Scrambling for Dollars: International Liquidity, Banks and Exchange Rates *

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Abstract

We develop a theory of exchange rate fluctuations arising from the demand of financial institutions for liquid assets. Financial flows are unpredictable and may leave banks "scrambling for dollars." Because of settlement frictions in interbank markets, a precautionary demand for dollar reserves emerges and gives rise to an endogenous convenience yield on the dollar. When the risk of dollar funding increases, the dollar appreciates as banks increase their demand for dollar assets. We show evidence of the relationship between exchange rate fluctuations for the G10 currencies and the quantity of dollar liquidity that is consistent with the theory.

Keywords: Exchange rates, liquidity premia, monetary policy

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1 Introduction

The well-known "disconnect" in international finance holds that foreign exchange rates show little empirical relationship to the macro variables, such as interest rates and output (Obstfeld and Rogoff, 2000). More recent work contends that the source of the disconnect is in financial markets (Itskhoki and Mukhin, 2021a). Moreover, there has long been evidence of time-varying expected excess returns in foreign exchange markets, and furthermore, it appears that the US dollar is particularly special, as dollar assets offer lower average returns relative to the rest of the major currencies when measured on historical data (Gourinchas and Rey, 2007). To account for the exchange rate disconnect and associated puzzles, the literature has turned to models with currency excess returns as the potential "missing link." The source or sources of these excess returns, however, remain elusive.

In this paper, we develop a theory of exchange rate fluctuations arising from the liquidity demand by financial institutions within an imperfect interbank market. We build on two observations of the international monetary system. First, US dollars are the dominant source of foreign currency funding. According to the BIS locational banking statistics, in March 2021, the global banking and non-bank financial sector had cross-border dollar liabilities of over $11 trillion. Second, there is an inherent instability of dollar funding. As documented, for example, in Acharya, Afonso and Kovner (2017), banks are occasionally subject to large funding uncertainty or interbank market freezing, which can leave them "scrambling for dollars." Narrative discussions attribute fluctuations in the US dollar exchange rate to such vicissitudes in the short-term international money markets. A contribution of our paper is to develop a formal framework to articulate this channel. We further provide quantitative analysis that estimates series for global dollar funding levels and uncertainty shocks. We provide evidence of a positive and significant statistical relationship between bank liquid dollar holdings and exchange-rate fluctuations and use our model to interpret that feature as evidence of the importance of dollar funding uncertainty shocks.

In our framework, financial institutions—hereafter referred to simply as "banks"—manage assets and liabilities in two currencies. Banks face the risk of sudden outflows of liabilities. If a bank ends up short of liquid assets to settle those flows, it needs to find a counterparty that provides the liquidity. However, there are times when banks may lose confidence in each other and, as a result, face tighter frictions in the interbank market. As insurance against these outflows, banks maintain a buffer of liquid assets, especially dollar liquid assets, in line with the aforementioned observations on the international financial system. As funding risk and interbank market frictions fluctuate over time, they alter the relative demand for currencies, resulting in movements in the exchange rate.

The theory uncovers how frictions in the settlement of international deposit flows emerge as a dollar liquidity premium. This dollar liquidity premium generates a time-varying wedge in the interest parity condition, or "convenience yield," which plays a pivotal role in the determination of the exchange rate. Critically, the convenience yield is endogenous and depends on the quantity of outside money (liquid assets) and policy rates, the matching frictions in the interbank market, and the volatility of deposit flows in different currencies. Through this endogenous convenience yield, we link nominal exchange rates and the dollar liquidity premium to the reserve position of banks in different currencies, funding risk, and confidence in the interbank market.

On the surface, the model resembles the seminal monetary exchange rate model of Lucas (1982). In that model, the two currencies earn a liquidity premium over bonds because the goods in each country must be bought with the local currency. A money demand equation determines the price levels in both currencies, and relative prices determine the exchange rate. Our model shares Lucas's segmentation of transactions and exchange rate determination. However, in our model, the demand for reserves in either currency stems from the precautionary demand by banks. This implies different predictions of how the exchange rate reacts to aggregate shocks and policy.

In particular, we show how the model can rationalize why the dollar tends to appreciate in times of high volatility—a phenomenon that remains elusive for existing open-economy models. Models of excess currency returns based on risk premia can account for the excess dollar returns by positing that the dollar appreciates during global downturns. However, they do not explain why the dollar appreciates during global downturns in the first place. Models of financially constrained intermediaries provide an alternative channel for excess dollar returns. Yet, these models predict that the US dollar should depreciate in a downturn, as the United States withstands a larger share of losses in a global economic downturn. [1]

In terms of policy, we find that changes in policy rates induce an attenuation effect on exchange rates. To understand why, consider an initial situation where the two currencies have the same interest rate and the exchange rate is expected to be constant. Suppose that the dollar interest rate goes up. Given exchange rates, banks have incentives to shift their portfolio towards dollar. As banks become relatively more satiated with dollars, the convenience yield falls. No arbitrage then requires that the dollar appreciates but less than it would in the absence of the endogenous convenience yield.

We provide empirical evidence that supports the theoretical link between the balance sheet of the banking sector and the US dollar exchange rate. According to the theory, the financial sector increases its demand for liquid dollar assets relative to dollar funding—US government obligations, including Treasuries and reserves held at the Federal Reserve—when funding becomes more uncertain, and this, in turn, translates into an appreciation of the dollar. In particular, the theory provides a tight prediction for the results of a regression of the exchange rate on banks' liquidity ratio as a function of the underlying shocks. As a validation exercise, we conduct an equivalent regression with data for the G10 currencies and with simulated data from the model. We find that the relationship in the data aligns remarkably well with the model. Moreover, the findings are robust to multiple specifications, including controlling for VIX —- a variable that captures a broad measure of uncertainty and has been shown to have significant explanatory power for exchange rates (Brunnermeier, Nagel and Pedersen, 2008; Lilley, Maggiori, Neiman and Schreger, 2019)—and using an instrumental variable approach.

We calibrate and estimate our model, disciplining the parameters with banks' balance sheet data and observed exchange rate fluctuations and covered interest parity (CIP) deviations. Our counterfactual analysis shows that over the last 20 years, liquidity factors accounted for more than 1/3 of the variations in the euro-dollar exchange rate and more than 90% of the deviations from covered interest parity.

[1] Maggiori (2017) shows that if the US has a larger capacity to withstand risk in a global downturn, US households bear a larger share of losses relative to the rest of the world in a global downturn. With home bias, this means that the dollar must experience a real depreciation in a global downturn.

Literature Review. Our paper contributes to the literature on exchange rates. Empirically, numerous studies have highlighted various failures of the canonical international macro model built on uncovered interest parity. Most notably, these shortcomings include the disconnect that exists in the data between exchange rates and macroeconomic fundamentals (“exchange rate disconnect”), and the inconsistency between differences in nominal rates and expected exchange rate movements (”forward premium puzzle”).[2]

On the theoretical front, a voluminous literature has aimed to address the aforementioned puzzles. Broadly speaking, one can group this literature among three different strands. A first strand of the literature has introduced exogenous convenience yields, for example, by introducing bonds in the utility function. Examples following this route include Engel (2016), Valchev (2020), Jiang, Krishnamurthy and Lustig (2020), and Kekre and Lenel (2021). While this approach has proven useful to account for exchange rate fluctuations, it leaves unexplained the source of the convenience yield that causes the fluctuations in exchange rates.

A second strand of the literature has focused on risk premia as a key driver of deviations from uncovered interest parity. This includes work on disaster risk (Farhi and Gabaix, 2016), consumption habits (Verdelhan, 2010), or long-run risk (Bansal and Shaliastovich, 2013; Colacito and Croce, 2011; Colacito, Croce, Ho and Howard, 2018). As in the closed-economy equity-premium puzzle literature, departing from standard preferences for consumption allows the model to generate substantial risk premia and can help address some of the shortcomings of the canonical international macro model.

A third strand of the literature has turned to models with segmented markets and frictions on financial intermediaries, which give rise to limits to international arbitrage. One approach in this literature focuses on balance sheet constraints on intermediaries. For example, Gabaix and Maggiori (2015) considers a two-country model where households can only take positions in local currency and trade with global financial intermediaries that can take positions in both currencies. Because the intermediary is subject to a leverage constraint, UIP fails to hold and financial

[2] See Meese and Rogoff (1983); Fama (1984); Obstfeld and Rogoff (2003). An active literature revisiting these puzzles and other important features of exchange rates includes Hassan and Mano (2019); Kalemli-Özcan (2019); Kalemli-Özcan and Varela (2021); Brunnermeier et al. (2008); Lilley et al. (2019). See Engel (1996, 2014) for surveys of the literature.

shocks, driven by exogenous changes in noise traders' position, affect exchange rates.[3] A second approach in this literature considers risk-averse intermediaries. In Itskhoki and Mukhin (2021a), households in the two countries trade in the local currency bond and foreign intermediaries are subject to limited hedging opportunities and therefore require compensation to take a currency mismatch. Following this literature, Gourinchas, Ray and Vayanos (2021), and Greenwood, Hanson, Stein and Sunderam (2020) introduce maturity in models with downward sloping demands and examine the implications for yield curves and Koijen and Yogo (2020)–with a more empirical approach—build a demand system to estimate elasticities of exchange rates to capital flows.

Our paper offers an alternative source for deviations of uncovered interest parity. While our theory is in many ways complementary to those based on risk premia or financial constraints on intermediaries, we argue that it offers signature predictions for exchange rates. As previously mentioned, a key challenge faced by these theories is explaining the safe heaven nature of the US dollar—often referred to as the "reserve currency paradox" (Maggiori, 2017). From the risk-premium channel perspective, the dollar earns a lower expected return because it is perceived to appreciate during a global crisis. However, why the dollar appreciates is for the most part, left unexplained.[4] Moreover, a challenge for this literature is that if exchange rates were indeed primarily driven by risk considerations, an increased demand for dollars due to insurance motives (and an appreciation) should not occur during a global crisis, as the adverse shock has already materialized.[5] From the perspective of intermediary asset pricing models, if the US is endowed with a greater capacity to absorb losses during a global economic downturn, its real exchange rate should depreciate because consumption falls more sharply in the US compared to the rest of the world. Our model provides a natural explanation for

[3] See also Amador, Bianchi, Bocola and Perri (2020); Fanelli and Straub (2020) for other examples and Maggiori (2021) for a review of this literature. An early paper that considers segmentation in domestic markets is Alvarez, Atkeson and Kehoe (2009). Bacchetta and Van Wincoop (2010) considers constraints on portfolio choices to understand the forward discount puzzle.

[4] In Farhi and Gabaix (2016), the dollar experiences a real appreciation when the risk of a disaster goes up because the disaster is assumed to affect disproportionally the non-tradable sector of the US economy.

[5] Motivated by the "reserve currency paradox", Kekre and Lenel (2021) develops a rich two-country model with nominal rigidities, exogenous convenience yields, and Epstein-Zin preferences. In their model, however, the flight to safety is modeled exogenously as an increase in the preference for foreign currency bonds.

the US dollar's safe haven status. In times of heightened uncertainty, the liquidity properties of the US dollar become increasingly valuable, causing banks to scramble for dollars and leading to an appreciation of the dollar.

Our paper also relates to an emerging literature on interbank market frictions and monetary policy (see e.g. Bianchi and Bigio, 2021; De Fiore, Hoerova and Uhlig, 2018; Piazzesi and Schneider, 2021; and Weill, 2020 for a review) To the best of our knowledge, our paper provides the first attempt at incorporating these frictions in an open economy framework. Our model builds more closely on Bianchi and Bigio (2021), which we generalize by allowing for multiple currencies and aggregate risk.

Outline. The paper is organized as follows.Sections 2 and 3 present the model and theoretical analysis. Section 4 presents the quantitative and empirical analysis. Section 5 concludes. All proofs are in the Appendix.

2 A Model of Banking Liquidity and Exchange Rates

We present a dynamic equilibrium model of global banks that intermediate international financial flows and are subject to idiosyncratic liquidity shocks. The model has two economies, the EU and the US, with corresponding currencies and central banks. We label the euro as the domestic currency and the dollar as the foreign currency. There is a representative global household and a single final tradable good produced by a continuum of international multinational firms.

2.1 Environment

Timing. Time is discrete and has an infinite horizon. Every period is divided into two sub-stages: a lending stage and a balancing stage. In the lending stage, banks make their equity payout, Div_t, and portfolio decisions. In the balancing stage, banks face liquidity shocks and re-balance portfolios.

Notation. We use an asterisk to denote the foreign currency (i.e., the dollar) variable. The exchange rate is defined as the amount of euros necessary to purchase one dollar—hence, a higher e indicates an appreciation of the dollar. We index the vector of aggregate shocks by X and use i_t to denote an interest rate paid in period t (and determined in period $t-1$).

Preferences and budget constraint. Banks' payouts are distributed to households that own banks' shares and have linear utility with discount factor β. A bank's objective is to maximize shareholders' value, and therefore it maximizes the net present value of dividends:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \cdot Div_t. \tag{1}$$

Banks enter the lending stage with a portfolio of assets and liabilities. The portfolio includes liquid assets in euros and dollars, m_t and m_t^*, and loans b_t, which are denominated in consumption goods and pay a real return R_t^b. We will refer to liquid assets as "reserves" for simplicity, but this term should be understood as also encompassing government bonds—the critical property, as we will see, is that these are assets that can be used as settlement instruments.[6]

On the liability side, banks obtain funding via demand deposits, d_t and d_t^*, discount window loans, w_t and w_t^*, and net interbank loans, f_t and f_t^* (which are negative if the bank has lent funds). Deposit and interbank market loans have market returns given by i_t^d and i_t^f, while central banks set the corridor rates for reserves and the discount window, which are i_t^m and i_t^w, respectively.

The bank's budget constraint, expressed in dollars, is given by

$$\begin{aligned} P_t^* Div_t + \frac{m_{t+1} - d_{t+1}}{e_t} + b_{t+1} P_t^* + m_{t+1}^* - d_{t+1}^* \leq\; & P_t^* b_t R_t^b + m_t^*(1 + i_t^{m^*}) - d_t^*(1 + i_t^{d,*}) \\ - f_t^*(1 + i_t^{f,*}) - w_t^*(1 + i_t^{w,*}) + & \frac{m_t(1 + i_t^m) - d_t(1 + i_t^d) - f_t(1 + i_t^f) - w_t(1 + i_t^w)}{e_t}. \end{aligned} \tag{2}$$

At the beginning of each period, a bank pays the interest on its liabilities, collects the interest on its assets, issues new liabilities, and buys new assets.

Withdrawal shocks. In the balancing stage, banks are subject to random withdrawal of deposits in both currencies. As in Bianchi and Bigio (2021), withdrawals have

[6]That is, our analysis is not about the management of scarce reserves per se but more broadly about liquidity management. As it is often discussed, banks have had abundant excess reserves for the most part since the 2008 financial crisis—see however, the work by Copeland, Duffie and Yang (2021) showing that reserves at various points in the post-crisis period were not so ample owing to the series of new liquidity regulations. In any case, liquidity concerns have remained a first-order concern for financial institutions, as evidenced by observed measures of liquidity premia as well, as the Senior Financial Officer Survey.

zero mean—hence, deposits are reshuffled but preserved within the banking system. We allow for time-varying volatility of these shocks, which, as we will see, play an essential role in driving exchange rate fluctuation. We denote by ω the withdrawal shock and use ϕ_t and Φ_t to denote the density and CDF. When $\omega > 0$, a bank receives an inflow of deposits; when $\omega < 0$, a bank faces an outflow.

The inflow and outflow of deposits across banks generate a transfer of liabilities across banks. We assume that these transfers are settled using reserves of the corresponding currency. Importantly, reserves for individual banks must remain positive at the end of the period. We denote by s_t^j the euro reserve balances of a bank when faced with a withdrawal shock ω_t^j on its euro deposits. This balance is given by

$$s_t^j = m_{t+1} + \omega_t^j d_{t+1}.$$

Higher liquidity holdings m_{t+1} make the bank more likely to end with a surplus.[7] In particular, if a bank faces a withdrawal shock $\omega < -m_{t+1}/d_{t+1}$, it will end with a deficit reserve balance. Otherwise, the bank has a surplus. Similarly, for dollars, we have that

$$s_t^{j,*} = m_{t+1}^* + \omega_t^{j,*} d_{t+1}^*.$$

Interbank market. After withdrawal shocks are realized, there is a distribution of bank surplus and bank deficit balances in both currencies. We assume there is an interbank market for each currency, in which banks with a deficit balance in one currency borrow from those with a surplus balance. These two interbank markets behave symmetrically, so it suffices to show only how one of them works.[8]

We model the interbank market as an over-the-counter (OTC) market. Modeling the interbank market using search and matching is natural, considering that the interbank market is a credit market in which banks on different sides of the market—surplus and deficit—must find a counterparty they trust (see, Ashcraft and Duffie, 2007 and Afonso and Lagos, 2015). Our specific formulation follows Bianchi and

[7]We omit the superscript j from bank portfolio choices because it is without loss of generality that all banks make the same choices in the lending stage.

[8]We assume a stark form of segmented interbank markets: dollar surpluses cannot be used to patch euro deficits and vice versa. This assumption can be relaxed to some extent. Still, some form of asset market segmentation is necessary to obtain liquidity premia and rule out Kareken and Wallace (1981)'s exchange rate indeterminacy. Section 3.4 discusses an extension of the baseline model along these lines.

Bigio (2017; 2021) which, in turn, integrates elements from Atkeson, Eisfeldt and Weill (2015) and Afonso and Lagos (2015).

As a result of the matching frictions, only a fraction of an individual bank's surplus or deficit is transacted in the interbank market. A bank with surplus s^j is able to lend a fraction Ψ_t^+ to other banks while the remaining surplus is kept in reserves. Conversely, a bank that has a deficit can only secure a fraction Ψ_t^-. The remainder of the deficit is borrowed at the penalty rate i_t^w. The penalty rate can be interpreted as the discount window rate or an overdraft rate charged by correspondent banks with access to the Fed's discount window.

The fractions of balanced matched Ψ_t^+ and Ψ_t^- are endogenous objects that depend on the aggregate reserve deficit balances relative to surplus balances. Assuming a constant return to scale matching function, the probabilities are only a function of market tightness, which is defined as

$$\theta_t \equiv S_t^- / S_t^+,$$

where $S_t^+ \equiv \int_0^1 \max\left\{s_t^j, 0\right\} dj$ and $S_t^- \equiv -\int_0^1 \min\left\{s_t^j, 0\right\} dj$ denote the aggregate surplus and deficit, respectively. Notice that because $m \geq 0$ and $\mathbb{E}(\omega) = 0$, we have that in equilibrium, $\theta \leq 1$. That is, there is a relatively larger mass of banks in surplus than in deficit.

The interbank market rate is the outcome of a bargaining problem between banks in deficit and those in surplus. There are multiple trading rounds in which banks trade with each other. If banks are not able to match by the end of the trading rounds, they deposit the surplus of reserves at the central bank or borrow from the discount window. Throughout the trading, the terms of trade at which banks borrow and lend— the interbank market rate— depends on the probability of finding a match in the future rounds.[9] Notice that we used i^f in the budget constraint (2) to denote the average interbank market rate at which banks trade. Ultimately, we can define a liquidity yield function χ that captures the benefit of having a real surplus $\tilde{s}$ (or the

[9] Multiple trading rounds imply that interbank market rates vary with the tightness of the interbank market. With a single trading round, the interbank market rate would be a constant that depends on policy rates but not on the interbank-market tightness.

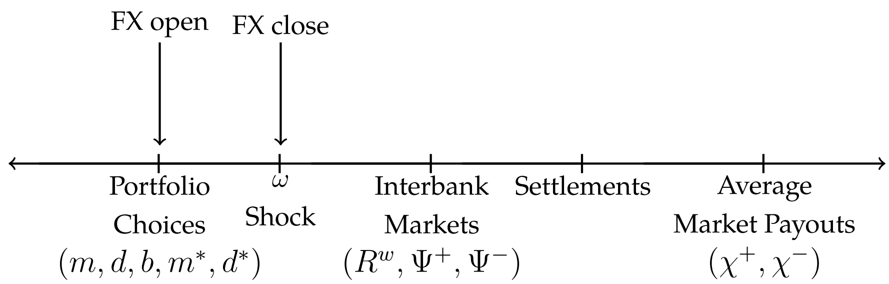

Figure 1: Timeline

cost of having a real deficit) upon facing the withdrawal shock as follows:

$$\chi(\theta, \tilde{s}; X, X') = \begin{cases} \chi^+(\theta; X, X')\tilde{s} & \text{if } \tilde{s} \geq 0, \\ \chi^-(\theta; X, X')\tilde{s} & \text{if } \tilde{s} < 0 \end{cases} \tag{3}$$

where χ^+ and $\chi^-$are given by

$$\chi^+(\theta; X, X') = \Psi^+(\theta)[R^f(X, X') - R^m(X, X')]. \tag{4}$$

$$\chi^-(\theta; X, X') = \Psi^-(\theta)[R^w(X, X') - R^f(X, X')] + (1 - \Psi^-(\theta))[R^w(X, X') - R^m(X, X')], \tag{5}$$

In these expressions, $R^y(X, X')$ denotes the expected real rate of return on an asset or liability y when the initial state is X and the next period state is X'. Note that the expected return depends on X' because the nominal rate is pre-determined, but the realized real return depends on the realized inflation rate. In particular, we have that $R^y(X, X') \equiv (1 + i^y(X))/(1 + \pi(X, X'))$, where $\pi(X, X') \equiv P(X')/P(X) - 1$ denotes the inflation rate. When it does not lead to confusion, we streamline the argument (X, X') in these expressions. We will also use 'bars' to denote expected returns. That is, $\bar{R}^y \equiv \mathbb{E}[R^y(X, X')|X]$ and $\bar{\chi} = \mathbb{E}[\chi(\theta, \tilde{s}; X, X')|X]$.

Equation (4) reflects that the benefit of lending in the interbank market in the case of surplus is $R^f - R^m$. By the same token, (5) reflects that borrowing from the interbank market and the discount window costs respectively $R^f - R^m$ and $R^w - R^m$.

Figure 1 presents a sketch of the timeline of decisions within each period. We next describe the bank optimization problem.

2.2 Banks' problem

The objective of a bank is to choose dividends and portfolios to maximize (1) subject to the budget constraint and the settlement frictions. Crucially, when choosing the portfolio, banks anticipate how withdrawal shocks may lead to a surplus or deficit of reserves and the associated costs and benefits of ending with these positions. We express the bank's optimization problem in terms of real portfolio holdings $\{\tilde{b}, \tilde{m}^{*}, \tilde{d}^{*}, \tilde{d}, \tilde{m}\}$ and real returns. Thus, we define $\tilde{x}_t \equiv x_t / P_{t-1}$. The individual state variable is net worth, n, defined as the value of real assets minus liabilities at the beginning of the period. Recursively, the bank problem is

$$v(n, X) = \max_{\{Div, \tilde{b}, \tilde{m}^{*}, \tilde{d}^{*}, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E}\left[v\left(n', X'\right)\right] \tag{6}$$

subject to the budget constraint

$$Div + \tilde{b} + \tilde{m}^{*} + \tilde{m} = n + \tilde{d} + \tilde{d}^{*}, \tag{7}$$

and the evolution of bank net worth

$$\begin{aligned} n' = & \underbrace{R^{b}(X)\tilde{b} + R^{m}(X, X')\tilde{m} + R^{m^{*}}(X, X')\tilde{m}^{*} - R^{d}(X, X')\tilde{d} - R^{d,*}(X, X')\tilde{d}^{*}}_{\text{Portfolio Returns}} \\ & + \underbrace{\chi^{*}(\theta^{*}(X), \tilde{m}^{*} + \omega^{*}\tilde{d}^{*}; X, X') + \chi(\theta(X), \tilde{m} + \omega\tilde{d}; X, X')}_{\text{Settlement Costs}}. \end{aligned} \tag{8}$$

The evolution of n depends on the realized return on assets, but also on the realized settlement costs.[10] Because of the linearity of bank payoffs and the objective function, the value function is linear in net worth. Anticipating that in general equilibrium, there is a finite demand for loans and deposits, we note that an equilibrium therefore requires that $R^{b}(X) = 1/\beta \geq R^{m}(X, X')$.[11] The next lemma is an intermediate step towards the solution of the bank under this condition.

[10] To obtain (8), we use the definition of χ as expressed in (3)-(5) and the real returns. Implicit in the law of motion is that when a bank borrows at the discount window or from other banks, it pays a high-interest rate but obtains the interest on reserves.

[11] If the return on loans were lower than $1/\beta$, banks would not invest in loans. Conversely, if the return on loans (or reserves) was higher than $1/\beta$, banks would inject infinite equity in the bank and the bank value would be infinite.

Lemma 1. *The solution to* (6) *is* $v(n, X) = n$*, and the optimal portfolio* $\{\tilde{m}, \tilde{d}, \tilde{m}^*, \tilde{d}^*\}$ *solves*

$$\Pi(X) = \max_{\{\tilde{m}, \tilde{d}, \tilde{m}^*, \tilde{d}^*\}} \mathbb{E}\Big\{ \left[R^b(X) - R^d(X, X')\right] \tilde{d} - \left[R^b(X) - R^m(X, X')\right] \tilde{m}$$
$$\left[R^b(X) - R^{d,*}(X, X')\right] \tilde{d}^* - \left[R^b(X) - R^{*,m}(X, X')\right] \tilde{m}^*$$
$$\left[\chi(\theta(X), \tilde{m} + \omega\tilde{d}; X, X')\right] + \left[\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*; X, X')\right]\Big] \Big\}. \quad (9)$$

The first two lines in (9) represent the direct portfolio payoffs and the third line constitutes the expected liquidity costs/benefits emerging from the settlement frictions. Notice that idiosyncratic shocks are only relevant for the latter term.

The bank portfolio problem is homogeneous of degree one. Thus, it must be that in general equilibrium, expected real returns are such that $\Pi(X) = 0$. This means that the scale of the individual bank portfolio is indeterminate at the individual bank level (although the aggregate one will be determined in equilibrium). On the other hand, the liquidity ratio is determined at the individual bank level. In effect, the kink in the liquidity cost function creates risk-averse behavior in the bank objective, pinning down the banks' ratios.

2.3 Non-financial sector

This section describes the non-financial block. This block comprises households that supply labor and save in deposits in both currencies. Some goods must be purchased only with dollar deposits and some with euro deposits. Firms are multinationals that use labor to produce the final good and are subject to working capital constraints, giving rise to a demand for loans. Goods trade is costless and, as a result, the law of one price holds. To further enhance tractability, we work with quasilinear preferences for households. As we show in Appendix C we obtain the following schedules for the real aggregate loan demand by firms, B_t^d, and real aggregate deposit supply for deposits in euros and dollars, D_t^s and $D_t^{*,s}$:

$$B_t^d = \Theta_t^b \left(R_{t+1}^b\right)^{\epsilon^b}, \qquad \epsilon^b < 0, \quad \Theta_t^b > 0, \tag{10}$$
$$D_{t+1}^s = \Theta_t^d \left(\bar{R}_{t+1}^d\right)^{\epsilon^d}, \qquad \epsilon^d > 0, \quad \Theta_t^d > 0, \tag{11}$$

$$D_{t+1}^{*,s} = \Theta_t^{d,*} \left(\bar{R}_{t+1}^{d,*}\right)^{\epsilon^{d^*}}, \qquad \epsilon^{d^*} > 0, \quad \Theta_t^{d,*} > 0, \tag{12}$$

where ϵ^b is the semi-elasticity of credit demand and $\left\{\epsilon^d, \epsilon^{d^*}\right\}$ are the semi-elasticities of the deposit supplies with respect to the real returns, while the Θ terms are scale coefficients. These parameters are linked to the production structure and preference parameters in the microfoundation.

2.4 Central Banks.

The two central banks choose the nominal rates for reserves, i_t^m, and the discount window, i_t^w, as well as the nominal supply of reserves $\left\{M_{t+1}, M_{t+1}^*\right\}$ and nominal discount window loans W_t. To balance the payments on reserves and the revenues from discount window loans, we assume that central banks passively adjust lump-sum taxes (or transfers). Because households have linear utility in the consumption good, these lump-sum taxes have no implications. We have the following budget constraint for the domestic central bank:

$$M_{t+1} + T_t - W_{t+1} = M_t(1 + i_t^m) - W_t(1 + i_t^w). \tag{13}$$

An identical budget constraint holds for the foreign central bank.

We only consider one type of government liability: we do not distinguish between government bonds and central bank reserves. However, our analysis can be immediately extended to allow for a distinction between reserves and government bonds, following Bianchi and Bigio (2021).

2.5 Competitive Equilibrium

We study recursive competitive equilibria in which all variables are indexed by the vector of aggregate shocks, X. We consider shocks to the nominal interest rates on reserves, the deposit supply, and the volatility of withdrawals. Without loss of generality, we restrict to a symmetric equilibrium, in which all banks choose the same portfolios.

Definition 1. Given central bank policies for both countries $\{M(X), i^m(X), i^w(X), W(X)\}$, $\{M^*(X), i^{m^*}(X), i^{w,*}(X), W^*(X)\}$, a recursive competitive equilibrium is a pair of price level functions $\{P(X), P^*(X)\}$, exchange rates $e(X)$, real returns for loans $R^b(X)$, nominal returns for deposits $\{i^d(X), i^{d,*}(X)\}$, an interbank market rate $i^f(X)$, market tightness $\theta(X)$, bank portfolios $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$, interbank and discount window loans $\{f(X), f^*(X), w(X), w^*(X)\}$, and aggregate quantities of loans $\{B(X)\}$ and deposits $\{D(X), D^*(X)\}$ such that

(i) Banks choose portfolios $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$ to maximize expected profits, as stated in (9).

(ii) Households are on their deposit supply, and firms are on their loan demand. That is, equations (10)-(11) are satisfied given real returns and quantities $\{B(X), D(X), D^*(X)\}$.

(iii) The law of one price holds $P(X) = P^*(X)e(X)$.

