Scrambling for Dollars: International Liquidity, Banks and Exchange Rates *

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Abstract

We develop a theory of exchange rate fluctuations arising from financial institutions' demand for liquid assets. Financial flows are unpredictable and may leave banks "scrambling for dollars." Because of settlement frictions in interbank markets, a precautionary demand for dollar reserves emerges, giving rise to an endogenous dollar convenience yield. We find that US dollar funding risk fluctuations contribute significantly to deviations from covered interest parity and exchange rate fluctuations. We show evidence of a tight connection between exchange rate fluctuations for the G10 currencies and the quantity of dollar liquidity consistent with the theory.

Keywords: Exchange rates, liquidity premia, monetary policy

JEL Classification: E44, F31, F41, G20

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1 Introduction

The well-known "disconnect" in international finance holds that foreign exchange rates show little empirical relationship to the macro variables, such as interest rates and output (Obstfeld and Rogoff, 2000). More recent work contends that the source of the disconnect stems from financial markets (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021). Moreover, there has long been evidence of time-varying expected excess returns in foreign exchange markets. Furthermore, the US dollar appears exceptional, as dollar-denominated assets offer lower average returns than the other major currencies when measured on historical data (Gourinchas and Rey, 2007). To account for the exchange rate disconnect and associated puzzles, the literature has turned to models with currency excess returns as the potential "missing link." The source or sources of these excess returns, however, remain elusive.

In this paper, we develop a theory of exchange rate fluctuations arising from the liquidity demand by financial institutions within an imperfect interbank market. We build on two observations of the international monetary system. First, US dollars are the dominant source of foreign currency funding. According to the BIS locational banking statistics, in March 2021, the global banking and non-bank financial sector had cross-border dollar liabilities of over $11 trillion. Second, there is an inherent instability of dollar funding. As documented, for example, in Acharya, Afonso and Kovner (2017), banks are occasionally subject to considerable funding uncertainty or interbank market freezing, which can leave them "scrambling for dollars." Narrative discussions attribute fluctuations in the US dollar exchange rate to such vicissitudes in the short-term international money markets. A contribution of our paper is to develop a formal framework to articulate this channel. We further provide quantitative analysis that estimates series for global dollar funding levels and uncertainty shocks. We provide evidence of a positive and significant statistical relationship between bank liquid dollar holdings and exchange-rate fluctuations and use our model to interpret that feature as evidence of the importance of dollar funding uncertainty shocks.

In our framework, financial institutions—hereafter referred to simply as "banks"—manage assets and liabilities in two currencies. Banks face the risk of sudden outflows of liabilities. If a bank is short of liquid assets to settle those flows, it needs to find a counterparty that provides the liquidity. However, there are times when banks may lose confidence in each other and, as a result, face tighter frictions in the interbank market. As insurance against these outflows, banks maintain a buffer of liquid assets, especially dollar liquid assets, in line with the aforementioned observations on the international financial system. As funding risk and interbank market frictions fluctuate over time, they alter the relative demand for currencies, resulting in movements in the exchange rate.

The theory uncovers how frictions in the settlement of international deposit flows project into a dollar liquidity premium. This dollar liquidity premium generates a time-varying wedge in the interest parity condition, or "convenience yield," which plays a pivotal role in the determination of the exchange rate. Critically, the convenience yield is endogenous and depends on the quantity of outside money (liquid assets) and policy rates, the matching frictions in the interbank market, and the volatility of deposit flows in different currencies. Through this endogenous convenience yield, we link nominal exchange rates and the dollar liquidity premium to the reserve position of banks in different currencies, funding risk, and confidence in the interbank market.

On the surface, the model resembles the seminal monetary exchange rate model of Lucas (1982). In that model, the two currencies earn a liquidity premium over bonds because the goods in each country must be bought with the local currency. A money demand equation determines the price levels in both currencies, and relative prices determine the exchange rate. Our model shares Lucas's segmentation of transactions and exchange rate determination. However, in our model, the demand for reserves in either currency stems from the precautionary demand by banks. This implies different predictions of how the exchange rate reacts to aggregate shocks and policy.

In particular, we show how the model can rationalize why the dollar tends to appreciate in times of high volatility—a phenomenon that remains elusive for existing open-economy models. Models of excess currency returns based on risk premia can account for the excess dollar returns by positing that the dollar appreciates during global downturns. However, they do not explain why the dollar appreciates during global downturns in the first place. Models of financially constrained intermediaries provide an alternative channel for excess dollar returns. Yet, these models predict that the US dollar should depreciate in a downturn, as the United States withstands a larger share of losses in a global economic downturn. [1]

We find that when there are changes in interest rates paid on reserves, the liquidity demand induces an attenuation effect on exchange rates. To understand why, consider an initial situation where the two currencies have the same interest rate, and the exchange rate is expected to be constant. Suppose that the dollar interest rate goes up. Given

[1] Maggiori (2017) shows that if the US has a larger capacity to withstand risk in a global downturn, US households bear a larger share of losses relative to the rest of the world in a global downturn. With home bias, the dollar must experience a real depreciation in a global downturn. Farhi and Gabaix (2016), Hassan (2013), and Richmond (2019) offer models in which nontraded goods in the U.S. may become relatively scarce during global downturns, leading to an appreciation.

exchange rates, banks have incentives to shift their portfolio towards the dollar. As banks become relatively more satiated with dollars, the convenience yield falls. No arbitrage then requires that the dollar appreciates but *less* than it would in the absence of the endogenous convenience yield.

We provide empirical evidence that supports the theoretical link between the balance sheet of the banking sector and the US dollar exchange rate. According to the theory, the financial sector increases its demand for liquid dollar assets relative to dollar funding—US government obligations, including Treasuries and reserves held at the Federal Reserve—when funding becomes more uncertain, and this, in turn, translates into an appreciation of the dollar. In particular, the theory provides a tight prediction for the results of a regression of the exchange rate on banks' liquidity ratio as a function of the underlying shocks. The relationship in the data aligns remarkably well with the model. Moreover, the findings are robust to multiple specifications, including controlling for VIX —- a variable that captures a broad measure of uncertainty and has been shown to have significant explanatory power for exchange rates (Brunnermeier, Nagel and Pedersen, 2008; Lilley, Maggiori, Neiman and Schreger, 2019). In contrast to, for example, Jiang, Krishnamurthy and Lustig (2021), and Engel and Wu (2023), we link a quantity-based measure of liquidity demand (the aggregate liquidity ratio of banks), rather than a market price (convenience yields) to the dollar exchange rate. This demonstrates that at least in one dimension, there is not a "disconnect" between the dollar exchange rate and aggregate economic variables.

We calibrate and simulate our model to evaluate the quantitative importance of funding risk in driving exchange rate fluctuations. Our approach consists of constructing a series for funding risk that is consistent with banks' balance sheets and observed spreads in the interbank market. Using the constructed series, we estimate a regime switching process and show how shocks to US dollar funding risk are important drivers of deviations from exchange rates and deviations from CIP.

Literature Review. Our paper contributes to the literature on exchange rates. Empirically, numerous studies have highlighted various failures of the canonical international macro model built on uncovered interest parity. Most notably, these shortcomings include the disconnect that exists in the data between exchange rates and macroeconomic fundamentals ("exchange rate disconnect"), and the inconsistency between differences in nominal rates and expected exchange rate movements (”forward premium puzzle”).[2]

On the theoretical front, a voluminous literature has aimed to address the aforementioned puzzles. Broadly speaking, one can group this literature among three different strands. A first strand of the literature has introduced exogenous convenience yields, for example, by introducing bonds in the utility function. Examples following this route include Engel (2016), Valchev (2020), Jiang, Krishnamurthy and Lustig (2023), and Kekre and Lenel (2024). While this approach has proven useful in accounting for exchange rate fluctuations, it leaves unexplained the source of the convenience yield that causes the fluctuations in exchange rates.

A second strand of the literature has focused on risk premia as a key driver of deviations from uncovered interest parity. This includes work on disaster risk (Farhi and Gabaix, 2016), consumption habits (Verdelhan, 2010), or long-run risk (Bansal and Shaliastovich, 2013; Colacito and Croce, 2011; Colacito, Croce, Ho and Howard, 2018). As in the closed-economy equity-premium puzzle literature, departing from standard preferences for consumption allows the model to generate substantial risk premia and can help address some of the shortcomings of the canonical international macro model.

A third strand of the literature has turned to models with segmented markets and frictions on financial intermediaries, which give rise to limits to international arbitrage. Gabaix and Maggiori (2015) develops a two-country model where households in each country trade local currency bonds with global financial intermediaries subject to a leverage constraint. They show how portfolio flows determine exchange rates and trace the implications for output and risk-sharing.[3] Itskhoki and Mukhin (2021) develop an international real business cycle model and show how financial shocks and conventional shocks can jointly account for exchange rates and business cycle moments. Following this literature, Gourinchas, Ray and Vayanos (2022), and Greenwood, Hanson, Stein and Sunderam (2023) introduce maturity in models with downward sloping demands and examine the implications for yield curves and Koijen and Yogo (2020)—with a more empirical approach—build a demand system to estimate elasticities of exchange rates to capital flows.

Our paper offers a distinct explanation for deviations from uncovered interest parity

[2]See Meese and Rogoff (1983); Fama (1984); Obstfeld and Rogoff (2003). An active literature revisiting these puzzles and other important features of exchange rates includes Hassan and Mano (2019); Kalemli-Özcan (2019); Kalemli-Özcan and Varela (2021); Brunnermeier, Nagel and Pedersen (2008); Lilley, Maggiori, Neiman and Schreger (2019). See Engel (1996, 2014) for surveys of the literature.

[3]See also Amador, Bianchi, Bocola and Perri (2020); Fanelli and Straub (2021) for related examples, and Maggiori (2022) for a comprehensive review of this literature. An early paper examining segmentation in domestic markets is Alvarez, Atkeson and Kehoe (2009). Bacchetta and Van Wincoop (2010) explores portfolio choice constraints to explain the forward discount puzzle.

compared to these strands of the literature, and we argue that it provides unique predictions for exchange rates. As previously mentioned, a key challenge existing theories face is explaining the safe-haven nature of the US dollar—often referred to as the *"reserve currency paradox"*(Maggiori, 2017). In terms of the risk-premia channel, the dollar earns a lower expected return because it is perceived to appreciate during a global crisis. However, the reasons behind the dollar's appreciation are largely left unexplained.[4] Moreover, a challenge for this literature is that if exchange rates were indeed primarily driven by risk considerations, increased demand for dollars due to insurance motives should not occur during a global crisis, since the adverse shock would have already materialized.[5] Regarding models with balance sheet constraints on intermediaries, if the US is endowed with a greater capacity to absorb losses during a global economic downturn, its real exchange rate should depreciate. This is because consumption would fall more sharply in the US compared to the rest of the world. Our model provides a natural explanation for the US dollar's safe-haven status. In times of heightened uncertainty, the liquidity properties of the US dollar become increasingly valuable, causing banks to scramble for dollars and leading to an appreciation of the dollar.

Our paper also relates to an emerging literature on interbank market frictions and monetary policy (see e.g. Bianchi and Bigio, 2022; De Fiore, Hoerova and Uhlig, 2018; Piazzesi and Schneider, 2021 and Weill, 2020 for a review). To the best of our knowledge, our paper provides the first attempt at incorporating the microstructure of interbank lending in an open economy framework. Specifically, our model builds on Bianchi and Bigio (2022), extending their closed economy framework to an open economy setting and introducing aggregate funding uncertainty to develop our theory of exchange rate fluctuations.

Outline. The paper is organized as follows. Sections 2 and 3 present the model and theoretical analysis. Section 4 presents the empirical analysis. Section 6 concludes. All proofs are in the Appendix.

[4] In Farhi and Gabaix (2016), the dollar experiences a real appreciation when the risk of a disaster increases because the disaster is assumed to disproportionately affect the non-tradable sector of the US economy. See also Hassan (2013) for an explanation based on country size, and Richmond (2019) for an explanation based on the centrality of the U.S. in exports of intermediate goods.

[5] Motivated by the "reserve currency paradox", Kekre and Lenel (2024) develops a rich two-country model with nominal rigidities, exogenous convenience yields, and Epstein-Zin preferences. In their model, however, the flight to safety is modeled exogenously as an increased preference for foreign currency bonds, and therefore uncertainty plays no role.

2 A Model of Banking Liquidity and Exchange Rates

We present a dynamic equilibrium model of global banks that intermediate international financial flows and are subject to idiosyncratic liquidity shocks. The model has two economies, the EU and the US, with corresponding currencies and central banks. We label the euro as the domestic currency and the dollar as the foreign currency. A representative global household and a single final tradable good is produced by a continuum of multinational firms.

2.1 Environment

Timing. Time is discrete and has an infinite horizon. Every period is divided into two sub-stages: the lending and balancing stages. In the lending stage, banks make their equity payout, Div_t, and portfolio decisions. In the balancing stage, banks face liquidity shocks and re-balance portfolios.

Notation. We use an asterisk to denote the foreign currency (i.e., the dollar) variable. The exchange rate is defined as the amount of euros necessary to purchase one dollar—hence, a higher e indicates an *appreciation* of the dollar. We index the vector of aggregate shocks by X and use i_t to denote an interest rate paid in period t (and determined in period $t-1$).

Preferences and budget constraint. Banks' payouts are distributed to households that own banks' shares and have linear utility with discount factor β. A bank's objective is to maximize shareholders' value, and therefore it maximizes the net present value of dividends:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \cdot Div_t. \tag{1}$$

Banks enter the lending stage with a portfolio of assets and liabilities. The portfolio includes liquid assets in euros and dollars, m_t and m_t^*, and loans b_t, which are denominated in consumption goods and pay a real return R_t^b. We will refer to liquid assets as "reserves" for simplicity, this term should be understood as also encompassing government bonds—the critical property, as we will see, is that these are assets that can be used as settlement instruments.[6]

On the liability side, banks obtain funding via demand deposits, d_t and d_t^*, discount window loans, w_t and w_t^*, and net interbank loans, f_t and f_t^* (which are negative if the bank has lent funds). Deposit and interbank market loans have market returns given by i_t^d and i_t^f, while central banks set the corridor rates for reserves and the discount window, which are i_t^m and i_t^w, respectively.

The bank's budget constraint, expressed in dollars, is given by

$$P_t^* Div_t + \frac{m_{t+1} - d_{t+1}}{e_t} + b_{t+1}P_t^* + m_{t+1}^* - d_{t+1}^* \leq \; P_t^* b_t R_t^b + m_t^*(1 + i_t^{m^*}) - d_t^*(1 + i_t^{d,*})$$
$$- f_t^*(1 + i_t^{f,*}) - w_t^*(1 + i_t^{w,*}) + \frac{m_t(1 + i_t^m) - d_t(1 + i_t^d) - f_t(1 + i_t^f) - w_t(1 + i_t^w)}{e_t}. \quad (2)$$

At the beginning of each period, a bank pays the interest on its liabilities, collects the interest on its assets, issues new liabilities, and buys new assets.

Withdrawal shocks. In the balancing stage, banks are subject to random withdrawal of deposits in both currencies. As in Bianchi and Bigio (2022), withdrawals have zero mean—hence, deposits are reshuffled but preserved within the banking system. We allow for time-varying volatility of these shocks, which, as we will see, play an essential role in driving exchange rate fluctuations. We denote by ω the withdrawal shock and use ϕ_t and Φ_t to denote the density and CDF. When $\omega > 0$, a bank receives an inflow of deposits; when $\omega < 0$, a bank faces an outflow.

The inflow and outflow of deposits across banks generate a transfer of liabilities across banks. We assume that these transfers are settled using reserves of the corresponding currency. Importantly, reserves for individual banks must remain positive at the end of the period. We denote by s_t^j the euro reserve balances of a bank when faced with a withdrawal shock ω_t^j on its euro deposits. This balance is given by

$$s_t^j = m_{t+1} + \omega_t^j d_{t+1}.$$

[6] That is, our analysis is not about the management of scarce reserves per se but more broadly about liquidity management. As it is often discussed, banks have had abundant excess reserves for the most part since the 2008 financial crisis—see however, the work by Copeland, Duffie and Yang (2024) showing that reserves at various points in the post-crisis period were not so ample owing to the series of new liquidity regulations. In any case, liquidity concerns have remained a first-order concern for financial institutions, as evidenced by observed measures of liquidity premia as well as the Senior Financial Officer Survey.

Higher liquidity holdings m_{t+1} make the bank more likely to end with a surplus.[7] In particular, if a bank faces a withdrawal shock $\omega < -m_{t+1}/d_{t+1}$, it will end with a deficit reserve balance. Otherwise, the bank has a surplus. Similarly, for dollars, we have that

$$s_t^{j,*} = m_{t+1}^* + \omega_t^{j,*} d_{t+1}^*.$$

Interbank market. After withdrawal shocks are realized, there is a distribution of bank surplus and bank deficit balances in both currencies. We assume there is an interbank market for each currency, in which banks with a deficit balance in one currency borrow from those with a surplus balance. These two interbank markets behave symmetrically, so it suffices to show only how one of them works.[8] Figure 1 presents a sketch of the timeline of decisions within each period. We next describe the bank optimization problem.

We model the interbank market as an over-the-counter (OTC) market. Modeling the interbank market using search and matching is natural, considering that the interbank market is a credit market in which banks on different sides of the market—surplus and deficit—must find a counterparty they trust (see, Ashcraft and Duffie, 2007 and Afonso and Lagos, 2015). Our specific formulation follows Bianchi and Bigio (2024; 2022) which, in turn, integrates elements from Atkeson, Eisfeldt and Weill (2015) and Afonso and Lagos (2015).

As a result of the matching frictions, only a fraction of an individual bank's surplus or deficit is transacted in the interbank market. A bank with surplus s^j is able to lend a fraction Ψ_t^+ to other banks while the remaining surplus is kept in reserves. Conversely, a deficit bank can only secure a fraction Ψ_t^-. The remainder of the deficit is borrowed at the penalty rate i_t^w. The penalty rate can be interpreted as the discount window rate or an overdraft rate charged by correspondent banks with access to the Fed's discount window.

The fractions of balanced matched Ψ_t^+ and Ψ_t^- are endogenous objects that depend on the aggregate reserve deficit balances relative to surplus balances. Assuming a constant return to scale matching function, the probabilities are only a function of market tightness, which is defined as

$$\theta_t \equiv S_t^- / S_t^+, \tag{3}$$

[7] We omit the superscript j from bank portfolio choices because it is without loss of generality that all banks make the same choices in the lending stage.

[8] We assume a stark form of segmented interbank markets: dollar surpluses cannot be used to patch euro deficits and vice versa. This assumption can be relaxed to some extent. Still, some form of asset market segmentation is necessary to obtain liquidity premia and rule out Kareken and Wallace (1981)'s exchange rate indeterminacy. Section **??** discusses an extension of the baseline model.

where $S_t^+ \equiv \int_0^1 \max\left\{s_t^j, 0\right\} dj$ and $S_t^- \equiv -\int_0^1 \min\left\{s_t^j, 0\right\} dj$ denote the aggregate surplus and deficit, respectively. Notice that because $m \geq 0$ and $\mathbb{E}(\omega) = 0$, we have that in equilibrium, $\theta \leq 1$. That is, there is a relatively larger mass of banks in surplus than in deficit.

The interbank market rate results from a bargaining problem between banks in deficit and those in surplus. There are multiple trading rounds in which banks trade with each other. If banks cannot match by the end of the trading rounds, they deposit the surplus of reserves at the central bank or borrow from the discount window. Throughout the trading, the terms of trade at which banks borrow and lend— the interbank market rate— depends on the probability of finding a match in the future rounds.[9] Notice that we used i^f in the budget constraint (2) to denote the average interbank market rate at which banks trade. Ultimately, we can define a liquidity yield function χ that captures the benefit of having a real surplus $\tilde{s}$ (or the cost of having a real deficit) upon facing the withdrawal shock as follows:

$$\chi(\theta, \tilde{s}; X, X') = \begin{cases} \chi^+(\theta; X, X')\tilde{s} & \text{if } \tilde{s} \geq 0, \\ \chi^-(\theta; X, X')\tilde{s} & \text{if } \tilde{s} < 0 \end{cases} \tag{4}$$

where χ^+ and χ^- are given by

$$\chi^+(\theta; X, X') = \Psi^+(\theta)[R^f(X, X') - R^m(X, X')], \tag{5}$$

$$\chi^-(\theta; X, X') = \Psi^-(\theta)[R^f(X, X') - R^m(X, X')] + (1 - \Psi^-(\theta))[R^w(X, X') - R^m(X, X')]. \tag{6}$$

In these expressions, $R^y(X, X')$ denotes the expected real rate of return on an asset or liability y when the initial state is X and the next period state is X'. Note that the expected return depends on X' because the nominal rate is pre-determined, but the realized real return depends on the realized inflation rate. In particular, we have that $R^y(X, X') \equiv (1 + i^y(X))/(1 + \pi(X, X'))$, where $\pi(X, X') \equiv P(X')/P(X) - 1$ denotes the inflation rate. When it does not lead to confusion, we streamline the argument (X, X') in these expressions. We will also use 'bars' to denote expected returns. That is, $\bar{R}^y \equiv \mathbb{E}\left[R^y(X, X')|X\right]$ and $\bar{\chi} = \mathbb{E}\left[\chi(\theta, \tilde{s}; X, X')|X\right]$.

Equation (5) reflects that the benefit of lending in the interbank market in the case of surplus is $R^f - R^m$. By the same token, (6) reflects that borrowing from the interbank market and the discount window costs respectively $R^f - R^m$ and $R^w - R^m$.

[9]Multiple trading rounds imply that interbank market rates vary with the tightness of the interbank market. With a single trading round, the interbank market rate would be a constant that depends on policy rates but not on the interbank-market tightness.

