

Optimal Time-Consistent Macroprudential Policy *

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Abstract

Collateral constraints widely used in models of financial crises feature a pecuniary externality, because agents do not internalize how future collateral prices respond to collective borrowing decisions, particularly when binding collateral constraints trigger a crisis. We show that agents in a competitive equilibrium borrow “too much” during credit expansions compared with a macro-prudential financial regulator who internalizes the externality. Under commitment, however, this regulator would face a time inconsistency problem: It promises low future consumption to prop up current asset prices when collateral constraints bind, but this is not optimal ex post. Instead, we study a time-consistent optimal policy of a regulator who cannot commit to future policies. Quantitative analysis shows that this policy reduces the incidence and magnitude of financial crises, removes fat tails from the distribution of returns and reduces risk premia. A state-contingent tax on debt of about 1 percent on average decentralizes the regulator’s allocations, but simpler policies implemented timely to preempt overborrowing can also produce gains.

KEYWORDS: Financial crises, macroprudential policy, systemic risk, collateral constraints

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1 Introduction

The cross-country analysis of credit boom episodes by Mendoza and Terrones (2012) shows that credit booms in advanced and emerging economies are relatively rare, occurring at a frequency of 2.8 percent in a sample of 61 countries spanning the period 1960-2010. They also found, however, that when they occur they display a clear cyclical pattern of economic expansion in the upswing followed by a steep contraction in the downswing. Strikingly, 1/3rd of these credit booms are followed by full blown financial crises, and this frequency is about the same in advanced and emerging economies. From this perspective, and with all dimensions properly taken, what happened in 2008 in the United States is a recurrent event.

The realization that credit booms are rare but perilous events that often end in financial crises and deep recessions has resulted in a strong push for implementing a new form of financial regulation. As described in the early work by Borio (2003) or a recent exposition by Bernanke (2011), the objective of this macro-prudential approach to regulation is to take a macroeconomic perspective of credit dynamics, with a view to defusing credit booms in their early stages as a prudential measure to prevent them from turning into crises. The efforts to move financial regulation in this direction, however, have moved faster and further ahead than our understanding of how financial policies influence the transmission mechanism driving financial crises, particularly in the context of quantitative macroeconomic models that can be used to design and evaluate the performance of these policies.

This paper aims to fill this gap by answering three key questions: First, can a credit friction affecting individual borrowers turn into a significant macroeconomic problem, in terms of both producing financial crises with quantitative features similar to those we see in the data and influencing ordinary business cycles? Second, what is the optimal design of macro-prudential policy taking into account commitment issues, which if ignored result in the classic time-inconsistency problem that undermines standard optimal policy arguments? Third, how powerful is this policy for affecting the incentives of private credit market participants and in terms of its effect on the magnitude and incidence of financial crises?

This paper proposes answers to these questions based on the quantitative predictions of a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We start by using quantitative methods to show that, in the absence of macro-prudential policy, the model's financial amplification mechanism produces financial crises with realistic features. Then we characterize and solve for the optimal, time-consistent macro-prudential policy of a financial regulator who lacks the ability to commit to future policies, and show that a state-contingent schedule of debt taxes can support the allocations of this policy in a decentralized equilibrium.

A central feature of the framework we develop is a pecuniary externality in a similar vein of those used in the related literature on credit booms and macro-prudential policy (e.g. Lorenzoni, 2008, Korinek, 2009, Bianchi, 2011, Stein, 2012): Individual agents facing a collateral constraint on their ability to borrow do not internalize how their borrowing decisions in "good times" affect the market price of collateral, and hence the aggregate borrowing capacity, in "bad times" in which the collateral constraint binds. This creates a form of market failure that results in equilibria that can be improved upon by a financial regulator who faces the same credit frictions but internalizes the externality.

In the model, the collateral constraint is in the form of an occasionally-binding limit on the total amount of debt (one-period debt and working capital loans) as a fraction of the market value of physical assets that can be posted as collateral, which are in fixed aggregate supply. This constraint introduces the above pecuniary externality as a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by the regulator. In addition, the constraint serves as the engine of the mechanism by which the model can produce financial crises with realistic features as an equilibrium outcome. This is because, when the constraint binds, Irving Fisher's classic debt-deflation financial amplification mechanism is set in motion. The result is a financial crisis driven by a nonlinear feedback loop between asset fire sales and borrowing ability.

The interaction of the pecuniary externality with Fisherian amplification implies that in this setting the externality has an intertemporal dimension: What private agents fail to internalize when making their borrowing plans taking collateral prices as given is that, in the future, if the collateral constraint binds fire sales of assets will cause a Fisherian debt-deflation spiral that will cause asset prices to decline sharply and the economy's overall borrowing ability to shrink.¹ Moreover, when the constraint binds, in our setup production plans are also affected, because working capital financing is needed in order to pay for a fraction of factor costs, and working capital loans are also subject to the collateral constraint. This results in a sudden increase in effective factor costs and a fall in output when the credit constraint binds. In turn, this affects dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the pecuniary externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions either.

We study the optimal policy problem of a financial regulator that chooses the level of credit to maximize the private agents' utility subject to resource and credit constraints and with two key features: First, the regulator internalizes the pecuniary externality. Second, the

¹For this reason, the literature also refers to this externality as a systemic risk externality, because individual agents contribute to the risk that a small shock can lead to large macroeconomic effects, or as a fire-sale externality, because as collateral prices drop, agents fire-sale the goods or assets that serve as collateral to meet their financial obligations.

regulator cannot commit to future policies. The first feature leads the regulator to impute a higher social marginal cost to choosing higher debt and leverage in good times, because the regulator takes into account that higher leverage can cause a Fisherian asset price deflation in bad times. The second feature implies that the regulator’s optimal policy is time-consistent, in contrast with the time-inconsistent policy chosen by a regulator acting under commitment. Under commitment, if the collateral constraint binds, it is optimal for the regulator to make promises of lower future consumption with the aim to prop up current asset prices, but renegeing is optimal ex post. Hence, if effective commitment devices are hard to come by, this policy strategy is not credible. Instead, we explicitly model the regulator’s inability to commit to future policies, and solve for optimal time-consistent macro-prudential policy as part of a Markov perfect equilibrium in which the effect of current optimal plans of the regulator on future plans is taken into account.

The paper develops some theoretical results and conducts a quantitative analysis in a version of the model calibrated to data for industrial economies. The theoretical analysis keeps the model tractable by abstracting from production and working capital, assuming that borrowing ability depends on the aggregate supply of assets, instead of individual asset holdings, and modeling exogenous dividend shocks as the only underlying shocks hitting the economy. These three assumptions are relaxed in the quantitative analysis.

The quantitative results show that financial crises in the competitive equilibrium are significantly more frequent and more severe than in the equilibrium attained by the regulator. The incidence of financial crises is about three times larger. Asset prices drop about 30 percent in a typical crisis in the decentralized equilibrium, versus 5 percent in the regulator’s equilibrium. Output drops about 20 percent more, because the fall in asset prices reduces access to working capital financing. The more severe asset price collapses also generate an endogenous “fat tail” in the distribution of asset returns in the decentralized equilibrium, which causes the price of risk to rise 1.5 times and excess returns to rise by 5 times, in both tranquil times and crisis times. The regulator can replicate exactly its equilibrium allocations as a decentralized equilibrium by imposing a state-contingent tax on debt of about 1 percent on average and positively correlated with leverage.

This paper contributes to the growing literature in the intersection of Macroeconomics and Finance by developing a non-linear quantitative framework suitable for the normative analysis of macro-prudential policy. The non-linear global methods are necessary in order to quantify accurately the macro implications of occasionally binding collateral constraints in models with incomplete asset markets and subject to aggregate shocks. This is important for determining whether the model provides a reasonable approximation to the non-linear macroeconomic features of actual financial crises, and thus whether it is a useful laboratory

for policy analysis, and also for capturing the prudential aspect of macro-prudential policy, which works by altering the incentives of economic agents to engage in precautionary behavior in “good times,” when credit and leverage are building up. Moreover, using non-linear global methods is also key for solving the Markov perfect equilibrium that characterizes optimal time-consistent macro-prudential policy in our framework.

Most of the recent Macro/Finance literature, including this article, follows in the vein of the research program on fire sales and financial accelerators initiated by [Bernanke and Gertler \(1989\)](#), [Kiyotaki and Moore \(1997\)](#). In particular, we follow [Mendoza \(2010\)](#) in the analysis of non-linear dynamics. He focused only on positive analysis to show how an occasionally binding collateral constraint generates financial crises with realistic features that are nested within regular business cycles as a result of shocks of standard magnitudes. We focus instead on normative analysis, and develop a framework for designing optimal, time-consistent macro-prudential regulation that can reduce the risk of financial crises and improve welfare.

As noted earlier, the pecuniary externality at work in our model is related to those examined in the theoretical work of [Caballero and Krishnamurthy \(2001\)](#), [Lorenzoni \(2008\)](#), and [Korinek \(2009\)](#), which arises because private agents do not internalize the amplification effects caused by financial constraints that depend on market prices.² There are also studies of this externality with a quantitative focus similar to ours. In particular, [Bianchi \(2011\)](#) makes a quantitative assessment of a prudential tax on borrowing, but in a setting in which the borrowing capacity is linked to the real exchange rate. In a similar model, [Benigno, Chen, Otrok, Rebucci, and Young \(2013\)](#) show that there can also be a role for ex-post policies in addition to prudential ones when the planner can reallocate labor from the non-tradables sector to the tradables sector.

This paper differs from the above quantitative studies in that it focuses on asset prices as a key factor driving debt dynamics and the pecuniary externality, instead of the relative price of nontradable goods. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Moreover, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent with a credit constraint linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. This also leads to the time consistency issues that we tackle in this study and that were absent from previous work. In addition, our model

²For a generic result on constrained inefficiency in incomplete markets see e.g. [Geneakoplos and Polemarchakis \(1986\)](#).

differs because it introduces working capital financing subject to the collateral constraint, which allows the externality to affect adversely production, factor allocations and dividend rates, and thus again asset prices. In contrast, [Bianchi \(2011\)](#) studies an endowment economy and in [Benigno, Chen, Otrok, Rebucci, and Young \(2013\)](#) employment and production rise when the collateral constraint binds, because of the higher shadow value of supplying labor to produce nontradable goods and thereby relax the credit limit.

This paper is also related to [Jeanne and Korinek \(2010\)](#) who study the quantitative effects of macroprudential policy in a model in which assets serve as collateral. In their model, however, output follows an exogenous Markov-switching process and individual credit is limited to the sum of a fraction of aggregate, rather than individual, asset holdings plus a constant term. Since in their calibration this second term dwarfs the first, and the probability of crises matches the exogenous probability of a low-output regime, the debt tax they examine has no effect on the frequency of crises and has small effects on their magnitude. In contrast, in our model both the probability of crises and output dynamics are endogenous, and macroprudential policy reduces sharply the incidence and magnitude of crises. Our approach also differs from Jeanne and Korinek in that they impose restrictions on the ability of the planner to distort asset prices when the collateral constraint binds, which bypasses time consistency problem. In particular, they assume that the planner takes as given an asset pricing function consistent with the Euler equation of asset holdings of the decentralized equilibrium.³ In contrast, we study a Markov perfect equilibrium taking into account explicitly the inability of the planner to commit to future policies. This also allows us to provide a clear analytical characterization of the pecuniary externality.⁴

Our analysis is also related to other recent studies exploring alternative theories of inefficient borrowing and their policy implications. For instance, [Schmitt-Grohé and Uribe \(2012\)](#) and [Farhi and Werning \(2012\)](#) examine the use of prudential capital controls as a tool for smoothing aggregate demand in the presence of nominal rigidities and a fixed exchange rate regime. In earlier work, [Uribe \(2006\)](#) examined an environment in which agents do not internalize an aggregate borrowing limit and yet borrowing decisions are the same as in an environment in which the borrowing limit is internalized.⁵ Our analysis differs in that the

³We followed a similar approach in [Bianchi and Mendoza \(2010\)](#) by setting up the optimal policy problem of the planner in recursive form using the asset pricing function of the unregulated decentralized equilibrium to value collateral.

⁴In particular, we show that the optimal state contingent tax is positive in states in which the collateral constraint is not binding, which rationalizes the lean-against-the-wind argument of macro-prudential policy. By contrast, Jeanne and Korinek provide an expression for the tax that depends on equilibrium objects with a potentially ambiguous sign.

⁵He provided analytical results for a canonical endowment economy model with a credit constraint where there is an exact equivalence between the two sets of allocations. In addition, he examined a model in which this exact equivalence does not hold, but still overborrowing is negligible.

regulator internalizes not only the borrowing limit but also the price effects that arise from borrowing decisions. Still, our results showing small differences in average debt ratios across competitive and regulated equilibria are in line with his findings.

The literature on participation constraints in credit markets initiated by [Kehoe and Levine \(1993\)](#) is also related to our work, because it examines the role of inefficiencies that result from endogenous borrowing limits. In particular, [Jeske \(2006\)](#) showed that if there is discrimination against foreign creditors, private agents have a stronger incentive to default than a planner who internalizes the effects of borrowing decisions on the domestic interest rate, which affects the tightness of the participation constraint. [Wright \(2006\)](#) then showed that as a consequence of this externality, subsidies on capital flows restore constrained efficiency.

This work also aims to make a methodological contribution by developing methods to solve for Markov perfect equilibria in models with occasionally binding collateral constraints and a social planner who faces forward-looking implementability constraints. In this regard, our paper relates to the literature on the use of Markov perfect equilibria to solve for time-consistent optimal fiscal policy, particularly [Klein, Krusell, and Rios-Rull \(2008\)](#) on government expenditures and [Klein, Quadrini, and Rios-Rull \(2005\)](#) on international taxation.

The rest of the paper is organized as follows: Section 2 presents the simple version of the model used for the analytical work and characterizes the unregulated competitive equilibrium. Section 3 conducts the normative analysis of the simple model, including the optimization problem of a constrained-efficient financial regulator who cannot commit to future policies. Section 4 extends the model for the quantitative analysis by endogenizing production, introducing working capital financing, allowing borrowing capacity to depend on individual asset holdings, and adding interest-rate shocks and shocks to leverage limits. Section 5 calibrates the model and discusses the quantitative findings. Section 6 provides conclusions.

2 A Simple Fisherian Model of Financial Crises

This Section characterizes the decentralized competitive equilibrium of a simple model of financial crises driven by a collateral constraint. We use this model to develop the normative analysis of the pecuniary externality and optimal time-consistent macro-prudential policy in a tractable way. The main features of this analysis will be preserved in the more general model that we use for the quantitative analysis later in the paper.

2.1 Economic Environment

Consider an economy inhabited by a representative agent with preferences given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (1)$$

In this expression, $\mathbb{E}(\cdot)$ is the expectations operator, β is the subjective discount factor. The utility function $u(\cdot)$ is a standard concave, twice-continuously differentiable function that satisfies the Inada condition. k_t denotes the agent's holdings of an asset that pays a random dividend z_t each period, with known probability distribution function. The asset is in fixed unit supply, so that the market clearing condition in the asset market is simply $k_t = 1$.

