Appendix to "Overborrowing and Systemic Externalities in the Business Cycle" *

Javier Bianchi
University of Maryland

1 Sensitivity Analysis

We continue here the sensitivity analysis presented in the body of the paper. We discuss separately the effects of varying each of the parameter values of the model, using the analysis of the externality term and the elasticity decomposition studied in the body of the paper. The main quantitative results of all experiments are shown in Table 4. The table shows for each experiment the average welfare loss, the average implied tax on debt, the relative volatility of consumption, the probability of a financial crisis for the decentralized equilibrium and constrained-efficient allocations, and the effects of a median crisis in consumption, the real exchange rate and the current account for the two equilibria.

Discount Factor (β).— An increase in the discount factor leads to a shift of the distribution of bond holdings towards a lower amount of debt, leading to less frequent binding

*Contact details: Department of Economics, University of Maryland, 3105 Tydings Hall, College Park MD 20742. Email: bianchi@econ.umd.edu. This paper is based on my dissertation at the University of Maryland. I am indebted to my advisors Enrique Mendoza, Anton Korinek, Carlos Vegh and John Shea. For useful comments and suggestions, I thank VV. Chari, Pablo D'Erasmo Juan Dubra, Bora Durdu, Emmanuel Farhi, Bertrand Gruss, Tim Kehoe, Alessandro Rebucci, Carmen Reinhart, Horacio Sapriza and participants at seminars at the Atlanta Fed, Board of Governors of the Federal Reserve, Boston Fed, Brown, Chicago Booth School of Business, Columbia Business School, LSE, New York Fed, NYU, UCL, Universidad de Montevideo, University of Maryland, University of Wisconsin and at the following conferences: 2010 AEA meetings, Central Bank of Uruguay Annual Conference, XV Workshop on Dynamic Macroeconomics, XII Workshop in International Economics and Finance, 2009 Latin American Econometric Society meetings, 2010 Society of Economic Dynamics (SED) meetings, IV Washington University Graduate Student Conference. I am grateful to the Federal Reserve Bank of Atlanta and the Board of Governors of the Federal Reserve for their hospitality.

constraints and causing the distribution of the externality term to concentrate higher probability in a region where its value is zero. This effect leads to smaller effects from the externality. There is an opposite effect from an increase in the discount factor. Recall that the maximum welfare gains from the externality arise in relatively tranquil times because of the reduction in future vulnerability to financial crises. Hence, a higher discount factor makes the economy value relatively more the benefits from a reduction in future variability, which should lead to higher welfare effects from correcting the externality. Quantitatively, we find that the first effect is more important. Increasing the discount factor by 0.02 reduces the average implied tax on debt to 3.3 percent, although large differences remain in the probability of financial crises: the probability of a crisis is 0.2 percent for the social planner and 4.1 percent for the decentralized equilibrium.

Interest Rate (r).— An increase in the interest rate has effects similar to an increase in the discount factor since both reduce the willingness to borrow and shifts the economy away from binding constraints. For a given amount of debt, however, a higher interest rate implies an increase in the debt service, which causes a larger depreciation of the real exchange rate. Quantitatively, we find that increasing the interest rate 100bps reduces the implied tax on debt from 5.2 percent to 4.4 percent, but the effects on the incidence and severity of financial crises remain very similar.

Risk Aversion (σ).— An increase in the risk aversion implies a higher disutility from consumption variability. This implies that a large drop in consumption generates a higher shadow value from relaxing the credit constraint at a given state where the constraint binds; therefore, this yields a higher externality term. At the same time, an increase in risk aversion makes both the social planner and private agents accumulate more precautionary savings making the constraint less likely to bind and shifting away the distribution of the externality term towards zero. Quantitatively, as shown in Table 4, we find that the effects of the externality decreases (increases) modestly when we consider $\sigma = 5$ ($\sigma = 1$).

