implies that the marginal value of additional liquidity, $\omega^d\mathbb{E}_{\omega'}\left[R_c^{\chi}(1,L,\omega')\right]$, equals the expected interest payments from the interbank market. Finally, recall that the lemma above implies the following

Corollary 1 *If there is no spread in the corridor system, $r^{ER}=r^{DW}$, then $r^{FF}=r^{ER}=r^{DW}$, and the marginal value of liquidity is constant and equal to $R_c^{\chi}=r^{FF}$.*

We will use this corollary and the previous lemma to derive additional results below.

3.2 Limit Case I: Risk-Neutral Banks ($\gamma=0$).

For $\gamma=0$, the bank's objective is to maximize expected returns. Thus, for this case:

$$\Omega=R^B+\max_{\{w_d,w_c\}}\left(R^B-R^D\right)w_d-\left(R^B-R^C\right)w_c-\mathbb{E}_{\omega'}\left[R^{\chi}(w_d,w_c)\right].$$

By Lemma 1, we can factor w_d and transform the problem above into:

$$\Omega = R^B+\max_{w_d}\underbrace{w_d}_{\substack{\text{Leverage}\\ \text{Choice}}}\left(\left(R^B-R^D\right)+\underbrace{\max_{L}\left\{-\left(R^B-R^C\right)L-\mathbb{E}_{\omega'}\left[\tilde{R}^{\chi}(1,L)\right]\right\}}_{\text{Liquidity Management}}\right)$$

subject to $\omega^d \in [0,\kappa]$ and $L\in\left[0,\dfrac{1+\omega^d}{\omega^d}\right]$.

This reformulation shows that the portfolio problem of risk-neutral bankers can be separated into two. First, the bank must solve an optimal liquidity management problem. Second, given a choice for L, the return per unit of leverage becomes linear, and the bank must choose a leverage scale.

The choice of leverage obeys the following trade-off. Issuing deposits yields a direct return of $\left(R^B-R^D\right)$. However, the L fraction of deposits is used to purchase reserves optimally. The optimal reserve ratio trades off the opportunity cost of obtaining liquidity against the reduction in the expected illiquidity cost. Let L^* be the optimal reserve ratio. L^* satisfies

$$\underbrace{\left(R^B-R^C\right)}_{\text{Liquidity Premium}}=-\underbrace{\mathbb{E}_{\omega'}\left[R_c^{\chi}(1,L^*)\right]}_{\text{Direct Liquidity Effect}},\tag{13}$$

which is consistent with the first-order condition (11) when $m=1$. Given L^*, the problem is linear in w^d if $L^*<\frac{1+w^d}{w^d}$. In equilibrium, $L\leq\frac{1+w^d}{w^d}$ is non-binding, otherwise an equilibrium features no loans. This, in turn, is ruled out by the shape of the loan demand. Since $-\mathbb{E}_{\omega'}\left[R_c^{\chi}\left(1,L,\omega'\right)\right] \in \left[r_t^{ER}, r_t^{DW}\right]$, the first-order condition above implies a relationship between the liquidity premium and the rates of the corridor system:

Proposition 5 *In equilibrium,* $R^C + r_t^{ER} \leq R^B \leq R^C + r_t^{DW}$.

The proposition shows that the Fed's corridor rates impose restrictions on the equilibrium spread between loans and reserves. In particular, this spread is bounded by the width of the corridor rates.[26] Several insights follow from the proposition. First, equation (13) captures a first-order effect of monetary policy. The choice of reserve holdings affects the expected penalties incurred in the interbank market. Thus, although risk aversion may reinforce this effect, monetary policy has effects in a risk-neutral environment through this channel. Second, if $r^{DW} = r^{ER}$, the marginal value of liquidity is independent of ω. This implies that under risk neutrality, changes in second, or higher order moments of F_t do not affect portfolio choices. Moreover, the proposition also underscores the role of the kink in χ : when $r^{DW} = r^{ER}$, χ has no kink. This means that the Fed cannot target R^B and R^C simultaneously because the bank's portfolio and all interest rates are determined uniquely by the choice of $r^{DW} = r^{ER}$. There is no scope for open-market operations.

Now, defining the return to an additional unit of leverage—the bank's leveraged return is:

$$R^{L^*} \equiv \underbrace{\left(R^B - R^D\right)}_{\text{Arbitrage on Loans}} - \left(\underbrace{\left(R^B - R^C\right)L^* + \mathbb{E}_{\omega'}\left[R^{\chi}\left(1,L^*\right)\right]}_{\text{Optimal Liquidity Ratio Cost}}\right).$$

An equilibrium for $\gamma = 0$ is characterized by:

Proposition 6 (Linear Characterization) *When* $\gamma = 0$, *in equilibrium,* $\Omega = R^B + \max\left\{\kappa R^{L^*}, 0\right\}$, *and*

$$w^{*d} = \begin{cases} 0 \text{ if } R^{L^*} < 0 \\ [0,\kappa] \text{ if } R^{L^*} = 0 \\ \kappa \text{ if } R^{L^*} > 0 \end{cases} \text{ and } \quad \text{div} = \begin{cases} 0 \text{ if } \beta v \Omega > 1 \\ [0,1] \text{ if } \beta v \Omega = 1 \\ 1 \text{ if } \beta v \Omega = 1 \end{cases}$$

In a steady state, $\beta v \Omega = 1, div = \Omega - 1$. *A steady state falls into one of the following cases:*[27]

Case 1 (non-biding leverage constraint steady state $(\mu = 0)$***).*** *The steady-state value of equity,* E_{ss}, *is sufficiently large such that* $R_{ss}^B = 1/\beta$ *is feasible and the following conditions hold:*

$$R^{L^*} = \left(1/\beta - 1/\beta^D\right) - \left(\left(1/\beta - R^C\right)L^* + R^{\chi}\left(1,L^*\right)\right) = 0, R^E = \frac{1}{\beta}.$$

[26]Under risk aversion, a risk premium adjustment would emerge and the loan-reserve spread could exceed the width of the bands. However, the corridor system would still impose bounds on the interest spread because the liquidity risk premium is also affected by the width of the bands.

[27]Unless leverage constraints are binding, a transition toward a steady state is instantaneous as in other models with linear bank objectives (see e.g. Bigio, 2014). If dividends cannot be negative and equity is low, banks would retain earnings until they reach a steady state value of equity E_{ss}, consistent with proposition 6.

Case 2 (binding leverage contraint steady state $(\mu > 0)$***).*** *E^{ss} is such that for $w^{*d} = \kappa$: $R^B_{ss} > 1/\beta$, and*

$$R^{L^*} = \left(R^B - 1/\beta^D\right) - \left(\left(R^B - R^C\right) L^* + R^{\chi}\left(1, L^*\right)\right) > 0, \quad \left(R^B + \kappa R^{L^*}\right) = \frac{1}{\beta}. \tag{14}$$

Proposition 6 characterizes two potential classes of steady states. If at steady state, capital requirements do not bind, the choice of $\left\{r^{ER}_{ss}, r^{DW}_{ss}\right\}$ and $M0_{ss}$ can affect R^C but not R^B. If instead capital requirements are binding, different combinations of $\left\{r^{ER}_{ss}, r^{DW}_{ss}\right\}$ and $M0_{ss}$ can affect R^C separately from R^B, as long as these rates satisfy (14).

3.3 Limit Case II: No Withdrawal Shocks $(\Pr(\omega = 0) = 1)$.

A special case that provides additional insights is one in which there are no withdrawal shocks, $\Pr[\omega = 0] = 1$. For this case, there is no difference between the portfolio decisions of risk-neutral and a risk averse banker—although their dividend policies may differ because the intertemporal elasticity of substitution may vary. Without uncertainty, the value of the portfolio problem is:

$$R^B + \max_{w_d \in [0,\kappa]} w_d \left[\left(R^B - R^D\right) + \left(\max_{L \in \left[0, \frac{1+\omega^d}{\omega^d}\right]} - \left(\left(R^B - R^C\right) L + \chi\left(L - \rho\right)\right)\right)\right].$$

An equilibrium with deterministic shocks satisfies the following analogue of Proposition 6:

Proposition 7 *In equilibrium, $R^C + r^{ER}_t \leq R^B \leq R^C + r^{DW}_t$. Moreover, in an equilibrium with strictly positive reserve holdings, $L^* = \rho$, the value of the bank's portfolio is given by $\Omega = R^B + \kappa \max\left\{\left(\left(R^B - R^D\right) - \left(R^B - R^C\right)\rho\right), 0\right\}$ and the banker's policies are as follows:*

$$w^{d*} = \begin{cases} 0 \text{ if } R^B < R^D + \left(R^B - R^C\right)\rho \\ [0,\kappa] \text{ if } R^B = R^D + \left(R^B - R^C\right)\rho \\ \kappa \text{ if } R^B > R^D + \left(R^B - R^C\right)\rho \end{cases} \quad \text{and } w^{c*} = \rho w^{d*}.$$

According to this proposition, in a monetary equilibrium, i.e., $M0_t > 0$, a banker sets the reserve ratio to ρ.[28] Since $L^* = \rho$ is independent of $\left\{r^{ER}_t, r^{DW}_t\right\}$, as long as this implementability constraint is satisfied, changes in $\left\{r^{ER}_t, r^{DW}_t\right\}$ have no effects on allocations. This is an important observation because it underscores the role of liquidity risk: the corridor rates affects equilibrium allocations only if there is liquidity risk because r^{DW}_t (r^{ER}_t) acts like a penalty (prize) for holding reserves below (above) ρ. Without risk, increasing r^{DW}_t is like increasing the penalty of a constraint that is already satisfied for a lower punishment. A similar insight holds for r^{ER}_t.

[28]When shocks are deterministic, banks control the amount of liquidity holdings by the end of the period. In that case, they choose zero holdings of reserves if they can either borrow them cheaply from the discount window, $r^{DW}_t \leq R^B_t - R^C_t$, or would not hold loans if the interest rate on excess reserves exceeds $R^B - R^C$. In equilibrium, reserves and loans are made so $r^{ER}_t \leq R^B_t - R^C_t \leq r^{DW}_t$ is an implementability condition for the Fed's policy.

Overall, for this limit case, since banks hold a liquidity ratio of $L^* = \rho$ per deposit, reserve requirements act like a tax on financial intermediation: for every deposit, banks must maintain ρ in reserves, which earn no return, as opposed to loans. The rest of the equilibrium is characterized by Propositions 3 and 4.

3.4 Limit Case III: Zero Lower Bound ($r^{DW} = r^{ER} = 0$).

Consider the ZLB as states that have no liquidity risk, i.e., $\chi_t(\cdot) = 0$. We focus on the case in which $r_t^{DW} = r_t^{ER} = 0$.[29,30]

Thus, Ω becomes:

$$\Omega = R^B + \max_{w_d} w_d \left(\left(R^B - R^D\right) + \left(\max_{L \in \left[0, \frac{1+\omega^d}{\omega^d}\right]} \left(R^B - R^C\right) L \right) \right).$$

An equilibrium with strictly positive holdings of both loans and reserves requires $R^B = R^C$, as reserves are only valued because of their monetary return. Because the risk of withdrawals plays no role, the asset composition of the individual bank's balance sheet is indeterminate. If, in addition, capital requirements do not bind, then $R^B = R^C = R^D$ so $\Omega = R^B + \kappa \max\left\{R^B - R^D, 0\right\}$. In summary, we have the following proposition.

Proposition 8 *A monetary equilibrium at the ZLB, $r_t^{DW} = r_t^{ER} = 0$, satisfies, $R_t^B = R_t^C \geq R_t^D$. The inequality is strict if and only if capital requirements are binding.*

Notice that at the ZLB, the Fed has effects on lending *if* the capital requirement is binding. By carrying out open-market operations and varying the relative return on reserves, the Fed can affect lending.

4 Calibration

4.1 Dispersion of Deposit Growth (F_t)

Our model requires a specification of the random withdrawal process for deposits, F_t. To obtain an empirical counterpart for this distribution, we use information from individual US commercial bank Call Reports. The Call Reports contain balance sheet information obtained from regulatory filings

[29] Absence of liquidity risk also arises when banks are not subject to capital requirements, $R^C \geq R^D$ and $r^{ER} = 0$. In this case, banks accumulate enough reserves so that they can fully cover deposits withdrawals. As long as deposits have a higher return or capital requirements bind, banks remain exposed to liquidity risk and individual banks have a determined portfolio, unlike standard monetary models (see e.g., Buera and Nicolini (2013) for a model with financial frictions and a cash in advance constraint on households' consumption).

[30] The bounds on $r_t^{DW}, r_t^{ER} \geq 0$ arise naturally in this setup. If $r_t^{DW} = r_t^{ER}$, one could argue that banks could request to hold currency—as opposed to electronic reserves. If $r^{DW} < 0$, banks would make infinite profits by borrowing from the Fed.

collected by the Federal Deposit Insurance Corporation (FDIC). This information is compiled quarterly, so we define a period in our model as one quarter. We use information from 1990 Q1–2010 Q4.

In our model, all banks experience the same expected growth rates in deposits during the lending stage. Deviations from average growth during the lending stage are directly associated with ω, the withdrawal shocks in the model. Hence, the distribution of the deviations from average deposit growth rates is directly associated with F_t. Thus, we calibrate F_t to that distribution.

Deposits in our model have no obvious empirical counterpart. In our model, demand deposits are the only liability whereas in practice commercial banks have other liabilities that include bonds and interbank loans, and long-term deposits such as time and savings deposits, in addition to demand deposits. To obtain an empirical counterpart of F_t, we use total deposits which include time and saving deposits and demand deposits. We make this choice for several reasons. The first reason is practical: total deposits feature a trend that is similar to the growth of all bank liabilities, unlike demand deposits. A second reason is that we do not want to attribute all deposit funding to demand deposits. Demand deposits feature substantially more dispersion than total deposits, which could exaggerate the liquidity costs associated with monetary policy changes.

The histogram in Figure 1 reports the empirical frequencies of the cross-sectional deviations of growth rates from the mean growth rates of the cross-section, for each bank-quarter observation. The bars in Figure 1 report the pre-crisis frequencies for the 2000 Q1–2007 Q4 sample of cross-sectional dispersion in deposit growth rates. The solid curve is the analogue for a post-crisis sample, 2008 Q1–2010 Q4. The dispersion in growth rates in Figure 1 suggests that total deposits are consistent with substantial liquidity risk, according to our model. However, the comparison among both samples shows only a minor change in the distribution during the crises—with a slightly more concentrated mass on the left.[31]

Given the constructed empirical distribution, we fit a logistic distribution $F(\omega, \mu^{\omega}, \sigma^{\omega})$ with $\mu = -0.0029$ and $\sigma^{\omega} = 0.022$. We conduct a Kolmogorov-Smirnov goodness-of-fit hypothesis test. We cannot reject that the empirical distribution is logistic—with 50 percent confidence. Appendix G provides additional details on how we construct the empirical distribution of deposit growth-rate deviations. That appendix also investigates the empirical soundness of other features of our model.[32]

[31]We use this information and a shutdown in the interbank market to study when we investigate hypothesis 3.

[32]Our model predicts that the growth of equity is highly correlated—though not perfectly correlated—with the behavior of deposits. In Appendix G, we analyze this correlation in the data and find a positive correlation of about 0.17. This should be expected since our model does not capture credit risks , variations in security prices, differences in dividend policies, or shifts in operating costs. We also discuss the validity of the time-independence of ω. We show that our deposit growth measures show a positive but small autocorrelation of about 0.17.

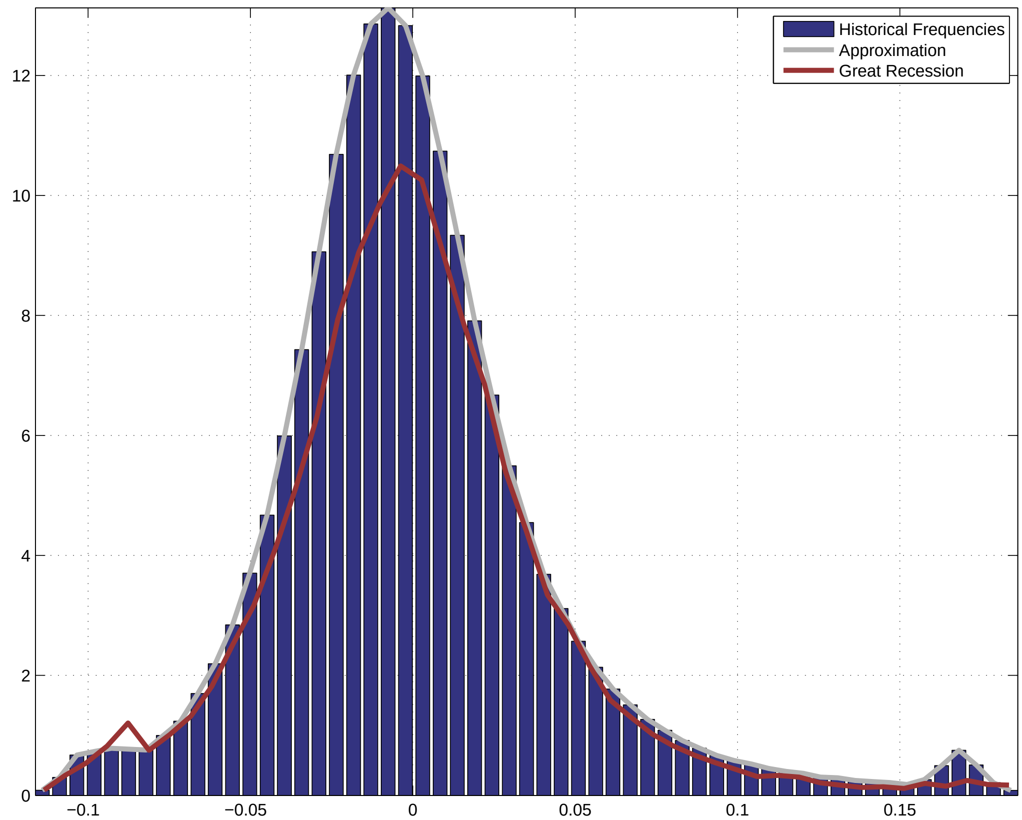

Figure 1: Histogram of Deviations from Cross-Sectional Mean Growth Rates for Total Deposits. For every bank-quarter observation, the histogram reports frequencies for deviations of the growth rate of total deposits relative to the cross-sectional average growth of total deposits in a given quarter.

4.2 Parameter Values

The values of all parameters are listed in Table 1. We need to assign values to the following parameters $\{\kappa, \rho, \beta, \delta, \xi, \gamma, \epsilon, r^{ER}, r^{DW}, R^d\}$. We set the capital requirement, $\kappa = 15$, and the reserve requirement, $\rho = 0.05$, to be consistent with actual regulatory parameters: this choice corresponds to a required capital ratio of 9 percent and a reserve ratio of 5 percent. We set $\delta = 0$ so that loans become one-period loans. We set risk aversion to $\gamma = 0.5$.

The value of the loan demand elasticity given by the inverse of ϵ is set to 1.8, which is an estimate of the loan demand elasticity by Bassett et al. (2010).[33] Finally, we set the discount factor so as to match a return on equity of 8 percent a year. This implies $\beta = 0.985$. The interest rate on deposits is set to $R^D = 1$. We set the value of $\xi = 0.5$ so that the Fed funds rate is in the middle of the corridor rate, as usually occurs in practice. We set $r^{ER} = 0$, which is the

[33] This value for the elasticity of loan demand is consistent with the microfoundation provided in Appendix D, based on estimates of the elasticity of labor supply in the lower range.