The agent's budget constraint is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R} = k_t(z_t + q_t) + b_t \quad (2)$$

where b_t denotes the holdings of one-period, non-state-contingent discount bonds at the beginning of date t , q_t is the market price of capital, and R is an exogenous real interest rate. This last assumption can be interpreted as implying that the economy is a price taker in world financial markets, which is a reasonable assumption for most of the advanced economies considered in the quantitative experiments of Section 5.⁶

The representative agent is also subject to a credit constraint by which it cannot borrow more than a fraction κ of the market value of the economy's aggregate quantity of assets:

$$\frac{b_{t+1}}{R} \geq -\kappa q_t \quad (3)$$

The assumption that borrowing ability depends on the aggregate market value of assets simplifies the analytical expressions that characterize the planner's problem of the next Section, but is not necessary in general. Hence, in Section 4 we extend the model for the quantitative analysis by assuming the more realistic scenario in which individual asset holdings determine borrowing capacity.

The agent chooses consumption, asset holdings and bond holdings to maximize (1) subject to the budget constraint (2) and the collateral constraint (3). This maximization problem

⁶An alternative assumption that yields an equivalent formulation is to assume deep-pockets, risk-neutral lenders that discount future utility at the rate $\beta^* = 1/R$.

yields the following first-order conditions for c_t , b_{t+1} and k_{t+1} respectively:

$$\lambda_t = u'(c_t) \quad (4)$$

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t \quad (5)$$

$$q_t \lambda_t = \beta \mathbb{E}_t [\lambda_{t+1} (z_{t+1} + q_{t+1})] \quad (6)$$

where $\lambda_t > 0$ and $\mu_t \geq 0$ are the Lagrange multipliers on the budget constraint and collateral constraint respectively. Condition (4) is standard. Condition (5) is the Euler equation for bonds. When the collateral constraint binds, this condition implies that the effective marginal cost of borrowing for additional consumption today exceeds the expected marginal utility cost of repaying R units of goods tomorrow by an amount equal to the shadow value of the credit constraint (i.e. the household faces an effective real interest rate higher than R). Condition (6) is the Euler equation for assets, which equates the marginal cost and benefit of holding them. Since the collateral constraint depends on the aggregate quantity of assets, this condition is not affected by μ_t .

The interaction between the collateral constraint and asset prices at work in this simple model can be illustrated by studying how standard asset pricing conditions are altered by the constraint. In particular, combining (5), (6) and the definition of asset returns ($R_{t+1}^q \equiv \frac{z_{t+1} + q_{t+1}}{q_t}$), it follows that the expected excess return on assets relative to bonds (i.e. the equity premium, $R_t^{ep} \equiv \mathbb{E}_t(R_{t+1}^q - R)$), satisfies the following condition:

$$R_t^{ep} = \frac{\mu_t}{u'(c_t) \mathbb{E}_t m_{t+1}} - \frac{\text{cov}_t(m_{t+1}, R_{t+1}^q)}{\mathbb{E}_t m_{t+1}} \quad (7)$$

Following Mendoza and Smith (2006), we can denote the first term in the right-hand-side of (7) as a direct (first-order) effect of the collateral constraint, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell assets, depressing the current price and increasing excess returns.⁷ There is also an indirect (second-order) effect given by the fact that $\text{cov}_t(m_{t+1}, R_{t+1}^q)$ is likely to become more negative, because the collateral constraint makes it harder for agents to smooth consumption.

Condition (6) yields a seemingly standard-looking forward solution for asset prices:

$$q_t = \mathbb{E}_t \sum_{j=1}^{\infty} m_{t,t+j} z_{t+j}, \quad m_{t,t+j} \equiv \frac{\beta^j u'(c_{t+j})}{u'(c_t)} \quad (8)$$

⁷When we extend the model in Section 4 to assume that individual asset holdings at the beginning of the period are posted as collateral, this direct effect is weaker because the agent also attaches additional value to holding assets as collateral.

But again following Mendoza and Smith, we can use the definition of asset returns to rewrite this pricing condition as follows:

$$q_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^j \mathbb{E}_{t+i} R_{t+1+i}^q \right)^{-1} z_{t+j+1}, \quad (9)$$

Expressed in this form, and taking into account condition (7), it follows that a binding collateral constraint at date t increases expected excess returns and lowers asset prices at t . This mechanism is at the core of the model's pecuniary externality: larger levels of debt lead to more frequent fire sales, driving excess returns up and depressing asset prices, which in turn reduce the borrowing capacity of the economy as a whole. Moreover, because expected returns rise whenever the collateral constraint is expected to bind at any future date, condition (9) also implies that asset prices at t are affected by collateral constraints not just when the constraints binds at t , but whenever it is expected to bind at any future date along the equilibrium path. Hence, expectations about future excess returns and risk premia feed back into current asset prices, and this interaction will be important for the analysis of macro-prudential policy, as shown in the next Section.

2.2 Recursive Competitive Equilibrium

We now characterize the competitive equilibrium in recursive form. Since the representative agent acting atomistically takes all prices as given, the recursive formulation separates individual bond holdings b that are under the control of the agent at date t from the economy's aggregate bond position B on which all prices depend. Hence, the state variables for the agent's problem are the individual states (b, k) and the aggregate states (B, z) . Aggregate capital is not carried as a state variable because it is in fixed supply. In order to be able to form expectations of future prices, the agent also needs to take as given a perceived law of motion governing the evolution of the economy's bond position $B' = \Gamma(B, z)$ as well as a conjectured asset pricing function $q(B, z)$.

For given $B' = \Gamma(B, z)$, the agent's recursive optimization problem is:

$$\begin{aligned} V(b, k, B, z) &= \max_{b', k', c} u(c) + \beta \mathbb{E}_{z'|z} V(b', k', B', z') & (10) \\ \text{s.t.} \quad q(B, z)k' + c + \frac{b'}{R} &= k(q(B, z) + z) + b \\ -\frac{b'}{R} &\leq \kappa q(B, z) \end{aligned}$$

The solution to this problem is characterized by the decision rules $\hat{b}(b, k, B, z)$, $\hat{k}(b, k, B, z)$,

$\hat{c}(b, k, B, z)$, $\hat{n}(b, k, B, z)$, $\hat{m}(b, k, B, z)$ and $\hat{h}(b, k, B, z)$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}(B, 1, B, z)$. In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium is defined by an asset pricing function $q(B, z)$, a perceived law of motion for aggregate bond holdings $\Gamma(B, z)$, and a set of decision rules $\{\hat{b}'(b, k, B, z), \hat{k}'(b, k, B, z), \hat{c}(b, k, B, z)\}$ with associated value function $V(b, k, B, z)$ such that:*

1. $\{\hat{b}(b, k, B, z), \hat{k}(b, k, B, z), \hat{c}(b, k, B, z)\}$ and $V(b, k, B, z)$ solve the agent's recursive optimization problem, taking as given $Q(B, z)$ and $\Gamma(B, z)$.
2. The perceived law of motion for aggregate bonds is consistent with the actual law of motion: $\Gamma(B, z) = \hat{b}(B, 1, B, z)$.
3. Asset prices satisfy:

$$Q(B, z')(u'(\hat{c})) = \beta \mathbb{E}_{z'|z} \left\{ u'(\hat{c}(b, k, B, z')) (Q(\hat{b}, z') + z') + \kappa' Q(\Gamma(B, \mathbf{s}), z') \hat{\mu}(b', z') \right\}$$

4. Goods and asset markets clear: $\frac{\hat{b}'(B, 1, B, z)}{R} + \hat{c}(B, 1, B, z) = z + B$ and $\hat{k}(B, 1, B, z) = 1$

3 Normative Analysis in the Simple Model

In this Section, we conduct a normative analysis of the model we just laid out. First we make a brief comparison of the competitive equilibrium with an efficient equilibrium in which there is no collateral constraint. Then we study a constrained-efficient social planner's (or financial regulator's) problem in which the regulator chooses the bond position for the private agent while lacking the ability to commit to future policies. Finally, we show that the allocations of this planner's problem can be decentralized with state-contingent taxes on borrowing.

3.1 Equilibrium without collateral constraint

In the absence of the collateral constraint (3), the competitive equilibrium allocations can be represented as the solution to the following standard planning problem:

$$\begin{aligned} H(B, z) &= \max_{B', c} u(c) + \beta \mathbb{E}_{z'|z} H(B', z') \\ \text{s.t.} \quad c + \frac{B'}{R} &= z + B \end{aligned}$$

and subject also to either this problem's *natural debt limit*, which is defined by $B' \geq -\min(z)/(R-1)$, or a tighter ad-hoc time- and state-invariant debt limit. Note that this problem is analogous to the problem solved by individual agents in standard heterogeneous agents models of savings under incomplete markets.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (e.g. [Mendoza, 2010](#)) is to compare the allocations of the competitive equilibrium with the collateral constraint with those arising by the above problem without collateral constraint. Private agents borrow *less* in the former because the collateral constraint limits the amount they can borrow, and also because they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint. Compared with the constrained-efficient allocations we examine next, however, we will show that the competitive equilibrium with collateral constraints displays *overborrowing*. Hence, the competitive equilibrium of the economy with the collateral constraint features *underborrowing* relative to the equilibrium without collateral constraints but *overborrowing* relative to the constrained-efficient equilibrium with the collateral constraint.

3.2 A Constrained Efficient, Time-Consistent Planner

Consider now a constrained-efficient social planner who makes the choice of debt for the representative agent subject to the same collateral constraint and lacking the ability to commit to future policies. A key assumption in defining this planner's problem relates to how the equilibrium price of collateral is determined, because this determines the economy's borrowing capacity and is closely related to the time-consistency issues discussed later in this Section. We assume that these prices are determined in competitive markets, and hence the social planner cannot control them directly (i.e. agents retain access to asset markets). The planner does, however, internalize how its borrowing decisions affect asset prices, and makes

optimal use of its debt policy to influence them.⁸

3.2.1 Private Agents Optimization Problem

Since the government chooses bond holdings, the optimization problem faced by the private agent reduces to choosing consumption and asset holdings taking as given a government transfer T_t , which corresponds to the resources added or subtracted by the planner's debt choices:

Problem 2 (The agent's Problem in Constrained Efficient Equilibrium)

$$\begin{aligned} \max_{\{c_t, k_{t+1}\}_{t \geq 0}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + q_t k_{t+1} = k_t(q_t + z_t) + T_t \end{aligned}$$

The first-order condition of this problem with respect to assets is standard:

$$q_t u'(c_t) = \beta \mathbb{E}_t [u'(c_{t+1}) (z_{t+1} + q_{t+1})] \quad (11)$$

This condition enters as an implementability constraint in the planner's problem. This is a key constraint, because as we explain in the analysis below, it links the planner's policy rules with the market price of assets. In particular, it drives the mechanism by which these rules influence the relationship between expectations about future consumption and asset prices and today's asset prices. As we explain below, this mechanism causes a planner assumed to be committed to future policies to display a time-inconsistency problem, which motivates our interest in formulating optimal macro-prudential policy as a time-consistent problem of a planner that lacks the ability to commit.⁹

⁸Because our focus is on macro-prudential policy, which by definition aims to prevent crises by altering behavior in pre-crisis times, this notion of constrained efficiency leaves out policies that may relax directly the credit constraint and make crisis less severe ex-post. These policies are examined by [Benigno, Chen, Otrok, Rebucci, and Young \(2013\)](#) in the context of a model in which the collateral constraint depends on goods prices.

⁹The time inconsistency problem does not arise in [Lorenzoni \(2008\)](#)'s classic model of fire sales because the asset price is determined by a static condition linking relative productivity of households and entrepreneurs, rather than expectations about future marginal utility as in our setup. Similarly, in [Bianchi \(2011\)](#) borrowing capacity is determined by a static price of non-tradable goods. [Bianchi and Mendoza \(2010\)](#) and [Jeanne and Korinek \(2010\)](#) impose time-consistency by construction in models with asset prices by imposing pricing conditions as explained in the Introduction.

3.2.2 Social Planner's Optimization Problem

As in Klein et al. (2005), we focus on Markov stationary policy rules, which set the values of bond holdings, consumption and asset prices as functions of the payoff-relevant state variables (b, z) . Since the planner is unable to commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future planners' decisions, and a Markov perfect equilibrium is characterized by a fixed point in these policy rules. At this fixed point, the policy rules of future planners that the current planner takes as given to solve its optimization problem match the policy rules that the current planner finds optimal to choose. Hence, the planner does not have the incentive to deviate from other planner's policy rules, thereby making these rules time-consistent.

Let $\mathcal{B}(b, z)$ be the policy rule for bond holdings of future planners that the planner takes as given, and $\mathcal{C}(b, z)$ and $\mathcal{Q}(b, z)$ the associated recursive functions that return the private consumption allocations and the market price of assets under that policy rule. Given these functions, we can use the fact that the first-order condition of the households' problem is an implementability constraint in the planner's problem to illustrate how by choosing b' the planner affects the stochastic discount factor that determines current asset prices. In particular, the implementability constraint (11) can be rewritten by replacing private consumption using the budget constraint of private agents evaluated at equilibrium together with the planner's budget constraint ($T_t = b_t - \frac{b_{t+1}}{R}$). The resulting expression indicates that the equilibrium asset price must satisfy:

$$q_t = \frac{\beta \mathbb{E}_t \left[u' \left(b_{t+1} + z_{t+1} - \frac{\mathcal{B}(b_{t+1}, z_{t+1})}{R} \right) (z_{t+1} + q_{t+1}) \right]}{u' \left(b_t + z_t - \frac{b_{t+1}}{R} \right)} \quad (12)$$

The right-hand-side of this expression shows that the debt choice of the planner affects asset prices directly, by inducing agents to reallocate consumption between t and $t + 1$ which affects the stochastic discount factor, and indirectly by affecting the bond holdings chosen by future governments, which also affects c_{t+1} . These effects will be reflected in the optimality conditions that characterize the social planner's equilibrium. This equilibrium can be defined in recursive form as follows.

Problem 3 (Recursive Representation of the Planner's Problem) *Given an initial state (b, z) , the policy rule of future planners $\mathcal{B}(b, z)$, and the associated consumption allocations $\mathcal{C}(b, z)$ and asset prices $\mathcal{Q}(b, z)$ the planner's problem is characterized by the following*

Bellman equation:

$$\begin{aligned}
\mathcal{V}(b, z) &= \max_{c, b', q} u(c) + \beta \mathbb{E}_{z'|z} \mathcal{V}(b', z') & (13) \\
c + \frac{b'}{R} &= b + z \\
\frac{b'}{R} &\geq -\kappa q \\
u'(c)q &= \beta \mathbb{E}_{z'|z} \left[u' \left(b' + z' - \frac{\mathcal{B}(b', z')}{R} \right) (\mathcal{Q}(b', z') + z') \right]
\end{aligned}$$

In the above problem, the planner chooses $b'(b, z)$ optimally to maximize the household's utility subject to three constraints: First, the economy's resource constraint (with Lagrange multiplier λ), which states that the consumption plan must be consistent with what private agents choose optimally given their budget constraint, market clearing in the asset market, and the planner's transfer. Second, the collateral constraint (with Lagrange multiplier μ), which the planner faces just like private agents. Third, the implementability constraint (with Lagrange multiplier ξ), which requires that the asset price be consistent with the optimality condition that holds in the private asset market.

