

Bank Runs, Fragility, and Credit Easing

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Motivation

- Financial crises typically involve bank runs
- Short-term debt can make a bank vulnerable to a self-fulfilling run
- Empirically, runs more likely with weak aggregate fundamentals
 - General equilibrium feedbacks potentially important

★ Macroeconomic model essential to understand feedbacks

Q: What are the implications for government policy?

A Macroeconomic Model of Bank Runs

- Limited commitment and endogenous default
 - Defaults triggered by fundamentals or runs
- Dynamic portfolio and equity payout decisions
- Asset prices endogenously determined in equilibrium
 - Fragility linked to individual and aggregate fundamentals
- Self-fulfilling runs occur despite assets being liquid
- Normative analysis of credit easing policies

Preview of Main Normative Results

- Desirability of **credit easing** depends on source of the crisis
 - Bad if driven by fundamentals. Good if driven by runs

Preview of Main Normative Results

- Desirability of **credit easing** depends on source of the crisis
 - Bad if driven by fundamentals. Good if driven by runs
- Marginal banks are **net buyers** during fundamental crises, but **net sellers** during run driven crises
 - ⇒ Increases in asset prices hurt repaying banks in a fundamental driven crisis, but benefit them in the case of runs

Contribution

Literature on bank runs: (Diamond and Dybvig 1983; Allen and Gale 2000; Ennis and Keister 2009; Uhlig 2010; Gertler and Kiyotaki 2015; Gertler, Kiyotaki and Prestipino 2020; etc).

- We embed runs in dynamic macro model
 - Key differences with bank panics in [Gertler-Kiyotaki](#):
 - Coordination problem between depositors of individual banks
 - Fragility linked to short-term debt maturity
- Runs occur despite perfectly liquid assets
- Credit easing desirable if crisis driven by runs

Also build on literature on limited commitment and sovereign default: (Thomas and Worrall 1994; Albuquerque and Hopenhayn 2004; Kehoe and Levine 1993, Alvarez and Jermann 2000; [Cole-Kehoe 2000](#))

Environment

- Discrete time, infinite horizon, no aggregate risk
- Technology
 - Production linear in capital
 - Capital in fixed supply \bar{K}
- Continuum of banks, preferences $\mathbb{E} \sum_{t=0}^{\infty} \beta^t \log(c_t)$.
- Continuum of creditors, linear utility and discount rate R
- Banks are ex-ante identical
 - Start at $t = 0$ with portfolio ($b_0 = B_0, k_0 = \bar{K}$)
 - At $t = 0$, idiosyncratic shock and possibility of runs

Outline

1. Bank problem in partial equilibrium
2. General equilibrium: market clearing for capital
3. Credit policy analysis: government purchases of capital

Banks' Budget Constraints

- If repay at time t :

$$c_t = (z_t + p_t)k_t - Rb_t + q_t(b_{t+1}, k_{t+1})b_{t+1} - p_t k_{t+1}.$$

- q_t price of deposits • p_t price of capital (liquid market)
- Productivity
 - At $t = 0$, iid shock productivity z_0 , drawn cdf F
 - $z_t = z$ for all $t \geq 1$

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Bank Optimization

$$V_t(b_t, k_t) = \max \left\{ \max_{k_{t+1}, b_{t+1}, c} \log(c_t) + \beta V_{t+1}(b_{t+1}, k_{t+1}), V_t^D(k_t) \right\}$$

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For default, assume

- Permanent financial exclusion $b' = 0$
- Productivity loss $y = z^D k$

$$V_t^D(k_t) = \max_{c_t, k_{t+1}} \log(c_t) + \beta V_{t+1}^D((p_{t+1} + z^D)k_{t+1})$$

$$\text{s.t. } c_t = (p + z^D)k_t - p_t k_{t+1}$$

Equilibrium Consistent Borrowing Limit

- Equilibrium default only at $t = 0$

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Otherwise, $q = 0$.

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- Guess and verify borrowing constraint

$$b_{t+1} \leq \gamma_t p_{t+1} k_{t+1}$$

where $\{\gamma_t\}$ is an eqm. object to be determined

The Value of Default

Denote $n_t^D = (z^D + p_t)k_t$

$$V_t^D(n_t^D) = A + \frac{1}{1-\beta} \log(n_t^D) + \frac{\beta}{1-\beta} \sum_{\tau \geq t} \beta^{\tau-t} \log(R_{\tau+1}^D),$$

where the return on capital under default

$$R_{t+1}^D = \frac{z^D + p_{t+1}}{p_t}$$

and $A \equiv \frac{1}{1-\beta} \left[\log(1-\beta) + \frac{\beta}{1-\beta} \log(\beta) \right]$

$$C_t^D(k) = (1-\beta)n_t^D, \quad \mathcal{K}_{t+1}^D(k) = \beta \frac{n_t^D}{p_t}$$