(iv) Markets clear for deposits $\tilde{d}(X) = D^s(X)$ and $\tilde{d}^*(X) = D^{s,*}(X)$; reserves $\tilde{m}\ (X)P(X) = M(X)$ and $\tilde{m}^*(X)P^*(X) = M^*(X)$; loans $\tilde{b}(X) = B(X)$; and the interbank markets $\Psi^+(X)S^+ = \Psi^-(X)S^-$ and $\Psi^{+,*}(X)S^{+,*} = \Psi^{-,*}(X)S^{-,*}$.

(v) For both currencies, market tightness $\theta(X)$ is consistent with the portfolios and the distribution of withdrawals, while the matching probabilities $\{\Psi^+(X), \Psi^-(X)\}$ and interbank market rates $i^f(X)$ are consistent with market tightness $\theta(X)$.

2.6 Discussion on interbank markets

A central ingredient of our framework is that financial institutions are subject to liquidity mismatch and when they are short of liquidity they trade in an OTC interbank market. Moreover, we assume that the dollar and euro interbank markets are segmented. While our model can capture various assumptions regarding the differences in the two markets, we will focus on a situation where funding risk is higher in the dollar market. This assumption is in line with the observation that the dollar serves as the leading funding currency, especially for short-term cross-border bank loans. As Ivashina, Scharfstein and Stein (2015) note. "[European] banks rely on wholesale dollar funding while raising more of their euro funding through insured retail deposits" (p. 1241), implying that dollar funding is more volatile and more unstable. Moreover, as the 2020 BIS working group report (Davies and Kent, 2020, p. 29) puts it,

> US dollar funding is channeled through the global financial system, involving entities across multiple sectors and jurisdictions. Participants in these markets face financial risks typically associated with liquidity, maturity, currency, and credit transformation. What makes global US dollar funding markets special is the broad participation of non-US entities worldwide. These participants are often active in US dollar funding markets without access to a stable US dollar funding base or to standing central bank facilities that can supply US dollars during episodes of market stress.

McGuire and Von Peter (2009) and IMF (2019) provide evidence and discussion of the dominance of the dollar for short-term funding in the world banking system and its attendant volatility. Bohorquez (2023) reports that 70 percent of all liabilities at non-U.S. BIS reporting banks are less-volatile demand deposits, while only 30 percent of dollar funding takes this shape.

3 Theoretical Characterization

3.1 Liquidity Premia and Exchange Rates

We first describe exchange rate determination. We combine both reserve-market clearing conditions, the law of one price, and deposit clearing conditions to arrive at a condition for the determination of the nominal exchange rate:

$$e(X) = \frac{P(X)}{P^{*}(X)} = \frac{M(X)/\tilde{m}(X)}{M^{*}(X)/\tilde{m}^{*}(X)}. \tag{14}$$

Condition (14) is a Lucas-style exchange rate determination equation, but rather than following from cash-in-advance constraints, it is derived from banks' liquidity management decisions. Given a real demand for reserves in euros and dollars that emerges from the bank portfolio problem (9), the dollar will be stronger (i.e., higher e) the larger is the nominal supply of euro reserves relative to that of dollar reserves.

Similarly, for given nominal supplies of euro and dollar reserves, the dollar will be stronger as the relative demand for real dollar reserves increase. The novelty relative to the canonical Lucas-style model is that liquidity factors play a role in the real demand for currencies and, therefore, affect the value of the exchange rate. We now turn to analyzing the determinants of the real demand for reserves in each currency.

To understand how liquidity factors affect the exchange rate through the demand for reserve balances, let us inspect the portfolio problem (9). We denote by $\mu = \tilde{m}/\tilde{d}$ the banks' liquidity ratio and note that $s^j < 0$ if and only if $\omega^j < -\mu$. Using the expression for the liquidity yield function (3), and recalling that 'bars' denote expected returns, we can express the first-order condition with respect to $\tilde{m}$ as

$$R^b - \bar{R}^m = (1 - \Phi(-\mu))\bar{\chi}^+(\theta) + \Phi(-\mu)\bar{\chi}^-(\theta). \tag{15}$$

At the optimum, banks equate the expected real marginal return on loans, R^b, with the expected real marginal return on reserves. The latter is given by the expected real interest on reserves $\bar{R}^m$ plus their marginal liquidity value. If the bank ends up in surplus, which occurs with probability $1 - \Phi(-\mu)$, the expected real marginal value is $\bar{\chi}^+$. If the bank ends up in deficit, which occurs with probability $\Phi(-\tilde{m}/\tilde{d})$, the expected real marginal value is $\bar{\chi}^-$. We label the difference in yields as the bond premium, $\mathcal{BP} \equiv R^b - \bar{R}^m$ and similarly $\mathcal{BP}^* \equiv R^b - \bar{R}^{m^*}$.

We have an analogous condition for m^*:

$$R^b - \bar{R}^{m^*} = (1 - \Phi^*(-\mu^*))\bar{\chi}^{+,*}(\theta^*) + \Phi^*(-\mu^*)\bar{\chi}^{-,*}(\theta^*). \tag{16}$$

Combining (15) and (16) and using the law of one price $1 + \pi = \mathbb{E}\left[(1 + \pi^*)e'/e\right]$, we obtain a *liquidity premium adjusted interest parity condition*. In particular, denoting the total derivative of $\bar{\chi}$ with respect to m (i.e., the right-hand side of eq. (15)) by $\bar{\chi}_m(s;\theta)$, we have that

$$\mathbb{E}_t\left\{\frac{1}{1+\pi_{t+1}}\left[1 + i_t^m - (1 + i_t^{m^*}) \cdot \frac{e_{t+1}}{e_t}\right]\right\} \quad = \quad \underbrace{\mathbb{E}\left[\bar{\chi}_{m^*}(s^*;\theta^*) - \bar{\chi}_m(s;\theta)\right]}_{\mathcal{DLP}}. \tag{17}$$

This equation establishes that the difference in the real return on reserves in the two currencies is equal to the difference in the marginal liquidity values. We refer to the difference in marginal liquidity values as the dollar liquidity premium, which we denote by $\mathcal{DLP}$.

In the absence of a liquidity premium, (17) would reduce to a canonical UIP that equates to a first order, the difference in nominal returns to the expected depreciation. However, whenever the marginal liquidity value of a dollar is larger than that of a euro (i.e., when $\mathcal{DLP} > 0$), a lower nominal interest rate in dollars than in euros is consistent with equilibrium, even if the exchange rate is expected to be constant. Note that because banks are risk neutral, there is no risk premium, and the deviation from UIP emerges entirely through liquidity.

Finally, we have the first-order conditions with respect to deposits in both currencies:

$$R^d = \bar{R}^m + \mathbb{E}_\omega \left[\bar{\chi}_m \left(s;\theta\right) + \bar{\chi}_d \left(s;\theta\right)\right]; \qquad \bar{R}^{d,*} = \bar{R}^{m^*} + \mathbb{E}_{\omega^*} \left[\bar{\chi}_{m^*} \left(s^*;\theta^*\right) + \bar{\chi}_{d^*} \left(s^*;\theta^*\right)\right]. \tag{18}$$

where $\bar{\chi}_d$ denotes the partial derivative of $\bar{\chi}$ with respect to d, the product of the derivate of the average settlement costs with respect to the average position s times the derivative of s with d—not the total derivative that would include the effect on θ. Like (17), these conditions imply that the expected real return on dollar and euro deposits may not be equated. In particular, a higher marginal liquidity cost of dollar deposits will be a force towards a lower real return of dollar deposits.

3.2 Funding Shocks

We now examine how funding shocks alter the exchange rate and liquidity premia. For analytical tractability, we assume that the supply of deposits is perfectly inelastic in both currencies. This assumption sharpens the results but does not alter the essence of the mechanism, as we will then show numerically.

We focus on shocks to dollar funding. The same shocks to the euro will have opposite effects on the exchange rate. Notice that because $R^b = 1/\beta$ is in equilibrium, the fact that deposit supplies are inelastic implies that shocks to the dollar funding will not affect $\mathcal{BP}$. Thus, $\mathcal{DLP}$ will move one to one with $\mathcal{BP}^*$, a result that speaks directly to the empirical literature connecting the liquidity premium of dollar-denominated assets to the exchange rate (Liao, 2020; Jiang, Krishnamurthy and Lustig, 2021; Engel and Wu, 2023).

A key object to characterize the effects of various shocks is the derivative of $\mathcal{DLP}$ with respect to the dollar liquidity ratio μ^*:

$$\mathcal{DLP}_{\mu^*} = \underbrace{\left[(1-\Phi^*(-\mu^*)) \cdot \bar{\chi}^{+^*}_{\theta^*} + \Phi^*(-\mu^*) \cdot \bar{\chi}^{-^*}_{\theta^*}\right]}_{\text{effect on average interbank rates}} \cdot \frac{\partial \theta^*}{\partial \mu^*} - \underbrace{\phi^*(-\mu^*) \cdot \left(\bar{\chi}^{-^*} - \bar{\chi}^{+^*}\right)}_{\text{liquidity risk exposure}} < 0.$$

This expression illustrates how a change in the dollar liquidity ratio must impact the dollar liquidity premium in equilibrium. There are two key terms. First, a higher liquidity ratio reduces the interbank-market tightness θ, thus easing the settlement frictions and reducing the average interbank rates. This general equilibrium effect reduces the liquidity premium. Second, a higher liquidity ratio reduces the probability that an individual bank ends up with a deficit. This partial equilibrium effect also reduces the liquidity premium because the cost of deficits is higher than the benefit of surpluses.

With this expression in hand, we can characterize the effects of different shocks.

Supply of dollar funding. The first question we explore is what are the effects of an increase in the supply of dollar funding?

Proposition 1 (Funding level shock)**.** *Consider an increase in the real supply for dollar deposits $\Theta^{d,*}$. We have the following:*

1) *If the shock is i.i.d, then the shock appreciates the dollar, reduces the dollar liquidity ratio μ^*, and raises $\mathcal{DLP}$. In particular,*

$$\frac{d\log e}{d\log D^*} = -\frac{\mathcal{DLP}_{\mu^*}}{R^b - \mathcal{DLP}_{\mu^*}\mu^*} \in (0,1)\,, \quad \frac{d\log \mu^*}{d\log D^*} = -\frac{R^b}{R^b - \mathcal{DLP}_{\mu^*}\mu^*} \in (-1,0)\,,$$

and $d\mathcal{DLP} = \bar{R}^{m^*} d\log e > 0$.

If the shock is permanent, then the shock appreciates the dollar one for one, and does not change the liquidity ratio μ^* nor $\mathcal{DLP}$:

$$\frac{d\log e^*}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1, \quad \textit{and} \quad d\mu^* = d\mathcal{DLP} = 0.$$

Proposition 1 establishes that a higher supply of dollar deposits appreciates the dollar regardless of whether the shock is temporary or permanent. The logic is simple: a higher amount of real dollar deposits increases the demand for real dollar reserves. As banks have more dollar liabilities, there is a higher marginal value from dollar reserves. Given a fixed nominal supply of reserves, the increase in demand leads to an appreciation of the dollar.

At the same time, the increase in the supply of dollar deposits has different implications for liquidity premia, depending on whether the shock is temporary or permanent. When the shock is temporary, the exchange rate is expected to revert to a lower initial value in the following period. Given nominal rates, this reduces the expected real return of holding dollar reserves, and the demand for dollar reserves falls for an individual bank. In equilibrium, dollar reserves must have a higher marginal liquidity value, and there is a rise in $\mathcal{DLP}$. Overall, we then have that in response to a temporary increase in the supply of dollar deposits, the dollar appreciates, the dollar liquidity ratio falls, and $\mathcal{DLP}$ increases.

When the shock is permanent, the effect on the exchange rate is also expected to be permanent. In the absence of any expected depreciation effects, $\mathcal{DLP}$ must remain constant. Thus, in equilibrium, the outcome is that banks increase their holdings of dollar reserves in real terms in proportion to the increase in the supply of deposits. Since the supply of M^* is fixed, the amount of goods that can be bought with one dollar must increase, and this appreciates the dollar.[12]

Dollar funding risk. Next, we characterize the effects of a rise in funding risk. For that purpose, it is useful to index Φ by a parameter that captures the volatility of withdrawals, σ. We make the following assumption:

Assumption 1. *The CDF for the distribution of withdrawal shocks satisfy $\Phi^*(\omega;\sigma^*)$ satisfies $\Phi^*_{\sigma^*}(\omega;\sigma^*) > 0$ for any $\omega < 0$.*

The implication is that as we increase σ^*, the risk of ending with a reserve deficit increases for any μ^*. Hence, a shock to σ^* captures greater funding risk. We then have the following result:

Proposition 2 (Funding risk shock)**.** *Consider an increase in the dollar funding risk, σ^*. Suppose that Assumption 1 holds. Then,*

[12] Constant returns to scale in the interbank matching technology is key for this result. As banks proportionally scale dollar deposits and reserves, given the same real returns on dollar and euro reserves, the original liquidity ratio remains consistent with the new equilibrium. See Coppola, Krishnamurthy and Xu (2023) for a recent study allowing for increasing returns to scale.

1) If the shock is i.i.d, then the shock appreciates the dollar, raises the dollar liquidity ratio μ^*, and increases $\mathcal{DLP}$. In particular,

$$\frac{d\log e}{d\log\sigma^*} = \frac{d\log\mu^*}{d\log\sigma^*} = \frac{\mathcal{DLP}_{\sigma^*}\sigma^*}{R^b - \mathcal{DLP}_{\mu^*}\mu^*} > 0, \quad \textit{and} \qquad d\mathcal{DLP} = \bar{R}^{m^*} d\log e > 0.$$

2) If the shock is permanent, then the shock appreciates the dollar, raises the liquidity ratio, and $\mathcal{DLP}$ remains constant. In particular,

$$\frac{d\log e}{d\log\sigma^*} = \frac{d\log\mu^*}{d\log\sigma^*} = -\frac{\mathcal{DLP}_{\sigma^*}\sigma^*}{\mathcal{DLP}_{\mu^*}\mu^*} > 0 \quad \textit{and} \qquad d\mathcal{DLP} = 0.$$

Proposition 2 presents a central result. In response to an increase in the risk of funding the dollar, the dollar appreciates, and there is an increase in the dollar liquidity ratio and $\mathcal{DLP}$. Intuitively, with a larger dollar funding risk, banks demand a greater amount of real dollar reserves. With the nominal supplies given, this must lead to an appreciation of the dollar. Again, there is a relevant distinction between temporary and permanent shocks. When the shock is temporary, the expected depreciation of the dollar reduces the expected real return of holding dollar reserves. Given the nominal rates, this implies that $\mathcal{DLP}$ must be higher in equilibrium for (17) to hold. When the shock is permanent, the volatility shock appreciates the dollar without any effects on $\mathcal{DLP}$. Unlike the case of the shock to the scale of dollar funding, in this case, the liquidity ratio increases together with the exchange rate. In equilibrium, therefore, the increase in the liquidity ratio offsets the higher volatility, and that is why $\mathcal{DLP}$ remains constant. The magnitude of the response is proportional to the magnitude of the response of the dollar liquidity premium to σ^*, $\mathcal{DLP}_{\sigma^*} > 0$.

Proposition 2 characterizes the effects of changes in volatility for i.i.d. or permanent shocks and perfectly inelastic deposit supply schedules. Yet, the results hold for mean-reverting processes and general elasticities for deposits, as we show numerically, based on a calibration described below. Figure 2 presents the results. As the figure shows, a higher funding risk appreciates the dollar (panel a) and lowers the expected return on dollar bonds relative to euros (panel b), reflecting the larger dollar liquidity premium. In addition, we can see an increase in the differential rate on deposits (panel c). That is, the rate on euro deposits increases relative to the

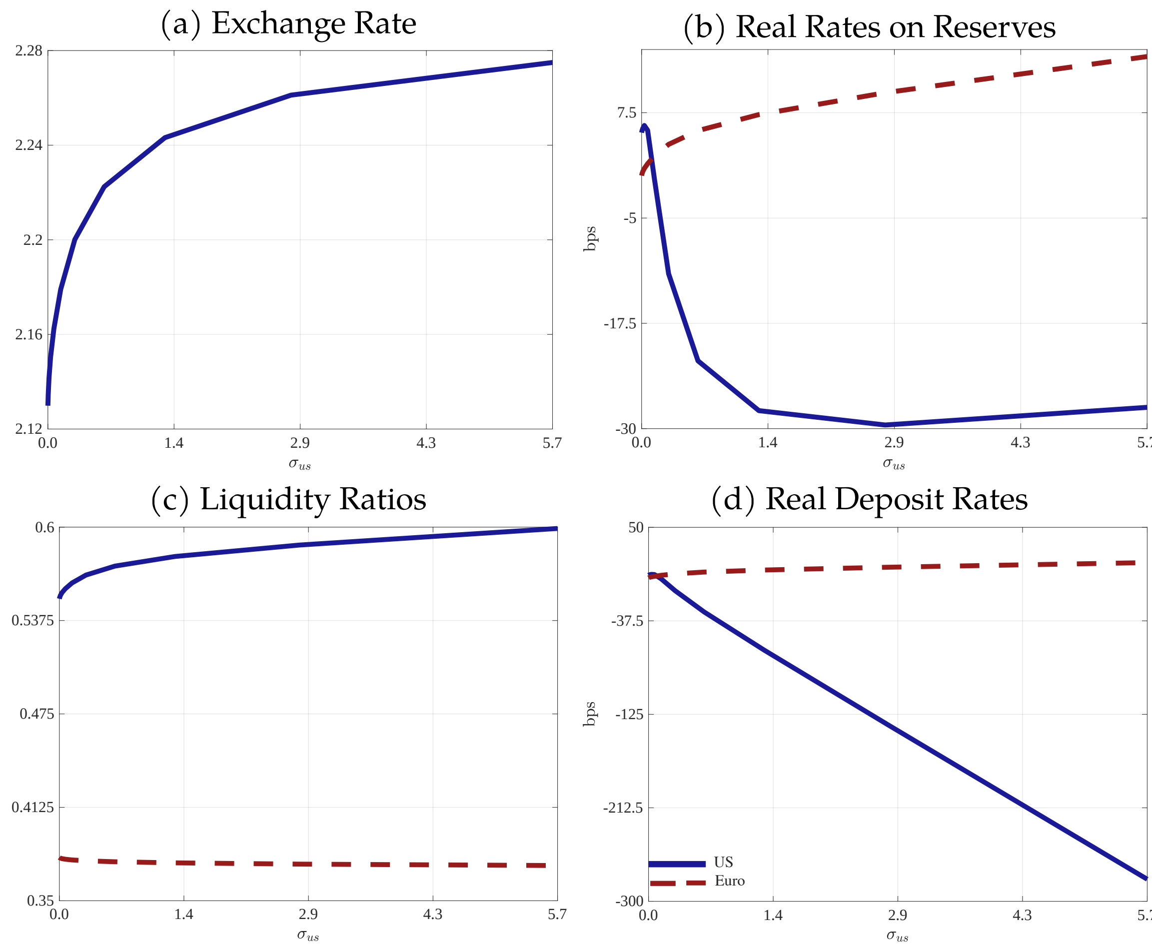

Figure 2: Equilibrium as function of dollar funding risk

dollar rate as the rise in volatility makes euro deposits more attractive. Finally, we also see an increase in the dollar liquidity ratio concomitant with a reduction in the euro liquidity ratio (panel d).

Notice that other things equal, a higher variance in the dollar funding risk makes dollar deposits less desirable for banks. This leads in equilibrium to a lower return on dollar deposits, in line with the data. While this may seem to call for a lower quantity of dollar deposits, of course, on the other side of the market are depositors who may have a preference for dollar deposits. This can be made explicit in our model by allowing for a larger scale of the supply of dollar deposits. Thus, the model can account at the same time why dollar deposits are prevalent, why banks are willing to hold dollar reserves even though the expected return is lower, and why the dollar tends to appreciate in times of heightened uncertainty.

The theoretical results of Propositions 1 and 2 link the funding level and the funding risk shocks to the liquidity ratio and the exchange rate. The results suggest that dollar funding risk may drive a positive correlation between the liquidity ratio and the exchange rate, whereas dollar funding level shocks predict the opposite correlation. Toward shedding further light on the connection between the theoretical and empirical results, we generalize the results in Propositions 1 and 2 to mean-reverting shocks. We assume that shocks to $\{D_t^*, \sigma_t^*\}$ follow an AR(1) process with autocorrelation $\rho^{D^*}, \rho^{\sigma^*}$ and standard deviation $\Sigma^{D^*}, \Sigma^{\sigma^*}$.

Lemma 2 (Persistent shocks)**.** *The first-order effects of shocks around the steady state are as follows:*

i) *In response to a small deviation to D^* near the steady state:*

$$\epsilon^e_{D^*} \equiv \frac{\log e - \log e_{ss}}{\log D^* - \log D^*_{ss}} \approx \frac{-\mathcal{DLP}^*_{\mu^*}\mu^*}{(1-\rho^{D^*})R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} \in (0,1)$$

and

$$\epsilon^{\mu^*}_{D^*} \equiv \frac{\log \mu^* - \log \mu^*_{ss}}{\log D^* - \log D^*_{ss}} \approx -\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} \in (-1,0)\,.$$

ii) *Suppose that Assumption 1 holds. In response to a small deviation near σ^*_{ss}:*

$$\epsilon^{\mu^*}_{\sigma^*} \equiv \frac{\log \mu^* - \log \mu^*_{ss}}{\log \sigma^* - \log \sigma^*_{ss}} = \epsilon^e_{\sigma^*} \equiv \frac{\log e^* - \log e^*_{ss}}{\log \sigma^* - \log \sigma^*_{ss}} \approx \frac{\mathcal{DLP}^*_{\sigma^*}d\sigma^*}{\left(1-\rho^{\sigma^*}\right)R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} > 0.$$

In the empirical analysis, we will use these results to infer from the data the key shocks driving exchange rate fluctuations.

3.3 Monetary Policy

Nominal Rates. We now study how monetary policy affects the exchange rate. We start by considering the effect of a change in the policy rates.

Proposition 3 (Attenuation of changes in policy rates)**.** *Consider an increase in the interest rate on dollar reserves, i^{m^*}, holding fixed the policy spread, $i^{w^*} - i^{m^*}$.*

1) *If the shock is i.i.d, the shock appreciates the dollar less than one for one, raises the liquidity*

ratio, and reduces $\mathcal{DLP}$:

$$\frac{d\log e}{d\log\left(1+i^{m^*}\right)} = \frac{d\log\mu}{d\log\left(1+i^{m^*}\right)} = \frac{\bar{R}^{m^*}}{R^b - \mathcal{DLP}_{\mu^*}\cdot\mu^*} \in (0,1) \quad \textit{and}$$

$d\mathcal{DLP} = \bar{R}^{m^*}\left(d\log e - d\log\left(1+i^{m^*}\right)\right) < 0.$

2) *If the shock is permanent, the shock appreciates the dollar raises the liquidity ratio, and reduces $\mathcal{DLP}$:*

$$\frac{d\log e}{d\log\left(1+i^{m^*}\right)} = \frac{d\log\mu^*}{d\log\left(1+i^{m^*}\right)} = -\frac{\bar{R}^{m^*}}{\mathcal{DLP}_{\mu^*}\cdot\mu^*} > 0, \quad \textit{and}$$

$d\mathcal{DLP} = -\bar{R}^{m^*}d\log(1+i^{m^*}) < 0.$

Proposition 3 establishes that in response to an increase in the US nominal rate, the dollar appreciates, the liquidity ratio increases and the liquidity premium falls. This occurs regardless of whether the shock is temporary or permanent. The appreciation of the dollar follows a standard effect: a higher nominal rate leads to a larger demand for dollars, which in equilibrium requires a dollar appreciation. In turn, given a fixed nominal supply of dollar reserves, there is an increase in the real amount of reserves. In the absence of liquidity premia, the difference in nominal returns across currencies would be exactly offset by the expected depreciation of the dollar, following the current revaluation. With a liquidity premium, however, the expected depreciation is not one-for-one: given the larger abundance of real dollar reserves, there is a decrease in the marginal value of dollar reserves, together with a reduction in dollar liquidity premium.

This result breaks the tight connection between interest-rate differentials and expected depreciation, which is at the heart of models featuring the Fama (1984) puzzle. In models where uncovered interest parity holds an increase in the dollar interest rate leads to a one-for-one expected dollar depreciation, and no change in the expected excess return on euro reserves. Here, the exogenous increase in the dollar interest rate reduces the dollar liquidity premium, attenuating the effects on the exchange rate.

Open-Market Operations. Finally, we consider open-market operations. In the model description, the expansions in M are implemented with transfers—that is, helicopter drops. In practice, central banks conduct open-market operations, purchasing assets and issuing central bank liabilities. Given that we interpret M as central bank and government liabilities, we are interested in unconventional open market operations. For that, we now modify the model by introducing an outstanding amount of private securities S_t that can be held by households, S_t^h, and by the central bank, S_t^g. We assume that these securities are perfect substitutes for private deposits. The joint demand for the sum of deposits and securities is given by

$$D_t + S_t^h = \Theta_t^d \left(\bar{R}_{t+1}^d\right)^{\epsilon^d}.$$

The government budget constraint is modified by adding $\left(1+i_t^d\right) S_t^g$ to the sources of funds and S_{t+1}^g to the uses, to the right-hand side and left-hand side of (13) respectively. We have analogous conditions for dollars.

Proposition 4 (Effects of open-market operations)**.** *Consider a purchase of private securities financed with reserves by the US central bank. Let $\Upsilon^* = \frac{P^* S^{G,*}}{M^*}$ denote the initial value of the securities as a function of reserves.*

1) If the change in the balance sheet is reversed in the following period, the shock depreciates the dollar, raises the liquidity ratio, and reduces $\mathcal{DLP}$:

$$\frac{d\log e}{d\log S_t^{*,g}} = \frac{\mathcal{DLP}^*_{\mu^*}\mu^*\left(1-\mu^*\right)\mathcal{BP}^*}{R^b - \left(1-\Upsilon^*\right)\mathcal{DLP}^*_{\mu^*}\mu^*} < 0,$$

$$\frac{d\log\mu^*}{d\log S_t^{*,g}} = \frac{R^b\Upsilon^*\left(1-\mu^*\right)}{R^b - \left(1-\Gamma^*\right)\mathcal{DLP}^*_{\mu^*}\mu^*} > 0, \quad \text{and} \quad d\mathcal{DLP} = \bar{R}^{m^*} d\log e < 0.$$

2) If the change in the balance sheet is permanent, the shock depreciates the dollar and does not change the liquidity ratio nor the dollar liquidity premium:

$$\frac{d\log e}{d\log S^{*,G}} = -\left(1-\mu^*\right)\frac{\Upsilon^*}{1-\Upsilon^*} \leq 0.$$

Proposition 4 establishes that a temporary open market operation—by increasing the nominal supply of reserves—leads to a temporary depreciation of the dollar. Given nominal rates on reserves, the expected appreciation leads to an increase in the expected return on dollar reserves and an increase in the real holdings of dollar reserves. In equilibrium, there is an increase in the liquidity ratio and a decrease in $\mathcal{DLP}$. For permanent shocks, instead, we find that the exchange rate depreciates permanently, and there are no effects on the liquidity ratio nor on $\mathcal{DLP}$.[13]

3.4 Discussion and Extensions

The theory we presented articulates the idea that banks' liquidity needs affect the exchange rate by altering their demand for liquid assets in different currencies, and can be extended along several directions.

Dollars as Collateral. In the model, the critical asymmetry needed to deliver a positive $\mathcal{DLP}$ is that the dollar funding risk must be greater, an assumption consistent with the prevalence of the US dollar in short-term liability funding. The model, however, can be adapted to allow for an asymmetry in settlement frictions in a way that also generates a positive dollar liquidity premium without differences in the dollar and euro funding risk. Specifically, assuming that banks can transfer dollar reserves to settle withdrawals of euro deposits, it follows that if banks are in deficit of euro reserves, they use dollar reserves before going to the euro interbank market. With this, the return on euro reserves must exceed the one on dollar reserves, even if the funding risks are the same in both currencies.

Risk Premia. To focus squarely on liquidity and highlight the novel channels of our theory, we have assumed risk-neutral banks. Here, we extend our framework to allow for risk premia and show that this yields an interesting interaction between risk premia and liquidity premia.

We now assume that banks maximize profits using a stochastic discount factor $\Lambda(X, X')$, which captures the risk aversion of shareholders. Incorporating this feature in the portfolio problem (9), we have that

$$\mathbb{E}\left[R^m_{t+1} - R^{m^*}_{t+1}\right] = \mathbb{E}\left[\chi_{m^*}\left(s^*;\theta^*\right) - \chi_m\left(s;\theta\right)\right] + \frac{\mathbb{COV}\left(\Lambda_{t+1}, \chi_{m^*}\left(s^*;\theta^*\right) - \chi_m\left(s;\theta\right)\right) + \mathbb{COV}\left(\Lambda_{t+1}, R^{m^*}_{t+1} - R^m_{t+1}\right)}{\mathbb{E}\left[\Lambda_{t+1}\right]}. \quad (19)$$

Relative to (17), eq. (19) has a risk premium associated with the liquidity premium, the first covariance term. In addition, there is now an additional "safety premium" term driving the difference between expected returns on the dollar and

[13] Our model also has predictions for the effects of foreign exchange interventions, where a central bank swaps domestic liabilities for foreign assets. We leave this for future research.

euro liquid assets. The safety premium represents the covariance between the difference in the realized yields of dollar and euro liquid assets and the stochastic discount factor. If the dollar tends to appreciate in bad times (i.e., when marginal utility is high), this implies that dollar assets will have a lower expected rate of return.