Table 1: Parameter Values

	Value
Capital requirement	$\kappa = 15$
Discount factor	$\beta = 0.985$
Risk aversion	$\gamma = 0.5$
Loan maturity	$\delta = 0$
Bargaining parameter	$\xi = 0.5$
Reserve requirement	$\rho = 0.05$
Loan demand elasticity	$1/\epsilon = 1.8$
Discount window rate (annual)	$r^{DW} = 2.5$
Interest on reserves (annual)	$r^{ER} = 0.$

pre-crisis interest rate on reserves paid by the Federal Reserve. The discount window rate is set to $r^{DW} = 2.5$ percent expressed at annualized rates. These choices deliver a fed funds rate of $r^{FF} = 1.25$ percent.[34] Finally, we assume that the Fed targets price stability so $R^C = 1$.

4.3 Steady-State Equilibrium Portfolio

We start with an analysis of the equilibrium portfolio at steady state and investigate the effects of withdrawal shocks on banks' balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation (1) evaluated at the loan price that clears the loans market, according to condition (10), and the equilibrium probability of matching in the interbank market.

The left panel of Figure 2 shows the probability distribution of the reserve deficits during the balancing stage and the penalty associated with each deficit; the mass of the probability distribution is rescaled to fit in the same plot. The penalty function χ has a kink at zero, because $r^{DW} > r^{ER}$. Notice that the distribution of the reserve deficits inherits the distribution of the withdrawal shock, since the reserve deficit depends linearly on the withdrawal realization. Because in equilibrium, there is an average excess surplus, the distribution's mean is above zero.

The right panel of Figure 2 shows the distribution of equity growth as a function of ω. In equilibrium, banks that experience deposit inflows will increase their equity, whereas those that experience outflows see their equity shrink. Because the penalty inflicts relatively higher losses to outflows than to the benefits from inflows, the distribution of equity growth is skewed to the left. In particular, there is a fat tail with probabilities of losing more than 1 percent of equity in

[34] Since we consider a steady state without inflation, this is also the real interest rate.

a given period, while the probability of growing more than 0.8 percent in a period is close to nil.

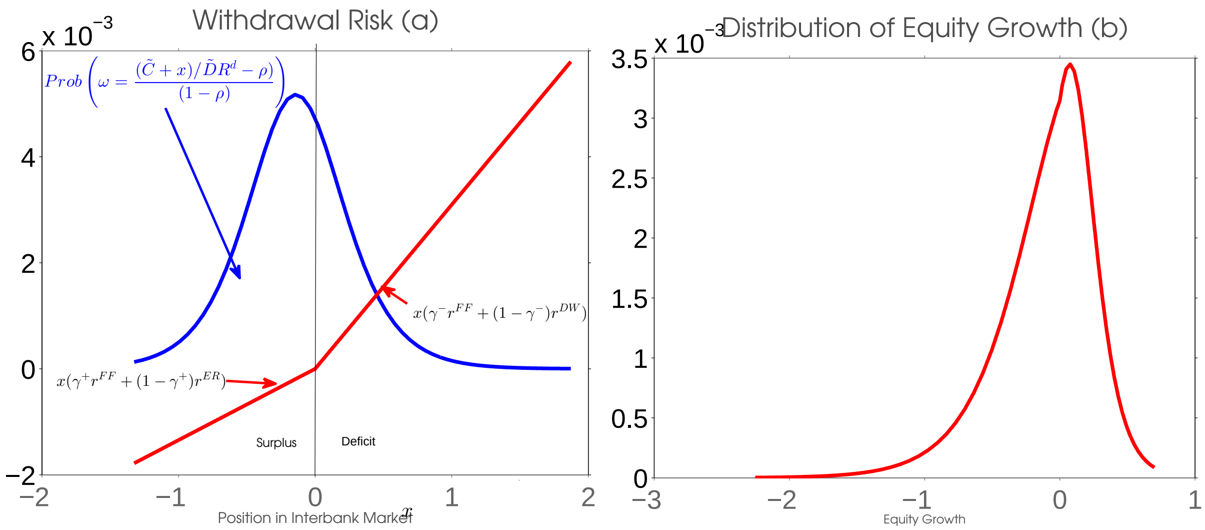

Figure 2: Portfolio Choices and Effects of Withdrawal Shocks

4.4 Policy Functions at Given Prices

We start with a partial equilibrium analysis of the model by showing banks' policy functions for different loan prices. Figure 3 reports decisions for reserves, loans and dividends, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risks, expected returns, and expected equity growth rates for different loan prices q. The policies correspond to the solution to the Bellman equation (4) for different values of q and fixing the probability of a match in the interbank markets at steady state. The solid dots in Figure 3 are the values associated with the equilibrium q.

As Figure 3 shows, the supply of loans is decreasing in q —i.e., increasing in the return on loans. Instead, reserve rates are *increasing* in q. As the loan prices decrease, loans become more profitable which leads banks to keep a lower fraction of their assets in low return assets, i.e., reserves.

In addition, dividends are increasing in q due to a substitution effect: when returns on loans are high, banks cut dividend payments to allocate more funds to profitable lending—recall that we have assumed that $\gamma < 1$ so that the substitution effect dominates the wealth effect. Exposure to liquidity risk, measured as the standard deviation of the cost of rebalancing the portfolio $\chi(x)x$, is also decreasing in loan prices, reflecting the fact that banks' asset portfolios become relatively more illiquid when loan prices decrease.

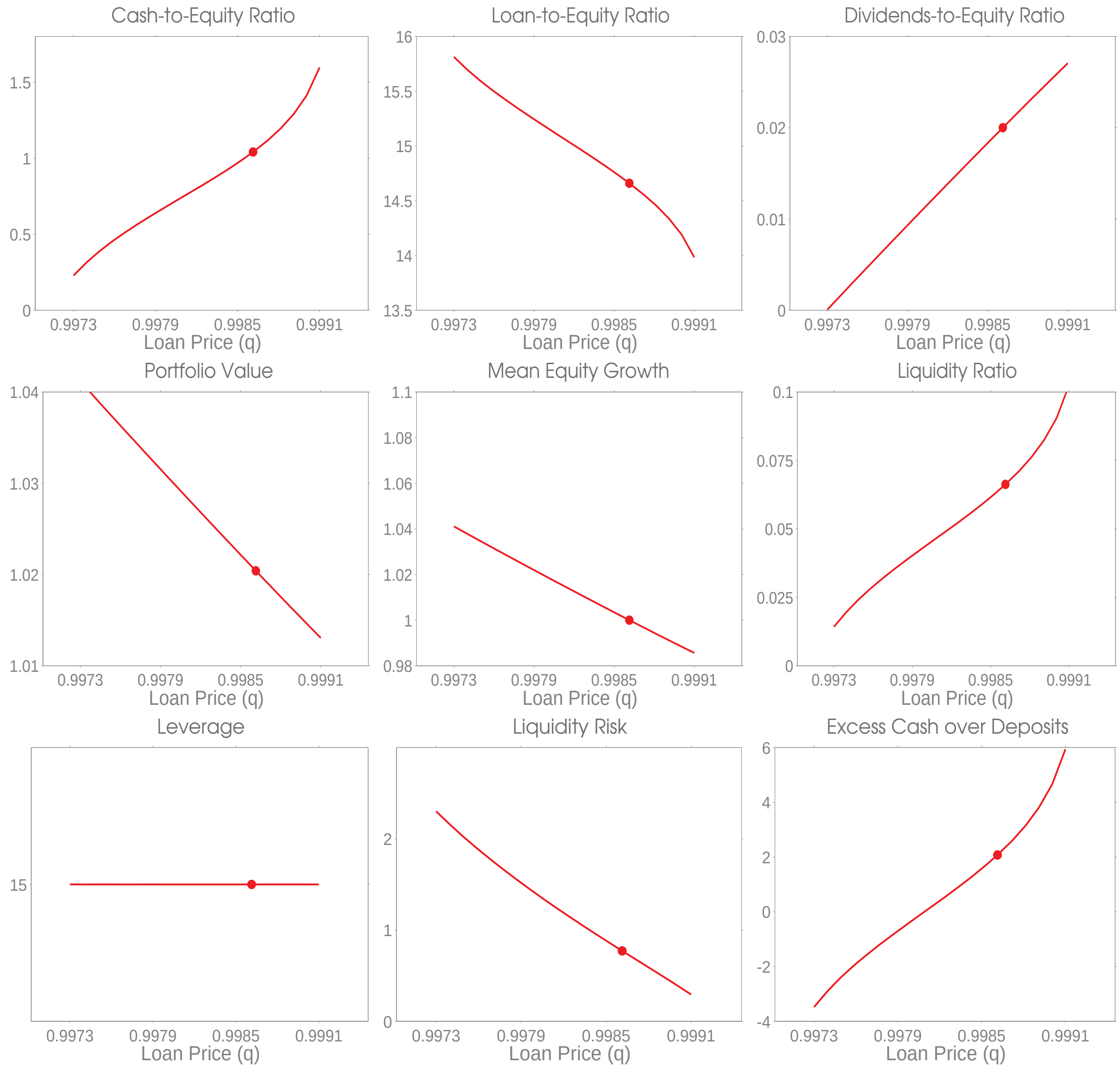

Figure 3: Policy Function for Different Loan Prices. The probability of matching in the interbank market is fixed at the steady-state value.

5 Transitional Dynamics

This section studies the transitional dynamics of the economy in response to different shocks associated with Hypotheses 1–5. The shocks we consider are equity losses, a tightening of capital requirements, an increase in the dispersion of withdrawals, a shutdown of the interbank market, credit demand shocks, and changes in the discount window and interest rate on reserves. Shocks are unanticipated upon arrival at $t = 0$, but their paths are deterministic for $t > 0$. In all cases, we assume that the shock follows $\varepsilon_t = \rho \varepsilon_{t-1} \forall t \geq 1$, where ε denotes the deviation from the steady state. We set $\rho = 0.8$, so that the half-life of the shock is about three years.[35] Throughout the experiments, we consider a monetary policy regime such that the Fed has a zero inflation target $R^C = 0$— i.e., the Fed performs open-market operations altering $M0_t$—to maintain price stability: $p_t = p$. [36]

5.1 Equity Losses

We begin with a shock that translates into a sudden, unexpected decline in bank equity. This shock captures an unexpected rise in non-performing loans, security losses, or off-balance-sheet losses left out of the model.[37] Figure 4 illustrates how bank balance sheets shrink in response to 2 percent equity losses. The top panels show the evolution of total lending, total reserves, and liquidity risk, and the bottom panel shows the level of equity, return on loans and the dividend rate.

To understand these dynamics, recall that all bank policy functions are linear in equity. Thus, holding prices fixed, a loss in equity should lead to a proportional 2 percent decline in loans and reserves. However, the contraction in loan supply also generates a drop in loan prices on impact —through movement along the loan demand curve. The reduction in q leads to an increase in loan returns through the transition. As a consequence of the higher profitability on loans, reserve holdings fall relatively more than loans. Banks shift their portfolios toward loans exposing themselves to more liquidity risk. The overall return to the banks' portfolios also increases after the shock. With this, dividends fall as their opportunity cost increases. The increase in bank returns and lower dividends leads to a gradual recovery of initial equity losses. As equity recovers, the economy converges to the initial steady state and the transition is quick; the effects of the shock cannot be observed after six quarters.

When $\delta > 0$, there is an additional amplification effect not shown here. The reduction in the supply of credit further lowers q, and this in turn, lowers marked-to-market equity, E, beyond the initial impact of the shock. All other responses are therefore amplified.

[35]The assumption of unanticipated shocks is mainly for pedagogical purposes. In fact, it is relatively straightforward to compute the model to allow for aggregate shocks, which are anticipated. Due to scale invariance, we would not have to keep track of the cross-sectional distribution of equity anyway.

[36]We assume this not only for illustrative reasons but also because in the context of the Great Recession, the core personal consumption expenditures index (PCE) remained close to 1 percent. It is straightforward to consider alternative monetary policy regimes.

[37]One way to incorporate this explicitly in the model would be to consider specific shocks to loan default rates. To the extent that equity is the only state variable, the analysis of the transitional dynamics is analogous to studying the evolution of the model under a richer structure for loans.

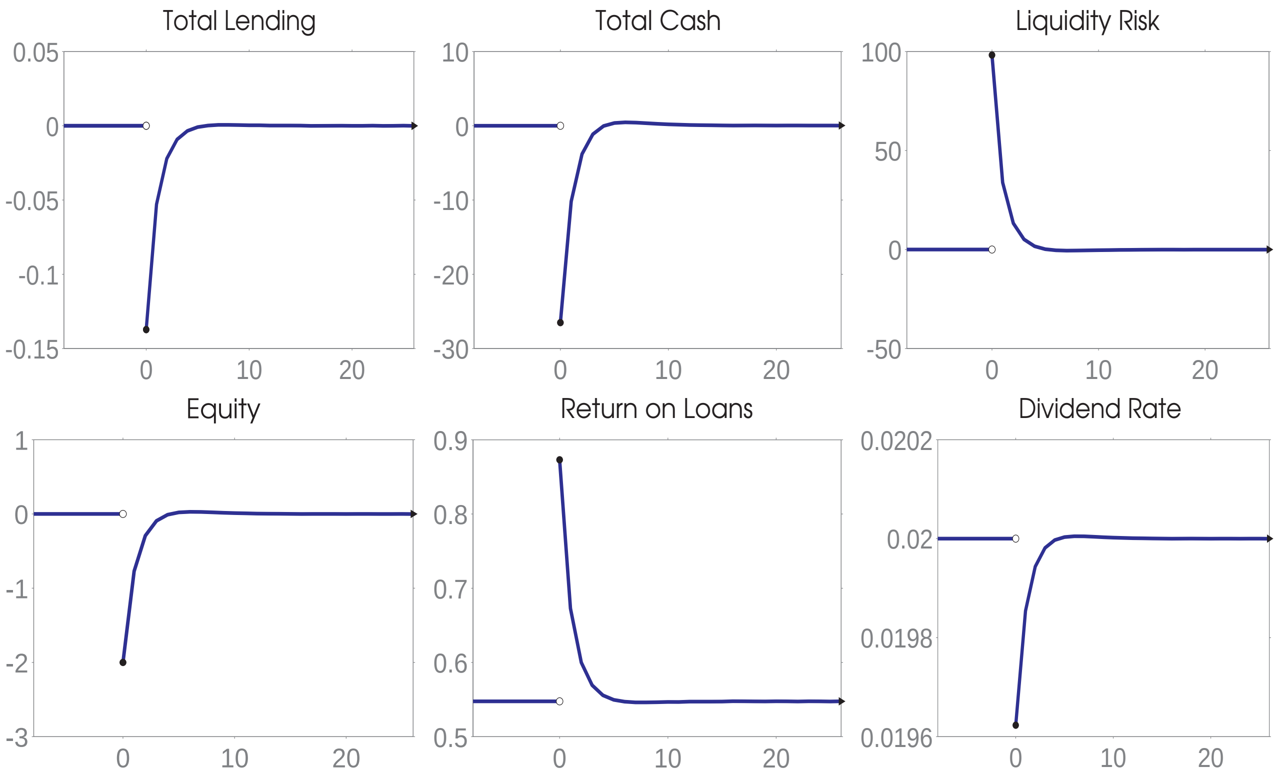

Figure 4: Impulse Response to Equity Losses

5.2 Capital Requirements

The effects of a sudden and transitory a reduction in κ, are shown in Figure 5. The shock is a 10 percent decrease in κ which is associated with a 1 percent increase in the capital ratio of banks for the calibrated level of leverage. Notice that because the capital requirement is binding at the steady state, the reduction in κ implies a tightening of the capital requirement.

The short-run behavior of the transition is very similar to the behavior after equity losses. As with equity losses, the contraction in capital requirements reduces the supply of loans because as the constraint gets tighter, banks must operate as if they had less equity. As a result, the return on loans increase and the liquidity ratio is reduced. Notice that because of the marked-to-market capital requirement constraint, the initial tightening in capital requirements is tightened further due to the general equilibrium effect on loan prices.

In the medium term, equity begins to exceed its steady-state value. This happens because the return on loans increases and banks pay fewer dividends. Eventually, the increase in equity overcomes the increase in capital requirements. Ultimately, the economy converges back to a steady-state level of equity as the capital requirement shock converges back to its original level.

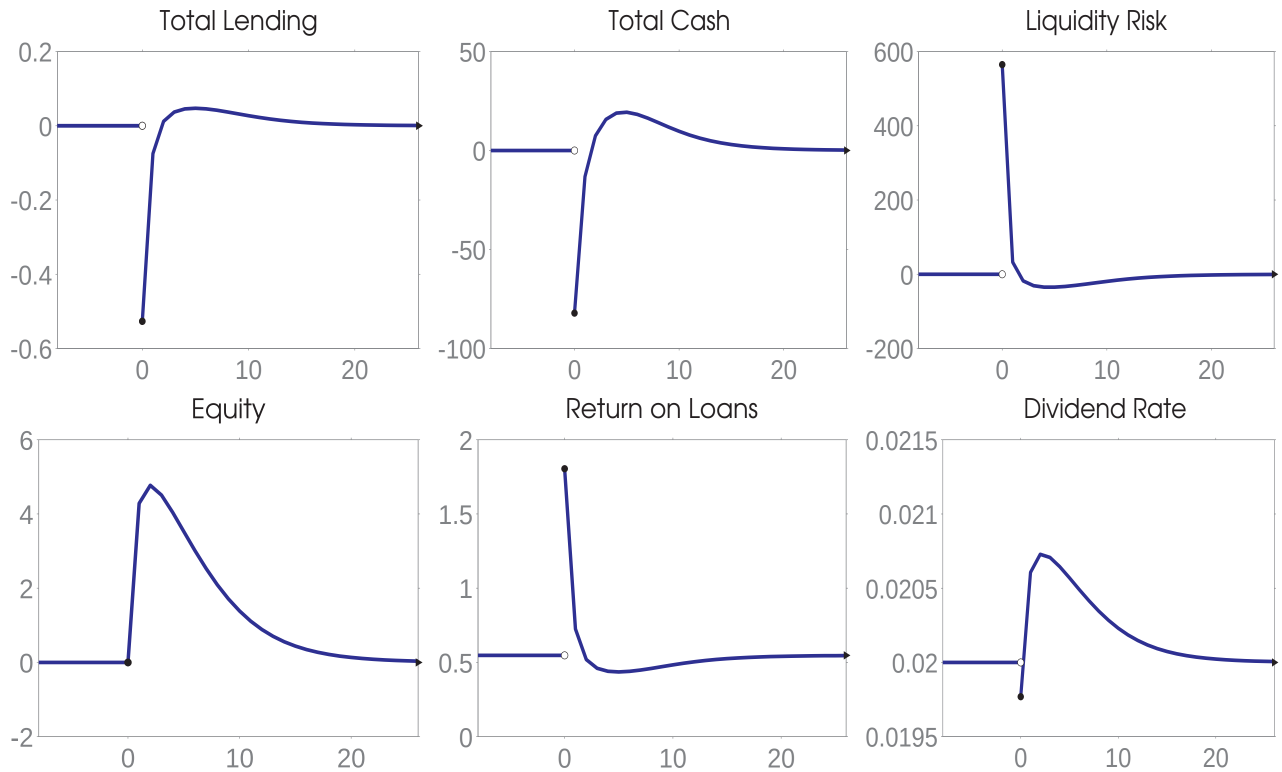

Figure 5: Impulse Response to a Tightening in Capital Requirements

5.3 Interbank Market Shocks

5.3.1 Bank-Run Shocks

Here we study the possibility of a bank run. We consider a 5 percent probability that all the deposits are withdrawn from a given bank—i.e., $\omega = 1$. These bank runs are on individual institutions, since we maintain the assumption that deposits are not withdrawn from the banking system as a whole.[38] The effects of this shock are illustrated in Figure 6.

