Helicopter Drops and Liquidity Traps

Manuel Amador   Javier Bianchi

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Impulse and Propagation, Summer Institute

This Paper

Theory:

- Helicopter drops can be useful during a liquidity trap

When the Central Bank

- faces balance sheet constraints, and
- cannot commit

Key mechanism: $\uparrow i$ are restricted by central bank net worth

- Cannot reduce M without assets

Helps with the time inconsistency problem at the ZLB

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Key mechanism: $\uparrow i$ are restricted by central bank net worth

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Helps with the time inconsistency problem at the ZLB

Complementary to other policies, such as QE

- Bhattarai, Eggertsson and Gafarov, 2022

Monetary Authority

- Accumulates risk-free assets A
- Issues monetary liabilities M
- Transfers τ_t to fiscal authority

$$M_{t-1} + \frac{A_t}{1+\iota_t} + \tau_t = M_t + A_{t-1}.$$

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Define nominal operating profits as

$$\tau^\star(A, \iota) \equiv \frac{\iota}{1+\iota} A$$

Define nominal networth as $N_t \equiv A_t - M_t$.

<u>Remark:</u> $N_t < N_{t-1}$ if and only if $\tau_t > \tau^\star(A_t, \iota_t)$

Monetary Authority (ctd)

Two *balance-sheet constraints*:

(1) Remittance constraint: $\tau_t \geq \tau^{\star}(A_t, \iota_t) \Rightarrow N_t \leq N_{t-1}$

(2) Non-negative asset holdings: $A_t \geq 0$

Fiscal Authority

- B_t: Nominal debt
- T_t: Lump-sum transfer to households
- Budget constraint:

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Let $i_t \equiv \log(1 + \iota_t)$. $y_t \equiv \log(Y_t/\bar{Y})$, $\pi_t \equiv \log(P_t/P_{t-1})$

Households / Firms

Households

- Issues bonds A_t, accumulates gov. bonds B_t
- CRRA, with money in the utility function, separable.
- ξ_t discount rate shock.

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Firms

- Phillips curve:

$$\pi_t = \beta\pi_{t+1} + \kappa y_t,$$

Private Sector Equilibrium (PSE)

PSE: A sequence $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$

$$y_t = y_{t+1} - \sigma(i_t - \pi_{t+1} - \rho - \xi_t)$$

$$i_t \geq 0$$

$$\frac{M_t}{P_t} \geq L(y_t, i_t); \text{ with equality if } i_t > 0$$

$$\pi_t = \beta\pi_{t+1} + \kappa y_t$$

Budget constraints hold, and HH transversality:

$$\lim_{t\to\infty} \frac{B_t - A_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} = 0$$

Helicopter Drops

PSE consistent with balance sheet constraints if:

$$\tau_t \geq \tau^\star(A_t, i_t)$$
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Definition: Helicopter drop at time t if $\tau_t > \tau^\star(A_t, i_t)$

- Equivalent: transfer τ^* to FA and remaining to households

Helicopter Drops

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MA budget constraint:

$$M_t - M_{t-1} = \overbrace{(A_t - A_{t-1})}^{\text{open mkt op}} + \overbrace{(\tau_t - \tau^\star(A_t, i_t))}^{\text{helicopter drop}}$$

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$$\geq -A_{t-1}$$

$$\boxed{\Rightarrow M_t \geq -N_{t-1}}$$

Towards the Policy Game

- Monetary policy objective:

$$\max \sum_{t=0}^{\infty} e^{-\rho t} W(\pi_t, y_t)$$

where

$$W(\pi, y) = -\left[(1-\varphi)\pi^2 + \varphi y^2\right]$$

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- Fiscal policy: choose $\{B_t, T_t\}$ such that FA budget holds and

$$\lim_{t\to\infty} \frac{B_t + M_t - A_t}{\prod_{s=0}^{t}(1+\iota_s)} = 0,$$

A Liquidity Trap Scenario

One-period discount rate shock:

$$\xi_t = \begin{cases} \tilde{\xi} < 0; & \text{if } t = 0, \\ 0; & \text{otherwise.} \end{cases}$$

Start with $N_{-1} > 0$.

