The Prudential Use of Capital Controls and Foreign Currency Reserves

Javier Bianchi [1] and Guido Lorenzoni [2]

[1]Federal Reserve Bank of Minneapolis
[2]Northwestern

April 2022

Advances in Theory and Practice

- Capital flow management has become part of accepted toolbox
- Growing literature on second-best use of capital controls
  - Pecuniary and aggregate demand externalities

    (Bianchi 2011; Schmitt-Grohé-Uribe 2016; Farhi-Werning 2016; etc)
- Focus on "prudential use"
- Similar developments for foreign currency reserves

This Paper

- Revisits literature using a unified framework
- A few themes:
  - Monetary policy dilemma for emerging economies
  - Pecuniary and aggregate demand externalities
  - Capital controls ex ante and ex post
  - Role of upward supply of funds
  - A reconciliation of two views on reserve accumulation
  - Controls on outflows as crisis management
- Also in the chapter:
  - Review of stylized facts, effectiveness, chronology of policies

Model Ingredients:

- Small open economy
- Tradable/Non-Tradable structure
- Incomplete financial markets
- Sticky wages
- Upward supply of funds from international investors
- Fear of floating

Model Ingredients:

- Small open economy
- Tradable/Non-Tradable structure
- Incomplete financial markets
- Sticky wages
- Upward supply of funds from international investors
- Fear of floating

Framework related to Basu, Boz, Gopinath, Roch, and Unsal (2020)

The Model

Preferences and Technology

- Representative consumer:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^T, c_t^N\right)$$

$$U\left(c_t^T, c_t^N\right) = \frac{1}{1-\sigma}\left(\phi^\rho \left(c_t^T\right)^{1-\rho} + (1-\phi)^\rho \left(c_t^N\right)^{1-\rho}\right)^{\frac{1-\sigma}{1-\rho}}$$

- ρ: inv-elasticity of subst. T-NT
- σ: inv-intertemp. elasticity

Preferences and Technology

- Representative consumer:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^T, c_t^N\right)$$

$$U\left(c_t^T, c_t^N\right) = \frac{1}{1-\sigma}\left(\phi^\rho \left(c_t^T\right)^{1-\rho} + (1-\phi)^\rho \left(c_t^N\right)^{1-\rho}\right)^{\frac{1-\sigma}{1-\rho}}$$

  - ρ: inv-elasticity of subst. T-NT
  - σ: inv-intertemp. elasticity

- T Endowment: y_t^T
  - Law of one price: $p_t^T = e_t p_t^*$ and $p_t^* = 1$

Preferences and Technology

- Representative consumer:

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t^T, c_t^N\right)$$

$$U\left(c_t^T, c_t^N\right) = \frac{1}{1-\sigma}\left(\phi^\rho \left(c_t^T\right)^{1-\rho} + (1-\phi)^\rho \left(c_t^N\right)^{1-\rho}\right)^{\frac{1-\sigma}{1-\rho}}$$

  - ρ: inv-elasticity of subst. T-NT
  - σ: inv-intertemp. elasticity

- T Endowment: y_t^T
  - Law of one price: $p_t^T = e_t p_t^*$ and $p_t^* = 1$

- N output: $y_t^N = n_t$
  - Supply of hours $\bar{n}$ (no labor disutility)

Consumers' Budget Constraint

- Position in pesos a_t (zero domestic net supply)
- Long position in dollars a_t^*
- Borrowing in dollars b_t^*

$$p_t^T c_t^T + p_t^N c_t^N + \frac{1}{1+i_t} a_{t+1} + \frac{1}{1+i_t^*} e_t a_{t+1}^* + e_t b_t^* \leq$$
$$e_t y_t^T + w_t n_t + a_t + e_t a_t^* + \frac{1}{1+\hat{i}_t^*} e_t b_{t+1}^*$$

In eqm.: either $b_t^* > 0$ and $a_t^* = 0$, or $a_t^* > 0$ and $b_t^* = 0$

- Interest rates:
  - i_t set by central bank; i_t^* exogenous; $\hat{i}_t^*$ endogenous

Firms and Nominal Rigidity

- Firms' problem: $\max_n p_t^N n - w_t n$

  $\Rightarrow p_t^N = w_t$ in equilibrium

Firms and Nominal Rigidity

- Firms' problem: $\max_n p_t^N n - w_t n$

  $\Rightarrow p_t^N = w_t$ in equilibrium

- Downward nominal wage rigidity (Schmitt-Grohé and Uribe)

- Employment demand determined if market clearing w below $\underline{w}$

$$w_t \geq \underline{w}, \quad n_t \leq \bar{n},$$

  with at least one equality

Prelude: Equilibrium Employment

Household optimality

$$\frac{p_t^N}{e_t} = \left(\frac{1-\phi}{\phi} \frac{c_t^T}{c_t^N} \right)^{\rho}$$

Market clearing

$$c_t^N = y_t^N$$

Prelude: Equilibrium Employment

Household optimality

$$\frac{p_t^N}{e_t} = \left(\frac{1-\phi}{\phi} \frac{c_t^T}{c_t^N} \right)^{\rho}$$

Market clearing

$$c_t^N = y_t^N$$

Assuming binding wage rigidity and using $w_t = p_t$

$$n_t = \frac{1-\phi}{\phi} \left(\frac{\underline{w}}{e_t} \right)^{-\frac{1}{\rho}} c_t^T$$

Employment increasing in e and c^T

Supply of Funds

- International investors face cost $\frac{1}{\omega}\Phi(\cdot)$ of taking dollar position in the country
- Borrow at rate $1 + i^*$ and lend at rate $1 + \hat{i}^*$
- Profits at $t + 1$ from buying $\hat{b}^*_{t+1}$ loans:

$$\hat{b}^*_{t+1} - \frac{1 + i^*_t}{1 + \hat{i}^*_t}\hat{b}^*_{t+1} - \frac{1}{\omega_t}\Phi\left(\frac{\hat{b}^*_{t+1}}{1 + \hat{i}^*_t}\right).$$

Supply of Funds

- International investors face cost $\frac{1}{\omega}\Phi(\cdot)$ of taking dollar position in the country
- Borrow at rate $1+i^*$ and lend at rate $1+\hat{i}^*$
- Profits at $t+1$ from buying $\hat{b}^*_{t+1}$ loans:

$$\hat{b}^*_{t+1} - \frac{1+i^*_t}{1+\hat{i}^*_t}\hat{b}^*_{t+1} - \frac{1}{\omega_t}\Phi\left(\frac{\hat{b}^*_{t+1}}{1+\hat{i}^*_t}\right).$$

- With quadratic Φ, optimality:

$$\frac{\hat{b}^*_{t+1}}{1+\hat{i}^*_t} = \omega_t\left(\hat{i}^*_t - i^*_t\right)$$

Supply of Funds

- International investors face cost $\frac{1}{\omega}\Phi(\cdot)$ of taking dollar position in the country
- Borrow at rate $1 + i^*$ and lend at rate $1 + \hat{i}^*$
- Profits at $t+1$ from buying $\hat{b}^*_{t+1}$ loans:

$$\hat{b}^*_{t+1} - \frac{1 + i^*_t}{1 + \hat{i}^*_t}\hat{b}^*_{t+1} - \frac{1}{\omega_t}\Phi\left(\frac{\hat{b}^*_{t+1}}{1 + \hat{i}^*_t}\right).$$

- With quadratic Φ, optimality:

$$\frac{\hat{b}^*_{t+1}}{1 + \hat{i}^*_t} = \omega_t\left(\hat{i}^*_t - i^*_t\right)$$

Supply of Funds

- International investors face cost $\frac{1}{\omega}\Phi(\cdot)$ of taking dollar position in the country
- Borrow at rate $1 + i^*$ and lend at rate $1 + \hat{i}^*$
- Profits at $t+1$ from buying $\hat{b}^*_{t+1}$ loans:

$$\hat{b}^*_{t+1} - \frac{1 + i^*_t}{1 + \hat{i}^*_t}\hat{b}^*_{t+1} - \frac{1}{\omega_t}\Phi\left(\frac{\hat{b}^*_{t+1}}{1 + \hat{i}^*_t}\right).$$