The risk of a bank run generates an increase in liquidity risk, leading banks to hoard reserves. Because reserve requirements are constant, this means that banks accumulate more excess reserves. Notice that the liquidity risk is still about three times larger than in the steady state although banks hold more reserves. Since the Fed's objective is a zero inflation target, the Fed supplies reserves to meet this target. Naturally, higher liquidity costs induce a decline in the supply of loans, since banks substitute loans for reserves. In equilibrium, this leads to an increase in the price of loans and a decline in the aggregate volume of lending.

In tandem, banks respond to the risk of a bank run by cutting dividend payments. Although higher liquidity costs are associated with lower returns, the contraction in loan supply generates a more-than-compensating increase in expected bank returns. This leads to an increase in equity

[38]Thus, we adjust F accordingly by assuming a 5 percent probability of a large inflow of deposits. It is also possible to extend the model to study system-wide bank runs, as in Uhlig (2010) (see also Robatto, 2013).

over time. As equity grows, this mitigates the fall in lending ratios. Eventually, lending rises above its steady-state value. This is because several quarters after the shock is realized, the effect on bank equity compensates for the portfolio effect as the shock begins to decay.

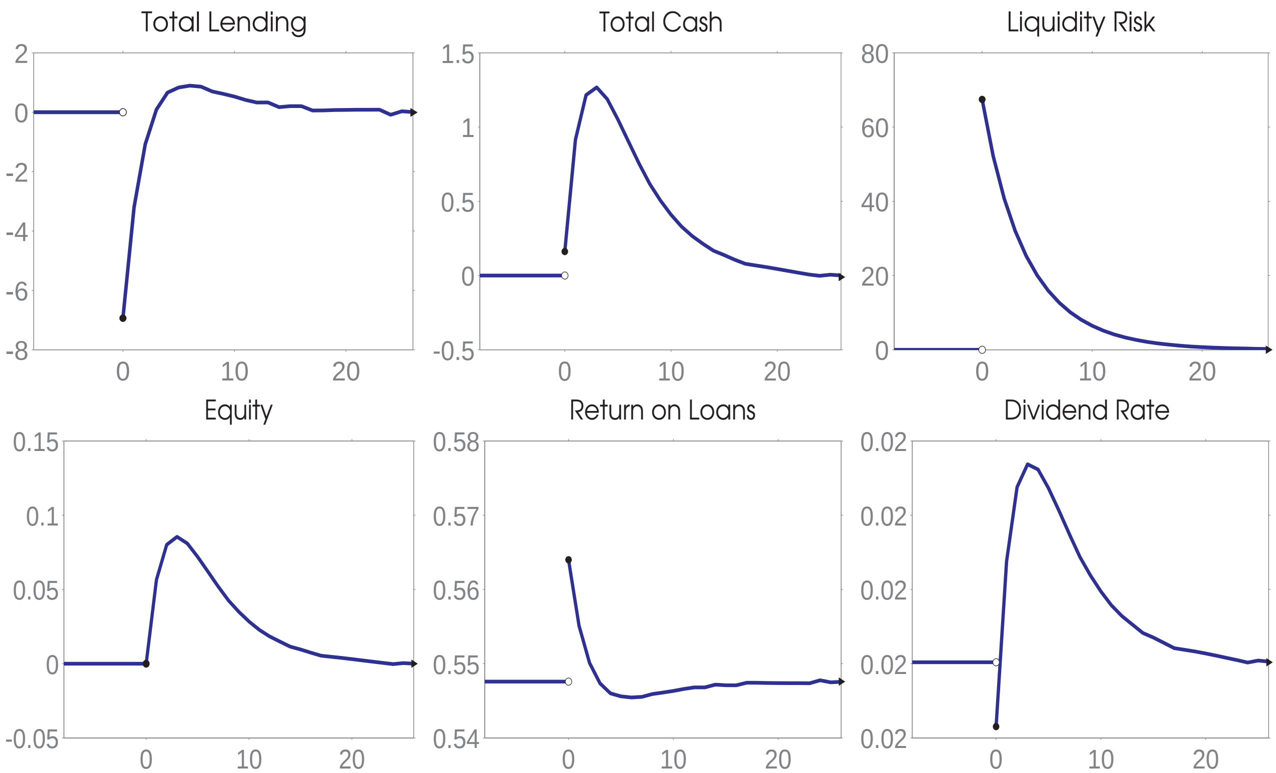

Figure 6: Impulse Response to a Bank-Run Shock

5.3.2 Interbank Market Shutdown

Disruptions in the interbank market can be studied through shocks that drive the probability of a match in the interbank market to zero.[39] Hence, reserves are borrowed (lent) only from (to) the Fed. Because banks that face a reserve deficit borrow directly at r^{DW}, liquidity risk increases. The effects of the interbank market freeze are shown in Figure 7. Overall, the effects are similar to the bank-run shock we describe earlier.

[39]A recent macroeconomic model of endogenous interbank market freezing due to asymmetric information with one-period lived banks is Boissay et al. (2013).

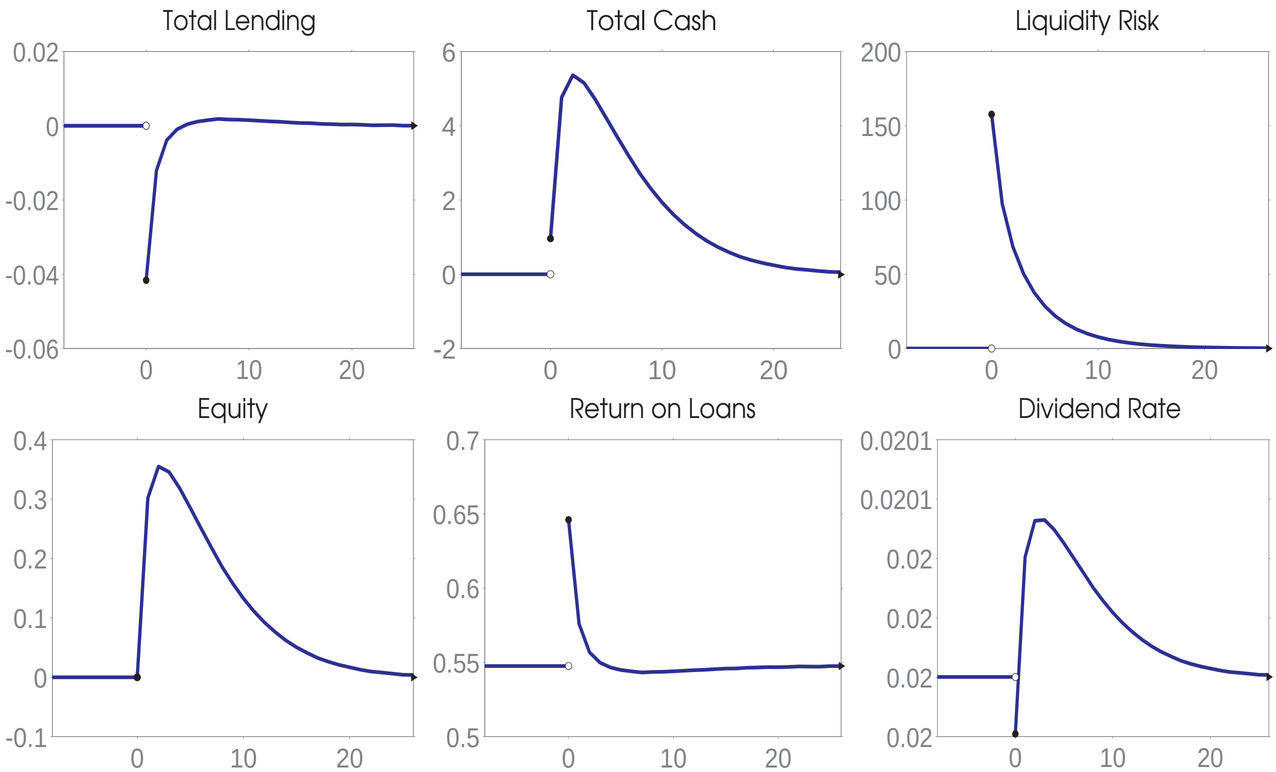

Figure 7: Impulse Response to Freezing in Interbank Markets

5.4 Credit Demand

The effects of negative credit demand shocks are captured through a decline in Θ_t. Figure 8 illustrates the effects of a negative temporary shock to Θ_t.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above, because all of the prior shocks cause a contraction in the supply of loans and an increase in the return on loans. In contrast, demand shocks cause a decline in the return on loans and a shift along the supply curve. As a result, banks shift their portfolios toward reserves as the opportunity cost of holding reserves lowers. The liquidity risk almost vanishes. Initially, banks respond by paying higher dividends due to the overall decline in their portfolio returns. The reduction in returns and dividend increments brings equity below the steady state. As the shock decays around a year and a half later, the economy follows a transition similar as to the shock to equity, slowly increasing lending rates and reducing dividend rates until equity returns to the steady state.

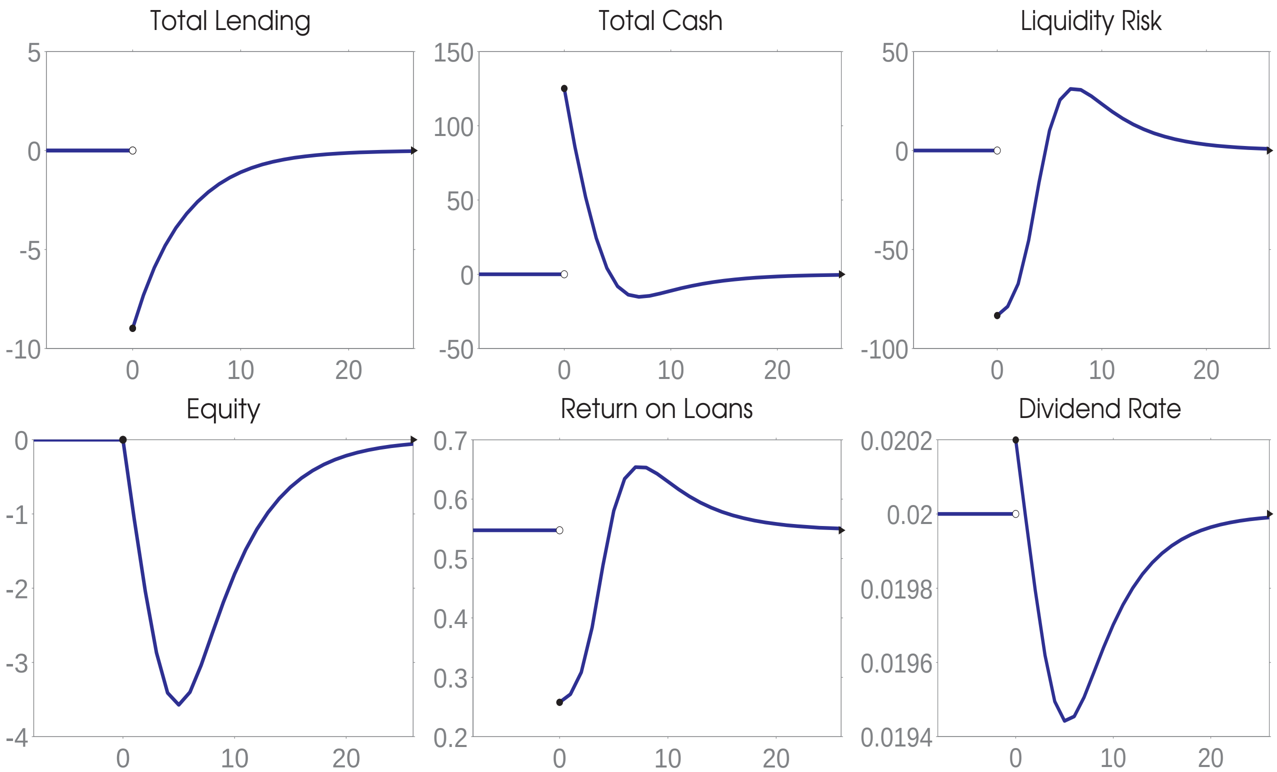

Figure 8: Impulse Response to Credit Demand Shock

5.5 Policy Rates

5.5.1 Discount Window

We now analyze the effects of interest rate policy shocks, depicted in Figure 9. In the experiment, we study a positive shock of 100 basis points (bps) to the discount window rate, expressed at annualized rates. Banks respond to this increase by reducing lending. Policy effects are similar to the effects of shocks that increase liquidity costs. In addition, there is a high pass-through from the policy rate to the return on loans.

5.5.2 Interest on Reserves

A shock to the interest on excess reserves works similarly to an increase in the discount window rate, since both increase the return of holding reserves. We study a shock that raises this rate from 0 bps to 100 bps (annualized), a shock that corresponds to the recent Fed policy of remunerating excess reserves. The effect of this policy is illustrated in Figure 10. The shock makes reserves relatively more attractive. In response, banks reallocate their portfolio from loans toward reserves.[40]

[40] Notice that liquidity risk does not decline despite the increase in cash holdings by banks. This occurs because the increase in the interest rate on excess reserves leads to larger differences in returns between banks with a surplus and banks with a deficit of reserves.

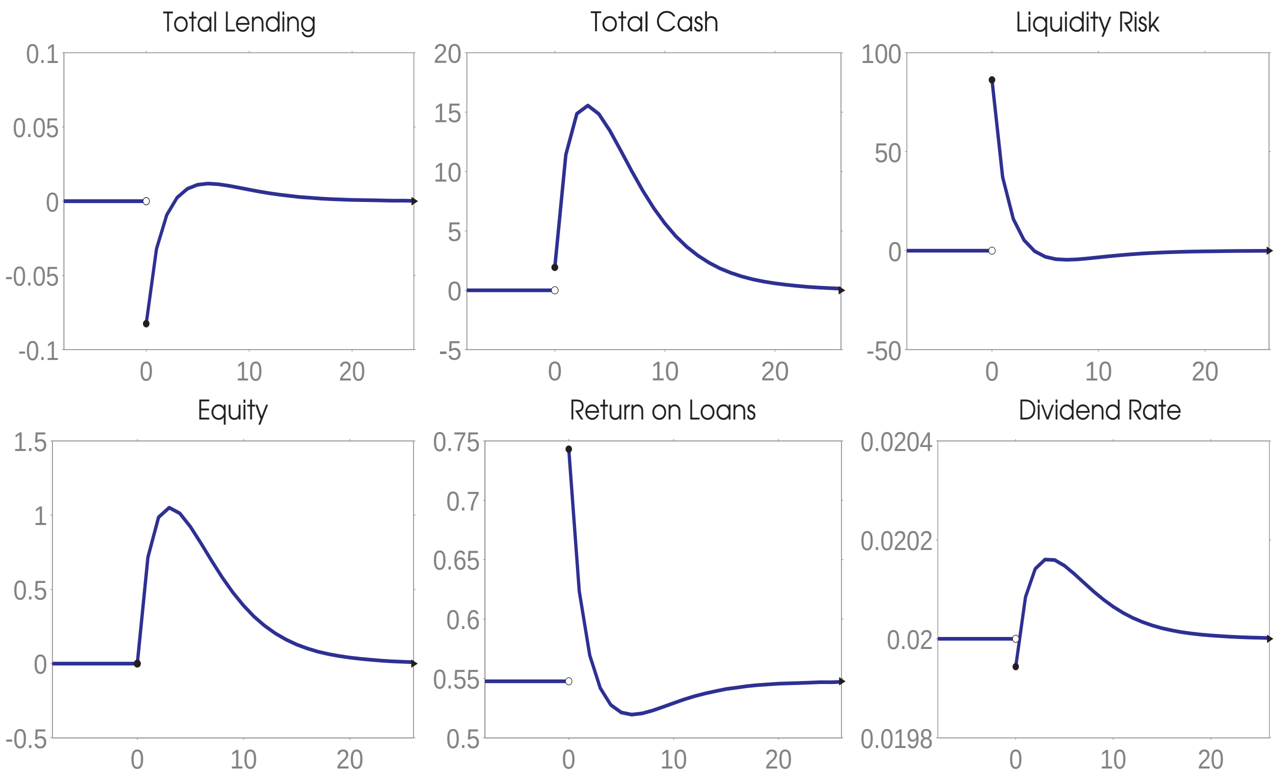

Figure 9: Impulse Response to a Rise in the Discount Window Rate

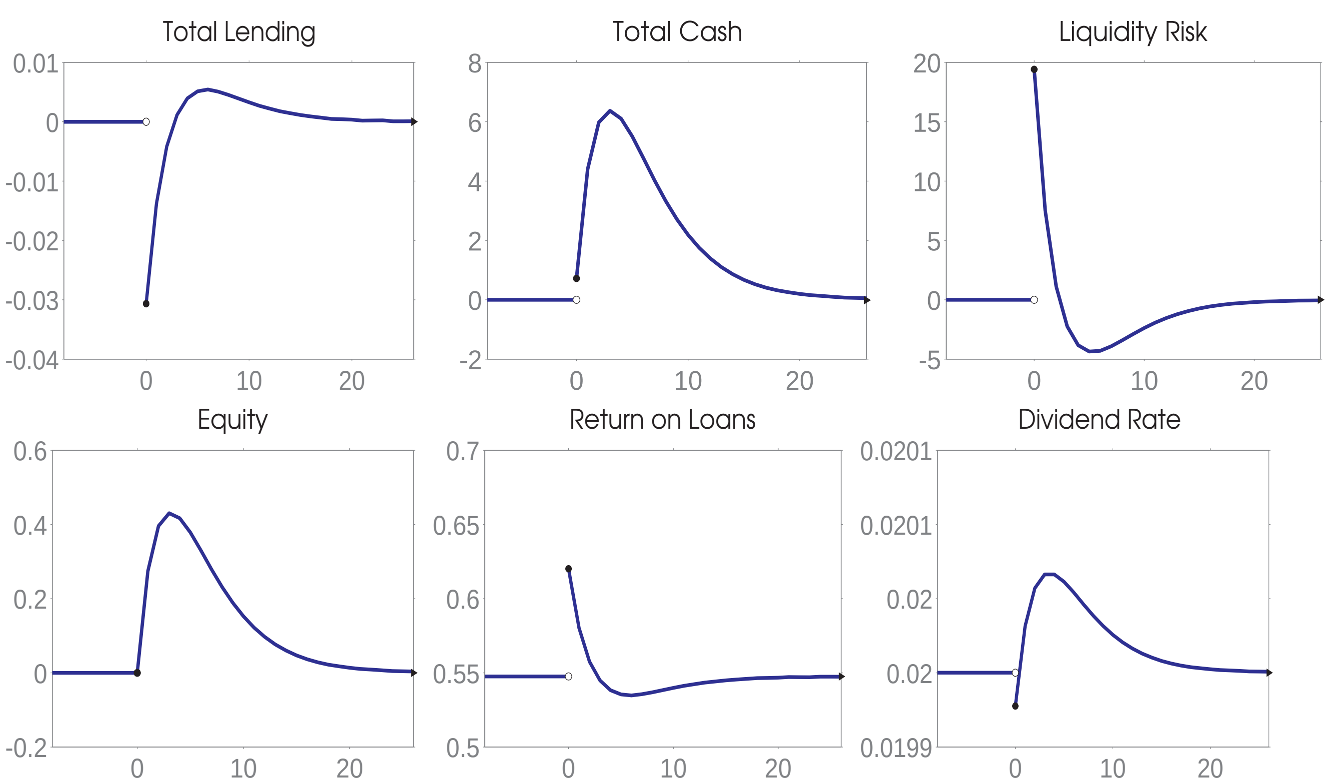

Figure 10: Impulse Response to a Shock to the Interest Rate on Excess Reserves

5.6 Unconventional Open-Market Operations

Finally, we study loan purchases by the Fed. We study the effects of loan purchases amounting to 2 percent of the outstanding stock of loans at the steady state. We also assume that the Fed gradually reverses the operation in about four years. Unconventional open-market operations boost total lending in the economy, as shown in Figure 11. However, there is a partial crowding-out effect. Fed purchases lower the return on loans, which in turn leads private banks to lend less. In equilibrium, banks also hold more reserves. As a result, the transitions are similar to the transitions after a negative credit demand shock, with the difference that total bank lending increases because of the Fed's holdings. The reason being that the Fed's OMO reduce the effective demand for loans that banks face, as the Fed takes over part of their activity.