The two premia, the liquidity premium and the safety premium, interact. As explained in Proposition 2, the dollar appreciates in response to a rise in the volatility of withdrawal shocks through a higher $\mathcal{DLP}$. To the extent that the rise in funding risk coincides with a high value for $\Lambda(X, X')$, a volatility shock would further increase the safety premium as the return on dollar assets increases when payoffs are more valuable. Thus, modeling an endogenous liquidity premium is likely to enhance the importance of the risk premium as a driver of exchange rates, a point that may resolve the reserve currency paradox in risk-premium models documented in Maggiori (2017).[14]

CIP Deviations. Our baseline model does not include an explicit forward market. To speak to the observed deviations from covered interest parity (CIP), we now allow for a forward market, which we assume to be perfectly competitive. A forward contract traded at time t promises to exchange one dollar for $\hat{e}_{t,t+1}$ euros in the lending stage in the following period.[15] The first-order condition with respect to the quantity of forwards purchased can be expressed as

$$0 = \mathbb{E}\left[\frac{\Lambda_{t+1}}{1+\pi_{t+1}} \cdot (e_{t+1} - \hat{e}_{t,t+1})\right]. \tag{20}$$

Let us examine the CIP deviation constructed using reserves. In the literature, this is often referred to as the "Treasury basis"—that is, the yield on an actual US Treasury (the analog of reserves in our model) minus the yield on an equivalent synthetic US Treasury. Denoting by $\mathcal{CIP}$ the deviation from covered interest parity,

[14]Despite the prevalent view that risk premia may make the dollar a safe haven, Maggiori (2017) shows, in a standard model of financial intermediation, that the dollar depreciates in times of negative aggregate shocks. The reason is that US households are optimally more exposed to aggregate risk and, therefore face larger losses in bad times relative to those faced by the rest of the world.

[15]Notice that since there are no aggregate shocks in the balancing stage, it is equivalent to pricing the forward in the lending or the balancing stage.

we have that by definition

$$\mathcal{CIP} = (1 + i_t^m) - \left(1 + i_t^{m^*}\right) \left(\frac{\hat{e}_{t+1}}{e_t}\right). \tag{21}$$

Replacing the forward rate $\hat{e}_{t,t+1}$ from (20) into (21) and using (19), we can obtain

$$\mathcal{CIP} = \frac{\mathbb{E}\left[\Lambda_{t+1}\left(\chi_{m^*} - \chi_m\right)\right]}{\mathbb{E}\left(\Lambda_{t+1}\left(1+\pi_{t+1}^*\right)^{-1}\right)}. \tag{22}$$

That is, according to our model, the CIP deviation is given by the nominal risk-adjusted dollar liquidity premium. Accordingly, in the quantitative analysis, we will use the empirical time series of the CIP deviation to discipline the calibration of the model.

In addition, using (19) and (22), we obtain that the deviation from uncovered interest parity, $\mathcal{UIP}$, is given by the deviation from CIP plus the safety premium:

$$\mathcal{UIP} = \mathcal{CIP}\frac{\mathbb{E}\left(\frac{\Lambda_{t+1}}{1+\pi_{t+1}^*}\right)}{\mathbb{E}\left[\Lambda_{t+1}\right]} + \frac{\mathbb{COV}\left(\Lambda_{t+1}, R_{t+1}^{m^*} - R_{t+1}^m\right)}{\mathbb{E}\left[\Lambda_{t+1}\right]}.$$

That is, banks are willing to hold dollar reserves at a lower return, either because they are a good hedge or because they provide superior liquidity value.

Empirically, deviations from UIP and CIP are well documented (see, in particular, Kalemli-Özcan and Varela, 2021 and Du, Tepper and Verdelhan, 2018). Here, the wedge between the CIP and UIP deviations is due to the safety premium, but, in practice, there can be other forces, including borrowing constraints and regulatory constraints.[16]

An alternative, perhaps more common, measurement of CIP deviation is performed using the interbank market rate (LIBOR) rather than the rate on government bonds. An interesting prediction of our model is that the two deviations from CIP are tightly linked.[17]

[16]In the quantitative analysis that follows, we will consider an exogenous wedge as a stand-in for these factors, using a risk-neutral version of the model. An alternative that we leave for future research is to take an explicit stochastic discount factor.

[17]In particular, we can show that under risk neutrality, the difference between the interbank market-based CIP and the government bond-based CIP depends on the endogenous liquidity objects and is given by $\chi_t^+/\Psi_t^+ - \chi_t^{+,*}/\Psi_t^{+,*}$.

Finally, we highlight an interesting observation made by Du et al. (2018) regarding the cross-sectional implications of the CIP deviation. Since the global financial crisis, low-interest-rate currencies have experienced a high risk-free excess return relative to that of the high-interest-rate currencies, a pattern that contrasts sharply with the carry-trade phenomenon. As we showed earlier, a decrease in the nominal interest rate increases $\mathcal{DLP}$ and thus raises the deviation from the CIP. Our model is thus consistent with this pattern.[18]

Real exchange rate. We have considered a model with a single tradable good and assumed that the law of one price holds. An implication is that the real exchange rate is constant and that the exchange rate moves one-to-one with the domestic price level for a given price in foreign currency. However, it is straightforward to allow for non-tradable goods or deviations from the law of one price to incorporate fluctuations in the real exchange rate. Extending the model in this direction would allow us, for example, to speak to the positive co-movement between the nominal exchange rate and the real exchange rate. This extension can be used to confront the Mussa facts (see e.g., Itskhoki and Mukhin, 2021b).[19]

4 Quantitative Analysis

We now conduct a quantitative analysis of the model. We linearize the equilibrium conditions of the model around the deterministic steady state. Thus, the method preserves the nonlinearity introduced by the portfolio choice and the idiosyncratic risk. We use data time series on the euro-dollar exchange rate, liquidity ratios, policy variables, and observed premia to calibrate the model and estimate the shocks. In what follows, we replace the $\{*\}$ notation with $i \in \{us, eu\}$ because we introduce additional currencies.

[18] An alternative explanation provided by Amador et al. (2020) highlights central bank policies of resisting an appreciation at the zero lower bound. In their framework, deviations from the CIP arise when a central bank purchases foreign reserves, and the bank's financial constraint prevents an arbitrage between domestic and foreign assets.

[19] In Itskhoki and Mukhin (2021b), when noisy traders demand more Euro bonds, the Euro appreciates and consumption increases in Europe, yielding a real exchange rate appreciation through consumption smoothing effects. Funding risk in our model is a natural candidate to drive the co-movement between nominal and real exchange rates once the model is extended with multiple goods.

4.1 Additional Features

Before we proceed with the calibration, we explain additional features that improve the mapping from the model to the data.

Limited equity. In the baseline model, banks have unlimited access to equity financing. This assumption fixes the return on loans to $1/\beta$. Under this assumption all the movements in the bond premium, $\mathcal{BP}$, follow from changes in the real expected return to dollar reserves, but not on the loan rate. To generate endogenous variations in the loan rate, and an additional source of movements in the bond premium $\mathcal{BP}$, we now allow for limited equity financing. Specifically, we assume that banks pay out their realized previous-period profits as dividends—or raise equity to finance their losses.

Open-market operations. Since unconventional open-market operations have been prevalent since the 2008 crisis, we assume that all deviations in the money supply away from the steady state are due to increases in the stock of securities held by the central bank. Thus, we have $M_t^{us} - M_{ss}^{us} = S_t^{g,us}$, where $S_t^{g,us}$ are the security holdings of the central bank, as described in Section 3.3. We use analogous equations for the euro.

CIP-UIP wedge. In the data, the deviation from UIP exceeds the deviations from CIP (see, e.g., Kalemli-Özcan and Varela 2021). In our baseline model with risk neutrality, UIP and CIP coincide. Section 3.4 shows that the model can be extended to allow risk premia. In what follows, we introduce and measure a "risk-premium wedge," denoted by ξ_t, to express the difference between the UIP and CIP deviations, $\mathcal{UIP}_t = \mathcal{CIP}_t + \xi_t$.

Pricing additional currencies. Our baseline model features only two currencies. However, we can easily price additional currencies. A simple way to do so is to assume that the deposit funding scales and reserves in all additional currencies approach zero.[20] Although its components approach zero, we can obtain a liquidity ratio consistent with an exchange rate given policy rates in each currency.

[20]In the baseline model, the funding scale of different currencies is not relevant given the linearity of the model. In the quantitative section, the assumption of limited equity implies that funding scales do matter.

4.2 Calibration - Estimation

We divide the parameters into three subsets. The first subset is the parameters associated with the exogenous monetary policy variables which we calibrate by normalizing their means and to match the observed autocorrelation and variances of their data counterparts. The second subset is the parameters associated with the preferences and technologies in the model, which are calibrated externally, borrowing from the literature. The third subset is the parameters associated with processes of the exogenous shocks to the model. We calibrate their means to match steady-state moments and estimate their autocorrelation and variances using the Kalman filter.

We assume that Φ, the distributions of funding risk shocks in each country, is a two-sided mean-zero exponential distribution indexed by a σ volatility parameter.[21]

Parameter values are listed in Table D.1. A period in the model represents a month. Unless otherwise noted, we employ monthly data from 2001m1 to 2016m12. Figure D.1 presents the data series.

Calibration of monetary policy variables. The exogenous policy variables are the interest on reserves $1 + i_t^m$ and the supply of liquid assets M_t in both currencies. We assume each of these variables follows a log AR(1) process. As data counterparts for $1 + i_t^m$, we use the three-month US and German government bond rates for the interest on reserves. Because in the model average inflation is zero, we calibrate the mean of $1 + i_t^m$ in the model to be the historical average of the data counterpart minus the average inflation in the country during the sample period. As data counterparts for M_t^{us} we take the sum of reserves held at Federal Reserve banks and government securities (Treasury and agency) held by commercial banks (the sum of TOTRESNS and USGSEC from FRED, which are found in the Fed's H.6 and H.8 releases, respectively.) For M_t^{eu} we use the sum of holdings of Euro Area Government Issued securities and Cash held by Monetary Financial Institutions (MFI). We assume that the supply of nominal reserves assets are stationary so that the average inflation rate and the exchange rate are stationary. Moreover, we nommrmalize M_t^{eu} to obatain a nominal steady-state exchange rate of 1—see Appendix

[21]This distribution is convenient because it allows for a continuum of shocks but renders closed-form solutions for the conditional expectations below a threshold. This allows us to compute the reserve deficit probability analytically, which is convenient for the computations.

F.1.

External calibration. We set the parameters of the interbank market matching process (i.e., the penalty rate i^w and the interbank-market efficiency and the bargaining powers) embedded in (3), to the values from Bianchi and Bigio (2021).[22] Importantly, the penalty rate i^w exceeds the discount window rate set by central banks because we interpret it more broadly as capturing stigma or collateral costs. Following the estimation in Bianchi and Bigio (2021), we set this value to 10% annually.

The semi-elasticity of the loan demand is set to 25, as in Bianchi and Bigio (2021). The loan demand scale Θ^b is set to one as a normalization. For the deposit supply schedules, we treat the euro area and the US funding supplies as symmetric and set both semi-elasticities to 1.

Shock processes. Next, we turn to the parameters that govern the shocks to the level and volatility of funding, and the risk-premium wedge, $\left\{\sigma_t^{us}, \Theta_t^{d,us}, \sigma_t^{eu}, \Theta_t^{d,eu}, \xi_t\right\}$. For each shock, we assume an $\text{AR}(1)$ process. We aid the estimation by setting their means to match average data targets and estimate their autocorrelation and variance coefficients. We use data counterparts for $\{\mathcal{DLP}, \mathcal{BP}^*, e, \mu^{us}, \mu^{eu}\}$ which are equilibrium objects in the model.

As a data counterpart for $\mathcal{DLP}$, we use CIP deviations, in line with the analysis in Section 3.4. To construct a series for CIP deviations, we use the mid-point quotes for spot, the counterpart for e_t, and forward exchange rates from Bloomberg and the nominal rates on reserves as data counterparts of the terms in equation (17). For the reference period, we obtain a value of 12 basis points, in line with Du et al. (2018). For the data counterpart of $\mathcal{BP}^*$, we use a measure of liquidity proposed by Stock and Watson (1989) and Friedman and Kuttner (1993), the commercial paper spread, the difference between the three-month spread between the AAA commercial paper and the three-month Treasury bill, our analog for the policy rate. We add 200bps to this difference to consider the safety premium of AAA commercial paper over the typical bank asset.

We construct a data counterpart for μ^{eu}, μ^{us} by dividing our data counterparts for M^{eu} and M^{us} by data counterparts for D^{eu} and D^{us}, and scaling these ratios. Regarding dollar funding, D^{us}, we use the sum of two sources of short-term funding

[22] See Appendix A for the mapping between the efficiency parameter λ and bargaining powers η to the probabilities of matching and the interbank market rate.

of all financial intermediaries. The first, used by Adrian, Etula and Shin (2010), is the US dollar financial commercial paper, series DTBSPCKFM from Federal Reserve Economic Data (FRED). Another major source of short-term funding to US banks is demand deposits, measured by DEMDEPSL from FRED (from the Fed's H.6 statistical release). We include funding to other financial institutions because these are key indicators of how the demand for dollars is affected by the financial sector's demand for liquid assets when funding risk increases. In terms of D^{eu}, we use the monthly deposits redeemable at notice and deposits with agreed maturity held by (MFI) between February 2001 to July. Since we include the funding of all US financial institutions but only the liquid holdings of banks, we normalize μ^{us} and μ^{EU} so that the average is 0.2 as obtained in Bianchi and Bigio (2021) by using data from individual Call Reports.

Steady state. We next describe the parameterization of the steady-state values of the shock processes $\{\sigma^{us}_{ss}, \sigma^{eu}_{ss}, \xi_{ss}, \Theta^{d,us}_{ss}, \Theta^{d,eu}_{ss}\}$. We assume symmetry in the funding scales $\Theta^{d,us}_{ss} = \Theta^{d,eu}_{ss}$. We use the following sequential procedure to calibrate the steady-state parameters. First, we solve for σ^{us}_{ss} from $\mathcal{BP}^*_{ss} = \mathbb{E}\left[\chi_m\left(\mu^{us}_{ss}, \sigma^{us}_{ss}\right)\right]$: taking μ^{us}_{ss} and $\mathcal{BP}^*_{ss}$ from the data, σ^{us}_{ss} is the only unknown in this equation. Second, once we have a value for σ^{us}_{ss}, we similarly obtain σ^{eu}_{ss} by solving it from DLP.[23] At steady-state, we obtain that σ^{us}_{ss} is almost four times the σ^{eu}_{ss} consistent with our notion that the volatility of dollar flows is larger because of their more ample use in short-term funding market. Third, the average risk-premium, ξ_{ss}, is obtained from $\mathcal{UIP}_{ss} = \mathcal{DLP}_{ss} + \xi_{ss}$, where we use $\mathcal{UIP}_{ss} = R^m_{ss} - R^{m^*}_{ss}$, constructed by using the historical interest differentials and average inflation rates.[24] Finally, we obtain $\Theta^{d,us}_{ss}$, which equals $\Theta^{d,eu}_{ss}$, from the bank's budget constraint: we re-arrange the budget constraint to obtain a value for $\Theta^{d,us}_{ss}$, given $\mathcal{BP}^*_{ss}, \mu^{us}_{ss}, \mu^{eu}_{ss}, \sigma^{us}_{ss}, \sigma^{eu}_{ss}$.[25]

Filtering. We have already calibrated the means of the exogenous processes. We now use the Kalman filter to infer the shocks to the level and volatility of funding,

[23] That is, $\mathcal{DLP}_{ss} = \mathbb{E}\left[\chi_m\left(\mu^{eu}_{ss}, \sigma^{eu}\right)\right] - \mathbb{E}\left[\chi_m\left(\mu^{us}_{ss}, \sigma^{us}_{ss}\right)\right]$.

[24] Kalemli-Özcan and Varela (2021) provide a comprehensive comparative analysis of different ways to measure $\mathcal{UIP}$.

[25] We obtain loans using $b = \Theta^b(R^{m,us}_{ss} + \mathcal{BP}_{ss})^{-\epsilon^b}$ from loan market clearing. Similarly, we find values for d^{us}_{ss} and d^{eu}_{ss} from their corresponding clearing conditions, using the values $\sigma^{us}_{ss}, \sigma^{eu}_{ss}$. Setting $\Theta^{d,us}_{ss} = \Theta^{d,eu}_{ss}$, and substituting $\{b_{ss}, d^{us}_{ss}, d^{eu}_{ss}, \mu^{us}_{ss}, \mu^{eu}_{ss}\}$ into the bank's budget constraint, we obtain a single equation for $\Theta^{d,us}_{ss}$. Appendix F.1 provides a detailed discussion.

and the risk-premium wedge, $\left\{\sigma_t^{us}, \Theta_t^{d,us}, \sigma_t^{eu}, \Theta_t^{d,eu}, \xi_t\right\}$ from which we estimate their autocorrelations and variances. The priors and posteriors that result from the estimation step are reported in Table D.2.

Let us discuss which data series are informative about which inferred series. As with the steady state, the observed bond premium is informative about the funding risk σ_t^{us}, given that $\mathcal{BP}_t^* = \mathbb{E}\left[\chi_m\left(\mu_t^{us}, \sigma^{us}\right)\right]$. In turn, the CIP deviation is informative about σ_t^{eu}, since $\mathcal{CIP}_t = \mathbb{E}\left[\chi_m\left(\mu_t^{eu}, \sigma_t^{eu}\right)\right] - \mathcal{BP}_t$. However, within a transition, the values of $\{\rho^x\}$ and the internal structure lead to a forecast of the expected future exchange rate, $\mathbb{E}\left[e_{t+1}|\xi_t, \sigma_t^{us}, \Theta_t^{d,us}, \sigma_t^{eu}, \Theta_t^{d,eu}\right]$. Thus, given the policy rates, the observed exchange rate is informative about the risk premium, ξ_t, given the observed deviations of the UIP and CIP. Finally, the liquidity ratios are informative about $\left\{\Theta_t^{d,us}, \Theta_t^{d,eu}\right\}$ in line with the results from Proposition 1.

Figure 3 presents the shocks we inferred from the Kalman filter. From panel (a), we obtain that the dollar funding risk σ_t^{us} is close to steady state before 2007 but increases sharply during the financial crisis and remains higher past that period. As Figure D.1 shows, the US bond premium $\mathcal{BP}^*$ increases during the financial crisis, while the US liquidity ratio μ_t^{us} remains consistently high after the financial crisis. To reconcile a higher liquidity ratio with a $\mathcal{BP}^*$ that returns close to steady state, the model needs a persistently high dollar funding risk, σ_t^{us}. The spikes in σ_t^{us} that we observe between 2007-2009 likely reflect a dysfunctional dollar interbank market. In turn, the persistent rise of σ_t^{us} after the crisis can be attributed to several factors, including the increase in counterparty risk and stricter liquidity regulation, which included the liquidity coverage ratio and the net stable funding ratio (Copeland et al., 2021).

Panel (b) presents the inferred euro funding risk σ_t^{eu}. The euro funding risk is substantially lower and more stable compared to the dollar's. There are a couple of dips and reversals around 2008 and the 2012 European debt crisis, which the estimation picks up from fluctuations in the CIP deviations. Panels (c) and (d) show the path of the scale of dollar and euro funding. The two variables exhibit low-frequency movements that reflect only secular trends in funding by currency.[26] The risk premium wedge in panel (e) features a sizable negative value in early 2000, which we attribute to difficulties in forecasting the euro-dollar exchange rate during

[26]For the dollar, we see a secular decline up to 2008 and a subsequent increase. For euros, we see an increase up to the 2012 crisis and then a decrease.

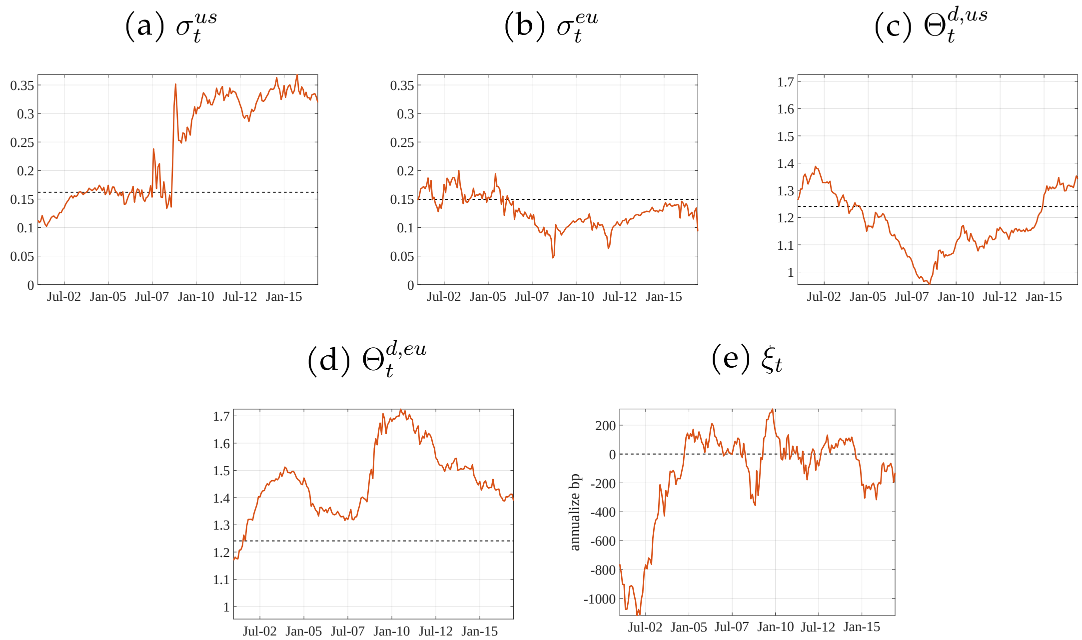

Figure 3: Estimated Shocks using the Kalman filter

the early inception period of the euro.[27]

By construction, the model reproduces the target moments. Table D.3 shows that, in addition, the estimated shocks deliver an exchange-rate persistence and standard deviation close to the data.

4.3 The role of liquidity

The particularly volatile behavior of the filtered shocks to the dollar funding risk, σ_t^{us}, hints that liquidity factors are important drivers of exchange rate fluctuations. We now pay special attention to the contribution of funding risk shocks in explaining the unconditional moments and historical patterns of exchange rates and premia.

How important are liquidity factors? We begin with a variance decomposition. We group the shocks to the funding scale, Θ_t^x, the funding risk, σ_t^x, and the supply of reserve assets, M_t^x, for $x \in \{us, eu\}$ into the liquidity group. Shocks to the policy rates in both countries and the risk-premium wedge form corresponding groups.

[27] This negative value captures that the euro was stronger than predicted from (17). After that initial transition, we see that the wedge fluctuates around zero with some spikes around 2008 and 2012.

Table 1: Variance Decomposition

Shock	e	$\mathcal{BP}^*$	$\mathcal{CIP}$
Liquidity	35%	99%	94%
Risk Premium	35%	<1%	3%
Policy Rates	30%	<1%	3%

Note: The variance decomposition is obtained from the model's estimated reduced form representation.

The results are presented in Table 1. Liquidity factors account for 35% of the variance of the euro-dollar exchange rate (panel a), 99% of the variance of the dollar bond premium (panel b), and 94% of the variance of the CIP deviation (panel c). When we further decompose the variance of the exchange rate by the contribution of the constituents of the liquidity factors, following the classification outlined above, funding risk turns out to be the main factor. Alone, funding risk drives 50% of the contribution of liquidity factors to the variance in the euro-dollar exchange rate, 88% of the variance of the dollar bond premium, and 84% of the contribution to the variance of the CIP deviation.

Historical Decomposition and Counterfactuals. Next, we perform a historical decomposition of the evolution of the euro-dollar exchange rate during our sample period. Figure 4 presents the results. The solid series is the percentage deviation of the euro-dollar exchange rate from the steady state. The vertical bars are the contribution of each group of shocks. For this exercise, we unpack the contribution of the liquidity factor into its components (liquidity risk, liquidity scale, and reserve supply) and present these together with the contribution of policy rate shocks (policy) and the risk premium wedge. To compute the contribution of each shock, we turn off the shock in each period by setting the value equal to the steady state and keeping the rest of the shocks on. The main takeaway is that since the 2008 financial crisis, funding risk has played a prominent role in accounting for a stronger dollar, an effect that has partially been offset by the large increase in the supply of dollar reserves.

To zoom in on the role of funding risk during the period, in Figure 5, we present the time series of the exchange rate together with two measures of CIP deviations—in terms of interbank and bond-market rates. We present the data, the model, and the counterfactual without the US funding risk shock (panels a, b, and c respectively). Consistent with the analysis, the counterfactual shows a much lower deviation from CIP (using both interest rates on reserves and the rates in the interbank market, as explained in Section 3.4) and a more depreciated dollar. Of course, the period post-2008 has also been a period of ample reserves, which offsets the effect of a greater funding risk.

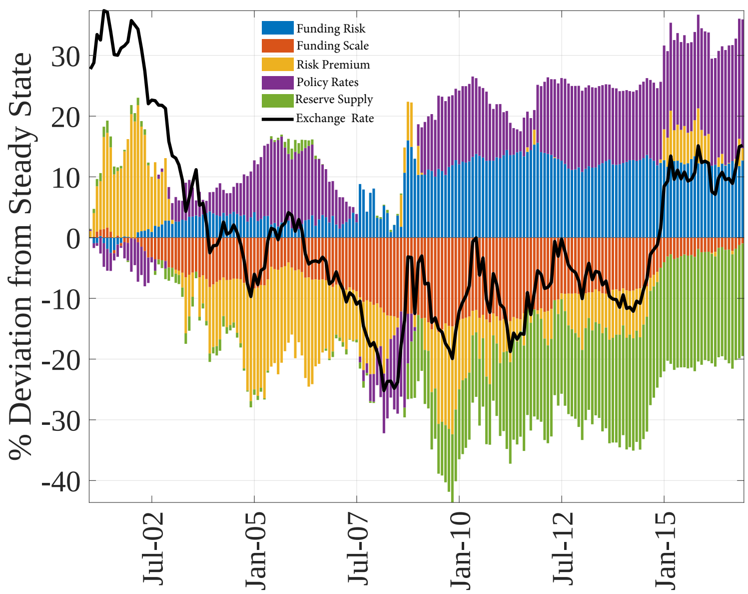

Figure 4: Shock Decomposition

Note: The figure reports the contribution of different groups of shocks in explaining the exchange rate.

The counterfactual exercises speak to a burgeoning literature on convenience yield on government bonds and the implications for foreign exchange markets (Jiang et al. 2021; Engel and Wu, 2023). To see this more clearly, Panel (d) shows the counterfactual value for the $\mathcal{BP}^*$ series, our measure of the convenience yields. Without the shock, convenience yields would have been compressed, considering the expansion in reserves that occurred during the period. Panels (e) and (f) show untargeted series that are, again, consistent with the pattern. Panel (e) shows the TED spread deviation, the difference between the interbank rate (the data

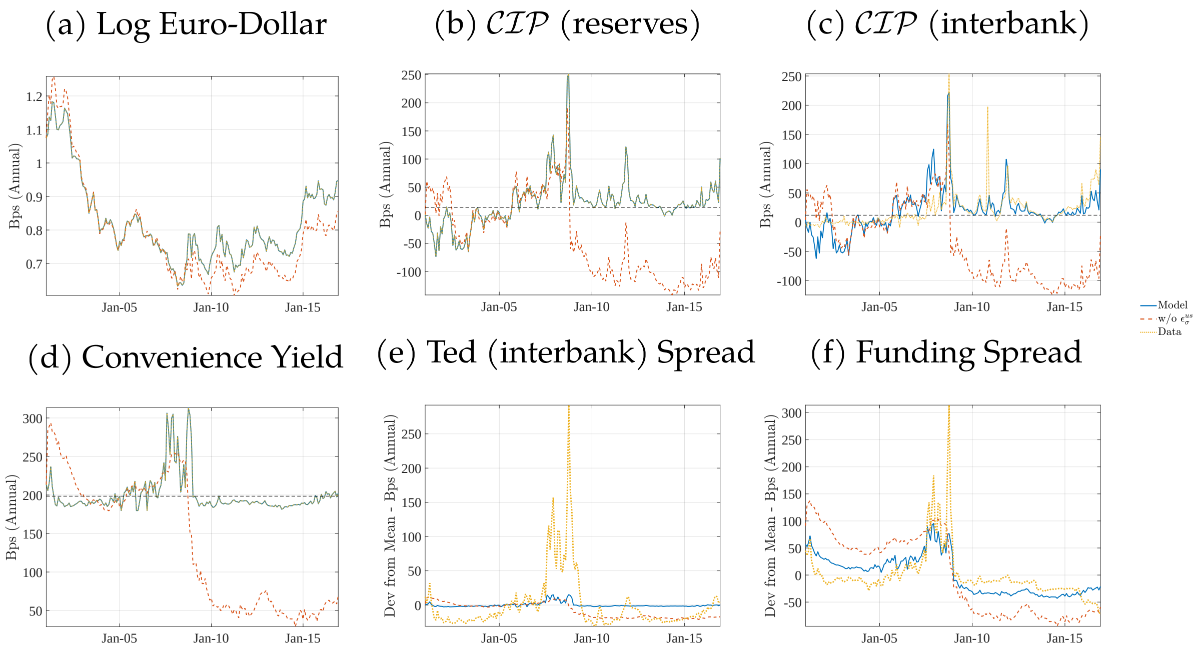

Figure 5: Counterfactual without σ^{us}

Note: The figure reports the predicted series of the model with all shocks and a counterfactual without σ_t^{us}. The funding spread, the TED spread, and the convenience yield are interpreted as $R^{d,us} - R^{m,us}$, $R^{f,us} - R^{m,us}$, $R^b - R^m$ respectively. The Ted and Funding spreads are reported as deviations from the historical mean.

counterpart is the effective Federal Funds rate) and the policy rate. The model and the data show a consistent pattern of large deviations from the mean during the crisis, with a normalization period starting in 2010, although clearly, the scale of the deviations in the data is much larger than in our model. Panel (f) shows the funding spread: the difference between the deposit rate—the data counterpart is the 4-week certificate deposit—and the policy rate. While untargeted, the fit to this series is also very good. The counterfactual shows that offered savings rates would have been lower without the liquidity risk shock post-crisis.

Taking Stock. Our findings show that liquidity factors are just as important as changes in risk compensation and policy rates, which previous research has mainly focused on. We view these results as reflecting a connection between the dollar's dominance in the international payments system and the associated bank funding risks. In particular,

1. The model suggests that the dollar liquidity premium is linked to increased funding risk. Our interpretation is that because the dollar is widely used in international transactions, this leads to a larger flow of dollar deposits relative to euros. From the perspective of banks, this means greater funding risks.
2. Many studies have argued that the dollar is a safe haven for investors during times of financial turmoil. We interpret the rise in observed funding risk during the crisis as a sign of the safety of the dollar and the prevalence of the dollar in the short-term funding market.
3. Finally, there has been an increase in funding risk after the global financial crisis as indicated by the rise in CIP deviations. We interpret this as emerging from the rise in liquidity and capital regulations post 2008 that generated a rise in the dollar liquidity premium.