Independent shocks.— We model tradable and nontradable endowment shocks as independent AR(1) processes and analyze the effects over the externality. When shocks are correlated, both tradable and nontradable shocks typically fall during financial crises. The fact that nontradables fall, however, mitigates the fall in the price of nontradables and the tightening of financial constraints. This channel suggests that making the two shocks independent should reduce the effects of the externality. There is another channel, however, by which making the shocks independent causes the externality to have higher effects. For the baseline calibration, the risk aversion and the elasticity of substitution between tradables and nontradables are such that tradable and nontradable goods are Edgeworth substitutes. As a result, a fall in the endowment of nontradables when the credit constraint binds, increases the marginal utility from tradable consumption, which increases the desire to borrow and increases the shadow value from relaxing the credit constraint. Quantitatively, we find that the effects over the shadow value from relaxing the credit constraint are stronger than those affecting the price effects, so that the differences in severity of financial crises become even stronger.

Volatility and Persistance ($Cov(\varepsilon)$).— An increase in the volatility of endowment shocks increases the severity of financial crises, in terms of the amplification effects and the disutility cost from a binding constraint. This effect increases the externality term. At the same time, private agents have an incentive to increase relatively more precautionary savings in response to the increase in volatility. This occurs because the concavity of the utility function implies that a given increase in variability is more costly in the decentralized equilibrium compared to the constrained-efficient allocations. In fact, when we vary simultaneously the volatility of the shocks to the endowment processes by 15 percent, we find that the externality decreases modestly with a higher volatility.

An increase in persistence leads to a higher probability of financial crises for a given level of precautionary savings although it does not alter the size of the shocks and the severity of financial crises. When we vary the autocorrelation of the endowment shocks by 15 percent, we find that a higher autocorrelation is associated with larger effects from the externality. In fact, the experiment with higher autocorrelation yields larger differences in the incidence and severity of financial crises, and this leads to larger welfare effects.

Elasticity of Substitution$(1/(1+\eta))$.— As explained in the paper, the elasticity of substitution between tradables and nontradables determines the debt service elasticity of the real exchange rate, which is in turn a key component of the externality term. Moreover, the elasticity of substitution also affects the incentive to accumulate precautionary savings: the lower the elasticity of substitution the higher the disutility from drops in consumption during financial crises. This second channel is similar to the increase in the risk aversion, but we find that the channel affecting directly the price effects are quantitatively more important.

Share of tradables (ω).— As explained in the paper, the weight of tradables in the utility function determines the borrowing limit elasticity of the real exchange rate and is key for the effects on the externality. There is another effect of this parameter. A higher share of tradables in the utility function implies that large drops in tradables consumption during financial crises are more costly, causing an increase in precautionary savings. As explained before, this second channel becomes qualitatively ambiguous, but we find that the price effects, which unambiguously increase the externality, are more significant.

Credit Coefficient (κ).— We set $\kappa^T = \kappa^N = \kappa$. An increase in κ has two effects. First, it increases directly the externality term, because for a given drop in the price of nontradables the effects over the borrowing ability are directly proportional to κ. Second, it makes the constraint less likely to bind, hence reducing the effects of the externality. On one hand, when κ is 0, there is no borrowing; therefore an increase in κ raises the effects of the externality. On the other hand, for a very large κ, the credit constraint never binds and there are no effects from the externality in the long run. Quantitatively, we find that increasing κ from 0.32 to 0.36 increases the welfare effects of the externality to 0.22 percentage points of permanent consumption. In addition, consumption during a median crisis drops almost three times as much in the decentralized equilibrium compared to the constrained-efficient equilibrium. Reducing κ to 0.28 reduces also slightly the effects of the externality but crises in the decentralized equilibrium remain ten times more likely than in the constrained-efficient equilibrium.