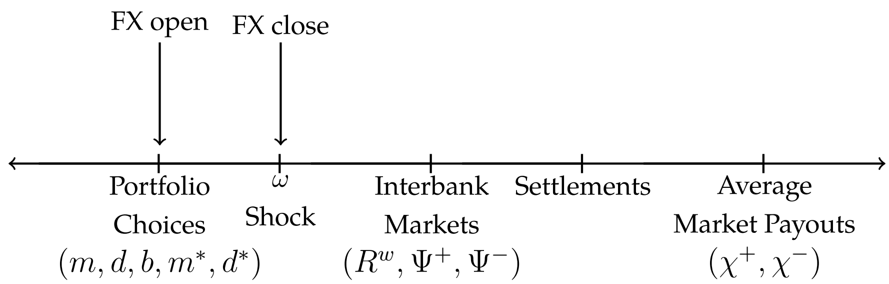

Figure 1: Timeline

Bank problem. The objective of a bank is to choose dividends and portfolios to maximize (1) subject to the budget constraint and the settlement frictions. Crucially, when choosing the portfolio, banks anticipate how withdrawal shocks may lead to a surplus or deficit of reserves and the associated costs and benefits of ending with these positions. We express the bank's optimization problem in terms of real portfolio holdings $\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$ and real returns. Thus, we define $\tilde{x}_t \equiv x_t/P_{t-1}$. The individual state variable is net worth, n, defined as the value of real assets minus liabilities at the beginning of the period. Recursively, the bank problem is

$$v(n, X) = \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E}\left[v(n', X')\right] \tag{7}$$

subject to the budget constraint

$$Div + \tilde{b} + \tilde{m}^* + \tilde{m} = n + \tilde{d} + \tilde{d}^*, \tag{8}$$

and the evolution of bank net worth

$$n' = \underbrace{R^b(X)\tilde{b} + R^m(X, X')\tilde{m} + R^{m^*}(X, X')\tilde{m}^* - R^d(X, X')\tilde{d} - R^{d,*}(X, X')\tilde{d}^*}_{\text{Portfolio Returns}}$$
$$+ \underbrace{\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*; X, X') + \chi(\theta(X), \tilde{m} + \omega\tilde{d}; X, X')}_{\text{Settlement Costs}}. \tag{9}$$

The evolution of n depends on the realized return on assets, but also on the realized settlement costs.[10] Because of the linearity of bank payoffs and the objective function, the value function is linear in net worth. Anticipating that in general equilibrium, there is a finite demand for loans and deposits, we note that an equilibrium therefore requires that

[10] To obtain (9), we use the definition of χ as expressed in (4)-(6) and the real returns. Implicit in the law of motion is that when a bank borrows at the discount window or from other banks, it pays a high-interest rate but obtains the interest on reserves.

$R^b(X) = 1/\beta \geq R^m(X, X')$.[11] The next lemma is an intermediate step towards the solution of the bank under this condition.

Lemma 1. *The solution to* (7) *is* $v(n, X) = n$*, and the optimal portfolio* $\{\tilde{m}, \tilde{d}, \tilde{m}^*, \tilde{d}^*\}$ *solves*

$$\Pi(X) = \max_{\{\tilde{m},\tilde{d},\tilde{m}^*,\tilde{d}^*\}} \mathbb{E}\Big\{ \left[R^b(X) - R^d(X,X')\right]\tilde{d} - \left[R^b(X) - R^m(X,X')\right]\tilde{m}$$
$$\left[R^b(X) - R^{d,*}(X,X')\right]\tilde{d}^* - \left[R^b(X) - R^{*,m}(X,X')\right]\tilde{m}^*$$
$$\Big[\chi(\theta(X), \tilde{m} + \omega\tilde{d}; X, X')\Big] + \Big[\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*; X, X')\Big]\Big] \Big\}. \quad (10)$$

The first two lines in (10) represent the direct portfolio payoffs and the third line constitutes the expected liquidity costs/benefits emerging from the settlement frictions. Notice that idiosyncratic shocks are only relevant for the latter term.

The bank portfolio problem is homogeneous of degree one. Thus, it must be that in general equilibrium, expected real returns are such that $\Pi(X) = 0$. This means that the scale of the individual bank portfolio is indeterminate at the individual bank level (although the aggregate one will be determined in equilibrium). On the other hand, the liquidity ratio is determined at the individual bank level. In effect, the kink in the liquidity cost function creates risk-averse behavior in the bank objective, pinning down the banks' ratios.

Non-financial sector. This section describes the non-financial block, which comprises households that supply labor and save in deposits in both currencies. Some goods must be purchased only with dollar deposits and some with euro deposits. Multinational firms use labor to produce the final good and are subject to working capital constraints, giving rise to a demand for loans. Goods trade is costless and, as a result, the law of one price holds. To further enhance tractability, we work with quasilinear preferences for households. As we show in the Online Addendum, we obtain the following schedules for the real aggregate loan demand by firms, B_t^d, and real aggregate deposit supply for deposits in euros and dollars, D_t^s and $D_t^{*,s}$:

$$B_t^d = \Theta_t^b \left(R_{t+1}^b\right)^{\epsilon^b}, \qquad \epsilon^b < 0, \quad \Theta_t^b > 0, \quad (11)$$
$$D_{t+1}^s = \Theta_t^d \left(\bar{R}_{t+1}^d\right)^{\epsilon^d}, \qquad \epsilon^d > 0, \quad \Theta_t^d > 0, \quad (12)$$
$$D_{t+1}^{*,s} = \Theta_t^{d,*} \left(\bar{R}_{t+1}^{d,*}\right)^{\epsilon^{d^*}}, \qquad \epsilon^{d^*} > 0, \quad \Theta_t^{d,*} > 0, \quad (13)$$

[11] If the return on loans were lower than $1/\beta$, banks would not invest in loans. Conversely, if the return on loans (or reserves) was higher than $1/\beta$, banks would inject infinite equity in the bank and the bank value would be infinite.

where ϵ^b is the semi-elasticity of credit demand and $\left\{\epsilon^d, \epsilon^{d^*}\right\}$ are the semi-elasticities of the deposit supplies with respect to the real returns, while the Θ terms are scale coefficients. These parameters are linked to the production structure and preference parameters in the microfoundation.

Central Banks. The two central banks choose the nominal rates for reserves, i_t^m, and the discount window, i_t^w, as well as the nominal supply of reserves $\left\{M_{t+1}, M_{t+1}^*\right\}$ and nominal discount window loans W_t. To balance the payments on reserves and the revenues from discount window loans, we assume that central banks passively adjust lump-sum taxes (or transfers). Because households have linear utility in the consumption good, these lump-sum taxes have no implications. We have the following budget constraint for the domestic central bank:

$$M_{t+1} + T_t - W_{t+1} = M_t(1 + i_t^m) - W_t(1 + i_t^w). \tag{14}$$

An identical budget constraint holds for the foreign central bank. We note that we only consider one type of government liability, and effectively consolidate government bonds and central bank reserves.[12]

2.2 Competitive Equilibrium

We study recursive competitive equilibria in which the vector of aggregate shocks, X, indexes all variables. We consider shocks to the nominal interest rates on reserves, the deposit supply, and the volatility of withdrawals. Without loss of generality, we restrict attention to a symmetric equilibrium in which all banks choose the same portfolios.[13]

Definition 1. Given central bank policies for both countries $\{M(X), i^m(X), i^w(X), W(X)\}$, $\{M^*(X), i^{m^*}(X), i^{w,*}(X), W^*(X)\}$, a recursive competitive equilibrium is a pair of price level functions $\{P(X), P^*(X)\}$, exchange rates $e(X)$, real returns for loans $R^b(X)$, nominal returns for deposits $\{i^d(X), i^{d,*}(X)\}$, an interbank market rate $i^f(X)$, market tightness $\theta(X)$, bank portfolios $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$, interbank and discount window loans $\{f(X), f^*(X), w(X), w^*(X)\}$, and aggregate quantities of loans $\{B(X)\}$ and deposits $\{D(X), D^*(X)\}$ such that

[12] See footnote 7. The analysis can be immediately extended to allow for a distinction between reserves and government bonds, following Bianchi and Bigio (2022).

[13] However, because banks are subject to idiosyncratic withdrawal shocks, there is an active interbank market trade and an endogenous distribution of assets and liabilities.

(i) Banks choose portfolios $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$ to maximize expected profits, as stated in (10).

(ii) Households are on their deposit supply, and firms are on their loan demand. That is, equations (11)-(12) are satisfied given real returns and quantities $\{B(X), D(X), D^*(X)\}$.

(iii) The law of one price holds $P(X) = P^*(X)e(X)$.

(iv) Markets clear for deposits $\tilde{d}(X) = D^s(X)$ and $\tilde{d}^*(X) = D^{s,*}(X)$; reserves $\tilde{m}\,(X)P(X) = M(X)$ and $\tilde{m}^*(X)P^*(X) = M^*(X)$; loans $\tilde{b}(X) = B(X)$; and the interbank markets $\Psi^+(X)S^+ = \Psi^-(X)S^-$ and $\Psi^{+,*}(X)S^{+,*} = \Psi^{-,*}(X)S^{-,*}$.

(v) For both currencies, market tightness $\theta(X)$ is consistent with the portfolios and the distribution of withdrawals, while the matching probabilities $\{\Psi^+(X), \Psi^-(X)\}$ and interbank market rates $i^f(X)$ are consistent with market tightness $\theta(X)$.

2.3 Discussion on interbank markets

A central ingredient of our framework is that financial institutions are subject to liquidity mismatch, and when they are short of liquidity, they trade in an OTC interbank market. Moreover, we assume that the dollar and euro interbank markets are segmented. While our model can capture various assumptions regarding the differences in the two markets, we will focus on a situation where funding risk is higher in the dollar market. This assumption aligns with the observation that the dollar serves as the leading funding currency, especially for short-term cross-border bank loans. As Ivashina, Scharfstein and Stein (2015) note. "[European] banks rely on wholesale dollar funding while raising more of their euro funding through insured retail deposits" (p. 1241), implying that dollar funding is more volatile and unstable. Moreover, as the 2020 BIS working group report (Davies and Kent, 2020, p. 29) puts it,

> US dollar funding is channeled through the global financial system, involving entities across multiple sectors and jurisdictions. Participants in these markets face financial risks typically associated with liquidity, maturity, currency, and credit transformation. What makes global US dollar funding markets special is the broad participation of non-US entities worldwide. These participants are often active in US dollar funding markets without access to a stable US dollar funding base or to standing central bank facilities that can supply US dollars during episodes of market stress.

McGuire and Von Peter (2009) and IMF (2019) provide evidence and discussion of the dollar's dominance for short-term funding in the world banking system and its attendant volatility. Bohorquez (2023) reports that 70 percent of all liabilities at non-U.S. BIS-reporting banks are less-volatile demand deposits, while only 30 percent of dollar funding takes this shape.

3 Theoretical Characterization

3.1 Liquidity Premia and Exchange Rates

We first describe exchange rate determination. We combine both reserve-market clearing conditions, the law of one price, and deposit clearing conditions to arrive at a condition for the determination of the nominal exchange rate:

$$e(X) = \frac{P(X)}{P^{*}(X)} = \frac{M(X)/\tilde{m}(X)}{M^{*}(X)/\tilde{m}^{*}(X)}. \tag{15}$$

Condition (15) is a Lucas-style exchange rate determination equation, but rather than following from cash-in-advance constraints, it is derived from banks' liquidity management decisions. Given a real demand for reserves in euros and dollars, that emerges from the bank portfolio problem (10), the dollar will be stronger (i.e., higher e) the larger is the nominal supply of euro reserves relative to that of dollar reserves. Similarly, for given nominal supplies of euro and dollar reserves, the dollar will be stronger as the relative demand for real dollar reserves increase. The novelty relative to the canonical Lucas-style model is that liquidity factors play a role in the real demand for currencies and, therefore, affect the value of the exchange rate. We now turn to analyzing the determinants of the real demand for reserves in each currency.

To understand how liquidity factors affect the exchange rate through the demand for reserve balances, let us inspect the portfolio problem (10). We denote by $\mu = \tilde{m}/\tilde{d}$ the banks' liquidity ratio and note that $s^{j} < 0$ if and only if $\omega^{j} < -\mu$. Using the expression for the liquidity yield function (4), and recalling that 'bars' denote expected returns, we can express the first-order condition with respect to $\tilde{m}$ as

$$R^{b} - \bar{R}^{m} = (1 - \Phi(-\mu))\bar{\chi}^{+}(\theta) + \Phi(-\mu)\bar{\chi}^{-}(\theta). \tag{16}$$

At the optimum, banks equate the expected real marginal return on loans, R^{b}, with the expected real marginal return on reserves. The latter is given by the expected real interest on reserves $\bar{R}^m$ plus their marginal liquidity value. If the bank ends up in surplus, which occurs with probability $1-\Phi(-\mu)$, the expected real marginal value is $\bar{\chi}^+$. If the bank ends up in deficit, which occurs with probability $\Phi(-\tilde{m}/\tilde{d})$, the expected real marginal value is $\bar{\chi}^-$. We label the difference in yields as the bond premium, $\mathcal{BP} \equiv R^b - \bar{R}^m$ and similarly $\mathcal{BP}^* \equiv R^b - \bar{R}^{m^*}$.

Using 16, the analogous condition for euros and using the law of one price $1+\pi = \mathbb{E}\left[(1+\pi^*)e'/e\right]$, we obtain a *liquidity premium adjusted interest parity condition*. In particular, denoting the total derivative of $\bar{\chi}$ with respect to m (i.e., the right-hand side of eq. (16)) by $\bar{\chi}_m\left(s;\theta\right)$, we have that

$$\mathbb{E}_t\left\{\frac{1}{1+\pi_{t+1}}\left[1+i_t^m-(1+i_t^{m^*})\cdot\frac{e_{t+1}}{e_t}\right]\right\} \;=\; \underbrace{\mathbb{E}\left[\bar{\chi}_{m^*}\left(s^*;\theta^*\right)-\bar{\chi}_m\left(s;\theta\right)\right]}_{\mathcal{DLP}}. \tag{17}$$

This equation establishes that the difference in the real return on reserves in the two currencies equals the difference in the marginal liquidity values. We refer to the difference in marginal liquidity values as the dollar liquidity premium, which we denote by $\mathcal{DLP}$.

In the absence of a liquidity premium, (17) would reduce to a canonical UIP that equates to a first order, the difference in nominal returns to the expected depreciation. However, whenever the marginal liquidity value of a dollar is larger than that of a euro (i.e., when $\mathcal{DLP} > 0$), a lower nominal interest rate in dollars than in euros is consistent with equilibrium, even if the exchange rate is expected to be constant. Note that because banks are risk neutral, there is no risk premium, and the deviation from UIP emerges entirely through liquidity.

Finally, we have the first-order conditions with respect to deposits in both currencies:

$$R^d = \bar{R}^m + \mathbb{E}_\omega\left[\bar{\chi}_m\left(s;\theta\right)+\bar{\chi}_d\left(s;\theta\right)\right]; \qquad \bar{R}^{d,*} = \bar{R}^{m^*} + \mathbb{E}_{\omega^*}\left[\bar{\chi}_{m^*}\left(s^*;\theta^*\right)+\bar{\chi}_{d^*}\left(s^*;\theta^*\right)\right]. \tag{18}$$

where $\bar{\chi}_d$ denotes the partial derivative of $\bar{\chi}$ with respect to d, the product of the derivative of the average settlement costs with respect to the average position s times the derivative of s with d—not the total derivative that would include the effect on θ. Like (17), these conditions imply that the expected real return on dollar and euro deposits may not be equated. In particular, a higher marginal liquidity cost of dollar deposits will be a force towards a lower real return of dollar deposits.

3.2 Funding Shocks

We now examine how funding shocks alter the exchange rate and liquidity premia. For analytical tractability, we assume that the supply of deposits is perfectly inelastic in both currencies. This assumption sharpens the results but does not alter the essence of the mechanism, as we will then show numerically.

We focus on shocks to dollar funding. The same shocks to the euro will have opposite effects on the exchange rate. Notice that because $R^b = 1/\beta$ is in equilibrium, the fact that deposit supplies are inelastic implies that shocks to the dollar funding will not affect $\mathcal{BP}$. Thus, $\mathcal{DLP}$ will move one to one with $\mathcal{BP}^*$, a result that speaks directly to the empirical literature connecting the liquidity premium of dollar-denominated assets to the exchange rate (Liao, 2020; Jiang et al., 2021; Engel and Wu, 2023).

A key object to characterize the effects of various shocks is the derivative of $\mathcal{DLP}$ with respect to the dollar liquidity ratio μ^*:

$$\mathcal{DLP}_{\mu^*} = \underbrace{\left[(1-\Phi^*(-\mu^*)) \cdot \bar{\chi}^{+^*}_{\theta^*} + \Phi^*(-\mu^*) \cdot \bar{\chi}^{-^*}_{\theta^*}\right]}_{\text{effect on average interbank rates}} \cdot \frac{\partial \theta^*}{\partial \mu^*} - \underbrace{\phi^*(-\mu^*) \cdot \left(\bar{\chi}^{-^*} - \bar{\chi}^{+^*}\right)}_{\text{liquidity risk exposure}} < 0.$$

This expression illustrates how a change in the dollar liquidity ratio must impact the dollar liquidity premium in equilibrium. There are two key terms. First, a higher liquidity ratio reduces the interbank-market tightness θ, thus easing the settlement frictions and reducing the average interbank rates. This general equilibrium effect reduces the liquidity premium. Second, a higher liquidity ratio reduces the probability that an individual bank ends up with a deficit. This partial equilibrium effect also reduces the liquidity premium because the cost of deficits is higher than the benefit of surpluses.

With this expression in hand, we can characterize the effects of different shocks.

Supply of dollar funding. The first question we explore is: what are the effects of an increase in the supply of dollar funding?

Proposition 1 (Funding level shock)**.** *Consider an increase in the real supply for dollar deposits $\Theta^{d,*}$. We have the following:*

i) If the shock is i.i.d, then the shock appreciates the dollar, reduces the dollar liquidity ratio μ^*, and raises $\mathcal{DLP}$. In particular,

$$\frac{d \log e}{d \log D^*} = -\frac{\mathcal{DLP}_{\mu^*}}{R^b - \mathcal{DLP}_{\mu^*} \mu^*} \in (0,1)\,, \quad \frac{d \log \mu^*}{d \log D^*} = -\frac{R^b}{R^b - \mathcal{DLP}_{\mu^*} \mu^*} \in (-1,0)\,,$$

and $d\mathcal{DLP} = \bar{R}^{m^*} d\log e > 0$.

ii) *If the shock is permanent, then the shock appreciates the dollar one for one, and does not change the liquidity ratio* μ^* *nor* $\mathcal{DLP}$*:*

$$\frac{d\log e^*}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1, \quad \textit{and} \quad d\mu^* = d\mathcal{DLP} = 0.$$

Proposition 1 establishes that a higher supply of dollar deposits appreciates the dollar regardless of whether the shock is temporary or permanent. The logic is simple: higher real dollar deposit amounts increase the demand for real dollar reserves. As banks have more dollar liabilities, there is a higher marginal value from dollar reserves. Given a fixed nominal supply of reserves, the increase in demand leads to an appreciation of the dollar.

At the same time, the increase in the supply of dollar deposits has different implications for liquidity premia, depending on whether the shock is temporary or permanent. When the shock is temporary, the exchange rate is expected to revert to a lower initial value in the following period. Given nominal rates, this reduces the expected real return of holding dollar reserves, and the demand for dollar reserves falls for an individual bank. In equilibrium, dollar reserves must have a higher marginal liquidity value, and there is a rise in $\mathcal{DLP}$. Overall, we then have that in response to a temporary increase in the supply of dollar deposits, the dollar appreciates, the dollar liquidity ratio falls, and $\mathcal{DLP}$ increases.

When the shock is permanent, the effect on the exchange rate is also expected to be permanent. In the absence of any expected depreciation effects, $\mathcal{DLP}$ must remain constant. Thus, in equilibrium, the outcome is that banks increase their holdings of dollar reserves in real terms in proportion to the increase in the supply of deposits. Since the supply of M^* is fixed, the amount of goods that can be bought with one dollar must increase, which appreciates the dollar.[14]

Dollar funding risk. Next, we characterize the effects of a rise in funding risk. For that purpose, it is useful to index Φ by a parameter that captures the volatility of withdrawals, σ. We make the following assumption:

Assumption 1. *The CDF for the distribution of withdrawal shocks satisfy* $\Phi^*(\omega;\sigma^*)$ *satisfies* $\Phi^*_{\sigma^*}(\omega;\sigma^*) > 0$ *for any* $\omega < 0$.

[14]Constant returns to scale in the interbank matching technology is key for this result. As banks proportionally scale dollar deposits and reserves, given the same real returns on dollar and euro reserves, the original liquidity ratio remains consistent with the new equilibrium. See Coppola, Krishnamurthy and Xu (2023) for a recent study allowing for increasing returns to scale.

The implication is that as we increase σ^*, the risk of ending with a reserve deficit increases for any μ^*. Hence, a shock to σ^* captures greater funding risk. We then have the following result:

Proposition 2 (Funding risk shock)**.** *Consider an increase in the dollar funding risk, σ^*. Suppose that Assumption 1 holds. Then,*

1) *If the shock is i.i.d, then the shock appreciates the dollar, raises the dollar liquidity ratio μ^*, and increases $\mathcal{DLP}$. In particular,*

$$\frac{d\log e}{d\log \sigma^*} = \frac{d\log \mu^*}{d\log \sigma^*} = \frac{\mathcal{DLP}_{\sigma^*}\sigma^*}{R^b - \mathcal{DLP}_{\mu^*}\mu^*} > 0, \quad \textit{and} \qquad d\mathcal{DLP} = \bar{R}^{m^*} d\log e > 0.$$

2) *If the shock is permanent, then the shock appreciates the dollar, raises the liquidity ratio, and $\mathcal{DLP}$ remains constant. In particular,*

$$\frac{d\log e}{d\log \sigma^*} = \frac{d\log \mu^*}{d\log \sigma^*} = -\frac{\mathcal{DLP}_{\sigma^*}\sigma^*}{\mathcal{DLP}_{\mu^*}\mu^*} > 0 \quad \textit{and} \qquad d\mathcal{DLP} = 0.$$

Proposition 2 presents a central result. In response to an increase in the risk of funding the dollar, the dollar appreciates, and there is an increase in the dollar liquidity ratio and $\mathcal{DLP}$. Intuitively, with a larger dollar funding risk, banks demand a greater amount of real dollar reserves. With the nominal supplies given, this must lead to an appreciation of the dollar. Again, there is a relevant distinction between temporary and permanent shocks. When the shock is temporary, the expected depreciation of the dollar reduces the expected real return of holding dollar reserves. Given the nominal rates, this implies that $\mathcal{DLP}$ must be higher in equilibrium for (17) to hold. When the shock is permanent, the volatility shock appreciates the dollar without any effects on $\mathcal{DLP}$. Unlike the case of the shock to the scale of dollar funding, in this case, the liquidity ratio increases with the exchange rate. In equilibrium, therefore, the increase in the liquidity ratio offsets the higher volatility, and that is why $\mathcal{DLP}$ remains constant. The magnitude of the response is proportional to the magnitude of the response of the dollar liquidity premium to σ^*, $\mathcal{DLP}_{\sigma^*} > 0$. [15]

3.3 Monetary Policy

In this section, we examine the effects of a change in the interest rate on dollar reserves.