Assuming that the equilibrium policy functions and the value function are differentiable, we can apply the standard Envelope theorem results to the first-order conditions of the planner's problem in order to recover the corresponding optimality conditions for c_t , b_{t+1} and q_t in sequential form. These optimality conditions are:

$$c_t :: \quad \lambda_t = u'(c_t) - \xi_t u''(c_t) q_t \quad (14)$$

$$b_{t+1} :: \quad u'(c_t) - \xi_t u''(c_t) q_t = \beta R \mathbb{E}_t \{ u'(c_{t+1}) - \xi_{t+1} u''(c_{t+1}) q_{t+1} + \quad (15)$$

$$\xi_t [u'(\mathcal{C}(b_{t+1}, z_{t+1})) \mathcal{B}_b(q_{t+1} + z_{t+1}) + \mathcal{Q}_b(b_{t+1}, z_{t+1}) u'(\mathcal{C}(b_{t+1}, z_{t+1}))]\}$$

$$q_t :: \quad \xi_t = \frac{\kappa \mu_t}{u'(c_t)} \quad (16)$$

The key differences between the unregulated competitive equilibrium and the financial regulator's equilibrium can be described intuitively by comparing the above optimality conditions with those of the decentralized competitive equilibrium. Compare first condition (14) with the analogous condition in the decentralized equilibrium, equation (4). Condition (4) states that for private agents the shadow value of wealth is equal to the marginal utility

of consumption, but (14) shows that for the regulator it equals the marginal utility of consumption plus the effect by which an increase in consumption relaxes the implementability constraint.¹⁰ Moreover, condition (16) shows that the planner sees a social benefit from relaxing the implementability constraint if and only if the collateral constraint is currently binding, i.e., $sign(\mu_t) = sign(\xi_t)$. Hence, when the collateral constraint binds, having an additional unit of wealth has a social benefit derived from how an increase in consumption raises equilibrium asset prices, which in turn relaxes the collateral constraint. This is clearer if we use (16) to rewrite the additional shadow value of wealth for the planner as $-u''(c_t)q_t \frac{\kappa\mu_t}{u'(c_t)}$. If the collateral constraint does not bind, $\mu_t = \xi_t = 0$ and the shadow values of wealth of the regulator and private agents in the decentralized equilibrium coincide.

Compare next the planner's Generalized Euler equation for bonds (15) with the analogous Euler equation in the competitive equilibrium (5). These equations differ in two key respects: First, condition (15) reflects the fact that the differences identified above in the valuation of bond holdings of the regulator and the private agents "ex post," when the collateral constraint binds, also result in valuation differences "ex ante," when the constraint is not binding, which arise because both the regulator and the agents are forward looking. In particular, if $\mu_t = 0$, the marginal cost of increasing debt at date t for private agents in the competitive equilibrium is simply $\beta R \mathbb{E}_t u'(c_{t+1})$. In contrast, the second term in the right-hand-side of (15) shows that the regulator attaches a higher social marginal cost to borrowing, because it internalizes the effect by which the larger debt at t reduces tomorrow's borrowing ability if the credit constraint binds then.¹¹ In other words, because the planner values more consumption when the constraint binds ex-post compared to private agents, it borrows less ex-ante. Moreover, this mechanism captures the standard pecuniary externality of the related literature, because it reflects the response of the regulator who takes into account how equilibrium asset prices tomorrow respond to the debt choice of today if the constraint becomes binding tomorrow. Since asset prices are determined in private markets, the equilibrium response is captured by the changes in the pricing kernel reflected in $u''(c_{t+1})$.

The second difference between the two Euler equations for bonds is in that condition (15) includes additional dynamic effects from current borrowing choices that the planner faces due to its inability to commit to future decisions. Because of this, the planner aims to influence future outcomes optimally by changing the endogenous state variable of the next-period's social planner, as reflected in the derivatives of the future policy rule and pricing function with respect to b inside the square bracket in the right-hand-side of (15). These incentives

¹⁰Note that $-\xi_t u''(c_t)q_t > 0$ because $u''(c_t) < 0$ and $\xi_t > 0$, as condition (16) implies. Hence, $\lambda_t > u'(c_t)$.

¹¹We can use again (16) to make this more evident mathematically by rewriting the second term in the right-hand-side of (15) as $-u''(c_{t+1})q_{t+1} \frac{\kappa\mu_{t+1}}{u'(c_{t+1})}$, which is positive for $\mu_{t+1} > 0$.

are only relevant, however, if the borrowing constraint is binding at t , because otherwise they vanish when $\xi_t = 0$.

We can now define the constrained-efficient equilibrium formally:

Definition 4 *The recursive constrained-efficient equilibrium is defined by the policy rule $b(b, z)$ with associated consumption plan $c(b, z)$, pricing function $q(b, z)$ and value function $\mathcal{V}(b, z)$, and the conjectured functions characterizing the policy rule of future planners $\mathcal{B}(b, z)$ and its associated consumption allocations $\mathcal{C}(b, z)$ and asset prices $\mathcal{Q}(b, z)$, such that the following conditions hold:*

1. *Planner's optimization: $\mathcal{V}(b, z), b'(b, z), c(b, z)$ and $q(b, z)$ solve the Bellman equation defined in Problem (3) given $\mathcal{B}(b, z), \mathcal{C}(b, z), \mathcal{Q}(b, z)$.*
2. *Time consistency (Markov stationarity): The conjectured policy rule, consumption allocations, and pricing function that represent optimal choices of future planners match the corresponding recursive functions that represent optimal plans of the current government: $b'(b, z) = \mathcal{B}(b, z)$, $c(b, z) = \mathcal{C}(b, z)$, $q(b, z) = \mathcal{Q}(b, z)$.*

Note that the requirements that the consumption allocation must be an optimal choice for households according to (2) and that the asset price satisfies the households Euler equation for assets are redundant, because the former is implied by the planner's resource constraint and the latter is the planner's implementability constraint.

3.3 Decentralization

We show now that a state-contingent tax on debt can decentralize the constrained-efficient, time-consistent allocations.¹² With a tax τ_t on borrowing, the budget constraint of private agents in the regulated competitive equilibrium becomes:¹³

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R(1 + \tau_t)} = k_t(z_t + q_t) + b_t + T_t \quad (17)$$

where T_t represents lump-sum transfers by which the government rebates all its tax revenue. The agents' Euler equation for bonds becomes:

$$u'(c_t) = \beta R(1 + \tau_t) \mathbb{E}_t [u'(c_{t+1})] + \mu_t \quad (18)$$

¹²It is also possible to decentralize the planner's problem using regulatory measures targeted to financial intermediaries by using capital and reserve requirements (see Bianchi, 2011). In addition, we can show that reducing loan-to-value ratios when constrained-efficiency calls for a strictly positive tax on borrowing can also achieve the constrained-efficient allocations.

¹³The tax can also be expressed as a tax on the price of bonds (i.e. on the income generated by borrowing), so that the post-tax price would be $(1 - \tau^R)(1/R)$. The two treatments are equivalent if we set $\tau^R = \tau/(1 + \tau)$.

Analyzing the optimality conditions of the planner's problem together with those of the regulated and unregulated decentralized equilibria leads to the following proposition:

Proposition 1 (Decentralization) *Assuming that the tax revenue from the debt tax is rebated to private agents as a lump-sum transfer, the constrained-efficient equilibrium can be decentralized by setting the tax to the following state-contingent rate:*

$$\tau_t = \frac{\beta R \mathbb{E}_t \left\{ -\xi_{t+1} u''(c_{t+1}) q_{t+1} + \xi_t \{ u'(\mathcal{C}(b_{t+1}, z_{t+1})) \mathcal{B}_b(\mathcal{Q}(b_{t+1}, z_{t+1}) + z_{t+1}) + \right.}{\beta \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))} \quad (19)$$

$$\left. \frac{\mathcal{Q}_b(b_{t+1}, z_{t+1}) u'(\mathcal{C}(b_{t+1}, z_{t+1})) \}}{\beta \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))} + \mu_t + \xi_t u''(c_t) q_t$$

Proof: See Appendix

The macro-prudential element of the above tax rule can be isolated by examining the optimal tax that applies when the collateral constraint is not binding at t . In this case, the tax on debt reduces to:

$$\tau_t = \frac{-\beta R \mathbb{E}_t \kappa \mu_{t+1} u''(\mathcal{C}(b_{t+1}, z_{t+1})) q_{t+1}}{\beta \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))^2} \quad (20)$$

This tax is strictly positive, since $u' > 0$, $u'' < 0$ and as shown above $\xi \geq 0$. In particular, the tax is strictly positive whenever there is a positive probability that the collateral constraint (or equivalently the implementability constraint, given condition (16)) can become binding at $t + 1$.

If the collateral constraint is binding at t , the optimal debt tax prescribed by the result in (19) is not very intuitive. It follows from our analysis of the planner's optimality conditions, however, that the same incentives influencing the planner's optimal debt choice are reflected in the design of the tax instrument that the planner uses to induce private agents to choose the same bond holdings. Hence, the optimal debt tax includes the terms that represent the higher shadow value of wealth of the planner when the constraint binds at t , the prudential effect resulting from internalizing the pecuniary externality in assessing the cost of borrowing at t if the constraint may bind at $t + 1$, and the effects resulting from the aim to influence the behavior of future planners by altering the bond holdings they receive. Moreover, it turns out that in numerical simulations of this simple model, it is possible to set $\tau = 0$ without affecting equilibrium allocations and prices when $m u_t > 0$. As shown in the Appendix, the role of the tax when $\mu_t > 0$ and the probability of $\mu_{t+1} > 0$ is zero is only to implement the planner's shadow value from relaxing the collateral constraint, and since this shadow value in the decentralization is affected by τ even if the constraint is binding, the analytical

expression for the tax can be positive or negative. The allocations and prices are not affected by the value τ because private agents borrow the maximum amount, which is independent of the tax.

3.4 Comparison with Commitment Case

We close this Section with some brief remarks highlighting how the constrained-efficient, time-consistent social planner's problem we proposed here differs from the analogous problem when the regulator is assumed to be able to commit to future policies. This is useful because, as we mentioned earlier, our interest in studying time-consistent macro-prudential policy is motivated in part by the fact that under commitment the planner's optimal policies are time-inconsistent.¹⁴ Hence, the focus of these remarks is on showing how this time inconsistency problem emerges.

Under commitment, the planner chooses at time 0 its policy rules in a once-and-for-all fashion. The first-order conditions of the planner's problem in sequential form are ($\forall t > 0$):

$$\lambda_t = u'(c_t) - \xi_t q_t u''(c_t) + u''(c_t) \xi_{t-1} (q_t + z_t) \quad (21)$$

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t \quad (22)$$

$$\xi_t = \xi_{t-1} + \frac{\mu_t \kappa K}{u'(c_t)} \quad (23)$$

The time inconsistency problem is evident from the presence of the lagged multipliers in these optimality conditions. According to (21), the planner internalizes how a change in consumption at time t helps relax the borrowing constraint at time t and makes it tighter at $t - 1$. As (23) shows, this implies that the Lagrange multiplier on the implementability constraint ξ_t follows a positive, non-decreasing sequence, which increases every time the constraint binds. Intuitively, when the constraint binds at t , this planner likes to promise lower future consumption so as to prop up asset prices and borrowing capacity at t , but ex post when $t + 1$ arrives it would be sub-optimal to keep this promise. We can show that under commitment, a state contingent tax on borrowing is also sufficient to implement the constrained-efficient solution, except that again it would be a non-credible policy because of the planner's incentives to deviate from announced policy rules ex post.

¹⁴As noted earlier, in [Bianchi and Mendoza \(2010\)](#) we followed an ad-hoc approach to construct a time-consistent macro-prudential policy, by proposing a conditionally-efficient planner restricted to value collateral using the same pricing function of the unregulated competitive equilibrium. Decentralizing this planner's allocations requires, in addition to the debt tax, a state-contingent tax on dividends.

4 Model for Quantitative Analysis

The remainder of the paper focuses on studying the model’s quantitative predictions. Before proceeding, however, we introduce three modifications that are important for enabling the model to produce financial crises episodes in line with the features of actual financial crises, so that the model can be viewed as a sound benchmark from which to conduct quantitative policy assessments. First, we introduce production and factor demands using a working capital channel which creates a link between financial amplification and the supply-side of the economy. Second, we modify the collateral constraint so that borrowing capacity is limited by individual asset holdings, instead of the aggregate supply of assets. Third, we introduce shocks to the interest rate and to the collateral constraint to incorporate additional exogenous driving forces of business cycles and financial crises. In the preceding analytical sections we abstracted from these features to keep the model tractable, and while these changes introduce effects that obviously interact with the pecuniary externality, the main features of macro-prudential regulation highlighted in the normative analysis are still present.

4.1 Firm-Households Optimization Problem

We follow Mendoza (2010) to add production into the model by replacing the representative agent of the simple model with a representative firm-household, which we also refer to as an agent. This agent makes both production plans and consumption-savings choices.

The agent’s preferences are given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \right] \quad (24)$$

where n_t is the agent’s labor supply. The argument of $u(\cdot)$ is the composite commodity $c_t - G(n_t)$ defined by Greenwood, Hercowitz, and Huffman (1988). $G(n)$ is a convex, strictly increasing and continuously differentiable function that measures the disutility of working. This formulation of preferences removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only. This is a common assumption in the literature but it is not innocuous, because the wealth effect would induce a counterfactual increase in labor supply and the equilibrium allocation of labor during financial crisis (when consumption is very low).

The representative firm-household combines physical assets, imported intermediate goods (m_t), and domestic labor services (h_t) to produce final goods using a production technology such that $y = z_t F(k_t, h_t, m_t)$, where F is a twice-continuously differentiable, concave produc-

tion function. Imported inputs are purchased in competitive world markets at a constant exogenous price p_m in terms of the domestically produced goods (i.e. p_m can be interpreted as the terms of trade). Hence, z_t is now a standard productivity shock instead of an exogenous dividends process. This shock has compact support and follows a finite-state, stationary Markov process.

The profits of the agent are given by $z_t F(k_t, h_t, m_t) - w_t h_t - p_m m_t$, and the agent's budget constraint can be written as:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + w_t n_t + [z_t F(k_t, h_t, m_t) - w_t h_t - p_m m_t] \quad (25)$$

where w_t is the wage rate and R_t is the real interest rate, which we continue to treat as exogenous. Like the productivity shocks, interest rate shocks also follow a finite-state, stationary Markov process with compact support. The two shocks can be modeled as correlated or independent processes.

As noted earlier, the assumption that the interest rate is exogenous is equivalent to assuming that the economy is a price-taker in world credit markets, as in other studies of the U.S. financial crisis like those of [Boz and Mendoza \(2010\)](#), [Corbae and Quintin \(2009\)](#) and [Howitt \(2001\)](#), or alternatively it implies that the model can be interpreted as a partial-equilibrium model of the household sector. This assumption is adopted for simplicity, but is also in line with evidence indicating that the observed decline in the U.S. risk-free rate in the era of financial globalization has been driven largely by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. [Warnock and Warnock \(2009\)](#) provide econometric evidence of the significant downward pressure exerted by foreign capital inflows on U.S. T-bill rates since the mid 1980s. [Mendoza and Quadrini \(2009\)](#) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents. From this perspective, assuming a fluctuating R_t around a constant mean is actually conservative, as in reality the pre-crisis boom years were characterized by a *falling* real interest rate, which would strengthen our results. Still, we study later in the sensitivity analysis how our quantitative results vary if we relax this assumption and consider instead an exogenous inverse supply-of-funds curve, which allows the real interest rate to increase as debt rises.