The theory thus calls for a further examination of the dollar's international dominance, in particular regarding the role of banks' funding risks and the impact of regulation. The notable "dash for cash" experienced in March 2020 came to a halt only after the Federal Reserve implemented a series of liquidity programs. In fact, transaction data, as analyzed by Cesa-Bianchi and Eguren-Martin (2021) revealed that the surge in demand for cash was predominantly a rush for dollars.

Next, we test our theory by providing evidence that a quantity variable, the liquidity ratio, is indeed correlated with various exchange rates, as predicted by the model.

4.4 Empirical test and model validation

The previous section suggests that liquidity funding risk played a prominent role in determining exchange rates during the sample period. In Section 3, we argue that the nature of shocks and their persistence drive the statistical relationship between the liquidity ratio and the exchange rate. A signature test of our theory and, in particular, of the relevance of liquidity funding risk is to obtain significant and positive regression coefficients in the data when regressing changes in the exchange rate to changes in the liquidity ratio. Before proceeding with the test, we formalize this insight in the following proposition the theoretical prediction of the model.

Proposition 5 (Regression coefficients)**.** *Consider a steady state. Then, up to first order, the regression coefficient of the change in the exchange rate against the change in the liquidity ratio is:*

$$\beta^e_{\mu^*} = \sum_{x \in \{\sigma^*, D^*\}} \frac{\epsilon^e_x}{\epsilon^{\mu^*}_x} \cdot \boldsymbol{w}_x,$$

where

$$\frac{\epsilon^e_{\sigma^*}}{\epsilon^{\mu^*}_{\sigma^*}} = 1 \text{ and } \frac{\epsilon^e_{D^*}}{\epsilon^{\mu^*}_{D^*}} = \frac{\mathcal{DLP}^*_{\mu^*}\mu^*}{(1-\rho^{D^*})R^b} < 0,$$

with weights given by

$$\boldsymbol{w}^{\sigma^*} = \frac{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2 \left(1-\left(\rho^{D^*}\right)^2\right)}{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2 \left(1-\left(\rho^{D^*}\right)^2\right) + \left(\epsilon^{\mu^*}_{D^*}\Sigma^{D^*}\right)^2 \left(1-\left(\rho^{\sigma^*}\right)^2\right)} = 1 - \boldsymbol{w}^{D^*}.$$

The proposition shows that the regression coefficient of the liquidity ratio against the exchange rate is a weighted average of the effects of changes in volatility and deposit supply on the correlation between the changes in the exchange rate and the liquidity ratio.

Test results. To conduct our test on multiple currencies, we further collect data for the price of US dollar against the other nine G10 currencies and their corresponding inflation rates and interest-rate levels.[28]

We test our theory by estimating the following regression for each currency i:

$$\Delta e_t = \alpha + \beta_1 \Delta Liq_t + \beta_2(\pi_t - \pi^*_t) + \beta_3 Liq_{t-1} + \epsilon_t. \tag{23}$$

In this regression, $\Delta(x_t)$ is the change from $t-1$ to t in the variable x_t; e_t is the log of the exchange rate expressed as the G10 currency price of a US dollar; Liq_t is the log of the liquidity ratio described above; $\pi_t - \pi^*_t$ is the difference between year-on-year inflation rates in each of the nine countries against the US inflation.

Our regression specification also includes the year-on-year inflation rate. Central banks may follow a policy rule that sets the policy instrument in response to inflation, so higher home-country inflation leads to tighter monetary policy and an appreciated

[28]The other currencies are the Australian dollar, the Canadian dollar, the Japanese yen, the New Zealand dollar, the Norwegian krone, the Swedish krona, the Swiss franc, and the UK pound. The inflation for Australia and New Zealand is reported only quarterly.

currency. Our model, however, abstracts from such a monetary policy rule.

Table 2 reports the results of the regression for the nine exchange rates. The coefficient of interest is β_1. If the dollar funding risk drives Liq_t, then we also expect a positive relationship between this variable and e_t; that is, β_1 should be positive. With the exception of Japan, the liquidity ratio variable has the expected positive sign and is statistically significant at the 1% level for all exchange rates.

Table 2: Exchange Rates and Liquidity Ratio: Feb. 2001 – July 2021

	EUR	AUS	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.227***	0.256***	0.127***	-0.134***	0.287***	0.187***	0.212***	0.141***	0.165***
	(4.839)	(4.106)	(2.723)	(-2.846)	(4.458)	(3.125)	(3.754)	(2.724)	(3.529)
$\pi_t - \pi_t^*$	-0.800***	-0.657***	-0.407**	0.011	-0.726***	-0.126	-0.465**	-0.565***	-0.335**
	(-3.972)	(-2.998)	(-1.982)	(0.084)	(-3.299)	(-0.873)	(-2.530)	(-2.644)	(-1.985)
Liq_{t-1}	0.008*	0.005	0.007	0.002	0.004	0.010*	0.006	0.005	0.008
	(1.890)	(0.796)	(1.554)	(0.307)	(0.696)	(1.730)	(1.109)	(0.985)	(1.628)
Constant	-0.010***	-0.002	-0.006*	-0.001	-0.005	-0.007	-0.008**	-0.015***	-0.006
	(-3.097)	(-0.595)	(-1.877)	(-0.120)	(-1.188)	(-1.618)	(-1.978)	(-2.966)	(-1.569)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.11	0.07	0.03	0.02	0.09	0.03	0.06	0.03	0.04

Note: t statistics in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table 3: Model Regression Coefficients

	EUR	AUS	CAN	JPN	NZ	NWY	SWE	SWZ	UK
$\Delta(\text{Liq}_t)$	0.136**	0.179	0.188***	0.188***	0.182***	0.171***	0.179**	0.186***	0.182***
	(2.021)	(1.591)	(3.0813)	(3.0813)	(2.166)	(2.671)	(2.35)	(3.049)	(2.637)
$\pi_t^i - \pi_t^*$	0.208*	0.236*	0.197*	0.197*	0.2262**	0.203*	0.218*	0.198*	0.212*
	(1.667)	(1.255)	(1.890)	(01.890)	(1.975)	(1.780)	(1.602)	(1.767)	(1.683)
Liq_{t-1}	0.006	0.009	0.008	0.008	0.008	0.008	0.008	0.008	0.008
	(0.235)	(0.255)	(0.388)	(0.388)	(0.351)	(0.439)	0.312)	(0.360)	(0.339)
Constant	0.010	0.014	0.011	0.011	0.013	0.011	0.012	0.011	0.012
	(0.273)	(0.252)	(0.336)	(0.249)	(0.291)	(0.367)	(0.290)	(0.334)	(0.375)
N	234	234	234	234	234	234	234	234	234
adj. R^2	0.045	0.030	0.067	0.066	0.044	0.058	0.050	0.066	0.056

Note: The coefficients reported are averages over 4273 simulations of 234 periods. Standard deviations are in parenthesis.

The significance of the coefficient is indeed a validation of our theory and, concretely, of the role of funding risk. We should stress that ΔLiq_t, is not a price variable but a quantity variable. Finding significant statistical relations between exchange rates and quantity variables has been elusive in international finance analysis. The significance of the coefficients for most currencies shows that liquidity can "explain" exchange rate movements without relying on a measure of liquidity that is contaminated with exchange rates.[29] The sign of the coefficient indicates a prevalent role of funding risk, as theoretically shown in Proposition 5.

Finally, we observe that the results also show a negative relationship between the change in a country's inflation rate and its exchange rate. This result is in line with much of the empirical literature and is consistent with monetary policy rules that follow inflation targeting, as noted previously. Another point worth highlighting is that our regressions are about current realized depreciation, not forecast future depreciation. We do not control for nominal interest rates in our baseline regressions as in standard UIP regressions, which are predictive regressions. However, our results are very similar if we add these controls, and the interest rates are not significant during this period.

Robustness of the Regression Analysis. We also investigate whether the data test results survive under different specifications. For starters, several asset-pricing studies have found that VIX–which is a measure of market uncertainty—has explanatory power in accounting for the movements of many asset prices. One possible concern is that if we were to include VIX, this would reduce the significance of liquidity and thus reduce the importance of funding risk in our regressions. In Table G1, we include the change in *VIX* along with the other variables. As expected, *VIX* has positive coefficients in all cases (except again for Japan) and is statistically significant (i.e., an increase in *VIX* is associated with an appreciation of the dollar). However, the regressions show that including the VIX does not reduce the significance of the liquidity ratio for any of the countries, and it has only a minor effect on the magnitude of the coefficient. This suggests that funding risk, as measured by the liquidity ratio, contributes to explaining the exchange rate beyond market uncertainty.

In the appendix, we also perform a number of additional robustness exercises. We show that our results are very similar when we use alternative liquidity measures that include broader measures of short-term funding. We also demonstrate that when we use balance sheet data exclusively from foreign-related banks in the US, the relationship between the liquidity ratio of dollar liquid assets to dollar funding

[29] This happens in recent empirical studies that use the share of dollar assets as an explanatory variable (see, e.g., Adrian and Xie, 2020.).

and dollar exchange rates still holds. Moreover, we also show that when we break the sample, we find larger coefficients during the pre-European debt crisis, but for most countries, liquidity remains significant after the crisis.

Finally, one important point to highlight is that the regressions should only be interpreted as a summary statistic of the comovement between liquidity and exchange rates—in line with the theory above. This is because the liquidity ratio is not exogenous. A plausible exogenous source of variation that is correlated with the liquidity ratio is the monthly average of the intra-daily Fed Funds spread—i.e., the difference between the high and low Fed funds rate transacted on each day. This measures interbank market disturbances and to the extent that the effects on the exchange rate occur primarily through banks' liquidity management, we can use an instrumental variable approach to isolate the effects of funding risk on exchange rates. As we show in Appendix G, our main conclusions remain robust in this case as well.

We do not further explore the relationship between measures of the convenience yield on government liquid liabilities and exchange rates, as that has already been extensively documented in studies such as Jiang et al. (2021) and Engel and Wu (2023). The appendix does present evidence of a significant statistical relationship between our measures of the liquidity ratio and the relative dollar convenience yield used in these previous studies.

Model vs. data. The empirical tests are motivated by our theoretical results regarding the relationship between the liquidity ratio and exchange rates fleshed out in Proposition 5. Although our filtering exercise is meant to match the historical series for the exchange rate and the liquidity ratio, there is no reason why the underlying estimated stochastic processes for shocks should allow the model to reproduce the regression coefficients of the empirical tests. Next, we use the model estimates and conduct Monte Carlo simulations to compare regressions in the model with the data counterparts.

For the simulations, we assume that the liquidity and risk premia shocks to all currencies other than the dollar are the same as those of the euro. In turn, we estimate the processes for the countries' nominal policy rates, which we feed directly from the data.[30] We then simulate the corresponding shocks in each currency and obtain a predicted exchange rate for each currency. We then contrast the empirical relationship between the dollar liquidity ratio and the different exchange rates in the data and those predicted by the model. We simulated 1 million observations, which we split into sub-samples of 246 periods corresponding to the data sample.

We report the simulation-based regression coefficients for all sub-samples in Table 3. The main takeaway is that the simulation-based regression delivers a positive value and is similar to the data coefficients for the effect of the liquidity ratio on the exchange rate. A discrepancy worth highlighting is that the Japanese yen delivers a negative coefficient in the data and a positive one in the model. We reconcile this discrepancy by allowing funding shocks to the Yen that are correlated with the US funding risk shocks.[31]

5 Conclusions

This study develops a theory of exchange rate determination emerging from financial institutions' demand for liquid dollar assets. Periods of increased funding volatility generate a "scrambling for dollars" effect that raises liquidity premia and appreciates the dollar. In line with the theory, we document that a higher liquidity ratio in the financial system is associated with a stronger dollar. We also use the model as a quantitative laboratory to decompose the different forces driving exchange rate fluctuations. We conclude from our analysis that funding risk is a crucial factor driving fluctuations in exchange rates.

Our framework can be extended in several directions. For example, it would be interesting to allow for a richer production structure and nominal rigidities to analyze conventional channels of monetary policy. In addition, our model offers a framework to study foreign exchange interventions, swap lines, and other less conventional policies. We leave this for future research.

[30] Thus, the only difference between the euro-dollar and other exchange rates stem from interest-rate differentials.

[31] In Appendix E we present a robustness exercise, where we allow demand shifters of the Yen and Swiss Franc funding correlated with funding risk that can replicate the more tenuous statistical relationship that holds for these two currencies in the broader set of regressions we display in the appendix. Another discrepancy is that we obtain a positive coefficient for inflation in the model and a negative in the data. In the data, the currency appreciates when past inflation is high due to central banks targeting inflation. We do not model an inflation targeting rule in our model, and instead have an exogenous money supply, so higher inflation leads to a depreciation.

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Online Appendix to "Scrambling for Dollars: International Liquidity, Banks and Exchange Rates"

By Javier Bianchi, Saki Bigio and Charles Engel

A Expressions for $\{\Psi^+, \Psi^-, \chi^+, \chi^-\}$

Here we reproduce formulas derived from Proposition 1 in Bianchi and Bigio (2017). That proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market.

The average interbank rate is:

$$R^f = (1 - \bar{\eta}(\theta))R^w + \bar{\eta}(\theta) R^m$$

where $\bar{\eta}(\theta)$ is an endogenous bargaining power given by

$$\bar{\eta}(\theta) \equiv \begin{cases} \frac{\theta}{\theta-1}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta > 1 \\ \eta & \text{if } \theta = 1 \\ \frac{\theta(1-\bar{\theta})-\bar{\theta}}{\bar{\theta}(1-\theta)}\left(\left(\frac{\bar{\theta}}{\theta}\right)^{\eta} - 1\right)(\exp(\lambda) - 1)^{-1} & \text{if } \theta < 1 \end{cases}.$$

and η is a parameter associated with the bargaining power of banks with reserve deficits in each trade—a Nash bargaining coefficient. In addition, $\bar{\theta}$ represents the market tightness after the interbank-market trading session is over:

$$\bar{\theta} = \begin{cases} 1 + (\theta - 1)\exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ \left(1 + \left(\theta^{-1} - 1\right)\exp(\lambda)\right)^{-1} & \text{if } \theta < 1 \end{cases}.$$

The parameter λ captures the matching efficiency of the interbank market. Trading probabilities are given by

$$\Psi^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta\left(1 - e^{-\lambda}\right) & \text{if } \theta < 1 \end{cases}, \qquad \Psi^- = \begin{cases} \left(1 - e^{-\lambda}\right)\theta^{-1} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}. \tag{A.1}$$

Finally, using 4 and 5, we arrive at the parameters of the liquidity yield function χ:

$$\bar{\chi}^+ = (R^w - R^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \bar{\chi}^- = (R^w - R^m)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \tag{A.2}$$

B Proofs

B.1 Preliminaries

Here we provide some intermediate results that we use to prove the propositions.

Recall that the liquidity ratio is denoted by $\mu \equiv m/d$ and $\theta = S^{-}/S^{+}$ where $S^{-} = -\int \min\{s,0\}\, d\Phi(\omega)$, $S^{+} = \int \max\{s,0\}\, d\Phi(\omega)$ and $s = m + \omega d$. Then,

$$\begin{aligned}\theta &= -\frac{\int_{\{s<0\}} s \cdot d\Phi(\omega;\sigma)}{\int_{\{s>0\}} s \cdot d\Phi(\omega;\sigma)},\\ &= -\frac{m\Phi(\{s<0\};\sigma) + d\int_{\{s<0\}} \omega \cdot d\Phi(\omega;\sigma)}{m(1-\Phi(\{s>0\};\sigma)) + d\int_{\{s\geq 0\}} \omega \cdot d\Phi(\omega;\sigma)}.\end{aligned}$$

Note that $s<0$ occurs when $\omega < -\mu$. Therefore, we express the interbank market tightness as:

$$\theta = -\frac{\int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}. \tag{B.1}$$

With abuse of notation, define $\theta(\mu,\sigma)$ as the function that maps μ and σ into a value of θ (thus, in equilibrium, $\theta = \theta(\mu,\sigma)$). We have the following Lemma:

Lemma B.1. *Interbank market tightness is decreasing in the liquidity ratio. That is, $\frac{d\theta}{d\mu} < 0$. Moreover, $\theta \in [0,1]$.*

Proof. From (C.1), using Leibniz rule, we obtain

$$\frac{d\theta}{d\mu} = \theta\left(\frac{\Phi(-\mu;\sigma)}{\int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma)} - \frac{1-\Phi(-\mu;\sigma)}{\int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}\right). \tag{B.2}$$

By definition of conditional expectation:

$$\mathbb{E}[\mu+\omega|\omega<-\mu] = \int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma) / \Phi(-\mu;\sigma),$$

and

$$\mathbb{E}[\mu+\omega|\omega>-\mu] = \int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma) / (1-\Phi(-\mu;\sigma)).$$

Replacing these definitions into (C.2), we obtain:

$$\frac{d\theta}{d\mu} = \theta \cdot \left(\frac{1}{\mathbb{E}[\mu+\omega|\omega<-\mu]} - \frac{1}{\mathbb{E}[\mu+\omega|\omega>-\mu]}\right) < 0,$$

where the inequality follows because $\mathbb{E}[\mu+\omega|\omega<-\mu] < 0$ and $\mathbb{E}[\mu+\omega|\omega>-\mu] > 0$.

Finally, the bounds on θ follow because $\lim_{\mu\to\infty} \theta = 0$ and $\theta = 1$ if $\mu = 0$. □

Next, we obtain the derivative of interbank market tightness with respect to σ.

Lemma B.2. *Under Assumption 1, we have that* $\frac{\partial\theta}{\partial\sigma} > 0$.

Proof. Passing the differential operator inside the integrals in the numerators, we have that:

$$\begin{aligned}\frac{\partial\theta}{\partial\sigma} &= \theta\cdot\left(\frac{\int_{-\infty}^{-\mu}(\mu+\omega)\,\phi_\sigma d\omega}{\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)} - \frac{\int_{-\mu}^{\infty}(\mu+\omega)\,\phi_\sigma d\omega}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right)\\ &= \theta\cdot\left(\frac{\partial}{\partial\sigma}\left[\log\left(\frac{\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right)\right]\right).\end{aligned}$$

Since the withdrawal shock is zero mean,

$$\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma) + \int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma) = \mu.$$

Therefore, identity this condition into the derivative just above we obtain:

$$\frac{\partial\theta}{\partial\sigma} = \log\left(\frac{\mu - \int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right).$$

Therefore, $\frac{\partial\theta}{\partial\sigma} > 0$ holds if and only if:

$$\frac{\partial}{\partial\sigma}\left[\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)\right] < 0.$$

Using the integration by parts formula:

$$\begin{aligned}\int_{-\infty}^{-\mu}(\mu+\omega)\,\phi_\sigma(\omega;\sigma)\,d\omega &= (\mu+\omega)\,\Phi_\sigma(\omega;\sigma)\,|_{-\infty}^{-\mu} - \int_{-\infty}^{-\mu}\Phi_\sigma(\omega;\sigma)\,d\omega\\ &= -\int_{-\infty}^{-\mu}\Phi_\sigma(\omega;\sigma)\,d\omega < 0\end{aligned}$$

where the last equality follows from $\lim_{\omega\to-\infty}((\mu+\omega))\,\Phi_\sigma(\omega;\sigma) = \frac{\partial}{\partial\sigma}\left[\lim_{\omega\to-\infty}((\mu+\omega))\,\Phi(\omega;\sigma)\right] = 0$ and the strict inequality follows from Assumption 1. We conclude that, $\frac{\partial\theta}{\partial\sigma} > 0$. □

We will also use the results from the following Lemma.

Lemma B.3. *The liquidity coefficients have the following derivatives:*

$$\frac{\partial\chi^+}{\partial\mu} = \frac{\partial\chi^+}{\partial\theta}\frac{\partial\theta}{\partial\mu} < 0 \quad \textit{and} \quad \frac{\partial\chi^-}{\partial\mu} = \frac{\partial\chi^-}{\partial\theta}\frac{\partial\theta}{\partial\mu} < 0, \tag{B.3}$$

$$\frac{\partial\chi^+}{\partial\sigma} = \frac{\partial\chi^+}{\partial\theta}\frac{\partial\theta}{\partial\sigma} > 0 \quad \textit{and} \quad \frac{\partial\chi^-}{\partial\mu} = \frac{\partial\chi^-}{\partial\theta}\frac{\partial\theta}{\partial\sigma} > 0, \tag{B.4}$$

$$\frac{\partial \bar{\chi}^+}{\partial P_t} = \frac{\bar{\chi}^+}{P_t} \quad \textit{and} \quad \frac{\partial \bar{\chi}^-(\theta)}{\partial P_t} = \frac{\bar{\chi}^-}{P_t}. \tag{B.5}$$

Proof. Notice first that $\frac{\partial \chi^+}{\partial \theta} > 0$ and $\frac{\partial \chi^-}{\partial \theta} > 0$ is an immediate result from their definitions in equations (A.2). Applying Lemmas C.1 and C.2, we obtain respectively (C.3) and (C.4).

In addition, we can express (A.2) as

$$\bar{\chi}^+ = \frac{P_t}{P_{t+1}} \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right), \ \bar{\chi}^- = \frac{P_t}{P_{t+1}} \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right) \tag{B.6}$$

Equation (C.5) follows immediately. □

It is useful to define $\mathcal{L}(\mu, \sigma, P)$ to be the bond liquidity premium as a function of the liquidity ratio, the index σ and the current price level. That is,

$$\mathcal{L}(\mu, \sigma, P) = (1 - \Phi(-\mu, \sigma)) \cdot \bar{\chi}^+ \left(\theta(\mu, \sigma), P\right) + \Phi(-\mu, \sigma) \cdot \bar{\chi}^- \left(\theta(\mu, \sigma), P\right) \tag{B.7}$$

In equilibrium $\mathcal{L}(\mu, \sigma, P) = R^b - R^m$. We have the following result.

Lemma B.4. *The liquidity bond premium is decreasing in the liquidity ratio and increasing in volatility. That is, $\mathcal{L}_\mu < 0$ and $\mathcal{L}_\sigma > 0$. In addition, $\mathcal{L}_P = -\mathcal{L}/P$.*

Proof. From (C.7), differentiating $\mathcal{L}$ with respect to μ:

$$\mathcal{L}_\mu = \left[(1 - \Phi(-\mu, \sigma)) \cdot \chi_\theta^+ + \Phi(-\mu, \sigma) \cdot \chi_\theta^-\right] - \left(\bar{\chi}^- - \bar{\chi}^+\right) \phi\left(-\mu, \sigma\right). \tag{B.8}$$

Using that $\frac{\partial \theta}{\partial \mu} < 0$ from Lemma C.1 and that $\bar{\chi}^- > \bar{\chi}^+$, we arrive at $\mathcal{L}_\mu < 0$.

From (C.7), differentiating $\mathcal{L}$ with respect to σ yields:

$$\mathcal{L}\mathcal{P}_\sigma = \frac{\partial \theta}{\partial \sigma} \left[(1 - \Phi(-\mu, \sigma)) \cdot \chi_\theta^+ + \Phi(-\mu, \sigma) \cdot \chi_\theta^-\right] + \left(\bar{\chi}^- - \bar{\chi}^+\right) \Phi_\sigma\left(-\mu, \sigma\right). \tag{B.9}$$

Using that $\frac{\partial \theta}{\partial \sigma} > 0$ from Lemma C.2 and that $\bar{\chi}^- > \bar{\chi}^+$, we conclude that $\mathcal{L}_\sigma > 0$. Finally, the expression for $\mathcal{L}_P$ follows directly from differentiating $\mathcal{L}$ with respect to P in (C.5). □

We now proceed with the proofs and use that these properties apply for both euros and dollars.

B.2 Proof of Proposition 1

Proof. **Part i).** By definition, the liquidity ratio μ^* is given by

$$\mu^*(P^*, D^*) = \frac{M^*/P^*}{D^*} \tag{B.10}$$

where we made explicit the dependence of μ^* on (P^*, D^*). Using that M^* is exogenously given, totally differentiating (C.10) yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right). \tag{B.11}$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, D^*), P^*). \tag{B.12}$$

Totally differentiating (C.12) with respect to P^* and D^*, and using (C.11), we obtain:

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) = -\mathcal{L}^*_{\mu^*} \left[\mu \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right) \right] + \mathcal{L}^*_P dP^* \tag{B.13}$$

where $\mathbb{E}[P^*(X')]$ remains constant because the shock is i.i.d. and the loan rate is constant at $R^b = 1/\beta$.

Using $\mathcal{L}^*_{P^*} = \frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b = R^{m,*} + \mathcal{L}^*$ and replacing in (C.13), we arrive to

$$\frac{d \log P^*}{d \log D^*} = \frac{\mathcal{L}^*_{\mu^*} \mu^*}{R^b - \mathcal{L}^*_{\mu^*} \mu^*} \in (-1, 0)\,. \tag{B.14}$$

The bounds follows immediately because $\mathcal{L}^*_\mu < 0$ as established in Lemma C.4 and from $R^b > 0$.

Notice also that the euro bond premium remains constant. To see this, we can replace $\mu = (M/P)/D$ in (15) and use (C.1) to obtain

$$R^b - (1 + i^m) \frac{P}{\mathbb{E}[P(X')]} = \left(1 - \Phi \left(-\frac{M/P}{D} \right) \right) \bar{\chi}^+(\theta((M/P)/D, \sigma)) + \\ \Phi \left(-\frac{M/P}{D} \right) \bar{\chi}^-(\theta((M/P)/D, \sigma)). \tag{B.15}$$

From (C.15), it follows that P must be constant and thus μ and $\mathcal{L}$ are also constant. As a result, $d\mathcal{L}^* = d\mathcal{DLP}$, $d\mathcal{L}^*_{\mu^*} = d\mathcal{DLP}_{\mu^*}$.

By the law of one price and using that P remains constant, we then have $\frac{d \log e}{d \log D^*} = -\frac{\mathcal{L}^*_\mu \mu^*}{R^b - \mathcal{L}^*_\mu \mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (C.13) as $\bar{R}^{m,*} (d \log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation remains constant. Moreover, given that nominal policy rates and expected inflation are constant, we have from (16) that $\mathcal{L}^*$ is constant. Hence, $\mathcal{DLP}$ is constant. Furthermore, the fact that $\mathcal{L}^*$ is constant, implies that μ must also be constant. Thus, using that (C.11) and that M^* is constant, we have from the law of one price that:

$$\frac{d\log e}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1.$$

$\square$

B.3 Proof of Proposition 2

Proof. **Part i).** Totally differentiating (C.10) with respect to P^* yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*}\right). \tag{B.16}$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*})\frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, \sigma^*), P^*). \tag{B.17}$$

Totally differentiating (C.17) with respect to P^* and σ^* and using (C.16) yields:

$$-R^{m,*}\left(\frac{dP^*}{P^*}\right) = -\mathcal{L}^*_\mu\left[\mu\left(\frac{dP^*}{P^*}\right)\right] + \mathcal{L}^*_{\sigma^*}d\sigma^* + \mathcal{L}^*_P dP^* \tag{B.18}$$

where we used that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b = 1/\beta$.

Using $\mathcal{L}^*_{P^*} = \frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b = R^{m,*} + \mathcal{L}^*$, and replacing in (C.16) , we obtain

$$\frac{d\log P^*}{d\log\sigma^*} = -\frac{\mathcal{L}^*_{\sigma^*}\sigma}{R^b - \mathcal{L}^*_\mu\mu^*} < 0 \tag{B.19}$$

where the sign follows from Lemma C.4. Notice also that the euro bond premium remains constant, and so do P, μ and $\mathcal{L}$, as demonstrated in the proof of Proposition 1.

By the law of one price, and using that P remains constant, we then have $\frac{d\log e}{d\log D^*} = \frac{\mathcal{L}^*_\sigma\sigma^*}{R^b - \mathcal{L}^*_\mu\mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (C.18) as $\bar{R}^{m,*}(d\log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation is constant. Given that nominal policy rates are constant, $\mathcal{L}^*$ and $\mathcal{DLP}$ are constant. Thus,

$$\mathcal{L}^*_{\mu^*}d\mu^* + \mathcal{L}^*_{\sigma^*}d\sigma^* = 0 \tag{B.20}$$

and so

$$\frac{d\log\mu^*}{d\log\sigma^*} = -\frac{\mathcal{L}^*_{\sigma^*}\sigma^*}{\mathcal{L}^*_{\mu^*}\mu^*} > 0 \tag{B.21}$$

where the sign follows from $\mathcal{L}^*_{\mu^*} < 0$ and $\mathcal{L}^*_{\sigma^*} > 0$ from Lemma C.4. Using that $d\log\mu^* = -d\log P^*$, from the law of one price, $\frac{d\log e^*}{d\log\sigma^*} = \frac{d\log\mu^*}{d\log\sigma^*}$. $\square$

B.4 Approximation to Mean Reverting Shocks

Proof. We now derive approximate analogues to propositions 1 and 2 for cases where shocks are mean reverting. In particular, shocks follow a log AR(1) process:

$$\log(x_t) = (1-\rho^x)\log(x_{ss}) + \rho^x \cdot \log(x_{t-1}) + \Sigma^x \varepsilon_t^x. \tag{B.22}$$

We have the following result. We use x_{ss} to refer to the deterministic steady-state value of any variable x. The proof extends the results in Propositions 1 and 2. We first show this intermediate result. In the model, prices are a function of the aggregate state, X. Thus, an equilibrium will feature a function $P^*(X_t)$ such that $P_t^* = P^*(X_t)$. Then, near the steady state, using a Taylor expansion of first-order with respect to the variable x. We have that:

$$\log P_t^* \approx \log P_{ss}^* + \frac{P_x^*(x_{ss})\,x_{ss}}{P_{ss}^*}\frac{x_t - x_{ss}}{x_{ss}}.$$

Thus, we have that for small deviations around the steady state:

$$d\log P_t^* \approx \frac{P_x^*(x_{ss})\,x_{ss}}{P_{ss}^*} d\log x_t. \tag{B.23}$$

Shifting this condition forward:

$$d\log P_{t+1}^* \approx \frac{P_x^*(x_{ss})\,x_{ss}}{P_{ss}^*} d\log x_{t+1}$$

Taking expectations:

$$\mathbb{E}\left[d\log P_{t+1}^*\right] \approx \frac{P_x^*(x_{ss})\,x_{ss}}{P_{ss}^*}\rho^x d\log x_t. \tag{B.24}$$

Dividing the left-hand side of (C.24) by (C.23),

$$\frac{\mathbb{E}\left[d\log P_{t+1}^*\right]}{d\log P_t^*} = \rho^x d\log x_t. \tag{B.25}$$

Next, we proof the main items of the propositions. The proof uses that for either currency:

$$\frac{\partial\bar{\chi}^+}{\partial P_{t+1}} = -\frac{\bar{\chi}^+}{\mathbb{E}[P_{t+1}]},\text{ and }\frac{\partial\bar{\chi}^-(\theta)}{\partial P_{t+1}} = -\frac{\bar{\chi}^-}{\mathbb{E}[P_{t+1}]}. \tag{B.26}$$

Hence:

$$\mathcal{L}^*_{P^*_{t+1}} = -\frac{\mathcal{L}^*}{P^*_{t+1}}$$

Recall that the dollar liquidity premium can be expressed as

$$R^b - (1+i^{m,*})\frac{P_t^*}{\mathbb{E}\left[P_{t+1}^*\right]} = \mathcal{L}^*(\mu^*(P^*, D^*), P_t^*, P_{t+1}^*), \tag{B.27}$$

where we now make explicit that $\mathcal{L}^*$ depends on both P_t and P_{t+1}.