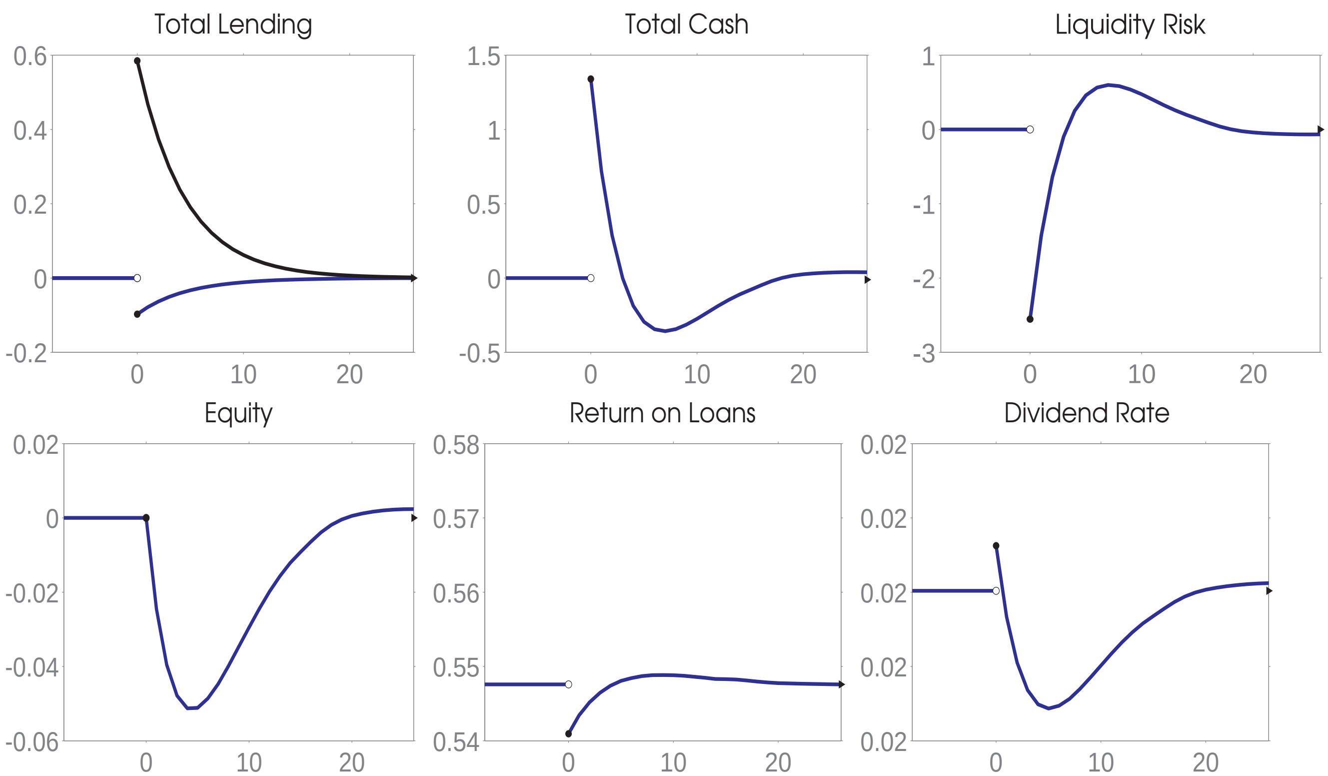

Figure 11: Impulse Response to Unconventional Monetary Policy

6 Application: Which Hypotheses Fit the Facts?

This section explores the possible driving forces that explain the holdings of excess reserves without a corresponding increase in lending by banks during the US financial crisis. Here, we discuss how the different shocks we studied in the preceding section fit the patterns we observe for the data. We first revisit some key facts about monetary policy, monetary aggregates and banking indicators during the recession that motivate our application.

6.1 Monetary Facts

Fact 1: Anomalous Fed Funds Rate Behavior. Panel (a) of Figure 12 plots the daily series for the overnight discount rate, the interest rate on reserves, the fed funds rate and the target rate. This figure shows that during the midst of the crisis, the fed funds rate exceeded both the target and the discount rate during several days. Later, at the beginning of the recession, the fed funds rate dropped to its lowest historical level for almost five years.

Fact 2: Fed Balance Sheet Expansion. As panel (b) shows, there has been a substantial increase in the assets held by the Fed, which corresponds to the various large-scale open-market operations programs carried out after the collapse of Lehman Brothers. Panel (c) shows the increase in the Fed's assets from direct lending to banks and mortgage-backed securities (MBS). This series is reported as a fraction of total bank credit (see the Appendix G for more details). This series was close to 0 percent before the crisis and reached 18 percent by the middle of 2014.

Fact 3: Excess-Reserve Holdings. The counterpart of fact 2 is a significant increase in the Fed's liabilities, especially reserves, as shown in panel (d). Notably, whereas prior to the crisis there were virtually no excess reserves, during its aftermath, excess reserves amount to almost 16 times greater than the amount of required reserves.

Fact 4: Depressed Lending Activities. Panel (e) shows the decline in commercial and industrial (C&I) lending during the crisis.

Fact 5: Drop in Money Multiplier. The large drop in the money multiplier for M1 summarizes facts 2, 3, and 4. This is shown in panel (f).

6.2 Banking Facts

Fact 6: Decline in Book-Value Leverage. Panel (a) of Figure 13 shows the decline in the tangible leverage —a measure that subtracts tangible assets from the book value of equity. From its peak at the middle of the crisis to 2010Q4, the average tangible leverage falls from 16 to about 12.

Fact 7: Increase Liquidity Ratio. Panel (b) of Figure 13 shows the behavior of the liquidity ratio, the ratio of liquid assets over total assets. Here, we take liquid assets to be the sum of reserves plus Treasury bills. The data show an increase from 6 percent to 12 percent for the same period. Interestingly, this finding implies that the increase in reserves, highlighted in fact 3, was not offset by the reduction in other liquid assets.

Fact 8: Bank Equity Losses. Panel (c) of Figure 13 shows the behavior of the realized returns on equity. This figure shows that at the beginning of the crisis, banks suffer large losses in equity.

Fact 9: Dividends. Panel (d) of Figure 13 shows a sharp decline in banks' dividend rates.

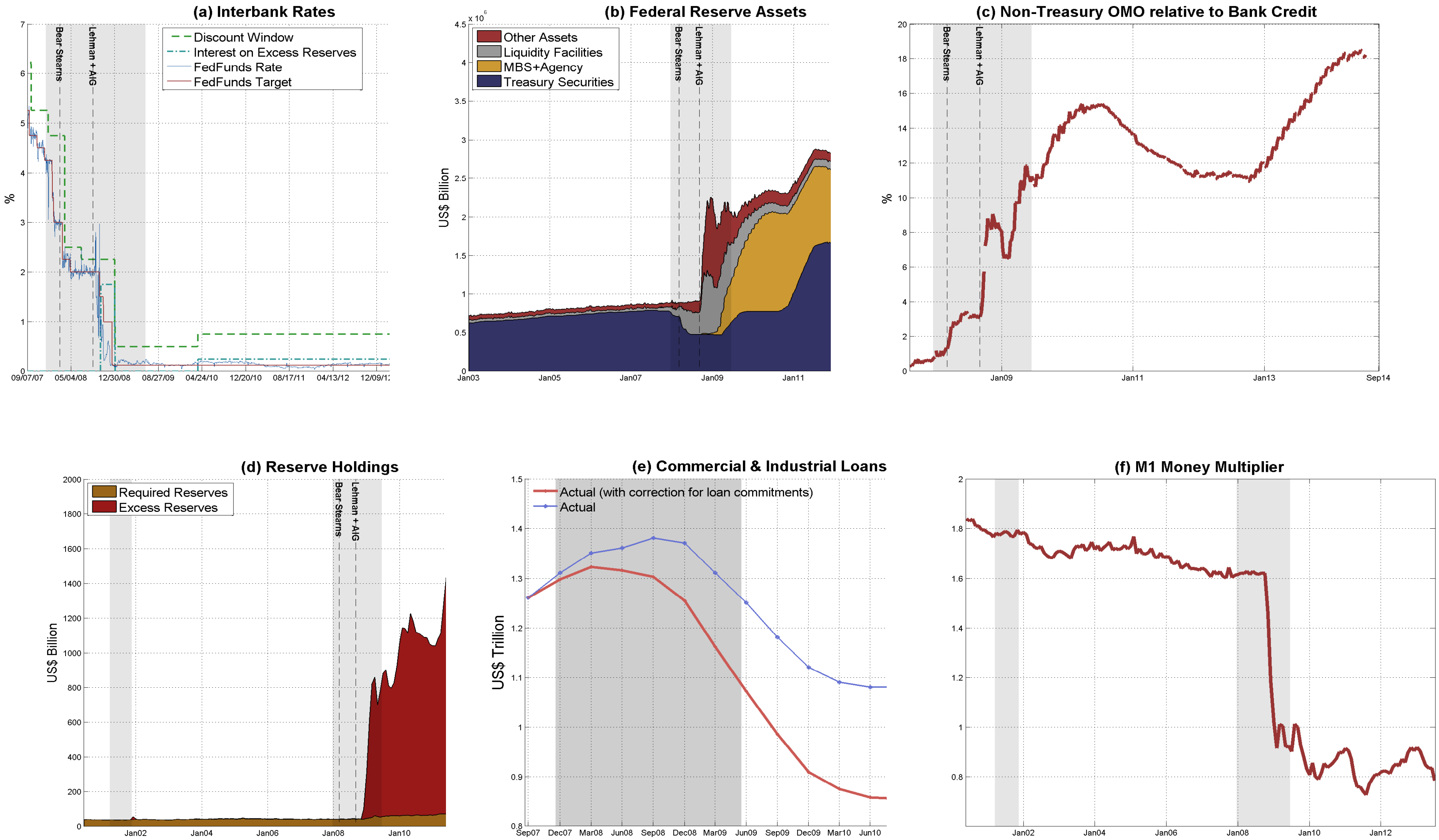

Figure 12: Monetary Facts

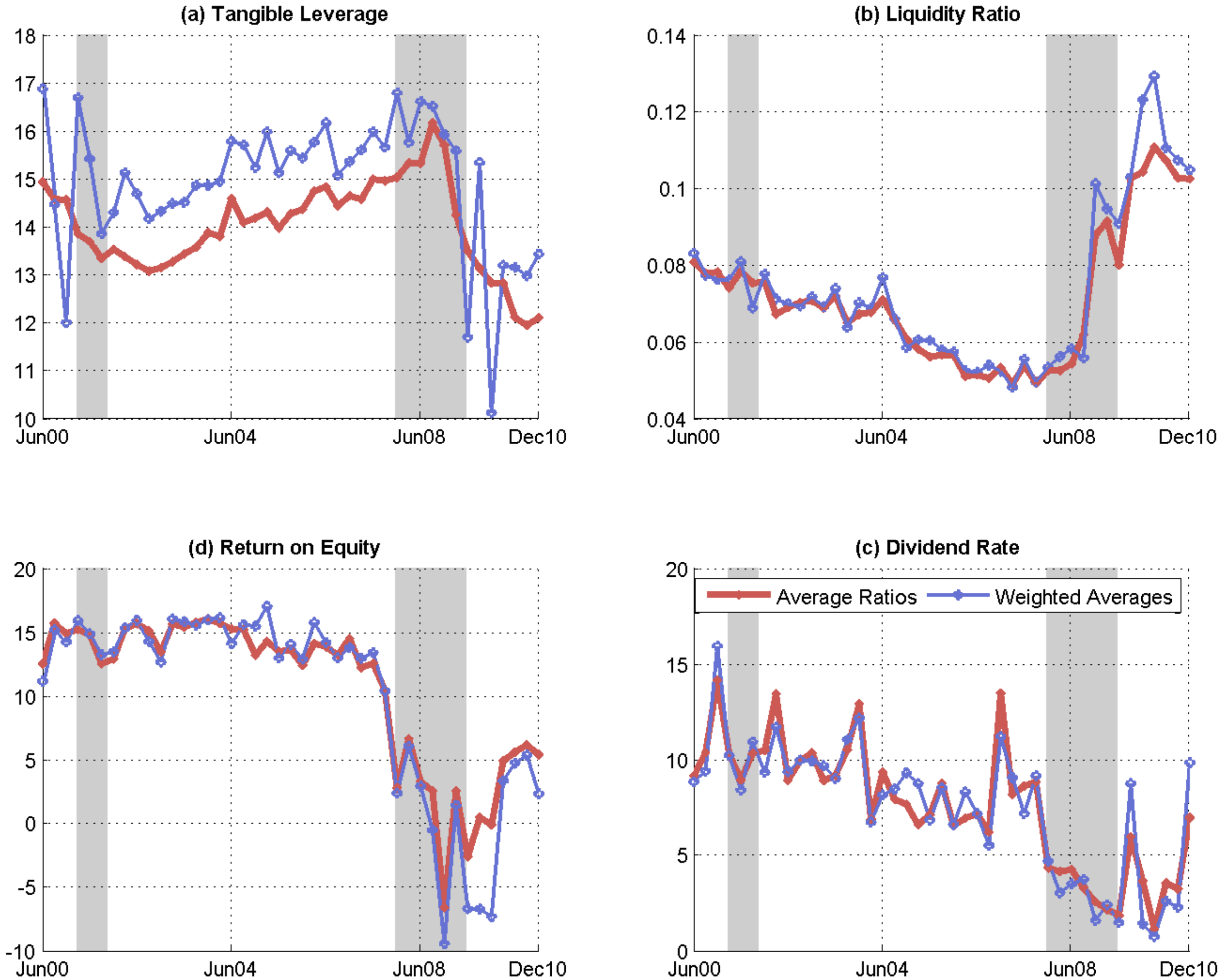

Figure 13: Banking Indicators: The figure reports four indicators of banking activity for the universe of commercial banks in the United States. All the series correspond to ratios of variables reported by computing simple averages and averages weighted by assets. Tangible leverage in panel (a) is total liabilities relative to equity minus intangible assets. The liquidity ratio in panel (b) is the sum of vault cash, reserves, and Treasury securities relative to total assets. The return on equity in panel(c) is the ratio of net operating revenues over equity. The dividend ratio in (d) is the sum of common and preferred dividends over equity. More details are found in Appendix G.

6.3 Which Hypotheses Fit the Facts?

We can now use our model as a laboratory in which to investigate the importance of Hypotheses 1–5 in explaining the facts described above. In particular, we seek to shed light on the possible drivers of the increase in liquidity and the persistent drop in lending that occurred in October 2008 after the failure of Lehman Brothers (facts 3,4 and 7). We first describe the qualitative prediction of our model and then turn to a quantitative evaluation.

6.3.1 Discussion

We can classify the shocks that we study above into supply and demand categories. Within supply shocks, there are two classes of shocks. First, equity losses and increments in capital requirements constrain the entire bank's portfolio. As analyzed above, by reducing the supply of loans and raising the return on loans relative to reserves, banks substitute reserves for lending. Hence, these two hypotheses can explain the collapse in lending but are inconsistent with the observed increase in banks' liquidity ratios. The second class of supply shocks are the shocks that disrupt the interbank market: the increase in the dispersion of the withdrawal process F_t, the shutdown of the interbank market, and changes in corridor rates. These shocks do not affect the funding capacity of the bank but reduce the relative return on loans. As a result, banks substitute lending for reserves and, in addition, pay fewer dividends. This pattern is consistent with what we see during the midst of the crisis. These shocks could have played a prominent role early during the crisis. However, from December 2008 onwards, the greater liquidity costs associated with these shocks were probably offset by the sharp reduction in the discount window rates.

In contrast, demand shocks reduce the return on loans and lead banks to substitute loans for reserves. Thus, these shocks are consistent with the decline in aggregate lending and the increase in reserves holdings. Moreover, the low lending rates of 2010 also suggest the hypothesis of a persistent negative credit demand shock.[41] The only counterfactual pattern is that dividends were sharply reduced at the beginning of the crisis, and demand shocks deliver the opposite prediction. However, the path for dividends in the data was also potentially influenced by government policies for those financial institutions that participated in government recapitalization programs.

To summarize, in our view, the model supports the hypothesis of strong disruptions in the interbank markets followed by a persistent negative credit demand shock. The short-run effects after each shock are summarized by the arrows in Table 2 below.

[41] Some caution must be exercised when using low lending rates as evidence for credit demand shocks, for at least two reasons. First, there seems to be a change in the composition of credit toward less risky loans. Second, besides interest rates, banks seem to have tightened lending standards, e.g., by requiring higher down-payments on mortgages.

Table 2: Summary of Effects on Impact

	Loans	Cash	Div.	Equity
Equity loss	↓	↓	↓	↓
Capital requirement	↓	↓	↓	↑
Uncertainty	↓	↑	↓	↑
Credit demand	↓	↑	↑	↓
Interest on reserves	↓	↑	↓	↑
Data	↓	↑	↓	↓

6.3.2 Quantitative Evaluation

The goal of this section is to assess the model's ability to quantitatively account for the monetary and banking facts we described in the previous section. For this purpose, we compute the model's transitional dynamics after a sequence of deterministic shocks assuming the economy was in steady state in 2007 Q3. In this experiment, we feed into the model a sequence of supply shocks that we interpret as observables and treat demand shocks as a residual to match the decline in lending.

Shocks. We feed the following shocks into the model, in line with the hypothesis discussed above. First, we consider a 2 percent shock to bank equity losses, which is equivalent to 0.2 percent of total assets; this shock assumes a corresponding decline in loans, keeping liabilities constant. The magnitude of this shock corresponds to the unexpected losses of AAA-rated subprime MBS tranches, estimated by Park (2011). Second, we consider a shock that anticipates higher capital requirements along the new prescriptions of Basel III. In particular, we assume that agents anticipate that the maximum leverage ratio will be permanently reduced from $\kappa = 15$ to $\kappa = 12$ starting in 2013 Q1. This is in line with new regulations that require a gradual increase in capital requirements of 2 percent between 2013 Q1 and 2015 Q4 (see for International Settlements, 2010). Third, we consider an interbank market freeze during 2008 Q3 and 2008 Q4, in line with the evidence of a sharp decrease in the fed funds market and the increase in discount window operations which reached \$400 billion at the peak of the crisis.[42] As explained above, this implies that the probability of a match in the interbank markets during the balancing stage becomes zero, so that banks only trade with the Fed.