- With quadratic Φ, optimality:

$$\frac{\hat{b}^*_{t+1}}{1 + \hat{i}^*_t} = \omega_t \left(\hat{i}^*_t - i^*_t\right)$$

Study equilibrium where households borrow $\Rightarrow \hat{i}^*_t > i^*_t$

Definition of Equilibrium

A competitive equilibrium, given an initial asset position $\{b^*_{t-1}, a^*_{t-1}, a_{t-1}\}$ and a process for the nominal interest rate $\{i_t\}^{\infty}_{s=t-1}$, is a sequence of prices $\{p^N_{s-1}, e_{s-1}\}^{\infty}_{s=t}$, allocations $\{c^T_s, c^N_s, n_s\}^{\infty}_{s=t-1}$, bond positions for consumers $\{b^*_{s+1}, a^*_{s+1}, a_{s+1},\}^{\infty}_{s=t}$ and investors $\{\hat{b}^*_{s+1}\}^{\infty}_{s=t}$ such that:

(i) Consumers, firms, and investors optimize

(ii) Market clearing for non-tradables

$$c^N_s = y^N_s$$

and for domestic bonds and foreign bonds

$$a_s = 0, \qquad b^*_s = \hat{b}^*_s$$

(iii) Labor market: $n_t \leq \bar{n}, w_t \geq \underline{w}$, $(n_t - \bar{n})(w_t - \underline{w}) = 0$

Simplifying Assumptions

- Constant tradable endowment y^T
- Economy starts at $t-1$ (with zero assets).
- Capital flight shock ω at time t
- From $t+1, t+2....$ all prices flexible, $\omega_t = \infty$ and $i_t^* = \frac{1}{\beta} - 1$

Steady state equilibrium in period $t+1$

- $c_{t+1}^N = c_{t+2}^N ... = \bar{n}, \qquad c_{t+1}^T = c_{t+2}^T ... = y^T - (1-\beta)b_{t+1}^*$

Simplifying Assumptions

- Constant tradable endowment y^T
- Economy starts at $t-1$ (with zero assets).
- Capital flight shock ω at time t
- From $t+1, t+2....$ all prices flexible, $\omega_t = \infty$ and $i_t^* = \frac{1}{\beta} - 1$

Steady state equilibrium in period $t+1$

- $c_{t+1}^N = c_{t+2}^N ... = \bar{n}, \qquad c_{t+1}^T = c_{t+2}^T ... = y^T - (1-\beta)b_{t+1}^*$
- Monetary policy neutral for $t+1...$
  - Assume exchange is kept constant for $t+1...$ at $\bar{e}$

Demand for Funds

- Intertemporal Euler

$$U_T\left(c_t^T, c_t^N\right) = \beta\left(1+\hat{i}_t^*\right) U_T\left(c_{t+1}^T, \bar{n}\right)$$

- Resource constraint for T

$$c_t^T = y^T + a_t^* - b_t^* + \frac{b_{t+1}^*}{1+\hat{i}_t^*}, \qquad c_{t+1}^T = y^T - (1-\beta)\, b_{t+1}^*.$$

Demand for Funds

- Intertemporal Euler

$$U_T\left(c_t^T, c_t^N\right) = \beta\left(1+\hat{i}_t^*\right) U_T\left(c_{t+1}^T, \bar{n}\right)$$

- Resource constraint for T

$$(1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) + \frac{1}{1+\hat{i}_t^*}\left(c_{t+1}^T - y^T\right) = 0$$

Demand for Funds

- Intertemporal Euler

$$U_T\left(c_t^T \quad\right) = \beta\left(1+\hat{i}_t^*\right) U_T\left(c_{t+1}^T \quad\right)$$

- Resource constraint for T

$$(1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) + \frac{1}{1+\hat{i}_t^*}\left(c_{t+1}^T - y^T\right) = 0$$

- Under separable prefs., borrowing demand simplifies.

Demand for Funds

- Intertemporal Euler

$$U_T\left(c_t^T \qquad\right) = \beta\left(1+\hat{i}_t^*\right)U_T\left(c_{t+1}^T \qquad\right)$$

- Resource constraint for T

$$(1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) + \frac{1}{1+\hat{i}_t^*}\left(c_{t+1}^T - y^T\right) = 0$$

- Denoting demand for funds by $\mathcal{D}(\hat{i}^*, a_t^* - b_t^*) = \frac{b_{t+1}^*}{1+\hat{i}_t^*}$

Demand for Funds

- Intertemporal Euler

$$U_T\left(c_t^T \quad\right) = \beta\left(1+\hat{i}_t^*\right) U_T\left(c_{t+1}^T \quad\right)$$

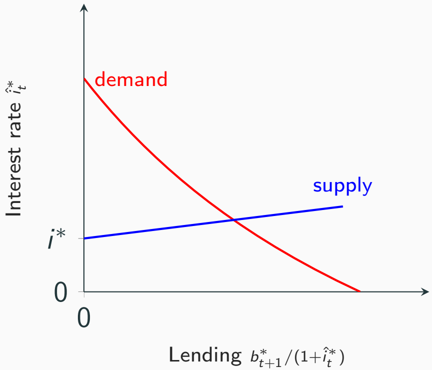
- Resource constraint for T

$$(1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) + \frac{1}{1+\hat{i}_t^*}\left(c_{t+1}^T - y^T\right) = 0$$

- Denoting demand for funds by $\mathcal{D}(\hat{i}^*, a_t^* - b_t^*) = \frac{b_{t+1}^*}{1+\hat{i}_t^*}$

$$\mathcal{D}(\hat{i}^*, a_t^* - b_t^*) = \frac{\left[1-\left[\beta\left(1+\hat{i}_t^*\right)\right]^{1/\sigma}\right] y^T - \left[\beta\left(1+\hat{i}_t^*\right)\right]^{1/\sigma}(a_t^* - b_t^*)}{(1-\beta)\left(1+\hat{i}_t^*\right) + \left[\beta\left(1+\hat{i}_t^*\right)\right]^{1/\sigma}}$$

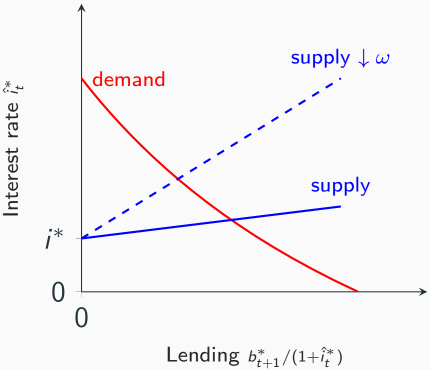
International Loan Market



$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t\left(\hat{i}_t^* - i_t^*\right)$$

International Loan Market

Capital Flight



$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t \left(\hat{i}_t^* - i_t^*\right)$$

Equilibrium in Period t

- Given monetary policy, system is block recursive in $(\hat{i}_t^*, e_t, n_t)$
  - Non-separable case ▸ General

$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t\left(\hat{i}_t^* - i_t^*\right)$$ international loan market

$$\frac{\bar{e}}{e_t}\left(1 + \hat{i}_t^*\right) = 1 + i_t$$ domestic/internat. borrowing indifference

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}}\left[y^T + a_t^* - b_t^* + \mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*)\right]$$ domestic demand