The Value of Repayment

Denote $n_t^R = (z + p_t)k_t - b_t R$

$$V_t^R(n_t^R) = A + \frac{1}{1 - \beta} \log(n_t^R) + \frac{\beta}{1 - \beta} \sum_{\tau \geq t} \beta^{\tau - t} \log(R_{\tau+1}^e),$$

where returns are

$$R_{t+1}^e = R_{t+1}^k + (R_{t+1}^k - R) \frac{\gamma_t p_{t+1}}{p_t - \gamma_t p_{t+1}} \quad R_{t+1}^k \equiv \frac{z + p_{t+1}}{p_t},$$

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Policies:

$$c_t^R(n_t) = (1 - \beta)n_t^R$$

$$B_{t+1}^R(n) = \gamma_t p_{t+1} \mathcal{K}_{t+1}^R(n^R), \quad \mathcal{K}_{t+1}^R(n) = \frac{\beta n}{p_t - \gamma_t p_{t+1}} \quad \text{if } R_{t+1}^k > R$$

Equilibrium Consistent Borrowing Limit

- Given a sequence of prices, a bank is indifferent between repaying and defaulting for if $V_{t+1}^R(n_{t+1}^R) = V_{t+1}^D(n_{t+1}^D)$

$$((z + p_t)k_t - R\gamma_{t-1}p_t k_t) \prod_{s=t+1}^{\infty} (R_s^e)^{\beta^{s-t}} = (z^D + p_t)k_t \prod_{s=t+1}^{\infty} (R_s^D)^{\beta^{s-t}}$$

for $t \geq 0$

⇒ Equilibrium consistent borrowing limit

$$\frac{z + p_{t+1}(1 - \gamma_t R)}{z^D + p_{t+1}} = \left(1 - \gamma_{t+1} \frac{p_{t+2}}{p_{t+1}}\right)^{\beta}$$

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- Potentially many solutions for γ_t , but show that only one consistent with NPG

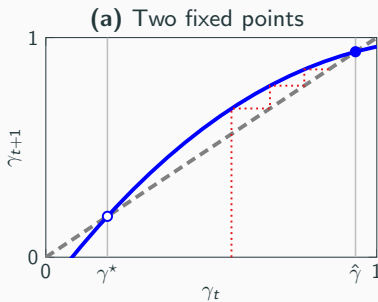
Solving for γ_t for Constant Price

$$\gamma_{t+1} = 1 - \left(\frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)$$

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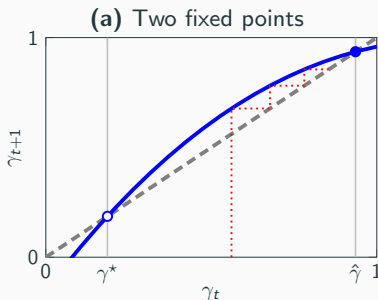
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 - First fixed point unstable \Rightarrow pins down $\gamma_t = \hat{\gamma}$



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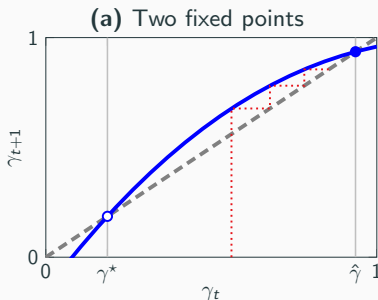
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 - First fixed point unstable \Rightarrow pins down $\gamma_t = \gamma^*$
 - γ^* is increasing in (β, z) and decreasing in (R, z^D, R^k)



Time 0 default thresholds: with and without runs

Time 0 Default Thresholds: The Case of Fundamentals

- Indifference condition at $t = 0$ given by

$$((\hat{z}^F + p_0)k_0 - Rb_0) \prod_{t=1}^{\infty} (R_t^e)^{\beta t} = (z^D + p_0)k_0 \prod_{t=1}^{\infty} (R_t^D)^{\beta t}.$$

- If $z_0 < \hat{z}^F$, a bank defaults at $t = 0$

Time 0 Default Thresholds: The Case of Fundamentals

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- If $z_0 < \hat{z}^F$, a bank defaults at $t = 0$