Proposition 3 (Attenuation of changes in policy rates)**.** *Consider an increase in the interest rate on dollar reserves, i^{m^*}, holding fixed the policy spread, $i^{w^*} - i^{m^*}$.*

[15] In the appendix, we show a generalization to mean-reverting shocks.

1) *If the shock is i.i.d, the shock appreciates the dollar less than one for one, raises the liquidity ratio, and reduces $\mathcal{DLP}$:*

$$\frac{d\log e}{d\log\left(1+i^{m^*}\right)} = \frac{d\log \mu}{d\log\left(1+i^{m^*}\right)} = \frac{\bar{R}^{m^*}}{R^b - \mathcal{DLP}_{\mu^*}\cdot\mu^*} \in (0,1) \quad \textit{and}$$

$d\mathcal{DLP} = \bar{R}^{m^*}\left(d\log e - d\log\left(1+i^{m^*}\right)\right) < 0.$

2) *If the shock is permanent, the shock appreciates the dollar, raises the liquidity ratio, and reduces $\mathcal{DLP}$:*

$$\frac{d\log e}{d\log\left(1+i^{m^*}\right)} = \frac{d\log \mu^*}{d\log\left(1+i^{m^*}\right)} = -\frac{\bar{R}^{m^*}}{\mathcal{DLP}_{\mu^*}\cdot\mu^*} > 0, \quad \textit{and}$$

$$d\mathcal{DLP} = -\bar{R}^{m^*} d\log(1+i^{m^*}) < 0.$$

Proof. In Appendix **??** □

Proposition 3 establishes that in response to an increase in the US IOR, the dollar appreciates, the liquidity ratio increases, and the liquidity premium falls. This occurs regardless of whether the shock is temporary or permanent. The appreciation of the dollar follows a standard effect: a higher nominal rate leads to a larger demand for dollars, which in equilibrium requires a dollar appreciation. In turn, given a fixed nominal supply of dollar reserves, there is an increase in the real amount of reserves. In the absence of liquidity premia, the difference in nominal returns across currencies would be exactly offset by the expected depreciation of the dollar, following the current revaluation. With a liquidity premium, however, the expected depreciation is not one-for-one: given the larger abundance of real dollar reserves, there is a decrease in the marginal value of dollar reserves, together with a reduction in dollar liquidity premium.

This result breaks the tight connection between interest-rate differentials and expected depreciation, which is at the heart of models featuring the Fama (1984) puzzle. In models where uncovered interest parity holds an increase in the dollar interest rate leads to a one-for-one expected dollar depreciation, and no change in the expected excess return on euro reserves. Here, the exogenous increase in the dollar interest rate reduces the dollar liquidity premium, attenuating the effects on the exchange rate.[16]

[16] It is worth highlighting that the implications are not inconsistent with Nagel (2016), which found that an increase in the interest rate lowers liquidity premia. The difference is that the analysis in that paper is about varying the interest rate on non-monetary assets. In contrast, here, we vary the interest rate on monetary assets. We see our formulation as closer to the current monetary framework where the Federal Reserve and the European Central Bank pay interest on reserves.

4 Empirical Analysis

Prelude. The theory predicts a tight link between banks' liquidity ratio and exchange rates. On one hand, Proposition 2 shows that an increase dollar funding risk generates an increase an appreciation of the dollar and an increase in the liquidity ratio. At the same time, Proposition 1 shows that an increase in deposits dollar funding generates an appreciation of the dollar and a decrease in the liquidity ratio. In the proposition below, we show how the sign of a regression that relates the exchange rate to liquidity ratio depends on the relative variance and correlations of funding level and funding risk shocks.

Proposition 4 (Regression coefficients)**.** *Assume that shocks to $\{D_t^*, \sigma_t^*\}$ follow an AR(1) process with autocorrelation $\rho^{D^*}, \rho^{\sigma^*}$ and standard deviation $\Sigma^{D^*}, \Sigma^{\sigma^*}$. Then, up to first order, the univariate OLS regression coefficient of the change in the exchange rate against the change in the liquidity ratio is:*

$$\beta^e_{\mu^*} = \boldsymbol{w}_{\sigma^*} + \frac{\mathcal{DLP}^*_{\mu^*}\mu^*}{(1-\rho^{D^*})R^b} \cdot \boldsymbol{w}_{D^*},$$

where the weights are given by

$$\boldsymbol{w}_{\sigma^*} = \frac{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2 \left(1-\left(\rho^{D^*}\right)^2\right)}{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2 \left(1-\left(\rho^{D^*}\right)^2\right) + \left(\epsilon^{\mu^*}_{D^*}\Sigma^{D^*}\right)^2 \left(1-\left(\rho^{\sigma^*}\right)^2\right)} = 1 - \boldsymbol{w}_{D^*}.$$

and the elasticities are given by

$$\epsilon^{\mu^*}_{D^*} \equiv \frac{\log\mu^* - \log\mu^*_{ss}}{\log D^* - \log D^*_{ss}} \approx -\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} \in (-1, 0)\,,$$

$$\epsilon^{\mu^*}_{\sigma^*} \equiv \frac{\log\mu^* - \log\mu^*_{ss}}{\log\sigma^* - \log\sigma^*_{ss}} \approx \frac{\mathcal{DLP}^*_{\sigma^*}d\sigma^*}{(1-\rho^{\sigma^*})\,R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} > 0.$$

The regression coefficient of the liquidity ratio on the exchange rate is computed as a weighted average—where the weights reflect the volatility and persistence of shocks—of the effects that shocks to dollar funding risk and scale have on the correlation between changes in the exchange rate and the liquidity ratio. Hence, a positive coefficient implies that funding risk plays a significant role in driving exchange rates.

Data and baseline regression. Building on the previous result, we test the importance of dollar funding risk, using data on the liquidity ratio for the US banking system. We note that because detailed data is not readily available for the global financial system, we use the US data as a proxy for the dollar-denominated elements of the global banking balance sheets. That is, we assume implicitly that when faced with uncertainty about dollar funding, foreign banks' demand for liquid dollar assets responds similarly to US based institutions (including US-based subsidiaries of foreign banks).[17] We consider two short-term funding measures for financial intermediaries, namely US dollar financial commercial paper, as in Adrian et al. (2010), and demand deposits. Our liquidity ratio is the sum of reserves held at Federal Reserve banks and government securities (both Treasury and agency) by commercial banks relative to their short-term funding.[18]

We conduct our test on multiple currencies. We collect data for the price of the US dollar against the other nine G10 currencies and their corresponding inflation rates and interest-rates.[19] We test our theory by estimating the following regression for each currency i:

$$\Delta e_t = \alpha + \beta_1 \Delta Liq_t + \beta_2(\pi_t - \pi_t^*) + \beta_3 Liq_{t-1} + \epsilon_t. \tag{19}$$

In this regression, $\Delta(x_t)$ is the change from $t-1$ to t in the variable x_t; e_t is the log of the exchange rate expressed as the G10 currency price of a US dollar; Liq_t is the log of the liquidity ratio described above; $\pi_t - \pi_t^*$ is the difference between year-on-year inflation rates in each of the nine countries against the US inflation.[20] As the model is estimated in first-differences of the exchange rate, it measures the effect of changes in liquidity demand on changes in exchange rates at a monthly frequency.

Empirical results. Table 1 reports the regression results for the nine currencies. The coefficient of interest is β_1. If the dollar funding risk drives liquidity holdings, as predicted by the theory, we expect a positive relationship coefficient forLiq_t. With the exception of Japan, the regression results show that β_1 is indeed positive and statistically significant at the 1% level for all currencies.

We highlight that ΔLiq_t, is not a "price variable" but rather a "quantity variable." Finding significant statistical relationships between exchange rates and quantities has proven challenging in international finance. The significant coefficients for most currencies suggest that the quantity of liquidity can account for exchange rate movements without relying

[17]This approach is also followed by Adrian, Etula and Shin (2010).

[18]The series, obtained from FRED, are: DTBSPCKFM for US dollar financial commercial paper, DEMDEPSL for demand deposits, and TOTRESNS and USGSEC for reserves held at Federal Reserve banks and government securities.

[19]The other currencies are the Australian dollar, the Canadian dollar, the Japanese yen, the New Zealand dollar, the Norwegian krone, the Swedish krona, the Swiss franc, and the UK pound. The inflation for Australia and New Zealand is reported only quarterly.

[20]Our regression specification includes the year-on-year inflation rate to account for central banks policy rules that respond to inflation. Our model, however, abstracts from such policy rules.

on a liquidity measure contaminated by exchange rates.[21] In addition, we note that our regressions examine *current* realized depreciation, not forecast future depreciation.

Robustness. Our results hold under different specifications. Several asset-pricing studies have found that the Volatility index VIX has explanatory power in accounting for the movements of many asset prices. One possible concern is that if we included VIX, this would reduce the significance of liquidity in our regressions. In Table 2, we include the change in *VIX* a regressor. As expected, an increase in *VIX* is associated with an appreciation of the dollar. Crucially, the regressions show that including the VIX does not reduce the significance of the liquidity ratio for any of the countries, and it has only a minor effect on the magnitude of the coefficient. This suggests that funding risk, measured by the liquidity ratio, explains the exchange rate beyond market uncertainty.

We present a battery of additional robustness exercises in Appendix D. In one set, we use the average daily volatility of the Fed funds rate (measured as the spread between high and low rate) as an instrumental variable for the liquidity ratio. This establishes a link from funding volatility to the liquidity ratio in the first stage, and in the second stage, a robust and significant relationship between the liquidity ratio and dollar exchange rates is found, even while controlling for changes in VIX.[22] Our results are also very similar when we use alternative liquidity measures that include broader short-term funding measures. One measure includes in liabilities ”net financing” of broker-dealers as defined in Adrian and Fleming (2005) (see Tables D2-D4.) Another takes a broader definition of liabilities, including M2 less currency in circulation (Tables D5-D7). We also demonstrate that when we use balance sheet data exclusively from foreign-related banks in the US, the relationship between the liquidity ratio of dollar liquid assets to dollar funding and dollar exchange rates still holds (Table D16). Moreover, we also show that when we break the sample, we find more significant coefficients during the pre-European debt crisis, but for most countries, liquidity remains significant after the crisis (Tables D10-D15).

We conclude the discussion of the empirical results with two observations: First, an extensive empirical literature relates convenience yields to exchange rates. Our empirical findings demonstrate that demand for liquidity by financial intermediaries is associated with movements in the dollar exchange rates. And, as we have noted, we have found a relationship between a quantity variable (the liquidity ratio), rather than an asset price (the

[21]This happens in recent empirical studies that use the share of dollar assets as an explanatory variable (see, e.g., Adrian and Xie, 2020.).

[22]See Tables D1, D4, and D7 for the full sample for each measure of the liquidity ratio (see below for different measures.) Table D8 has the results of the first-stage regressions for the full sample. Tables D12 and D15 have the IV regressions for the split sample mentioned below.

convenience yield), and exchange rates. [23] Second, we highlight that the regressions should be interpreted as a summary statistic regarding the comovement between liquidity and exchange rates rather than as evidence of a causal relationship. As is clear from the theory, the liquidity ratio is not exogenous. In the next section, we will calibrate and simulate our model to shed further light on the relationship between the liquidity ratio and the exchange rate.

Table 1: Exchange Rates and Liquidity Ratio: Feb. 2001 – July 2021

	EUR	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.227***	0.256***	0.127***	-0.134***	0.287***	0.187***	0.212***	0.141***	0.165***
	(4.839)	(4.106)	(2.723)	(-2.846)	(4.458)	(3.125)	(3.754)	(2.724)	(3.529)
$\pi_t - \pi_t^*$	-0.800***	-0.657***	-0.407**	0.011	-0.726***	-0.126	-0.465**	-0.565***	-0.335**
	(-3.972)	(-2.998)	(-1.982)	(0.084)	(-3.299)	(-0.873)	(-2.530)	(-2.644)	(-1.985)
Liq_{t-1}	0.008*	0.005	0.007	0.002	0.004	0.010*	0.006	0.005	0.008
	(1.890)	(0.796)	(1.554)	(0.307)	(0.696)	(1.730)	(1.109)	(0.985)	(1.628)
Cons.	-0.010***	-0.002	-0.006*	-0.001	-0.005	-0.007	-0.008**	-0.015***	-0.006
	(-3.097)	(-0.595)	(-1.877)	(-0.120)	(-1.188)	(-1.618)	(-1.978)	(-2.966)	(-1.569)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.11	0.07	0.03	0.02	0.09	0.03	0.06	0.03	0.04

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table 2: Exchange Rates and Liquidity Ratio and VIX: Feb. 2001–July 2021

	EUR	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.198***	0.178***	0.082*	-0.119**	0.225***	0.137**	0.175***	0.126**	0.141***
	(4.283)	(3.250)	(1.935)	(-2.528)	(3.762)	(2.429)	(3.206)	(2.426)	(3.044)
$\pi_t - \pi_t^*$	-0.668***	-0.386**	-0.278	-0.007	-0.523**	-0.051	-0.407**	-0.517**	-0.271
	(-3.354)	(-2.008)	(-1.491)	(-0.050)	(-2.561)	(-0.382)	(-2.315)	(-2.411)	(-1.622)
ΔVIX_t	0.124***	0.344***	0.220***	-0.077**	0.285***	0.239***	0.189***	0.064*	0.099***
	(3.841)	(8.995)	(7.399)	(-2.350)	(6.792)	(6.116)	(4.991)	(1.784)	(3.163)
Liq_{t-1}	0.009**	0.006	0.007*	0.002	0.005	0.010*	0.007	0.005	0.008
	(2.059)	(1.204)	(1.770)	(0.306)	(0.995)	(1.877)	(1.330)	(1.047)	(1.562)
Cons.	-0.010***	-0.004	-0.006*	-0.001	-0.006	-0.007*	-0.008**	-0.015***	-0.005
	(-3.056)	(-1.104)	(-1.963)	(-0.193)	(-1.445)	(-1.697)	(-2.052)	(-2.833)	(-1.471)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.16	0.30	0.21	0.04	0.23	0.16	0.14	0.04	0.08

Notes: VIX is taken from the VIXCLS series on FRED. t statistics in parentheses. p<0.1, ** p<0.05, *** p<0.01

[23] Table D9 in the appendix presents evidence of a significant statistical relationship between our liquidity ratio measures and the relative dollar convenience yield used in these previous studies.

5 Quantitative Analysis

The previous section provides evidence of a robust positive correlation between changes in the US liquidity ratio and the dollar's strength. As the model indicates, dollar funding risk is a central variable driving this relationship. Here, we calibrate and solve a non-linear version of the model focusing on understanding the importance of funding risk in driving exchange rate fluctuations.

5.1 Calibration

We assume that the only aggregate shock is funding risk and that all nominal policy variables are constant. As we explain below, we use the structure of the model to filter out a series for funding risk shocks and estimate a Markov switching regime.

Steady state parameters. Table 3 displays the parameter values. We need to calibrate three parameters of the interbank market in each currency, λ, η, and the penalty rate $i^w - i^m$. For parsimony, we use the same calibration for each country's interbank market. We set λ to 3.5, consistent with a ratio of discount-window loans to the interbank-market volume of 3.1%, in line with what is observed in the data.[24] We set η to 0.5, assuming symmetric bargaining. Following Bianchi and Bigio (2022), we set the nominal discount window rate so that the average spread between the real rate on discount window loans and the real rate on reserves equals 10% annually. This value exceeds typical central bank spreads to rationalize observed interbank rates above the discount window- and can be interpreted as capturing stigma and collateral costs–see Armantier, Cipriani and Sarkar (2024) for a recent discussion.

For simplicity, we assume that each currency's deposit supply schedules are inelastic and normalize their real aggregate quantity to one. A key object in the model is the difference between the dollar and euro reserve yields. In Appendix C, we show that the empirical counterpart of the $\mathcal{DLP}$ is the deviation from covered interest parity (CIP). We also show that this applies in an extension of the model with risk premia. We set the nominal return on dollars such that the average $R_t^{m,*}$ equals -0.58% annualized, which is the monthly average of 1-month Treasuries between January 2001 and June 2020. We set the nominal

[24] For the analog of the discount window, we used the FRED series BORROW, which accounts for total borrowings from the Federal Reserve and borrowings from the discount window's primary, secondary, and seasonal credit programs and other borrowings from emergency lending facilities. For the interbank market, we use the Federal Funds volume series provided by the Federal Reserve Bank of New York.

return in Euros so that the CIP deviation is 20bps on average, in line with Du, Tepper and Verdelhan (2018). [25] This yields $R^m_t = -0.36\%$.

Funding Risk. Next, we describe how we back out series for the funding risk shocks. Following the results from Section 2, we can express market tightness, as defined in (3), as a function of the liquidity ratio and the distribution of withdrawal shocks Φ:

$$\theta(\mu_t, \sigma^*_t) = -\frac{\int_{-\infty}^{-\mu_t} \mu_t + \omega\Phi_t(d\omega;\sigma^*_t)}{\int_{-\mu}^{\infty} \mu_t + \omega\Phi_t(d\omega;\sigma^*_t)}.$$

Here, we assume that Φ, the distributions of withdrawal shocks in each country, is a symmetric Laplace distribution indexed by a single parameter, σ, the dispersion parameter.[26]

The above expression maps the liquidity ratio and the dispersion parameter σ^*_t into a market tightness for each interbank market. In the data, we observe the liquidity ratio but we do not observe directly θ_t. The theory, however, provides a mapping between market tightness and the spread between the interbank rate and the rate on reserves, also known as the "TED spread." In particular, as we show in Appendix A we have that

$$\mathcal{TED}(\mu_t, \sigma_t) = (R^w_t - R^m_t) \cdot \eta(\theta(\mu_t, \sigma_t)), \tag{20}$$

where the function $\eta(\theta(\mu, \sigma))$ is an endogenous bargaining power characterized in equation (A.2). Thus, given the observed TED spreads and liquidity ratios, we will back out the series for the funding risk shocks.[27]

To construct the TED spread, we use the one-month Libor rates minus the inflation average for the period and subtract the data counterparts for R^m_t. For the dollar liquidity ratio, we take the series described in Section 4. To construct the series for the Euro, we use the sum of Euro Area government-issued securities and cash holdings by Monetary Financial Institutions (MFIs) as liquid assets and monthly deposits redeemable at notice

[25]To construct a series for CIP deviations, we use the mid-point quotes for the spot exchange rate and forward exchange rates from Bloomberg and the nominal rates on 3-month US and German government bonds.

[26]The Laplace distribution is convenient because it allows for a continuum of shocks while rendering closed-form solutions for the conditional expectations below a threshold.

[27]As discussed in Bianchi and Bigio (2022), a key challenge in the procedure is that, in practice, some institutions (e.g., GSE) in the interbank market do not have access to interest on reserves. Consequently, the Fed funds rate may fall below the interest on reserves or Treasury yields. Because we abstract from these institutions, we truncate the TED spread in the data at a non-negative bound, which enables the model—with our specified functional forms for the matching function and the value of λ—to replicate the observed spreads.

and deposits with agreed maturity held by MFIs from February 2001 to July for deposits. Since we account for the funding of all U.S. financial institutions but only the liquid holdings of banks, we normalize μ and μ^* to a sample mean of 0.2, the average liquidity ratio reported in individual bank Call Reports.

Figure 2 presents the inferred series for funding risk in both currencies (panel [a]). The series for dollar funding risk is particularly volatile and faces sudden transitions from tranquil times to highly volatile episodes. The results highlight key differences in funding risk in the two currencies. The volatility of dollar funding is nearly four times higher than the one in euros, which rationalizes why the $\mathcal{DLP}$ is positive on average.

Figure 2 also shows that in episodes where there is a spike in dollar funding risk, there is an increase in US discount window loans and higher dispersion in interbank market rates (panels [e] and [f], respectively). As further validity, we report the bond premia in the model and data, in Panel (c) for the US and (d) for the Euro.[28] The quantitative fit is remarkable if we consider that we only feed funding risk shock.

Regime switching The inferred series for σ_t^* suggest the presence of a regime-switching behavior for the dollar, which we proceed to estimate. Given the stability of the euro funding risk, we assume that it is constant for the rest of the section. We assume that the log of σ_t^*, which we denote by $\hat{\sigma}$, follows an AR(1) process with time-varying coefficients:

$$\hat{\sigma}_t = \hat{\sigma}_{ss}\left(Z_t\right) + \rho^{\sigma,us}\left(Z_t\right)\left(\hat{\sigma}^u s_{t-1} - \hat{\sigma}_{ss}\left(Z_t\right)\right) + \Sigma^{\sigma,us}\left(Z_t\right)\varepsilon_t^{\sigma,us}. \tag{21}$$

The transition probabilities for the Markov chain are defined as $\Pr(Z_t = j \mid Z_{t-1} = i) = p_{ij}, \quad$ for $i, j \in \{1, 2\}$, where $P = \{p_{ij}\}$ is the 2×2 transition probability matrix. We estimate this Markov-switching with smoothed probabilities following Hamilton (1989) and Kim (1994).[29] The estimation results are reported in Figure 3. The shaded areas in all panels in Figure 2 correspond to events where the probability of being in the the scrambling-for-dollar regime is above 50%.