[42] The amount of fed funds sold was reduced by a factor of 4 in 2008 Q3, and have never recovered from pre-crisis levels. We consider only two periods for this shock because, as described in Section 6.1, interbank market rates normalized following various policies by the Fed. Moreover, because discount window rates were significantly reduced in 2009 Q3, results would be very similar with more persistent shocks.

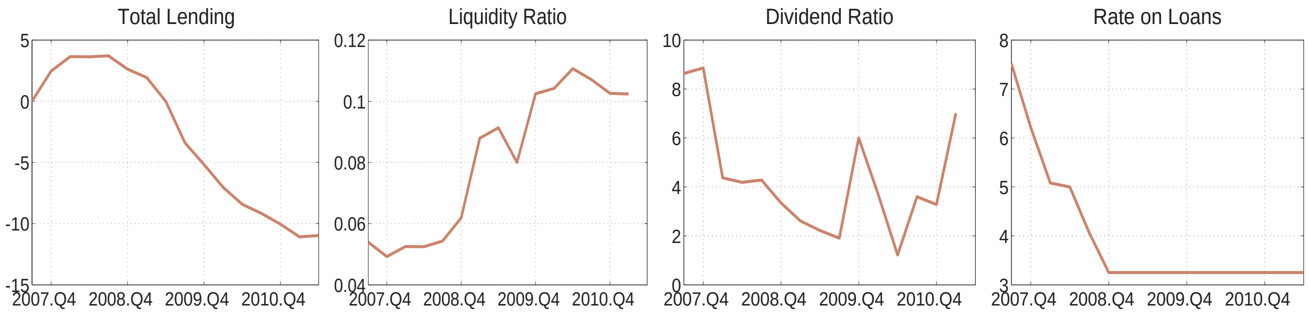

Figure 14: Data

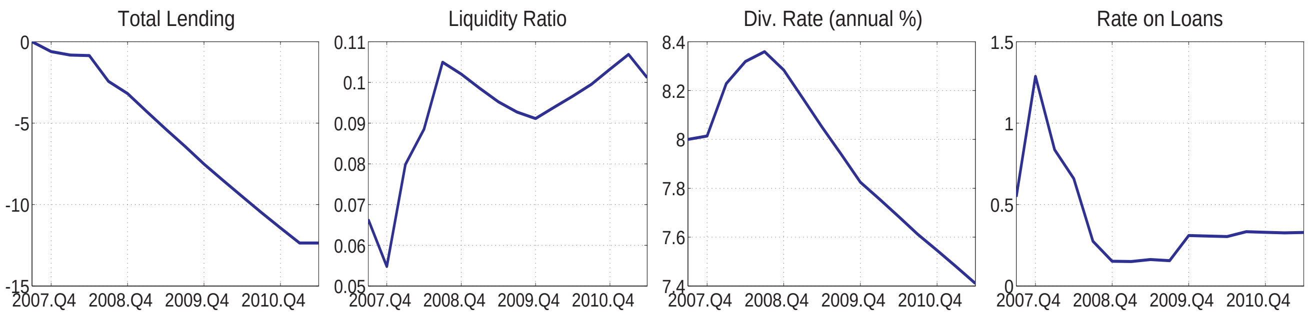

Figure 15: All Shocks

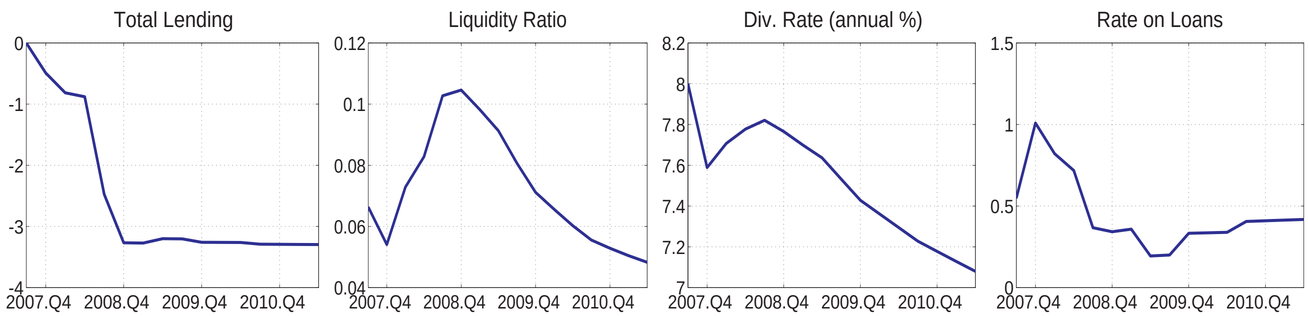

Figure 16: No Credit Demand Shocks

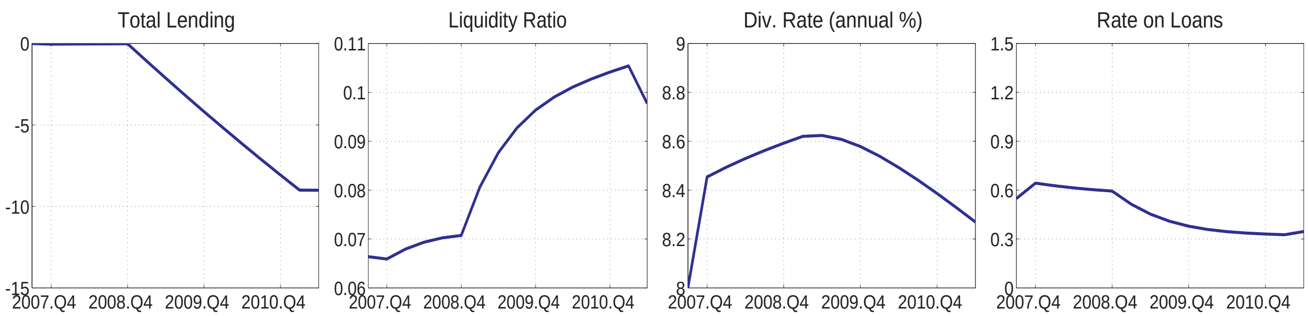

Figure 17: Only Credit Demand Shocks

The two remaining observable shocks are policy responses by the Fed. First, we feed in the sequence for discount window rates and interest on excess reserves shown in panel (a) of Figure 12. Second, we feed in the sequence of loan purchases as part of the unconventional OMO carried out by the FED, as described in Fact 2. We assume that these operations are gradually reversed starting in 2020, except for the interest on reserves, which we assume it stays at a constant level of 0.2 percent (annualized).

Finally, we estimate a credit demand shock. Given the observed time series for loans shown in the first panel of Figure 14, we consider a gradual shock to Θ_t to match the decline in lending. Specifically, Θ_t^{-1} is reduced by about 1 percent in every quarter from 2008 Q3 to 2010 Q4 and stays permanently at a level of 10 percent below the steady state.[43]

Results. Figures 14–17 show the evolution of loans, liquidity ratios, dividend rates, and the return on loans in the data and in the model for different simulation scenarios.[44] As Figure 14 shows, the model that includes all shocks can account reasonably well for the key patterns in the data. The model predicts a simultaneous sharp and persistent drop in lending, a substantial increase in liquidity, and a drop in the money multiplier. As it turns out, the credit demand shock is the most important shock. To see this, consider the third panel from Figure 15, which feeds in all shocks considered except for the credit demand shock. In this case, the model successfully predicts a spike in reserve holdings and a decline in lending at the beginning of the crisis, but fails to predict the persistent increase in reserves and the magnitude of the fall in lending. On the other hand, when we feed in only a credit demand shock, the model predicts a large increase in dividend payments, which is inconsistent with the patterns of dividend payments and equity issuances observed during the crisis, as shown in Figure 17.

7 Conclusions

Modern monetary macro models have developed independently from banking models. The recent crises in the United States, Europe and Japan, however, have revealed the need for a model to study monetary policy in conjunction with the banking system. Such a model would be useful in addressing many issues that emerge in current policy and academic debates.

This paper presents a dynamic macro model to study the implementation of monetary policy

[43]As we noted above, the estimation of the demand shock is obtained by residual. If one were to consider other shocks, this would affect the estimation. For example, incorporating credit risk so that a fraction of the loans of the bank defaults would reduce our estimation of the size of the demand shock. However, considering that default rates rose by about 2 percent implies that demand shocks are likely to remain important to explain the levels in the data.

[44]For the model simulations, variations in lending are taken with respect to the steady state. For the data, the percentage change in lending takes September 2007 as the basis value. Return on loans in the data correspond to the real return on one-year mortgage rates minus the fed funds rate. The measure of bank lending in the data is from Bassett et al. (2010) and constitutes the sum of commercial and industrial loans, loans secured by real estate, and consumer loans.

through the liquidity management of banks. We have used the model to understand the effects of various shocks to the banking system. As an application, we employ the model to contribute to one policy question: why have banks held on to so many reserves and not expanded their lending activities? We argue that an early interbank market freeze may have been important at the beginning of the Great Recession. However, a persistent decline in demand seems the most plausible explanation for the increase in reserve holdings and the decline in lending from 2008 onward. This result is suggestive of phenomena in which an initial contraction in the supply of loans eventually translates into a subsequent strong and permanent contraction in the demand for credit.

We believe the model can be used to answer a number of other questions present in policy debates. For example, the model can be used to study the Fed's exit strategy and its fiscal implications. In addition, it can also be used to evaluate alternative policy tools and targets. Other relevant extensions of the model are also possible. An extension that breaks down aggregation would allow us to study the cross-sectional responses of banks to different shocks depending on their liquidity and leverage ratios. Moreover, the model can also be extended to investigate the role of monetary policy as a macroprudential tool. We hope that the model we propose here can serve as a good starting point for such investigations.

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Appendix

A Proofs

A.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straightforward by noticing that once E is determined, the banker does not care how he came up with those resources. The proof of Propositions 2 and 3 is presented jointly, and the strategy is guess and verify. Let X be the aggregate state. We guess the following. $V(E;X) = v(X)E^{1-\gamma}$, where $v(X)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: $DIV(E;X) = div(X)E$, $\tilde{B}(E;X) = \tilde{b}(X)E$, $\tilde{D}(E;X) = \tilde{d}(X)E$ and $\tilde{C}(X) = \tilde{c}(X)E$, for $div(X), \tilde{b}(X), \tilde{d}(X)$ and $\tilde{c}(X)$ policy functions that are independent of E.

A.1.1 Proof of Proposition 2

Given the conjecture for the functional form of the value function, the value function satisfies

$$\begin{aligned}
V(E;X) &= \max_{\{DIV,\tilde{C},\tilde{B},\tilde{D}\}\in\mathbb{R}} U(DIV) + \beta\mathbb{E}\left[v(X')(E')^{1-\gamma})|X\right] \\
\text{Budget Constraint} &: \quad E = q\tilde{B} + \tilde{C}p + DIV - \frac{\tilde{D}}{R^D} \\
\text{Evolution of Equity} &: \quad E' = (q'\delta + (1-\delta))\tilde{B} + \tilde{C}p' - \tilde{D} - \chi\left((\rho + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - p\tilde{C}\right) \\
\text{Capital Requirement} &: \quad \frac{\tilde{D}}{R^D} \le \kappa\left(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D}\right)
\end{aligned}$$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of E. Dividing all of the constraints by E, we obtain

$$\begin{aligned}
1 &= div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^d} \\
E'/E &= (q'\delta + 1 - \delta)\tilde{b} + \tilde{c}p' - \tilde{d} - \chi((\rho + \omega'(1-\rho))\tilde{D} - p\tilde{C}) \\
\frac{\tilde{d}}{R^D} &\le \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})
\end{aligned}$$

where $div = DIV/E$,$\tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since E is given at the time of the decisions of B,C,D and DIV, we can express the value function in terms of choice of these ratios.

Substituting the evolution of E' into the objective function, we obtain

$$\begin{aligned} V(E;X) &= \max_{\{w_b,w_c,w_d,div\}\in} U(divE) + \beta\mathbb{E}\left[v(X')\left(R(\omega,X,X')E\right)^{1-\gamma})|X\right] \\ 1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\ \frac{\tilde{d}}{R^D} &\leq \kappa\left(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D}\right) \end{aligned}$$

where we use the fact that E' can be written as

$$E' = R(\omega', X, X')E,$$

, where $R(\omega', X, X')$ is the realized return to the bank's equity and defined by:

$$R(\omega', X, X') \equiv (q(X')\delta + (1-\delta))\tilde{b} + p(X')\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - p(X)\tilde{c}).$$

We can do this factorization for E because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in χ. To see this, observe that $\chi\left((\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - p\tilde{C}\right) = \chi\left((\rho+\omega'(1-\rho))\frac{\tilde{d}E}{R^D} - \tilde{c}E\right)$ by definition of $\left\{\tilde{d},\tilde{c}\right\}$. Since $E \geq 0$ always, we have that

$$(\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C} \leq 0 \leftrightarrow \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) \leq 0.$$

Thus, by definition of χ,

$$\begin{aligned} \chi((\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C}) &= \begin{cases} E\underline{\chi}\left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) \leq 0 \\ E\overline{\chi}\left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) > 0 \end{cases}. \\ &= E\chi\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right). \end{aligned}$$

Hence, the evolution of $R(\omega', X, X')$ is a function of the portfolio ratios b, c and d but not of the level of E. With these properties, we can factor out $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$\begin{aligned} V(E;X) &= E^{1-\gamma}\left[\max_{\{w_b,w_c,w_d,div\}\in} U(div) + \beta\mathbb{E}\left[v(X')R(\omega,X,X')^{1-\gamma})|X\right]\right] \quad (15) \\ 1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\ \tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{d}). \quad (16) \end{aligned}$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\begin{aligned}
\tilde{v}(X) &= \max_{\{w_b,w_c,w_d,div\}\in} U(div) + \beta\mathbb{E}\left[\tilde{v}(X')\,R(\omega,X,X')^{1-\gamma})|X\right] \\
1 &= div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^D} \\
\frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D}).
\end{aligned}$$

We now show that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (15) allows us to write $V(E;X) = \tilde{v}(X)\,E^{1-\gamma}$. Note this is true because maximizing over $div, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. This also shows that $div, \tilde{c}, \tilde{b}, \tilde{d}$ are independent of E, and $DIV = divE$, $\tilde{B} = \tilde{b}E$, $\tilde{C} = \tilde{c}E$, and $\tilde{D} = \tilde{d}E$.

A.1.2 Proof of Proposition 3

We have from Proposition 2 that

$$\begin{aligned}
v(X) &= \max_{\{w_b,w_c,w_d,div\}^4_+} U(div) + \beta\mathbb{E}\left[v(X')\,|X\right]... \\
&\quad \mathbb{E}_{\omega'}\left((q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\,\tilde{d}/R^D - p\tilde{c})\right)^{1-\gamma}
\end{aligned}$$

subject to

$$\begin{aligned}
1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D} \\
\frac{\tilde{d}}{R^D} &\leq \kappa\left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D}\right).
\end{aligned}$$

Now define

$$w_b \equiv \frac{\tilde{b}q}{(1-div)},\ w_c \equiv \frac{\tilde{c}p}{(1-div)} \text{ and } w_d \equiv \frac{\tilde{d}}{R^D(1-div)},$$

and collecting terms on $1 = q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D}$, we obtain:

$$div + (1-div)(w_b + w_c + w_d) = 1 \Leftrightarrow .w_b + w_c - w_d = 1.$$

Then using the definition of w_b, w_c, w_d have that $v(X)$

$$
\begin{aligned}
v(X) &= \max_{\{w_b,w_c,w_d,div\}\in\mathbb{R}^4_+} U(div) + \beta\mathbb{E}\left[v\left(X'\right)|X\right](1-div)^{1-\gamma}... \\
& \mathbb{E}_{\omega'}\left\{\frac{q'\delta+(1-\delta)}{q}w_b + p'w_c - \ w_d(R^D) - \chi((\rho+\omega'(1-\rho)) \ w_d - w_c \ )\right\}^{1-\gamma} \\
& s.t. \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa\left(w_b + w_c - w_d\right).
\end{aligned}
$$

Using the definition of returns, we can define the portfolio value as

$$
\begin{aligned}
\Omega^*(X) &\equiv \max_{\{w_b,w_c,w_d,div\}\in} \left\{\mathbb{E}_{\omega'}\left(R^B w_b + R^C w_c - \ w_d R^D - R^{\chi}(w_d, w_c)\ \right)^{1-\gamma}\right\}^{\frac{1}{1-\gamma}} \\
& s.t. \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa\left(w_b + w_c - w_d\right).
\end{aligned}
$$

Since the solution to $\Omega(X)$ is the same for any div and using the fact that X is deterministic, we have that

$$
v(x) = \max_{\{w_b,w_c,w_d,div\}^4_+} U(div) + \beta\mathbb{E}\left[v\left(X'\right)|X\right](1-div)^{1-\gamma}\Omega^*(X)^{1-\gamma},
$$

which is the formulation in Proposition 3.

For $\gamma \to 1$, the objective becomes:

$$
\Omega(X) = \exp\left\{\mathbb{E}_\omega\left[\log\left(R\left(\omega, X, X'\right)\right)\right]\right\},
$$

and for $\gamma \to 0$,

$$
\Omega(X) = \mathbb{E}_\omega\left[R\left(\omega, X, X'\right)\right].
$$

A.2 Proof of Proposition 4

Taking first-order conditions on (3) and using the CRRA functional form for $U(\cdot)$, we obtain

$$
div = \left(\beta\mathbb{E}v\left(X'\right)|X\right)^{-1/\gamma}\Omega^*(X)^{-(1-\gamma)/\gamma}(1-div)(1-\gamma),
$$

and therefore we obtain:

$$
div = \frac{1}{1+\left[\beta\mathbb{E}\left[v\left(X'\right)|X\right](1-\gamma)\Omega^*(X)\right)^{1-\gamma}\right]^{1/\gamma}}.
$$

Substituting this expression for dividends, we obtain a functional equation for the value function

$$\begin{aligned} v(X) &= \frac{1}{(1-\gamma)}\left(1+\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma} \\ &\quad +\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\left[\frac{\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)}\right]^{1-\gamma}. \end{aligned}$$

Therefore, we obtain the following functional equation:

$$v(X) = \frac{1}{1-\gamma}\left[1+\left(\beta(1-\gamma)\Omega^*(X)^{1-\gamma}\,\mathbb{E}\left[v(X')\,|X\right]\right)^{\frac{1}{\gamma}}\right]^{\gamma}.$$

We can treat the right-hand side of this functional equation as an operator. This operator will be a contraction depending on the values of $\left(\beta(1-\gamma)(\Omega^*(X))^{1-\gamma}\right)^{\frac{1}{\gamma}}$. Theorems in Alvarez and Stokey (1998) guarantee that this operator satisfies the dynamic programming arguments.