Time 0 Default Thresholds: The Case of Runs

When a bank faces a run and repays, its value is:

$$V_0^{Run}(n_0) = \max_{k_1 \geq 0, c_0 > 0} \log(c_0) + \beta V_1^R((z + p_1)k_1),$$

subject to

$$c_0 = n_0 + \overset{0 \text{ Deleveraging}}{\cancel{b}} - p_0 k_1.$$

Run Thresholds, Spreads and Franchise Value

$$\hat{z}^{Run} = (z^D + p_0) \left(\frac{R_1^D}{R_1^k} \right)^\beta \times \prod_{t=2}^{\infty} \left(\frac{R_t^D}{R_t^e} \right)^{\beta^t} - p_0 \left(1 - R \frac{b_0}{p_0 k_0} \right)$$

- Run threshold higher than fundamental one if $R_1^k > R$:
 - Inability to leverage reduces franchise value \Rightarrow **Runs with liquid assets if $R^k > R$**
- If $R_1^k = R$ (or $\gamma_0 = 0$), thresholds coincide
 - Access to spot market renders runs irrelevant

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 - Access to spot market renders runs irrelevant
- Marginal bank at run threshold: lower net purchases of capital than marginal bank at fundamental threshold
 - In particular, $\hat{k}_1^{Run} < k_1^D < \hat{k}_1^F$

General Equilibrium

- Market clearing for capital

$$K_t^D + K_t^R = \bar{K}$$

Consider two cases: (i) all banks subject to runs if vulnerable $\hat{z} = \hat{z}^{Run}$. (ii) only fundamental defaults $\hat{z} = \hat{z}^F$

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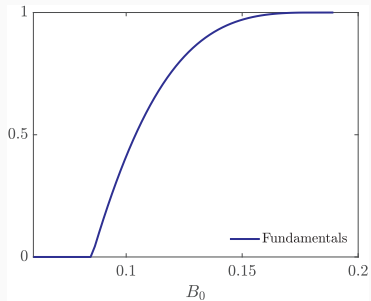
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A competitive equilibrium is a sequence of prices of capital, $\{p_t\}_{t=0}^{\infty}$, a sequence of borrowing limits, $\{\gamma_t\}_{t=0}^{\infty}$, a sequence of net worths, debt and capital holdings, $\{N_t, N_t^D, B_t, K_t^R, K_t^D\}_{t=0}^{\infty}$, and a default threshold \hat{z} , such that

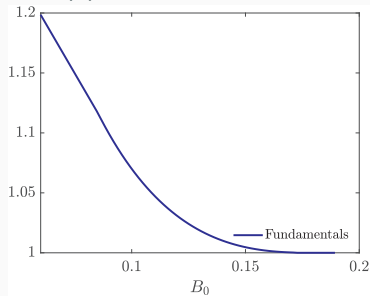
- (i) Aggregates are consistent with banks' policies
- (ii) The borrowing limits γ_t are equilibrium consistent
- (iii) Markets clear for capital
- (iv) The thresholds satisfy corresponding conditions

Equilibrium as a function of B_0

(a) Fraction of defaults $F(\hat{z})$

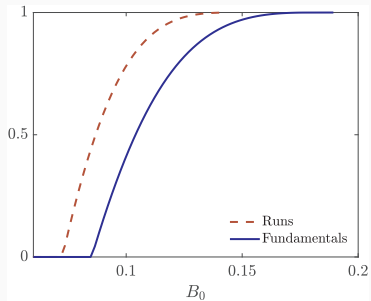


(b) Price of Capital p_0

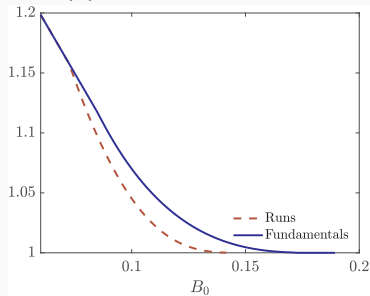


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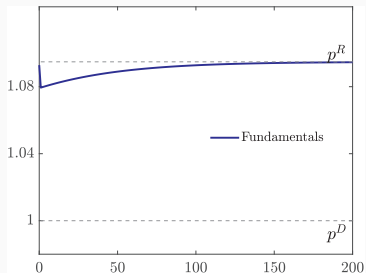


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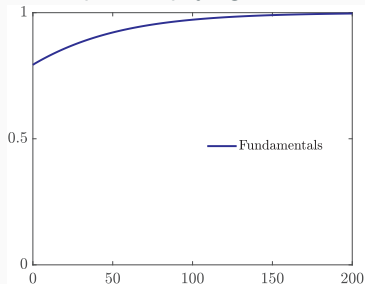