The estimation and the filtered probability confirm what we anticipate from a visual inspection: the estimation identifies two very distinct regimes for the funding risk: an

[28]In the model, the bond premium is defined as $\mathcal{BP}(\mu, \sigma) = \mathbb{E}\left[\chi(\theta(\mu, \sigma), \sigma)\right]$. For the U.S. data counterpart, we use $\mathcal{BP}_t^{obs,us}$, measured by the liquidity premium suggested in Stock and Watson (1989) and Friedman and Kuttner (1993): the commercial paper spread, defined as the difference between the three-month AAA commercial paper rate and the three-month Treasury bill rate, which serves as our proxy for the policy rate. For the euro, we use the convenience yield series from Diamond and Tassel (2022).

[29]The Markov Switching Model with two regimes was estimated using the package provided by Dadej (2023).

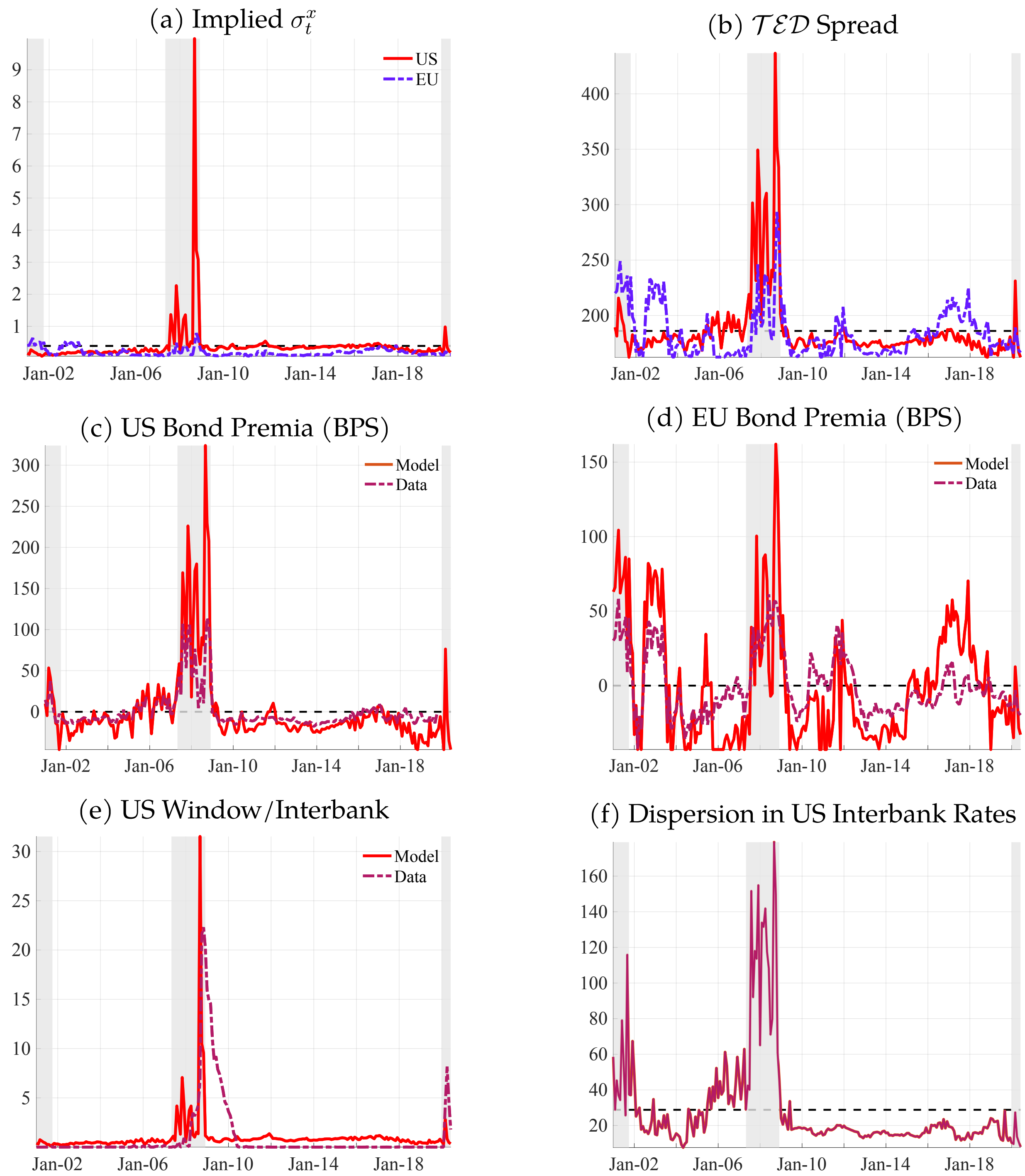

Figure 2: Data Variables

Note: The shaded regions correspond to a probability of being in the scrambling dollar regime exceeding 50%.

infrequent *scrambling-for-dollars* regime, with an expected duration of 16 months and a more frequent *normal times* regime, with an average duration of 5 years. While transitions from

Table 3: Parameters

Parameter	Value	Description	Target/Reference
λ	3.5	matching efficiency	Fed Funds / Disc. Window Volume
R^w-R^m	10%	policy corridor	Bianchi and Bigio (2022)
η	1/2	borrower bargain power	Symmetry
$\bar{R}^{m,us}$	-0.58%	US real rate on reserves	average one-month T-Bill real rate
$\bar{R}^{m,*}$	-0.36%	EU real rate on reserves	CIP deviation =20bps

Within Regime Processes

Coefficient	Scrambling	Normal
$\hat{\sigma}_{ss}$	-0.8	-1.3
	(0.2)	(0.0)
$\rho^{\sigma,us}$	0.6	0.9
	(0.2)	(0.0)
$\Sigma^{\sigma,us}$	0.9	0.2
	(0.1)	(0.0)

Transition Matrix

	Scrambling	Normal
$\Pr(Z_t = Z_{t-1})$	93.9%	98.4%

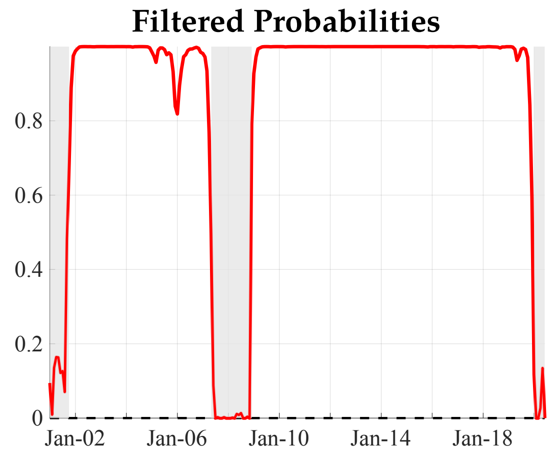

Figure 3: Estimation and Filtered probabilities of the states

normal times to scrambling-for-dollar regimes are rare—the probability is 1.6 percent—funding risk increases and becomes extremely volatile. Within the scrambling-for-dollars regime, the process for the log of funding risk is less persistent ($\rho^{\sigma,us} = 0.6$ vs. $\rho^{\sigma,us} = 0.9$) but substantially more volatile ($\Sigma^{\sigma,us} = 0.9$ vs. $\Sigma^{\sigma,us} = 0.2$). Importantly, expressed in levels, we obtain that the average funding risk, σ_t^*, is three times higher during scrambling for dollar episodes, 0.84 versus 0.3 of normal times. Moreover, we also obtain that funding risk shocks are more skewed toward higher values during scrambling for dollar episodes.

5.2 Model Evaluation

We now feed the regime-switching process for the dollar funding risk estimated above. We assume this is the only aggregate shock (but banks still face idiosyncratic funding risk in euros). We emphasize that the goal is not to replicate observed fluctuations in the data, but to evaluate the quantitative significance of funding risk shocks for understanding the dynamics of exchange rages.

To that end, we compute the model's nonlinear solution. The state variables are the cross-sectional variance of dollar deposit growth in the balancing stage, σ_t^*, and the regime Z. Given central bank policies, the model maps the state variables to banks' liquidity choices, the exchange rate, expected real rates, and liquidity premia. This mapping, in turn, depends on expectations about future exchange rates. Appendix **??** describes the algorithm used to solve the fixed-point problem that characterizes the rational-expectations equilibrium.

Figure 4 presents several key equilibrium objects as a function of σ^* for both the normal and scrambling-for-dollars regimes. Panel (a) plots the ergodic distribution of $sigma^*$ for each regime: the scrambling-for-dollars regime exhibits a larger variance and is positively skewed, indicating a higher likelihood of events with substantially greater dollar funding risk. Panels (b), (c), and (d) show that, in response to higher dollar funding risk, the dollar appreciates, banks' dollar liquidity ratio increases, and the *DLP* increases. These results are in line with the analytical results in Proposition 2, which considered the special cases of *iid* shocks and permanent shocks.

The figure also shows that, conditional on funding risk, a switch to the scrambling-for-dollars regime typically leads to a dollar appreciation. Because the estimated persistence is higher for the normal regime, if volatility significantly exceeds the means under both regimes, $\mathcal{DLP}$ is expected to remain higher relative to the scrambling-for-dollars regime; hence, a switch to the scrambling-for-dollars regime would lead to a depreciation. Note, though, that such an event has a very low probability under the ergodic distribution.[30]

Model vs. data. Table 4 compares moments for the *DLP* in the model with those of the CIP, its counterpart in the data. Recall that we associate scrambling-for-dollars regime in the data with periods when the estimated probability exceeds 50%. While the model is calibrated to match the mean of the CIP, the model is able to capture well conditional moments. In particular, the difference between the average CIP during scrambling-for-dollars regimes and during normal times is 39.1 basis points, closely matching the 38.0 basis points observed in the data. Moreover, the model successfully explains the autocorrelation of the CIP unconditionally and conditional on the regime. While the model overstates the unconditional standard deviation of the CIP, it accurately captures the increase in volatility during scrambling-for-dollars regimes; specifically, the ratio of standard deviations (scrambling-for-dollars over normal regimes) is 3.5 in the data compared to 2.1 in the

[30] We note also that in panel (d) $\mathcal{DLP}$ differ in the two regimes, but the differences are small.

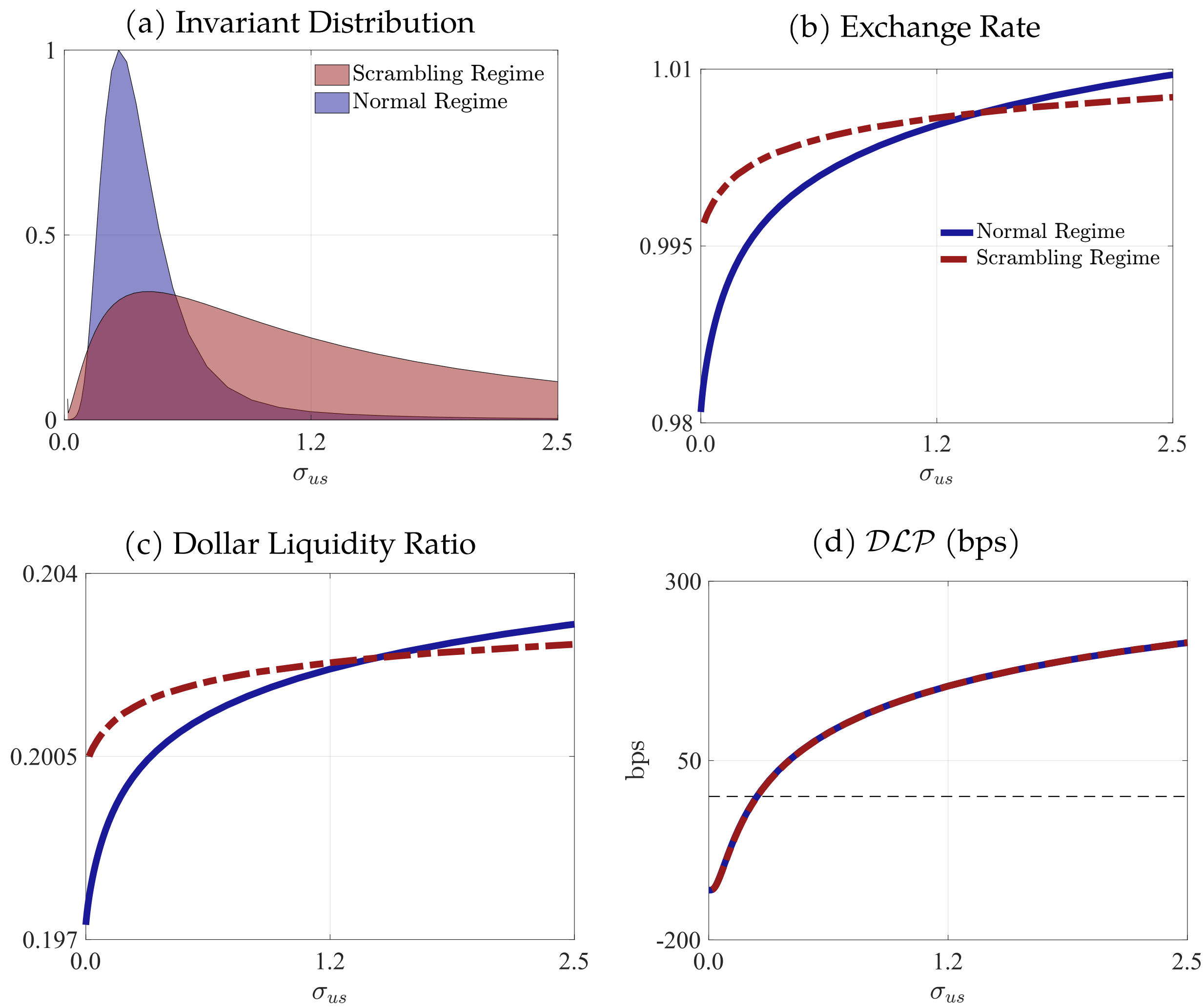

Figure 4: Equilibrium as function of dollar funding risk

model.

The fact that the model can account for untargeted moments of the CIP deviation indicates that it successfully accounts for a key factor driving exchange rates. However, we should note that many additional forces—e.g., interest-rate risk or time-varying effective risk premia—are omitted. Consequently, it is not surprising that the exchange rate in the data is substantially more persistent (0.97 in the data versus 0.89 in the model) and an order of magnitude more volatile. On the other hand, the model captures a significant portion of the dollar appreciation during regime transitions: when transitioning from a normal to a scrambling-for-dollars regime, the dollar appreciates by 0.65% in the model, whereas it appreciated by 1.3% for the two transitions identified over the sample period.

Table 4: $\mathcal{DLP}$: Data vs. Model

	Moment					
	Unconditional			Scrambling for Dollars vs. Normal Regimes		
	Mean	AutoCorr.	Std.	Mean Diffs.	AutoCorr.	Relative Stds.
$\mathcal{DLP}$(data)	20.6	0.80	24.2	38.0	{ 0.52, 0.66 }	3.5
$\mathcal{DLP}$ (model)	21.9	0.75	65.0	39.1	{ 0.58, 0.87 }	2.1

Note: Model moments are computed from the moment's invariant distribution (unconditional and conditional on a regime). Data conditional moments are computed from events with probabilities of being in the scrambling regime above and below 0.5. Mean Diffs. represent differences in conditional averages (scrambling minus normal regimes). The auto-correlations in brackets are conditional on regime (scrambling regime first, normal regime second). Relative Stds. are the ratios of standard deviations, (scrambling regime over normal regime).

Taking Stock. Our findings demonstrate that liquidity factors significantly contribute to exchange rate dynamics. We emphasize that the model endogenously explains why the dollar appreciates during a rise in funding risk—a result that has been difficult to reconcile in previous models. In fact, despite the prevalent view that risk premia may render the dollar a safe haven, Maggiori (2017) show, within a standard model of financial intermediation, that the dollar depreciates in response to negative aggregate shocks. The rationale is that U.S. households are optimally more exposed to aggregate risk and, consequently, incur larger losses in downturns than the rest of the world. Given the dominance of dollars in short-term funding markets, we contend that the scrambling-for-dollars phenomenon is a plausible explanation for the reserve currency paradox.

6 Conclusions

We develop a theory of exchange rate determination rooted in financial institutions' demand for liquid dollar assets. Central to the theory is a bank's liquidity management problem arising from the risk of sudden deposit withdrawals and the necessity to trade in an interbank market subject to OTC frictions. We show that the theory generates a "scrambling for dollars" effect that can help explain why the dollar tends to appreciate during periods of high volatility—a phenomenon known as the reserve currency paradox that has remained elusive in the literature. We also provide empirical evidence that, in line with the theory, a higher liquidity ratio in the financial system is associated with a stronger dollar, and demonstrate that funding risk is a quantitatively important factor driving exchange rate fluctuations.

Our framework can be extended in several directions. One could expand the model to incorporate a production structure with multiple goods, which would allow to study real exchange rate fluctuations and introduce nominal rigidities to analyze conventional monetary policy channels. In addition, we have designed the model so that the only asymmetry across currencies is the process for funding risk. Moreover, allowing dollars to have an advantage in settlements in the interbank market would further raise the dollar liquidity premia. Finally, our model provides a framework to study foreign exchange interventions, swap lines, and other unconventional policies. Exploring these topics is an exciting avenue for future research.

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A Expressions for $\{\Psi^{+}, \Psi^{-}, \chi^{+}, \chi^{-}, , R^{f}\}$

Here, we reproduce formulas derived from Proposition 1 in Bianchi and Bigio (2024), applied to the case where $\theta < 1$, as occurs in this paper. The average interbank rate is:

$$R^{f}(\theta) = (1 - \bar{\eta}(\theta))R^{w} + \bar{\eta}(\theta)\, R^{m}, \tag{A.1}$$

where $\bar{\eta}(\theta)$ is an endogenous bargaining power is given by

$$\bar{\eta}(\theta) \equiv \frac{\theta(1-\bar{\theta}) - \bar{\theta}}{\bar{\theta}\,(1-\theta)} \left(\left(\frac{\bar{\theta}}{\theta} \right)^{\eta} - 1 \right) (\exp(\lambda) - 1)^{-1} \tag{A.2}$$

and η is a parameter associated with the bargaining power of banks with reserve deficits in each trade—a Nash bargaining coefficient. In addition, $\bar{\theta}$ represents the market tightness after the interbank-market trading session is over:

$$\bar{\theta} = \left(1 + \left(\theta^{-1} - 1\right) \exp(\lambda)\right)^{-1}.$$

The parameter λ captures the matching efficiency of the interbank market. Trading probabilities are given by

$$\Psi^{+} = \theta \left(1 - e^{-\lambda}\right) \qquad \Psi^{-} = 1 - e^{-\lambda} \tag{A.3}$$

Using (5) and (6), we arrive at the parameters of the liquidity yield function χ:

$$\bar{\chi}^{+} = (R^{w} - R^{m}) \left(\frac{\bar{\theta}}{\theta}\right)^{\eta} \left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \text{ and } \bar{\chi}^{-} = (R^{w} - R^{m}) \left(\frac{\bar{\theta}}{\theta}\right)^{\eta} \left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \quad \text{(A.4)}$$

B Proofs

Preliminary Steps. In this section, we provide some intermediate results that we use to prove the propositions. Recall that the liquidity ratio is denoted by $\mu \equiv m/d$ and $\theta = S^{-}/S^{+}$ where $S^{-} = -\int \min\{s, 0\}\, d\Phi(\omega)$, $S^{+} = \int \max\{s, 0\}\, d\Phi(\omega)$ and $s = m + \omega d$. Then,

$$\theta = -\frac{\int_{\{s<0\}} s \cdot d\Phi(\omega; \sigma)}{\int_{\{s>0\}} s \cdot d\Phi(\omega; \sigma)}.$$

Note that $s < 0$ if $\omega < -\mu$. Therefore, we express the interbank market tightness as:

$$\theta = -\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}. \quad \text{(B.1)}$$

With abuse of notation, define $\theta(\mu, \sigma)$ as the function that maps μ and σ into the equilibrium value of θ. We have the following Lemma:

Lemma B.1. *Interbank market tightness is decreasing in the liquidity ratio. That is, $\frac{d\theta}{d\mu} < 0$. Moreover, $\theta \in [0, 1]$.*

Proof. From (B.1), using Leibniz rule, we obtain

$$\frac{d\theta}{d\mu} = \theta \left(\frac{\Phi(-\mu; \sigma)}{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} - \frac{1 - \Phi(-\mu; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}\right). \quad \text{(B.2)}$$

By definition of conditional expectation:

$$\mathbb{E}[\mu + \omega | \omega < -\mu] = \int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma) / \Phi(-\mu; \sigma),$$

and

$$\mathbb{E}[\mu + \omega | \omega > -\mu] = \int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma) / (1 - \Phi(-\mu; \sigma)).$$

Replacing these definitions into (B.2), we obtain:

$$\frac{d\theta}{d\mu} = \theta \cdot \left(\frac{1}{\mathbb{E}\left[\mu + \omega | \omega < -\mu\right]} - \frac{1}{\mathbb{E}\left[\mu + \omega | \omega > -\mu\right]} \right) < 0,$$

where the inequality follows because $\mathbb{E}\left[\mu + \omega | \omega < -\mu\right] < 0$ and $\mathbb{E}\left[\mu + \omega | \omega > -\mu\right] > 0$. The bounds on θ follow because $\lim_{\mu \to \infty} \theta = 0$ and $\theta = 1$ if $\mu = 0$. □

Next, we obtain the derivative of interbank market tightness with respect to σ.

Lemma B.2. *Under Assumption 1, we have that* $\frac{\partial \theta}{\partial \sigma} > 0$.