In a non stochastic steady state, we obtain

$$v^{ss} = \frac{1}{1-\gamma}\left(\frac{1}{1-\left(\beta\Omega^{*1-\gamma}\right)^{\frac{1}{\gamma}}}\right)^{\gamma}$$

and

$$\text{div}^{ss} = 1-\beta^{\frac{1}{\gamma}}\Omega^{*1/\gamma-1}.$$

A.3 Proof of Lemma 1

Define the threshold $\bar{\omega}$ shock that determines whether the bank has a reserve deficit or surplus, i.e., the shock that solves $(\rho+(1-\rho)\bar{\omega})\,w_d = w_c$. This shock is

$$\bar{\omega}(1,L) = \bar{\omega}(w_d,w_c) \equiv \frac{w_c/w_d-\rho}{(1-\rho)} = \frac{L-\rho}{(1-\rho)},$$

where L is the reserve ratio. We can express the expected liquidity cost in terms of $\bar{\omega}$:

$$\begin{aligned} \mathbb{E}_{\omega'}\left[R^{\chi}(w_d,w_c)\right] &= \chi_l\left[\int_{\bar{\omega}(w_d,w_c)}^{1}\left((\rho+(1-\rho)\,\omega')\,w_d-w_c\right)f(\omega')\,d\omega'\right] \\ &\quad +\chi_b\left[\int_{-\infty}^{\bar{\omega}(w_d,w_c)}\left((\rho+(1-\rho)\,\omega')\,w_d-w_c\right)f(\omega')\,d\omega'\right]. \end{aligned}$$

We separate the integral into terms that depend on ω' and the independent terms. We obtain that the expected liquidity cost:

$$
\begin{aligned}
\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right] &= \left(\rho w_d-w_c\right)\left[\chi_l\left(1-F\left(\bar{\omega}\left(w_d,w_c\right)\right)\right)+\chi_b F\left(\bar{\omega}\left(w_d,w_c\right)\right)\right] \\
&\quad +\left(1-\rho\right) w_d(\chi_l\left(1-F\left(\bar{\omega}\left(w_d,w_c\right)\right)\right)\mathbb{E}_{\omega'}\left[\omega'|\omega'>\bar{\omega}\left(w_d,w_c\right)\right] \\
&\quad +\left(1-\rho\right) w_d(\chi_b\left(F\left(\bar{\omega}\left(w_d,w_c\right)\right)\right)\mathbb{E}_{\omega'}\left[\omega'|\omega'\leq\bar{\omega}\left(w_d,w_c\right)\right].
\end{aligned}
$$

From here, we can factor ω^d from all of the terms in the expression above:

$$
\begin{aligned}
\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right] &= w_d((\rho-L)\left[\chi_l\left(1-F\left(\bar{\omega}\left(1,L\right)\right)\right)+\chi_b F\left(\bar{\omega}\left(1,L\right)\right)\right]... \\
&\quad +\left(1-\rho\right)\left(\chi_l\left(1-F\left(\bar{\omega}\left(1,L\right)\right)\right)\mathbb{E}_{\omega'}\left[\omega'|\omega'>\bar{\omega}\left(1,L\right)\right]... \\
&\quad +\left(1-\rho\right)\left(\chi_b\left(F\left(\bar{\omega}\left(1,L\right)\right)\right)\mathbb{E}_{\omega'}\left[\omega'|\omega'\leq\bar{\omega}\left(1,L\right)\right]\right) \\
&= w_d\mathbb{E}_{\omega'}\left[\tilde{R}^{\chi}\left(1,L\right)\right].
\end{aligned}
$$

From the expression above, we find that if we multiply $\{w_d, w_c\}$ by any constant, the expected liquidity cost increases by that same constant. Thus, $\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right]$ is homogeneous of degree 1 in $\{w_d, w_c\}$.

A.4 Proof of Lemma 2

The closed form expression for $\mathbb{E}_{\omega'}\left[R^{\chi}_c\left(1,L\right)\right]$ is obtained as follows. Given an ω', the reserve surplus per unit of deposit is $\left(\rho-L+\left(1-\rho\right)\omega\right)$:

$$
\mathbb{E}_{\omega'}\left[R^{\chi}\left(1,L\right)\right]=\chi_l\int_{\frac{L-\rho}{(1-\rho)}}^{1}\left(\rho-L+\left(1-\rho\right)\omega'\right)f\left(\omega\right)d\omega+\chi_b\int_{-\infty}^{\frac{L-\rho}{(1-\rho)}}\left(\rho-L+\left(1-\rho\right)\omega'\right)f\left(\omega\right)d\omega.
$$

Taking the derivative with respect to L yields:

$$
\begin{aligned}
\mathbb{E}_{\omega'}\left[R^{\chi}_c\left(1,L\right)\right] &= \underbrace{\left(\chi_b-\chi_l\right)\left\{\left(\rho-L+\left(1-\rho\right)\omega'\right)f\left(\omega\right)_{\frac{L-\rho}{(1-\rho)}}\right.}_{=0}... \\
&\quad -\left(\chi_b F\left(\frac{L-\rho}{(1-\rho)}\right)+\chi_l\left(1-F\left(\frac{L-\rho}{(1-\rho)}\right)\right)\right).
\end{aligned}
$$

A.5 Proof of Proposition 6

Since the objective is linear, the solution to the leverage decision is:

$$w^{*d} = \begin{cases} 0 \text{ if } R^{L^*} < 0 \\ [0,\kappa] \text{ if } R^{L^*} = 0 \\ \kappa \text{ if } R^{L^*} > 0 \end{cases}.$$

Substituting this result implies that the return to the bank's equity is $R^E = R^b + \max\left\{\kappa R^{L^*}, 0\right\}$. Thus, the bank's dividend decision is:

$$\text{div} = \begin{cases} 0 \text{ if } \beta R^E > 1 \\ [0,1] \text{ if } \beta R^E = 1 \\ 1 \text{ if } \beta R^E < 1 \end{cases}.$$

In any steady state, it must be that $\beta R^E = 1$ and $div = R^E - 1$, because otherwise equity is not constant. If the leverage constraint is non-binding in the steady state, then by the condition above $R^{L^*} = 0$, and therefore $R^B = 1/\beta$. Otherwise, there is a positive spread. The statement in the proposition follows.

A.6 Proof of Proposition 7

Since the objective of the liquidity management subproblem is linear, we have that its value is:

$$\max\left(\underbrace{-\chi_b\rho}_{L=0}, \underbrace{-\left(R^B - R^C\right)\rho}_{L=\rho}, \underbrace{-\left(\left(R^B - R^C\right) - \chi_l\right)\frac{1+\omega^d}{\omega^d} - \chi_l\rho}_{L=\frac{1+\omega^d}{\omega^d}}\right).$$

Here we study the equilibrium in the interbank market. An equilibrium is studied as the Nash equilibrium of a game, that is we study the choice of L of a given bank, given a choice $\tilde{L}$ by other banks.

Case 1 ($\tilde{L} = 0$). Assume all banks choose $\tilde{L} = 0$. If an individual bank chooses $L \leq \rho$, the cost of reserve deficits equals the discount window rate $\chi_b = r^{DW}$ because there are no other banks to borrow reserves from. Therefore, we have that $\left(R^B - R^C\right) < r^{DW}$ is necessary and sufficient to guarantee that $L = 0$ is not an equilibrium, because we require positive reserves in a monetary equilibrium.

Case 2 ($\tilde{L} = \rho$). If all banks choose $\tilde{L} = \rho$, a bank deviating to $L = 0$ would pay $r^{DW} > \left(R^B - R^C\right)$, because, again, no banks would lend reserves to that bank. Thus, $L = \rho$ dominates

$L = 0$ when other banks choose $L = \rho$ and $r^{DW} > \left(R^B - R^C\right)$. This shows that $\left(R^B - R^C\right) < r^{DW}$ is necessary and sufficient to guarantee that $L = 0$ is not an equilibrium when $\tilde{L} = \rho$.

So far, $\left(R^B - R^C\right) < r^{DW}$ is enough to argue that $\tilde{L} \geq \rho$ in a symmetric Nash equilibrium. Assume now that also $\left(R^B - R^C\right) > r^{ER}$ holds.

Case 3 ($\tilde{L} = \frac{1+\omega^d}{\omega^d}$)**.** If all banks set $\tilde{L} = \frac{1+\omega^d}{\omega^d} > \rho$, no bank will be short of reserves. Thus, $\chi_l = r^{ER}$ since $\gamma^+ = 0$. Thus, an individual bank is better off deviating by reducing L to ρ.

Case 4 ($\tilde{L} = \rho$)**.** Instead, if all banks set $\tilde{L} = \rho$, then, $\chi_l = r^{ER}$ since again $\gamma^+ = 0$. Thus, $L = \rho$ is an optimal choice because deviating to $\frac{1+\omega^d}{\omega^d}$ is not profitable.

Hence, $r_t^{ER} < R^B - R^C < r_t^{DW}$ will hold in any equilibrium with positive reserves, and this implies $L^* = \rho$.

B Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded, and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel σ-algebra on the positive real line. Then, define $Q_t(e, \boldsymbol{E})$ as the probability that an individual bank with current equity e transits to the set $\boldsymbol{E}$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \rightarrow [0, 1]$, and

$$Q(e, \boldsymbol{E}) = \int_{-1}^{1} \mathbb{I}\left\{e_t\left(\omega\right) e \in \boldsymbol{E}\right\} F\left(d\omega\right),$$

where $\mathbb{I}$ is the indicator function of the event in brackets. Then Q is a transition function and the associated T^* operator for the evolution of bank equity is given by

$$\Gamma_{t+1}\left(\boldsymbol{E}\right) = \int_{0}^{1} Q(e, \boldsymbol{E})\Gamma_{t+1}\left(e\right) de.$$

The distribution of equity is fanning out, and the operator is unbounded. Gibrat's law shows that for t large enough, Γ_{t+1} is approximated by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}\left(\boldsymbol{E}\right)$. We will use these properties in the calibrated version of the model.

C Algorithm

C.1 Steady State

1. Guess prices for loans q and for the probability of a match in the interbank market γ^{-}, γ^{+}.
2. Solve banks' optimization problem 3. Compute value of the bank and dividend payments.
3. Compute associated average equity growth and average surpluses in the interbank market.
4. If equity growth equals zero and the conjectured probability of a match in the interbank market is consistent with the average surplus, stop. Otherwise, adjust and continue iterating.

Algorithm to solve transition dynamics in baseline model

C.2 Transitional Dynamics

1. Guess a sequence of loan prices q_t and for the probability of a match in the interbank market $\gamma_t^{-}, \gamma_t^{+}$.
2. Solve banks's dynamic programming problem. Use (3) for banks' portfolio and (4) for the value function and dividend rates.
3. Compute growth rate of equity and average surplus in interbank markets.
4. Compute price implied by the aggregate sequence of loans resulting from (2) and (3), and the probability of a match according to average surpluses computed in (3).
5. If the conjectured prices equal effective price from (4) and the average surpluses computed in (4) are consistent with the guessed sequences, stop. Otherwise, continue iterating until convergence.

D Microfoundation for Loan Demand and Deposit Supply

Introducing a demand for loans and a supply of deposits can be done in multiple ways. Here, the demand for loans emerges from firms who borrow working capital from banks and the supply of deposits from the Households' savings decision. With working capital constraints, a low price for loans, q_t, translates immediately into labor market distortions and, therefore, has real output effects. This formulation is borrowed from the classic setup of Christiano and Eichenbaum (1992). To keep the model simple, we deliberately model the real sector so that loan demand is static—in the sense that it does not depend on future outcomes— and the supply of loans is perfectly elastic.

D.1 Households

Households' Problem. Households obtain utility by consuming and disutility from providing labor. They work during the lending stage and consume during the balancing stage. This distinction is irrelevant for households but matters for the sequence of events that we describe later. Households have quasi-linear utility in consumption and have a convex cost of providing labor given by $\frac{h_t^{1+\nu}}{1+\nu}$. The only savings instruments available to households are bank deposits and their holding of shares of firms. Households solve the following recursive problem:

$$W\left(s_t, d_t; X_t\right) = \max_{\{c_t \geq 0, h_t, d_{t+1} \geq 0\}} c_t - \frac{h_t^{1+\nu}}{1+\nu} + \beta^D \mathbb{E}\left[W\left(s_{t+1}, d_{t+1}; X_{t+1}\right) | X_t\right]$$

subject to the budget constraint:

$$d_{t+1} + c_t + p_t^s s_{t+1} = s_t(z_t + p_t^s) + w_t h_t + R_t^D d_t + T_t.$$

Here, β^D is the Households' discount factor and ν the inverse of the Frisch elasticity. In the budget constraint, d_t are deposits in banks that earn a real rate of R^D, h_t are hours worked that earn a wage of w_t, and s_t are shares of productive firms. The price of shares is p_t^s, and these pay z_t dividends per share. Finally, c_t is consumption and T is lump-sum transfers from the government.

The first-order conditions for the households' problem yield the following labor supply:

$$w_t^{\frac{1}{\nu}} = h_t.$$

This supply schedule is static and only a function of real wages. Hence, the total wage income for the household is $w_t^{\frac{1+\nu}{\nu}}$. In turn, substituting the optimality condition in this problem and using the fact that in equilibrium $s_{t+1} = s_t$, we can solve for the optimal policy decisions, $\{c, d\}$, independently from the labor choice. The solution is immediate and given by,

$$\{c,d\} = \begin{cases} c_t = w_t^{\frac{1+\nu}{\nu}} + R^D d_t + T; d_{t+1} = 0 & if \quad R^D < 1/\beta^D, \\ c_t \in [0, y_t], d_{t+1} = y_t - c_t & if \quad R^D = 1/\beta^D \\ c_t = 0; d' = w_t^{\frac{1+\nu}{\nu}} + R^D d_t + T_t & if \quad R^D > 1/\beta^D \end{cases}$$

These two results imply that households consume all cash on hand in the period they receive it if the interest is very low and they do not save, or carry real balances. If $R^D = 1/\beta^D$, they are indifferent between consuming or savings. Otherwise, they either do not consume or save all of their resources. We will consider parameterizations where in equilibrium $R^D = 1/\beta^D$.

D.2 Firms' Problem

Firms. Firms maximize $\mathbb{E}\left[\sum_{t=0}^{\infty} m_t z_t\right]$ where z_t is dividend payouts from the firms and μ_t is the stochastic discount factor of the representative household. Given the linearity of the households' objective, the discount factor is equivalent to $m_t = (\beta^D)^t$.

Timing. A continuum of firms of measure one is created at the lending stage of every period. Firms choose a production scale together with a loan size during their arrival period. In periods after this scale choice is decided, firms produce, and pay back loans to banks; the residual is paid in dividends.

Production Technology. A firm created in period t uses labor h_t, to produce output according to $f_t(h_t) \equiv A_t h_t^{1-\alpha}$. The scale of production is decided during the lending stage of the period when the firm is created. Although the scale of production is determined immediately at the time of creation, output takes time to be realized. In particular, the firm produces $\delta^s (1-\delta) f_t(h_t)$ of its output during the $s-th$ balancing stage after its scale was decided.

Labor is also employed when the firm is created, and workers are required to be paid at that moment.[45] Since firms do not possess the cash flow to pay their workers—no equity injections are possible—firms need to borrow from banks to finance the payroll. Firms issue liabilities to the banking sector—loans— by, l_t, in exchange for deposits—bank liabilities—, $q_t l_t$, which firms can use immediately to pay workers. The repayment of those loans occurs over time. In particular, firms repay $\delta^s (1-\delta) l_t$ during the $s-th$ lending stage after the loan was made. Notice that the repayment rate δ coincides exactly with the δ rate of sales. This delivers a problem for firms similar to the one in Christiano and Eichenbaum (1992), with maturity. Taking as given a labor

[45] This constraint emerges if it is possible that the firm defaults on this promise and defaults on its payroll (see Bigio et al., 2011). The implicit assumption is that banks have a special advantage of monitoring loans compared with households.

tax τ^l, and the loan price q_t, the problem of the firm created during the period t is:

$$\max_{\{h_t,l_t\}} \sum_{s=1}^{\infty} (\beta^D)^{s-1} z_{t+s-1}$$

subject to:

$$z_{t+s-1} = \delta^s (1-\delta) A_t f_t (h_t) - \delta^s (1-\delta) l_t$$

and

$$\left(1+\tau_t^l\right) w_t h_t = q_t l_t.$$

Substituting z_{t+s-1} into the objective function, and substituting the working capital loan yields a static maximization problem for firms:

$$\max_{\{h_t\}} A_t h_t^{1-\alpha} - \left(1+\tau_t^l\right) w_t h_t / q_t.$$

Taking first-order conditions from this problem and the household's first-order condition, $w_t h_t = h_t^{1+\nu}$, yields an allocation for labor

$$h_t = \left[\frac{q_t A_t (1-\alpha)}{\left(1+\tau_t^l\right)}\right]^{\frac{1}{\alpha+\nu}}.$$

Now, using the working capital constraint, $\left(1+\tau_t^l\right) w_t h_t = l_t q_t$, and the expression above:

$$\left(1+\tau_t^l\right) \left[\frac{q_t A_t (1-\alpha)}{\left(1+\tau_t^l\right)}\right]^{\frac{1+\nu}{\alpha+\nu}} = l_t q_t.$$

Clearing q_t from this expression yields:

$$l_t = (A_t (1-\alpha))^{\frac{1+\nu}{\alpha+\nu}} \left(\left(1+\tau_t^l\right) q_t\right)^{\frac{1-\alpha}{\alpha+\nu}}.$$

This is the expression in equation (8) and proves the following proposition:

Proposition 9 *The demand for loans takes the form*

$$q_t = \Theta_t I_t^{\epsilon},$$

where

$$\Theta_t = \left(1+\tau_t^l\right) [A_t (1-\alpha)]^{-\frac{1+\nu}{1-\alpha}} \text{ and } \epsilon = \frac{(\alpha+\nu)}{(1-\alpha)}.$$

Standard calibrations assume $\alpha = \frac{1}{3}$ and some $\nu \in \left[\frac{1}{3}, 2\right]$. This provides the bounds $\epsilon \in [1.0, 3.5]$.

E Environment with Finite Size Orders

This section explains how the Fed funds rate obtained as the solution to the objective of Problem 1 is the limit of a sequence of Nash bargaining problems between a borrowing order and a lending order when the size of the transaction converges to zero. We do this gradually by first describing precisely the environment and then providing the proof in several steps.

Balancing Stage Interbank Markets. Recall that by the end of every lending stage, after the realization of the ω withdrawal shocks, there is an distribution across banks of reserve deficits equal. Deficits are denoted by x—where x is negative for surplusses. After banks observe x, they must obtain x funds in order to meet their reserve requirements. Recall that the interbank market for reserves is directed and over-the-counter market: banks in surplus place lending orders and banks in deficit place borrowing orders. Each type of orders are sent to either side of the market and matched.

Fixed Order Sizes. Assume that banks that wish to lend dollars in excess can place lending orders of a fixed size, Δ, in the OTC market. A bank that needs to borrow a dollar to patch its reserve deficit can place a borrowing order, also for fixed size Δ.[46] We adopt the convention that banks cannot place more than $\eta(x, \Delta) \equiv \lfloor |x| / \Delta \rfloor$ orders; here, $\lfloor z \rfloor$ stands for the floor function understood as the largest integer not greater than z.[47] The motivation behind this convention is that if a lender bank places lending orders that exceed his excess reserves, there is a chance it will not be able to have the funds to transfer to the borrowing bank.[48] Because banks can only place integer numbers of orders, typically, there will be a remainder amount of reserves deficits (surpluses) that will not be placed as orders. These residuals can be borrowed (lent) from the Fed at the discount window rated r^{DW} (excess reserve rate r^{ER}) directly. Mathematically, this residual is exactly $\phi(x, \Delta) = |x| - |x| \, mod(\Delta)$.