Repaying banks increase capital over time

Price of Capital p_t

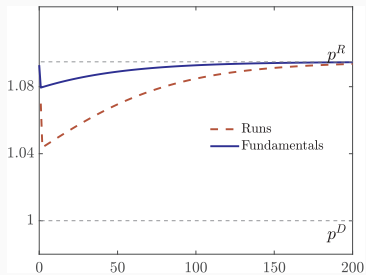


Capital Repaying Banks

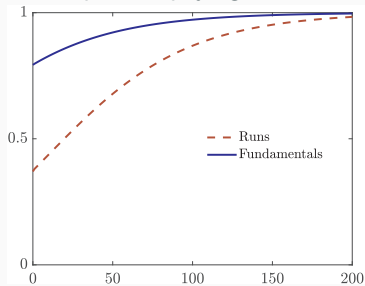


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Capital Repaying Banks



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$$K_t^D + K_t^R + K^g = \bar{K}$$

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- Govt. budget constraints at $t = 0, 1$

$$p_0 K^g = \tau(N_0 + N_0^D) + B_1^g, \quad RB_1^g = (z^g + p_1) K^g$$

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- Focus on govt. return $R^g = \frac{p_1 + z^g}{p_0} < R$:

$$\tau(N_0 + N_0^D) = \frac{p_0 K^g}{R} [R - R^g] > 0,$$

⇒ Government needs to tax to cover losses

Credit Easing

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- If investors have same return as govt., they would not buy

Q: How do government purchases affect welfare?

$$W(\hat{z}) \equiv \int_{\hat{z}}^{\bar{z}} V_0^R((z_0 + p_0)\bar{K} - RB_0)dF(z_0) + F(\hat{z})V_0^D((z^D + p_0)\bar{K})$$

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The effect of a change in the default threshold

$$W'(\hat{z}) = -f(\hat{z}) \underbrace{\left[V_0^R((\hat{z} + p_0)\bar{K} - RB_0) - V_0^D((z^D + p_0)\bar{K}) \right]}_{>0 \text{ under runs}}$$

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- A reduction in default threshold improves welfare under runs

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- A reduction in default threshold improves welfare under runs
 - Crucial: how credit easing affects default thresholds?

The Effects on Run Threshold

$$\hat{V}((1 - \tau_0)((\hat{z} + p_0)\bar{K} - RB_0)) = V_0^D((1 - \tau_0)(z^D + p_0)\bar{K})$$

- With proportional tax to wealth, effects of K^g operate exclusively through $\{p_t\}$
- Assuming only p_0 changes, we can show that $\frac{d\hat{z}^{Run}}{dp_0} < 0$
 - Banks facing a run are net sellers of capital and thus benefit from a rise in asset prices
 - Defaulting banks may also benefit but to lower extent

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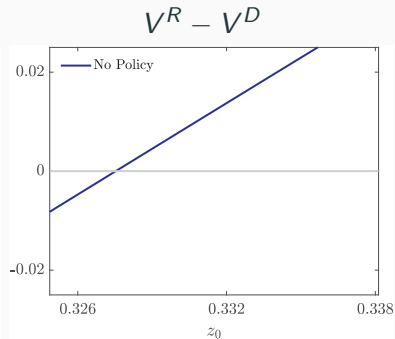
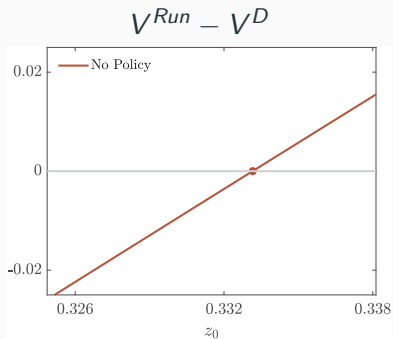
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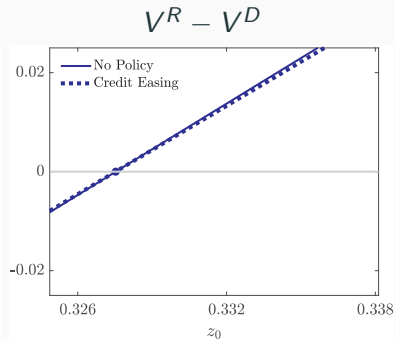
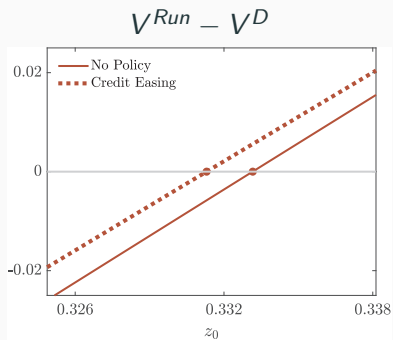
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 - To the extent that marginal bank is a net buyer, $\frac{d\hat{z}^F}{dp_0} > 0$
- If credit easing raises asset prices, positive effects on banks' welfare under runs