Policy Analysis

Fear of Floating Ingredient

- Policy maker's objective:

$$(1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, c_{t+1}^N\right)$$

where $\Psi\left(e_t\right)$ is a convex function with a minimum at $\bar{e}$

Fear of Floating Ingredient

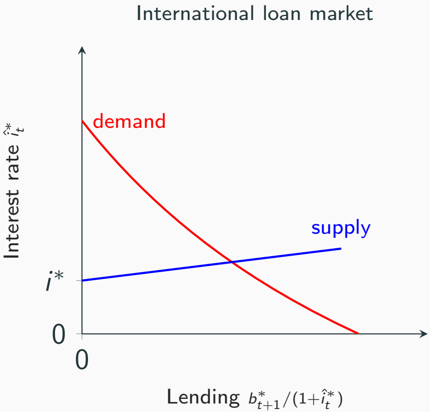
- Policy maker's objective:

$$(1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, c_{t+1}^N\right)$$

where $\Psi\left(e_t\right)$ is a convex function with a minimum at $\bar{e}$

- Possible microfoundations:
  - Balance sheet constraints
  - Reputation

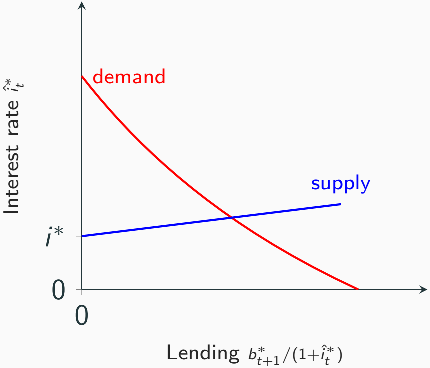
Monetary Policy Dilemma



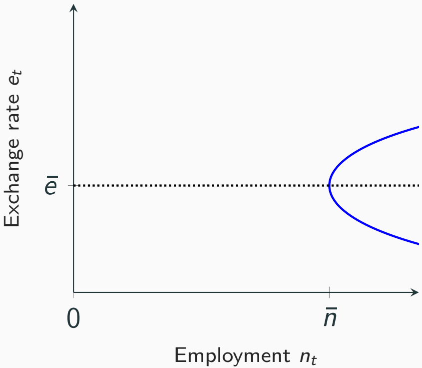
$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t\left(\hat{i}_t^* - i_t^*\right)$$

Monetary Policy Dilemma

International loan market

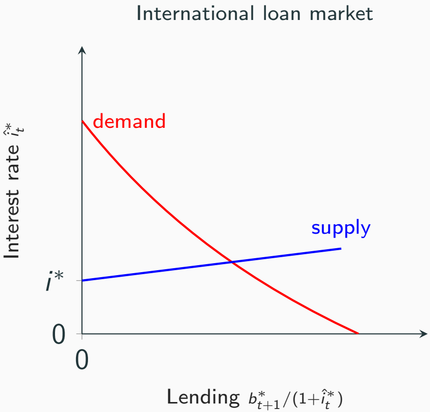


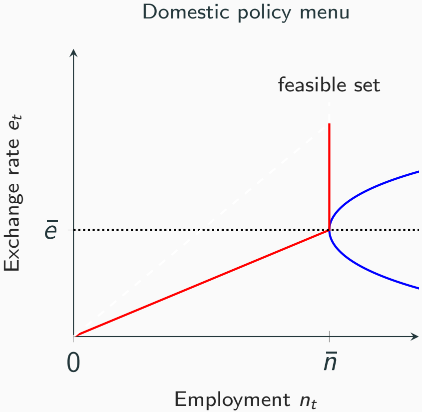
Domestic policy menu



$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t \left(\hat{i}_t^* - i_t^*\right)$$

Monetary Policy Dilemma

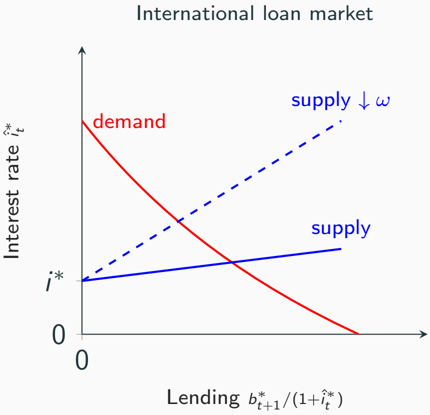


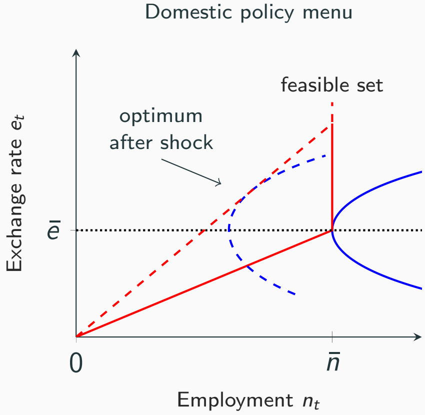


$$\mathcal{D}\left(\hat{i}^*_t, a^*_t - b^*_t\right) = \omega_t\left(\hat{i}^*_t - i^*_t\right)$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}}\left[y^T + a^*_t - b^*_t + \mathcal{D}(\hat{i}^*_t, a^*_t - b^*_t)\right]$$

Monetary Policy Dilemma





$$\mathcal{D}\left(\hat{i}^*_t, a^*_t - b^*_t\right) = \omega_t \left(\hat{i}^*_t - i^*_t\right)$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}}\left[y^T + a^*_t - b^*_t + \mathcal{D}(\hat{i}^*_t, a^*_t - b^*_t)\right]$$

Timeline

t-1

Ex Ante:

Financial decisions

- Monetary Policy
- Capital Controls
- FX Intervention

t

Ex Post:

Realization of ω_t

- Monetary policy
- Capital controls

t+1

Long run

$\omega_{t+1}=\infty$

$i^*_{t+1}=1-\frac{1}{\beta}$

Flexible wage

Timeline

t-1 | t | t+1

Ex Ante:

Financial decisions

- Monetary Policy
- Capital Controls
- FX Intervention

Ex Post:

Realization of ω_t

- Monetary policy
- Capital controls

Long run

$\omega_{t+1} = \infty$

$i^*_{t+1} = 1 - \frac{1}{\beta}$

Flexible wage

Optimal ex-post policy

$$V_t\left(a_t^* - b_t^*, \omega_t\right) = \max_{\substack{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t \\ , w_t, \tau_t}} (1-\beta)\, U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) \leq \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right)$$

$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t\left(\hat{i}_t^* - i_t^*\right)$$

$$c_t^N \leq \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T$$

$$c_t^N \leq \bar{n}, \quad w_t \geq \underline{w}$$

$$U_T(c_t^T, c_t^N) = \beta(1-\tau_t)(1+i_t^*)U_T(c_{t+1}^T, \bar{n})$$

Optimal ex-post policy

$$V_t\left(a_t^* - b_t^*, \omega_t\right) = \max_{\substack{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t \\ , w_t}} (1-\beta)\, U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) \leq \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right)$$

$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t\left(\hat{i}_t^* - i_t^*\right)$$

$$c_t^N \leq \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T$$

$$c_t^N \leq \bar{n}, \quad w_t \geq \underline{w}$$

Optimal ex-post policy

Assume wage rigidity binds

$$V_t\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t} (1-\beta)\, U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) \leq \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right)$$

$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t\left(\hat{i}_t^* - i_t^*\right)$$

$$c_t^N \leq \frac{1-\phi}{\phi}\left(\frac{\underline{w}}{e_t}\right)^{-\frac{1}{\rho}} c_t^T$$

Optimal ex-post policy

$$V_t\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, \hat{i}_t^*, e_t} (1-\beta)\, U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y^T - a_t^* + b_t^*\right) \leq \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right) \qquad [\lambda_t]$$

$$c_t^T - y^T - a_t^* + b_t^* \leq \omega_t\left(\hat{i}_t^* - i_t^*\right) \qquad [\mu_t]$$

$$c_t^N \leq \frac{1-\phi}{\phi}\left(\frac{\underline{w}}{e_t}\right)^{-\frac{1}{\rho}} c_t^T \qquad [\nu_t]$$

Optimal ex-post policy

$$V_t\left(a_t^*-b_t^*,\omega_t\right)=\max_{c_t^T,c_{t+1}^T,c_t^N,\hat{i}_t^*,e_t}(1-\beta)\,U\left(c_t^T,c_t^N\right)-\Psi\left(e_t\right)+\beta U\left(c_{t+1}^T,\bar{n}\right)$$

$$\text{s.t.}\quad (1-\beta)\left(c_t^T-y^T-a_t^*+b_t^*\right)\leq\frac{1}{1+\hat{i}_t^*}\left(y^T-c_{t+1}^T\right)\qquad[\lambda_t]$$

$$c_t^T-y^T-a_t^*+b_t^*\leq\omega_t\left(\hat{i}_t^*-i_t^*\right)\qquad[\mu_t]$$

$$c_t^N\leq\frac{1-\phi}{\phi}\left(\frac{\underline{w}}{e_t}\right)^{-\frac{1}{\rho}}c_t^T\qquad[\nu_t]$$

First-order conditions:

$$(1-\beta)\,U_T\left(c_t^T,c_t^N\right)+\nu_t\frac{c_t^N}{c_t^T}=(1-\beta)\,\lambda_t+\mu_t$$