Proof. Passing the differential operator inside the integrals in the numerators, we have that:

$$\begin{aligned} \frac{\partial \theta}{\partial \sigma} &= \theta \cdot \left(\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \phi_\sigma d\omega}{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} - \frac{\int_{-\mu}^{\infty} (\mu + \omega) \phi_\sigma d\omega}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right) \\ &= \theta \cdot \left(\frac{\partial}{\partial \sigma} \left[\log \left(\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right) \right] \right). \end{aligned}$$

Since the withdrawal shock is zero mean, their average is μ and, hence,:

$$\frac{\partial \theta}{\partial \sigma} = \log \left(\frac{\mu - \int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right).$$

Therefore, $\frac{\partial \theta}{\partial \sigma} > 0$ holds if and only if:

$$\frac{\partial}{\partial \sigma} \left[\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma) \right] < 0.$$

Using the integration by parts:

$$\begin{aligned} \int_{-\infty}^{-\mu} (\mu + \omega) \phi_\sigma(\omega; \sigma) d\omega &= (\mu + \omega) \Phi_\sigma(\omega; \sigma) |_{-\infty}^{-\mu} - \int_{-\infty}^{-\mu} \Phi_\sigma(\omega; \sigma) d\omega \\ &= - \int_{-\infty}^{-\mu} \Phi_\sigma(\omega; \sigma) d\omega < 0, \end{aligned}$$

where we used $\lim_{\omega \to -\infty} ((\mu + \omega)) \Phi_\sigma(\omega; \sigma) = \frac{\partial}{\partial \sigma} \left[\lim_{\omega \to -\infty} ((\mu + \omega)) \Phi(\omega; \sigma) \right] = 0$ and Assumption 1. We conclude that, $\frac{\partial \theta}{\partial \sigma} > 0$. □

We use the results from the following Lemma.

Lemma B.3. *The liquidity coefficients have the following derivatives:*

$$\frac{\partial \chi^+}{\partial \mu} = \frac{\partial \chi^+}{\partial \theta}\frac{\partial \theta}{\partial \mu} < 0 \quad \text{and} \quad \frac{\partial \chi^-}{\partial \mu} = \frac{\partial \chi^-}{\partial \theta}\frac{\partial \theta}{\partial \mu} < 0, \tag{B.3}$$

$$\frac{\partial \chi^+}{\partial \sigma} = \frac{\partial \chi^+}{\partial \theta}\frac{\partial \theta}{\partial \sigma} > 0 \quad \text{and} \quad \frac{\partial \chi^-}{\partial \mu} = \frac{\partial \chi^-}{\partial \theta}\frac{\partial \theta}{\partial \sigma} > 0, \tag{B.4}$$

$$\frac{\partial \bar{\chi}^+}{\partial P_t} = \frac{\bar{\chi}^+}{P_t} \quad \text{and} \quad \frac{\partial \bar{\chi}^-}{\partial P_t} = \frac{\bar{\chi}^-}{P_t}. \tag{B.5}$$

Proof. Notice first that $\frac{\partial \chi^+}{\partial \theta} > 0$ and $\frac{\partial \chi^-}{\partial \theta} > 0$ is an immediate result from their definitions in equations (A.4). Applying Lemmas B.1 and B.2, we obtain respectively (B.3) and (B.4).

In addition, we can express (A.4) as

$$\bar{\chi}^+ = \frac{P_t}{P_{t+1}}\left(i^w - i^m\right)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right), \; \bar{\chi}^- = \frac{P_t}{P_{t+1}}\left(i^w - i^m\right)\left(\frac{\bar{\theta}}{\theta}\right)^{\eta}\left(\frac{\theta^{\eta}\bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right) \tag{B.6}$$

Equation (B.5) follows immediately. □

Define $\mathcal{L}(\mu, \sigma, P)$ to be the bond liquidity premium as a function of the liquidity ratio, the volatility σ and the current price level. That is,

$$\mathcal{L}(\mu, \sigma, P) = \left(1 - \Phi(-\mu, \sigma)\right) \cdot \bar{\chi}^+\left(\theta(\mu, \sigma), P\right) + \Phi(-\mu, \sigma) \cdot \bar{\chi}^-\left(\theta(\mu, \sigma), P\right) \tag{B.7}$$

In equilibrium $\mathcal{L}(\mu, \sigma, P) = R^b - R^m$. We have the following result.

Lemma B.4. *The liquidity bond premium is decreasing in the liquidity ratio and increasing in volatility. That is, $\mathcal{L}_\mu < 0$ and $\mathcal{L}_\sigma > 0$. In addition, $\mathcal{L}_P = \mathcal{L}/P$.*

Proof. From (B.7), differentiating $\mathcal{L}$ with respect to μ:

$$\mathcal{L}_\mu = \left[\left(1 - \Phi(-\mu, \sigma)\right) \cdot \chi^+_\theta + \Phi(-\mu, \sigma) \cdot \chi^-_\theta\right] - \left(\bar{\chi}^- - \bar{\chi}^+\right)\phi\left(-\mu, \sigma\right). \tag{B.8}$$

Using that $\frac{\partial \theta}{\partial \mu} < 0$ from Lemma B.1 and that $\bar{\chi}^- > \bar{\chi}^+$, we arrive at $\mathcal{L}_\mu < 0$.

From (B.7), differentiating $\mathcal{L}$ with respect to σ yields:

$$\mathcal{LP}_\sigma = \frac{\partial \theta}{\partial \sigma}\left[\left(1 - \Phi(-\mu, \sigma)\right) \cdot \chi^+_\theta + \Phi(-\mu, \sigma) \cdot \chi^-_\theta\right] + \left(\bar{\chi}^- - \bar{\chi}^+\right)\Phi_\sigma\left(-\mu, \sigma\right). \tag{B.9}$$

Using that $\frac{\partial \theta}{\partial \sigma} > 0$ from Lemma B.2 and that $\bar{\chi}^- > \bar{\chi}^+$, we conclude that $\mathcal{L}_\sigma > 0$. Finally, the expression for $\mathcal{L}_P$ follows directly from differentiating $\mathcal{L}$ with respect to P in (B.5). □

We now proceed with the main proofs.

B.1 Proof of Proposition 1

Proof. **Part i).** By definition, the liquidity ratio μ^* is given by

$$\mu^*(P^*, D^*) = \frac{M^*/P^*}{D^*} \tag{B.10}$$

where we made explicit the dependence of μ^* on (P^*, D^*). Using that M^* is exogenously given, totally differentiating (B.10) yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right). \tag{B.11}$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, D^*), P^*). \tag{B.12}$$

Totally differentiating (B.12) with respect to P^* and D^*, and using (B.11), we obtain:

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) = -\mathcal{L}^*_{\mu^*} \left[\mu \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right) \right] + \mathcal{L}^*_P dP^* \tag{B.13}$$

where $\mathbb{E}[P^*(X')]$ remains constant because the shock is i.i.d. and the loan rate is constant at $R^b = 1/\beta$.

Using $\mathcal{L}^*_{P^*} = \frac{\mathcal{L}^*}{P^*}$ from Lemma B.4, $R^b = R^{m,*} + \mathcal{L}^*$ and replacing in (B.13), we arrive to

$$\frac{d \log P^*}{d \log D^*} = \frac{\mathcal{L}^*_{\mu^*} \mu^*}{R^b - \mathcal{L}^*_{\mu^*} \mu^*} \in (-1, 0). \tag{B.14}$$

The bounds follows immediately because $\mathcal{L}^*_\mu < 0$ as established in Lemma B.4 and from $R^b > 0$.

The euro bond premium remains constant: replace $\mu = (M/P)/D$ in (16) and use (B.1) to obtain

$$R^b - (1+i^m)\frac{P}{\mathbb{E}[P(X')]} = \left(1 - \Phi\left(-\frac{M/P}{D}\right)\right)\bar{\chi}^+(\theta((M/P)/D,\sigma)) + \Phi\left(-\frac{M/P}{D}\right)\bar{\chi}^-(\theta((M/P)/D,\sigma)). \quad \text{(B.15)}$$

From (B.15), it follows that P must be constant and thus μ and $\mathcal{L}$ are also constant. As a result, $d\mathcal{L}^* = d\mathcal{DLP}$, $d\mathcal{L}^*_{\mu^*} = d\mathcal{DLP}_{\mu^*}$.

By the law of one price and using that P remains constant, we then have $\frac{d\log e}{d\log D^*} = -\frac{\mathcal{L}^*_\mu \mu^*}{R^b - \mathcal{L}^*_\mu \mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (B.13) as $\bar{R}^{m,*}(d\log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation remains constant. Moreover, given that nominal policy rates and expected inflation are constant, we have from (**??**) that $\mathcal{L}^*$ is constant. Hence, $\mathcal{DLP}$ is constant. Furthermore, the fact that $\mathcal{L}^*$ is constant, implies that μ must also be constant. Thus, using that (B.11) and that M^* is constant, we have from the law of one price that:

$$\frac{d\log e}{d\log D^*} = -\frac{d\log P^*}{d\log D^*} = 1.$$

□

B.2 Proof of Propositions 2 and 3

The proofs of follow the same steps as the Proof of Proposition 1. Thus, we direct readers to the working paper version of this paper Bianchi, Bigio and Engel (2023).

C Mapping the model to the data: CIP and $\mathcal{DLP}$

A data counterpart for the $\mathcal{DLP}$ is crucial to map the model to the data. We show now that once we include an explicit forward market, the $\mathcal{DLP}$ maps to deviations from covered interest parity (CIP).

With a forward market available, banks can create synthetic dollar assets by buying euros and converting their returns into dollars through the forward and spot markets. However, these synthetic dollars do not meet the liquidity needs of banks, as they must settle obligations with depositors in actual dollars. For example, during the March 2020 "dash for cash," the heightened demand for liquid dollar assets by financial intermediaries resulted in lower returns on actual dollars compared to synthetic dollars.

We assume that the forward market is perfectly competitive and for generality, and for generality, we allow for risk premia. In particular, we assume that banks maximize profits using a stochastic discount factor $\Lambda(X, X')$, which captures the risk aversion of shareholders.[31] A forward contract traded at time t promises to exchange one dollar for $\hat{e}_{t,t+1}$ euros in the lending stage in the following period.[32] The first-order condition with respect to the volume of forwards purchased can be expressed as

$$0 = \mathbb{E}\left[\frac{\Lambda_{t+1}}{1+\pi_{t+1}} \cdot (e_{t+1} - \hat{e}_{t,t+1})\right]. \tag{C.1}$$

By definition, the $\mathcal{CIP}$ deviation (or Treasury basis) is given by

$$\text{CIP} = (1 + i_t^m) - \left(1 + i_t^{m^*}\right)\left(\frac{\hat{e}_{t+1}}{e_t}\right). \tag{C.2}$$

Replacing the forward rate $\hat{e}_{t,t+1}$ from (C.1) into (C.2) and using (17), we obtain

$$\text{CIP} = \frac{\mathbb{E}\left[\Lambda_{t+1}\left(\bar{\chi}_{m^*} - \bar{\chi}_m\right)\right]}{\mathbb{E}\left[\frac{\Lambda_{t+1}}{1+\pi_{t+1}}\right]}. \tag{C.3}$$

That is, according to our model, the CIP deviation is given by the nominal risk-adjusted dollar liquidity premium. Accordingly, in the quantitative analysis, we use the time series of the CIP deviation to as a counterpart of the $\mathcal{DLP}$.

[31]Incorporating this feature in the portfolio problem (10) yields that the difference between expected returns on the dollar and euro liquid assets is driven by the sum of the *DLP* and a standard risk premium.

[32]Notice that since there are no aggregate shocks in the balancing stage, it is equivalent to pricing the forward in the lending or the balancing stage.

Online Appendix to "Scrambling for Dollars: International Liquidity, Banks and Exchange Rates"

By Javier Bianchi, Saki Bigio and Charles Engel

D Additional Tables: Empirical Analysis

Table D1: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	U.K.
$\Delta(\text{Liq}_t)$	0.370***	0.553***	0.441***	-0.247**	0.440***	0.394**	0.373***	-0.006	0.463***
	(2.997)	(3.573)	(3.702)	(-2.122)	(2.843)	(2.587)	(2.634)	(-0.042)	(3.189)
$\pi_t - \pi_t^*$	-0.834***	-0.733***	-0.534**	0.051	-0.662***	-0.194	-0.519***	-0.371	-0.634***
	(-3.567)	(-3.018)	(-2.364)	(0.359)	(-2.884)	(-1.208)	(-2.631)	(-1.451)	(-2.642)
Liq_{t-1}	0.011**	0.012*	0.014***	-0.001	0.008	0.016**	0.010*	0.004	0.017**
	(2.380)	(2.038)	(2.706)	(-0.161)	(1.423)	(2.418)	(1.779)	(0.703)	(2.553)
ΔVIX_t	0.104***	0.296***	0.183***	-0.066*	0.260***	0.212***	0.168***	0.077**	0.064*
	(2.901)	(6.584)	(5.106)	(-1.885)	(5.665)	(4.883)	(4.092)	(1.970)	(1.701)
Constant	-0.012***	-0.008*	-0.012***	0.002	-0.008*	-0.011**	-0.012**	-0.011*	-0.012**
	(-3.329)	(-1.838)	(-2.988)	(0.317)	(-1.880)	(-2.288)	(-2.514)	(-1.723)	(-2.513)
N	245	245	245	245	245	245	245	245	245

Notes: FFundsSpread is the monthly average of the intra-daily Fed Funds spread: the difference between the high and low Fed funds rate transacted on each day. StDev(XRate), lagged FFundsSpread and lagged Δ(Liq) instrument for Δ(Liq)., StDev(Inf). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01

Table D2: Exchange Rates and Alternative Measure of Liquidity Ratio

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.099***	0.117***	0.069***	-0.012	0.125***	0.101***	0.088***	0.079***	0.103***
	(3.774)	(3.298)	(2.65)	(-0.451)	(3.416)	(3.081)	(2.791)	(2.765)	(4.026)
$\pi_t - \pi_t^*$	-0.836***	-0.612***	-0.393*	-0.082	-0.667***	-0.125	-0.441**	-0.635***	-0.334**
	(-3.661)	(-2.635)	(-1.823)	(-0.541)	(-2.939)	(-0.839)	(-2.261)	(-2.716)	(-1.998)
Liq2_{t-1}	0.004	0.003	0.005*	0.004	0.002	0.006*	0.004	0.003	0.003
	(1.357)	(0.989)	(1.724)	(1.220)	(0.476)	(1.720)	(1.355)	(1.105)	(1.207)
Constant	-0.005***	0.000	-0.002	-0.002	-0.002	-0.001	-0.005*	-0.014***	-0.001
	(-2.625)	(0.079)	(-1.270)	(-0.661)	(-1.005)	(-0.573)	(-1.794)	(-3.172)	(-0.352)
N	235	235	235	235	235	235	235	235	235
adj. R^2	0.09	0.05	0.03	-0.00	0.06	0.04	0.04	0.04	0.06

Note: The alternative measure of the liquidity ratio includes as liabilities "net financing" of broker-dealer banks, as defined by Adrian and Fleming (2005). t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D3: Exchange Rates and Alternative Measure of Liquidity Ratio with VIX

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.090***	0.092***	0.058**	-0.007	0.106***	0.088***	0.078***	0.074***	0.0965***
	(3.538)	(3.064)	(2.503)	(-0.278)	(3.194)	(2.874)	(2.612)	(2.606)	(3.800)
$\pi_t - \pi_t^*$	-0.649***	-0.332*	-0.269	-0.112	-0.452**	-0.051	-0.380**	-0.568**	-0.261
	(-2.881)	(-1.164)	(-1.389)	(-0.743)	(-2.181)	(-0.364)	(-2.062)	(-2.428)	(-1.583)
ΔVIX_t	0.145***	0.382***	0.240***	-0.092**	0.324***	0.246***	0.218***	0.081**	0.109***
	(4.179)	(9.527)	(7.674)	(-2.586)	(7.281)	(6.071)	(5.443)	(2.157)	(3.300)
Liq2_{t-1}	0.004	0.005	0.005**	0.004	0.003	0.006*	0.005	0.003	0.003
	(1.613)	(1.539)	(1.998)	(1.265)	(0.911)	(1.856)	(1.631)	(1.194)	(1.187)
Constant	-0.004**	-0.001	-0.002	-0.003	-0.003	-0.001	-0.004*	-0.013***	-0.001
	(-2.347)	(-0.464)	(-1.258)	(-0.830)	(-1.183)	(-0.592)	(-1.736)	(-2.923)	(-0.322)
N	235	235	235	235	235	235	235	235	235
adj. R^2	0.15	0.32	0.23	0.02	0.23	0.16	0.15	0.06	0.10

t statistics in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table D4: Exchange Rates and Alternative Measure of Liquidity Ratio Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq2}_t)$	0.370***	0.511***	0.365***	-0.242*	0.389**	0.458***	0.365**	0.017	0.545***
	(2.764)	(2.904)	(3.058)	(-1.882)	(2.440)	(2.625)	(2.543)	(0.133)	(2.697)
$\pi_t - \pi_t^*$	-1.103***	-0.936***	-0.374	0.109	-0.736**	-0.331	-0.542**	-0.474	-1.038**
	(-3.106)	(-2.559)	(-1.436)	(0.521)	(-2.585)	(-1.502)	(-2.314)	(-1.575)	(-2.419)
Liq2_{t-1}	0.005	0.007	0.007**	0.000	0.004	0.011**	0.007*	0.003	0.010*
	(1.600)	(1.630)	(2.008)	(0.076)	(1.021)	(2.237)	(1.779)	(1.074)	(1.950)
ΔVIX_t	0.113**	0.328***	0.213***	-0.072*	0.292***	0.211***	0.192***	0.086**	0.056
	(2.477)	(5.609)	(4.978)	(-1.676)	(5.423)	(3.845)	(3.905)	(2.136)	(1.007)
Constant	-0.007***	-0.001	-0.004*	0.003	-0.004	-0.003	-0.007**	-0.011*	-0.003
	(-2.681)	(-0.289)	(-1.674)	(0.509)	(-1.478)	(-1.213)	(-2.173)	(-1.914)	(-1.226)
N	234	234	234	234	234	234	234	234	234

Notes: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq2) instrument for Δ(Liq2). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01.

Table D5: Exchange Rates and Alternative Measure of Liquidity Ratio (M2-Currency)

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.084	0.114	0.099	-0.243***	0.214**	0.086	0.074	0.016	0.134**
	(1.285)	(1.318)	(1.574)	(-3.899)	(2.333)	(1.061)	(0.949)	(0.232)	(2.116)
$\pi_t - \pi_t^*$	-0.607***	-0.520**	-0.363*	-0.023	-0.699***	-0.067	-0.360*	-0.417*	-0.217
	(-2.901)	(-2.315)	(-1.792)	(-0.180)	(-3.007)	(-0.467)	(-1.933)	(-1.938)	(-1.385)
Liq3_{t-1}	0.006	0.005	0.008	0.006	0.003	0.010	0.006	0.004	0.005
	(1.170)	(0.789)	(1.503)	(1.131)	(0.424)	(1.565)	(1.080)	(0.803)	(0.930)
Constant	0.003	0.006	0.007	0.008	0.001	0.012	0.004	-0.005	0.005
	(0.474)	(0.777)	(1.157)	(1.285)	(0.111)	(1.482)	(0.553)	(-0.573)	(0.829)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.03	0.02	0.01	0.06	0.03	0.00	0.01	0.01	0.01

Note: The alternative measure of the liquidity ratio includes as liabilities M2 less currency in circulation. t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D6: Exchange Rates and Alternative Measure of Liquidity Ratio (M2-Currency) with VIX

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.085	0.117	0.108*	-0.247***	0.209**	0.095	0.083	0.018	0.135**
	(1.352)	(1.580)	(1.906)	(-4.018)	(2.519)	(1.262)	(1.120)	(0.251)	(2.192)
$\pi_t - \pi_t^*$	-0.473**	-0.287	-0.258	-0.037	-0.507**	-0.015	-0.320*	-0.374*	-0.172
	(-2.320)	(-1.484)	(-1.415)	(-0.299)	(-2.390)	(-0.115)	(-1.816)	(-1.744)	(-1.125)
ΔVIX_t	0.147***	0.365***	0.229***	-0.091***	0.309***	0.253***	0.208***	0.078**	0.115***
	(4.471)	(9.539)	(7.795)	(-2.863)	(7.344)	(6.511)	(5.441)	(2.177)	(3.677)
Liq3_{t-1}	0.007	0.008	0.009*	0.006	0.006	0.011*	0.008	0.005	0.005
	(1.501)	(1.429)	(1.884)	(1.088)	(0.890)	(1.858)	(1.402)	(0.930)	(1.020)
_cons	0.005	0.009	0.009	0.007	0.004	0.014*	0.006	-0.003	0.006
	(0.863)	(1.294)	(1.544)	(1.187)	(0.537)	(1.770)	(0.878)	(-0.380)	(0.926)
N	246	246	246	246	246	246	246	246	246
adj. R^2	0.10	0.28	0.21	0.08	0.21	0.15	0.11	0.02	0.06

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D7: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio (M2-Currency) Instrumental Variable Regression: Feb. 2001–July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq3}_t)$	0.180	0.367**	0.272**	-0.220*	0.546***	0.264*	0.338**	-0.080	0.361***
	(1.522)	(2.566)	(2.572)	(-1.950)	(3.242)	(1.868)	(2.424)	(-0.608)	(2.929)
$\pi_t - \pi_t^*$	-0.515**	-0.450**	-0.313*	-0.043	-0.737***	-0.063	-0.388**	-0.308	-0.307*
	(-2.443)	(-2.149)	(-1.666)	(-0.336)	(-3.058)	(-0.452)	(-2.101)	(-1.361)	(-1.796)
Liq3_{t-1}	0.008	0.010*	0.010**	0.006	0.007	0.013**	0.009	0.005	0.008
	(1.556)	(1.658)	(2.117)	(1.128)	(1.047)	(2.046)	(1.615)	(0.908)	(1.463)
ΔVIX_t	0.147***	0.360***	0.229***	-0.092***	0.305***	0.254***	0.208***	0.077**	0.114***
	(4.419)	(9.164)	(7.621)	(-2.860)	(6.984)	(6.425)	(5.296)	(2.118)	(3.544)
Constant	0.005	0.011	0.009*	0.007	0.004	0.015*	0.007	-0.002	0.008
	(0.837)	(1.450)	(1.658)	(1.211)	(0.545)	(1.892)	(0.892)	(-0.202)	(1.228)
N	245	245	245	245	245	245	245	245	245

Notes: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq3) instrument for Δ(Liq3). t statistics in parentheses, * p<0.1, ** p<0.05, *** p<0.01