The agent also faces a working capital constraint which requires it to pay for a fraction of the cost of input purchases in advance of production using foreign financing. In particular, a foreign working capital loan is used to pay for a fraction θ of the cost of imported inputs $p_m m_t$ at the beginning of the period and repaid at the end of the period. In the conventional

working capital setup, a cash-in-advance-like motive for holding funds to pay for inputs implies that the effective marginal cost of inputs carries a financing cost determined by R_t . In contrast, here we simply assume that working capital funds are within-period loans so that the interest rate on working capital is effectively zero. We follow this approach so as to show that the effects of working capital in our analysis hinge only on the need to provide collateral for working capital, as explained below, and not on the effect of interest rate fluctuations on effective factor costs, which is the standard mechanism in business cycle models with working capital (e.g. [Uribe and Yue, 2006](#)). Moreover, we consider only imported inputs in the working capital constraint because this constraint relates to external financing, which is natural to connect to inputs acquired abroad (i.e. we view working capital as akin to trade credit from foreign suppliers of intermediate goods). Labor can be viewed as using working capital loans from domestic creditors that do not require collateral without altering our setup.

The agent faces a collateral constraint that limits total debt, including both intertemporal debt and atemporal working capital loans, not to exceed a possibly stochastic fraction κ_t of the market value of beginning-of-period asset holdings (i.e. κ_t imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta p_t m_t \leq \kappa_t q_t k_t \quad (26)$$

We interpret shocks to κ_t as financial shocks that lead creditors to adjust collateral requirements on borrowers. Note, however, that neither the nature of the amplification mechanism nor the normative arguments stated in the previous Section rely on κ_t being stochastic, and that even with κ constant models in this class can produce crises dynamics with realistic features (see [Mendoza \(2010\)](#) and [Bianchi and Mendoza \(2010\)](#)). Fluctuations in κ_t are useful for improving the model's ability to match the co-movements linking financial flows and business cycles (see [Jermann and Quadrini \(2012\)](#)) and to generate sharp credit expansions in pre-crisis periods (see ([Boz and Mendoza, 2010](#))).

Collateral constraints similar to the one proposed above are often defined using k_{t+1} as collateral (e.g. [Kiyotaki and Moore \(1997\)](#) and [Aiyagari and Gertler \(1999\)](#)) instead of k_t . This is immaterial for the equilibrium amount of assets that can be pledged as collateral in this model, because assets remain in fixed unit aggregate supply. As we show below, the assumption does affect the timing of the shadow values of binding collateral constraints in the agent's Euler equation for assets, but what is critical is that the collateral constraint depends on asset prices. As long as this is the case, the intuition for the role that the collateral constraint plays and the incentives of the planner to manage the pecuniary externality are qualitatively the same. We used k_t as collateral for tractability and because it facilitates providing a contractual foundation for the existence of the constraint. In particular, we

show in the Appendix that the collateral constraint (26) can be obtained as an implication of incentive compatibility constraints on the part of borrowers in an environment in which limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the k_t owned by a defaulting debtor.

4.2 Unregulated Decentralized Equilibrium

In the unregulated decentralized competitive equilibrium (DE), agents maximize (24) subject to (25) and (26) taking asset and factor prices as given. Also, the markets of goods, assets and factors of production clear.

The maximization problem of the agent yields the following sequence of optimality conditions for each date t :

$$w_t = G'(n_t) \tag{27}$$

$$z_t F_h(k_t, h_t, m_t) = w_t \tag{28}$$

$$z_t F_m(k_t, h_t, m_t) = p_m [1 + \theta \mu_t / u'(t)] \tag{29}$$

$$u'(t) = \beta R_t \mathbb{E}_t [u'(t+1)] + \mu_t \tag{30}$$

$$q_t u'(t) = \beta \mathbb{E}_t [u'(t+1) (z_{t+1} F_k(k_{t+1}, h_{t+1}, m_{t+1}) + q_{t+1}) + \kappa_{t+1} \mu_{t+1} q_{t+1}] \tag{31}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint and $u'(t)$ denotes $u'(c_t - G(h_t))$.

Condition (27) is the agent's labor supply condition, which equates the marginal disutility of labor with the wage rate. Condition (28) is the standard labor demand condition equating the marginal productivity of labor with the wage rate. Condition (29) is a similar condition setting the demand for imported inputs by equating their marginal productivity with their marginal cost. Note, however, that there is key difference in the latter, because the marginal cost of imported inputs includes the extra financing cost $\theta \mu_t / u'(t)$ which is incurred in states of nature in which the collateral constraint binds.

The last two conditions are the Euler equations for bonds and assets respectively, and they yield similar implications for the effects of binding collateral constraints as in the simple model. When the collateral constraint binds, condition (30) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (31) equates the marginal cost of an extra unit of assets with its marginal gain. The fact that assets serve as collateral increases the benefits of holding the assets by $\beta \mathbb{E}_t \kappa_{t+1} \mu_{t+1} q_{t+1}$.

Proceeding again as we did with the simple model, we can combine the Euler equations for bonds and assets to derive this model's expression for the equity premium:

$$R_t^{ep} = \frac{\mu_t}{u'(t)\mathbb{E}_t m_{t+1}} - \frac{\text{cov}_t(m_{t+1}, R_{t+1}^q)}{\mathbb{E}_t[m_{t+1}]} - \frac{\mathbb{E}_t(\phi_{t+1}m_{t+1})}{\mathbb{E}_t m_{t+1}}$$

Notice that with individual asset holdings serving as collateral, we now have a *collateral effect* that contributes to reduce excess returns by the marginal gain in borrowing ability at $t + 1$ that investing in an extra unit of assets at t allows if the collateral constraint binds:

$$\phi_{t+1} \equiv \kappa_{t+1} \frac{\mu_{t+1}}{u'(c_t)} \frac{q_{t+1}}{q_t}.^{15}$$

The collateral effect can in turn be decomposed into a first-order effect and a risk (or second-order) term:

$$\frac{\mathbb{E}_t(\phi_{t+1}m_{t+1})}{\mathbb{E}_t m_{t+1}} = \frac{\text{cov}_t(\phi_{t+1}, m_{t+1})}{\mathbb{E}_t m_{t+1}} - \mathbb{E}_t \phi_{t+1}$$

Notice that $\text{cov}_t(m_{t+1}, R_{t+1}^q)$ and $\text{cov}_t(\phi_{t+1}, m_{t+1})$ have opposite signs as collateral is most valued when the household is more constrained, which coincides with high marginal utility.

Given the definitions of the Sharpe ratio ($S_t \equiv R_t^{ep}/\sigma_t(R_{t+1}^q)$) and the price of risk ($s_t \equiv \sigma_t(m_{t+1})/\mathbb{E}_t m_{t+1}$), we can rewrite the expected excess return and the Sharpe ratio as:

$$R_t^{ep} = S_t \sigma_t(R_{t+1}^q), \quad S_t = \frac{\mu_t - \mathbb{E}_t(\phi_{t+1}m_{t+1})}{u'(t)\mathbb{E}_t m_{t+1} \sigma_t(R_{t+1}^q)} - \rho_t(R_{t+1}^q, m_{t+1}) s_t \quad (32)$$

where $\sigma_t(R_{t+1}^q)$ is the date- t conditional standard deviation of asset returns and $\rho_t(R_{t+1}^q, m_{t+1})$ is the conditional correlation between R_{t+1}^q and m_{t+1} . Thus, the collateral constraint has direct and indirect effects on the Sharpe ratio analogous to those it has on the equity premium. The indirect effect reduces to the usual expression in terms of the product of the price of risk and the correlation between asset returns and the stochastic discount factor. The direct effect is normalized by the variance of returns. These relationships will be used later to quantify the effects of the credit friction and the macro-prudential policy on asset pricing conditions.

The solution method that we implement in the next Section works using the recursive representation of the competitive equilibrium. To simplify notation, we denote by \mathbf{s} the vector of current aggregate shocks, i.e., $\mathbf{s}_t = \{z_t, \kappa_t, R_t\}$. The individual states are still b, k and the aggregate states are B, \mathbf{s} . Denoting by $\Gamma(B, \mathbf{s})$ the agent's perceived law of motion of aggregate bonds and $q(B, \mathbf{s})$ and $w(B, \mathbf{s})$ the pricing functions for assets and labor

¹⁵A similar effect is present when k_{t+1} serves as collateral instead of k_t , but its timing also changes. In this case, the marginal benefit of holding more assets as collateral shows up as the term $-\mu_t \kappa_t$ in the equity premium expression (see [Mendoza and Smith, 2006](#) and [Bianchi and Mendoza, 2010](#))

respectively, the representative firm-household's recursive optimization problem is:

$$\begin{aligned}
V(b, k, B, \mathbf{s}) &= \max_{b', k', c, n, h} u(c - G(n)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}} V(b', k', B', \mathbf{s}') & (33) \\
\text{s.t.} \quad q(B, \mathbf{s})k' + c + \frac{b'}{R} &= q(B, \mathbf{s})k + b + w(B, \mathbf{s})n + [zF(k, h, m) - w(B, \mathbf{s})h - p_m m] \\
-\frac{b'}{R} + \theta p_m m &\leq \kappa q(B, \mathbf{s})k
\end{aligned}$$

given the law of motion $B' = \Gamma(B, \mathbf{s})$. The solution to this problem is characterized by the decision rules $\hat{b}(b, k, B, \mathbf{s})$, $\hat{k}(b, k, B, \mathbf{s})$, $\hat{c}(b, k, B, \mathbf{s})$, $\hat{n}(b, k, B, \mathbf{s})$, $\hat{m}(b, k, B, \mathbf{s})$ and $\hat{h}(b, k, B, \mathbf{s})$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}(B, 1, B, \mathbf{s})$.

Definition 5 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium is defined by an asset pricing function $q(B, \mathbf{s})$, a pricing function for labor $w(B, \mathbf{s})$, a perceived law of motion for aggregate bond holdings $\Gamma(B, \mathbf{s})$, and a set of decision rules $\{\hat{b}(b, k, B, \mathbf{s}), \hat{k}(b, k, B, \mathbf{s}), \hat{c}(b, k, B, \mathbf{s}), \hat{n}(b, k, B, \mathbf{s}), \hat{h}(b, k, B, \mathbf{s})\}$ with associated value function $V(b, k, B, \mathbf{s})$ such that:*

1. $\{\hat{b}(b, k, B, \mathbf{s}), \hat{k}(b, k, B, \mathbf{s}), \hat{c}(b, k, B, \mathbf{s}), \hat{n}(b, k, B, \mathbf{s}), \hat{h}(b, k, B, \mathbf{s}), \hat{m}(b, k, B, \mathbf{s}), \hat{\mu}(b, k, B, \mathbf{s})\}$ and $V(b, k, B, \mathbf{s})$ solve the agents' recursive optimization problem, taking as given $Q(B, \mathbf{s}), w(B, \mathbf{s})$ and $\Gamma(B, \mathbf{s})$.
2. The perceived law of motion for aggregate bonds is consistent with the actual law of motion: $\Gamma(B, \mathbf{s}) = \hat{b}(B, 1, B, \mathbf{s})$.
3. Wages satisfy $w(B, \mathbf{s}) = G'(\hat{n}(B, 1, B, \mathbf{s}))$ and asset prices satisfy:

$$\begin{aligned}
Q(B, \mathbf{s}') (u'(\hat{c}) - G'(\hat{h})) &= \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left\{ u'(\hat{c}(b, k, B, \mathbf{s}')) (Q(\hat{b}, \mathbf{s}') + z' F_k(1, \hat{h}(B', 1, B, \mathbf{s}'), \hat{m}(B', 1, B, \mathbf{s}')) \right. \\
&\quad \left. \kappa' Q(\Gamma(B, \mathbf{s}), \mathbf{s}') \hat{\mu}(\Gamma(B, \mathbf{s}), \mathbf{s}') \right\}
\end{aligned}$$

4. Goods, labor and asset markets clear: $\frac{\hat{b}(B, 1, B, \mathbf{s})}{R} + \hat{c}(B, 1, B, \mathbf{s}) = zF(1, \hat{n}(B, 1, B, \mathbf{s})) + B - p_m \hat{m}(b, 1, B, \mathbf{s})$, $\hat{n}(B, 1, B, \mathbf{s}) = \hat{h}(B, 1, B, \mathbf{s})$ and $\hat{k}(B, 1, B, \mathbf{s}) = 1$

4.3 Planner's Problem and Macroprudential Policy

As in the simple model, we continue to assume that a constrained-efficient, time-consistent planner (SP) chooses directly the amount of debt, while consumption, asset holdings, and now production, labor, and imported inputs are chosen competitively by private agents. As

shown in the Appendix, taking as given a policy rule for bond holdings of future governments \mathcal{B} and the associated recursive functions governing labor \mathcal{H} , consumption \mathcal{C} , imported inputs \mathcal{M} , and land prices Q , the current planner's optimization problem can be written as the following Bellman equation:

$$\begin{aligned}
\mathcal{V}(b, \mathbf{s}) &= \max_{c, b', \mu, h, m} u(c - G(h)) + \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} \mathcal{V}(b', \mathbf{s}') & (34) \\
c + \frac{b'}{R} &\leq b + z_t F(1, h, m) - pm \\
z_t F_h(1, h, m) &= G'(h) \\
z_t F_m(1, h, m) &= p_m \left(1 + \frac{\theta \mu}{u'(c - G(h))} \right) \\
\frac{b'}{R} - p_m m &\geq -\kappa q \\
\mu \left(\frac{b'}{R} - p_m m + \kappa q \right) &= 0 \\
q u'(c - G(h)) &= \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} \left\{ u'(\mathcal{C}(b', \mathbf{s}') - G'(\mathcal{H}(b, \mathbf{s}))) (\mathcal{Q}(b', \mathbf{s}') + z' F_k(1, \mathcal{H}(b', \mathbf{s}'), \mathcal{M}(b', \mathbf{s}')) \right. \\
&\quad \left. + \kappa' \mu(b', \mathbf{s}') \mathcal{Q}(b', \mathbf{s}') \right\} & (35)
\end{aligned}$$

Finally, following Proposition 1, the constrained efficient allocations can again be decentralized as a tax on borrowing. In particular, we show in the Appendix that when the collateral constraint is not currently binding the optimal tax is:

$$\tau_t = \frac{1}{\beta \mathbb{E}_t u'(t+1)} \beta RE_t \left\{ \frac{\zeta_{t+1}^m p_m \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) q_{t+1} \right\}$$

where ζ_t^m is the shadow value of relaxing the implementability constraint associated with the private agents' limited access to working capital financing for purchasing imported inputs. As with the simple model, the tax is zero if there is zero probability of a binding collateral constraint tomorrow. On the other hand, when there is a positive probability of hitting the constraint at $t+1$ the optimal tax can no longer be unambiguously signed because there is an additional term that captures the incentive compatibility constraint associated with the choice of imported inputs with ambiguous sign (the first term in the right-hand-side of (4.3)). Quantitatively, however, because the implementability constraint associated with intermediate inputs is not strongly binding and the fact that θ is a small fraction the second term is large, we obtained positive tax rates in all our numerical experiments.