Table 1: Sensitivity Analysis

						Severity of Financial Crises					
				Probab. Crisis		Consumption		RER		Current Account	
	Welfare	Tax on Debt	$\sigma_{c^{de}}/\sigma_{c^{sp}}$	DE	SP	DE	SP	DE	SP	DE	SP
baseline	0.13	5.0	1.13	5.5	0.4	-16.7	-10.0	19.2	1.1	7.9	0.0
$\beta = 0.93$	0.08	3.2	1.08	4.1	0.2	-13.0	-9.2	17.6	7.7	5.8	1.3
$\beta = 0.89$	0.19	7.0	1.18	6.4	0.4	-17.9	-10.0	22.1	1.0	9.5	-0.2
$r = 0.05$	0.11	4.2	1.11	4.7	0.3	-16.1	-10.2	17.6	1.3	6.9	-0.1
$r = 0.03$	0.16	5.8	1.16	6.0	0.4	-17.3	-10.1	20.7	1.1	8.8	-0.1
$\sigma = 5$	0.11	4.5	1.06	2.4	0.2	-12.8	-7.9	17.2	4.3	5.5	-0.2
$\sigma = 1$	0.15	5.1	1.21	6.4	0.4	-19.4	-10.1	25.7	1.1	11.6	-0.1
Independent shocks	0.15	4.8	1.2	4.9	0.3	-12.4	-2.2	35.4	14.7	13.0	1.0
Volatility ε (15 % less)	0.17	5.1	1.21	4.9	0.0	-18.9	-10.1	23.9	0.2	10.9	-0.1
Volatility ε (15 % more)	0.13	4.8	1.10	5.5	0.4	-10.5	-6.5	15.3	5.1	6.6	2.1
Autocorrelation ε (15 % less)	0.12	4.9	1.12	6.0	0.4	-15.4	-9.6	17.4	1.6	6.7	-0.2
Autocorrelation ε (15 % more)	0.16	5.0	1.17	4.9	0.1	-18.6	-10.5	23.0	1.2	10.0	0.0
$\kappa = 0.36$	0.21	5.5	1.15	4.3	0.4	-14.5	-5.6	28.0	7.3	10.4	0.0
$\kappa = 0.28$	0.09	4.5	1.10	6.4	0.6	-13.5	-8.2	18.9	5.1	6.5	0.3
$\omega = 0.34$	0.08	4.2	1.11	6.5	0.9	-13.8	-9.9	17.2	8.5	6.6	2.0
$\omega = 0.29$	0.22	5.6	1.14	4.1	0.0	-22.3	-10.2	35.6	1.6	15.9	0.0
$1/(\eta + 1) = 1.0$	0.07	4.1	1.10	7.7	1.3	-9.8	-6.3	12.7	4.9	6.3	2.3
$1/(\eta + 1) = 0.7$	0.19	5.6	1.15	4.5	0.5	-17.0	-8.0	30.0	5.2	11.2	0.0
Intermediate inputs	0.10	5.2	1.13	5.5	0.5	-22.0	-16.3	27.5	16.9	7.1	0.0
$\kappa^N/\kappa^T = 0.5$	0.04	3.7	1.08	7.0	2.0	-17.1	-13.8	11.4	1.9	5.3	0.0

Note: 'DE' represents the decentralized equilibrium,'SP' represents the social planner. Tax on debt is the unconditional average of the optimal tax on debt. Welfare represent the unconditional average of the welfare gains from correcting the externality. $\sigma_{c_{de}}/\sigma_{c_{sp}}$ represents the relative volatility in consumption. The tax on debt, welfare and the probability of a crisis are expressed in percentage. Consumption and the depreciation of the real exchange rate (RER) are expressed as percentage deviations from long run values during a median crisis (see footnote 12 in the body of the paper). Current account is expressed as a percentage of GDP.