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Commitment Solution

- Given $N_{-1} > 0$, find optimal $\{y_t, \pi_t, i_t\}$, ignoring balance sheet constraints. ▸ Details
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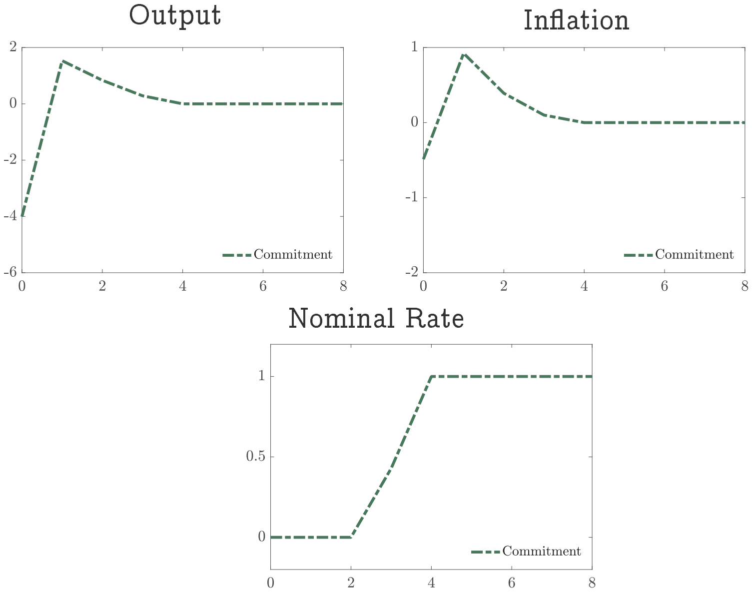
- Helicopter drops irrelevant Details
- If ZLB binds, maintain zero nominal rates for longer.

Commitment Solution



▸ Calibration

Problem: commitment solution is not time-consistent

- After $t \geq 1$, optimal to set $y_t = 0, \pi_t = 0, i_t = \rho$

(Krugman, 1998; Eggertsson and Woodford, 2003; Jung, Teranishi and Watanabe, 2005; Werning, 2011)

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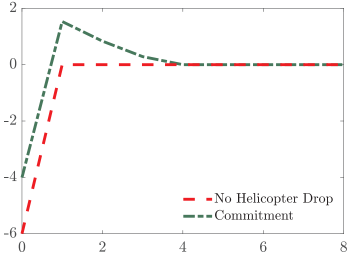
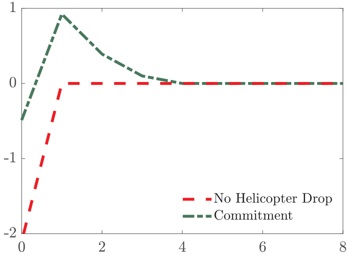
Focus on Markov equilibria

No Commitment: No Balance Sheet Constraints

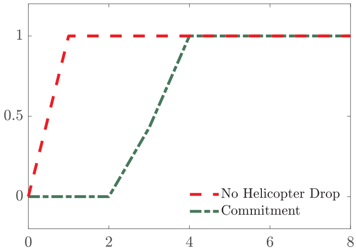
Output



Inflation



Nominal Rate



No Commitment: Balance Sheet Constraints

- Implementing $(0, 0)$ alloc. at $t = 1$ requires M_1 such that:

$$M_1 = P_0 L(e^0, \rho)$$

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- But balance sheet constraints require:

$$M_1 \geq -N_0$$

- MA at $t = 0$ can choose N_0 sufficiently low to violate this
  - Making $(0, 0)$ not possible at $t = 1$!