After orders are directed to their corresponding sides, orders from lending side are randomly matched with orders from the borrowing side. As in the main text, the corresponding matching probabilities are given by M^{+} and M^{-}, the masses of lending and borrowing orders. In particular, the probability that a borrowing order finds a lending order is given by $\gamma^{-} = \min(1, M^{+}/M^{-})$. Conversely, the probability that a lending order finds a borrowing order is $\gamma^{+} = \min(1, M^{-}/M^{+})$. Note that as the order sizes converges to zero, M^{-} and M^{+} converge to the mass of dollars in deficit and in surplus. The reason is that $\phi(x, \Delta)$ converges to 0 as Δ goes to 0—as we have in

[46] A protocol for a fixed size of trades seems a natural assumption that avoids leaving left overs from different matches.

[47] The absolute value, $|x|$, allows us to talk about surplusses and deficits.

[48] If a lending bank lacks the funds to transfer to the bank in deficit, this would constitute a default. For sufficiently high penalties, no bank will ever place lending orders above the amounts they hold. For borrowing orders this is not so clear. A bank in excess of their reserve deficits would have to pay an interest for a loan they didn't need if the place additional orders than what they require. However, this additional cost may be compensated by the additional probabilities of success. If all banks in deficit behave the same way, matching probabilities would be unaffected leaving only the risk of placing too many borrowing orders.

the main text.

Interbank Loan Contract. When two orders find each other, and a loan contract is agreed upon, it is executed within the balancing stage. If an agreement is reached, the bank placing the lending order agrees to transfer Δ reserves to the bank short of reserves. These reserves will be returned by the beginning of the next lending stage, but only after the Fed counts the reserve balance. Thus, borrowed reserves count as part of the reserve balance of the borrower bank. Since this is a transfer that reversed by the end of the period, the principal of the loan has no effect on the value of the bank. The interest rate is an amount, $r^{FF}\Delta$, of deposits to be transferred from the lending bank to the borrowing bank. In other words, the borrowing bank will absorb $r^{FF}\Delta$ of the liabilities of the lending bank.

If two matched orders cannot agree on a contract, thet lending order is automatically placed as a lending order earning the Fed's excess reserve rate, r^{ER}, and the borrowing order is funded by the Fed at the discount window rate, r^{DW}.

Recall that in the main text, $\chi(x)$ is a deterministic function of the deterministic reserve balance x. However, when trade sizes are fixed, $\chi(x)$ are the total interests paid out—or received in the case of a bank in surplus—at the interbank market given the reserve balance x. Thus, $\chi(x)$ is a random variable that converges to a deterministic function as $\Delta \to 0$. This is a result we prove next.

Under a fixed order size, this total payments are random and take the form:

$$\chi(x) = \underbrace{\left[-r^{ER}\mathbb{I}_{[x\leq 0]} + r^{DW}\mathbb{I}_{[x>0]}\right]}_{\text{direct payments to the Fed}} \underbrace{\phi\left(x,\Delta\right)}_{\text{Residual orders}} + \sum_{o=1}^{\eta(x,\Delta)} \Delta \underbrace{r\left(o\right)}_{\text{Interest paid on the o-th order}} . \tag{17}$$

These interest payments have the following interpretation: Since orders can be made only at fixed sizes, out of a deficit (surplus) x there will be a little left over that will have to be borrowed (lent) directly from the Fed. This is the amount $\phi\left(x,\Delta\right)$, which will pay an interest rate r^{DW} if in deficit (or receive r^{ER} if in surplus). The rest of the reserve—deficits or surplusses—will be placed in $\eta\left(x,\Delta\right)$ orders sent to the interbank market. These will amount to a total of x modulo Δ dollars. These orders can be enumerated, $o = \{1 : \eta\left(x,\Delta\right)\}$. For the computation of payments, this order of course, does not matter. What matters is rate $r\left(o\right)$, which stands for the interest paid (or earned) by the o-th order. For example, if the o-th order was a lending order that did not find a match, then r(o) $= r^{ER}$. If it found a match, $r\left(o\right)$ equals the terms of the agreement. Of course, since the bargaining depends on the characteristics of the banks that sent the orders, $r\left(o\right)$ is a random variable. Let's now turn to the bargaining problem of two bank orders that are matched.

Bargaining Problem. Consider two banks, z and $\tilde{z}$, such that only z is in deficit so $x\left(z\right) > 0$ and $x\left(\tilde{z}\right) < 0$. Then, suppose that a lending order from bank $\tilde{z}$ is paired with a borrowing order from z. We adopt the convention, as in Atkeson et al. (2012), that in the computation of the surplus in the bargaining among the two orders, the interest paid by other orders at both banks are taken as given—that is, as a purely exogenous random variable. Their bargaining problem consists of solving the expected surplus resulting from a match:

$$\begin{aligned} B\left(z, z', \Delta, X'\right) &= \max_{r^{FF}} \mathbb{E}\left(\left[\bar{V}^b(z, r^{FF}, \Delta, X') - \bar{V}^b(z, r^{DW}, \Delta, X')\right]\right)^{\xi} \times ... \\ &\quad \left(\left[\bar{V}^b(\tilde{z}, -r^{FF}, \Delta, X') - \bar{V}^b(\tilde{z}, -r^{ER}, \Delta, X')\right]\right)^{1-\xi}. \end{aligned}$$

The generic value function $\bar{V}^b(\bar{z}, \bar{r}, \Delta, X')$ is the value of bank $\bar{z}$ at the balancing stage given a contract for a loan size Δ at rate $\bar{r}$ for the loan contract of the o-th order. This expression takes the form:

$$\begin{aligned} \bar{V}^b(\bar{z}, \bar{r}, \Delta, X') &\equiv V^l\left(\tilde{C}\left(\bar{z}\right) - \frac{\omega\tilde{D}\left(\bar{z}\right)}{p}, \tilde{B}\left(\bar{z}\right), D'\left(\bar{z}\right); X'|\tilde{X}\right) \\ D'\left(\bar{z}\right) &= \tilde{D}\left(\bar{z}\right)\left(1-\omega\right) + \left[-r^{ER}\mathbb{I}_{[x(\bar{z})\leq 0]} + r^{DW}\mathbb{I}_{[x(\bar{z})>0]}\right]\phi\left(x\left(\bar{z}\right), \Delta\right) ... \\ &\quad + \Delta \sum_{o=1}^{\eta(x(\bar{z}),\Delta)-1} r\left(o\right) + \Delta\bar{r} \qquad (18) \\ x\left(\bar{z}\right) &= \rho\left(\frac{1-\omega}{R^D}\right)\tilde{D}\left(\bar{z}\right) - \left(\tilde{C}\left(\bar{z}\right)p - \frac{\omega\tilde{D}\left(\bar{z}\right)}{R^D}\right) \end{aligned}$$

for arbitrary $\bar{z}$,$\bar{r}$ and Δ. This value function takes the same form as the value function at the balancing stage except that $\chi(x)$ is replaced by the expression (17). Also, notice that in this definition, the orders are reorganized so that the contract that is being negotiated is affecting the interest payment of the the last order —this is without loss of generality. Moreover, from the perspective of the $\eta\left(x\left(\bar{z}\right), \Delta\right) - th$ order, all the other rates obtained by other orders $r\left(o\right)$, are random variables. Thus, the expectation $\mathbb{E}$ is also with respect to the sequence of $r\left(o\right)$. To spare notation, we suppress all arguments that involve the aggregate state.

E.1 Interbank Market with Infinitesimal Orders

Result. We now turn to the result of interest. The main point we want to show is that the limit of the solutions of $B\left(z, z', \Delta\right)$ as $\Delta \searrow 0$ converges to the solution of Problem 1 in the main text**.** Formally, we want to show:

Proposition 10 (Limit of Bargaining Problems) *The solution to $B\left(z, z', \Delta\right)$ as $\Delta \searrow 0$ is also the solution to:*

$$m_b \times m_l \max_{r^{FF}} \left(r_t^{DW} - r^{FF}\right)^{\xi} \left(r^{FF} - r_t^{ER}\right)^{1-\xi} \qquad (19)$$

where $m_b \times m_l$ are arbitrary constants that don't affect the solution.

The proof involves two steps. First, we begin with the observation that the solution to $B(z,z',\Delta)$ is the same as the solution to $B(z,z',\Delta)/\Delta$, for any constant Δ. Then,

$$\lim_{\Delta\searrow 0}\frac{B(z,z',\Delta)}{\Delta} = \lim_{\Delta\searrow 0}\max_{r^{FF}}\mathbb{E}\frac{\left(\bar{V}^b(z,r^{FF},\Delta)-\bar{V}^b(z,r^{DW},\Delta')\right)^{\xi}}{\Delta^{\xi}}\times\ldots \quad \frac{\left(\bar{V}^b(\tilde{z},-r^{FF},\Delta)-\bar{V}^b(\tilde{z},-r^{ER},\Delta)\right)^{1-\xi}}{\Delta^{\xi}}. \tag{20}$$

The first step in the proof of Proposition 10 is to show that we can pass the limit inside the max operator. That is, we need to show that:

Proposition 11 *The limit of the solutions to the sequence of problems $\frac{B(z,z',\Delta)}{\Delta}$ as $\Delta\searrow 0$ equals the solution of the bargaining problem of the limit of the objective function as $\Delta\searrow 0$. That is*

$$\lim_{\Delta\searrow 0}\max_{r^{FF}}\mathbb{E}\frac{\left(\bar{V}^b(z,r^{FF},\Delta)-\bar{V}^b(z,r^{DW},\Delta)\right)^{\xi}}{\Delta^{\xi}}\times\ldots \quad \frac{\left(\bar{V}^b(\tilde{z},-r^{FF},\Delta)-\bar{V}^b(\tilde{z},-r^{ER},\Delta)\right)^{1-\xi}}{\Delta^{\xi}}$$

$$= \max_{r^{FF}}\lim_{\Delta\searrow 0}\mathbb{E}\frac{\left(\left[\bar{V}^b(z,r^{FF},\Delta)-\bar{V}^b(z,r^{DW},\Delta)\right]\right)^{\xi}}{\Delta^{\xi}}\times\ldots \quad \frac{\left(\bar{V}^b(\tilde{z},-r^{FF},\Delta)-\bar{V}^b(\tilde{z},-r^{ER},\Delta)\right)^{1-\xi}}{\Delta^{\xi}}$$

The main complication in this proof is that $B(z,z',\Delta)$ has a countable number of discontinuities in Δ. This results from the discontinuities in $\phi(x,\Delta)$. Because, $B(z,z',\Delta)$ is not continuous, we cannot employ the Theorem of the Maximum and use the fact that $B(z,z',\Delta)$ is continuous to pass the limit inside. However, we can use the same steps of the proof the Theorem of the Maximum to show the desired result—thus, it only applies for $\Delta=0$.

The second step in the proof is to show that the bargaining problem at the limit $\Delta\searrow 0$ is the objective in Problem 1. Formally, we have:

Proposition 12 *The following problems are equivalent:*

$$\max_{r^{FF}}\lim_{\Delta\searrow 0}\mathbb{E}\frac{\left(\left[\bar{V}^b(z,r^{FF},\Delta)-\bar{V}^b(z,r^{DW},\Delta)\right]\right)^{\xi}}{\Delta^{\xi}}\times\ldots \quad \frac{\left(\bar{V}^b(\tilde{z},-r^{FF},\Delta)-\bar{V}^b(\tilde{z},-r^{ER},\Delta)\right)^{1-\xi}}{\Delta^{\xi}}$$

and

$$m^l\times m^b\times\max_{r^{FF}}\left(r^{FF}-r^{DW}\right)^{\xi}\left(r^{FF}-r^{ER}\right)^{1-\xi}.$$

In the proofs, we use two useful calculations that we summarize in the following Lemma:

Lemma 3 *Suppose $r(o)$ is a bounded random variable with know mean $\mathbb{E}[r(o)]$. The limit of interest payments by all orders except the last satisfies:*

$$\lim_{\Delta\searrow 0}\Delta\sum_{o=1}^{\eta(x(\bar{z}),\Delta)-1} r(o) = x(\bar{z})\,\mathbb{E}[r_o].$$

In addition, the limit of the interest payments on the residual converges to 0:

$$\lim_{\Delta\searrow 0}\left[-r^{ER}\mathbb{I}_{[x\leq 0]}+r^{DW}\mathbb{I}_{[x>0]}\right]\phi(x,\Delta)=0.$$

A Corollary of that follows from this Lemma and the propositions above is that:

Corollary 2 *Moreover, the function*

$$\chi(x)=\left[-r^{ER}\mathbb{I}_{[x\leq 0]}+r^{DW}\mathbb{I}_{[x>0]}\right]\phi(x,\Delta)+\Delta\sum_{o=1}^{\eta(x,\Delta)} r(o)$$

converges to

$$\chi(x)=\begin{cases}\gamma^{-}\bar{r}^{FF}+(1-\gamma^{-})\,r^{DW} & \text{for } x>0\\ \gamma^{+}\bar{r}^{FF}+(1-\gamma^{-})\,r^{ER} & \text{for } x<0\end{cases}$$

where $\bar{r}^{FF}$ is the solution to (19).

The proof is immediate once we observe that the solution to (19) is a constant r^{FF}. This result implies that $r_o=\bar{r}^{FF}$ if a match is successful and r_o is r^{DW} or r^{ER} if the borrowing and lending orders are not matched. The Lemma above and the Law of Large numbers implies the limit of $\chi(x)$.

F Proof of Propositions 10 and 11 and Lemma 3

We proceed by backward induction. We first prove Lemma 3, then Proposition 11 and finally Proposition 10. We use these results sequentially in the proofs.

F.1 Proof of Lemma 3

To prove the first result we show that $\Delta\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o)$ is bounded by two numbers tha converge to the same limit as Δ tends to 0. First, notice that by property of the floor function, $\left(\frac{x(z)-2\Delta}{\Delta}\right)\left(\frac{1}{\eta(x(z),\Delta)-1}\right)<1<\left(\frac{x(z)-\Delta}{\Delta}\right)\left(\frac{1}{\eta(x(z),\Delta)-1}\right)$.

Thus, we have the following bounds:

$$\frac{x(z)-2\Delta}{\Delta}\Delta\frac{\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o)}{\eta(x(z),\Delta)-1}\leq\Delta\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o)\leq\frac{x(z)-\Delta}{\Delta}\Delta\frac{\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o)}{\eta(x(z),\Delta)-1}.$$

Taking the limit of the upper bound we have:

$$\begin{aligned}
\lim_{\Delta\searrow 0}\frac{x(z)-\Delta}{\Delta}\Delta\frac{\sum_{o=1}^{\eta(x(z),\Delta)-1}r(o)}{\eta(x(z),\Delta)-1} &= \lim_{\Delta\searrow 0}\left(x(z)-\frac{\Delta^2}{\Delta}\right)\lim_{\Delta\searrow 0}\frac{\sum_{o=1}^{\eta(x(z),\Delta)-1}r(o)}{\eta(x(z),\Delta)-1}\\
&= \lim_{\Delta\searrow 0}\left(x(z)-2\frac{\Delta^2}{\Delta}\right)\lim_{\Delta\searrow 0}\frac{\sum_{o=1}^{\eta(x(z),\Delta)-1}r(o)}{\eta(x(z),\Delta)-1}\\
&= x(z)\,\mathbb{E}\left[r_o\right].
\end{aligned}$$

The first equality uses that the limit of the product of two convergent sequences is the product of their limits. The second line uses that both, $\frac{\Delta^2}{\Delta}$ and $2\frac{\Delta^2}{\Delta}$ converge to 0. Thus, the lower bound shares the same limit. The third line follows from the Law of Large Numbers applied to the limit of the sequence of random variables $r(o)$. Because both bounds converge to the same number, this is enough to show that:

$$\lim_{\Delta\searrow 0}\Delta\sum_{o=1}^{\eta(x(z),\Delta)-1}r(o)=x(z)\,\mathbb{E}\left[r_o\right].$$

For the second result, $\lim_{\Delta\searrow 0}\left[-r^{ER}\mathbb{I}_{[x\leq 0]}+r^{DW}\mathbb{I}_{[x>0]}\right]\phi(x,\Delta)=0$, observe that $0\leq\phi(x,\Delta)<\Delta$. Thus, $\lim_{\Delta\searrow 0}\phi(x,\Delta)=0$.

F.2 Proof of Proposition 11

We want to show that the limit as $\Delta\searrow 0$ of the objecitve of the bargaining problem is the objective in Problem 1 in the main text. We begin with a useful factorization:

$$\begin{aligned}
&\max_{r^{FF}}\lim_{\Delta\searrow 0}\mathbb{E}\frac{\left(\bar{V}^b(z,r^{FF},\Delta)-\bar{V}^b(z,r^{DW},\Delta)\right)^{\xi}}{\Delta^{\xi}}\times\ldots\\
&\frac{\left(\bar{V}^b(\tilde{z},-r^{FF},\Delta)-\bar{V}^b(\tilde{z},-r^{ER},\Delta)\right)^{1-\xi}}{\Delta^{\xi}}\\
=\;&\max_{r^{FF}}\mathbb{E}\left(\lim_{\Delta\searrow 0}\left[\frac{\bar{V}^b(z,r^{FF},\Delta)}{\Delta}-\frac{\bar{V}^b(z,r^{DW},\Delta)}{\Delta}\right]\right)^{\xi}\times ..\\
&\left(\lim_{\Delta\searrow 0}\left[\frac{\bar{V}^b(\tilde{z},-r^{FF},\Delta)}{\Delta}-\frac{\bar{V}^b(\tilde{z},-r^{ER},\Delta)}{\Delta}\right]\right)^{1-\xi}.
\end{aligned}$$

The second line factors in Δ inside the surpluss of both orders. In addition, we pass limits inside the expectations operator. We can do this because all values are bounded and the expectations are with respect to a discrete random variable.

Next, we compute the value of $\lim_{\Delta\searrow 0}\frac{\bar{V}^b(z,r^{FF},\Delta,X')}{\Delta}-\frac{\bar{V}^b(z,r^{DW},\Delta,X')}{\Delta}$ and the corrsesponding limit for the surplus of the lending order. We will show this limit only for the borrowing order. The limit for the surplus of the lending is obtained following the same steps.