Part (i). We present here the proof for item (i). Totally differentiating (C.27) with respect to P_t, P_{t+1}, and D^* and using (C.11) near the steady state, we obtain

$$-R^{m,*}\left(\frac{dP_t^*}{P_t^*}\right)+R^{m,*}\frac{\mathbb{E}\left[dP_{t+1}^*\right]}{\mathbb{E}\left[P_{t+1}^*\right]}=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right)+\mathcal{L}_{P_t^*}^*dP_t^*-\mathcal{L}_{P_{t+1}^*}^*\mathbb{E}\left[dP_{t+1}^*\right]. \tag{B.28}$$

Then, collecting terms:

$$-\left(R^{m,*}+\mathcal{L}^*\right)\left(1-\frac{\mathbb{E}\left[d\log P_{t+1}^*\right]}{d\log P_t^*}\right)d\log P_t^*=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right). \tag{B.29}$$

Substituting $R^b=R^{m,*}+\mathcal{L}^*$ and (C.25), we obtain:

$$R^b\left(1-\rho^{D^*}\right)d\log P_t^*\approx\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right).$$

Thus, we obtain

$$\frac{d\log P^*}{d\log D^*}\approx\frac{\mathcal{L}_{\mu^*}^*\mu^*}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}<0.$$

Then, it follows from the law of one price and the differential form of μ that

$$\epsilon_{D^*}^e\equiv\frac{d\log e}{d\log D^*}\approx-\frac{\mathcal{L}_{\mu^*}^*\mu^*}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(0,1),$$

and

$$\epsilon_{\mu^*}^e\equiv\frac{d\log\mu}{d\log D^*}\approx-\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(-1,0).$$

Part (ii). We present here the proof for item (ii). It follows the same steps as in Part (i): We totally differentiate (C.27) with respect to P_t, P_{t+1}, and σ^* and using (C.11) for the case where $dD^*=0$. We obtain:

$$-R^{m,*}\left(\frac{dP_t^*}{P_t^*}\right)+R^{m,*}\frac{\mathbb{E}\left[dP_{t+1}^*\right]}{\mathbb{E}\left[P_{t+1}^*\right]}=\mathcal{L}_{\sigma^*}^*d\sigma^*-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}\right)+\mathcal{L}_{P_t^*}^*dP_t^*-\mathcal{L}_{P_{t+1}^*}^*\mathbb{E}\left[dP_{t+1}^*\right]. \tag{B.30}$$

Collecting terms and using the same identities that we use to derive C.29, we arrive at:

$$\left(R^b\left(1-\rho^{\sigma^*}\right)-\mathcal{L}_{\mu^*}^*\mu^*\right)d\log P_t^*\approx-\mathcal{L}_{\sigma^*}^*d\sigma^*.$$

Therefore, we obtain:

$$\frac{d\log P^*}{d\log\sigma^*}\approx\frac{-\mathcal{L}_{\sigma^*}^*d\sigma^*}{\left(1-\rho^{\sigma^*}\right)R^b-\mathcal{L}_{\mu^*}^*\mu^*}<0.$$

Then using that $\mu^*=M^*/\left(P^*D^*\right)$ and that $e=P/P^*$ and that P,M^* and D^* are constant, we arrive at:

$$\frac{d\log\mu^*}{d\log\sigma^*} \approx \frac{d\log e}{d\log\sigma^*} = -\frac{d\log P^*}{d\log\sigma^*} = \frac{\mathcal{L}^*_{\sigma^*} d\sigma^*}{(1-\rho^{\sigma^*})\, R^b - \mathcal{L}^*_{\mu^*}\mu^*} > 0.$$

□

B.5 Proof of Proposition 5

We again consider that any shock x follows a log AR(1) process:

$$\log(x_t) = (1-\rho^x)\log(x_{ss}) + \rho^x \cdot \log(x_{t-1}) + \Sigma^x \varepsilon_t^x.$$

We consider only shocks to dollar funding risk and the dollar funding scale and that $Var(\varepsilon_t^x) = 1$ for all shocks. Thus

$$Var(x_t) = \frac{(\Sigma^x)^2}{\left(1-(\rho^x)^2\right)}. \tag{B.31}$$

Consider a univariate linear regression of $\Delta\log e^*$ against $\Delta\log\mu^*$ where $\Delta x_t = x_t - x_{t-1}$. The regression coefficient is a function of two moments:

$$\gamma^{e^*}_{\mu^*} = \frac{CoV(\Delta\log e^*, \Delta\log\mu^*)}{Var(\Delta\log\mu^*)}. \tag{B.32}$$

Consider an endogenous variable Y_t in the model. An equilibrium will feature a function $Y(X_t)$ such that $Y_t = Y(X_t)$, where X_t is the exogenous state. Then, using a first-order Taylor expansion:

$$\log Y_t \approx \log Y_{ss} + \sum_{x\in X} \frac{Y_x(x_{ss})\cdot x_{ss}}{Y_{ss}} \frac{x_t - x_{ss}}{x_{ss}} \text{ for } x \in X.$$

Therefore, we have that:

$$\Delta\log Y_t \quad \approx \quad \sum_{x\in X} \frac{Y_x(x_{ss})\cdot x_{ss}}{Y_{ss}} \left(\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}}\right).$$

Near a steady state:

$$\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}} \approx \Delta\log(x_t) = \rho^x \cdot (\log(x_{t-1}) - \log(x_{ss})) + \Sigma^x \varepsilon_t^x.$$

Using this identity,

$$\Delta\log Y_t \quad \approx \textstyle\sum_{x\in X} \frac{Y_x(x_{ss})\cdot x_{ss}}{Y_{ss}} \quad (\rho^x \cdot (\log(x_{t-1}) - \log(x_{ss})) + \Sigma^x \varepsilon_t^x).$$

Then, for small shocks the log-deviation from steady-state is approximately the elasticity near steady state.

$$\frac{Y_x(x)\cdot x}{Y_{ss}}=\epsilon_x^Y.$$

Hence, we have that $\Delta\log e_t^*$ and $\Delta\log\mu_t^*$ follow:

$$\begin{aligned}\Delta\log e_t^* &= \epsilon_{\sigma^*}^{e^*}\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)... \\ &+\epsilon_{D^*}^{e^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \end{aligned} \tag{B.33}$$

Likewise, for the dollar liquidity ratio:

$$\begin{aligned}\Delta\log \mu_t^* &= \epsilon_{\sigma^*}^{\mu^*}\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)... \\ &+\epsilon_{D^*}^{\mu^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \end{aligned} \tag{B.34}$$

From, (C.34) variance of the change in the liquidity ratio is:

$$Var\left(\Delta\log\mu^*\right)=\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\left(\Sigma^{D^*}\right)^2\right)^2.$$

Substituting (C.31) into the equation above:

$$\begin{aligned} Var\left(\Delta\log\mu^*\right) &= \left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}+\left(\Sigma^{D^*}\right)^2\right) \\ &= \left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{\sigma^*}^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}. \end{aligned}$$

Provided that the shocks to σ^* and D^* are orthogonal, from (C.33) and (C.34), we have that following covariance between the change in the exchange rate and the change in the dollar liquidity ratio:

$$\begin{aligned} Cov(\Delta\log e^*,\Delta\log\mu^*) &\approx \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\Sigma^{D^*}\right)^2 \\ &= \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\Sigma^{D^*}\right)^2 \\ &= \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}. \end{aligned}$$

Thus, substituting the approximations to $Var\left(\Delta\log\mu^*\right)$ and $Cov(\Delta\log e^*,\Delta\log\mu^*)$ back into (C.32), we obtain that the univariate regression coefficient is approximately:

$$\begin{aligned}\beta_{\mu^*}^{e^*} &\approx \frac{\epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\cdot Var\left(\sigma^*\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}Var\left(D^*\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 Var\left(\sigma^*\right)+\left(\epsilon_{\sigma^*}^{D^*}\right)^2 Var\left(D^*\right)}.\\ &= \frac{\epsilon_{\sigma^*}^{e^*}}{\epsilon_{\sigma^*}^{\mu^*}}\cdot\boldsymbol{w}^{\sigma^*}+\frac{\epsilon_{D^*}^{e^*}}{\epsilon_{D^*}^{\mu^*}}\boldsymbol{w}^{D^*}.\end{aligned}$$

where:

$$\boldsymbol{w}^{\sigma^*}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{1-\left(\rho^{\sigma^*}\right)^2}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{D^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\Sigma^{D^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}.$$

and

$$\boldsymbol{w}^{D^*}=\frac{\left(\epsilon_{D^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{\sigma^*}^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\Sigma^{D^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}.$$

B.6 Proof of Proposition (3)

Proof. **Part i)** Totally differentiating (C.10) with respect to P^* yields

$$d\mu^*=-\mu^*\left(\frac{dP^*}{P^*}\right). \tag{B.35}$$

The dollar liquidity premium is

$$R^b-(1+i^{m,*})\frac{P^*}{\mathbb{E}[P^*(X')]}=\mathcal{L}^*(\mu^*(P^*),P^*) \tag{B.36}$$

Totally differentiating (C.36) with respect to P^* and $(1+i^{m,*})$, and using (C.35), we obtain

$$-R^{m,*}\left(\frac{dP^*}{P^*}\right)-\frac{P^*}{\mathbb{E}[P^*(X')]}d\left(1+i^{m,*}\right)=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP^*}{P^*}\right)+\mathcal{L}_{P^*}^*dP^* \tag{B.37}$$

where notice that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b=1/\beta$.

Using $\mathcal{L}_{P^*}^*=\frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b=R^{m,*}+\mathcal{L}^*$, and $\bar{R}^m=P^*(1+i^{m,*})/\mathbb{E}[P^*(X')]$, and replacing these equalities in (C.37), we obtain:

$$\frac{d\log P^*}{d\log\left(1+i^{m,*}\right)}=-\frac{\bar{R}^m}{R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(-1,0) \tag{B.38}$$

where the sign follows from Lemma C.4. The upper bound follows because $R^b > \bar{R}^m$.

Notice also that the euro bond premium remains constant, and so do P, μ and $\mathcal{L}$, as demonstrated in the proof of Proposition 1. This implies that $d\mathcal{L}^* = d\mathcal{DLP}$, $d\mathcal{L}^*_{\mu^*} = d\mathcal{DLP}_{\mu^*}$.

By the law of one price, we then have $\frac{d\log e^*}{d\log(1+i^{m,*})} = \frac{\bar{R}^m}{R^b - \mathcal{L}^*_{\mu}\mu^*}$ which implies an appreciation of the dollar.

Finally, we can rewrite (C.37) as

$$R^{m,*}\left(d\log e - d\log\left(1+i^{m,*}\right)\right) = d\mathcal{L}^* = d\mathcal{DLP} < 0 \tag{B.39}$$

where the sign follows from the bounds on (C.38).

Part ii). When the shock is permanent, expected inflation is constant. From (16), it follows that the increase in $1+i^{m,*}$ leads to a decrease in $\mathcal{L}^*$ and a reduction in $\mathcal{DLP}$. Total differentiation of (C.36) with respect to $1+i^{m,*}$ and μ^* yields

$$-\bar{R}^{m,*} d\log(1+i^{m,*}) = \mathcal{L}^*_{\mu}\mu^* d\log\mu^*, \tag{B.40}$$

and thus

$$\frac{d\log\mu^*}{d\log(1+i^{m,*})} = -\frac{\bar{R}^{m,*}}{\mathcal{L}^*_{\mu^*}\mu^*} > 0. \tag{B.41}$$

where the sign follows from Lemma C.4. Using that $d\log\mu^* = -d\log P^*$ when M^* and D^* are constant, we have from the law of one price that $\frac{d\log e^*}{d\log 1+i^{m,*}} = \frac{\bar{R}^{m,*}}{-\mathcal{L}^*_{\mu}\mu^*}$. Finally

$$d\mathcal{DLP} = -\bar{R}^{m,*} d\log(1+i^{m,*})$$

. □

Proofs of Proposition 4 (Open-Market Operations)

Preliminary Observations. We make two assumptions: first, deposits and securities are perfect substitutes, but the demand for the sum of deposits and securities is perfectly inelastic. Second, the supply of securities is fixed. Let $S^{H,*}$ indicate the household holding of dollar securities and $S^{G,*}$ the central bank's holdings of dollar securities. Thus, we have $S^{H,*} + S^{G,*} = S^*$ where S^* is a fixed supply of securities.

Consider a purchase of securities with reserves. The central banks' budget constraint in this case is modified to:

$$M^*_t + T^*_t + W^*_{t+1} + \left(1 + i^{d,*}_t\right) \cdot P^*_{t-1} S^{G,*}_{t-1} = P^*_t \cdot S^{G,*}_t + M^*_{t-1}(1 + i^{m,*}_t) + W^*_t(1 + i^{w,*}_t).$$

As in earlier proofs, we avoid time subscripts. Consider a small change in the holdings of central bank securities purchased with reserves. We obtain:

$$dM^* = S^{G,*}dP^* + P^* dS^{G,*} = P^* S^{G,*}\frac{dP^*}{P^*} + P^* S^{G,*}\frac{dS^{G,*}}{S^{G,*}}. \tag{B.42}$$

Assuming that the central bank has a balance sheet such that Υ of its liabilities are backed with securities,

$$\Upsilon^* = \frac{P^* S^{G,*}}{M^*},$$

we modify (C.42) to obtain:

$$\frac{dM^*}{M^*} = \frac{P^* S^{G,*}}{M}\left(\frac{dP^*}{P^*} + \frac{dS^{G,*}}{S^{G,*}}\right) = \Upsilon^*\left(\frac{dP^*}{P^*} + \frac{dS^{G,*}}{S^{G,*}}\right).$$

Thus, expressed in logs, this condition is:

$$d\log M^* = \Upsilon^*\left(d\log P^* + d\log S^{G,*}\right). \tag{B.43}$$

The equation accounts for the fact that the growth in the money supply needed to finance the open-market operation has consider the change in the price level.

Next, since households are inelastic regarding the some of securities and deposits, it must be that $dD^* = -dS^{H,*}$. Since the supply of the security is fixed $-dS^{H,*} = dS^{G,*}$. Hence,

$$dD^* = dS^{G,*}.$$

Next, we express the change in the liquidity ratio in its differential form:

$$\begin{aligned} d\mu^* &= \mu^*\left(\frac{dM^*}{M^*} - \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*}\right)\right), \\ &= \mu^*\left(\frac{dM^*}{M^*} - \left(\frac{dP^*}{P^*} + \mu^*\Upsilon^*\frac{dS^{G,*}}{S^{G,*}}\right)\right), \end{aligned}$$

where the second line applies the definitions of μ^* and Υ^*.

In log terms, the last equation is:

$$d\log\mu^* = \left(d\log M^* - \left(d\log P^* + \Upsilon^*\mu^*\cdot d\log S^{G,*}\right)\right).$$

Substituting (C.43) we obtain:

$$\begin{aligned} d\log\mu^* &= \left(\Upsilon^*\left(d\log P^* + d\log S^{G,*}\right) - d\log P^* - \Upsilon^*\mu^*\cdot d\log S^{G,*}\right) \\ &= -\left(1-\Upsilon^*\right)d\log P^* + \Upsilon^*\left(1-\mu^*\right)\cdot d\log S^{G,*}. \end{aligned} \tag{B.44}$$

Proof. **Item (i).**

We now derive the main results, following the earlier proofs. Totally differentiating the liquidity premium with respect to μ^* and P^*, we obtain:

$$\bar{R}^{m,*}d\log P^* + \mathcal{L}^* d\log P^* + \mathcal{L}^*_{\mu^*}\mu^* d\log\mu^* = 0. \tag{B.45}$$

Substituting (C.44) and collecting terms we obtain:

$$\left(\bar{R}^{m,*} + \mathcal{L}^* - \left(1-\Upsilon^*\right)\mathcal{L}\mathcal{P}^*_{\mu^*}\mu^*\right)d\log P^* + \mathcal{L}^*_{\mu^*}\mu^*\Upsilon^*\left(1-\mu^*\right)\cdot d\log S^{G,*} = 0. \tag{B.46}$$

Thus, we obtain:

$$\frac{d\log P^*}{d\log S^{G,*}} = \frac{-\mathcal{LP}^*_{\mu^*}\mu^*\left(1-\mu^*\right)\Upsilon^*}{R^b - \left(1-\Upsilon^*\right)\mathcal{LP}^*_{\mu^*}\mu^*} > 0.$$

Substituting this expression in (C.44) we obtain:

$$\frac{d\log \mu^*}{d\log S^{G,*}} = \frac{R^b\Upsilon^*\left(1-\mu^*\right)}{R^b - \left(1-\Upsilon^*\right)\mathcal{LP}^*_{\mu^*}\mu^*} > 0.$$

Finally, by the law of one price:

$$d\log e = -d\log P^* = \frac{\mathcal{L}^*_{\mu^*}\mu^*\left(1-\mu^*\right)\Upsilon^*}{R^b - \left(1-\Upsilon^*\right)\mathcal{L}^*_{\mu^*}\mu^*} < 0.$$

Finally, the excess-bond premium and the dollar liquidity premium is:

$$d\mathcal{L}^* = d\mathcal{DLP}^* = -R^{*,m}d\log P^* < 0.$$

Item (ii).

If the shock is permanent expected inflation does not change. Since nominal rates are fixed, we have that the dollar liquidity ratio must remain constant:

$$d\log \mu^* = 0. \tag{B.47}$$

Moreover, $d\mathcal{BP}^* = d\mathcal{DLP}^* = 0$. From (C.44)

$$\frac{d\log P^*}{d\log S^{G,*}} = \frac{\Upsilon^*}{\left(1-\Upsilon^*\right)}\left(1-\mu^*\right) > 0\cdot$$

By the law of one price then:

$$\frac{d\log e}{d\log S^{G,*}} = -\frac{d\log P^*}{d\log S^{G,*}} = -\frac{\Upsilon^*}{\left(1-\Upsilon^*\right)}\left(1-\mu^*\right).$$

□

C Microfoundations - Deposit and Loan Schedules

C.1 Preliminaries

Here we provide some intermediate results that we use to prove the propositions.

Recall that the liquidity ratio is denoted by $\mu \equiv m/d$ and $\theta = S^{-}/S^{+}$ where $S^{-} = -\int \min\{s,0\}\, d\Phi(\omega)$, $S^{+} = \int \max\{s,0\}\, d\Phi(\omega)$ and $s = m + \omega d$. Then,

$$\begin{aligned}\theta &= -\frac{\int_{\{s<0\}} s \cdot d\Phi(\omega;\sigma)}{\int_{\{s>0\}} s \cdot d\Phi(\omega;\sigma)}, \\ &= -\frac{m\Phi(\{s<0\};\sigma) + d\int_{\{s<0\}} \omega \cdot d\Phi(\omega;\sigma)}{m\left(1-\Phi(\{s>0\};\sigma)\right) + d\int_{\{s\geq 0\}} \omega \cdot d\Phi(\omega;\sigma)}.\end{aligned}$$

Note that $s<0$ occurs when $\omega < -\mu$. Therefore, we express the interbank market tightness as:

$$\theta = -\frac{\int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}. \tag{C.1}$$

With abuse of notation, define $\theta(\mu,\sigma)$ as the function that maps μ and σ into a value of θ (thus, in equilibrium, $\theta = \theta(\mu,\sigma)$). We have the following Lemma:

Lemma C.1. *Interbank market tightness is decreasing in the liquidity ratio. That is,* $\frac{d\theta}{d\mu} < 0$. *Moreover,* $\theta \in [0,1]$.

Proof. From (C.1), using Leibniz rule, we obtain

$$\frac{d\theta}{d\mu} = \theta\left(\frac{\Phi(-\mu;\sigma)}{\int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma)} - \frac{1-\Phi(-\mu;\sigma)}{\int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma)}\right). \tag{C.2}$$

By definition of conditional expectation:

$$\mathbb{E}[\mu+\omega|\omega<-\mu] = \int_{-\infty}^{-\mu} (\mu+\omega) \cdot d\Phi(\omega;\sigma) / \Phi(-\mu;\sigma),$$

and

$$\mathbb{E}[\mu+\omega|\omega>-\mu] = \int_{-\mu}^{\infty} (\mu+\omega) \cdot d\Phi(\omega;\sigma) / \left(1-\Phi(-\mu;\sigma)\right).$$

Replacing these definitions into (C.2), we obtain:

$$\frac{d\theta}{d\mu} = \theta \cdot \left(\frac{1}{\mathbb{E}[\mu+\omega|\omega<-\mu]} - \frac{1}{\mathbb{E}[\mu+\omega|\omega>-\mu]}\right) < 0,$$

where the inequality follows because $\mathbb{E}[\mu+\omega|\omega<-\mu] < 0$ and $\mathbb{E}[\mu+\omega|\omega>-\mu] > 0$.

Finally, the bounds on θ follow because $\lim_{\mu\to\infty} \theta = 0$ and $\theta = 1$ if $\mu = 0$. □

Next, we obtain the derivative of interbank market tightness with respect to σ.

Lemma C.2. *Under Assumption 1, we have that* $\frac{\partial\theta}{\partial\sigma} > 0$.

Proof. Passing the differential operator inside the integrals in the numerators, we have that:

$$\begin{aligned}\frac{\partial\theta}{\partial\sigma} &= \theta\cdot\left(\frac{\int_{-\infty}^{-\mu}(\mu+\omega)\,\phi_\sigma d\omega}{\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)} - \frac{\int_{-\mu}^{\infty}(\mu+\omega)\,\phi_\sigma d\omega}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right)\\ &= \theta\cdot\left(\frac{\partial}{\partial\sigma}\left[\log\left(\frac{\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right)\right]\right).\end{aligned}$$

Since the withdrawal shock is zero mean,

$$\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma) + \int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma) = \mu.$$

Therefore, identity this condition into the derivative just above we obtain:

$$\frac{\partial\theta}{\partial\sigma} = \log\left(\frac{\mu - \int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}{\int_{-\mu}^{\infty}(\mu+\omega)\cdot d\Phi(\omega;\sigma)}\right).$$

Therefore, $\frac{\partial\theta}{\partial\sigma} > 0$ holds if and only if:

$$\frac{\partial}{\partial\sigma}\left[\int_{-\infty}^{-\mu}(\mu+\omega)\cdot d\Phi(\omega;\sigma)\right] < 0.$$

Using the integration by parts formula:

$$\begin{aligned}\int_{-\infty}^{-\mu}(\mu+\omega)\,\phi_\sigma(\omega;\sigma)\,d\omega &= (\mu+\omega)\,\Phi_\sigma(\omega;\sigma)\,|_{-\infty}^{-\mu} - \int_{-\infty}^{-\mu}\Phi_\sigma(\omega;\sigma)\,d\omega\\ &= -\int_{-\infty}^{-\mu}\Phi_\sigma(\omega;\sigma)\,d\omega < 0\end{aligned}$$

where the last equality follows from $\lim_{\omega\to-\infty}((\mu+\omega))\,\Phi_\sigma(\omega;\sigma) = \frac{\partial}{\partial\sigma}\left[\lim_{\omega\to-\infty}((\mu+\omega))\,\Phi(\omega;\sigma)\right] = 0$ and the strict inequality follows from Assumption 1. We conclude that, $\frac{\partial\theta}{\partial\sigma} > 0$. □

We will also use the results from the following Lemma.

Lemma C.3. *The liquidity coefficients have the following derivatives:*

$$\frac{\partial\chi^+}{\partial\mu} = \frac{\partial\chi^+}{\partial\theta}\frac{\partial\theta}{\partial\mu} < 0 \quad \textit{and} \quad \frac{\partial\chi^-}{\partial\mu} = \frac{\partial\chi^-}{\partial\theta}\frac{\partial\theta}{\partial\mu} < 0, \tag{C.3}$$

$$\frac{\partial\chi^+}{\partial\sigma} = \frac{\partial\chi^+}{\partial\theta}\frac{\partial\theta}{\partial\sigma} > 0 \quad \textit{and} \quad \frac{\partial\chi^-}{\partial\mu} = \frac{\partial\chi^-}{\partial\theta}\frac{\partial\theta}{\partial\sigma} > 0, \tag{C.4}$$

$$\frac{\partial \bar{\chi}^+}{\partial P_t} = \frac{\bar{\chi}^+}{P_t} \quad \textit{and} \quad \frac{\partial \bar{\chi}^-(\theta)}{\partial P_t} = \frac{\bar{\chi}^-}{P_t}. \tag{C.5}$$

Proof. Notice first that $\frac{\partial \chi^+}{\partial \theta} > 0$ and $\frac{\partial \chi^-}{\partial \theta} > 0$ is an immediate result from their definitions in equations (A.2). Applying Lemmas C.1 and C.2, we obtain respectively (C.3) and (C.4).

In addition, we can express (A.2) as

$$\bar{\chi}^+ = \frac{P_t}{P_{t+1}} \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right), \; \bar{\chi}^- = \frac{P_t}{P_{t+1}} \left(i^w - i^m\right) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right) \tag{C.6}$$

Equation (C.5) follows immediately. □

It is useful to define $\mathcal{L}(\mu, \sigma, P)$ to be the bond liquidity premium as a function of the liquidity ratio, the index σ and the current price level. That is,

$$\mathcal{L}(\mu, \sigma, P) = (1 - \Phi(-\mu, \sigma)) \cdot \bar{\chi}^+ \left(\theta(\mu, \sigma), P\right) + \Phi(-\mu, \sigma) \cdot \bar{\chi}^- \left(\theta(\mu, \sigma), P\right) \tag{C.7}$$

In equilibrium $\mathcal{L}(\mu, \sigma, P) = R^b - R^m$. We have the following result.

Lemma C.4. *The liquidity bond premium is decreasing in the liquidity ratio and increasing in volatility. That is, $\mathcal{L}_\mu < 0$ and $\mathcal{L}_\sigma > 0$. In addition, $\mathcal{L}_P = -\mathcal{L}/P$.*

Proof. From (C.7), differentiating $\mathcal{L}$ with respect to μ:

$$\mathcal{L}_\mu = \left[(1 - \Phi(-\mu, \sigma)) \cdot \chi^+_\theta + \Phi(-\mu, \sigma) \cdot \chi^-_\theta\right] - \left(\bar{\chi}^- - \bar{\chi}^+\right) \phi\left(-\mu, \sigma\right). \tag{C.8}$$

Using that $\frac{\partial \theta}{\partial \mu} < 0$ from Lemma C.1 and that $\bar{\chi}^- > \bar{\chi}^+$, we arrive at $\mathcal{L}_\mu < 0$.

From (C.7), differentiating $\mathcal{L}$ with respect to σ yields:

$$\mathcal{LP}_\sigma = \frac{\partial \theta}{\partial \sigma} \left[(1 - \Phi(-\mu, \sigma)) \cdot \chi^+_\theta + \Phi(-\mu, \sigma) \cdot \chi^-_\theta\right] + \left(\bar{\chi}^- - \bar{\chi}^+\right) \Phi_\sigma\left(-\mu, \sigma\right). \tag{C.9}$$

Using that $\frac{\partial \theta}{\partial \sigma} > 0$ from Lemma C.2 and that $\bar{\chi}^- > \bar{\chi}^+$, we conclude that $\mathcal{L}_\sigma > 0$. Finally, the expression for $\mathcal{L}_P$ follows directly from differentiating $\mathcal{L}$ with respect to P in (C.5). □

We now proceed with the proofs and use that these properties apply for both euros and dollars.

C.2 Proof of Proposition 1

Proof. **Part i).** By definition, the liquidity ratio μ^* is given by

$$\mu^*(P^*, D^*) = \frac{M^*/P^*}{D^*} \tag{C.10}$$

where we made explicit the dependence of μ^* on (P^*, D^*). Using that M^* is exogenously given, totally differentiating (C.10) yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right). \tag{C.11}$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, D^*), P^*). \tag{C.12}$$

Totally differentiating (C.12) with respect to P^* and D^*, and using (C.11), we obtain:

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) = -\mathcal{L}^*_{\mu^*} \left[\mu \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right) \right] + \mathcal{L}^*_P dP^* \tag{C.13}$$

where $\mathbb{E}[P^*(X')]$ remains constant because the shock is i.i.d. and the loan rate is constant at $R^b = 1/\beta$.