5 Quantitative Analysis

This section describes the baseline calibration of the model's parameters and exogenous stochastic process and discusses the main quantitative findings.

5.1 Calibration

We calibrate the model to annual frequency using data from advanced economies. For some variables (i.e. value of housing wealth, utilization-adjusted TFP and Frisch elasticity of labor supply), data limitations forced us to use U.S. data only, but we examine the implications of parameter variations in the sensitivity analysis.

The functional forms for preferences and technology are the following:

$$u(c - G(n)) = \frac{\left(c - \chi \frac{n^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1 - \sigma} \quad \omega > 0, \sigma > 1$$

$$F(k, h, m) = zk^{\alpha_K} m^{\alpha_m} h^{\alpha_h}, \quad \alpha_K, \alpha_m, \alpha_h \geq 0 \quad \alpha_K + \alpha_m + \alpha_h < 1$$

We set $\sigma = 1.5$, which is in the range of commonly used values in open-economy DSGE models. The Frisch elasticity of labor supply ($1/\omega$) is set equal to 1, in line with evidence for the United States by [Kimball and Shapiro \(2008\)](#). The parameter χ is inessential and is set so that mean hours are equal to 1, which requires $\chi = 0.64$.

The production function of gross output is Cobb-Douglas. To calibrate the share of imported inputs, we use data reported by [Goldberg and Campa \(2010\)](#) on the average ratio of imported to domestic intermediate goods for 16 advanced economies. The average ratio across them is 25 percent. At an average ratio of total intermediate goods to gross output of 45 percent, the implied share of imported inputs in gross output is $\alpha_m = 0.124$. The factor share of labor is then set so that in terms of value added we obtain the standard share of 0.64, which is similar across industrial countries (see [Stockman and Tesar \(1995\)](#)). This implies $\alpha_h = 0.64^*(1 - \alpha_m) = 0.56$. Since capital in the model is in fixed supply, we do not set the capital share to the standard 1/3rd of GDP, because this factor share measures capital income accrued to the entire capital stock. Instead, we set α_K so that the model matches an estimate of the ratio of capital in fixed supply to GDP based on the value of the housing stock. Consistent data across countries on this component of household wealth are not available, so we measured the ratio for the United States using data from the *Flow of Funds* database of the Federal Reserve. In particular we used the ratio as of 2007, which was about 1.3, because it is in the last year before the start of the 2008 financial crisis. The

model matches this ratio, given the other parameter values, if we set $\alpha_K = 0.05$.¹⁶ Notice this implies that production effectively has decreasing returns to scale, but this is not critical for the results because of the unit supply of capital and because profits return to private agents as income.

We follow [Schmitt-Grohe and Uribe \(2007\)](#) in taking M1 money balances in possession of firms as a proxy for working capital. Based on the observations that in the United States about two-thirds of M1 are held by firms ([Mulligan, 1997](#)) and that M1 was 10 percent of GDP in 2007, we calibrate the working capital-GDP ratio to match $(2/3)*0.1 = 0.066$. Given $\alpha_m = 0.124$ and that the ratio of GDP to gross output is $1 - \alpha_m$, and assuming also that the collateral constraint does not bind, the value of theta is solved for as $\theta = 0.066[(1 - \alpha_m)/\alpha_m]$, which is about 0.5.

The value of β is set to 0.96, which is also a standard value, but in addition it supports an average household debt-income ratio in a range that is in line with U.S. data from the *Flow of Funds* database. Before the mid-1990s this ratio was stable at about 30 percent. Since then and until just before the 2008 crisis, it rose steadily to a peak of almost 70 percent. By comparison, the average debt-income ratio in the stochastic steady-state of the model with the baseline calibration is 38 percent. A mean debt ratio of 38 percent is sensible because 70 percent was an extreme at the peak of a credit boom and 30 percent is an average from a period before the substantial financial innovation of recent years.

We calibrate the shocks to the interest rate and productivity to fit a discrete approximation to estimates of the following VAR process:

$$\begin{pmatrix} z_t \\ R_t \end{pmatrix} + \rho \begin{pmatrix} z_{t-1} \\ R_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t} \\ \epsilon_{R,t} \end{pmatrix},$$

where $(\epsilon_{z,t}, \epsilon_{R,t})$ follow a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix V . The interest rate is measured as the ex-post real interest rate on 3 month U.S. Treasury Bills, which is the standard measure of the exogenous world real interest rate in international Macro models. TFP is measured using the TFP estimates adjusted for changes in utilization and input relative prices constructed by [Fernald \(2012\)](#). The VAR estimation yields the following autocorrelation and variance-covariance matrices:

$$\rho = \begin{bmatrix} 0.755972 & -0.030037 \\ -0.074327 & 0.743032 \end{bmatrix}, \quad \mathbf{Cov} = \begin{bmatrix} 0.0000580 & -0.0000107 \\ -0.0000107 & 0.0001439 \end{bmatrix}.$$

¹⁶Estimates of the value of capital in fixed supply vary depending on whether they include land used for residential or commercial purposes, or owned by government at different levels. We used an estimate based on residential property because it is a closer match to the structure of the model (for example, the discount factor was set to match the household debt ratio).

Table 1: Calibration

<i>Description</i>	<i>Value</i>	<i>Source / target</i>
Risk aversion	$\sigma = 1.5$	Standard value
Discount factor	$\beta = 0.96$	Standard value
Share of imported inputs	$\alpha_m = 0.124$	cross-country data from Goldberg and Campa (2010)
Share of labor	$\alpha_n = 0.56$	0.64 cross-country data from Stockman and Tesar (1995)
Share of assets	$\alpha_K = 0.05$	U.S. housing stock/GDP $q/GDP = 1.3$
Labor disutility coefficient	$\chi = 0.64$	Normalization
Frisch elasticity parameter	$\omega = 1$	Kimball and Shapiro (2008)
Working capital coefficient	$\theta = 0.5$	2007 U.S. firms' $M1/GDP = 0.1$
Financial shock	$\kappa^L = 0.23$	Average leverage = 30%
	$P_{HH} = 0.9$	Probability of a crisis = 3 percent
	$P_{LL} = 0.15$	Duration of credit crunch = 3 years
TFP and Interest Rate Shocks	see text	

The discrete approximation of the VAR process is constructed using the Tauchen-Hussey quadrature method to produce a Markov chain with 3 realizations in each shock. Their algorithm uses as inputs $\boldsymbol{\rho}$ and \mathbf{Cov} , and yields the Markov realization and transition probability matrices, which are available from the authors on request.

The regime-switching process of κ is modeled as independent from the bivariate process of (z_t, R_t) . In particular, κ_t follows a two-state Markov chain with values given by $\{\kappa^L, \kappa^H\}$ and transition matrix

$$P = \begin{bmatrix} P_{L,L} & 1 - P_{L,L} \\ 1 - P_{H,H} & P_{H,H} \end{bmatrix},$$

This is a process analogous to a standard regime-switching Markov process. There are hard- and easy-credit regimes, each defined by the low and high values of κ , with mean durations of $1/(1 - P_{L,L})$ and $1/(1 - P_{H,H})$ respectively.

The values of κ^H and κ^L , as well as the transition matrix, are difficult to calibrate directly from the data because of the model's high level of aggregation and the wide dispersion in loan-to-value restrictions and ability to leverage across households and firms of various characteristics in the data. Our calibration follows [Bianchi \(2012\)](#). We set κ^H high enough so that the collateral constraint does not bind with strictly positive probability when the financial shock takes this value. The value of κ^L is set to 0.30, which delivers a value of mean leverage of 30 percent consistent with measures of household and corporate leverage,

which were respectively 0.2 and 0.45 at the onset of the 2007 financial crisis.¹⁷ The transition matrix for the financial shock is set to target the frequency and the duration of financial crises, defined in a similar way as in the empirical literature (see e.g. (Reinhart and Rogoff (2009))).¹⁸ In particular, we target an incidence of 3 crises every 100 years and an average duration of 3 years. $P_{L,L}$, which mostly affects the duration of crises, equals 0.2 and $P_{H,H}$, which primarily affects the long-run probability of a crisis, equals 0.9.

The model is solved using a global, nonlinear solution method that takes into account the occasionally binding, stochastic credit constraint. In SP problem, the algorithm also takes into account the need to solve for the Markov stationary policy rules that support time-consistency. Further details are provided in the Appendix.

5.2 Frequency and Severity of Financial Crises

The first important result of this quantitative analysis is that the time-consistent macroprudential policy reduces significantly the frequency and severity of financial crises. With regard to the former, recall that we calibrated κ so that the DE has financial crises with the 3-percent frequency observed in the data. Under the same baseline calibration, the SP economy experiences financial crises with a long-run frequency of only 0.2 percent. Thus, in the absence of regulation, the pecuniary externality results in financial crises that are 15 times more likely.¹⁹

The reduced severity of financial crises in the SP economy is illustrated by conducting an event analysis of crisis episodes using simulated data for the DE and SP economies. The results are presented in Figure 1, which shows nine-year event windows for total credit (bonds plus working capital), asset prices, output, and consumption, all expressed as deviations from long-run averages in the DE equilibrium, as well as two windows that show the evolution of

¹⁷These leverage ratios were computed using asset and liabilities data from the *Flow of Funds* (total assets and credit market debt outstanding of households and nonprofit organizations, and total assets and debt outstanding of the domestic nonfinancial business sector). The resulting leverage ratios are lower than maximum loan-to-value ratios in home mortgages, which peaked above 95 percent in the sub-prime market at the peak of the housing boom, but the lower ratio in the *Flow of Funds* data suggests that this is not a representative figure for the broader housing sector.

¹⁸Following Forbes and Warnock’s (2012) methodology to identify the timing and duration of sharp changes in international capital flows, we defined a financial crisis as an event in which credit falls by at least two standard deviations, with a starting date set by the year within the previous two years in which credit first fell below one standard deviation. In addition, we also require that the collateral constraint be binding.

¹⁹We identify financial crises for the SP using the credit thresholds of the DE in levels. Re-computing the thresholds using the standard deviation of credit in the SP equilibrium, which is smaller, the frequency of crises for the regulator rises to 0.6 percent, which is still much lower than in DE. The big gap between the two is also robust to using alternative crisis identification filters.

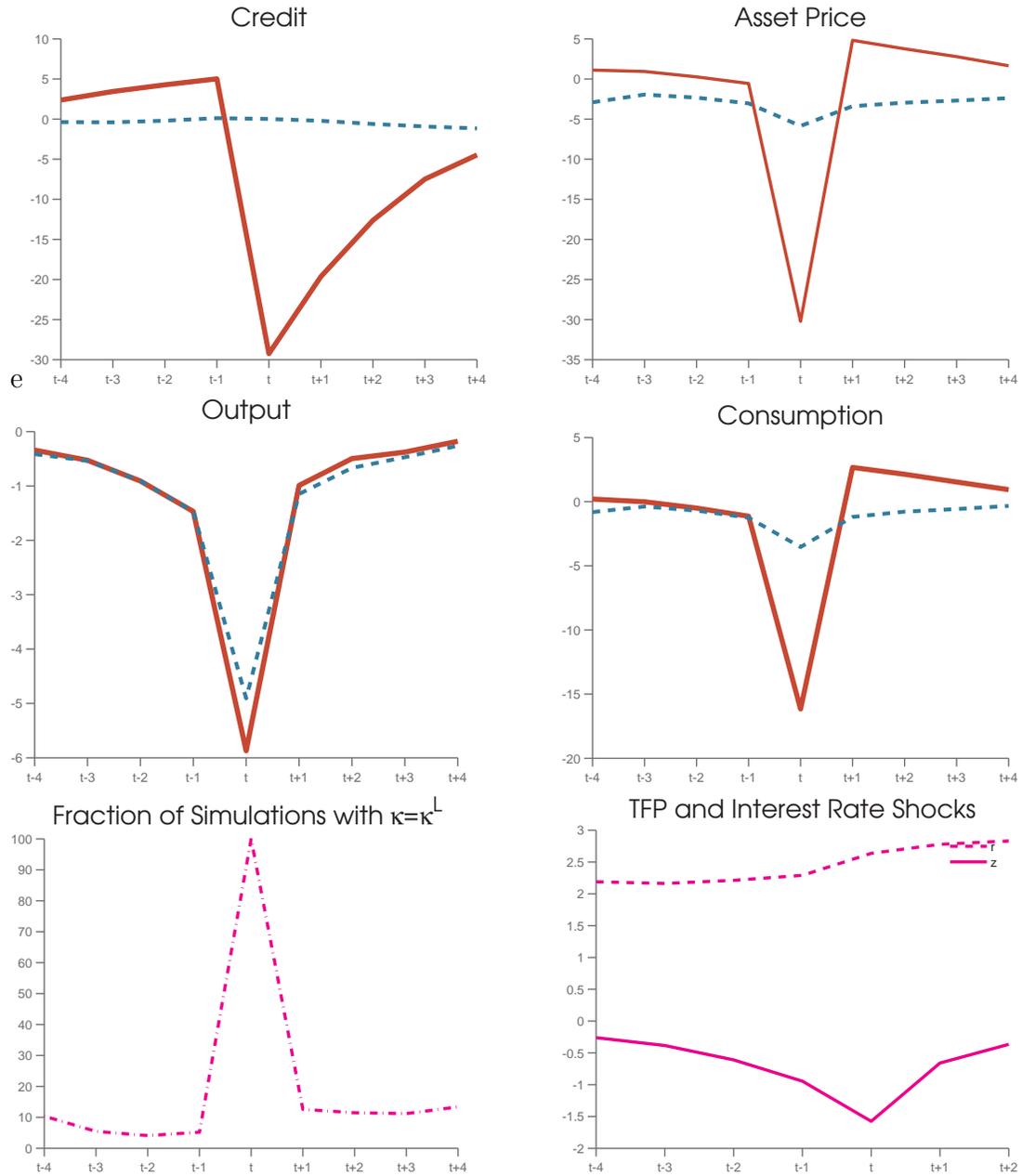


Figure 1: Event Analysis: percentage differences relative to unconditional averages of the decentralized equilibrium

the model's exogenous shocks.²⁰

We construct comparable event windows for the two economies following this procedure: First we simulate the DE for 100,000 periods and identify financial crisis events using the event-study methodology we borrowed from the empirical literature as described in footnote 17. Second, we construct nine-year event windows centered in the crisis year by computing the averages of all endogenous variables across the cross section of crisis events at each date. This produces the DE dynamics plotted as the red, continuous lines in Figure 1. For TFP and interest rate shocks we also show the cross sectional average in each year, but for the financial shock we show the fraction of the time the κ^L regime was observed in the cross section of each year. We do this because, as mentioned earlier, the value of κ^H was set high enough so that it does not bind with positive probability, and hence its actual value does not matter for the equilibrium dynamics. Third, we take the initial bond position at $t - 4$ of the DE and the sequences of shocks it went through in the nine years of each crisis event, and then pass them through the decisions rules of the SP. Finally, we average in each date the cross-sectional sample of the SP allocations and asset prices to generate the averages shown as the blue, dashed line in Figure 1.