By definition of $\bar{V}^b(z,\bar{r},\Delta)$, this limit also equals:

$$\lim_{\Delta\searrow 0}\frac{V^l\left(C'(z),\tilde{B}(z),D'\left(z,r^{FF},\Delta\right)\right)}{\Delta}-\frac{V^l\left(C'(z),\tilde{B}(z),D'\left(z,r^{DW},\Delta\right)\right)}{\Delta} \tag{21}$$

where we are defining

$$\begin{aligned} D'(z,\bar{r},\Delta) \equiv & \tilde{D}(z)(1-\omega)+\left[-r^{ER}\mathbb{I}_{[x(z)\leq 0]}+r^{DW}\mathbb{I}_{[x(z)>0]}\right]\phi(x(z),\Delta)+... \\ & +\Delta\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o)+\Delta\bar{r}. \end{aligned}$$

Recall from Lemma 3 that,

$$\lim_{\Delta\searrow 0}\Delta\sum_{o=1}^{\eta(x(\bar{z}),\Delta)-1} r(o)=x(\bar{z})\,\mathbb{E}[r_o].$$

for the expectation of interest payments on other orders is $\mathbb{E}[r_o]$. Also recall that

$$\lim_{\Delta\searrow 0}\left[-r^{ER}\mathbb{I}_{[x(\bar{z})\leq 0]}+r^{DW}\mathbb{I}_{[x(\bar{z})>0]}\right]\phi(x(\bar{z}),\Delta)=0.$$

By this, the limit $\lim_{\Delta\searrow 0}D'(z,0,\Delta)$, equals:

$$\begin{aligned} & \lim_{\Delta\searrow 0}\tilde{D}(z)(1-\omega)+\left[-r^{ER}\mathbb{I}_{[x(z)\leq 0]}+r^{DW}\mathbb{I}_{[x(z)>0]}\right]\phi(x(z),\Delta)+\Delta\sum_{o=1}^{\eta(x(z),\Delta)-1} r(o) \\ = & \tilde{D}(z)(1-\omega)+x(z)\,\mathbb{E}[r_o] \\ \equiv & D'(z,0,0). \end{aligned}$$

The next step is to express the differences in (21) so that their limit equals a derivative that we can compute. We will use the definition of the Chain Rule. Thus, we divide and V^l evaluated at $D'(z,0,0)$, inside the parenthesis in equation (21). We have:

$$\begin{aligned} & \lim_{\Delta\searrow 0}\left(\frac{V^l\left(C'(z),\tilde{B}(z),D'\left(z,r^{FF},\Delta\right)\right)}{\Delta}-\frac{V^l\left(C'(z),\tilde{B}(z),D'(z,0,0)\right)}{\Delta}\right)... \\ & -\left(\frac{V^l\left(C'(z),\tilde{B}(z),D'\left(z,r^{DW},\Delta\right)\right)}{\Delta}-\frac{V^l\left(C'(z),\tilde{B}(z),D'(z,0,0)\right)}{\Delta}\right). \end{aligned}$$

Dividing and multiplying by two convenient constants, we have:

$$
\begin{aligned}
& \lim_{\Delta \searrow 0} \frac{V^l\left(C'(z), \tilde{B}(z), D'(z,\Delta)\right) - V^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right)}{\Delta} \frac{D'\left(z, r^{FF}, \Delta\right) - D'(z,0,0)}{D'\left(z, r^{FF}, \Delta\right) - D'(z,0,0)} \ldots \\
& - \frac{V^l\left(C'(z), \tilde{B}(z), D'(z,\Delta)\right) - V^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right)}{\Delta} \frac{D'\left(z, r^{DW}, \Delta\right) - D'(z,0,0)}{D'\left(z, r^{DW}, \Delta\right) - D'(z,0,0)} \\
= & \lim_{\Delta \searrow 0} \frac{V^l\left(C'(z), \tilde{B}(z), D'(z,\Delta)\right) - V^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right)}{D'\left(z, r^{FF}, \Delta\right) - D'(z,0,0)} \frac{D'\left(z, r^{FF}, \Delta\right) - D'(z,0,0)}{\Delta} \ldots \\
& - \lim_{\Delta \searrow 0} \frac{V^l\left(C'(z), \tilde{B}(z), D'(z,\Delta)\right) - V^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right)}{D'\left(z, r^{DW}, \Delta\right) - D'(z,0,0)} \frac{D'\left(z, r^{DW}, \Delta\right) - D'(z,0,0)}{\Delta} \\
= & V_D^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right) \lim_{\Delta \searrow 0} \frac{\Delta r^{FF}}{\Delta} \ldots \\
& - V_D^l\left(C'(z), \tilde{B}(z), D'(z,0,0)\right) \lim_{\Delta \searrow 0} \frac{\Delta r^{DW}}{\Delta} \\
= & V_D^l\left(r^{FF} - r^{DW}\right).
\end{aligned}
$$

The first equality rearranges terms. The second equality uses the definition of derivative and the limit of the product of convergent sequences. Thus, we are employing the Chain Rule of derivatives directly. The last line is immediate. Now, by the Envelope Theorem, we know that that $V_D^l(z) = U'(DIV'(z)) \equiv m^b$ and $V_D^l(z)\left(r^{FF} - r^{DW}\right) = m^b\left(r^{FF} - r^{DW}\right)$. Thus, the limit of the surplus of the borrower converges to $m^b\left(r^{FF} - r^{DW}\right)$. By analogy, the limit of the surplus of the lender converges to $m^l\left(r^{FF} - r^{ER}\right)$.

This establishes the desired result:

$$
\begin{aligned}
& \max_{r^{FF}} \lim_{\Delta \searrow 0} \mathbb{E} \frac{\left(\left[V^l(z, r^{FF}, \Delta, X') - V^l(z, r^{DW}, \Delta, X')\right]\right)^{\xi}}{\Delta^{\xi}} \times \ldots \\
& \frac{\left(V^l(\tilde{z}, -r^{FF}, \Delta, X') - V^l(\tilde{z}, -r^{ER}, \Delta, X')\right)^{1-\xi}}{\Delta^{\xi}} \\
= \; & m^l \times m^b \times \max_{r^{FF}} \left(r^{FF} - r^{DW}\right)^{\xi} \left(r^{FF} - r^{ER}\right)^{1-\xi}.
\end{aligned}
$$

F.3 Proof of Proposition 10

First, observe that the constraint set for r^{FF} is $\left[r^{ER}, r^{DW}\right]$. This set is independent of Δ, so the constraint correspondence is continuous. Since for any sequence of $r(o)$ the value of the objective is concave in r^{FF} and the constraint set is compact, there is a unique solution to r^{FF} for any Δ. Call the solution of r^{FF} given Δ, $r^{FF}(\Delta)$. Now, consider a sequence $\Delta_n \searrow 0$. We want to show that the solutions $r^{FF}(\Delta_n)$ converges to:

$$\bar{r}^{FF} = \arg\max_{r^{FF}} \left(m_b r_t^{DW} - m_b r^{FF}\right)^{\xi} \left(m_l r^{FF} - m_l r_t^{ER}\right)^{1-\xi}.$$

We use the same steps as in the proof of the Theorem of the Maximum: Since $\left[r^{ER}, r^{DW}\right]$ is compact, every sequence $r^{FF}(\Delta_n)$ is Cauchy, and thus has a convergent subsequence $r^{FF}(\Delta_{n_k})$. Call the convergence limit for the subsequence $\tilde{r}^{FF}$.

Suppose by contradiction that $\tilde{r}^{FF}$ differs from the solution $\bar{r}^{FF}$. Then,

$$\lim_{n\to\infty} \mathbb{E}\frac{\left(\left[\bar{V}^b(z, r^{FF}(\Delta_n), \Delta_n) - \bar{V}^b(z, r^{DW}, \Delta)\right]\right)^{\xi}}{\Delta_n^{\xi}} \times \frac{\left(\left[\bar{V}^b(\tilde{z}, r^{FF}(\Delta_n), \Delta_n) - \bar{V}^b(\tilde{z}, r^{ER}, \Delta_n)\right]\right)^{1-\xi}}{\Delta_n^{\xi}} \geq$$

$$\mathbb{E}\frac{\left(\left[\bar{V}^b(z, \bar{r}^{FF}, \Delta_n) - \bar{V}^b(z, r^{DW}, \Delta)\right]\right)^{\xi}}{\Delta_n^{\xi}} \times \frac{\left(\left[\bar{V}^b(\tilde{z}, \bar{r}^{FF}, \Delta_n) - \bar{V}^b(\tilde{z}, r^{ER}, \Delta_n)\right]\right)^{1-\xi}}{\Delta_n^{\xi}}.$$

The reason for this result is that the inequality holds pointwise and we know that $r^{FF}(\Delta_n)$ and Δ converge. Now, $\bar{V}^b(\bar{z}, \bar{r}, \Delta_n)$ is continuous in $\bar{r}$, but we have argued that it is not continuous in Δ. However, Proposition 2 shows that it converges as $\Delta_n \searrow 0$. Following the same steps as in the proof of Proposition 1, that is, subtracting and adding limiting terms for $D'(z)$ and $D'(\tilde{z})$, the term on the left of the inequality converges to:

$$\left(m_b r_t^{DW} - m_b \tilde{r}^{FF}\right)^{\xi} \left(m_l r^{FF} - m_l \tilde{r}^{FF}\right)^{1-\xi}.$$

By proposition 1, the terms at the right converges to:

$$\left(m_b r_t^{DW} - m_b \bar{r}^{FF}\right)^{\xi} \left(m_l \bar{r}^{FF} - m_l r_t^{ER}\right)^{1-\xi}.$$

Since the inequality holds pointwise, this implies:

$$\left(m_b r_t^{DW} - m_b \tilde{r}^{FF}\right)^{\xi} \left(m_l r^{FF} - m_l \tilde{r}^{FF}\right)^{1-\xi} \geq \left(m_b r_t^{DW} - m_b \bar{r}^{FF}\right)^{\xi} \left(m_l \bar{r}^{FF} - m_l r_t^{ER}\right)^{1-\xi},$$

which contradicts the fact that $\bar{r}^{FF}$ is a unique maximizer.

G Data Analysis

G.1 Aggregate Monetary Data

All the aggregate monetary time series are obtained from the Federal Reserve Bank of St. Louis Economic Research Database, FRED© .

These series are used in the construction of Figure 12.

Daily Series. The series for interest rates in panel (a) of Figure 12 are daily. We use the following data for the construction of policy rates:

Variable	Source Acronym	Source
Daily Fed Funds Rate	DFF	FRED
Daily Fed Funds Target Rate	DFEDTAR	FRED
Daily Fed Funds Target Rate Upper Limit	DFEDTARU	FRED
Daily Fed Funds Target Rate Lower Limit	DFEDTARL	FRED
Primary Credit Rate (Discount Window Rate)	DPCREDIT	FRED

To reconstruct a series for the fed funds target rate, we use the Daily Fed Funds Target Rate when this series is available. Otherwise, we take the average of the Daily Fed Funds Target Rate Upper Limit and Daily Fed Funds Target Rate Lower Limit.

Weekly Series. The data used to reconstruct the balance sheet components of the Fed is weekly. These series are used in the upper-middle panel of Figure 12. We use the following weekly data:

Variable	Source Acronym	Source
Weekly Fed Total Assets (Less of Consolidation)	$WALCL$	$FRED$
Securities Held Outright	$WSHOL$	$FRED$
Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans	$WSRLL$	$FRED$
Treasury Securities	$WSHOTS$	$FRED$
Federal Agency Debt	$WSHOFDSL$	$FRED$
Mortgage-Backed Securities	$WSHOMCB$	$FRED$
Bank Credit of All Commercial Banks	$TOTBKCR$	$FRED$

We directly plot the series for Treasury Securities. The series that corresponds to Mortgage-Backed Securities plus Agency Debt (MBS+Agency) is the difference between Securities Held Outright and Treasury Securities. We call liquidity facilities the series that includes Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans. All other assets correspond to the Weekly Fed Total Assets (Less of Consolidation) minus the sum of Securities Held Outright and Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans. The upper-right panel is constructed by dividing the Fed's Weekly Fed Total Assets by the series for Bank Credit of All Commercial Banks.

Monthly Series. Finally, we use monthly data to report excess and required reserves and the money multiplier. These series appear in the panels (d) and (f) of Figure 12. The series correspond to:

Variable	Source Acronym	Source
Excess Reserves	EXCRESNS	FRED
Required Reserves	REQRESNS	FRED
$M1$ Money Multiplier	MULT	FRED

G.2 Individual Bank Data

We use information on FDIC Call Reports for data on commercial banks in the construction of all the time series that are based on individual bank data. The industry experienced a considerable amount of mergers and acquisitions. Moreover, many US chartered banks report very small amounts of lending activities during certain periods relative to their assets— something that may underrepresent many of the ratios we discuss. To present a consistent view of bank ratios, commercial lending and the growth rates of different accounts, we follow Bigio and Majerovitz (2013) in the construction of the data we report in the paper and in this appendix. Bigio and Majerovitz (2013) use data filters based on those used by Kashyap and Stein (2000), and on den Haan et al. (2002). This filter gets rid of abnormal outliers and adjusts the data for mergers.

Filters. The details of the filters we use are provided in Bigio and Majerovitz (2013). In a nutshell, the first and last quarters when a bank is in the sample are dropped. All observations for which total loans, assets, or liabilities are zero are dropped. Those observations that are more than five—cross-sectional— standard deviations away from the cross-sectional mean for the quarter, in any of the aforementioned variables for which growth rates are calculated, are dropped. If a bank underwent a merger or acquisition—or a split, transfer of assets, and so on.–it is dropped from the panel data but not from the aggregate time series.

Seasonal Adjustments. Most series feature strong seasonal components. Moreover, we find seasonal components at the bank level. We use standard seasonal adjustment procedures to correct for seasonality at the bank level.

Series. Panel (e) of Figure 12 reports two time series for commercial and industrial loans (C&I loans). These series are constructed using the filters explained above and reported as percentage deviations from the value of the series during 2007Q4. The first series is the time series for commercial and industrial loans. The other series adjusts the original series for increases in lending that have to do with prior commitments. The series adjusted for prior commitments is constructed in the following way. First, we construct an upper bound for the use of loan commitments, subtracting the value of the stock of loan commitments at a given quarter from the stock during 2007Q4. Then, the adjusted series for C&I loans is the original series minus the series for the use of loan commitments.

The following section of this appendix describes some statistics for several bank balance-sheet accounts. That analysis guides our judgements of using total deposits to calibrate the withdrawal distribution, F_t, in our model. We narrow the analysis to the statistics of total deposits (TD), demand deposits (DD), total liabilities (TL), tangible equity (TE), equity (E), and loans net of unearned income (LNUI).

Bank Ratios. The bank ratios reported in Figure 13 are the following: Tangible leverage is the value of total liabilities minus intangibles over the value of equity minus intangibles. The liquidity ratio is constructed as the sum of reserves (cash) plus Treasury securities over total assets. The dividend rate is the value of dividends relative to equity. The series for the return on equity is income over the value of equity. We report the cross-sectional average for every bank and every quarter in the cross section. We report two averages, simple averages, and averages weighted by asset size.

Summary of Individual Variables. The summary of the series we use is found here:

Variable	Source Acronym	Source
total deposits (TD)	rcfd2200	Call Reports
demand deposits (DD)	rcfd2948	Call Reports
total liabilities (TL)	rcfd2948	Call Reports
intangible	rcfd2143	Call Reports
cash	rcfd0010	Call Reports
treasury holdings	rcfd0400+rcfd8634	Call Reports
tangible equity (TE)	Equity Intangible	Call Reports
equity (E)	TotalAssets-TotalLiability	Call Reports
total loans net of unearned income (LNUI)	rcfd2122	Call Reports
commercial and industrial loans	rcfd1766	Call Reports
commercial and industrial loans (commitments)	rcfd3816+rcfd6550	Call Reports
total assets	rcfd2170	Call Reports
income	riad4000	Call Reports
dividends	riad4460+riad4470	Call Reports

G.3 Data Analysis

1990-2010 Sample Averages. The summary statistics for the quarterly growth rate of the aggregate time series is presented in Table 3.

Table 3: Summary Statistics for Bank-Quarter Observations

Variable	**Mean**	**Std. Dev.**	**N**
TD	1.018	0.064	536074
DD	1.027	0.810	536074
TL	1.018	0.063	536074
TE	1.018	0.058	536074
E	1.019	0.067	536074
LNUI	1.022	0.061	536074

The data exhibit very similar patterns when we compare the average growth and standard deviation of the growth rate of total deposits and total liabilities. Demand deposits, on the contrary, are almost ten times as volatile as total deposits. This is one reason to use total deposits as our data counterpart to calibrate F_t. Although less volatile than demand deposits, total deposits still feature substantial volatility. The standard deviation of this series is 6.4 percent per quarter, and it is close to the volatility of total liabilities, 6.3 percent. Total deposits are also more correlated with equity growth—for both tangible and total equity. The correlation matrix of the variables in the analysis is reported in Table 4.

Table 4: Cross-sectional correlation for bank-quarter observations

Variables	TD	DD	TL	TE	E	LNUI
TD	1.000					
DD	0.059	1.000				
TL	0.286	0.050	1.000			
TE	0.117	0.005	0.102	1.000		
E	0.145	0.006	0.098	0.855	1.000	
LNUI	0.527	0.024	0.198	0.153	0.155	1.000

Quarterly Cross-Sectional Deviations. Part of the variation in the bank-quarter statistics presented above follows from the influence of aggregate trends and seasonal components. To decompose the variation of these liabilities into their common trend, we present the summary statistics in terms of deviations of these variables from their quarterly cross-sectional averages. Table 5 presents the summary for cross-sectional deviations.

Table 5: Summary Statistics for Cross-Sectional Deviations from Mean Growth

Variable	Mean	Std. Dev.	N
devTD	0	0.043	536074
devDD	0	0.123	536074
devTL	0	0.042	536074
devTE	0	0.039	536074
devE	0	0.041	536074
devLNUI	0	0.045	536074

A comparison between Tables 3 and 5 reveals that the series for deviations from the cross-sectional mean preserve much of the variation of the aggregated time series. This is evidence of a fair amount of idiosyncratic volatility in total deposit growth across banks. Table 6 shows the correlation in cross-sectional deviations from quarterly means across these variables. These correlations are almost identical to the correlations of historical growth rates. This implies that the idiosyncratic component is very important to explain the cross correlations, more so than common aggregate trends.

The correlation between the cross-sectional deviations of tangible equity growth and the counterpart for total deposits is 8.2 percent. In the model, this correlation is very high—though not one due to the kink in $\chi(\cdot)$— because deposit volatility is the only source of risk for banks. In practice, banks face other sources of risks that include loan risk, duration risk, and trading risk. This figure, however, suggests that deposit withdrawal risks are non negligible risks for banks. Figure 1, found in the body of the paper, reports the empirical histograms for every bank-quarter growth observation and decomposes the data into two samples, pre-crisis (1990Q1-2007Q4) and crisis (2008Q1-2010Q4). We use the empirical histogram of the quarterly deviations of total deposits to calibrate F_t, the process for withdrawal shocks.

Tests for Growth Independence. We have assumed that the withdrawal process is i.i.d. over time and across banks. This assumption is critical to solve the model without keeping track

Table 6: Correlation for Cross-Sectional Deviations from Means

Variables	dev TD	dev DD G	dev TL	dev TE	dev RTE	dev E	dev RE
dev T D	1.000						
dev DD	0.389	1.000					
dev TL	0.844	0.345	1.000				
dev TE	0.082	0.027	0.135	1.000			
dev RTE	0.050	0.016	0.097	0.854	1.000		
dev E	0.152	0.052	0.238	0.727	0.635	1.000	
dev RE	0.118	0.040	0.187	0.635	0.769	0.881	1.000

of distributions. This assumption implies that if we subtract the common growth rates of all the balance sheet variables in our model, the residual should be serially uncorrelated. We test the independence of the deviations-from-means quarterly growth rates using an OLS estimation procedure. We run the deviations in quarterly growth rates from the cross-sectional averages against their lags. The evidence from OLS autoregressions does not support the assumption that of time-independent growth because autocorrelations are significant. Table 7 reports the autocorrelation coefficients of all the variables in deviations. Though none are statistically equal to zero most of these autocorrelation coefficients are low. The low values of the autocorrelation coefficients are suggestive that assuming i.i.d. is a good approximation to the actual process.

Table 7: Autocorrelation coefficients for cross-sectional deviations from mean growth

Variable	**Coefficient**	**Std. Error**	**N**
devTD	0.171	(0.001)***	526641
devDD	-0.262	(0.001)***	526641
devTL	0.196	(0.001)***	526641
devTE	0.204	(0.001)***	526641
devE	0.225	(0.001)***	526641
devLNUI	0.376	(0.001)***	526641