2 Numerical Solution Method for Competitive Equilibrium

The computation of the competitive equilibrium requires solving for functions $\mathcal{B}(b,y)$, $\mathcal{P}^N(b,y)$,$\mathcal{C}^T(b,y)$ such that:

$$\mathcal{P}^N(b,y) = \left(\frac{1-\omega}{\omega}\right)\left(\frac{\mathcal{C}^T(b,y)}{y^N}\right)^{\eta+1} \tag{1}$$

$$u_T(\mathcal{C}^T(b,y), y^N\ ) \geq \beta(1+r)\mathbb{E}_{y'/y} u_T(\mathcal{C}^T(\mathcal{B}(b,y),y'), y^{N\prime}\ ) \tag{2}$$

$$\mathcal{B}(b,y) \geq -\left(\kappa^N \mathcal{P}^N(b,y) y^N + \kappa^T y^T\right) \qquad \text{with} = \text{if (25) holds with strict inequality} \tag{3}$$

$$\mathcal{B}(b,y) + \mathcal{C}^T(b,y) = b(1+r) + y^T \tag{4}$$

where $u_T(\mathcal{C}^T(b,y), y^N\ ) = u_C(\mathcal{C}(b,y))\mathcal{C}_T(b,y)$, $\mathcal{C}(b,y) = \left[\omega\left(\mathcal{C}^T(b,y)\right)^{-\eta} + (1-\omega)\left(y^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}$ and $y = (y^T, y^N)$.

The algorithm employed to solve for the competitive equilibrium is based on the time iteration algorithm modified to address the occasionally binding endogenous constraint. The algorithm follows these steps:[1]

1. Generate a discrete grid for the economy's bond position $G_b = \{b_{1,} b_2, ...b_M\}$ and the shock state space $G_Y = \{y_{1,} y_2, ...y_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 800 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture $\mathcal{P}^N_K(b,y)$, $\mathcal{B}_K(b,y)$, $\mathcal{C}^T_K(b,y)$ at time K $\forall b \in G_b$ and $\forall y \in G_Y$.

3. Set $j = 1$

4. Solve for the values of $\mathcal{P}^N_{K-j}(b,y), \mathcal{B}_{K-j}(b,y), \mathcal{C}^T_{K-j}(b,y)$ at time $K-j$ using (1),(2),(3),(4) and $\mathcal{B}_{K-j+1}(b,y), \mathcal{P}^N_{K-j+1}(b,y)$, $\mathcal{C}^T_{K-j+1}(b,y)$, $\forall b \in G_b$ and $\forall y \in G_Y$:

   (a) Set $\mathcal{B}_{K-j}(b,y) = -\left(\kappa^N \mathcal{P}^N_{K-j+1}(b,y) y^N + \kappa^T y^T\right)$ and compute $\mathcal{C}^T_{K-j}(b,y)$ from (4)

[1] For the social planner's allocations, we use a standard value function iteration algorithm.

(b) Compute

$$U = u_T(\mathcal{C}^T_{K-j}(b,y), y^N) - \beta(1+r)\mathbb{E}_{y'/y} u_T(\mathcal{C}^T_{K-j}(\mathcal{B}_{K-j}(b,y), y'), y^{N'})$$

(c) If $U > 0$, the credit constraint binds; move to (e).

(d) Solve for $\mathcal{B}_{K-j}(b,y), \mathcal{C}^T_{K-j}(b,y)$ using (2) and (4) with a root finding algorithm.

(e) Set $P^N_{K-j}(b,y) = \left(\frac{1-\omega}{\omega}\right)\left(\frac{\mathcal{C}^T_{K-j}(b,y)}{y^N}\right)^{\eta+1}$

5. Evaluate convergence. If $\sup_{b\in G_b, y\in G_Y} \|x_{K-j}(b,y) - x_{K-j+1}(b,y)\| < \varepsilon$ for $x = \mathcal{B}, \mathcal{C}^T, \mathcal{P}^N$ we have found the competitive equilibrium. Otherwise, set $x_{K-j}(b,y) = \alpha x_{K-j}(b,y) + (1-\alpha)x_{K-j+1}(b,y)$ and $j \rightsquigarrow j+1$ and go to step 4. We use values of α close to 1.