Figure 1 shows that the pecuniary externality results in higher borrowing in the DE in the periods preceding the crisis. While credit in the SP remains very close to the DE average throughout the nine periods of the window, credit in the DE is above its long run average by 2.5 to 5 percentage points from $t - 4$ to $t - 1$. The cumulative difference relative to the SP is close to 15 percent. As a result of this overborrowing, the DE builds up higher levels of leverage and experiences a larger collapse in credit when the financial crisis hits. Credit falls about 30 percentage points more, and, although it rises at a fast pace after the crisis, four years later it remains 5 percentage points below both its long-run average and the SP level.²¹

Asset prices, output and consumption also fall much more sharply in the DE than the SP. The declines in consumption and asset prices are particularly larger (-16 percent v. -4 percent for consumption and -30 percent v. -6 percent for asset prices). The asset price collapse plays an important role in explaining the more pronounced decline in credit in the DE, because it reflects the impact of the Fisherian deflation mechanism. Output falls 1 percentage point more in the DE than in the SP, because of the higher shadow price of imported inputs due to the effect of the tighter binding credit constraint on access to working capital.

²⁰We also produced event windows expressing the data as deviations from the long-run averages of each economy, instead of both in percent of DE means, but this did not result in significant differences.

²¹The model produces relatively large drops in credit (e.g. in the U.S. crisis, the through in credit in the third quarter of 2010 reached about -7 percent of GDP below the mean). This is partly due to the fact that intertemporal credit in the model is in the form of one-period bonds, whereas loans in the data have on average a much larger maturity. In addition, the DE does not take into account the strong policy intervention that took place since the end of 2008 aiming to prevent what would have been a larger credit crunch.

In the DE simulation, the economy spends about 90 percent of the time in the easy-credit financial regime, except in the period in which the crisis hits, it always coincides with a switch to the κ^L regime. TFP is declining on average before the financial crisis, and reaches a trough of about -1.5 percent below the mean when the crisis hits, and after that it recovers at a fast pace. The real interest rate is stable at about 2 percent in the years before the crisis, and when the crisis hits it rises about 50 basis points, and remains stable at 2.5 percent in the years that follow. Thus, in the DE financial crises are associated with adverse TFP, interest rate and financial shocks.

Overall, it is important to note that the DE simulation does a reasonable job at matching key features of actual financial crises, as in [Mendoza \(2010\)](#). This is an important first step in making the case for considering the normative implications of the model as relevant. While we did not build a rich equilibrium business cycle model in order to keep the analysis of the externality tractable, and hence our match to the data is imperfect, the model does produce financial crises with realistic features in terms of abrupt, large declines in credit, asset prices, consumption and output and it supports non-crisis output fluctuations roughly in line with those observed in the business cycles of advanced economies.

5.3 Borrowing Decisions and Amplification

The financial amplification mechanism and the pecuniary externality at work in the DE simulation, and the credit effects of the SP's macro-prudential policy, can be illustrated further by studying the differences in borrowing decisions in those two economies. These borrowing decisions are reflected in the policy functions for bonds of each economy ($\mathcal{B}^{DE}(B, \mathbf{s})$ and $\mathcal{B}^{SP}(B, \mathbf{s})$) shown in [Figure 2](#) as the red, continuous and blue, dashed curves respectively. These policy functions indicate the each economy's choice of bonds for $t + 1$ (B') in the vertical axis for the bond holdings at t (B) measured in the horizontal axis, and for the values of exogenous shocks in the triple \mathbf{s} set to κ^L , average R and a negative productivity shock with TFP about 1.5 standard deviations below its mean. The Figure also shows the limit on intertemporal borrowing that each economy faces ($\bar{\mathcal{B}}^{DE}(B, \mathbf{s})$ and $\bar{\mathcal{B}}^{SP}(B, \mathbf{s})$), computed using the asset pricing functions of each economy valued at the same \mathbf{s} .²²

An important result illustrated in this Figure is that the Fisherian deflation mechanism generates V-shaped bond policy functions, instead of the typical monotonically-increasing ones of both standard incomplete markets models and RBC models of the small open economy. The point at which the policy functions switch slope in corresponds to the value of B at which the collateral constraint holds with equality but does not bind in each economy. To the

²²The borrowing limits are defined as $-\kappa q^{DE}(B, \mathbf{s}) - p_m \hat{m}^{DE}(B, 1, B, \mathbf{s})$ and $-\kappa q^{SP}(B, \mathbf{s}) - p_m \hat{m}^{SP}(B, \mathbf{s})$

right of this point, the collateral constraint does not bind and the policy functions are upward sloping. To the left of this point, the policy functions are *decreasing* in B , because a reduction in B results in a sharp reduction asset prices, which tightens the borrowing constraint, thus increasing B' .²³ In line with this result, the policy functions lie above their corresponding borrowing limits to the right of the values of B at which the constraint becomes binding in the DE and SP, because at those higher values of B the credit constraint does not bind. Similarly, since to the left of those critical values the constraint binds, the decision rules are equal to the corresponding borrowing limits.

The bonds policy functions can be divided into three regions: a “constrained credit region,” a “positive crisis probability region” and a “stable credit region.” The “constrained credit region” is defined by the values of B with sufficiently high initial debt (low B) such that the collateral constraint binds for the regulator. This is the range with $B \leq -0.33$. In this region, the collateral constraint must bind also for private agents in the competitive equilibrium, because the externality affecting agents in this economy implies that the constraint starts binding at higher values of B in the DE than in the SP, as we show below.

The total debt (working capital plus debt in bonds) in the constrained credit region is very similar in the DE and SP outcomes. It is not identical because asset prices differ, as the SP is optimally using its policy function to influence asset prices and thus manage borrowing capacity, but within the limits of what its inability to commit allows. To see why asset prices differ, suppose that total debt was the same and hence asset prices were the same with the constraint binding. However, because future marginal utilities differ under the DE and SP, this would imply different asset prices. In addition, the bond choices are also similar (as shown in Figure (Figure 2)), because the choices of intermediate inputs are also similar in the two economies.²⁴

The positive crisis probability region is located to the right of the constrained region, and it includes the interval $-0.33 < B < -0.24$. Here, the regulator chooses uniformly higher B' (lower debt) than private agents, because of the effect that internalizing the externality has on the SP’s decisions when the constrained region is near. In fact, private agents hit the credit constraint at $B = -0.32$, while at this B the regulator still retains some borrowing capacity. Moreover, this region is characterized by “financial instability,” in the sense that the values of B' chosen in the DE are high enough so that adverse shocks at $t + 1$ can lead to

²³In [Bianchi \(2011\)](#), there is also a non-monotonicity in the bonds decision rule, but in that model it arises because of how consumption affects the atemporal marginal rate of substitution between tradables and non-tradables and thus the relative price of nontradables, rather than through the effects on a intertemporal marginal rate of substitution on asset prices as in this model.

²⁴Notice that the planner still treats asset prices as endogenous in these states, which could in principle lead it to consume more to raise asset prices and relax the constraint. However, doing this is not feasible because the increase in debt to sustain higher consumption violates the collateral constraint.

a binding credit constraint and a financial crisis. The financial amplification dynamics that can occur when the economy is in this region are discussed in further detail below. We will also show later that this is the region of the state space in which the regulator uses actively its macro-prudential policy.

The stable credit region is the interval for which $B \geq -0.24$. In this region, the probability of a binding constraint next period is zero for both DE and SP. The bond choices of the two still differ, however, because expected marginal utilities differ for under the two equilibria. But here the regulator does not set a tax on debt, because even negative shocks cannot lead to a binding credit constraint in the following period.

The larger debt (i.e. lower B') choices of private agents relative to the regulator, particularly in the positive crisis probability region, provide a measure of the overborrowing effect at work in the competitive equilibrium. The regulator accumulates extra precautionary savings above and beyond what private individuals consider optimal in order to self-insure against the risk of financial crises. This effect is small in terms of the difference between the two bond choices, but as we demonstrate below, this still leads to large differences in financial crises dynamics. Moreover, the event analysis of crisis shown in Figure 1 is also reflecting the large differences in macroeconomic outcomes when the constraint binds despite the small differences in bond choices when it does not.

The long-run probabilities with which the SP (DE) visit the three regions of the policy functions of bonds are 3 (2) percent for the constrained credit region, 86 (88) percent for the positive crisis probability region, and 11 (10) percent for the stable credit region. Both economies spend close to 90 percent of the time in the second region, but the prudential actions of the regulator reduce the probability of entering in the constrained region by a half. This is consistent with the finding mentioned earlier that financial crises are much less frequent in the SP equilibrium than in the unregulated DE.

The significantly nonlinear debt dynamics that result from the financial amplification mechanism, and the SP's ability to weaken them are illustrated in Figure 3. This Figure shows the policy functions for bonds of the DE and SP over the interval $[-0.4, -0.2]$ of their domain for two different triples of \mathbf{s} . One is labeled *positive shock*, which is a state with higher TFP and lower R, and the second is labeled *negative shock*, which has lower TFP and higher R.²⁵ The financial regime is kept at κ^L in both cases. Hence, the plot illustrates amplification dynamics in response to shocks of standard magnitudes to TFP and R and in the absence of changes in κ . The ray from the origin is the stationary choice line, where

²⁵In the positive shock state, TFP is above and R below their means by about 1.3 times their standard deviations, and in the negative shock state R reverts to its mean and TFP falls below its mean also by about 1.3 times its standard deviation.

$B' = B$. We use a narrower range of bond values than in Figure 2 to highlight the differences in policy functions in the region that is relevant for financial amplification.

The amplification dynamics to which the DE is exposed can be illustrated as follows. Assume the economy starts a hypothetical first period at point A , which is the intersection of DE's policy function with positive shock with the 45 degree line. At this point, the choice of B' in the DE is identical to the value of B , hence the economy ends the period with the same amount of bonds it started with. Assume then that the second period arrives and the realizations of TFP and R shocks shift to the negative shock state. The DE starts at point A but now the collateral constraint becomes binding, and the Fisherian deflation dynamics force a sharp, nonlinear upward adjustment of the bond position such that B' increases to point B' , which is an increase of about 1200 basis points (from -0.34 to -0.22).

Compare the above crisis dynamics with what the same experiment produces under the SP equilibrium. The planner starts also at point A , but its policy function for that initial condition indicates a value of B' that reduces its debt slightly below what the DE chooses (the planner's B' is just about 50 basis points higher than what the DE's chooses at point A). This occurs because the regulator builds precautionary savings and borrow less, since it faces the higher marginal cost of borrowing that results from internalizing the effects of the pecuniary externality due the positive probability that the credit constraint may bind in the second period. The second period arrives with the same shift to the negative shock state. The planner starts from point B because of its lower borrowing choice in the first period, and its policy function for the negative shock state indicates that its bond choice will increase to point B' . Hence, the slight difference in initial debt of the SP v. the DE in the second period results in a sharply smaller upward adjustment in B' for the planner (about 200 basis points v. 1200 in the DE). This illustrates the dynamics of the mechanism that results in the significantly smaller magnitude of the crisis episodes in the SP v. the DE show in Figure 1.

The differences in borrowing decisions are also reflected in differences in the long-run distributions of leverage of the DE and SP. Figure 4 shows the cumulative ergodic distributions of leverage ratios (measured as $\frac{-b_{t+1} + \theta p_m m_t}{q_t}$) in the two economies. The stronger precautionary savings motive of the SP results in an ergodic distribution of leverage that concentrates less probability at higher leverage ratios than in the DE. The maximum leverage ratio in both economies is given by κ but notice that the DE equilibrium concentrates higher probabilities at higher levels of leverage. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than 1 percent. Hence, overborrowing may seem a relatively minor problem when comparing unconditional long-run averages of leverage ratios.²⁶ Behind

²⁶Measuring "ex ante" leverage as $\frac{-b_t + \theta p_m m_t}{q_t K}$, we find that leverage ratios in the competitive equilibrium can exceed the maximum of those for the regulator 2.6 percent of the time and by up to 12 percentage points.

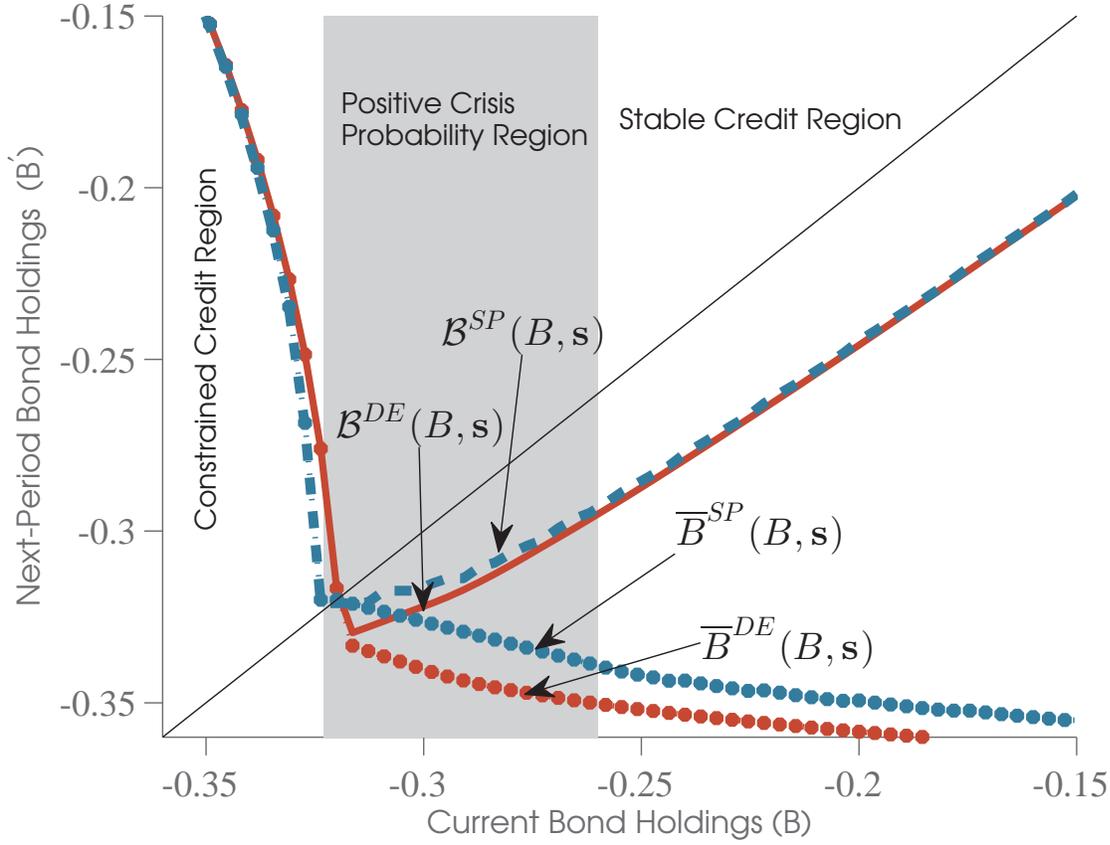


Figure 2: Policy functions for bonds in the decentralized and constrained-efficient equilibria. \bar{B} denotes borrowing limit defined as $-\kappa q'(B, s) - p_m \hat{m}(B, 1, B, s)$

this apparently negligible difference are hiding the large differences in crises probabilities and magnitudes that result from the strongly nonlinear dynamics of financial amplification in the DE. Note also that these same features result in a longer left tail in the DE's distribution of leverage. In the SP equilibrium the leverage ratio is never below 23 percent with positive probability, whereas in the DE leverage ratios near 16 percent are still positive-probability states.