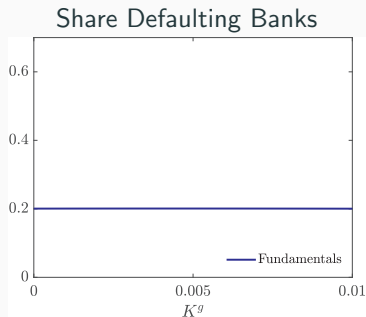
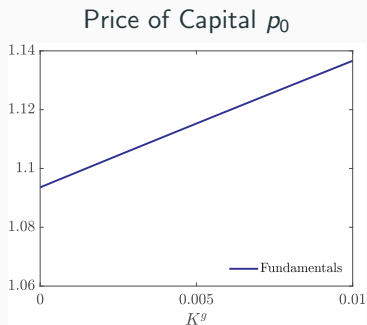
How credit easing affects default thresholds?



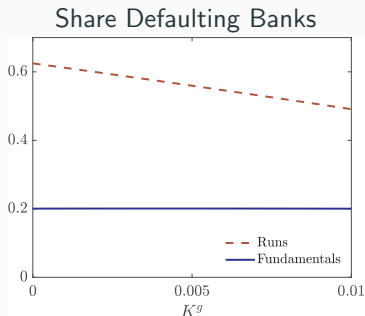
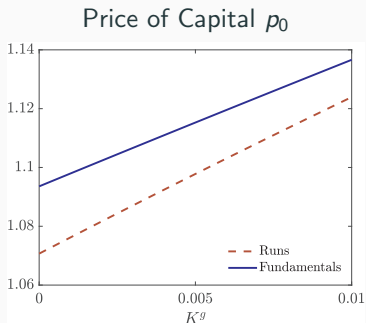
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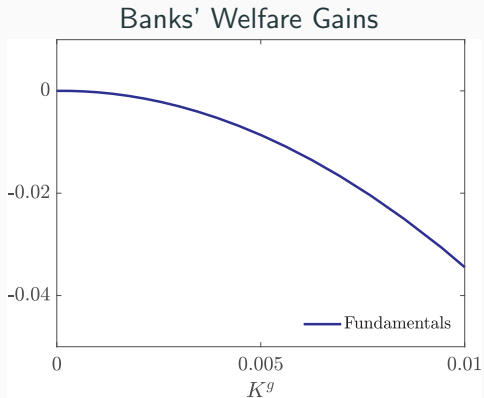
Credit easing under fundamentals



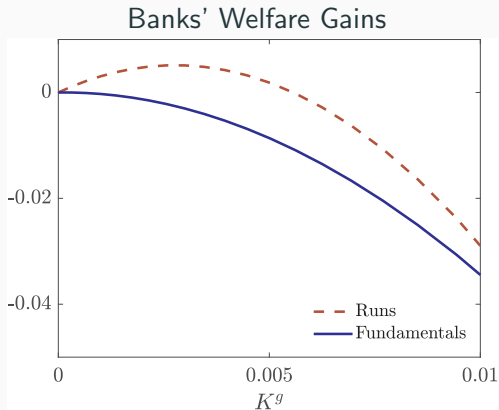
Credit easing reduces defaults under runs



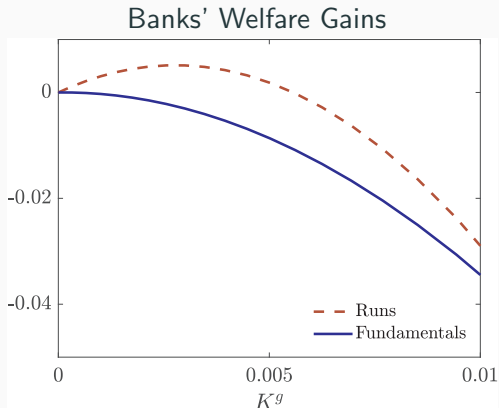
Credit Easing desirable under Runs



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Credit Easing desirable under Runs



Note: Creditors' welfare goes up as long as fewer banks default

- Sunspots
- Endogenous initial debt level B_0
- Alternative default values

Conclusions

- A dynamic macroeconomic model of self-fulfilling bank runs
- General equilibrium effects crucial to assess govt. policies
- Desirability of credit easing depends on whether a crisis is driven by fundamentals or self-fulfilling runs
- Agenda:
 - Anticipation effects of credit easing
 - Use framework for other policies, such as monetary and macroprudential