Optimal ex-post policy

$$V_t\left(a_t^*-b_t^*,\omega_t\right)=\max_{c_t^T,c_{t+1}^T,c_t^N,\hat{i}_t^*,e_t}(1-\beta)\,U\left(c_t^T,c_t^N\right)-\Psi\left(e_t\right)+\beta U\left(c_{t+1}^T,\bar{n}\right)$$

$$\text{s.t.}\quad (1-\beta)\left(c_t^T-y^T-a_t^*+b_t^*\right)\leq\frac{1}{1+\hat{i}_t^*}\left(y^T-c_{t+1}^T\right)\qquad[\lambda_t]$$

$$c_t^T-y^T-a_t^*+b_t^*\leq\omega_t\left(\hat{i}_t^*-i_t^*\right)\qquad[\mu_t]$$

$$c_t^N\leq\frac{1-\phi}{\phi}\left(\frac{\underline{w}}{e_t}\right)^{-\frac{1}{\rho}}c_t^T\qquad[\nu_t]$$

First-order conditions:

$$(1-\beta)\,U_T\left(c_t^T,c_t^N\right)+\nu_t\frac{c_t^N}{c_t^T}=(1-\beta)\,\lambda_t+\mu_t$$

$$\lambda_t=\beta(1+\hat{i}_t^*)U_T\left(c_{t+1}^T,\bar{n}\right),\quad \nu_t=(1-\beta)\,U_N\left(c_t^T,c_t^N\right),\quad \mu_t=-\frac{\lambda_t}{\omega_t}\frac{c_{t+1}^T-y^T}{(1+\hat{i}_t^*)^2}$$

Optimal ex-post policy

$$V_t\left(a_t^*-b_t^*,\omega_t\right)=\max_{c_t^T,c_{t+1}^T,c_t^N,\hat{i}_t^*,e_t}\left(1-\beta\right)U\left(c_t^T,c_t^N\right)-\Psi\left(e_t\right)+\beta U\left(c_{t+1}^T,\bar{n}\right)$$

$$\text{s.t.}\quad (1-\beta)\left(c_t^T-y^T-a_t^*+b_t^*\right)\leq\frac{1}{1+\hat{i}_t^*}\left(y^T-c_{t+1}^T\right)\qquad[\lambda_t]$$

$$c_t^T-y^T-a_t^*+b_t^*\leq\omega_t\left(\hat{i}_t^*-i_t^*\right)\qquad[\mu_t]$$

$$c_t^N\leq\frac{1-\phi}{\phi}\left(\frac{\underline{w}}{e_t}\right)^{-\frac{1}{\rho}}c_t^T\qquad[\nu_t]$$

First-order conditions:

$$(1-\beta)\,U_T\left(c_t^T,c_t^N\right)+\nu_t\frac{c_t^N}{c_t^T}=(1-\beta)\,\lambda_t+\mu_t$$

$$\lambda_t=\beta(1+\hat{i}_t^*)U_T\left(c_{t+1}^T,\bar{n}\right),\quad \nu_t=(1-\beta)\,U_N\left(c_t^T,c_t^N\right),\quad \mu_t=-\frac{\lambda_t}{\omega_t}\frac{(1-\beta)b_{t+1}^*}{(1+\hat{i}_t^*)^2}$$

Do ex post capital controls help?

- Not quite...

$$U_T\left(c_t^T, c_t^N\right) + \underbrace{U_N\left(c_t^T, c_t^N\right)\frac{c_t^N}{c_t^T}}_{\text{AD Externality}} =$$

$$\beta U_T\left(c_{t+1}^T, \bar{n}\right)\left(1+\hat{i}_t^* \qquad \underbrace{+\frac{1}{\omega_t}\frac{b_{t+1}^*}{1+\hat{i}_t^*}}_{\text{Borrowing prem. externality}}\right)$$

- Stimulating c^T raises employment but increases $\hat{i}_t^*$ paid to foreign investors

Do ex post capital controls help?

- Not quite...

$$U_T\left(c_t^T, c_t^N\right) + \underbrace{U_N\left(c_t^T, c_t^N\right)\frac{c_t^N}{c_t^T}}_{\text{AD Externality}} =$$

$$\beta U_T\left(c_{t+1}^T, \bar{n}\right)\left(1 + \hat{i}_t^* \qquad \underbrace{+\frac{1}{\omega_t}\frac{b_{t+1}^*}{1+\hat{i}_t^*}}_{\text{Borrowing prem. externality}}\right)$$

- Stimulating c^T raises employment but increases $\hat{i}_t^*$ paid to foreign investors

Timeline

t-1 | t | t+1

Ex Ante:	Ex Post:	Long run
Financial decisions	Realization of ω_t	$\omega_{t+1}=\infty$
		$i^*_{t+1}=1-\frac{1}{\beta}$
▶ Monetary Policy	▶ Monetary policy	Flexible wage
▶ Capital Controls	▶ Capital controls	
▶ FX Intervention		

Timeline

t-1 | t | t+1

Ex Ante:

Financial decisions

- Monetary Policy
- Capital Controls
- FX Intervention

$\Psi = 0$

Ex Post:

Realization of ω_t

- Monetary policy
- Capital controls

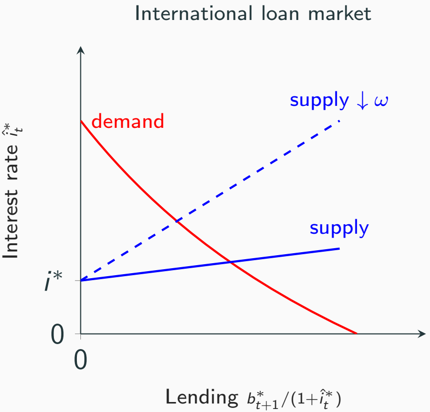
Long run

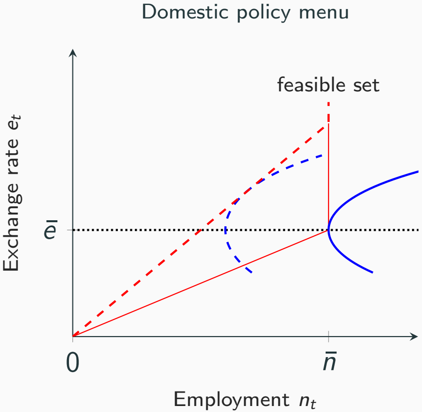
$\omega_{t+1}=\infty$

$i^*_{t+1}=1-\frac{1}{\beta}$

Flexible wage

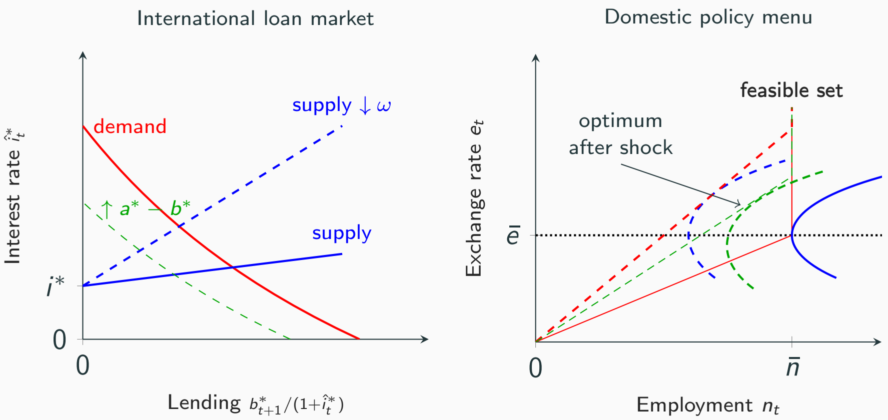
↑ NFA Ex Ante improves Policy Menu Ex Post





$$\mathcal{D}\left(\hat{i}_t^*, a_t^* - b_t^*\right) = \omega_t \left(\hat{i}_t^* - i_t^*\right) \qquad n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}}\left[y^T + a_t^* - b_t^* + \mathcal{D}(\hat{i}_t^*, a_t^* - b_t^*)\right]$$