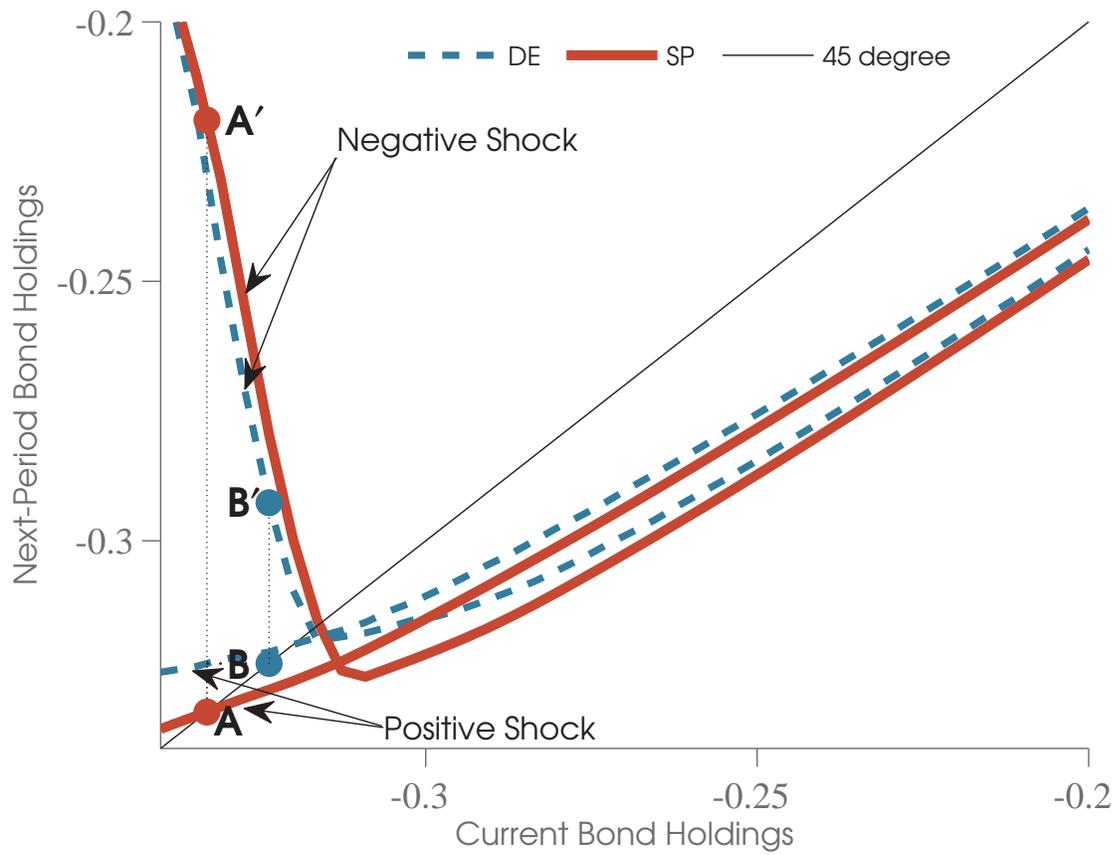


Figure 3: Amplification dynamics in response to adverse shocks.

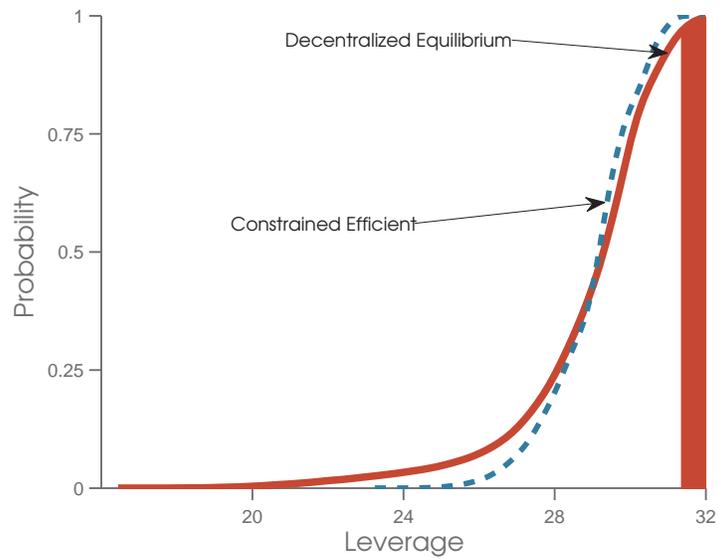


Figure 4: Cumulative Ergodic Distribution of Leverage $\left(\frac{-b_{t+1} + \theta p_m m_t}{q_t K} \right)$

5.4 Asset Pricing

We show that overborrowing has important quantitative implications for asset returns and their determinants.

5.4.1 Conditional Asset Prices

Figure 5 shows plots of six key asset pricing variables as functions of B in the DE and SP economies for the same state of exogenous shocks as the policy functions of Figure ???. The variables plotted are the expected return on assets, the price of assets, the Sharpe ratio, the volatility of returns, the risk premium, and the price of risk.

This figure shows how risk premium, sharpe ratios and return volatility increase with the level of debt, as the risk that an adverse macroeconomic shock triggers the Fisherian deflation mechanism increases. Accordingly, land prices are reduced with higher levels of debt, as future dividends are discounted at a higher rate. These plots provide further evidence of important non-linearities present in the model, now for the compensation for risk taking and the excess returns.

Figure 5 also shows that risk premium, sharpe ratios and return volatility is much higher for the decentralized equilibrium. This reflects the fact that the decentralized economy is more risky than the constrained efficient economy. While this pushes land prices down by reducing excess returns, there is, however, an additional channel that pushes land prices down in the constrained efficient equilibrium, which dominates quantitatively this effect. Because a tax on debt raises the required return on assets, this acts depressing asset prices as shown in Figure 5. The effects of taxes on asset prices will be important when we analyze the effects of fixed taxes below.

5.4.2 Distribution of Returns

Figure 6 shows the long-run distributions of realized equity returns for the competitive equilibrium and the regulator.

A key result from our analysis is that the distribution of equity returns for the competitive equilibrium displays fatter tails. In fact, the 99th percentile of returns is about -30.5 percent, v. -3.6 percent for the regulator. The fatter left tail in the competitive equilibrium corresponds to states in which a negative shocks hit when agents have a relatively high level of debt. Following a similar logic, the fatter right-tail in the distribution of returns of the competitive equilibrium corresponds to periods with positive positive shocks, which were preceded by unusually low asset prices due to fire sales.

We showed above that the fatter tails of the distribution of asset returns, and the associated

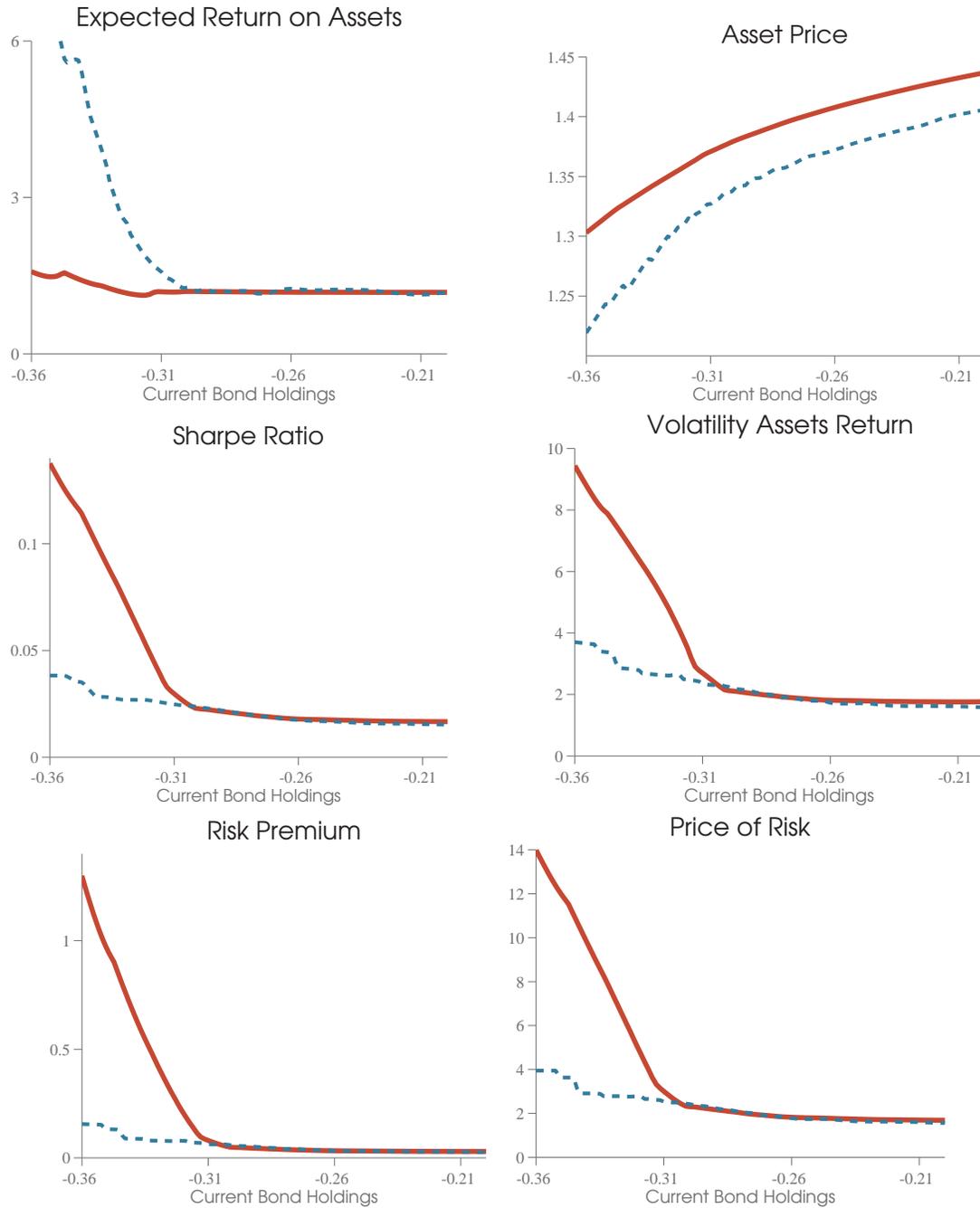


Figure 5: Asset Pricing Variables

time-varying risk of financial crises, have substantial effects on the risk premium. These features of our model are similar to those examined in the literature on asset pricing and “disasters” Note, however, that this literature generally treats financial disasters as resulting from exogenous stochastic processes with fat tails and time-varying volatility, whereas in our

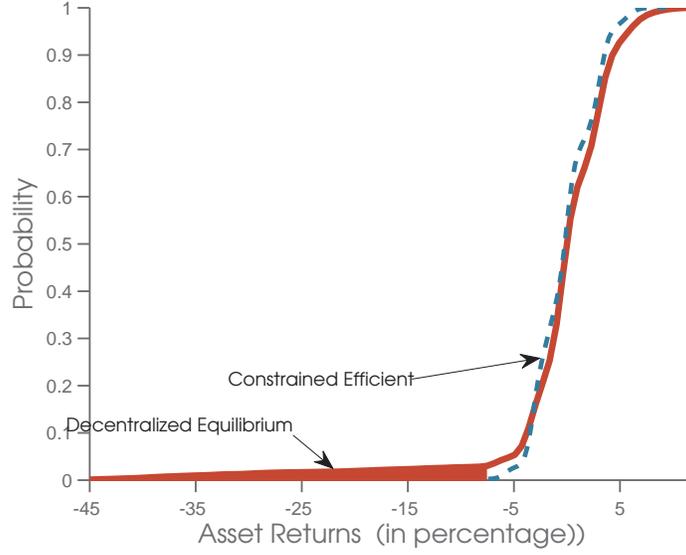


Figure 6: Ergodic Cumulative Distribution of Realized Asset Returns

Table 2: Asset Pricing Moments

	Expected Return	Risk-free Plus Tax	Equity Premium	Collateral Current	Collateral Expected	Risk Premium	s_t	$\sigma_t(R_{t+1}^q)$	S_t
DE	4.320	2.808	1.511	1.391	0.000	0.366	5.840	4.683	0.004
SP	3.959	3.947	0.012	0.13	0.190	0.068	2.583	2.577	-0.000

setup financial crises and their time-varying risk are both endogenous. ²⁷

5.4.3 Unconditional Moments

Table 2 reports statistics that characterize the main properties of asset returns for the regulator the competitive equilibrium. In particular, Table 2 lists expected excess returns, equity premium, first-order effects of the collateral constraint, risk premium, the log standard deviation of returns, i.e., the price of risk, and the Sharpe ratio. These moments are reported for the unconditional long-run distributions of each model economy

²⁷The literature on disasters typically uses Epstein-Zin preferences so as to be able to match the large observed equity premia. Here we use standard CRRA preferences with a risk aversion coefficient of 1.5, and as we show later, we can obtain larger risk premia than in the typical CRRA setup without credit frictions..

5.5 Macro-Prudential Policies

We now study the properties of macroprudential policy by examining the features of the tax on borrowing. In particular, we engage in discussion about the cyclicity of this tax.

Figure 7 shows the tax on borrowing that decentralizes the constrained efficient allocations for different values of bonds and different interest rate and financial shocks. When financial conditions are loose, higher taxes

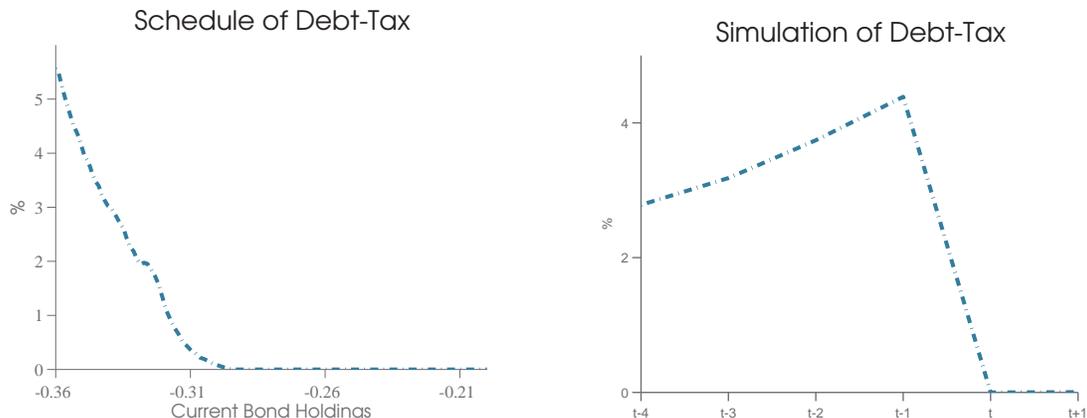


Figure 7: Tax on borrowing for positive and negative financial shocks. The left plot shows the tax on debt for different values of initial bonds and mean values of TFP and interest rate shocks and $\kappa = \kappa^H$. The right plot corresponds to the tax on debt that decentralized the constrained efficient allocations in the states given by the event analysis in section 5.2.

Table 4 shows the statistical moments that characterize the macro-prudential taxes on debt. To make the two comparable, we express the dividend tax as a percent of the price of assets.