Using $\mathcal{L}^*_{P^*} = \frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b = R^{m,*} + \mathcal{L}^*$ and replacing in (C.13), we arrive to

$$\frac{d \log P^*}{d \log D^*} = \frac{\mathcal{L}^*_{\mu^*} \mu^*}{R^b - \mathcal{L}^*_{\mu^*} \mu^*} \in (-1, 0) \,. \tag{C.14}$$

The bounds follows immediately because $\mathcal{L}^*_\mu < 0$ as established in Lemma C.4 and from $R^b > 0$.

Notice also that the euro bond premium remains constant. To see this, we can replace $\mu = (M/P)/D$ in (15) and use (C.1) to obtain

$$R^b - (1 + i^m) \frac{P}{\mathbb{E}[P(X')]} = \left(1 - \Phi \left(-\frac{M/P}{D} \right) \right) \bar{\chi}^+(\theta((M/P)/D, \sigma)) + \\ \Phi \left(-\frac{M/P}{D} \right) \bar{\chi}^-(\theta((M/P)/D, \sigma)). \tag{C.15}$$

From (C.15), it follows that P must be constant and thus μ and $\mathcal{L}$ are also constant. As a result, $d\mathcal{L}^* = d\mathcal{DLP}$, $d\mathcal{L}^*_{\mu^*} = d\mathcal{DLP}_{\mu^*}$.

By the law of one price and using that P remains constant, we then have $\frac{d \log e}{d \log D^*} = -\frac{\mathcal{L}^*_\mu \mu^*}{R^b - \mathcal{L}^*_\mu \mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (C.13) as $\bar{R}^{m,*} (d \log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation remains constant. Moreover, given that nominal policy rates and expected inflation are constant, we have from (16) that $\mathcal{L}^*$ is constant. Hence, $\mathcal{DLP}$ is constant. Furthermore, the fact that $\mathcal{L}^*$ is constant, implies that μ must also be constant. Thus, using that (C.11) and that M^* is constant, we have from the law of one price that:

$$\frac{d\log e}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1.$$

$\square$

C.3 Proof of Proposition 2

Proof. **Part i).** Totally differentiating (C.10) with respect to P^* yields

$$d\mu^* = -\mu^*\left(\frac{dP^*}{P^*}\right). \tag{C.16}$$

The dollar liquidity premium is

$$R^b - (1+i^{m,*})\frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*,\sigma^*),P^*). \tag{C.17}$$

Totally differentiating (C.17) with respect to P^* and σ^* and using (C.16) yields:

$$-R^{m,*}\left(\frac{dP^*}{P^*}\right) = -\mathcal{L}^*_\mu\left[\mu\left(\frac{dP^*}{P^*}\right)\right] + \mathcal{L}^*_{\sigma^*}d\sigma^* + \mathcal{L}^*_P dP^* \tag{C.18}$$

where we used that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b = 1/\beta$.

Using $\mathcal{L}^*_{P^*} = \frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b = R^{m,*} + \mathcal{L}^*$, and replacing in (C.16) , we obtain

$$\frac{d\log P^*}{d\log\sigma^*} = -\frac{\mathcal{L}^*_{\sigma^*}\sigma}{R^b - \mathcal{L}^*_\mu\mu^*} < 0 \tag{C.19}$$

where the sign follows from Lemma C.4. Notice also that the euro bond premium remains constant, and so do P, μ and $\mathcal{L}$, as demonstrated in the proof of Proposition 1.

By the law of one price, and using that P remains constant, we then have $\frac{d\log e}{d\log D^*} = \frac{\mathcal{L}^*_\sigma\sigma^*}{R^b - \mathcal{L}^*_\mu\mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (C.18) as $\bar{R}^{m,*}(d\log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation is constant. Given that nominal policy rates are constant, $\mathcal{L}^*$ and $\mathcal{DLP}$ are constant. Thus,

$$\mathcal{L}^*_{\mu^*}d\mu^* + \mathcal{L}^*_{\sigma^*}d\sigma^* = 0 \tag{C.20}$$

and so

$$\frac{d\log\mu^*}{d\log\sigma^*} = -\frac{\mathcal{L}^*_{\sigma^*}\sigma^*}{\mathcal{L}^*_{\mu^*}\mu^*} > 0 \tag{C.21}$$

where the sign follows from $\mathcal{L}^*_{\mu^*} < 0$ and $\mathcal{L}^*_{\sigma^*} > 0$ from Lemma C.4. Using that $d\log\mu^* = -d\log P^*$, from the law of one price, $\frac{d\log e^*}{d\log\sigma^*} = \frac{d\log\mu^*}{d\log\sigma^*}$. $\square$

C.4 Approximation to Mean Reverting Shocks

Proof. We now derive approximate analogues to propositions 1 and 2 for cases where shocks are mean reverting. In particular, shocks follow a log AR(1) process:

$$\log(x_t) = (1-\rho^x)\log(x_{ss}) + \rho^x \cdot \log(x_{t-1}) + \Sigma^x \varepsilon_t^x. \tag{C.22}$$

We have the following result. We use x_{ss} to refer to the deterministic steady-state value of any variable x. The proof extends the results in Propositions 1 and 2. We first show this intermediate result. In the model, prices are a function of the aggregate state, X. Thus, an equilibrium will feature a function $P^*(X_t)$ such that $P_t^* = P^*(X_t)$. Then, near the steady state, using a Taylor expansion of first-order with respect to the variable x. We have that:

$$\log P_t^* \approx \log P_{ss}^* + \frac{P_x^*(x_{ss})\, x_{ss}}{P_{ss}^*}\frac{x_t - x_{ss}}{x_{ss}}.$$

Thus, we have that for small deviations around the steady state:

$$d\log P_t^* \approx \frac{P_x^*(x_{ss})\, x_{ss}}{P_{ss}^*} d\log x_t. \tag{C.23}$$

Shifting this condition forward:

$$d\log P_{t+1}^* \approx \frac{P_x^*(x_{ss})\, x_{ss}}{P_{ss}^*} d\log x_{t+1}$$

Taking expectations:

$$\mathbb{E}\left[d\log P_{t+1}^*\right] \approx \frac{P_x^*(x_{ss})\, x_{ss}}{P_{ss}^*}\rho^x d\log x_t. \tag{C.24}$$

Dividing the left-hand side of (C.24) by (C.23),

$$\frac{\mathbb{E}\left[d\log P_{t+1}^*\right]}{d\log P_t^*} = \rho^x d\log x_t. \tag{C.25}$$

Next, we proof the main items of the propositions. The proof uses that for either currency:

$$\frac{\partial \bar{\chi}^+}{\partial P_{t+1}} = -\frac{\bar{\chi}^+}{\mathbb{E}\left[P_{t+1}\right]}, \text{ and } \frac{\partial \bar{\chi}^-(\theta)}{\partial P_{t+1}} = -\frac{\bar{\chi}^-}{\mathbb{E}\left[P_{t+1}\right]}. \tag{C.26}$$

Hence:

$$\mathcal{L}^*_{P^*_{t+1}} = -\frac{\mathcal{L}^*}{P^*_{t+1}}$$

Recall that the dollar liquidity premium can be expressed as

$$R^b - (1+i^{m,*})\frac{P_t^*}{\mathbb{E}\left[P_{t+1}^*\right]} = \mathcal{L}^*(\mu^*(P^*, D^*), P_t^*, P_{t+1}^*), \tag{C.27}$$

where we now make explicit that $\mathcal{L}^*$ depends on both P_t and P_{t+1}.

Part (i). We present here the proof for item (i). Totally differentiating (C.27) with respect to P_t, P_{t+1}, and D^* and using (C.11) near the steady state, we obtain

$$-R^{m,*}\left(\frac{dP_t^*}{P_t^*}\right)+R^{m,*}\frac{\mathbb{E}\left[dP_{t+1}^*\right]}{\mathbb{E}\left[P_{t+1}^*\right]}=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right)+\mathcal{L}_{P_t^*}^*dP_t^*-\mathcal{L}_{P_{t+1}^*}^*\mathbb{E}\left[dP_{t+1}^*\right]. \tag{C.28}$$

Then, collecting terms:

$$-\left(R^{m,*}+\mathcal{L}^*\right)\left(1-\frac{\mathbb{E}\left[d\log P_{t+1}^*\right]}{d\log P_t^*}\right)d\log P_t^*=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right). \tag{C.29}$$

Substituting $R^b=R^{m,*}+\mathcal{L}^*$ and (C.25), we obtain:

$$R^b\left(1-\rho^{D^*}\right)d\log P_t^*\approx\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}+\frac{dD_t^*}{D_t^*}\right).$$

Thus, we obtain

$$\frac{d\log P^*}{d\log D^*}\approx\frac{\mathcal{L}_{\mu^*}^*\mu^*}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}<0.$$

Then, it follows from the law of one price and the differential form of μ that

$$\epsilon_{D^*}^e\equiv\frac{d\log e}{d\log D^*}\approx-\frac{\mathcal{L}_{\mu^*}^*\mu^*}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(0,1),$$

and

$$\epsilon_{\mu^*}^e\equiv\frac{d\log\mu}{d\log D^*}\approx-\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(-1,0).$$

Part (ii). We present here the proof for item (ii). It follows the same steps as in Part (i): We totally differentiate (C.27) with respect to P_t, P_{t+1}, and σ^* and using (C.11) for the case where $dD^*=0$. We obtain:

$$-R^{m,*}\left(\frac{dP_t^*}{P_t^*}\right)+R^{m,*}\frac{\mathbb{E}\left[dP_{t+1}^*\right]}{\mathbb{E}\left[P_{t+1}^*\right]}=\mathcal{L}_{\sigma^*}^*d\sigma^*-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP_t^*}{P_t^*}\right)+\mathcal{L}_{P_t^*}^*dP_t^*-\mathcal{L}_{P_{t+1}^*}^*\mathbb{E}\left[dP_{t+1}^*\right]. \tag{C.30}$$

Collecting terms and using the same identities that we use to derive C.29, we arrive at:

$$\left(R^b\left(1-\rho^{\sigma^*}\right)-\mathcal{L}_{\mu^*}^*\mu^*\right)d\log P_t^*\approx-\mathcal{L}_{\sigma^*}^*d\sigma^*.$$

Therefore, we obtain:

$$\frac{d\log P^*}{d\log\sigma^*}\approx\frac{-\mathcal{L}_{\sigma^*}^*d\sigma^*}{(1-\rho^{\sigma^*})\,R^b-\mathcal{L}_{\mu^*}^*\mu^*}<0.$$

Then using that $\mu^*=M^*/\left(P^*D^*\right)$ and that $e=P/P^*$ and that P,M^* and D^* are constant, we arrive at:

$$\frac{d\log\mu^*}{d\log\sigma^*} \approx \frac{d\log e}{d\log\sigma^*} = -\frac{d\log P^*}{d\log\sigma^*} = \frac{\mathcal{L}^*_{\sigma^*} d\sigma^*}{\left(1-\rho^{\sigma^*}\right)R^b - \mathcal{L}^*_{\mu^*}\mu^*} > 0.$$

$\square$

C.5 Proof of Proposition 5

We again consider that any shock x follows a log AR(1) process:

$$\log\left(x_t\right) = \left(1-\rho^x\right)\log\left(x_{ss}\right) + \rho^x \cdot \log\left(x_{t-1}\right) + \Sigma^x \varepsilon^x_t.$$

We consider only shocks to dollar funding risk and the dollar funding scale and that $Var\left(\varepsilon^x_t\right) = 1$ for all shocks. Thus

$$Var\left(x_t\right) = \frac{\left(\Sigma^x\right)^2}{\left(1-\left(\rho^x\right)^2\right)}. \tag{C.31}$$

Consider a univariate linear regression of $\Delta\log e^*$ against $\Delta\log\mu^*$ where $\Delta x_t = x_t - x_{t-1}$. The regression coefficient is a function of two moments:

$$\gamma^{e^*}_{\mu^*} = \frac{CoV(\Delta\log e^*, \Delta\log\mu^*)}{Var\left(\Delta\log\mu^*\right)}. \tag{C.32}$$

Consider an endogenous variable Y_t in the model. An equilibrium will feature a function $Y\left(X_t\right)$ such that $Y_t = Y\left(X_t\right)$, where X_t is the exogenous state. Then, using a first-order Taylor expansion:

$$\log Y_t \approx \log Y_{ss} + \sum_{x\in X}\frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}}\frac{x_t - x_{ss}}{x_{ss}} \text{ for } x \in X.$$

Therefore, we have that:

$$\Delta\log Y_t \quad \approx \quad \sum_{x\in X}\frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}}\left(\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}}\right).$$

Near a steady state:

$$\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}} \approx \Delta\log\left(x_t\right) = \rho^x \cdot \left(\log\left(x_{t-1}\right) - \log\left(x_{ss}\right)\right) + \Sigma^x\varepsilon^x_t.$$

Using this identity,

$$\Delta\log Y_t \quad \approx \textstyle\sum_{x\in X}\frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}} \quad \left(\rho^x \cdot \left(\log\left(x_{t-1}\right) - \log\left(x_{ss}\right)\right) + \Sigma^x\varepsilon^x_t\right).$$

Then, for small shocks the log-deviation from steady-state is approximately the elasticity near steady state.

$$\frac{Y_x(x)\cdot x}{Y_{ss}} = \epsilon_x^Y.$$

Hence, we have that $\Delta \log e_t^*$ and $\Delta \log \mu_t^*$ follow:

$$\begin{aligned}\Delta \log e_t^* &= \epsilon_{\sigma^*}^{e^*}\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)\ldots \\ &+\epsilon_{D^*}^{e^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \qquad \text{(C.33)}\end{aligned}$$

Likewise, for the dollar liquidity ratio:

$$\begin{aligned}\Delta \log \mu_t^* &= \epsilon_{\sigma^*}^{\mu^*}\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)\ldots \\ &+\epsilon_{D^*}^{\mu^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \qquad \text{(C.34)}\end{aligned}$$

From, (C.34) variance of the change in the liquidity ratio is:

$$Var\left(\Delta\log\mu^*\right)=\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\left(\Sigma^{D^*}\right)^2\right)^2.$$

Substituting (C.31) into the equation above:

$$\begin{aligned}Var\left(\Delta\log\mu^*\right) &= \left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}+\left(\Sigma^{D^*}\right)^2\right) \\ &= \left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{\sigma^*}^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}.\end{aligned}$$

Provided that the shocks to σ^* and D^* are orthogonal, from (C.33) and (C.34), we have that following covariance between the change in the exchange rate and the change in the dollar liquidity ratio:

$$\begin{aligned}Cov(\Delta\log e^*,\Delta\log\mu^*) &\approx \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\Sigma^{D^*}\right)^2 \\ &= \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\Sigma^{D^*}\right)^2 \\ &= \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}.\end{aligned}$$

Thus, substituting the approximations to $Var\left(\Delta\log\mu^*\right)$ and $Cov(\Delta\log e^*,\Delta\log\mu^*)$ back into (C.32), we obtain that the univariate regression coefficient is approximately:

$$\begin{aligned}\beta_{\mu^*}^{e^*} &\approx \frac{\epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\cdot Var\left(\sigma^*\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}Var\left(D^*\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 Var\left(\sigma^*\right)+\left(\epsilon_{\sigma^*}^{D^*}\right)^2 Var\left(D^*\right)}.\\ &= \frac{\epsilon_{\sigma^*}^{e^*}}{\epsilon_{\sigma^*}^{\mu^*}}\cdot\boldsymbol{w}^{\sigma^*}+\frac{\epsilon_{D^*}^{e^*}}{\epsilon_{D^*}^{\mu^*}}\boldsymbol{w}^{D^*}.\end{aligned}$$

where:

$$\boldsymbol{w}^{\sigma^*}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{1-\left(\rho^{\sigma^*}\right)^2}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{D^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\Sigma^{D^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}.$$

and

$$\boldsymbol{w}^{D^*}=\frac{\left(\epsilon_{D^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\left(\epsilon_{\sigma^*}^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{\left(1-\left(\rho^{D^*}\right)^2\right)}}=\frac{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\Sigma^{D^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}.$$

C.6 Proof of Proposition (3)

Proof. **Part i)** Totally differentiating (C.10) with respect to P^* yields

$$d\mu^*=-\mu^*\left(\frac{dP^*}{P^*}\right). \tag{C.35}$$

The dollar liquidity premium is

$$R^b-(1+i^{m,*})\frac{P^*}{\mathbb{E}[P^*(X')]}=\mathcal{L}^*(\mu^*(P^*),P^*) \tag{C.36}$$

Totally differentiating (C.36) with respect to P^* and $(1+i^{m,*})$, and using (C.35), we obtain

$$-R^{m,*}\left(\frac{dP^*}{P^*}\right)-\frac{P^*}{\mathbb{E}[P^*(X')]}d\left(1+i^{m,*}\right)=-\mathcal{L}_{\mu^*}^*\mu^*\left(\frac{dP^*}{P^*}\right)+\mathcal{L}_{P^*}^*dP^* \tag{C.37}$$

where notice that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b=1/\beta$.

Using $\mathcal{L}_{P^*}^*=\frac{\mathcal{L}^*}{P^*}$ from Lemma C.4, $R^b=R^{m,*}+\mathcal{L}^*$, and $\bar{R}^m=P^*(1+i^{m,*})/\mathbb{E}[P^*(X')]$, and replacing these equalities in (C.37), we obtain:

$$\frac{d\log P^*}{d\log\left(1+i^{m,*}\right)}=-\frac{\bar{R}^m}{R^b-\mathcal{L}_{\mu^*}^*\mu^*}\in(-1,0) \tag{C.38}$$

where the sign follows from Lemma C.4. The upper bound follows because $R^b > \bar{R}^m$.

Notice also that the euro bond premium remains constant, and so do P, μ and $\mathcal{L}$, as demonstrated in the proof of Proposition 1. This implies that $d\mathcal{L}^* = d\mathcal{DLP}$, $d\mathcal{L}^*_{\mu^*} = d\mathcal{DLP}_{\mu^*}$.

By the law of one price, we then have $\frac{d\log e^*}{d\log(1+i^{m,*})} = \frac{\bar{R}^m}{R^b - \mathcal{L}^*_\mu \mu^*}$ which implies an appreciation of the dollar.

Finally, we can rewrite (C.37) as

$$R^{m,*}\left(d\log e - d\log\left(1+i^{m,*}\right)\right) = d\mathcal{L}^* = d\mathcal{DLP} < 0 \tag{C.39}$$

where the sign follows from the bounds on (C.38).

Part ii). When the shock is permanent, expected inflation is constant. From (16), it follows that the increase in $1+i^{m,*}$ leads to a decrease in $\mathcal{L}^*$ and a reduction in $\mathcal{DLP}$. Total differentiation of (C.36) with respect to $1+i^{m,*}$ and μ^* yields

$$-\bar{R}^{m,*} d\log(1+i^{m,*}) = \mathcal{L}^*_\mu \mu^* d\log\mu^*, \tag{C.40}$$

and thus

$$\frac{d\log\mu^*}{d\log(1+i^{m,*})} = -\frac{\bar{R}^{m,*}}{\mathcal{L}^*_{\mu^*}\mu^*} > 0. \tag{C.41}$$

where the sign follows from Lemma C.4. Using that $d\log\mu^* = -d\log P^*$ when M^* and D^* are constant, we have from the law of one price that $\frac{d\log e^*}{d\log 1+i^{m,*}} = \frac{\bar{R}^{m,*}}{-\mathcal{L}^*_\mu \mu^*}$. Finally

$$d\mathcal{DLP} = -\bar{R}^{m,*} d\log(1+i^{m,*})$$

. $\square$

Proofs of Proposition 4 (Open-Market Operations)

Preliminary Observations. We make two assumptions: first, deposits and securities are perfect substitutes, but the demand for the sum of deposits and securities is perfectly inelastic. Second, the supply of securities is fixed. Let $S^{H,*}$ indicate the household holding of dollar securities and $S^{G,*}$ the central bank's holdings of dollar securities. Thus, we have $S^{H,*} + S^{G,*} = S^*$ where S^* is a fixed supply of securities.

Consider a purchase of securities with reserves. The central banks' budget constraint in this case is modified to:

$$M^*_t + T^*_t + W^*_{t+1} + \left(1 + i^{d,*}_t\right) \cdot P^*_{t-1} S^{G,*}_{t-1} = P^*_t \cdot S^{G,*}_t + M^*_{t-1}(1 + i^{m,*}_t) + W^*_t(1 + i^{w,*}_t).$$

As in earlier proofs, we avoid time subscripts. Consider a small change in the holdings of central bank securities purchased with reserves. We obtain:

$$dM^* = S^{G,*} dP^* + P^* dS^{G,*} = P^* S^{G,*} \frac{dP^*}{P^*} + P^* S^{G,*} \frac{dS^{G,*}}{S^{G,*}}. \tag{C.42}$$

Assuming that the central bank has a balance sheet such that Υ of its liabilities are backed with securities,

$$\Upsilon^{*}=\frac{P^{*}S^{G,*}}{M^{*}},$$

we modify (C.42) to obtain:

$$\frac{dM^{*}}{M^{*}}=\frac{P^{*}S^{G,*}}{M}\left(\frac{dP^{*}}{P^{*}}+\frac{dS^{G,*}}{S^{G,*}}\right)=\Upsilon^{*}\left(\frac{dP^{*}}{P^{*}}+\frac{dS^{G,*}}{S^{G,*}}\right).$$

Thus, expressed in logs, this condition is:

$$d\log M^{*}=\Upsilon^{*}\left(d\log P^{*}+d\log S^{G,*}\right). \tag{C.43}$$

The equation accounts for the fact that the growth in the money supply needed to finance the open-market operation has consider the change in the price level.

Next, since households are inelastic regarding the some of securities and deposits, it must be that $dD^{*}=-dS^{H,*}$. Since the supply of the security is fixed $-dS^{H,*}=dS^{G,*}$. Hence,

$$dD^{*}=dS^{G,*}.$$

Next, we express the change in the liquidity ratio in its differential form:

$$\begin{aligned}
d\mu^{*} &= \mu^{*}\left(\frac{dM^{*}}{M^{*}}-\left(\frac{dP^{*}}{P^{*}}+\frac{dD^{*}}{D^{*}}\right)\right),\\
&= \mu^{*}\left(\frac{dM^{*}}{M^{*}}-\left(\frac{dP^{*}}{P^{*}}+\mu^{*}\Upsilon^{*}\frac{dS^{G,*}}{S^{G,*}}\right)\right),
\end{aligned}$$

where the second line applies the definitions of μ^{*} and Υ^{*}.

In log terms, the last equation is:

$$d\log\mu^{*}=\left(d\log M^{*}-\left(d\log P^{*}+\Upsilon^{*}\mu^{*}\cdot d\log S^{G,*}\right)\right).$$

Substituting (C.43) we obtain:

$$\begin{aligned}
d\log\mu^{*} &= \left(\Upsilon^{*}\left(d\log P^{*}+d\log S^{G,*}\right)-d\log P^{*}-\Upsilon^{*}\mu^{*}\cdot d\log S^{G,*}\right)\\
&= -\left(1-\Upsilon^{*}\right)d\log P^{*}+\Upsilon^{*}\left(1-\mu^{*}\right)\cdot d\log S^{G,*}.
\end{aligned} \tag{C.44}$$

Proof. **Item (i).**

We now derive the main results, following the earlier proofs. Totally differentiating the liquidity premium with respect to μ^{*} and P^{*}, we obtain:

$$\bar{R}^{m,*}d\log P^{*}+\mathcal{L}^{*}d\log P^{*}+\mathcal{L}^{*}_{\mu^{*}}\mu^{*}d\log\mu^{*}=0. \tag{C.45}$$

Substituting (C.44) and collecting terms we obtain:

$$\left(\bar{R}^{m,*}+\mathcal{L}^{*}-\left(1-\Upsilon^{*}\right)\mathcal{L}\mathcal{P}^{*}_{\mu^{*}}\mu^{*}\right)d\log P^{*}+\mathcal{L}^{*}_{\mu^{*}}\mu^{*}\Upsilon^{*}\left(1-\mu^{*}\right)\cdot d\log S^{G,*}=0. \tag{C.46}$$

Thus, we obtain:

$$\frac{d\log P^{*}}{d\log S^{G,*}}=\frac{-\mathcal{LP}^{*}_{\mu^{*}}\mu^{*}\left(1-\mu^{*}\right)\Upsilon^{*}}{R^{b}-\left(1-\Upsilon^{*}\right)\mathcal{LP}^{*}_{\mu^{*}}\mu^{*}}>0.$$

Substituting this expression in (C.44) we obtain:

$$\frac{d\log \mu^{*}}{d\log S^{G,*}}=\frac{R^{b}\Upsilon^{*}\left(1-\mu^{*}\right)}{R^{b}-\left(1-\Upsilon^{*}\right)\mathcal{LP}^{*}_{\mu^{*}}\mu^{*}}>0.$$

Finally, by the law of one price:

$$d\log e=-d\log P^{*}=\frac{\mathcal{L}^{*}_{\mu^{*}}\mu^{*}\left(1-\mu^{*}\right)\Upsilon^{*}}{R^{b}-\left(1-\Upsilon^{*}\right)\mathcal{L}^{*}_{\mu^{*}}\mu^{*}}<0.$$

Finally, the excess-bond premium and the dollar liquidity premium is:

$$d\mathcal{L}^{*}=d\mathcal{DLP}^{*}=-R^{*,m}d\log P^{*}<0.$$

Item (ii).

If the shock is permanent expected inflation does not change. Since nominal rates are fixed, we have that the dollar liquidity ratio must remain constant:

$$d\log \mu^{*}=0. \tag{C.47}$$

Moreover, $d\mathcal{BP}^{*}=d\mathcal{DLP}^{*}=0$. From (C.44)

$$\frac{d\log P^{*}}{d\log S^{G,*}}=\frac{\Upsilon^{*}}{\left(1-\Upsilon^{*}\right)}\left(1-\mu^{*}\right)>0\cdot$$

By the law of one price then:

$$\frac{d\log e}{d\log S^{G,*}}=-\frac{d\log P^{*}}{d\log S^{G,*}}=-\frac{\Upsilon^{*}}{\left(1-\Upsilon^{*}\right)}\left(1-\mu^{*}\right).$$

□

Here we provide the micro-foundations for the loan demand and deposit supply schedules in a deterministic version of the model. We consider a representative global household. The household saves in dollar and euro deposits, supplies labor to an international firm, holds shares of this firm and owns a diversified portfolio of banks.

C.7 The Non-Financial Sector

Global household problem. The household enters the periods with a portfolio of dollar and euro deposits, denoted by $\{D_t, D_t^{*}\}$, holds shares of a global firm, Σ_t, and shares in a perfectly diversified portfolio of global banks, ϑ_t. These shares entitle the household to the firm's and bank's profits. The financial wealth available to the household (expressed in euros) is given by:

$$P_t n_t^h \equiv \left(1+i_t^d\right) D_t + T_t + e_t \left(\left(1+i_t^{*,d}\right) D_t^* + T_t^*\right) + P_t \left(q_t + r_t^h\right) \Sigma_t + P_t \left(Q_t + div_t\right) \vartheta_t \tag{C.48}$$

where T_t and T_t^* represent euro and dollar central bank transfers, q_t is the price of the firm (in terms of goods), r_t^h is the profit of the international firm, and Q_t is the price of the bank portfolio and div_t the dividend payout of banks.

In addition, the household supplies h_t hours that are remunerated at z_t euros per hour. The household uses its wealth to purchase deposits, to buy shares, and to consume. There are three types of consumption goods: dollar goods, denoted by c_t^*, euro goods, denoted by c_t, and a linear good, denoted by c_t^h. The household's budget constraint is:

$$e_t P_t^* c_t^* + P_t c_t + P_t c_t^h + D_{t+1} + e_t D_{t+1}^* + P_t q_t \Sigma_{t+1} + P_t Q_t \vartheta_{t+1} = P_t n_t^h + z_t h. \tag{C.49}$$

Both dollar and euro consumption are subject to deposit-in-advance (DIA) constraints:

$$c_t \leq \left(1+i_t^d\right) \frac{D_t}{P_t}, \tag{C.50}$$

and

$$c_t^* \leq \left(1+i_t^d\right) \frac{D_t}{P_t}. \tag{C.51}$$

The period utility is

$$U^*\left(c_t^*\right) + U\left(c_t\right) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu},$$

where U^* and U are concave utility functions over both goods and $h_t^{1+\nu}/\left(1+\nu\right)$ is a labor dis-utility. To simplify the algebra of this section, we assume that $U_{c^*}^*\left(1\right) = U_c\left(1\right) = 1$.

The household's problem is:

$$\begin{aligned} V_t^h\left(D_t, D_t^*, \Sigma_t, \vartheta_t\right) = \max_{\left\{c_t, c_t^*, c_t^h, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right\}} & U^*\left(c_t^*\right) + U\left(c_t\right) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu} \ldots \\ & + \beta V_{t+1}^h\left(D_{t+1}, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right) \end{aligned} \tag{C.52}$$

subject to the budget constraint (C.49 and C.48) and the two DIA constraints (C.50-C.51).

Firm Problem. The firm produces all goods in the economy using the same production function

$$y_t = A_{t+1} h_t^{\alpha}.$$

The firm's output is divided into:

$$c_t^* + c_t + c_t^h = y_t. \tag{C.53}$$

The firm revenues come from selling goods in the dollar, euro, and linear good markets:

$$e_t P_t^* c_t^* + P_t c_t + P_t c_t^h = P_t y_t.$$

To produce positive amounts of all goods, the firm must be indifferent between selling in either market. Hence, the law of one price will hold in an equilibrium with positive consumption of all goods—the Inada conditions guarantee this is the case.

To maximize profits, the firm chooses borrowed funds B_{t+1}^d and labor h_t. The demand for loansemerges from a working capital constraint: $z_t h_t \leq B_{t+1}^d$. The firm saves in deposits whatever borrowings it doesn't spend in wages.