↑ NFA Ex Ante improves Policy Menu Ex Post



$$\mathcal{D}\left(\hat{i}^*_t, a^*_t - b^*_t\right) = \omega_t \left(\hat{i}^*_t - i^*_t\right) \qquad n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}}\left[y^T + a^*_t - b^*_t + \mathcal{D}(\hat{i}^*_t, a^*_t - b^*_t)\right]$$

Optimal Ex Ante Policy

$$\max_{c^T_{t-1},c^N_{t-1},\hat{i}^*_{t-1},a^*_t,b^*_t,e_{t-1},w_{t-1},\tau_{t-1}} (1-\beta)U\left(c^T_{t-1},c^N_{t-1}\right)+\beta\mathbf{E}_{t-1}\left[V_t\left(a^*_t-b^*_t,\omega_t\right)\right],$$

$$\text{s.t.}\quad c^T_{t-1}\leq y^T_{t-1}-\frac{1}{1+i^*_{t-1}}a^*_t+\frac{1}{1+\hat{i}^*_{t-1}}b^*_t$$

$$c^T_{t-1}-y^T_{t-1}\leq\omega_{t-1}\left(\hat{i}^*_t-i^*_t\right)$$

$$c^N_{t-1}=\frac{1-\phi}{\phi}\left(\frac{w_{t-1}}{e_{t-1}}\right)^{-\frac{1}{\rho}}c^T_{t-1}$$

$$c^N_{t-1}\leq\bar{n}\quad w_{t-1}\geq\underline{w}$$

$$U_T(c^T_{t-1},c^N_{t-1})=\beta(1-\tau_{t-1})(1+i^*_{t-1})\mathbf{E}_{t-1}U_T(c^T_t,n_t)$$

Optimal Ex Ante Policy

$$\max_{c^T_{t-1}, c^N_{t-1}, \hat{i}^*_{t-1}, a^*_t, b^*_t, e_{t-1}, w_{t-1}} (1-\beta) U\left(c^T_{t-1}, c^N_{t-1}\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(a^*_t - b^*_t, \omega_t\right)\right],$$

$$\text{s.t.} \quad c^T_{t-1} \leq y^T_{t-1} - \frac{1}{1+i^*_{t-1}} a^*_t + \frac{1}{1+\hat{i}^*_{t-1}} b^*_t$$

$$c^T_{t-1} - y^T_{t-1} \leq \omega_{t-1}\left(\hat{i}^*_t - i^*_t\right)$$

$$c^N_{t-1} = \frac{1-\phi}{\phi}\left(\frac{w_{t-1}}{e_{t-1}}\right)^{-\frac{1}{\rho}} c^T_{t-1}$$

$$c^N_{t-1} \leq \bar{n} \quad w_{t-1} \geq \underline{w}$$

Optimal Ex Ante Policy

$$\max_{c^T_{t-1}, c^N_{t-1}, \hat{i}^*_{t-1}, a^*_t, b^*_t, e_{t-1},} (1-\beta) U\left(c^T_{t-1}, \bar{n} \quad\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(a^*_t - b^*_t, \omega_t\right)\right],$$

$$\text{s.t.} \quad c^T_{t-1} \leq y^T_{t-1} - \frac{1}{1+i^*_{t-1}} a^*_t + \frac{1}{1+\hat{i}^*_{t-1}} b^*_t$$

$$c^T_{t-1} - y^T_{t-1} \leq \omega_{t-1}\left(\hat{i}^*_t - i^*_t\right)$$

Optimal Ex Ante Policy

$$\max_{c_{t-1}^T, c_{t-1}^N, \hat{i}_{t-1}^*, a_t^*, b_t^*, e_{t-1},} (1-\beta) U\left(c_{t-1}^T, \bar{n} \quad\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(a_t^* - b_t^*, \omega_t\right)\right],$$

$$\text{s.t.} \quad c_{t-1}^T \leq y_{t-1}^T - \frac{1}{1+i_{t-1}^*} a_t^* + \frac{1}{1+\hat{i}_{t-1}^*} b_t^*$$

$$c_{t-1}^T - y_{t-1}^T \leq \omega_{t-1}\left(\hat{i}_t^* - i_t^*\right)$$

Optimal Ex Ante Policy

$$\max_{c^T_{t-1}, c^N_{t-1}, \hat{i}^*_{t-1}, a^*_t, b^*_t, e_{t-1},} (1-\beta) U\left(c^T_{t-1}, \bar{n} \quad\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(a^*_t - b^*_t, \omega_t\right)\right],$$

$$\text{s.t.} \quad c^T_{t-1} \leq y^T_{t-1} - \frac{1}{1+i^*_{t-1}} a^*_t + \frac{1}{1+\hat{i}^*_{t-1}} b^*_t$$

$$c^T_{t-1} - y^T_{t-1} \leq \omega_{t-1}\left(\hat{i}^*_t - i^*_t\right)$$

First-order conditions:

$$(1-\beta) U_T(c^T_{t-1}, \bar{n}) + \mu_{t-1} = \lambda_{t-1}, \qquad \frac{\lambda_{t-1}}{1+\hat{i}^*_{t-1}} = V_{a^*}\left(a^*_t - b^*_t, \omega_t\right)$$

Optimal Ex Ante Policy

$$\max_{c^T_{t-1}, c^N_{t-1}, \hat{i}^*_{t-1}, a^*_t, b^*_t, e_{t-1},} (1-\beta) U\left(c^T_{t-1}, \bar{n} \quad\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(a^*_t - b^*_t, \omega_t\right)\right],$$

$$\text{s.t.} \quad c^T_{t-1} \leq y^T_{t-1} - \frac{1}{1+i^*_{t-1}} a^*_t + \frac{1}{1+\hat{i}^*_{t-1}} b^*_t$$

$$c^T_{t-1} - y^T_{t-1} \leq \omega_{t-1}\left(\hat{i}^*_t - i^*_t\right)$$

First-order conditions:

$$(1-\beta) U_T(c^T_{t-1}, \bar{n}) + \mu_{t-1} = \lambda_{t-1}, \qquad \frac{\lambda_{t-1}}{1+\hat{i}^*_{t-1}} = V_{a^*}\left(a^*_t - b^*_t, \omega_t\right)$$

Envelope conditions (from t): Problem period t

$$V_{a^*}\left(a^*_t - b^*_t, \omega_t\right) = (1-\beta)\left(U_T\left(c^T_t, c^N_t\right) + U_N\left(c^T_t, c^N_t\right)\frac{c^N_t}{c^T_t}\right)$$

Socially optimal choice of b_t^*

- Benefits of lower b_t^*: lower borrowing costs + higher demand menu)

$$U_T\left(c_{t-1}^T, \bar{n}\right) = \beta\left(1 + \hat{i}_{t-1}^* + \underbrace{\frac{1}{\omega_{t-1}}\frac{b_t^*}{1+\hat{i}_{t-1}^*}}_{\text{Borr. premium externality}}\right)$$

$$E_{t-1}\left[U_T\left(c_t^T, c_t^N\right) + \underbrace{\iota_{n_t<\bar{n}} U_N\left(c_t^T, c_t^N\right)\frac{c_t^N}{c_t^T}}_{\text{AD externality}}\right]$$

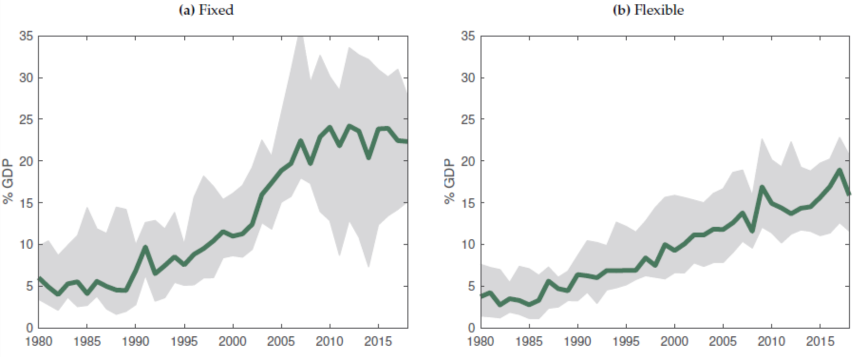
- Ex ante both externalities go in same direction