Table D8: First Stage Regressions for Each Measure of the Liquidity Ratio (ΔLiq)

	Constant	St.Dev Inf	St.Dev Dep	$\Delta(\text{Liq(-1)}_t)$	$\Delta(\text{FFSpread(-1)}_t)$
$\Delta(\text{Liq1}_t)$	-0.027***	0.39	1.396***	0.130**	0.035***
	(2.88)	(0.45)	(25.14)	(2.14)	(2.64)
$\Delta(\text{Liq2}_t)$	-0.035***	0.132	2.001***	-0.002	0.006
	(-1.86)	(0.08)	(4.11)	(-0.03)	(0.23)
$\Delta(\text{Liq3}_t)$	0.012*	0.299	0.605***	0.459***	0.038***
	(-1.94)	(0.57)	(3.62)	(8.13)	(4.16)

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D9: Correlation: Measures of Liquidity Ratio (Liq) and Convenience Yield ($ConvYd$) on 1-year U.S. Treasury notes in Dynamic Regression

	Const.	Δ(Conv Yd$_t$)	Δ(Conv Yd(-1)$_t$)	Δ(Conv Yd(-2)$_t$)	Δ(Liq(-1)$_t$)	Δ(Liq(-2)$_t$)
Δ(Liq1$_t$)	0.004	4.391*	9.370***	1.541	0.162**	0.170**
	(1.44)	(1.72)	(3.62)	(0.63)	(2.19)	(2.31)
Δ(Liq2$_t$)	0.005	3.649	9.658*	6.298	0.022	0.057
	(1.03)	(0.74)	(1.96)	(1.29)	(0.31)	(0.79)
Δ(Liq3$_t$)	0.002	-1.210	6.674***	5.000***	0.478***	-0.003
	(1.47)	(-0.75)	(4.07)	(2.99)	(6.80)	(-0.04)
Δ(Liq1$_t$)	0.004	-	8.708***	1.107	0.119*	0.163**
	(1.56)		(3.40)	(0.45)	(1.68)	(2.21)
Δ(Liq2$_t$)	0.005	-	9.132*	5.772	0.012	0.055
	(1.06)		(1.88)	(1.20)	(0.17)	(0.78)
Δ(Liq3$_t$)	0.002	-	6.916***	5.233***	0.483***	0.006
	(1.42)		(4.34)	(6.96)	(3.18)	(0.08)

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D10: Relationship of Exchange Rates and Liquidity Ratio Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
Δ(Liq$_t$)	0.240***	0.310***	0.143**	-0.144**	0.338***	0.203***	0.217***	0.198***	0.152**
	(3.781)	(3.816)	(2.380)	(-2.371)	(4.106)	(2.664)	(2.807)	(2.771)	(2.587)
$\pi_t - \pi_t^*$	-1.095***	-0.725**	-0.601**	-0.261	-0.615**	-0.199	-0.522**	-0.713**	-0.225
	(-3.565)	(-2.540)	(-2.372)	(-1.133)	(-2.255)	(-1.093)	(-2.125)	(-2.454)	(-1.041)
Liq$_{t-1}$	0.008	-0.009	-0.001	-0.002	-0.003	0.001	-0.002	-0.003	0.003
	(1.181)	(-1.054)	(-0.077)	(-0.300)	(-0.309)	(0.102)	(-0.282)	(-0.381)	(0.360)
Constant	-0.011***	-0.001	-0.006	-0.008	-0.005	-0.006	-0.008	-0.017***	-0.004
	(-2.811)	(-0.109)	(-1.633)	(-1.007)	(-1.009)	(-1.399)	(-1.565)	(-2.636)	(-0.890)
N	135	135	135	135	135	135	135	135	135
adj. R^2	0.13	0.11	0.05	0.04	0.12	0.03	0.05	0.06	0.03

Note: . t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D11: Exchange Rates and Liquidity Ratio with VIX Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.182***	0.176**	0.061	-0.114*	0.248***	0.134*	0.136*	0.169**	0.137**
	(2.856)	(2.467)	(1.102)	(-1.831)	(3.127)	(1.767)	(1.827)	(2.286)	(2.250)
$\pi_t - \pi_t^*$	-0.846***	-0.391	-0.362	-0.342	-0.443*	-0.109	-0.423*	-0.632**	-0.193
	(-2.765)	(-1.586)	(-1.563)	(-1.464)	(-1.724)	(-0.614)	(-1.831)	(-2.146)	(-0.879)
ΔVIX_t	0.175***	0.430***	0.266***	-0.089*	0.301***	0.212***	0.267***	0.087	0.044
	(3.284)	(7.211)	(5.700)	(-1.730)	(4.551)	(3.493)	(4.409)	(1.453)	(0.896)
Liq_{t-1}	0.009	-0.004	0.001	-0.003	-0.000	0.003	0.001	-0.002	0.003
	(1.422)	(-0.541)	(0.243)	(-0.406)	(-0.047)	(0.415)	(0.149)	(-0.216)	(0.348)
Constant	-0.010***	-0.003	-0.005	-0.010	-0.005	-0.006	-0.008	-0.016**	-0.003
	(-2.694)	(-0.679)	(-1.554)	(-1.283)	(-1.191)	(-1.395)	(-1.608)	(-2.444)	(-0.827)
N	135	135	135	135	135	135	135	135	135
adj. R^2	0.19	0.36	0.24	0.06	0.23	0.11	0.17	0.06	0.03

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D12: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: StDev(Inf), StDev(XRate), lagged FFundsSpread and lagged Δ(Liq) instrument for Δ(Liq) Feb. 2001 – April 2012

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.324**	0.579***	0.418***	-0.109	0.475***	0.495**	0.397**	0.142	0.498***
	(2.183)	(3.209)	(2.977)	(-0.736)	(2.647)	(2.442)	(2.169)	(0.803)	(2.889)
$\pi_t - \pi_t^*$	-1.019***	-0.767**	-0.662**	-0.343	-0.586**	-0.391	-0.634**	-0.591*	-0.643**
	(-2.887)	(-2.472)	(-2.314)	(-1.254)	(-2.064)	(-1.622)	(-2.269)	(-1.677)	(-2.019)
Liq_{t-1}	0.010	-0.002	0.004	-0.002	0.002	0.006	0.002	-0.001	0.013
	(1.497)	(-0.190)	(0.613)	(-0.355)	(0.188)	(0.664)	(0.306)	(-0.170)	(1.329)
ΔVIX_t	0.142**	0.331***	0.186***	-0.092	0.250***	0.133*	0.212***	0.091	-0.036
	(2.246)	(4.333)	(3.068)	(-1.526)	(3.257)	(1.715)	(2.945)	(1.298)	(-0.557)
Constant	-0.012***	-0.006	-0.011**	-0.010	-0.008	-0.012**	-0.012**	-0.015*	-0.011**
	(-2.816)	(-1.246)	(-2.438)	(-1.078)	(-1.596)	(-2.106)	(-2.136)	(-1.889)	(-1.983)
N	134	134	134	134	134	134	134	134	134

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D13: Relationship of Exchange Rates and Liquidity Ratio May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.208***	0.172*	0.126	-0.051	0.181*	0.218**	0.252***	0.043	0.184**
	(2.927)	(1.735)	(1.657)	(-0.624)	(1.663)	(2.077)	(2.894)	(0.583)	(2.290)
$\pi_t - \pi_t^*$	-0.451	-0.399	0.181	0.074	-0.798	-0.169	-0.273	-0.408	-0.660*
	(-1.551)	(-0.847)	(0.445)	(0.320)	(-1.365)	(-0.623)	(-0.940)	(-1.132)	(-1.802)
Liq_{t-1}	0.017	0.019	0.012	0.002	0.004	0.015	0.006	0.019	0.033
	(0.976)	(0.670)	(0.614)	(0.061)	(0.187)	(0.607)	(0.328)	(1.031)	(1.507)
Constant	-0.016	-0.013	-0.008	0.002	-0.004	-0.008	-0.004	-0.024	-0.027
	(-0.973)	(-0.498)	(-0.439)	(0.079)	(-0.190)	(-0.357)	(-0.248)	(-1.165)	(-1.386)
N	111	111	111	111	111	111	111	111	111
adj. R^2	0.06	0.00	0.01	-0.02	0.01	0.02	0.06	-0.01	0.04

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D14: Exchange Rates and Liquidity Ratio with VIX May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.206***	0.159*	0.125*	-0.050	0.174*	0.212**	0.251***	0.043	0.181**
	(2.940)	(1.807)	(1.790)	(-0.621)	(1.770)	(2.247)	(2.958)	(0.572)	(2.389)
$\pi_t - \pi_t^*$	-0.408	-0.090	0.103	0.072	-0.402	-0.057	-0.235	-0.393	-0.607*
	(-1.419)	(-0.214)	(0.274)	(0.314)	(-0.752)	(-0.233)	(-0.829)	(-1.086)	(-1.755)
ΔVIX_t	0.074**	0.255***	0.170***	-0.082*	0.265***	0.257***	0.116**	0.026	0.152***
	(1.992)	(5.472)	(4.565)	(-1.946)	(4.996)	(5.100)	(2.557)	(0.653)	(3.794)
Liq_{t-1}	0.016	0.008	0.015	0.001	0.006	0.012	0.007	0.019	0.032
	(0.950)	(0.321)	(0.836)	(0.051)	(0.289)	(0.541)	(0.358)	(1.015)	(1.548)
Constant	-0.015	-0.003	-0.011	0.002	-0.005	-0.006	-0.004	-0.024	-0.026
	(-0.933)	(-0.148)	(-0.649)	(0.090)	(-0.233)	(-0.293)	(-0.259)	(-1.138)	(-1.422)
N	111	111	111	111	111	111	111	111	111
adj. R^2	0.09	0.22	0.17	0.00	0.19	0.20	0.10	-0.02	0.15

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D15: Exchange Rates and Liquidity Ratio Instrumental Variable Regression: StDev(Inf), StDev(XRate), lagged FFundsSpread, lagged Δ(Liq), and Δ(USIndProd) instrument for Δ(Liq) May 2012 – July 2021

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	UK
$\Delta(\text{Liq}_t)$	0.380*	0.191	0.361	-0.236	0.461	0.687**	0.257	0.030	0.360
	(1.745)	(0.755)	(1.648)	(-0.945)	(1.498)	(2.185)	(1.027)	(0.139)	(1.546)
$\pi_t - \pi_t^*$	-0.489	-0.116	0.046	0.146	-0.495	-0.145	-0.234	-0.385	-0.713*
	(-1.574)	(-0.251)	(0.115)	(0.580)	(-0.878)	(-0.520)	(-0.825)	(-0.998)	(-1.887)
Liq_{t-1}	0.016	0.009	0.013	-0.002	0.002	0.009	0.007	0.019	0.033
	(0.893)	(0.338)	(0.684)	(-0.081)	(0.080)	(0.342)	(0.346)	(1.013)	(1.559)
ΔVIX_t	0.073*	0.255***	0.169***	-0.082*	0.262***	0.254***	0.116**	0.026	0.151***
	(1.902)	(5.444)	(4.328)	(-1.884)	(4.764)	(4.527)	(2.556)	(0.655)	(3.672)
Constant	-0.014	-0.004	-0.008	0.005	0.000	0.000	-0.004	-0.024	-0.027
	(-0.859)	(-0.160)	(-0.448)	(0.205)	(0.021)	(0.001)	(-0.244)	(-1.128)	(-1.396)
N	111	111	111	111	111	111	111	111	111

Note: t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

Table D16: Relationship of Exchange Rates and Liquidity Ratio in Foreign Related Banks

	Euro	AU	CAN	JPN	NZ	NWY	SWE	CH	U.K.
$\Delta(\text{LiqRatFRB}_t)$	0.139***	0.200***	0.161***	-0.137***	0.184***	0.151***	0.122***	0.039	0.171***
	(3.188)	(3.622)	(3.505)	(2.764)	(2.767)	(2.485)	(2.016)	(0.734)	(3.026)
$\pi_t - \pi_t^*$	-0.566***	-0.450***	-0.377*	-0.098	-0.577***	-0.049	-0.399**	-0.377***	-0.304*
	(-2.519)	(-2.053)	(-1.898)	(-0.737)	(-2.455)	(-0.347)	(-2.079)	(-1.735)	(-1.733)
LiqRatFRB_{t-1}	0.005*	0.006*	0.006*	0.004	0.003	0.007*	0.005*	0.003	0.004
	(1.782)	(1.902)	(2.207)	(1.366)	(0.881)	(2.177)	(1.792)	(1.167)	(1.431)
$\Delta(\text{VIX}_t)$	0.127***	0.311***	0.206***	-0.069**	0.279***	2.33***	0.190***	0.072*	0.088**
	(3.406)	(7.598)	(6.430)	(-1.973)	(5.982)	(5.507)	(4.593)	(1.936)	(2.501)
Constant	0.001	0.006	0.005	0.003	0.001	0.007	0.002	-0.006	0.004
	(0.333)	(1.452)	(1.583)	(0.860)	(0.156)	(1.771)	(0.453)	(-1.039)	(1.017)
N	245	245	245	245	245	245	245	245	245

Notes: The liquidity ratio is for foreign-related bank subsidiaries and branches located in the U.S. Liquid assets are the sum of cash assets (CASFRIW027SBOG from FRED) and Treasury and agency securities (TASFRIW027SBOG). Short-term liabilities are the sum of deposits (DPSFRIW027SBOG) and borrowings (H8B3094NFRD). t statistics in parentheses. * p<0.1, ** p<0.05, *** p<0.01

D.1 Proof of Proposition 4

We prove the proposition in two steps. First, we generalize the results of Propositions 1 and 2 to mean-reverting shocks using a first-order approximation. In particular, we assume that shocks to $\{D_t^*, \sigma_t^*\}$ follow an AR(1) process with autocorrelation $\rho^{D^*}, \rho^{\sigma^*}$ and standard deviation $\Sigma^{D^*}, \Sigma^{\sigma^*}$. We have the following lemma.

Lemma D.1 (Persistent shocks)**.** *The first-order effects of shocks around the steady state are as follows: i) In response to a small deviation to D^* near the steady state:*

$$\epsilon^{e}_{D^*} \equiv \frac{\log e - \log e_{ss}}{\log D^* - \log D^*_{ss}} \approx \frac{-\mathcal{DLP}^*_{\mu^*}\mu^*}{(1-\rho^{D^*})R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} \in (0,1)$$

and

$$\epsilon^{\mu^*}_{D^*} \equiv \frac{\log \mu^* - \log \mu^*_{ss}}{\log D^* - \log D^*_{ss}} \approx -\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} \in (-1,0)\,.$$

ii) Suppose that Assumption 1 holds. In response to a small deviation near σ^*_{ss}:

$$\epsilon^{\mu^*}_{\sigma^*} \equiv \frac{\log \mu^* - \log \mu^*_{ss}}{\log \sigma^* - \log \sigma^*_{ss}} = \epsilon^{e}_{\sigma^*} \equiv \frac{\log e^* - \log e^*_{ss}}{\log \sigma^* - \log \sigma^*_{ss}} \approx \frac{\mathcal{DLP}^*_{\sigma^*}d\sigma^*}{\left(1-\rho^{\sigma^*}\right)R^b - \mathcal{DLP}^*_{\mu^*}\mu^*} > 0.$$

We exploit this result to present the tests for the regression coefficients below. We first proof the result.

Proof. We now derive approximate analogues to Propositions 1 and 2 for cases where shocks are mean reverting. In particular, shocks follow a log AR(1) process:

$$\log\left(x_t\right) = \left(1-\rho^x\right)\log\left(x_{ss}\right) + \rho^x \cdot \log\left(x_{t-1}\right) + \Sigma^x \varepsilon^x_t. \tag{D.1}$$

We have the following result. We use x_{ss} to refer to the deterministic steady-state value of any variable x. The proof extends the results in Propositions 1 and B.2. We first show this intermediate result. In the model, prices are a function of the aggregate state, X. Thus, an equilibrium will feature a function $P^*\left(X_t\right)$ such that $P^*_t = P^*\left(X_t\right)$. Then, near the steady state, using a first-order Taylor expansion with respect to the variable x. We have that: for small deviations around the steady state:

$$d\log P^*_t \approx \frac{P^*_x\left(x_{ss}\right)x_{ss}}{P^*_{ss}} d\log x_t. \tag{D.2}$$

Shifting forward:

$$d\log P^*_{t+1} \approx \frac{P^*_x\left(x_{ss}\right)x_{ss}}{P^*_{ss}} d\log x_{t+1}$$

Taking expectations:

$$\mathbb{E}\left[d\log P^*_{t+1}\right] \approx \frac{P^*_x\left(x_{ss}\right)x_{ss}}{P^*_{ss}} \rho^x d\log x_t. \tag{D.3}$$

Combining (D.3) and (D.2),

$$\frac{\mathbb{E}\left[d\log P^*_{t+1}\right]}{d\log P^*_t} = \rho^x d\log x_t. \tag{D.4}$$

Next, we prove the main items of the propositions. The proof uses that for either currency:

$$\frac{\partial\bar{\chi}^+_t}{\partial P_{t+1}} = -\frac{\bar{\chi}^+_t}{\mathbb{E}\left[P_{t+1}\right]}, \text{ and } \frac{\partial\bar{\chi}^-_t}{\partial P_{t+1}} = -\frac{\bar{\chi}^-_t}{\mathbb{E}\left[P_{t+1}\right]}. \tag{D.5}$$

The negative sign follows from the fact that P_{t+1} divides each χ-term.

We thus have that:

$$\mathcal{L}^*_{P^*_{t+1}} = -\frac{\mathcal{L}^*}{P^*_{t+1}}$$

Recall that the dollar liquidity premium can be expressed as

$$R^b - (1+i^{m,*})\frac{P^*_t}{\mathbb{E}\left[P^*_{t+1}\right]} = \mathcal{L}^*(\mu^*(P^*, D^*), P^*_t, P^*_{t+1}), \tag{D.6}$$

where we now make explicit that $\mathcal{L}^*$ depends on both P_t and P_{t+1}.

Part (i). We present here the proof for item (i). Totally differentiating (D.6) with respect to P_t, P_{t+1}, and D^* and using (B.11) near the steady state, we obtain

$$-R^{m,*}\left(\frac{dP^*_t}{P^*_t}\right) + R^{m,*}\frac{\mathbb{E}\left[dP^*_{t+1}\right]}{\mathbb{E}\left[P^*_{t+1}\right]} = -\mathcal{L}^*_{\mu^*}\mu^*\left(\frac{dP^*_t}{P^*_t} + \frac{dD^*_t}{D^*_t}\right) + \mathcal{L}^*_{P^*_t}dP^*_t - \mathcal{L}^*_{P^*_{t+1}}\mathbb{E}\left[dP^*_{t+1}\right]. \tag{D.7}$$

Then, collecting terms:

$$-\left(R^{m,*} + \mathcal{L}^*\right)\left(1 - \frac{\mathbb{E}\left[d\log P^*_{t+1}\right]}{d\log P^*_t}\right) d\log P^*_t = -\mathcal{L}^*_{\mu^*}\mu^*\left(\frac{dP^*_t}{P^*_t} + \frac{dD^*_t}{D^*_t}\right). \tag{D.8}$$

Substituting $R^b = R^{m,*} + \mathcal{L}^*$ and (D.4), we obtain:

$$R^b \left(1 - \rho^{D^*}\right) d\log P_t^* \approx \mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*} + \frac{dD_t^*}{D_t^*}\right).$$

Thus, we obtain

$$\frac{d\log P^*}{d\log D^*} \approx \frac{\mathcal{L}_{\mu^*}^* \mu^*}{(1-\rho^{D^*})R^b - \mathcal{L}_{\mu^*}^* \mu^*} < 0.$$

Then, it follows from the law of one price and the differential form of μ that

$$\epsilon_{D^*}^e \equiv \frac{d\log e}{d\log D^*} \approx -\frac{\mathcal{L}_{\mu^*}^* \mu^*}{(1-\rho^{D^*})R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (0,1),$$

and

$$\epsilon_{\mu^*}^e \equiv \frac{d\log \mu}{d\log D^*} \approx -\frac{(1-\rho^{D^*})R^b}{(1-\rho^{D^*})R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (-1,0).$$

Part (ii). We present here the proof for item (ii). It follows the same steps as in Part (i): We totally differentiate (D.6) with respect to P_t, P_{t+1}, and σ^* and using (B.11) for the case where $dD^* = 0$. We obtain:

$$-R^{m,*}\left(\frac{dP_t^*}{P_t^*}\right) + R^{m,*}\frac{\mathbb{E}\left[dP_{t+1}^*\right]}{\mathbb{E}\left[P_{t+1}^*\right]} = \mathcal{L}_{\sigma^*}^* d\sigma^* - \mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*}\right) + \mathcal{L}_{P_t^*}^* dP_t^* - \mathcal{L}_{P_{t+1}^*}^* \mathbb{E}\left[dP_{t+1}^*\right]. \quad \text{(D.9)}$$

Collecting terms and using the same identities that we use to derive D.8, we arrive at:

$$\left(R^b \left(1-\rho^{\sigma^*}\right) - \mathcal{L}_{\mu^*}^* \mu^*\right) d\log P_t^* \approx -\mathcal{L}_{\sigma^*}^* d\sigma^*.$$

Therefore, we obtain:

$$\frac{d\log P^*}{d\log \sigma^*} \approx \frac{-\mathcal{L}_{\sigma^*}^* d\sigma^*}{\left(1-\rho^{\sigma^*}\right) R^b - \mathcal{L}_{\mu^*}^* \mu^*} < 0.$$

Then using that $\mu^* = M^*/\left(P^* D^*\right)$ and that $e = P/P^*$ and that P,M^* and D^* are constant, we arrive at:

$$\frac{d\log \mu^*}{d\log \sigma^*} \approx \frac{d\log e}{d\log \sigma^*} = -\frac{d\log P^*}{d\log \sigma^*} = \frac{\mathcal{L}_{\sigma^*}^* d\sigma^*}{\left(1-\rho^{\sigma^*}\right) R^b - \mathcal{L}_{\mu^*}^* \mu^*} > 0.$$

□

Importantly, we recover the expressions for the i.i.d. and permanent shock cases. We proceed to the regression coefficient results.