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But then, what happens? And what is the optimal N_0?

Markov Equilibrium with Balance Sheet Constraints

State variable: $n_{t-1} \equiv (A_{t-1} - M_{t-1})/P_{t-1}$

- Stationary case for $t \geq 1$: $\xi_t = 0$ for all t
- Liquidity trap problem at $t = 0$

Denote by $\mathcal{Y}(n)$ and $\Pi(n)$ private sector expectations

Markov equilibrium: The Stationary Case

Monetary Authority's Problem

$$V(n) = \max_{(y,\pi,i,n' \in \Omega)} W(\pi, y) + \beta V(n')$$

subject to:

$$y = \mathcal{Y}(n') - \sigma(i - \Pi(n') - \rho)$$

$$\pi = \beta \Pi(n') + \kappa y$$

$$i \geq 0$$

$$L(e^y, i) \geq -n' \text{ if } i > 0$$

$$n' \leq e^{-\pi} n \qquad \text{(strict} \Rightarrow \text{helicopter drop)}$$

Detals

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▸ Detals

V (weakly) increasing in n

Markov equilibrium

V, $\mathcal{Y}$, Π such that V is the value function in MA's problem given $\mathcal{Y}$ and Π; and $\mathcal{Y}$ and Π are themselves optimal policy functions.

High net worth

Recall first best: $\{y_t, \pi_t\} = (0, 0)$ and $m = L(e^0, \rho)$

Define $n^\star \equiv -L(e^0, \rho)$

Good equilibria

Suppose that for all $n \geq n^\star$, $\mathcal{Y}(n) = 0$ and $\Pi(n) = 0$.
Then for $n \geq n^\star$, $(0, 0)$ solve MA's problem

There are also deflationary trap equilibria (Benhabib et al. 2001; Armenter, 2018)

We will focus on the good equilibria   Paper discusses how the traps can be ruled out

Low net worth

For $n < n^\star$, the first-best outcome $\{0, 0\}$ is not feasible.

Low net worth

No Helicopter Drops in Stationary Case

Suppose that $\Pi(n)$, $\mathcal{Y}(n)$ are weakly decreasing and that $\Pi(n)$ is strictly decreasing for $n < n^\star$. For $n < n^\star$, the optimum features $n' = e^{-\pi}n$.

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$$y = \mathcal{Y}(n') - \sigma(\quad - \Pi(n') - \rho) > 0$$
$$\pi = \beta \Pi(n') + \kappa y_t > 0$$

If solution features $i = 0$:

- Solution features $\pi > 0, y > 0$
- Suppose $n' < e^{-\pi} n$. Then, $\uparrow n'$ lowers π, y (& increase $V(n')$) $\Rightarrow$ improves welfare

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Constrain future policies by MA without any benefits today

Low net worth (ctd)

Recall

$$L(e^y, i) \geq -n', \quad n' \leq e^{-\pi} n$$

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$$L(e^y, i) \geq -n', \quad n' = e^{-\pi} n$$

$$\boxed{e^{\pi} L(e^y, i) \geq -n}$$

Low net worth (ctd)

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- Lower n requires a combination of higher (π, y) and lower i

Linear equilibria

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- Modified state: $k \equiv -\log(-n) + \log(-n^{\star})$

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Linear equilibria:

for $k < 0$ (and such that $i > 0$):

$$\mathcal{Y}(k) = ak, \quad \Pi(k) = bk, \quad V(k) = -\nu k^2$$

for $k \geq 0$:

$$\mathcal{Y}(k) = \Pi(k) = V(k) = 0$$

- Cannot prove existence/uniqueness in general
  - But conditions are easy to check given parameters