The firm's problem is given by:

$$\begin{aligned} P_{t+1} r_{t+1}^h &= \max_{B_{t+1}^d \geq 0, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - \left(1 + i_{t+1}^b\right) B_{t+1}^d + \left(1 + i_{t+1}^d\right)\left(B_{t+1}^d - z_t h_t\right) \\ &= \max_{B_{t+1}^d \geq 0, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - \left(1 + i_{t+1}^b\right) z_t h_t - \left(i_{t+1}^b - i_{t+1}^d\right)\left(B_{t+1}^d - z_t h_t\right) \end{aligned} \tag{C.54}$$

Equilibrium. In the body of the paper we characterized the equilibrium in loan and deposit markets, taking as given the loan demand and deposit supply schedules, and the transfers rules. In addition to these financial markets, the non-financial sector features a labor market, firm shares market, bank shares market, and the three goods markets. Next, we derive the loan demand and deposit supply schedules and comment on how once these asset markets clear, all other markets clear.

C.8 Derivation of Deposit Supply and Loan Demand

Step 1 - deposit demand. We clear the linear good, c_t^h, from the household's budget constraint:

$$\begin{aligned} c_t^h &= \frac{P_t n^h + z_t h - \left(e_t P_t^* c^* + P_t c + D_{t+1} + e_t D_{t+1}^* + P_t\left(r_t + q_t\right)\Sigma_t + P_t\left(Q_t + div_t\right)\vartheta_t\right)}{P_t} \\ &= n^h + \frac{z_t}{P_t} h_t - \left(c_t^* + c_t + \left(r_t + q_t\right)\Sigma_t + \left(Q_t + div_t\right)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*}\right). \end{aligned} \tag{C.55}$$

where the second line uses the law of one price.

Substituting (C.55) into the objective of the household's problem (C.52) we obtain:

$$\begin{aligned} V_t^h\left(D_t, D_t^*, \Sigma_t, \vartheta_t\right) &= n_t^h + \max_{\left\{c_t, c_t^*, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right\}} U^*\left(c_t^*\right) + U\left(c_t\right) - \frac{h_t^{1+\nu}}{1+\nu} \ldots \\ &\quad + \frac{z_t}{P_t} h_t - \left(c_t^* + c_t + \left(r_t + q_t\right)\Sigma_t + \left(Q_t + div_t\right)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*}\right) \ldots \\ &\quad + \beta V_{t+1}^h\left(D_{t+1}, D_{t+1}, \Sigma_{t+1}, \vartheta_{t+1}\right) \end{aligned}$$

subject to the two DIA constraints (C.50-C.51).

We proceed to obtain the deposit supply.

Since $\left\{D_{t+1}, D^*_{t+1}\right\}$ enter symmetrically, we derive the deposit supply only for one currency. We take the first-order condition with respect to c_t and notice that if the DIA constraint does not bind, $U_c\left(c\right) = 1$. In turn, if the deposit in advance constraint indeed binds, then:

$$c = \left(1 + i^d_t\right) \frac{D_t}{P_t} = \frac{\left(1 + i^d_t\right)}{P_t/P_{t-1}} \frac{D_t}{P_{t-1}} = R^d_t \frac{D_t}{P_{t-1}}.$$

Thus, we can combine both cases, with and without the binding DIA constraint, to write down the optimal consumption rule:

$$c \quad = \quad \min\left\{\left(U_c\right)^{-1}\left(1\right), R^d_t \cdot \frac{D_t}{P_{t-1}}\right\}. \tag{C.56}$$

By analogy:

$$c^* = \min\left\{\left(U^*_{c^*}\right)^{-1}\left(1\right), R^{d,*}_t \cdot \frac{D^*_t}{P^*_{t-1}}\right\}.$$

It is convenient to treat c and c^* directly as functions of $\frac{D}{P_{t-1}}$ and $\frac{D^*}{P^*_{t-1}}$ in the next step.

Step 2 - deposit supply schedules. We replace the optimal euro and dollar consumption rules into the objective. We have:

$$\begin{aligned}
V^h_t\left(D_t, D^*_t, \Sigma_t, \vartheta_t\right) \quad = \quad & n^h + \max_{\left\{c_t, c^*_t, h_t, D_t, D^*_{t+1}, \Sigma_{t+1}, \vartheta_{t+1}\right\}} U^*\left(\min\left\{\left(U^*_{c^*}\right)^{-1}\left(1\right), R^{d,*}_t \cdot \frac{D^*}{P^*_{t-1}}\right\}\right)\ldots \\
+ \quad & U\left(\min\left\{\left(U_c\right)^{-1}\left(1\right), R^d_t \cdot \frac{D_t}{P_{t-1}}\right\}\right) - \frac{h^{1+\nu}}{1+\nu}\ldots \\
+ \quad & n^h + \frac{z_t}{P_t} h - \min\left\{\left(U^*_{c^*}\right)^{-1}\left(1\right), R^{d,*}_t \cdot \frac{D^*_t}{P^*_{t-1}}\right\}\ldots \\
- \quad & \min\left\{\left(U_c\right)^{-1}\left(1\right), R^d_t \cdot \frac{D_t}{P_{t-1}}\right\}... \\
- \quad & \left(\left(r_t + q_t\right)\Sigma_t + \left(Q_t + div_t\right)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D^*_{t+1}}{P^*_t}\right)\ldots \\
+ \quad & \beta V^h_{t+1}\left(D_{t+1}, D_{t+1}, \Sigma_{t+1}, \vartheta_{t+1}\right).
\end{aligned} \tag{C.57}$$

Next, we the derive deposit demand: We take the first-order conditions with respect to D_{t+1}/P_t to obtain:

$$1 \quad = \quad \beta \frac{\partial V^h_{t+1}}{\partial\left(D_{t+1}/P_t\right)}. \tag{C.58}$$

Next, we derive the envelope condition. We have two cases.

Case 1: binding DIA constraint the following period. For the case where

$$R^d_t \cdot D_t/P_{t-1} < 1$$

we have from (C.57) that:

$$\frac{\partial V_t^h}{\partial\left(D_t/P_{t-1}\right)} = U_c R_t^d - R_t^d + R_t^d = U_c R_t^d.$$

Case 1: non-binding DIA constraint the following period. For the case where $R_{t-1}^d \frac{D}{P_{t-1}} \geq 1$, we have from (C.57) that

$$\frac{\partial V_t^h}{\partial\left(D_t/P_{t-1}\right)} = R_t^d.$$

Thus, combining the two envelope conditions we obtain:

$$\frac{\partial V_t^h}{\partial\left(D_t/P_{t-1}\right)} = \begin{cases} U_c\left(R_t^d \cdot D_t/P_{t-1}\right) R_t^d & \text{for } R_t^d \cdot D_t/P_{t-1} < 1 \\ R_t^d & \text{otherwise.} \end{cases} \tag{C.59}$$

We shift (C.59) one period forward and substitute in (C.58) in the left-hand side to obtain:

$$\frac{1}{\beta} = \begin{cases} U_c\left(R_{t+1}^d \cdot D_{t+1}/P_t\right) R_{t+1}^d & R_{t+1}^d \cdot D_t/P_{t-1} < 1 \\ R_{t+1}^d & \text{otherwise.} \end{cases}$$

In the body of the paper, using the banks' problem, we show that $R_{t+1}^d < R^b = 1/\beta$. Thus, the only relevant portion to determine the demand condition is the one where there's no satiation of deposits. We hence, will only use this portion.

We now adopt power utility. Assume that $U = c^{1-\gamma}/\left(1-\gamma\right)$ and $U^* = \left(c^*\right)^{1-\gamma^*}/\left(1-\gamma^*\right)$. Then,

$$\frac{1}{\beta} = \left(R_{t+1}^d \cdot D_{t+1}/P_t\right)^{-\gamma} R_{t+1}^d.$$

We clear D/P_t to obtain the euro deposit supply schedule:

$$D_{t+1}/P_t = \beta^{1/\gamma}\left(R_{t+1}^d\right)^{\frac{1-\gamma}{\gamma}}.$$

By analogy, we have the dollar supply schedule:

$$D_{t+1}^*/P_t^* = \beta^{1/\gamma^*}\left(R_{t+1}^{*,d}\right)^{\frac{1-\gamma^*}{\gamma^*}}.$$

More generically, following the same states, if we introduce preference shocks to the utility specifications, as follows:

$$U_t = \left(c/x_t\right)^{1-\gamma}/\left(1-\gamma\right) \text{ and } U_t^* = \left(c^*/x_t^*\right)^{1-\gamma^*}/\left(1-\gamma^*\right),$$

the demand schedules generalize to:

$$D_{t+1}/P_t = x_t\beta^{1/\gamma}\left(R_t^d\right)^{\frac{1-\gamma^x}{\gamma^x}}.$$

By analogy, we have that

$$D_{t+1}^*/P_t^* = x_t^* \beta^{1/\gamma} \left(R_t^d\right)^{\frac{1-\gamma^x}{\gamma^x}}.$$

We obtain this conditions following exactly the same steps.

All in all, the demand schedules are akin to those in the body of the paper, where the reduced form coefficients are given by:

$$\Theta_t^d = x_t \beta^{1/\gamma^d} \text{ and } \epsilon^d = \frac{1}{\gamma^d} - 1,$$

and

$$\Theta_t^{*,d} = x_t^* \beta^{1/\gamma^{*,d}} \text{ and } \epsilon^{*,d} = \frac{1}{\gamma^{*,d}} - 1.$$

Next, we describe the labor supply schedule.

Step 3 - labor supply. The first-order condition with respect to h in the household's problem yields a labor supply that only depends on the real wage:

$$h_t^\nu = z_t/P_t. \tag{C.60}$$

Next, we move to the firm's problem to obtain the labor demand.

Step 4 - labor demand. Since from the bank's problem, it will be the case that $i_{t+1}^b > i_{t+1}^d$, then the working capital constraint in (C.54) is binding, $z_t h_t = B_{t+1}^d$. Thus, the firm's objective is to

$$\max_{h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - \left(1 + i_{t+1}^b\right) z_t h_t.$$

The first-order condition for labor h_t yields:

$$P_{t+1} \alpha A_{t+1} h_t^\alpha = \left(1 + i_{t+1}^b\right) z_t h_t.$$

Dividing both sides by P_t, we obtain

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^\alpha = \left(1 + i_{t+1}^b\right) \frac{z_t}{P_t} h_t. \tag{C.61}$$

Step 5 - loan demand. Next, we use the labor supply (C.60) and labor demand (C.61), to solve for labor as a function of the loans rate:

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^\alpha = \left(1 + i_{t+1}^b\right) h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1} h_t^\alpha}{h_t^{\nu+1}}. \tag{C.62}$$

Since the working capital constraint binds:

$$\frac{B_{t+1}^d}{P_t} = h_t \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{1}{\nu+1}}. \tag{C.63}$$

Thus, we can combine (C.62) and (C.63) to obtain the loans demand:

$$R_t^b = \alpha A_{t+1}\left(\frac{B_{t+1}^d}{P_t}\right)^{-1}\left(\frac{B_{t+1}^d}{P_t}\right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t\left(R_{t+1}^b\right)^{\epsilon^b}, \tag{C.64}$$

where the reduced form coefficients of the loans demand are:

$$\Theta_t^b = (\alpha A_{t+1})^{-\epsilon^b} \text{ and } \epsilon^b = \left(\frac{\nu+1}{\alpha-(\nu+1)}\right).$$

Step 6 - output, firm value and bank values. We replace the loans demand (C.64) into (C.63) to obtain the equilibrium labor as a function of the equilibrium loans rate:

$$h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}}\left(R_{t+1}^b\right)^{\frac{1}{\alpha-(\nu+1)}}.$$

We replace (C.63) into the production function to obtain:

$$y_{t+1} = A_{t+1}\left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}\left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \rightarrow y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}}\left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The profit of the international firm is given by:

$$r_{t+1}^h = y_{t+1} - R_{t+1}^b B_{t+1} \rightarrow r_{t+1}^h = A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}}\left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}}\right)\cdot\left(R_{t+1}^b\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The price of the firm is given by the first-order condition with respect to Σ. In that case, q_t must satisfy:

$$q_t = \beta\left(r_{t+1}^h + q_{t+1}\right) \rightarrow q_t = \sum_{\tau\geq 1}\beta^\tau\left(r_{t+\tau}^h\right).$$

With this, we conclude that output, hours, and the firm price are decreasing in current (and future) loans rate.

Finally, consider the price of the bank's shares. By the same token,

$$Q_t = \beta\left(div_t + Q_{t+1}\right),$$

Multiply both sides by $1/\beta$ and recall that $\vartheta_t = 1$. Thus, we have:

$$\frac{1}{\beta}Q_t = \left(div_t + \beta\frac{1}{\beta}Q_{t+1}\right).$$

By change of variables let $v_t \equiv \frac{1}{\beta}Q_t$. Therefore, the value of the firm is given by

$$v_t = div_t + \beta v_{t+1}.$$

Solving this condition from time zero implies that:

$$v_0 = \sum_{t \geq 0} \beta^t div_t.$$

Thus, the bank's objective in the body of the paper is consistent with maximizing the bank's value.

Remark. We priced the firms and banks so that they are held in equilibrium by households. Thus, the shares markets clear. Note that throughout the proof we use the labor market-clearing condition, (C.62). Hence, the labor market clears. Since in the body of the paper we deal with clearing in the loans and deposit markets, by Walras's law, this implies clearing in the three goods markets.

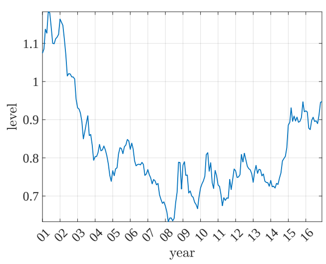
All in all, the equilibrium in the banking block is an autonomous system. As long as the loan and deposit markets clear, we have clearing in the non-financial sector: Once we compute equilibria taking the schedules as exogenous in the bank's problem, we obtain output and household consumption from the equilibrium loan and deposit rates.

Finally, we should note that in presence of aggregate risk (inflation risk in particular), the deposit demand schedules will feature a risk premium that we are not considering in the derivation. We ignore this terms.

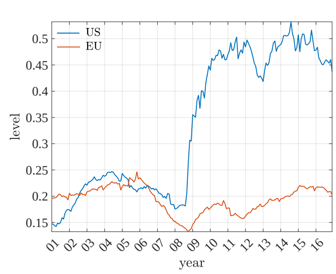
D Additional Figures and Tables

Figure D.1: Data series

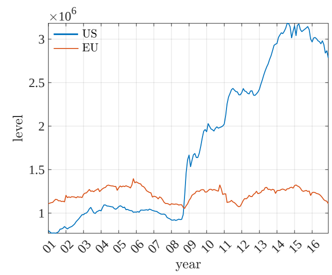
(a) Log Euro-Dollar



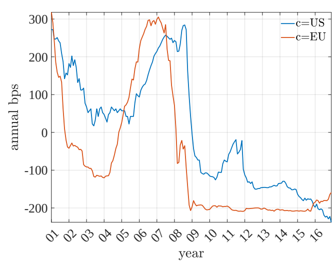
(b) Liquidity Ratio



(c) Money Supply



(d) Policy Rates



(e) Premia

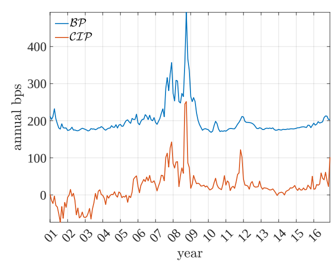


Table D.1: Calibrated Parameters

Parameter	Value	Description	Target
External Calibration			
M_{ss}^{us}/M_{ss}^{eu}	0.6841	relative money supply	normalized to exchange rate levels
Θ^{b}	1	global loan demand scale	normalization
ϵ^{b}	-35	loan demand elasticity	Bianchi and Bigio (2021)/symmetry
$\epsilon^{d,us}=\epsilon^{d,eu}$	1	US/EU deposit demand elasticity	normalization / symmetry
$\lambda^{us}=\lambda^{eu}$	1	US/EU interbank market matching efficiency	normalization
ι^{us}=ι^{eu}	10	US policy corridor spread	Bianchi and Bigio (2021)
$\eta^{us}=\eta^{eu}$	1/2	benchmark/symmetry	
Steady-State Estimation of Financial Variables			
$\Theta_{ss}^{d,*}$	1.0026	US deposit demand scale	To match steady-state moments
Θ_{ss}^{d}	1.0026	EU deposit demand scale	symmetry
σ_{ss}^{us}	76.8541	average US funding volatility	steady-state moment targets
σ_{ss}^{eu}	21.9133	average Euro funding volatility	steady-state moments targets
ξ_{ss}	-62.5845	average $\mathcal{CIP}$ and $\mathcal{UIP}$ wedge	steady-state moments targets
Data Estimates of US and EU policy variables			
$\mathbb{E}\left(i_t^{m,us}\right)$	0.9957	annualized US interest on reserves	
$\Sigma\left(i_t^{m,us}\right)$	0.0026	std annual US policy rate	
$\rho\left(i_t^{m,us}\right)$	0.9777	autocorrelation annual US policy rate	
$\mathbb{E}\left(i_t^{m,eu}\right)$	1.0027	average annual Euro policy rate	
$\Sigma\left(i_t^{m,eu}\right)$	0.0017	std annual Euro policy rate	
$\rho\left(i_t^{m,eu}\right)$	0.9894	autocorrelation annual Euro policy rate	
$\mathbb{E}\left(\ln M_t^{us}\right)$	6.7255	average monthly US liquid assets stock ($ Billion)	
$\Sigma\left(\ln M_t^{us}\right)$	0.0281	std monthly US liquid assets stock	
$\rho\left(\ln M_t^{us}\right)$	0.9894	autocorrelation monthly US liquid assets stock	
$\mathbb{E}\left(\ln M_t^{us}\right)$	6.7255	average monthly US liquid assets stock ($ Billion)	
$\Sigma\left(\ln M_t^{us}\right)$	0.0281	std monthly US liquid assets stock	
$\rho\left(\ln M_t^{us}\right)$	0.9894	autocorrelation monthly US liquid assets stock	

Table D.2: Estimated Process

Parameter	Prior	Prior Mean	Post. Mean	Post. Interval
$\Sigma^{\sigma^{us}}$	Inv. Gamma	0.150	0.092	0.085-0.100
$\Sigma^{\sigma^{eu}}$	Inv. Gamma	0.150	0.111	0.103-0.120
$\Sigma^{\Theta^{us}}$	Inv. Gamma	0.030	0.017	0.015-0.018
$\Sigma^{\Theta^{eu}}$	Inv. Gamma	0.020	0.015	0.013-0.017
Σ^{ξ}	Inv. Gamma	0.000	0.001	0.000-0.001
$\rho^{\sigma^{us}}$	Beta	0.930	0.990	0.987-0.991
$\rho^{\sigma^{eu}}$	Beta	0.930	0.953	0.929-0.972
$\rho^{\Theta^{us}}$	Beta	0.985	0.991	0.989-0.992
$\rho^{\Theta^{eu}}$	Beta	0.980	0.989	0.987-0.991
ρ^{ξ}	Beta	0.980	0.980	0.979-0.982

Table D.3: Model and Data Moments

Moment	Data	Simulated Moment
Steady-State Targets		
Mean of $\mathcal{CIP}$	13.407	11.015
Mean of $\mathcal{BP}$	194.605	192.575
Mean of μ^{us}	0.226	0.234
Mean of μ^{eu}	0.222	0.231
Mean of e	1.251	1.282
Untargetted Moments		
Std. of $\mathcal{CIP}$	42.672	121.603
Std. of $\mathcal{BP}$	42.271	103.001
Std. of μ^{us}	0.133	0.036
Std. of μ^{eu}	0.025	0.042
Std. of e	0.167	0.182
AR(1) Coef. of $\mathcal{CIP}$	0.801	0.947
AR(1) Coef. of $\mathcal{BP}$	0.879	0.954
AR(1) Coef. of μ^{us}	0.989	0.957
AR(1) Coef. of μ^{eu}	0.986	0.956
AR(1) Coef. of e	0.963	0.952

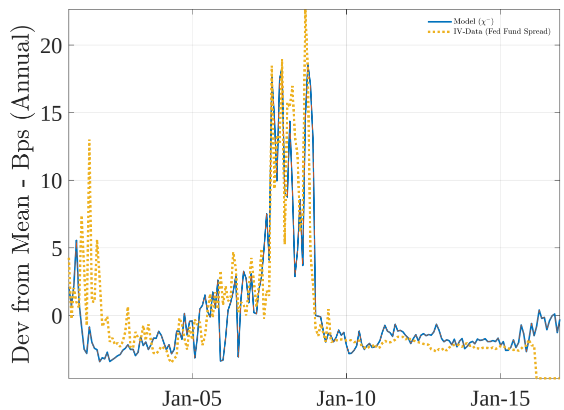


Figure D.2: Interbank Stress - Data and Model

E Why are Japan and Switzerland different?

One conclusion from Section 4.4 is that the dollar exchange rate against the Japanese Yen and, to a lesser extent, the Swiss Franc present a more tenuous relationship with the liquidity ratio. We can exploit the model to explain what could drive pattern: from the theory, we know that σ_t^{us} drives the correlation between any exchange rate with the dollar and the dollar liquidity ratio. Hence, any shock that increases the demand for the Yen and or the Swiss Franc that is correlated with σ_t^{us} will reduce the regression coefficients for these currencies. In particular, since we do control for policy variables, the model requires that the funding scale in Yen and Swiss Francs, Θ_t^d, to be correlated with σ_t^{us} to reduce the regression coefficients. In a robustness exercise, we estimate the following process for the demand shifters of the Yen and Swiss Franc funding:

$$\Theta_t^{d,x} = \left(1-\rho^{\Theta,x}\right)\Theta_{ss}^{d,x}\exp\left(\Gamma\left(\sigma_t^{us}/\sigma_{ss}^{us}-1\right)\right)+\rho^{\Theta,x}\Theta_{ss}^{d,x}+\varepsilon^{d,x}, \quad x\in\{jp,swz\},$$

using the Swiss Franc as a target. In this case, we obtain a posterior estimate for Γ of 0.72. In this case, as indicated in Table B4, the significance of the regressions for the Yen and the Swiss franc vanish, as they do in the data.

Table B4: Model Regression Coefficients - Correlated Funding Shocks in Japan and Switzerland

	EUR	AUS	CAN	JPN	NZ	NWY	SWE	SWZ	UK
$\Delta(\text{LiqRatio}_t)$	0.121*	0.168	0.170***	0.217	0.169*	0.156**	0.170*	0.156	0.163**
$\Delta(\pi_t^i-\pi_t^{us})$	0.243*	0.241	0.209*	0.228*	0.258*	0.219*	0.243*	0.218*	0.243*
LiqRatio(-1)	0.009	0.014	0.009	0.037	0.011	0.008	0.009	0.025	0.011
Constant	0.016	0.022	0.015	0.060	0.020	0.014	0.016	0.039	0.019
N	234	234	234	234	234	234	234	234	234
adj. R^2	0.046	0.029	0.066	0.037	0.045	0.059	0.053	0.031	0.056

Note: t statistics in parentheses. * p<0.1, ** p<0.05, ***p<0.01

F Computational Algorithms

F.1 Summary of Equilibrium Conditions

Steady State: equilibrium conditions. We solve the steady-state of the model where $n = 0$ every period.[32] Solving for steady-state equilibrium requires requires to solve for 11 variables, three interest rates, $\left\{R^{*,d}, R^d, R^b\right\}$, three prices $\{P, P^*, e\}$ and five quantities $\{m^*, m, d, d^*, b^*\}$. We summarize these conditions below and show that the system .

Prices given quantities are given by:

$$d^* = \Theta^{*,d}\left(R^{*,d}\right)^{\epsilon^{d^*}} \tag{F.1}$$

$$d = \Theta^d\left(R^d\right)^{\epsilon^d} \tag{F.2}$$

and

$$b^* = \Theta^b\left(R^b\right)^{\epsilon^b}. \tag{F.3}$$

The two prices are given by the equilibrium in the market for real dollar reserves,

$$m^* = \frac{M^*}{P^*}. \tag{F.4}$$

and by the equilibrium in the market for real euro reserves is for euro reserves:

$$m = \frac{M}{P}. \tag{F.5}$$

In turn, the exchange rate is obtained via the law of one price:

$$e = P/P^*.$$

Finally, we have four first-order conditions and the budget constraint to pin down the quantities:

a) the dollar liquidity premium:

$$R^{m,*} + \mathbb{E}\left[\chi^*_{m^*}\right] = R^m + \mathbb{E}\left[\chi_m\right]$$

b) the bond premium:

$$R^b = R^{m,*} + \mathbb{E}\left[\chi^*_{m^*}\right] \tag{F.6}$$

[32]When $n > 0$, $R^b = 1/\beta$ so we drop one variable and the budget constraint.

c-d) the two deposit premia

$$R^{d,*} + \mathbb{E}\left[\chi^{*}_{d^{*}}\right] = R^{m,*} + \mathbb{E}\left[\chi^{*}_{m^{*}}\right] \tag{F.7}$$

and

$$R^{d,*} + \mathbb{E}\left[\chi^{*}_{d^{*}}\right] \quad = \quad R^{m,*} + \mathbb{E}\left[\chi^{*}_{m^{*}}\right] \tag{F.8}$$

Finally, the budget constraint for $n = 0$ is:

$$b + m + m^{*} = d + d^{*}.$$

This is a system of 11 equations and 11 unknowns. Next, show how to solve the model in ratios.

Steady State: solving the model in ratios. Recall that liquidity ratios are given by:

$$\mu \equiv \frac{m}{d} \text{ and } \mu^{*} \equiv \frac{m^{*}}{d^{*}}.$$

We define the ratio of real euro to dollar funding as:

$$\upsilon \equiv \frac{d}{d^{*}}.$$

Once we obtain $\{d^{*}, \nu, \mu, \mu^{*}\}$, we obtain $\{m, m^{*}, d\}$ from these three definitions.

We have shown that the interbank market tightness in euros and dollars are given by:

$$\theta(\mu) = \max\left\{-\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega)}, 0\right\}$$

and

$$\theta^{*}(\mu^{*}) = \left\{-\frac{\int_{-\infty}^{-\mu^{*}} (\mu^{*} + \omega) \cdot d\Phi^{*}(\omega^{*})}{\int_{-\mu^{*}}^{\infty} (\mu + \omega) \cdot d\Phi^{*}(\omega^{*})}, 0\right\}.$$

Thus we have introduced three ratios. Notice that once we obtain $\{\mu, \mu^{*}\}$ we obtain $\{d, d^{*}\}$. Once we obtain υ, we have $\left\{e^{-1}\right\}$.

Furthermore, the budget constraint written in ratios is:

$$b^{*} \quad = \quad (\upsilon(1-\mu) + (1-\mu^{*}))\, d^{*}. \tag{F.9}$$

We substitute out $\left\{R^{b}, R^{d}, R^{*,d}\right\}$ and work directly with the market clearing conditions. We replace b^{*} from the budget constraint. If we substitute the ratios $\{\mu^{*}, \mu, \upsilon, d^{*}\}$ into the equilibrium conditions and, thus only have one quantity variable d^{*}, and the rest of the system is expressed in ratios.

Steady State: autonomous sub-system. We solve for $\{\mu^{*}, \mu, \upsilon, d^{*}\}$ using:

1) Bond premium:

$$\Theta^{b}\left(\left(\upsilon\left(1-\mu\right)+\left(1-\mu^{*}\right)\right)d^{*}\right)^{\epsilon^{b}} \quad = \quad R^{*,m}+\mathbb{E}\left[\chi_{m}\left(\mu^{*}\right)\right]. \tag{F.10}$$

2) The dollar liquidity premium:

$$R^{m}+\mathbb{E}\left[\chi_{m}\left(\mu\right)\right]=R^{*,m}+\mathbb{E}\left[\chi_{m}\left(\mu^{*}\right)\right].$$

3) The euro funding premium:

$$\Theta^{d}\left(\upsilon d^{*}\right)^{-\epsilon^{d}}+\mathbb{E}\left[\chi_{d}\left(\mu\right)\right]=R^{*,m}+\mathbb{E}\left[\chi_{m}\left(\mu^{*}\right)\right] \tag{F.11}$$

4) dollar funding premium:

$$\Theta^{*,d}\left(d^{*}\right)^{-\epsilon^{d^{*}}}+\mathbb{E}\left[\chi_{d^{*}}\left(\mu^{*}\right)\right] \quad = \quad R^{*,m}+\mathbb{E}\left[\chi_{m^{*}}\left(\mu^{*}\right)\right]. \tag{F.12}$$

These four equations provide us with a solution to $\{d^{*},\nu,\mu,\mu^{*}\}$.

Steady State: Solving the rest of the model. With the solution to $\{\mu^{*},\mu,\upsilon,d^{*}\}$ we obtain $\{m^{*},m,d\}$ using:

$$m=\mu d,\ m^{*}=\mu^{*}d^{*},\ \text{and}\ d=\nu d^{*}.$$

Then, we obtain the euro price from

$$P=\frac{M}{\mu\upsilon d^{*}},$$

the dollar price from

$$P^{*}=\frac{M^{*}}{\mu^{*}d^{*}},$$

and the exchange rate from

$$e=\frac{P}{P^{*}}.$$

F.2 Algorithm to obtain a Global Solution

Define $X\in\mathcal{X}=\{1,2,3,\ldots,N^{s}\}$ to be a finite set of states. We let X follow a Markov process with transition matrix Q. Thus, $X'\sim Q\left(X\right)$. That is, at each period, $X=\{\sigma^{*},\sigma,i^{*,m},i^{m}$, $M,M^{*},\Theta^{d},\Theta^{*,d}\}$ are all, potentially, functions of the state X.

The algorithm proceeds as follows. We define a "greed" parameter Δ^{greed} and a tolerance parameter ε^{tol}, and construct a grid for $\mathcal{X}$. We conjecture a price-level functions $\left\{p_{(0)}\left(X\right),p_{(0)}^{*}\left(X\right)\right\}$ which produces a price levels in both currencies as a function of the state. As an initial guess, we use $p_{(0)}\left(X\right)=p_{ss}^{*}$, and $p_{(0)}^{*}\left(X\right)=p_{ss}^{*}$ setting the exchange rate to its steady state level in all periods. We proceed by iterations, setting a tolerance count tol

to $tol > 2 \cdot \varepsilon^{tol}$.

Outerloop 1: Iteration of price functions. We iterate price functions until they converge. Let n be the $n-th$ step of a given iteration. Given a $p_{(n)}(X), p^{*}_{(n)}(X)$, we produce a new price level functions $p_{(n+1)}(X), p^{*}_{(n+1)}(X)$ if $tol > \varepsilon^{tol}$.