Connections

- Aggregate demand externalities with fixed exchange rates
  - Schmitt-Grohé and Uribe (2016); Farhi and Werning (2016); etc
- Pecuniary externalities and sudden stops
  - Caballero and Krishnamurthy (2003); Bianchi (2011); Korinek (2018); Bianchi and Mendoza (2018); Jeanne and Korinek (2019); etc
- Reduction in borrowing costs when intermediares are constrained
  - Amador, Bianchi, Bocola and Perri (2018); Fanelli and Straub (2021); Basu, Boz, Gopinath, Roch, and Unsal (2020); etc
- Weaker case for ex-post policies under upward supply of funds
  - Bianchi, Ottonello and Presno (2019)

Foreign Currency Reserve Interventions

Reserve Accumulation and Exchange Rate Regimes



▸ Data Details

Ilzetzki, Reinhart, and Rogoff (2019); Bianchi and Sosa Padilla (2020)

Two Views of Reserve Accumulation

- Precautionary view: protect spending in a crisis
- Exchange rate management view: prevent large fluctuations in e_t

Two Views of Reserve Accumulation

- Precautionary view: protect spending in a crisis
- Exchange rate management view: prevent large fluctuations in e_t

Two sides of the same coin:

- More resources to spend, lower contracting in spending

$$U_T\left(c_{t-1}^T, \bar{n}\right) = \beta\left(1+\hat{i}_{t-1}^* + \frac{1}{\omega_{t-1}}\frac{b_t^*}{1+\hat{i}_{t-1}^*}\right) E_{t-1}\left[U_T\left(c_t^T, c_t^N\right) + \iota_{n_t<\bar{n}} U_N\left(c_t^T, c_t^N\right)\frac{c_t^N}{c_t^T}\right]$$

- Intervention prevents a large depreciation

$$U_T\left(c_{t-1}^T, \bar{n}\right) = \beta\left(1+\hat{i}_{t-1}^* + \frac{1}{\omega_{t-1}}\frac{b_t^*}{1+\hat{i}_{t-1}^*}\right) E_{t-1}\left[U_T\left(c_t^T, c_t^N\right) + \Psi'\left(e_t\right)\rho\frac{e_t}{c_t^T}\right]$$

FX Interventions

- Government purchases foreign reserves A_t^* at $t-1$
- Financed with lump-sum taxes for simplicity (wlog)

- Questions:
  - Does this work to increase the country's NFA position?
  - What are the costs and benefits?

FX Interventions (ctd)

- Country budget constraint

$$\frac{1}{1+i^*_{t-1}}(A^*_t + a^*_t) - \frac{1}{1+\hat{i}^*_{t-1}} b^*_t + c^T_{t-1} = y^T_{t-1}.$$

- Assume initially households choose $a^*_t = 0$ and now govt. sets $A^*_t > 0$
- Keeping c^T_{t-1} requires higher b^*, but increase in $\hat{i}^*$ discourages households from borrowing
- Increase in A^* leads to higher $A^* - b^*$ but less than 1:1

▸ figure

FX Interventions (ctd)

- Country budget constraint

$$\frac{1}{1+i^*_{t-1}}(A^*_t + a^*_t) - \frac{1}{1+\hat{i}^*_{t-1}} b^*_t + c^T_{t-1} = y^T_{t-1}.$$

- Assume initially households choose $a^*_t = 0$ and now govt. sets $A^*_t > 0$
- Keeping c^T_{t-1} requires higher b^*, but increase in $\hat{i}^*$ discourages households from borrowing
- Increase in A^* leads to higher $A^* - b^*$ but less than 1:1

figure

FX Interventions (ctd)

- Country budget constraint

$$\frac{1}{1+i^*_{t-1}}(A^*_t + \quad) - \frac{1}{1+\hat{i}^*_{t-1}} b^*_t + c^T_{t-1} = y^T_{t-1}.$$

- Assume initially households choose $a^*_t = 0$ and now govt. sets $A^*_t > 0$
- Keeping c^T_{t-1} requires higher b^*, but increase in $\hat{i}^*$ discourages households from borrowing
- Increase in A^* leads to higher $A^* - b^*$ but less than 1:1

▸ figure

Optimal FX Policy

$$\max_{e_{t-1}, c^T_{t-1}, b^*_t, A^*_t} (1-\beta) U\left(c^T_{t-1}, c^N_{t-1}\right) + \beta \mathbf{E}_{t-1}\left[V_t\left(A^*_t - b^*_t, \omega_t\right)\right],$$

$$\text{s.t. } c^T_{t-1} \leq y^T_{t-1} - \frac{1}{1+i^*_{t-1}} A^*_t + \frac{1}{1+\hat{i}^*_{t-1}} b^*_t$$

$$c^T_{t-1} + \frac{A^*_t}{1+i^*_{t-1}} - y^T_{t-1} \leq \omega_t \left(\hat{i}^*_t - i^*_t\right)$$

$$c^N_{t-1} = \frac{1-\phi}{\phi} \left(\frac{w_{t-1}}{e_{t-1}}\right)^{-\frac{1}{\rho}} c^T_{t-1}$$

$$c^N_{t-1} \leq \bar{n} \quad w_{t-1} \geq \underline{w}$$

$$U_T(c^T_{t-1}, c^N_{t-1}) = \beta(1+\hat{i}^*_{t-1}) U_T(c^T_t, n_t)$$

Optimal FX Policy

$$\max_{e_{t-1},c^T_{t-1},b^*_t,A^*_t} (1-\beta)U\left(c^T_{t-1},c^N_{t-1}\right)+\beta\mathbf{E}_{t-1}\left[V_t\left(A^*_t-b^*_t,\omega_t\right)\right],$$

$$\text{s.t. } c^T_{t-1} \leq y^T_{t-1}-\frac{1}{1+i^*_{t-1}}A^*_t+\frac{1}{1+\hat{i}^*_{t-1}}b^*_t$$

$$c^T_{t-1}+\frac{A^*_t}{1+i^*_{t-1}}-y^T_{t-1} \leq \omega_t\left(\hat{i}^*_t-i^*_t\right)$$

$$U_T(c^T_{t-1},c^N_{t-1}) = \beta(1+\hat{i}^*_{t-1})U_T(c^T_t,n_t)$$

Capital Controls or FX Interventions?

- Here: capital controls $\succ$ FX interventions
- Both achieve increase in NFA, but FX carry losses
  - Equivalent if no losses (Arce, Bengui and Bianchi 2020)
- In practice, other costs from capital controls too
  - Circumvention: Bengui and Bianchi 2019

Connections

- Mechanics of FX interventions:
  - Gabaix and Maggiori (2015); Amador, Bianchi, Bocola and Perri (2020); Fanelli and Straub (2021)
- Prudential aspects:
  - Arce, Bengui and Bianchi (2019); Devereux, Davis and Yu (2020); Kim and Zhang (2020)
- Aggregate demand stabilization:
  - Bianchi and Sosa-Padilla (2020)

Controls on outflows

- So far, capital controls respect contractual returns

Controls on outflows

- So far, capital controls respect contractual returns
- A policy preventing investors from repatriating funds:

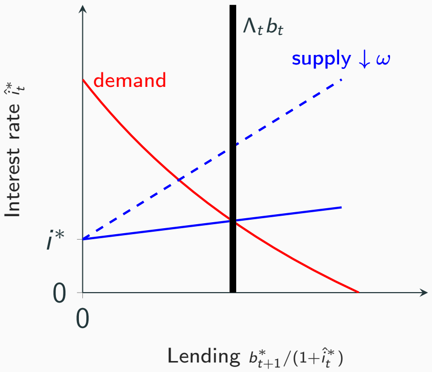
$$\max_{\{b^*_{t+1}\}} \sum_{t=0}^{\infty} \frac{1}{(1+i^*_t)^{t+1}} \left[b^*_{t+1} - \frac{1+i^*_t}{1+\hat{i}^*_t} b^*_{t+1} - \frac{1}{\omega_t} \Phi\left(\frac{b^*_{t+1}}{1+\hat{i}^*_t} \right) \right],$$