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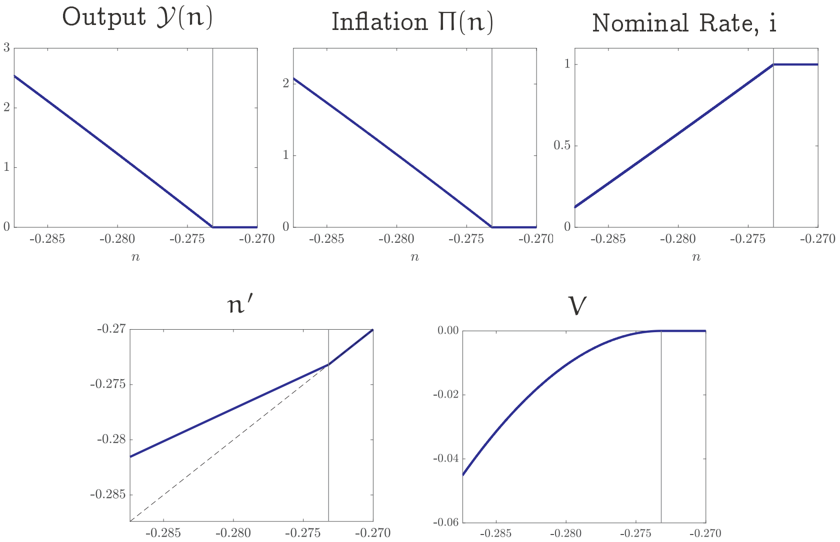
- Cannot prove existence/uniqueness in general
  - But conditions are easy to check given parameters
- For our parameterization:
  - Unique linear eqm with $a < 0$, $b < 0$, $\nu > 0$

Stationary Policies



Vertical line denotes $n^\star$

Markov Equilibrium: Period 0

Monetary Authority's Problem in a Liquidity Trap

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subject to:

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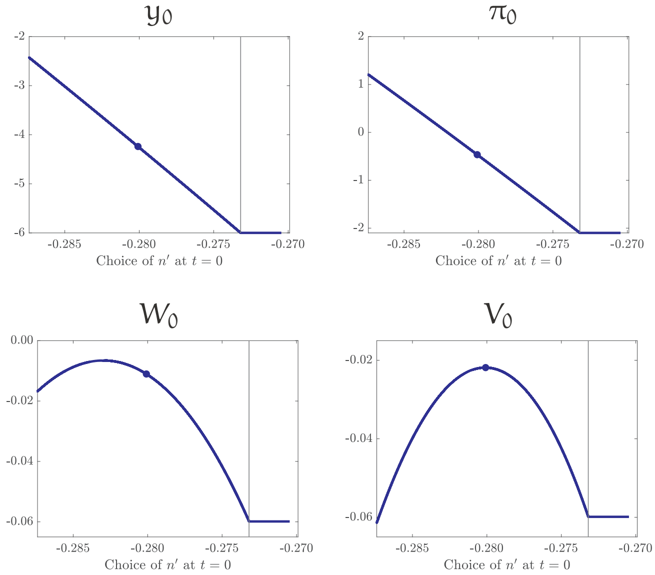
Initial values as a function of n_0 choice



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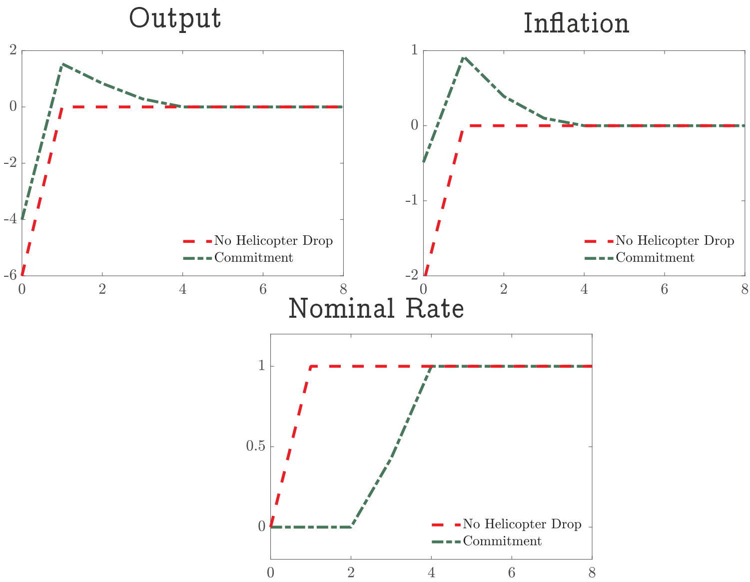
Simulation Comparison