Regression Coefficients. We can now consider any shock x that follows a log AR(1) process:

$$\log\left(x_t\right) = \left(1-\rho^x\right)\log\left(x_{ss}\right) + \rho^x \cdot \log\left(x_{t-1}\right) + \Sigma^x \varepsilon_t^x.$$

We consider only shocks to dollar funding risk and the dollar funding scale and that $Var\left(\varepsilon_t^x\right) = 1$ for all shocks. Thus

$$Var\left(x_t\right) = \frac{\left(\Sigma^x\right)^2}{\left(1-\left(\rho^x\right)^2\right)}. \tag{D.10}$$

Consider a univariate linear regression of $\Delta \log e^*$ against $\Delta \log \mu^*$ where $\Delta x_t = x_t - x_{t-1}$. The regression coefficient is a function of two moments:

$$\gamma_{\mu^*}^{e^*} = \frac{CoV(\Delta \log e^*, \Delta \log \mu^*)}{Var\left(\Delta \log \mu^*\right)}. \tag{D.11}$$

Consider an endogenous variable Y_t in the model. An equilibrium will feature a function $Y\left(X_t\right)$ such that $Y_t = Y\left(X_t\right)$, where X_t is the exogenous state. Then, using a first-order Taylor expansion:

$$\log Y_t \approx \log Y_{ss} + \sum_{x\in X} \frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}} \frac{x_t - x_{ss}}{x_{ss}} \text{ for } x \in X.$$

Therefore, we have that:

$$\Delta \log Y_t \approx \sum_{x\in X} \frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}} \left(\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}}\right).$$

Near a steady state:

$$\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}} \approx \Delta \log\left(x_t\right) = \rho^x \cdot \left(\log\left(x_{t-1}\right) - \log\left(x_{ss}\right)\right) + \Sigma^x \varepsilon_t^x.$$

Using this identity,

$$\Delta \log Y_t \approx \textstyle\sum_{x\in X} \frac{Y_x\left(x_{ss}\right)\cdot x_{ss}}{Y_{ss}} \left(\rho^x \cdot \left(\log\left(x_{t-1}\right) - \log\left(x_{ss}\right)\right) + \Sigma^x \varepsilon_t^x\right).$$

Then, for small shocks the log-deviation from steady-state is approximately the elasticity near steady state.

$$\frac{Y_x\left(x\right)\cdot x}{Y_{ss}} = \epsilon_x^Y.$$

Hence, we have that $\Delta \log e_t^*$ and $\Delta \log \mu_t^*$ follow:

$$\begin{aligned}\Delta \log e_t^* &= \epsilon_{\sigma^*}^{e^*}\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)\ldots\\ &\quad+\epsilon_{D^*}^{e^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \qquad \text{(D.12)}\end{aligned}$$

Likewise, for the dollar liquidity ratio:

$$\begin{aligned}\Delta \log \mu_t^* = \epsilon_{\sigma^*}^{\mu^*}&\left(\rho^{\sigma^*}\cdot\left(\log\left(\sigma_{t-1}^*\right)-\log\left(\sigma_{ss}^*\right)\right)+\Sigma^{\sigma^*}\varepsilon_t^{\sigma^*}\right)\\ &+\epsilon_{D^*}^{\mu^*}\left(\rho^{D^*}\cdot\left(\log\left(D_{t-1}^*\right)-\log\left(D_{ss}^*\right)\right)+\Sigma^{D^*}\varepsilon_t^{D^*}\right). \qquad \text{(D.13)}\end{aligned}$$

From, (D.13) variance of the change in the liquidity ratio is:

$$Var\left(\Delta\log\mu^*\right)=\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\left(\Sigma^{D^*}\right)^2\right)^2.$$

Substituting (D.10) into the equation above:

$$\begin{aligned}Var\left(\Delta\log\mu^*\right)&=\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\left(\left(\rho^{\sigma^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{1-\left(\rho^{\sigma^*}\right)^2}+\left(\Sigma^{\sigma^*}\right)^2\right)+\left(\epsilon_{D^*}^{\mu^*}\right)^2\left(\left(\rho^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{1-\left(\rho^{D^*}\right)^2}+\left(\Sigma^{D^*}\right)^2\right)\\ &=\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2\frac{\left(\Sigma^{\sigma^*}\right)^2}{1-\left(\rho^{\sigma^*}\right)^2}+\left(\epsilon_{\sigma^*}^{D^*}\right)^2\frac{\left(\Sigma^{D^*}\right)^2}{1-\left(\rho^{D^*}\right)^2}.\end{aligned}$$

Provided that the shocks to σ^* and D^* are orthogonal, from (D.12) and (D.13), we have that following covariance between the change in the exchange rate and the change in the dollar liquidity ratio:

$$\begin{aligned}Cov(\Delta\log e^*,\Delta\log\mu^*) &\approx \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\left(\left(\rho^{\sigma^*}\right)^2 Var\left(\sigma^*\right)+\left(\Sigma^{\sigma^*}\right)^2\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\left(\left(\rho^{D^*}\right)^2 Var\left(D^*\right)+\Sigma^{D^*}\right)^2\\ &= \epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\frac{\left(\Sigma^{\sigma^*}\right)^2}{\left(1-\left(\rho^{\sigma^*}\right)^2\right)}+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}\frac{\left(\Sigma^{D^*}\right)^2}{1-\left(\rho^{D^*}\right)^2}.\end{aligned}$$

Substituting the approximations to $Var\left(\Delta\log\mu^*\right)$ and $Cov(\Delta\log e^*,\Delta\log\mu^*)$ back into (D.11), we obtain that the univariate regression coefficient is approximately:

$$\begin{aligned}\beta_{\mu^*}^{e^*} &\approx \frac{\epsilon_{\sigma^*}^{e^*}\cdot\epsilon_{\sigma^*}^{\mu^*}\cdot Var\left(\sigma^*\right)+\epsilon_{D^*}^{e^*}\cdot\epsilon_{D^*}^{\mu^*}Var\left(D^*\right)}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 Var\left(\sigma^*\right)+\left(\epsilon_{\sigma^*}^{D^*}\right)^2 Var\left(D^*\right)}.\\ &= \frac{\epsilon_{\sigma^*}^{e^*}}{\epsilon_{\sigma^*}^{\mu^*}}\cdot\boldsymbol{w}^{\sigma^*}+\frac{\epsilon_{D^*}^{e^*}}{\epsilon_{D^*}^{\mu^*}}\boldsymbol{w}^{D^*}.\end{aligned}$$

where:

$$\boldsymbol{w}^{\sigma^*} = \frac{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{D^*}\right)^2\right)}{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2\left(1-(\rho^{D^*})^2\right)+\left(\epsilon^{\mu^*}_{D^*}\Sigma^{D^*}\right)^2\left(1-(\rho^{\sigma^*})^2\right)}.$$

and

$$\boldsymbol{w}^{D^*} = \frac{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2\left(1-\left(\rho^{\sigma^*}\right)^2\right)}{\left(\epsilon^{\mu^*}_{\sigma^*}\Sigma^{\sigma^*}\right)^2\left(1-(\rho^{D^*})^2\right)+\left(\epsilon^{\mu^*}_{D^*}\Sigma^{D^*}\right)^2\left(1-(\rho^{\sigma^*})^2\right)}.$$

E Microfoundations - Deposit and Loan Schedules

Here we provide the micro-foundations for the loan demand and deposit supply schedules in a deterministic version of the model. We consider a representative global household. The household saves in dollar and euro deposits, supplies labor to an international firm, holds shares of this firm and owns a diversified portfolio of banks.

E.1 The Non-Financial Sector

Global household problem. The household enters the periods with a portfolio of dollar and euro deposits, denoted by $\{D_t, D^*_t\}$, holds shares of a global firm, Σ_t, and shares in a perfectly diversified portfolio of global banks, ϑ_t. These shares entitle the household to the firm's and bank's profits. The financial wealth available to the household (expressed in euros) is given by:

$$P_t n^h_t \equiv \left(1+i^d_t\right) D_t + T_t + e_t\left(\left(1+i^{*,d}_t\right) D^*_t + T^*_t\right) + P_t\left(q_t + r^h_t\right)\Sigma_t + P_t\left(Q_t + div_t\right)\vartheta_t \quad \text{(E.1)}$$

where T_t and T^*_t represent euro and dollar central bank transfers, q_t is the price of the firm (in terms of goods), r^h_t is the profit of the international firm, and Q_t is the price of the bank portfolio and div_t the dividend payout of banks.

In addition, the household supplies h_t hours that are remunerated at z_t euros per hour. The household uses its wealth to purchase deposits, to buy shares, and to consume. There are three types of consumption goods: dollar goods, denoted by c^*_t, euro goods, denoted by c_t, and a linear good, denoted by c^h_t. The household's budget constraint is:

$$e_t P^*_t c^*_t + P_t c_t + P_t c^h_t + D_{t+1} + e_t D^*_{t+1} + P_t q_t \Sigma_{t+1} + P_t Q_t \vartheta_{t+1} = P_t n^h_t + z_t h. \quad \text{(E.2)}$$

Both dollar and euro consumption are subject to deposit-in-advance (DIA) constraints:

$$c_t \leq \left(1+i_t^d\right) \frac{D_t}{P_t}, \tag{E.3}$$

and

$$c_t^* \leq \left(1+i_t^d\right) \frac{D_t}{P_t}. \tag{E.4}$$

The period utility is

$$U^*\left(c_t^*\right)+U\left(c_t\right)+c_t^h-\frac{h_t^{1+\nu}}{1+\nu},$$

where U^* and U are concave utility functions over both goods and $h_t^{1+\nu}/\left(1+\nu\right)$ is a labor dis-utility. To simplify the algebra of this section, we assume that $U_{c^*}^*\left(1\right)=U_c\left(1\right)=1$.

The household's problem is:

$$V_t^h\left(D_t, D_t^*, \Sigma_t, \vartheta_t\right)=\max_{\left\{c_t, c_t^*, c_t^h, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right\}} U^*\left(c_t^*\right)+U\left(c_t\right)+c_t^h-\frac{h_t^{1+\nu}}{1+\nu} \ldots$$
$$+\beta V_{t+1}^h\left(D_{t+1}, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right) \tag{E.5}$$

subject to the budget constraint (E.2 and E.1) and the two DIA constraints (E.3-E.4).

Firm Problem. The firm produces all goods in the economy using the same production function

$$y_t=A_{t+1} h_t^\alpha.$$

The firm's output is divided into:

$$c_t^*+c_t+c_t^h=y_t. \tag{E.6}$$

The firm revenues come from selling goods in the dollar, euro, and linear good markets:

$$e_t P_t^* c_t^*+P_t c_t+P_t c_t^h=P_t y_t.$$

To produce positive amounts of all goods, the firm must be indifferent between selling in either market. Hence, the law of one price will hold in an equilibrium with positive consumption of all goods—the Inada conditions guarantee this is the case.

To maximize profits, the firm chooses borrowed funds B_{t+1}^d and labor h_t. The demand for loansemerges from a working capital constraint: $z_t h_t \leq B_{t+1}^d$. The firm saves in deposits whatever borrowings it doesn't spend in wages.

The firm's problem is given by:

$$\begin{aligned} P_{t+1}r_{t+1}^{h} &= \max_{B_{t+1}^{d}\geq 0,h_t\geq 0} P_{t+1}A_{t+1}h_t^{\alpha} - \left(1+i_{t+1}^{b}\right)B_{t+1}^{d} + \left(1+i_{t+1}^{d}\right)\left(B_{t+1}^{d}-z_t h_t\right) \\ &= \max_{B_{t+1}^{d}\geq 0,h_t\geq 0} P_{t+1}A_{t+1}h_t^{\alpha} - \left(1+i_{t+1}^{b}\right)z_t h_t - \left(i_{t+1}^{b}-i_{t+1}^{d}\right)\left(B_{t+1}^{d}-z_t h_t\right). \end{aligned} \tag{E.7}$$

Equilibrium. In the body of the paper we characterized the equilibrium in loan and deposit markets, taking as given the loan demand and deposit supply schedules, and the transfers rules. In addition to these financial markets, the non-financial sector features a labor market, firm shares market, bank shares market, and the three goods markets. Next, we derive the loan demand and deposit supply schedules and comment on how once these asset markets clear, all other markets clear.

E.2 Derivation of Deposit Supply and Loan Demand

Step 1 - deposit demand. We clear the linear good, c_t^h, from the household's budget constraint:

$$\begin{aligned} c_t^h &= \frac{P_t n^h + z_t h - \left(e_t P_t^* c^* + P_t c + D_{t+1} + e_t D_{t+1}^* + P_t\left(r_t+q_t\right)\Sigma_t + P_t\left(Q_t + div_t\right)\vartheta_t\right)}{P_t} \\ &= n^h + \frac{z_t}{P_t}h_t - \left(c_t^* + c_t + \left(r_t+q_t\right)\Sigma_t + \left(Q_t+div_t\right)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*}\right). \end{aligned} \tag{E.8}$$

where the second line uses the law of one price.

Substituting (E.8) into the objective of the household's problem (E.5) we obtain:

$$\begin{aligned} V_t^h\left(D_t, D_t^*, \Sigma_t, \vartheta_t\right) &= n_t^h + \max_{\left\{c_t, c_t^*, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right\}} U^*\left(c_t^*\right) + U\left(c_t\right) - \frac{h_t^{1+\nu}}{1+\nu}\dots \\ &\quad + \frac{z_t}{P_t}h_t - \left(c_t^* + c_t + \left(r_t+q_t\right)\Sigma_t + \left(Q_t+div_t\right)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*}\right)\dots \\ &\quad + \beta V_{t+1}^h\left(D_{t+1}, D_{t+1}, \Sigma_{t+1}, \vartheta_{t+1}\right) \end{aligned}$$

subject to the two DIA constraints (E.3-E.4).

We proceed to obtain the deposit supply.

Since $\left\{D_{t+1}, D_{t+1}^*\right\}$ enter symmetrically, we derive the deposit supply only for one currency. We take the first-order condition with respect to c_t and notice that if the DIA constraint does not bind, $U_c(c) = 1$. In turn, if the deposit in advance constraint indeed binds, then:

$$c = \left(1 + i_t^d\right) \frac{D_t}{P_t} = \frac{\left(1 + i_t^d\right)}{P_t/P_{t-1}} \frac{D_t}{P_{t-1}} = R_t^d \frac{D_t}{P_{t-1}}.$$

Thus, we can combine both cases, with and without the binding DIA constraint, to write down the optimal consumption rule:

$$c = \min\left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\}. \tag{E.9}$$

By analogy:

$$c^* = \min\left\{ (U_{c^*}^*)^{-1}(1), R_t^{d,*} \cdot \frac{D_t^*}{P_{t-1}^*} \right\}.$$

It is convenient to treat c and c^* directly as functions of $\frac{D}{P_{t-1}}$ and $\frac{D^*}{P_{t-1}^*}$ in the next step.

Step 2 - deposit supply schedules. We replace the optimal euro and dollar consumption rules into the objective. We have:

$$\begin{aligned}
V_t^h(D_t, D_t^*, \Sigma_t, \vartheta_t) &= n^h + \max_{\left\{c_t, c_t^*, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\right\}} U^*\left(\min\left\{ (U_{c^*}^*)^{-1}(1), R_t^{d,*} \cdot \frac{D^*}{P_{t-1}^*} \right\} \right) \ldots \\
&+ U\left(\min\left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\} \right) - \frac{h^{1+\nu}}{1+\nu} \ldots \\
&+ n^h + \frac{z_t}{P_t} h - \min\left\{ (U_{c^*}^*)^{-1}(1), R_t^{d,*} \cdot \frac{D_t^*}{P_{t-1}^*} \right\} \ldots \\
&- \min\left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\} \ldots \\
&- \left((r_t + q_t)\Sigma_t + (Q_t + div_t)\vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*} \right) \ldots \\
&+ \beta V_{t+1}^h(D_{t+1}, D_{t+1}, \Sigma_{t+1}, \vartheta_{t+1}).
\end{aligned} \tag{E.10}$$

Next, we the derive deposit demand: We take the first-order conditions with respect to D_{t+1}/P_t to obtain:

$$1 = \beta \frac{\partial V_{t+1}^h}{\partial (D_{t+1}/P_t)}. \tag{E.11}$$

Next, we derive the envelope condition. We have two cases.

Case 1: binding DIA constraint the following period. For the case where

$$R_t^d \cdot D_t/P_{t-1} < 1$$

we have from (E.10) that:

$$\frac{\partial V_t^h}{\partial \left(D_t/P_{t-1}\right)} = U_c R_t^d - R_t^d + R_t^d = U_c R_t^d.$$

Case 1: non-binding DIA constraint the following period. For the case where $R_{t-1}^d \frac{D}{P_{t-1}} \geq 1$, we have from (E.10) that

$$\frac{\partial V_t^h}{\partial \left(D_t/P_{t-1}\right)} = R_t^d.$$

Thus, combining the two envelope conditions we obtain:

$$\frac{\partial V_t^h}{\partial \left(D_t/P_{t-1}\right)} = \begin{cases} U_c \left(R_t^d \cdot D_t/P_{t-1}\right) R_t^d & \text{for } R_t^d \cdot D_t/P_{t-1} < 1 \\ R_t^d & \text{otherwise.} \end{cases} \tag{E.12}$$

We shift (E.12) one period forward and substitute in (E.11) in the left-hand side to obtain:

$$\frac{1}{\beta} = \begin{cases} U_c \left(R_{t+1}^d \cdot D_{t+1}/P_t\right) R_{t+1}^d & R_{t+1}^d \cdot D_t/P_{t-1} < 1 \\ R_{t+1}^d & \text{otherwise.} \end{cases}$$

In the body of the paper, using the banks' problem, we show that $R_{t+1}^d < R^b = 1/\beta$. Thus, the only relevant portion to determine the demand condition is the one where there's no satiation of deposits. We hence, will only use this portion.

We now adopt power utility. Assume that $U = c^{1-\gamma}/\left(1-\gamma\right)$ and $U^* = \left(c^*\right)^{1-\gamma^*}/\left(1-\gamma^*\right)$. Then,

$$\frac{1}{\beta} = \left(R_{t+1}^d \cdot D_{t+1}/P_t\right)^{-\gamma} R_{t+1}^d.$$

We clear D/P_t to obtain the euro deposit supply schedule:

$$D_{t+1}/P_t = \beta^{1/\gamma} \left(R_{t+1}^d\right)^{\frac{1-\gamma}{\gamma}}.$$

By analogy, we have the dollar supply schedule:

$$D_{t+1}^*/P_t^* = \beta^{1/\gamma^*} \left(R_{t+1}^{*,d}\right)^{\frac{1-\gamma^*}{\gamma^*}}.$$

More generically, following the same states, if we introduce preference shocks to the utility specifications, as follows:

$$U_t = (c/x_t)^{1-\gamma}/(1-\gamma) \text{ and } U_t^* = (c^*/x_t^*)^{1-\gamma^*}/(1-\gamma^*),$$

the demand schedules generalize to:

$$D_{t+1}/P_t = x_t \beta^{1/\gamma} \left(R_t^d\right)^{\frac{1-\gamma^x}{\gamma^x}}.$$

By analogy, we have that

$$D_{t+1}^*/P_t^* = x_t^* \beta^{1/\gamma} \left(R_t^d\right)^{\frac{1-\gamma^x}{\gamma^x}}.$$

We obtain this conditions following exactly the same steps.

All in all, the demand schedules are akin to those in the body of the paper, where the reduced form coefficients are given by:

$$\Theta_t^d = x_t \beta^{1/\gamma^d} \text{ and } \epsilon^d = \frac{1}{\gamma^d} - 1,$$

and

$$\Theta_t^{*,d} = x_t^* \beta^{1/\gamma^{*,d}} \text{ and } \epsilon^{*,d} = \frac{1}{\gamma^{*,d}} - 1.$$

Next, we describe the labor supply schedule.

Step 3 - labor supply. The first-order condition with respect to h in the household's problem yields a labor supply that only depends on the real wage:

$$h_t^\nu = z_t/P_t. \tag{E.13}$$

Next, we move to the firm's problem to obtain the labor demand.

Step 4 - labor demand. Since from the bank's problem, it will be the case that $i_{t+1}^b > i_{t+1}^d$, then the working capital constraint in (E.7) is binding, $z_t h_t = B_{t+1}^d$. Thus, the firm's objective is to

$$\max_{h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - \left(1 + i_{t+1}^b\right) z_t h_t.$$

The first-order condition for labor h_t yields:

$$P_{t+1}\alpha A_{t+1}h_t^{\alpha} = \left(1+i_{t+1}^{b}\right) z_t h_t.$$

Dividing both sides by P_t, we obtain

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha} = \left(1+i_{t+1}^{b}\right)\frac{z_t}{P_t}h_t. \tag{E.14}$$

Step 5 - loan demand. Next, we use the labor supply (E.13) and labor demand (E.14), to solve for labor as a function of the loans rate:

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha} \;=\; \left(1+i_{t+1}^{b}\right)h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1}h_t^{\alpha}}{h_t^{\nu+1}}. \tag{E.15}$$

Since the working capital constraint binds:

$$\frac{B_{t+1}^{d}}{P_t} \;=\; h_t\frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^{d}}{P_t}\right)^{\frac{1}{\nu+1}}. \tag{E.16}$$

Thus, we can combine (E.15) and (E.16) to obtain the loans demand:

$$R_t^b \;=\; \alpha A_{t+1}\left(\frac{B_{t+1}^{d}}{P_t}\right)^{-1}\left(\frac{B_{t+1}^{d}}{P_t}\right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^{d}}{P_t} = \Theta_t\left(R_{t+1}^{b}\right)^{\epsilon^b}, \tag{E.17}$$

where the reduced form coefficients of the loans demand are:

$$\Theta_t^b \;=\; \left(\alpha A_{t+1}\right)^{-\epsilon^b} \text{ and } \epsilon^b = \left(\frac{\nu+1}{\alpha-(\nu+1)}\right).$$

Step 6 - output, firm value and bank values. We replace the loans demand (E.17) into (E.16) to obtain the equilibrium labor as a function of the equilibrium loans rate:

$$h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}}\left(R_{t+1}^{b}\right)^{\frac{1}{\alpha-(\nu+1)}}.$$

We replace (E.16) into the production function to obtain:

$$y_{t+1} = A_{t+1}\left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}\left(R_{t+1}^{b}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} \rightarrow y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}}\left(R_{t+1}^{b}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The profit of the international firm is given by:

$$r_{t+1}^{h}=y_{t+1}-R_{t+1}^{b}B_{t+1}\rightarrow r_{t+1}^{h}=A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}}\left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}}-\alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}}\right)\cdot\left(R_{t+1}^{b}\right)^{\frac{\alpha}{\alpha-(\nu+1)}}.$$

The price of the firm is given by the first-order condition with respect to Σ. In that case, q_t must satisfy:

$$q_{t}=\beta\left(r_{t+1}^{h}+q_{t+1}\right)\rightarrow q_{t}=\sum_{\tau\geq1}\beta^{\tau}\left(r_{t+\tau}^{h}\right).$$

With this, we conclude that output, hours, and the firm price are decreasing in current (and future) loans rate.