subject to

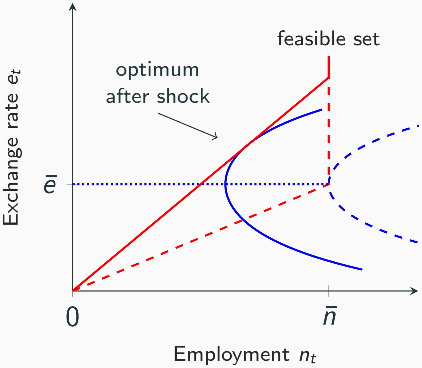
$$b_{t+1} \geq \Lambda_t b_t$$

Mechanics of Controls on Outflows

International loan market

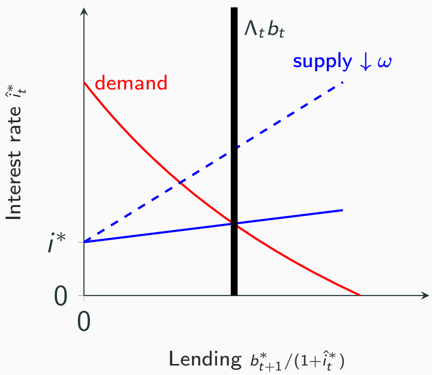


Domestic policy menu

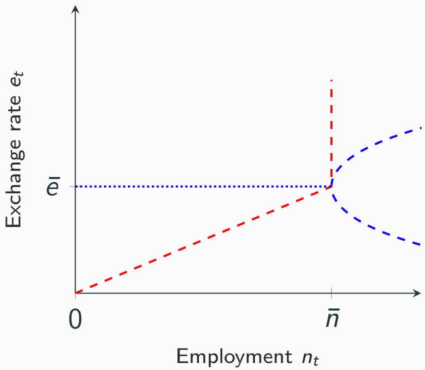


Mechanics of Controls on Outflows

International loan market



Domestic policy menu



- More similar to Malaysia in 1997 and Iceland 2011
- Costly ex ante once policy is anticipated

Balance-sheet effects and pecuniary externalities

Balance-Sheet Effects

- So far, scope for prudential policies due to rigidities in prices and monetary policy
- Similar macroeconomic externality with balance sheet constraints

$$\frac{b^*_{t+1}}{1+i^*_t} \leq \kappa_t (y^T_t + \frac{p^N_t}{e_t} y^N)$$

- Lower demand when credit constraint binds reduces p^N and further tightens constraint

Optimal Ex Post Policy

Assume collateral constraint binds at time t (and also $\Phi = \Psi = 0$)

$$V_t(a_t^* - b_t^*, \kappa_t) = \max_{c_t^T, c_t^N \leq \bar{n}, e_t, w_t} (1-\beta)\, U\left(c_t^T, c_t^N\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+i_t^*}\left(y^T - c_{t+1}^T\right),$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T,$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \kappa\left(y^T + \frac{w_t}{e_t} n_t\right) \quad [\mu_t]$$

Optimal Ex Post Policy

Assume collateral constraint binds at time t (and also $\Phi = \Psi = 0$)

$$V_t(a_t^* - b_t^*, \kappa_t) = \max_{c_t^T, c_t^N \leq \bar{n}, e_t, w_t} (1-\beta)\, U\left(c_t^T, c_t^N\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+i_t^*}\left(y^T - c_{t+1}^T\right),$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T,$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \kappa\left(y^T + \frac{w_t}{e_t} n_t\right) \quad [\mu_t]$$

- Depreciating may be costly $\Rightarrow$ endogenous Ψ
  - Farhi-Werning (2016); Ottonello (2021); Coulibaly (2021); Basu et al. (2021); Bianchi and Coulibaly (2022)

Optimal Ex Post Policy

Assume collateral constraint binds at time t (and also $\Phi = \Psi = 0$)

$$V_t(a_t^* - b_t^*, \kappa_t) = \max_{c_t^T, c_t^N \leq \bar{n}, e_t, w_t} (1-\beta) U\left(c_t^T, c_t^N\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+i_t^*}\left(y^T - c_{t+1}^T\right),$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T,$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \kappa\left(y^T + \frac{w_t}{e_t} n_t\right) \quad [\mu_t]$$

- Higher c^T ex post raises borrowing capacity $\Rightarrow$ ex ante macropru:

$$U_T\left(c_{t-1}^T, c_{t-1}^N\right) = \left(1 + i_{t-1}^*\right) \beta \mathbf{E}_{t-1}\left[U_T\left(c_t^T, c_t^N\right) + \kappa \frac{p_t^N c_t^N}{c_t^T} \rho \mu_t\right]$$

  - Bianchi (2011)

Conclusions

- Growing literature on prudential capital flow management policies
- Pecuniary and aggregate demand externalities will continue to be playing a central role
- Many open questions
    - Better understanding of the source of fear of floating
    - Sources of capital flights (credit demand/supply, liquidity/solvency)
    - Interactions of externalities and combination of policy tools (IPF)
    - International coordination
    - Empirical-theoretical connections
    - Quantitative models and implementation

Extra Slides

Capital Controls and Reserves-to-GDP Ratios

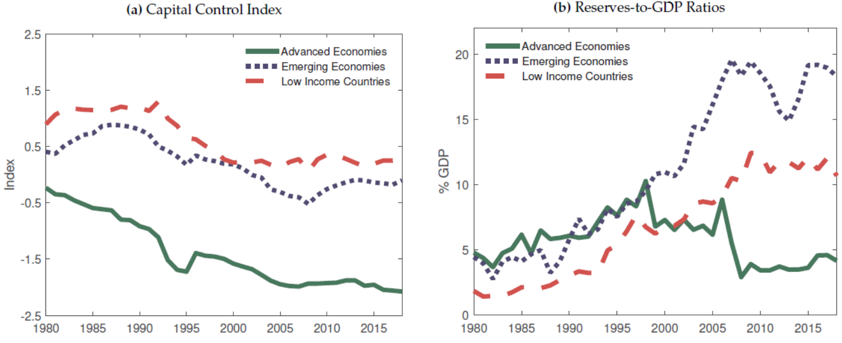


Figure 1: Capital Controls and Reserve Accumulation

Trends in International Reserves

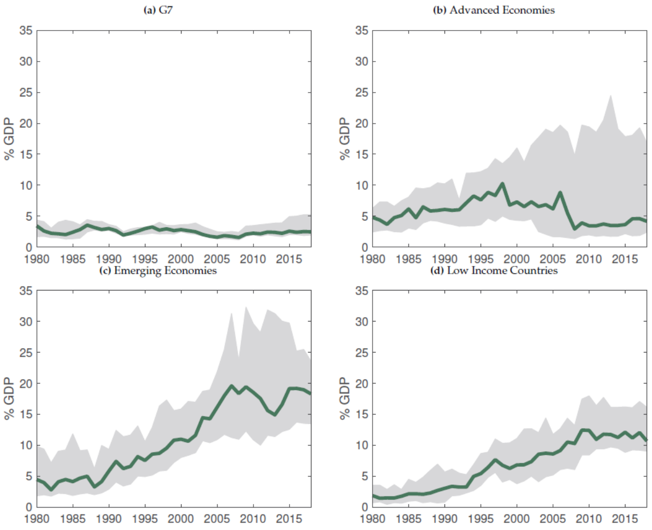


Figure 2: Trends in International Reserves

▸ back

4 Equations in $(c_t^T, \hat{i}_t^*, n_t, e_t)$: General Case

- Equilibrium for period t Back

$$U_T(c_t^T, n_t) = \beta(1+\hat{i}^*)U_T(y^T-(1+\hat{i}^*)(c^T-y^T-a_t^*+b_t^*), \bar{n})$$

$$c_t^T-y^T-a_t^*+b_t^* = \omega(\hat{i}_t^*-i^*)$$

$$\frac{e_{t+1}}{e_t}\left(1+\hat{i}_t^*\right) = 1+i_t$$

$$n_t = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}} c_t^T$$

Optimal ex-post policy

$$V\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t\left(\hat{i}_t^* - i_t^*\right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}},$$

Optimal ex-post policy

$$V\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t\left(\hat{i}_t^* - i_t^*\right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}},$$

Envelope and optimality:

$$V_{a^*}\left(a_t^* - b_t^*, \omega_t\right) = (1-\beta)\,\lambda_t + \mu_t$$

Optimal ex-post policy

$$V\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t\left(\hat{i}_t^* - i_t^*\right),$$

$$\frac{c_t^N}{c_t^T} = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}},$$