Innerloop 1: Solve for real policy rates. For each X in the grid for $\mathcal{X}$, we solve for

$$\left\{\bar{R}^{m}(X), \bar{R}^{*,m}(X), \bar{R}^{w}(X), \bar{R}^{*,w}(X)\right\}.$$

Let j be the $j-th$ step of a given iteration. Conjecture values

$$\left\{\bar{R}^{m}_{(0)}(X), \bar{R}^{*,m}_{(0)}(X), \bar{R}^{w}_{(0)}(X), \bar{R}^{*,w}_{(0)}(X)\right\}$$

We use $\left\{\bar{R}^{m}_{ss}, \bar{R}^{*,m}_{ss}, \bar{R}^{w}_{ss}, \bar{R}^{*,w}_{ss}\right\}$ as an initial guess. We then update

$$\left\{\bar{R}^{m}_{(j)}(X), \bar{R}^{*,m}_{(j)}(X), \bar{R}^{w}_{(j)}(X), \bar{R}^{*,w}_{(j)}(X)\right\}$$

until we obtain convergence:

2.a Given this guess, we solve for the liquidity ratios in Dollars and Euro $\left\{\mu, \mu^{*}, \bar{R}^{d}, \bar{R}^{*,d}\right\}$ as a function of the state using:

$$\bar{R}^{d} + \frac{1}{2}\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) = \bar{R}^{*,d} + \frac{1}{2}\left(\chi^{*,+}(\mu^{*}) - \chi^{*,-}(\mu^{*})\right)$$

$$\bar{R}^{m} + \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right) = \bar{R}^{*,m} + \frac{1}{2}\left(\chi^{*,+}(\mu^{*}) + \chi^{*,-}(\mu^{*})\right)$$

$$\Theta^{b}\left(\left(\upsilon(1-\mu) + (1-\mu^{*})\right) d^{*}\right)^{\epsilon^{b}} = \bar{R}^{*,d} + \frac{1}{2}\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right)$$

$$\bar{R}^{m} = \bar{R}^{*,d} + \frac{1}{2}\left(\chi^{+}(\mu) - \chi^{-}(\mu)\right) - \frac{1}{2}\left(\chi^{+}(\mu) + \chi^{-}(\mu)\right).$$

This step yields an update for $\left\{R^{d}(X), R^{*,d}(X)\right\}$.

2.b Given the solutions to $\left\{R^{d}(X), R^{*,d}(X)\right\}$, we solve $\{d^{*}, \upsilon\}$ using:

$$d^{*} = \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}}\right]^{1/\epsilon^{d*}}$$

$$\upsilon = \left[\frac{\bar{R}^{d}}{\Theta^{d}}\right]^{1/\epsilon^{d}} \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}}\right]^{1/\epsilon^{d*}}.$$

This step yields an update for $\{d^{*}(X), \upsilon(X)\}$.

2.c Given $\{d^*(X), \upsilon(X)\}$ we solve for prices $\{p, p^*, e\}$ using:

$$\mu \upsilon d^* = \frac{M}{p}$$

$$\mu^* d^* = \frac{e}{p} M^*$$

$$p^* = e^{-1} p.$$

2.d Finally, we update the real policy rates. For that we construct the expected inflation in each currency:

$$\mathbb{E}[\pi^*] = \frac{\sum_{s' \in S} Q(s'|s)\, p^*_{(n)}(s)}{p^*(s)}$$

and

$$\mathbb{E}[\pi] = \frac{\sum_{s' \in S} Q(s'|s)\, p_{(n)}(s)}{p(s)}.$$

We then update the policy rates by:

$$R^{*,a}_{(j+1)} = \frac{(1 + i^{*,a})}{(1 + \pi^*)} \text{ for } a \in \{m, w\}$$

and

$$R^{a}_{(j+1)} = \frac{(1 + i^{a})}{(1 + \pi)} \text{ for } a \in \{m, w\}.$$

2.e Repeat steps 2.a-2.d, unless

$$\left\{R^m_{(j)}(X), R^{*,m}_{(j)}(X), R^w_{(j)}(X), R^{*,w}_{(j)}(X)\right\}$$

is close to

$$\left\{R^m_{(j+1)}(X), R^{*,m}_{(j+1)}(X), R^w_{(j+1)}(X), R^{*,w}_{(j+1)}(X)\right\}.$$

If the real policy rates have converged, update prices according to

$$p^*_{(n+1)}(X) = \Delta^{greed} p^* + \left(1 - \Delta^{greed}\right) p^*_{(n)}(X)$$

and

$$p_{(n+1)}(X) = \Delta^{greed} p + \left(1 - \Delta^{greed}\right) p^*_{(n)}(X)$$

and proceed back to the outer-loop.

G Additional Tables: Empirical Analysis

Table G1: Exchange Rates and Liquidity Ratio and VIX: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	U.K.
$\Delta(\text{Liq}_t)$	0.198***	0.178***	0.082*	-0.119**	0.225***	0.137**	0.175***	0.126**	0.141***
	(4.283)	(3.250)	(1.935)	(-2.528)	(3.762)	(2.429)	(3.206)	(2.426)	(3.044)
$\pi_t - \pi_t^*$	-0.668***	-0.386**	-0.278	-0.007	-0.523**	-0.051	-0.407**	-0.517**	-0.271
	(-3.354)	(-2.008)	(-1.491)	(-0.050)	(-2.561)	(-0.382)	(-2.315)	(-2.411)	(-1.622)
ΔVIX_t	0.124***	0.344***	0.220***	-0.077**	0.285***	0.239***	0.189***	0.064*	0.099***
	(3.841)	(8.995)	(7.399)	(-2.350)	(6.792)	(6.116)	(4.991)	(1.784)	(3.163)
Liq_{t-1}	0.009**	0.006	0.007*	0.002	0.005	0.010*	0.007	0.005	0.008
	(2.059)	(1.204)	(1.770)	(0.306)	(0.995)	(1.877)	(1.330)	(1.047)	(1.562)
Constant	-0.010***	-0.004	-0.006*	-0.001	-0.006	-0.007*	-0.008**	-0.015***	-0.005
	(-3.056)	(-1.104)	(-1.963)	(-0.193)	(-1.445)	(-1.697)	(-2.052)	(-2.833)	(-1.471)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.16	0.30	0.21	0.04	0.23	0.16	0.14	0.04	0.08

Notes: VIX is taken from the VIXCLS series on FRED. t statistics in parentheses. $p<0.1$, ** $p<0.05$, *** $p<0.01$

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Table G2: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	U.K.
$\Delta(\text{Liq}_t)$	0.370***	0.553***	0.441***	-0.247**	0.440***	0.394**	0.373***	-0.006	0.463***
	(2.997)	(3.573)	(3.702)	(-2.122)	(2.843)	(2.587)	(2.634)	(-0.042)	(3.189)
$\pi_t - \pi_t^*$	-0.834***	-0.733***	-0.534**	0.051	-0.662***	-0.194	-0.519***	-0.371	-0.634***
	(-3.567)	(-3.018)	(-2.364)	(0.359)	(-2.884)	(-1.208)	(-2.631)	(-1.451)	(-2.642)
Liq_{t-1}	0.011**	0.012*	0.014***	-0.001	0.008	0.016**	0.010*	0.004	0.017**
	(2.380)	(2.038)	(2.706)	(-0.161)	(1.423)	(2.418)	(1.779)	(0.703)	(2.553)
ΔVIX_t	0.104***	0.296***	0.183***	-0.066*	0.260***	0.212***	0.168***	0.077**	0.064*
	(2.901)	(6.584)	(5.106)	(-1.885)	(5.665)	(4.883)	(4.092)	(1.970)	(1.701)
Constant	-0.012***	-0.008*	-0.012***	0.002	-0.008*	-0.011**	-0.012**	-0.011*	-0.012**
	(-3.329)	(-1.838)	(-2.988)	(0.317)	(-1.880)	(-2.288)	(-2.514)	(-1.723)	(-2.513)
N	245	245	245	245	245	245	245	245	245

Notes: FFundsSpread is the monthly average of the intra-daily Fed Funds spread: the difference between the high and low Fed funds rate transacted on each day. StDev(XRate), lagged FFundsSpread and lagged Δ(Liq) instrument for Δ(Liq)., StDev(Inf). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01

Table G3: Exchange Rates and Alternative Measure of Liquidity Ratio

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.099***	0.117***	0.069***	-0.012	0.125***	0.101***	0.088***	0.079***	0.103***
	(3.774)	(3.298)	(2.65)	(-0.451)	(3.416)	(3.081)	(2.791)	(2.765)	(4.026)
$\pi_t - \pi_t^*$	-0.836***	-0.612***	-0.393*	-0.082	-0.667***	-0.125	-0.441**	-0.635***	-0.334**
	(-3.661)	(-2.635)	(-1.823)	(-0.541)	(-2.939)	(-0.839)	(-2.261)	(-2.716)	(-1.998)
Liq2_{t-1}	0.004	0.003	0.005*	0.004	0.002	0.006*	0.004	0.003	0.003
	(1.357)	(0.989)	(1.724)	(1.220)	(0.476)	(1.720)	(1.355)	(1.105)	(1.207)
Constant	-0.005***	0.000	-0.002	-0.002	-0.002	-0.001	-0.005*	-0.014***	-0.001
	(-2.625)	(0.079)	(-1.270)	(-0.661)	(-1.005)	(-0.573)	(-1.794)	(-3.172)	(-0.352)
N	235	235	235	235	235	235	235	235	235
adj. R^2	0.09	0.05	0.03	-0.00	0.06	0.04	0.04	0.04	0.06

Note: The alternative measure of the liquidity ratio includes as liabilities "net financing" of broker-dealer banks, as defined by Adrian and Fleming (2005). t statistics in parentheses.
* p<0.1, ** p<0.05, *** p<0.01

Table G4: Exchange Rates and Alternative Measure of Liquidity Ratio with VIX

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.090***	0.092***	0.058**	-0.007	0.106***	0.088***	0.078***	0.074***	0.0965***
	(3.538)	(3.064)	(2.503)	(-0.278)	(3.194)	(2.874)	(2.612)	(2.606)	(3.800)
$\pi_t - \pi_t^*$	-0.649***	-0.332*	-0.269	-0.112	-0.452**	-0.051	-0.380**	-0.568**	-0.261
	(-2.881)	(-1.164)	(-1.389)	(-0.743)	(-2.181)	(-0.364)	(-2.062)	(-2.428)	(-1.583)
ΔVIX_t	0.145***	0.382***	0.240***	-0.092**	0.324***	0.246***	0.218***	0.081**	0.109***
	(4.179)	(9.527)	(7.674)	(-2.586)	(7.281)	(6.071)	(5.443)	(2.157)	(3.300)
Liq2_{t-1}	0.004	0.005	0.005**	0.004	0.003	0.006*	0.005	0.003	0.003
	(1.613)	(1.539)	(1.998)	(1.265)	(0.911)	(1.856)	(1.631)	(1.194)	(1.187)
Constant	-0.004**	-0.001	-0.002	-0.003	-0.003	-0.001	-0.004*	-0.013***	-0.001
	(-2.347)	(-0.464)	(-1.258)	(-0.830)	(-1.183)	(-0.592)	(-1.736)	(-2.923)	(-0.322)
N	235	235	235	235	235	235	235	235	235
adj. R^2	0.15	0.32	0.23	0.02	0.23	0.16	0.15	0.06	0.10

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table G5: Exchange Rates and Alternative Measure of Liquidity Ratio Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.370***	0.511***	0.365***	-0.242*	0.389**	0.458***	0.365**	0.017	0.545***
	(2.764)	(2.904)	(3.058)	(-1.882)	(2.440)	(2.625)	(2.543)	(0.133)	(2.697)
$\pi_t - \pi_t^*$	-1.103***	-0.936***	-0.374	0.109	-0.736**	-0.331	-0.542**	-0.474	-1.038**
	(-3.106)	(-2.559)	(-1.436)	(0.521)	(-2.585)	(-1.502)	(-2.314)	(-1.575)	(-2.419)
Liq2_{t-1}	0.005	0.007	0.007**	0.000	0.004	0.011**	0.007*	0.003	0.010*
	(1.600)	(1.630)	(2.008)	(0.076)	(1.021)	(2.237)	(1.779)	(1.074)	(1.950)
ΔVIX_t	0.113**	0.328***	0.213***	-0.072*	0.292***	0.211***	0.192***	0.086**	0.056
	(2.477)	(5.609)	(4.978)	(-1.676)	(5.423)	(3.845)	(3.905)	(2.136)	(1.007)
Constant	-0.007***	-0.001	-0.004*	0.003	-0.004	-0.003	-0.007**	-0.011*	-0.003
	(-2.681)	(-0.289)	(-1.674)	(0.509)	(-1.478)	(-1.213)	(-2.173)	(-1.914)	(-1.226)
N	234	234	234	234	234	234	234	234	234

Notes: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq2) instrument for Δ(Liq2). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01.

Table G6: Exchange Rates and Alternative Measure of Liquidity Ratio (M2-Currency)

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.084	0.114	0.099	-0.243***	0.214**	0.086	0.074	0.016	0.134**
	(1.285)	(1.318)	(1.574)	(-3.899)	(2.333)	(1.061)	(0.949)	(0.232)	(2.116)
$\pi_t - \pi_t^*$	-0.607***	-0.520**	-0.363*	-0.023	-0.699***	-0.067	-0.360*	-0.417*	-0.217
	(-2.901)	(-2.315)	(-1.792)	(-0.180)	(-3.007)	(-0.467)	(-1.933)	(-1.938)	(-1.385)
Liq3_{t-1}	0.006	0.005	0.008	0.006	0.003	0.010	0.006	0.004	0.005
	(1.170)	(0.789)	(1.503)	(1.131)	(0.424)	(1.565)	(1.080)	(0.803)	(0.930)
Constant	0.003	0.006	0.007	0.008	0.001	0.012	0.004	-0.005	0.005
	(0.474)	(0.777)	(1.157)	(1.285)	(0.111)	(1.482)	(0.553)	(-0.573)	(0.829)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.03	0.02	0.01	0.06	0.03	0.00	0.01	0.01	0.01

Note: The alternative measure of the liquidity ratio includes as liabilities M2 less currency in circulation. t statistics in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table G7: Exchange Rates and Alternative Measure of Liquidity Ratio (M2-Currency) with VIX

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.085	0.117	0.108*	-0.247***	0.209**	0.095	0.083	0.018	0.135**
	(1.352)	(1.580)	(1.906)	(-4.018)	(2.519)	(1.262)	(1.120)	(0.251)	(2.192)
$\pi_t - \pi_t^*$	-0.473**	-0.287	-0.258	-0.037	-0.507**	-0.015	-0.320*	-0.374*	-0.172
	(-2.320)	(-1.484)	(-1.415)	(-0.299)	(-2.390)	(-0.115)	(-1.816)	(-1.744)	(-1.125)
ΔVIX_t	0.147***	0.365***	0.229***	-0.091***	0.309***	0.253***	0.208***	0.078**	0.115***
	(4.471)	(9.539)	(7.795)	(-2.863)	(7.344)	(6.511)	(5.441)	(2.177)	(3.677)
Liq3_{t-1}	0.007	0.008	0.009*	0.006	0.006	0.011*	0.008	0.005	0.005
	(1.501)	(1.429)	(1.884)	(1.088)	(0.890)	(1.858)	(1.402)	(0.930)	(1.020)
_cons	0.005	0.009	0.009	0.007	0.004	0.014*	0.006	-0.003	0.006
	(0.863)	(1.294)	(1.544)	(1.187)	(0.537)	(1.770)	(0.878)	(-0.380)	(0.926)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.10	0.28	0.21	0.08	0.21	0.15	0.11	0.02	0.06

Note: t statistics in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table G8: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio (M2-Currency) Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.180	0.367**	0.272**	-0.220*	0.546***	0.264*	0.338**	-0.080	0.361***
	(1.522)	(2.566)	(2.572)	(-1.950)	(3.242)	(1.868)	(2.424)	(-0.608)	(2.929)
$\pi_t - \pi_t^*$	-0.515**	-0.450**	-0.313*	-0.043	-0.737***	-0.063	-0.388**	-0.308	-0.307*
	(-2.443)	(-2.149)	(-1.666)	(-0.336)	(-3.058)	(-0.452)	(-2.101)	(-1.361)	(-1.796)
Liq3_{t-1}	0.008	0.010*	0.010**	0.006	0.007	0.013**	0.009	0.005	0.008
	(1.556)	(1.658)	(2.117)	(1.128)	(1.047)	(2.046)	(1.615)	(0.908)	(1.463)
ΔVIX_t	0.147***	0.360***	0.229***	-0.092***	0.305***	0.254***	0.208***	0.077**	0.114***
	(4.419)	(9.164)	(7.621)	(-2.860)	(6.984)	(6.425)	(5.296)	(2.118)	(3.544)
Constant	0.005	0.011	0.009*	0.007	0.004	0.015*	0.007	-0.002	0.008
	(0.837)	(1.450)	(1.658)	(1.211)	(0.545)	(1.892)	(0.892)	(-0.202)	(1.228)
N	245	245	245	245	245	245	245	245	245

Notes: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq3) instrument for Δ(Liq3). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01

Table G9: First Stage Regressions for Each Measure of the Liquidity Ratio (ΔLiq)

	Constant	St.Dev Inf	St.Dev Dep	$\Delta(\text{Liq(-1)}_t)$	$\Delta(\text{FFSpread(-1)}_t)$
$\Delta(\text{Liq1}_t)$	-0.027***	0.39	1.396***	0.130**	0.035***
	(2.88)	(0.45)	(25.14)	(2.14)	(2.64)
$\Delta(\text{Liq2}_t)$	-0.035***	0.132	2.001***	-0.002	0.006
	(-1.86)	(0.08)	(4.11)	(-0.03)	(0.23)
$\Delta(\text{Liq3}_t)$	0.012*	0.299	0.605***	0.459***	0.038***
	(-1.94)	(0.57)	(3.62)	(8.13)	(4.16)

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G10: Correlation: Measures of Liquidity Ratio (Liq) and Convenience Yield ($ConvYd$) on 1-year U.S. Treasury notes in Dynamic Regression

	Const.	Δ(Conv Yd$_t$)	Δ(Conv Yd(-1)$_t$)	Δ(Conv Yd(-2)$_t$)	Δ(Liq(-1)$_t$)	Δ(Liq(-2)$_t$)
Δ(Liq1$_t$)	0.004	4.391*	9.370***	1.541	0.162**	0.170**
	(1.44)	(1.72)	(3.62)	(0.63)	(2.19)	(2.31)
Δ(Liq2$_t$)	0.005	3.649	9.658*	6.298	0.022	0.057
	(1.03)	(0.74)	(1.96)	(1.29)	(0.31)	(0.79)
Δ(Liq3$_t$)	0.002	-1.210	6.674***	5.000***	0.478***	-0.003
	(1.47)	(-0.75)	(4.07)	(2.99)	(6.80)	(-0.04)
Δ(Liq1$_t$)	0.004	-	8.708***	1.107	0.119*	0.163**
	(1.56)		(3.40)	(0.45)	(1.68)	(2.21)
Δ(Liq2$_t$)	0.005	-	9.132*	5.772	0.012	0.055
	(1.06)		(1.88)	(1.20)	(0.17)	(0.78)
Δ(Liq3$_t$)	0.002	-	6.916***	5.233***	0.483***	0.006
	(1.42)		(4.34)	(6.96)	(3.18)	(0.08)

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G11: Relationship of Exchange Rates and Liquidity Ratio Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
Δ(Liq$_t$)	0.240***	0.310***	0.143**	-0.144**	0.338***	0.203***	0.217***	0.198***	0.152**
	(3.781)	(3.816)	(2.380)	(-2.371)	(4.106)	(2.664)	(2.807)	(2.771)	(2.587)
$\pi_t - \pi_t^*$	-1.095***	-0.725**	-0.601**	-0.261	-0.615**	-0.199	-0.522**	-0.713**	-0.225
	(-3.565)	(-2.540)	(-2.372)	(-1.133)	(-2.255)	(-1.093)	(-2.125)	(-2.454)	(-1.041)
Liq$_{t-1}$	0.008	-0.009	-0.001	-0.002	-0.003	0.001	-0.002	-0.003	0.003
	(1.181)	(-1.054)	(-0.077)	(-0.300)	(-0.309)	(0.102)	(-0.282)	(-0.381)	(0.360)
Constant	-0.011***	-0.001	-0.006	-0.008	-0.005	-0.006	-0.008	-0.017***	-0.004
	(-2.811)	(-0.109)	(-1.633)	(-1.007)	(-1.009)	(-1.399)	(-1.565)	(-2.636)	(-0.890)
N	135	135	135	135	135	135	135	135	135
adj. R^2	0.13	0.11	0.05	0.04	0.12	0.03	0.05	0.06	0.03

Note: . t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G12: Exchange Rates and Liquidity Ratio with VIX Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.182***	0.176**	0.061	-0.114*	0.248***	0.134*	0.136*	0.169**	0.137**
	(2.856)	(2.467)	(1.102)	(-1.831)	(3.127)	(1.767)	(1.827)	(2.286)	(2.250)
$\pi_t - \pi_t^*$	-0.846***	-0.391	-0.362	-0.342	-0.443*	-0.109	-0.423*	-0.632**	-0.193
	(-2.765)	(-1.586)	(-1.563)	(-1.464)	(-1.724)	(-0.614)	(-1.831)	(-2.146)	(-0.879)
ΔVIX_t	0.175***	0.430***	0.266***	-0.089*	0.301***	0.212***	0.267***	0.087	0.044
	(3.284)	(7.211)	(5.700)	(-1.730)	(4.551)	(3.493)	(4.409)	(1.453)	(0.896)
Liq_{t-1}	0.009	-0.004	0.001	-0.003	-0.000	0.003	0.001	-0.002	0.003
	(1.422)	(-0.541)	(0.243)	(-0.406)	(-0.047)	(0.415)	(0.149)	(-0.216)	(0.348)
Constant	-0.010***	-0.003	-0.005	-0.010	-0.005	-0.006	-0.008	-0.016**	-0.003
	(-2.694)	(-0.679)	(-1.554)	(-1.283)	(-1.191)	(-1.395)	(-1.608)	(-2.444)	(-0.827)
N	135	135	135	135	135	135	135	135	135
adj. R^2	0.19	0.36	0.24	0.06	0.23	0.11	0.17	0.06	0.03

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G13: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq) instrument for Δ(Liq) Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.324**	0.579***	0.418***	-0.109	0.475***	0.495**	0.397**	0.142	0.498***
	(2.183)	(3.209)	(2.977)	(-0.736)	(2.647)	(2.442)	(2.169)	(0.803)	(2.889)
$\pi_t - \pi_t^*$	-1.019***	-0.767**	-0.662**	-0.343	-0.586**	-0.391	-0.634**	-0.591*	-0.643**
	(-2.887)	(-2.472)	(-2.314)	(-1.254)	(-2.064)	(-1.622)	(-2.269)	(-1.677)	(-2.019)
Liq_{t-1}	0.010	-0.002	0.004	-0.002	0.002	0.006	0.002	-0.001	0.013
	(1.497)	(-0.190)	(0.613)	(-0.355)	(0.188)	(0.664)	(0.306)	(-0.170)	(1.329)
ΔVIX_t	0.142**	0.331***	0.186***	-0.092	0.250***	0.133*	0.212***	0.091	-0.036
	(2.246)	(4.333)	(3.068)	(-1.526)	(3.257)	(1.715)	(2.945)	(1.298)	(-0.557)
Constant	-0.012***	-0.006	-0.011**	-0.010	-0.008	-0.012**	-0.012**	-0.015*	-0.011**
	(-2.816)	(-1.246)	(-2.438)	(-1.078)	(-1.596)	(-2.106)	(-2.136)	(-1.889)	(-1.983)
N	134	134	134	134	134	134	134	134	134

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G14: Relationship of Exchange Rates and Liquidity Ratio May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.208***	0.172*	0.126	-0.051	0.181*	0.218**	0.252***	0.043	0.184**
	(2.927)	(1.735)	(1.657)	(-0.624)	(1.663)	(2.077)	(2.894)	(0.583)	(2.290)
$\pi_t - \pi_t^*$	-0.451	-0.399	0.181	0.074	-0.798	-0.169	-0.273	-0.408	-0.660*
	(-1.551)	(-0.847)	(0.445)	(0.320)	(-1.365)	(-0.623)	(-0.940)	(-1.132)	(-1.802)
Liq_{t-1}	0.017	0.019	0.012	0.002	0.004	0.015	0.006	0.019	0.033
	(0.976)	(0.670)	(0.614)	(0.061)	(0.187)	(0.607)	(0.328)	(1.031)	(1.507)
Constant	-0.016	-0.013	-0.008	0.002	-0.004	-0.008	-0.004	-0.024	-0.027
	(-0.973)	(-0.498)	(-0.439)	(0.079)	(-0.190)	(-0.357)	(-0.248)	(-1.165)	(-1.386)
N	111	111	111	111	111	111	111	111	111
adj. R^2	0.06	0.00	0.01	-0.02	0.01	0.02	0.06	-0.01	0.04

Note: t statistics in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table G15: Exchange Rates and Liquidity Ratio with VIX May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.206***	0.159*	0.125*	-0.050	0.174*	0.212**	0.251***	0.043	0.181**
	(2.940)	(1.807)	(1.790)	(-0.621)	(1.770)	(2.247)	(2.958)	(0.572)	(2.389)
$\pi_t - \pi_t^*$	-0.408	-0.090	0.103	0.072	-0.402	-0.057	-0.235	-0.393	-0.607*
	(-1.419)	(-0.214)	(0.274)	(0.314)	(-0.752)	(-0.233)	(-0.829)	(-1.086)	(-1.755)
ΔVIX_t	0.074**	0.255***	0.170***	-0.082*	0.265***	0.257***	0.116**	0.026	0.152***
	(1.992)	(5.472)	(4.565)	(-1.946)	(4.996)	(5.100)	(2.557)	(0.653)	(3.794)
Liq_{t-1}	0.016	0.008	0.015	0.001	0.006	0.012	0.007	0.019	0.032
	(0.950)	(0.321)	(0.836)	(0.051)	(0.289)	(0.541)	(0.358)	(1.015)	(1.548)
Constant	-0.015	-0.003	-0.011	0.002	-0.005	-0.006	-0.004	-0.024	-0.026
	(-0.933)	(-0.148)	(-0.649)	(0.090)	(-0.233)	(-0.293)	(-0.259)	(-1.138)	(-1.422)
N	111	111	111	111	111	111	111	111	111
adj. R^2	0.09	0.22	0.17	0.00	0.19	0.20	0.10	-0.02	0.15

Note: t statistics in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table G16: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: StDev(Inf), StDev(XRate), lagged FFundsSpread, lagged Δ(Liq), and Δ(USIndProd) instrument for Δ(Liq) May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.380*	0.191	0.361	-0.236	0.461	0.687**	0.257	0.030	0.360
	(1.745)	(0.755)	(1.648)	(-0.945)	(1.498)	(2.185)	(1.027)	(0.139)	(1.546)
$\pi_t - \pi_t^*$	-0.489	-0.116	0.046	0.146	-0.495	-0.145	-0.234	-0.385	-0.713*
	(-1.574)	(-0.251)	(0.115)	(0.580)	(-0.878)	(-0.520)	(-0.825)	(-0.998)	(-1.887)
Liq_{t-1}	0.016	0.009	0.013	-0.002	0.002	0.009	0.007	0.019	0.033
	(0.893)	(0.338)	(0.684)	(-0.081)	(0.080)	(0.342)	(0.346)	(1.013)	(1.559)
ΔVIX_t	0.073*	0.255***	0.169***	-0.082*	0.262***	0.254***	0.116**	0.026	0.151***
	(1.902)	(5.444)	(4.328)	(-1.884)	(4.764)	(4.527)	(2.556)	(0.655)	(3.672)
Constant	-0.014	-0.004	-0.008	0.005	0.000	0.000	-0.004	-0.024	-0.027
	(-0.859)	(-0.160)	(-0.448)	(0.205)	(0.021)	(0.001)	(-0.244)	(-1.128)	(-1.396)
N	111	111	111	111	111	111	111	111	111

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table G17: Relationship of Exchange Rates and Liquidity Ratio in Foreign Related Banks

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	U.K.
$\Delta(\text{LiqRatFRB}_t)$	0.139***	0.200***	0.161***	-0.137***	0.184***	0.151***	0.122***	0.039	0.171***
	(3.188)	(3.622)	(3.505)	(2.764)	(2.767)	(2.485)	(2.016)	(0.734)	(3.026)
$\pi_t - \pi_t^*$	-0.566***	-0.450***	-0.377*	-0.098	-0.577***	-0.049	-0.399**	-0.377***	-0.304*
	(-2.519)	(-2.053)	(-1.898)	(-0.737)	(-2.455)	(-0.347)	(-2.079)	(-1.735)	(-1.733)
LiqRatFRB_{t-1}	0.005*	0.006*	0.006*	0.004	0.003	0.007*	0.005*	0.003	0.004
	(1.782)	(1.902)	(2.207)	(1.366)	(0.881)	(2.177)	(1.792)	(1.167)	(1.431)
$\Delta(\text{VIX}_t)$	0.127***	0.311***	0.206***	-0.069**	0.279***	2.33***	0.190***	0.072*	0.088**
	(3.406)	(7.598)	(6.430)	(-1.973)	(5.982)	(5.507)	(4.593)	(1.936)	(2.501)
Constant	0.001	0.006	0.005	0.003	0.001	0.007	0.002	-0.006	0.004
	(0.333)	(1.452)	(1.583)	(0.860)	(0.156)	(1.771)	(0.453)	(-1.039)	(1.017)
N	245	245	245	245	245	245	245	245	245

Note: The liquidity ratio is for foreign-related bank subsidiaries and branches located in the U.S. Liquid assets are the sum of cash assets (CASFRIW027SBOG from FRED) and Treasury and agency securities (TASFRIW027SBOG). Short-term liabilities are the sum of deposits (DPSFRIW027SBOG) and borrowings (H8B3094NFRD). t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01