Finally, consider the price of the bank's shares. By the same token,

$$Q_{t}=\beta\left(div_{t}+Q_{t+1}\right),$$

Multiply both sides by $1/\beta$ and recall that $\vartheta_t=1$. Thus, we have:

$$\frac{1}{\beta}Q_{t}=\left(div_{t}+\beta\frac{1}{\beta}Q_{t+1}\right).$$

By change of variables let $v_t\equiv\frac{1}{\beta}Q_t$. Therefore, the value of the firm is given by

$$v_{t}=div_{t}+\beta v_{t+1}.$$

Solving this condition from time zero implies that:

$$v_{0}=\sum_{t\geq0}\beta^{t}div_{t}.$$

Thus, the bank's objective in the body of the paper is consistent with maximizing the bank's value.

Remark. We priced the firms and banks so that they are held in equilibrium by households. Thus, the shares markets clear. Note that throughout the proof we use the labor market-clearing condition, (E.15). Hence, the labor market clears. Since in the body of the paper we deal with clearing in the loans and deposit markets, by Walras's law, this implies clearing in the three goods markets.

All in all, the equilibrium in the banking block is an autonomous system. As long as the loan and deposit markets clear, we have clearing in the non-financial sector: Once we compute equilibria taking the schedules as exogenous in the bank's problem, we obtain output and household consumption from the equilibrium loan and deposit rates.

Finally, we should note that in presence of aggregate risk (inflation risk in particular), the deposit demand schedules will feature a risk premium that we are not considering in the derivation. We ignore this terms.

F Computational Algorithms

F.1 Summary of Equilibrium Conditions

Steady State: equilibrium conditions. We solve the steady-state of the model where $n = 0$ every period.[33] Solving for steady-state equilibrium requires to solve for 11 variables, three interest rates, $\left\{R^{*,d}, R^{d}, R^{b}\right\}$, three prices $\{P, P^{*}, e\}$ and five quantities $\{m^{*}, m, d, d^{*}, b^{*}\}$. We summarize these conditions below:

Prices given quantities are given by:

$$d^{*} = \Theta^{*,d}\left(R^{*,d}\right)^{\epsilon^{d^{*}}} \tag{F.1}$$

$$d = \Theta^{d}\left(R^{d}\right)^{\epsilon^{d}} \tag{F.2}$$

$$b^{*} = \Theta^{b}\left(R^{b}\right)^{\epsilon^{b}}. \tag{F.3}$$

The two prices are given by the equilibrium in the market for real dollar and euro reserves,

$$m^{*} = \frac{M^{*}}{P^{*}} \quad m = \frac{M}{P}. \tag{F.4}$$

The exchange rate is obtained via the law of one price:

$$e = P/P^{*}.$$

Finally, we have four first-order conditions and the budget constraint to pin down the five quantities:

a) the dollar liquidity premium:

$$R^{m,*} + \mathbb{E}\left[\chi^{*}_{m^{*}}\right] = R^{m} + \mathbb{E}\left[\chi_{m}\right]$$

[33] When $n > 0$, $R^{b} = 1/\beta$ so we drop one variable and the budget constraint.

b) the bond premium:

$$R^b \quad = \quad R^{m,*} + \mathbb{E}\left[\chi^*_{m^*}\right] \tag{F.5}$$

c-d) the two deposit premia

$$R^{d,*} + \mathbb{E}\left[\chi^*_{d^*}\right] = R^{m,*} + \mathbb{E}\left[\chi^*_{m^*}\right] \tag{F.6}$$

and

$$R^{d,*} + \mathbb{E}\left[\chi^*_{d^*}\right] \quad = \quad R^{m,*} + \mathbb{E}\left[\chi^*_{m^*}\right]. \tag{F.7}$$

Finally, e) the budget constraint is:

$$b + m + m^* = d + d^*.$$

This is a system of 11 equations for 11 unknowns. Next, we solve the model in ratios.

Steady State: solving the model in ratios. Recall that liquidity ratios are given by:

$$\mu \equiv \frac{m}{d} \text{ and } \mu^* \equiv \frac{m^*}{d^*}.$$

We define the ratio of real euro to dollar funding as:

$$\upsilon \equiv \frac{d}{d^*}.$$

Once we obtain $\{d^*, \nu, \mu, \mu^*\}$, we obtain $\{m, m^*, d\}$ from these three definitions.

We have shown that the interbank market tightness in euros and dollars are given by:

$$\theta\left(\mu\right) = \max\left\{-\frac{\int_{-\infty}^{-\mu}\left(\mu+\omega\right)\cdot d\Phi\left(\omega\right)}{\int_{-\mu}^{\infty}\left(\mu+\omega\right)\cdot d\Phi\left(\omega\right)}, 0\right\}$$

and an analogous equation holding for the dollar. Thus, we have three ratios. Once we obtain $\{\mu, \mu^*\}$ we obtain $\{d, d^*\}$. Once we obtain υ, we obtain $\{e^{-1}\}$. The budget constraint written in ratios i:

$$b^* \quad = \quad \left(\upsilon\left(1-\mu\right) + \left(1-\mu^*\right)\right)d^*. \tag{F.8}$$

We substitute out $\left\{R^b, R^d, R^{*,d}\right\}$ and work directly with the market clearing conditions. We replace b^* from the budget constraint. If we substitute the ratios $\{\mu^*, \mu, \upsilon, d^*\}$ into the equilibrium conditions and, thus, only have one quantity variable d^*, and the rest of the system is expressed in ratios.

Steady State: autonomous sub-system. We solve for $\{\mu^*, \mu, \upsilon, d^*\}$ using:

1) Bond premium:

$$\Theta^b \left(\left(\upsilon \left(1-\mu\right) + \left(1-\mu^*\right)\right) d^*\right)^{\epsilon^b} \quad = \quad R^{*,m} + \mathbb{E}\left[\chi_m\left(\mu^*\right)\right]. \tag{F.9}$$

2) The dollar liquidity premium:

$$R^m + \mathbb{E}\left[\chi_m\left(\mu\right)\right] = R^{*,m} + \mathbb{E}\left[\chi_m\left(\mu^*\right)\right].$$

3) The euro funding premium:

$$\Theta^d \left(\upsilon d^*\right)^{-\epsilon^d} + \mathbb{E}\left[\chi_d\left(\mu\right)\right] = R^{*,m} + \mathbb{E}\left[\chi_m\left(\mu^*\right)\right] \tag{F.10}$$

4) dollar funding premium:

$$\Theta^{*,d} \left(d^*\right)^{-\epsilon^{d^*}} + \mathbb{E}\left[\chi_{d^*}\left(\mu^*\right)\right] \quad = \quad R^{*,m} + \mathbb{E}\left[\chi_{m^*}\left(\mu^*\right)\right]. \tag{F.11}$$

These four equations provide us with a solution to $\{d^*, \nu, \mu, \mu^*\}$.

Steady State: Solving the rest of the model. With the solution to $\{\mu^*, \mu, \upsilon, d^*\}$ we obtain $\{m^*, m, d\}$ using:

$$m = \mu d, \; m^* = \mu^* d^*, \; \text{and} \; d = \nu d^*.$$

Then, we obtain the euro and dollar prices and the exchange rate:

$$P = \frac{M}{\mu \upsilon d^*}, \quad P^* = \frac{M^*}{\mu^* d^*}, \quad e = \frac{P}{P^*}.$$

F.2 Algorithm to obtain a Global Solution

Define $X \in \mathcal{X} = \{1, 2, 3, \ldots, N^s\}$ to be a finite set of states. We let X follow a Markov process with transition matrix Q. Thus, $X' \sim Q\left(X\right)$. That is, at each period, $X = \{\sigma^*, \sigma, i^{*,m}, i^m,$ $M, M^*, \Theta^d, \Theta^{*,d}\}$ are all, potentially, functions of the state X.

The algorithm proceeds as follows. We define a "greed" parameter Δ^{greed} and a tolerance parameter ε^{tol}, and construct a grid for $\mathcal{X}$. We conjecture a price-level functions $\left\{p_{(0)}(X), p^*_{(0)}(X)\right\}$ which produces a price levels in both currencies as a function of the state. As an initial guess, we use $p_{(0)}(X) = p^*_{ss}$, and $p^*_{(0)}(X) = p^*_{ss}$ setting the exchange rate to its steady state level in all periods. We proceed by iterations, setting a tolerance count tol to $tol > 2 \cdot \varepsilon^{tol}$.

Outerloop 1: Iteration of price functions. We iterate price functions until they converge. Let n be the $n-th$ step of a given iteration. Given a $p_{(n)}(X), p^*_{(n)}(X)$, we produce a new price level functions $p_{(n+1)}(X), p^*_{(n+1)}(X)$ if $tol > \varepsilon^{tol}$.

Innerloop 1: Solve for real policy rates. For each X in the grid for $\mathcal{X}$, we solve for

$$\left\{\bar{R}^m(X), \bar{R}^{*,m}(X), \bar{R}^w(X), \bar{R}^{*,w}(X)\right\}.$$

Let j be the $j-th$ step of a given iteration. Conjecture values

$$\left\{\bar{R}^m_{(0)}(X), \bar{R}^{*,m}_{(0)}(X), \bar{R}^w_{(0)}(X), \bar{R}^{*,w}_{(0)}(X)\right\}$$

We use $\left\{\bar{R}^m_{ss}, \bar{R}^{*,m}_{ss}, \bar{R}^w_{ss}, \bar{R}^{*,w}_{ss}\right\}$ as an initial guess. We then update

$$\left\{\bar{R}^m_{(j)}(X), \bar{R}^{*,m}_{(j)}(X), \bar{R}^w_{(j)}(X), \bar{R}^{*,w}_{(j)}(X)\right\}$$

until we obtain convergence:

2.a Given this guess, we solve for the liquidity ratios in Dollars and Euro $\left\{\mu, \mu^*, \bar{R}^d, \bar{R}^{*,d}\right\}$ as a function of the state using:

$$\begin{aligned}
\bar{R}^d + \frac{1}{2}\left(\chi^+(\mu) - \chi^-(\mu)\right) =& \bar{R}^{*,d} + \frac{1}{2}\left(\chi^{*,+}(\mu^*) - \chi^{*,-}(\mu^*)\right) \\
\bar{R}^m + \frac{1}{2}\left(\chi^+(\mu) + \chi^-(\mu)\right) =& \bar{R}^{*,m} + \frac{1}{2}\left(\chi^{*,+}(\mu^*) + \chi^{*,-}(\mu^*)\right) \\
\Theta^b\left(\left(\upsilon(1-\mu) + (1-\mu^*)\right)d^*\right)^{\epsilon^b} =& \bar{R}^{*,d} + \frac{1}{2}\left(\chi^+(\mu) - \chi^-(\mu)\right) \\
\bar{R}^m =& \bar{R}^{*,d} + \frac{1}{2}\left(\chi^+(\mu) - \chi^-(\mu)\right) - \frac{1}{2}\left(\chi^+(\mu) + \chi^-(\mu)\right).
\end{aligned}$$

This step yields an update for $\left\{R^d(X), R^{*,d}(X)\right\}$.

2.b Given the solutions to $\left\{R^d(X), R^{*,d}(X)\right\}$, we solve $\{d^*, \upsilon\}$ using:

$$d^* = \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}}\right]^{1/\epsilon^{d*}}$$

$$\upsilon = \left[\frac{\bar{R}^d}{\Theta^d}\right]^{1/\epsilon^d} \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}}\right]^{1/\epsilon^{d*}}.$$

This step yields an update for $\{d^*(X), \upsilon(X)\}$.

2.c Given $\{d^*(X), \upsilon(X)\}$ we solve for prices $\{p, p^*, e\}$ using:

$$\mu\upsilon d^* = \frac{M}{p}\mu^* d^* = \frac{e}{p}M^* \tag{F.12}$$

$$p^* = e^{-1}p. \tag{F.13}$$

2.d Finally, we update the real policy rates. For that we construct the expected inflation in each currency:

$$\mathbb{E}[\pi^*] = \frac{\sum_{s'\in S} Q(s'|s)\, p^*_{(n)}(s)}{p^*(s)}$$

and

$$\mathbb{E}[\pi] = \frac{\sum_{s'\in S} Q(s'|s)\, p_{(n)}(s)}{p(s)}.$$

We then update the policy rates by:

$$R^{*,a}_{(j+1)} = \frac{(1+i^{*,a})}{(1+\pi^*)} \text{ for } a \in \{m, w\}$$

and

$$R^{a}_{(j+1)} = \frac{(1+i^{a})}{(1+\pi)} \text{ for } a \in \{m, w\}.$$

2.e Repeat steps 2.a-2.d, unless

$$\left\{R^m_{(j)}(X), R^{*,m}_{(j)}(X), R^w_{(j)}(X), R^{*,w}_{(j)}(X)\right\}$$

is close to

$$\left\{R^m_{(j+1)}(X), R^{*,m}_{(j+1)}(X), R^w_{(j+1)}(X), R^{*,w}_{(j+1)}(X)\right\}.$$

If the real policy rates have converged, update prices according to

$$p^*_{(n+1)}\left(X\right) = \Delta^{greed} p^* + \left(1 - \Delta^{greed}\right) p^*_{(n)}\left(X\right)$$

and

$$p_{(n+1)}\left(X\right) = \Delta^{greed} p + \left(1 - \Delta^{greed}\right) p^*_{(n)}\left(X\right)$$

and proceed back to the outer-loop.

G Additional Figures

The following figures plot the G-9 exchange rates and CIP deviations. Shaded areas correspond to the scrambling-for-dollars regime.

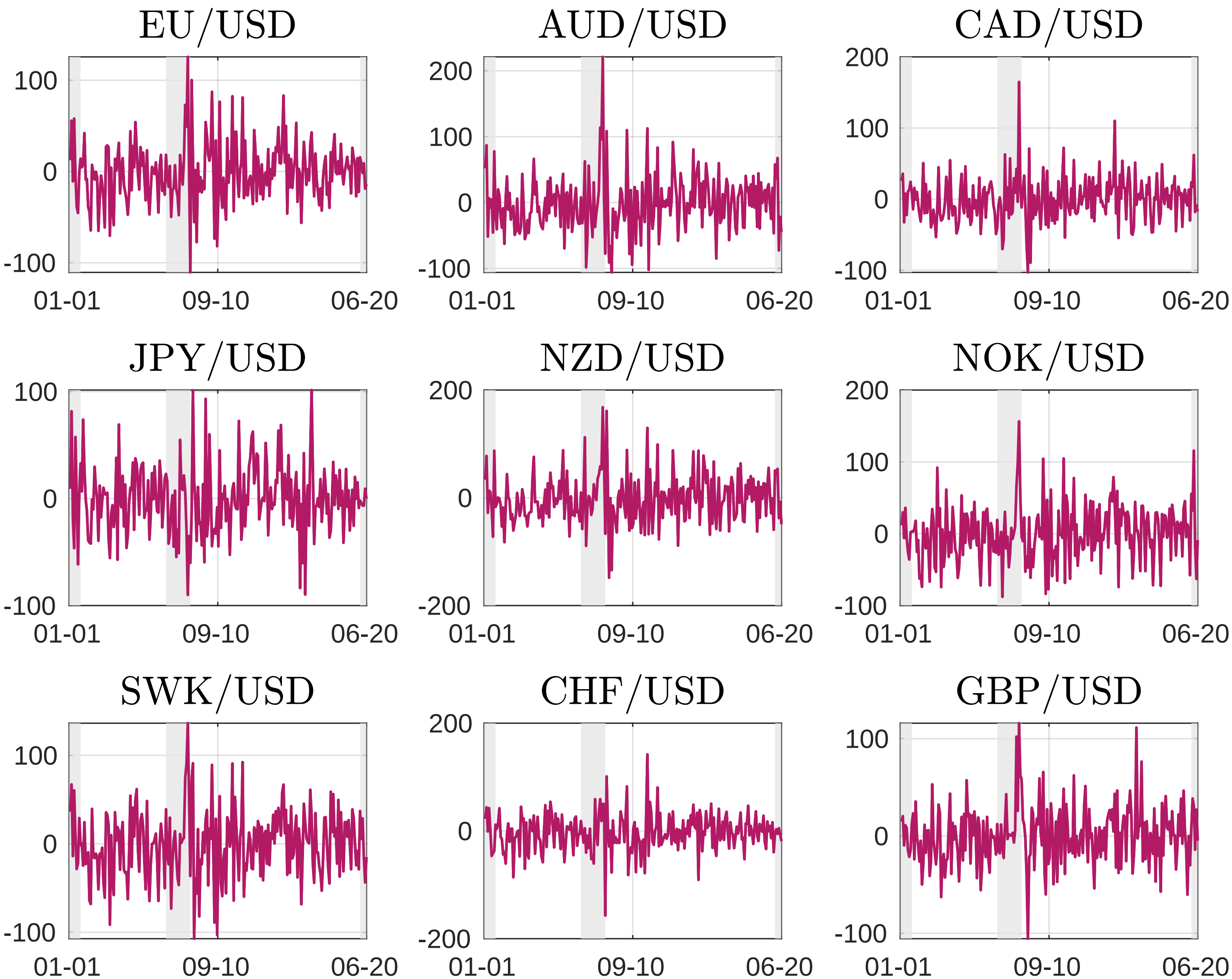

Figure G.1: Currency Devaluations

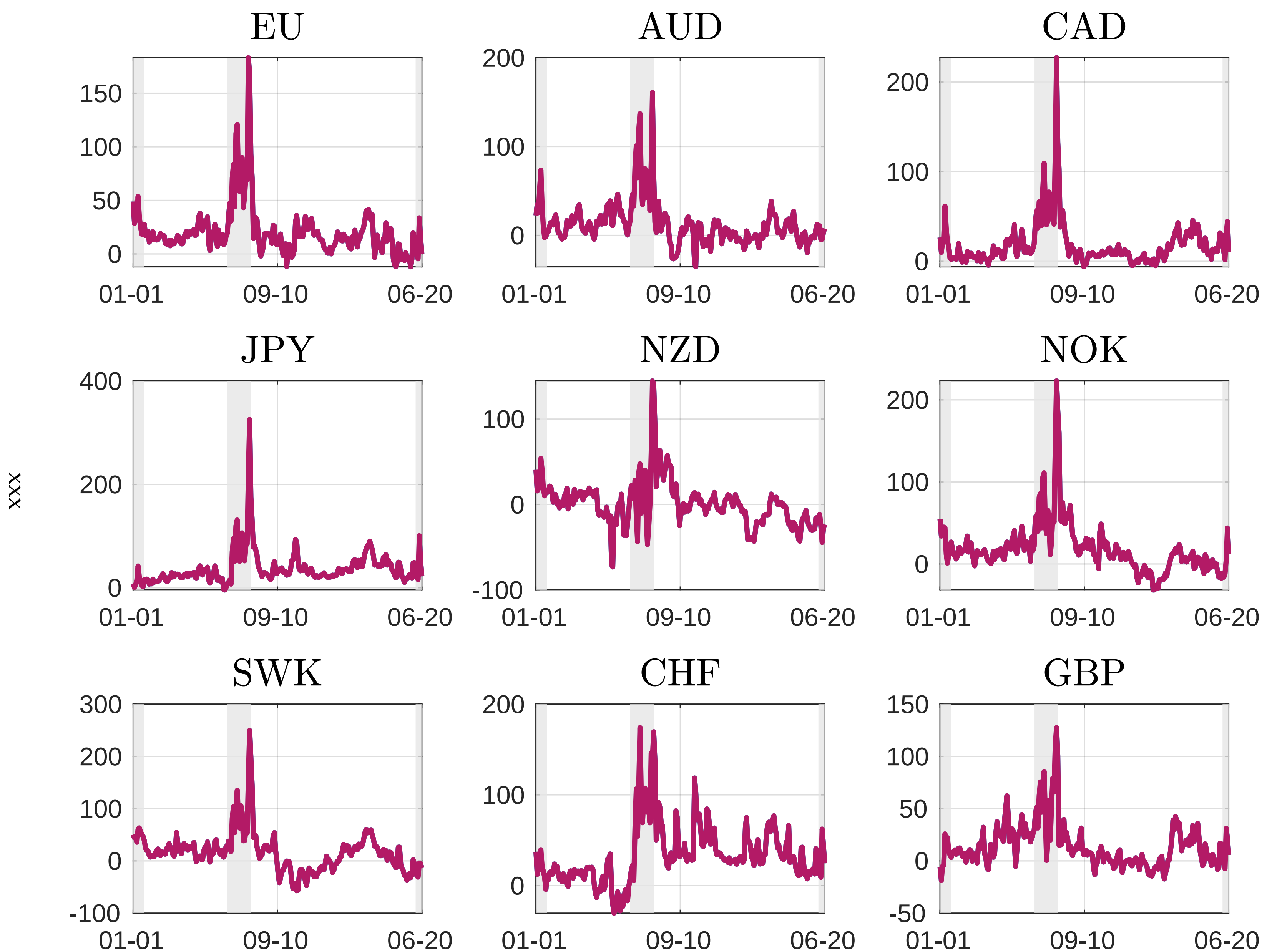

Figure G.2: CIP Deviations