Envelope and optimality:

$$V_{a^*}\left(a_t^* - b_t^*, \omega_t\right) = (1-\beta)\,\lambda_t + \mu_t$$

$$(1-\beta)\,U_T\left(c_t^T, c_t^N\right) + \nu_t \frac{c_t^N}{c_t^T} = (1-\beta)\,\lambda_t + \mu_t, \quad \nu_t = (1-\beta)\,U_N\left(c_t^T, c_t^N\right)$$

Optimal ex-post policy

$$V\left(a_t^* - b_t^*, \omega_t\right) = \max_{c_t^T, c_{t+1}^T, c_t^N, e_t, \hat{i}_t^*} (1-\beta)\left(U\left(c_t^T, c_t^N\right) - \Psi\left(e_t\right)\right) + \beta U\left(c_{t+1}^T, \bar{n}\right)$$

$$\text{s.t.} \quad (1-\beta)\left(c_t^T - y_t^T - a_t^* + b_t^*\right) = \frac{1}{1+\hat{i}_t^*}\left(y^T - c_{t+1}^T\right),$$

$$c_t^T - y_t^T - a_t^* + b_t^* = \omega_t\left(\hat{i}_t^* - i_t^*\right),$$

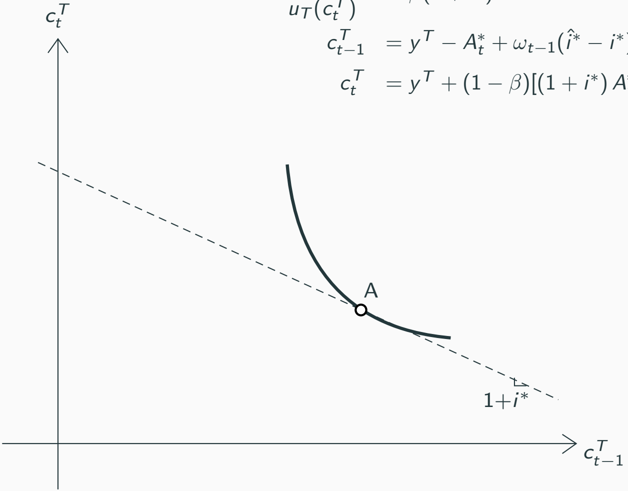
$$\frac{c_t^N}{c_t^T} = \frac{1-\phi}{\phi}\left(\frac{w_t}{e_t}\right)^{-\frac{1}{\rho}},$$

Envelope and optimality: ▸ Back

$$V_{a^*}\left(a_t^* - b_t^*, \omega_t\right) = (1-\beta)\left(U_T(c_t^T, c_t^N) + U_N(c_t^T, c_t^N)\right)$$

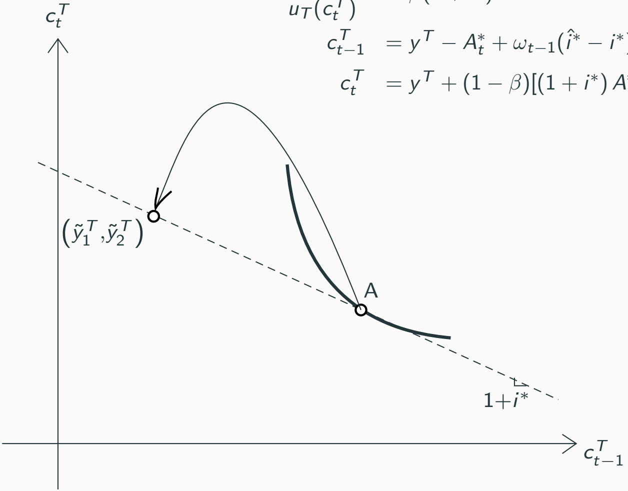
Mechanics of FX Interventions ▸ Back

$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1+\hat{i}^*)$$
$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$
$$c_t^T = y^T + (1-\beta)[(1+i^*)\,A_t^* - (1+i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$



Mechanics of FX Interventions ▸ Back

$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1+\hat{i}^*)$$
$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$
$$c_t^T = y^T + (1-\beta)[(1+i^*)\,A_t^* - (1+i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$

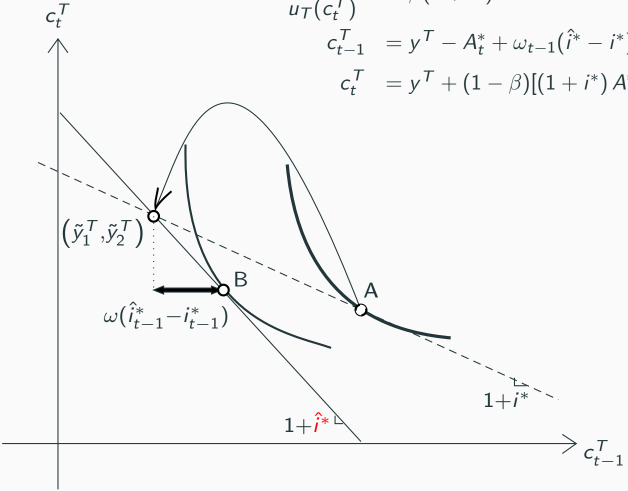


Mechanics of FX Interventions ‣ Back

$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1+\hat{i}^*)$$

$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$

$$c_t^T = y^T + (1-\beta)[(1+i^*)\,A_t^* - (1+i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$

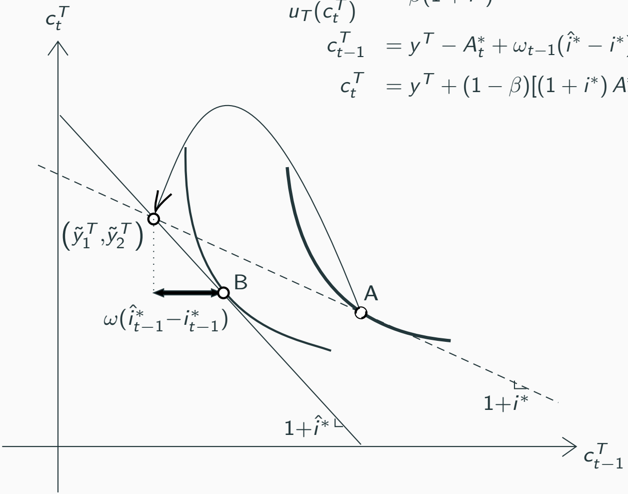


Mechanics of FX Interventions ▸ Back

$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1+\hat{i}^*)$$

$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$

$$c_t^T = y^T + (1-\beta)[(1+i^*)\,A_t^* - (1+i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$

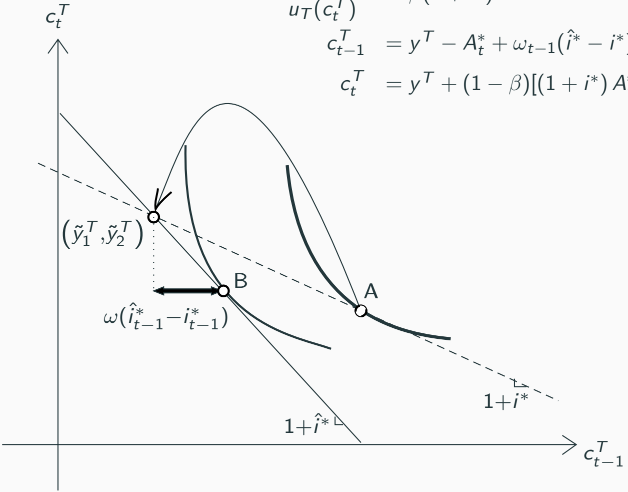


Amador; Bianchi; Bocola and Perri (2018)

Mechanics of FX Interventions ‣ Back

$\Delta c > 0$ desirable with agg. demand or pec. externalities

$$\frac{u_T(c_{t-1}^T)}{u_T(c_t^T)} = \beta(1+\hat{i}^*)$$
$$c_{t-1}^T = y^T - A_t^* + \omega_{t-1}(\hat{i}^* - i^*)$$
$$c_t^T = y^T + (1-\beta)[(1+i^*)\,A_t^* - (1+i^*)\omega_{t-1}(\hat{i}^* - i^*)]$$



Amador; Bianchi; Bocola and Perri (2018)