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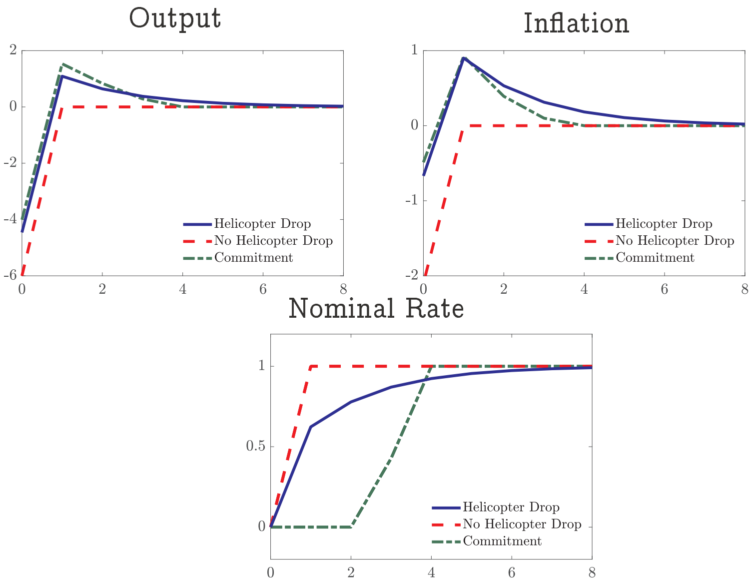

Conclusion

- Theory of helicopter drops as a commitment device
- In the model: useful during a liquidity trap

- Caveats:
  - Commitment vs Flexibility
  - Balance sheet constraints modeled ad-hoc
  - Absence of MA reserves

Numerical Simulations

Calibration: $\rho = 0.01$ $\sigma = 0.5$. $\kappa = 0.35$ $\varphi = 0.05$,

Money demand follows:

$$L(y, i) = \theta e^{y - \eta i}$$

Set $\eta = 0.5$ and $\theta = 0.10$ to match currency to GDP =10%

Set $\tilde{\xi} = -0.12$ to generate output drop of 6% in liquidity trap

▸ Back

Irrelevance

Helicopter drops do not enlarge the set of PSE consistent with balance sheet constraints:

- PSE with helicopter drop $\Rightarrow$ remove it and substitute with open mkt op.

  (converse is not true)

Back

Sufficient net worth

$N_{-1} \geq 0 \Rightarrow$ For any $\{y_t, \pi_t, i_t\}$ that satisfies Euler, PC and ZLB, there exists a policy such the allocations belong to a PSE with balance sheet constraints.

- Can set $\tau_t = \tau_t^* \Rightarrow$ net worth stays constant.
  $N_t \equiv A_t - M_t > 0$ imply $A_t > 0$

Back

Monetary authority's Problem

$n_{t-1} \equiv (A_{t-1} - M_{t-1})/P_{t-1}$

Two constraints:

$$A_t \geq 0 \text{ and } N_t \leq N_{t-1}$$

Using $N_t = A_t - M_t$

$$M_t \geq -N_t \Leftrightarrow m_t \geq -n_t$$
$$N_t \leq N_{t-1} \Leftrightarrow n_t \leq e^{-\pi_t} n_{t-1}$$

And $m_t = L(e^{y_t}, i_t)$ for $i_t > 0$.

Back