The dynamics of the debt taxes around crisis events are shown in the left panel of Figure 7. The debt tax is high relative to its average, at about 2.7 percent, at $t - 4$, $t - 3$, $t - 2$ and $t - 1$, and this again reflects the macro-prudential nature of these taxes: Their goal is to reduce borrowing so as to mitigate the magnitude of the financial crisis if bad shocks occur. At date t the debt tax falls to zero, and it rises again at $t + 1$ and $t + 2$ to about 2 percent. The latter occurs because this close to the crisis the economy still remains financially fragile (i.e. there is still a non-zero probability of agents becoming credit constrained next period).

The unconditional average of the debt tax is 1.07 percent. The debt tax fluctuates about 2/3rds as much as GDP and is positively correlated with leverage, i.e. $\frac{-b_{t+1} + \theta p_m m_t}{q_t K}$. This is consistent with the macro-prudential rationale behind the tax: The tax is high when leverage is building up and low when the economy is deleveraging. Note, however, that since leverage

Table 3: Long Run Moments of Macro-prudential Policies

	Mean	Standard Deviation	Correlation with Leverage
Tax	1.1	1.5	0.81

itself is negatively correlated with GDP, the tax also has a negative GDP correlation. Finally, the tax also has a positive correlation with credit conditions measured by κ , again providing foundations for macroprudential policy.

5.6 Welfare Effects

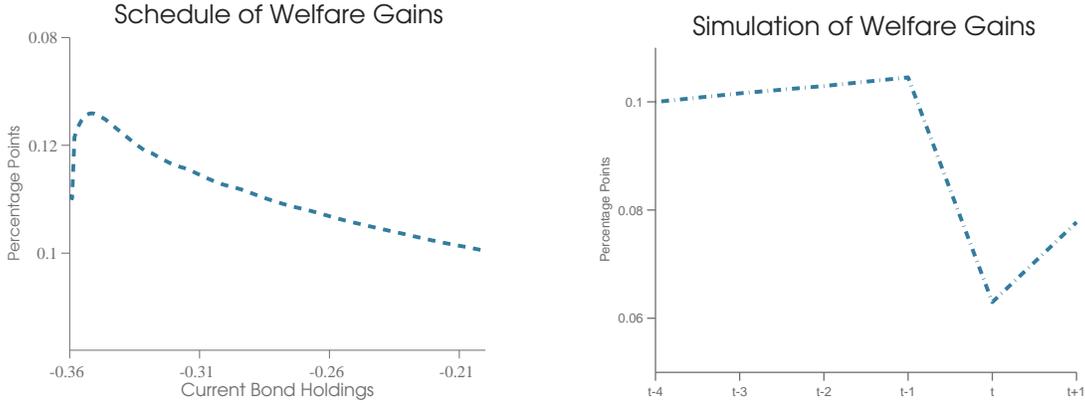


Figure 8: Welfare Gains from Macprudential Policy. The left plot shows the welfare gains for different values of initial bonds and mean values of TFP and interest rate shocks and $\kappa = \kappa^L$. The right plot corresponds to the welfare gains of implementing macroprudential policy in the states given by the event analysis in section x.

We move next to explore the welfare implications of the credit externality. To this end, we calculate welfare costs as compensating consumption variations for each state of nature that make agents indifferent between the allocations of the competitive equilibrium and those attained by the financial regulator. Formally, for a given initial state (B, \mathbf{s}) at date 0, the welfare cost is computed as the value of γ that satisfies this condition:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1 + \gamma_0) - G(n_t^{DE})) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(n_t^{SP})) \quad (36)$$

where the superscript DE denotes allocations in the decentralized competitive equilibrium and the superscript SP denotes the regulator's allocations. Note that these welfare costs

measure also the (negative of) the welfare gains that would be obtained by introducing the regulator's optimal tax on debt.

The welfare losses of the DE arise from two sources. The first source is the higher variability of consumption, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs due to the effect of the credit friction on working capital. Without the working capital constraint, the marginal disutility of labor equals the marginal product of labor. With the working capital constraint, however, the shadow cost of employing labor rises when the constraint binds, and this drives a wedge between the marginal product of labor and its marginal disutility. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss.

Figure 8 in the left panel plots the welfare gains from correcting the fire-sale externality as a function of b for a negative, two-standard-deviations TFP shock. These welfare costs are sharply increasing in b at the high debt levels of the constrained region and some of the borrowing-tax region, and after that become decreasing in b and with a much flatter slope. This pattern is due to the differences in the optimal plans of the regulator vis-a-vis private agents in the decentralized equilibrium. Recall that in the constrained region, the current allocations of the DE essentially coincide with those of the SP, as described in Figure 2. Therefore, in this region the welfare gains from implementing the regulator's allocations only arise from how future allocations will differ. On the other hand, in the borrowing-tax region, the regulator's allocations differ sharply from those of the DE, and this generally enlarges the welfare losses caused by the credit externality. Notice that, since the regulator's allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the regulator, but these are far outweighed by less vulnerability to sharp decreases in future consumption during financial crises. Finally, as the level of debt is decreased further and the economy enters the no-tax region, financial crises are unlikely and the welfare costs of the inefficiency decrease.

Figure 8 in the right panel also shows that gains from correcting the externality peak right before the crisis. The unconditional average welfare cost computed using the DE's ergodic distribution of bonds and exogenous shocks is 0.09 percentage points of permanent consumption.

The fact that welfare losses from the externality are small although the differences in consumption variability are large is related to the well-known Lucas result that models with CRRA utility, trend-stationary income, and no idiosyncratic uncertainty produce low welfare costs from consumption fluctuations. Moreover, although the efficiency loss in the supply-side can be relatively large, these losses occur only when the constraint binds produces, which is

a low probability event.

5.7 Simple Policies

The state-contingent nature of the macro-prudential taxes raises a familiar criticism posed in the context of Ramsey optimal taxation analysis: State-contingent policy schedules are impractical because of the limited flexibility of policy-making institutions to adhere to complex, pre-determined, time-varying rules for adjusting policy instruments.²⁸ In particular, we have shown that the optimal tax experience significant variability oscillating from zero to 6 percent. For this reason, we studied the performance of an alternative regulated decentralized equilibrium in which the policy rules are simple time and state-invariant taxes on debt, with tax rates set equal to their long-run averages under the optimal macro-prudential policy.

We show that the effects of a fixed prudential tax on borrowing has different welfare effects depending on whether the constraint is currently binding. In particular, there are welfare gains when the constraint is not currently binding, whereas there are welfare losses when the constraint is currently binding. The reason for this difference are due to the short-run effects of introducing taxes on borrowing. Introducing a tax on borrowing has a depressing effect on asset prices because the increase in the cost of borrowing shifts demand away from land towards bonds. Notice that there is also a positive effect on asset prices due to the reduction in the riskiness of land, but this effect is dominated by the first-order effect of taxes on the relative demand for bonds.

Figure 9 illustrates the effects of fixed taxes on welfare. We first compute the welfare effects as equation 36. Second, we compute

$$\int \max(\gamma_0, 0) dF(b, \mathbf{s}) \tag{37}$$

where $F(b, \mathbf{s})$ is the ergodic distribution of the economy with zero taxes. The max operator reflects the fact that the policy is implemented only when there are welfare gains. This corresponds to the broken line in Figure 9. The straight line corresponds to the standard average welfare gain calculation using the ergodic distribution of the decentralized equilibrium. The differences reflect that there are welfare losses in some states from introducing fixed taxes. As the right panel shows, this corresponds to states where the constraint is already binding.

The stabilizing effects of fixed taxes are quite powerful. The probability and severity of

²⁸A related criticism is that Ramsey optimal policies are time-inconsistent, but as we explained earlier, the financial regulator does not have commitment.

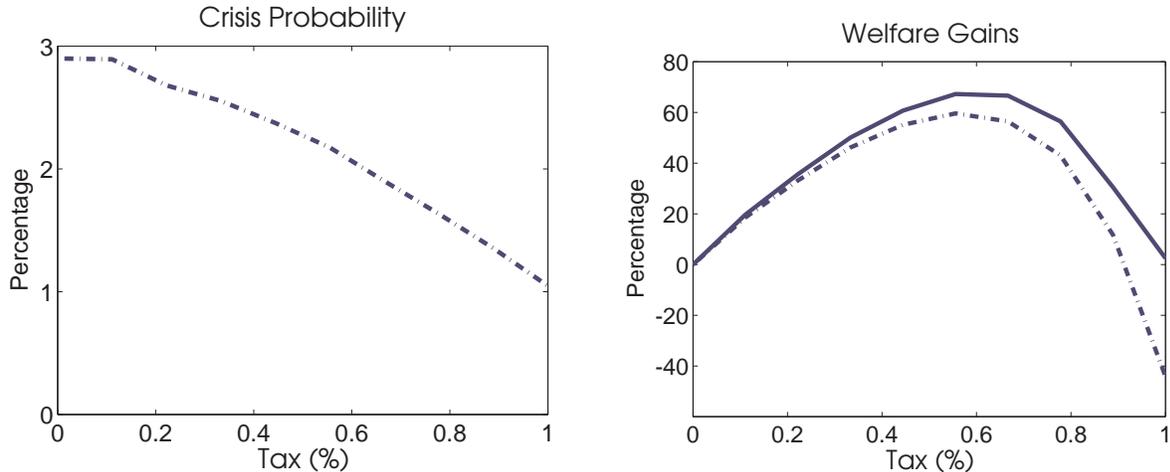


Figure 9: Welfare Effects of Fixed Taxes on Borrowing. The left panel shows the probability of a crisis as a function of the fixed tax on debt. The right panel shows the average welfare gains relative to the optimal state contingent policy.

financial crises still fall sharply relative to the competitive equilibrium, although relatively less than with the fully optimal policy (e.g. asset prices fall 7 percent for the optimal regulator v. 12 percent with fixed taxes, and 30 percent in the decentralized equilibrium). Partial use of this simple policy, by implementing only the fixed tax on debt, is again welfare reducing, and again the intuition is due to the fact that debt taxes have a depressing effect on asset prices, which tightens the collateral constraint in states where the collateral constraint already binds.

The results for the regulator’s optimal policy and the fixed-taxes policy show that, while our results may provide a justification for the use of macro-prudential policies, they also provide a warning. Introducing macro-prudential policy in crisis times can be welfare reducing. In this experiment, this happens because taxes on debt depress asset prices and because the constraint is initially binding, this makes the constraint tighter.²⁹

6 Conclusions

This paper examined the positive and normative effects of a pecuniary externality in a dynamic stochastic general equilibrium model in which a collateral constraint limits access to debt and working capital to a fraction of the market value of an asset in fixed supply. We compared the allocations and welfare attained by private agents in a competitive equilibrium, in which agents face this constraint taking prices as given, with those attained by a constrained-efficient

²⁹This contrasts sharply to the results in Bianchi (2011) where fixed taxes could improve welfare across all states. The stark difference arises because in our model a tax on borrowing at a given state has a direct effect of affecting the tightness of the constraint across other periods.

financial regulator who lacks the ability to commit to future policies. This regulator faces the same borrowing limits as private agents but takes into account how its current borrowing choices affect asset prices and the borrowing decisions of the regulators in future periods.

The regulator's inability to commit is important because it implies that the optimal macro-prudential policy we study is endogenously time-consistent. Under commitment, the regulator would be tempted to promise lower future consumption to prop up current asset prices when the constraint binds, and then renege when the future arrives. Up to now, time-inconsistency had been set aside in the literature on macro-prudential policy by either setting up problems without the forward-looking elements that drive time inconsistency under commitment (which emerge naturally in asset pricing models by their nature), or by imposing constraints on the ability of so-called conditionally efficient financial regulators to alter asset pricing functions of unregulated equilibria (e.g. Bianchi and Mendoza (2010) and Jeanne and Korinek (2010)).

The financial regulator internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs. Moreover, when the constraint binds, the regulator responds to the incentives implied by its inability to commit, thus internalizing how the debt choices of future regulators, and the associated future consumption allocations and asset prices, respond to current borrowing decisions.

We also show that the allocations of the optimal, time-consistent financial regulator can be decentralized by means of a macro-prudential policy in the form of a state-contingent debt tax. The tax is strictly positive in states in which the collateral constraint does not bind but may become binding in the following period. As Bianchi (2011) showed, however, policies like capital requirements or loan to value ratios can be used with similar results as debt taxes.

Our analysis quantifies the effects of the debt deflation process and the pecuniary externality in a setup in which the credit friction has effects on both aggregate demand and supply. On the demand side, consumption drops as access to debt becomes constrained, and this induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and demand for imported inputs are affected by the collateral constraint because firms buy the latter using working capital loans that are limited by the collateral constraint. Hence, when the constraint binds

the effective cost imported inputs rises, so the demand for these inputs and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the Macro/Finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing regulated v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to data for industrial countries. This analysis showed that, even though the externality results in only slightly larger average ratios of debt and leverage to output compared with the regulator's allocations, the externality does produce financial crises that are significantly more severe and more frequent than in the regulated equilibrium, and produces higher long-run business cycle variability. There are also important asset pricing implications. In particular, the externality and its associated higher macroeconomic volatility in the competitive equilibrium produce equity premia, Sharpe ratios, and market price of risk that are much larger than in the regulated economy.

In terms of the macro-prudential debt tax, our results show that the optimal, time-consistent taxes can fully neutralize the externality and increase social welfare. The tax is about 1 percent on average, and positively correlated with leverage. Hence, the tax is higher when the economy is building up leverage and becoming vulnerable to a financial crisis, but before a crisis actually occurs, so as to induce private agents to value more the accumulation of precautionary savings than they do in the competitive equilibrium without regulation. We also examine the implications of policies simpler than the state-contingent optimal tax and find that they can also have significant effects reducing the severity of frequency of crises. In particular, we found this to be the case for a time- and state-invariant tax set at the 1-percent average of the optimal tax policy.

We recognize that despite the positive findings of this paper about the potentially large benefits of macro-prudential policy in terms of the frequency and severity of financial crises, there are also important hurdles. One has to do with the complexity of actual financial markets vis-a-vis the simple structure of the model. In reality, there is a large set of financial constraints affecting borrowers in credit markets at the level of households, nonfinancial firms and financial intermediaries, and the regulator aiming for the optimal policy would thus be faced with severe informational requirements in terms of both coverage and timeliness of leverage and debt positions. A second important hurdle relates to incomplete information and financial innovation. Financial regulators need to be able to tell apart changes in leverage positions due to fluctuations in the credit cycle and pecuniary externalities from those due to deep-structure financial innovation. In [Bianchi, Boz, and Mendoza \(2012\)](#) we examined the

limitations of macro-prudential regulation in an environment in which financial innovation occurs but agents, including the regulator, are imperfectly informed about it.

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A Proofs

A.1 Proof of Proposition 1

Define the tax as:

$$\tau_t = \frac{\beta R \mathbb{E}_t \left\{ -\xi_{t+1} u''(c_{t+1}) q_{t+1} + \xi_t \{ u'(C(b_{t+1}, z_{t+1})) \mathcal{B}_b(\mathcal{Q}(b_{t+1}, z_{t+1}) + z_{t+1}) + \right.}{\beta \mathbb{E}_t u'(C(b_{t+1}, z_{t+1}))} \\ \left. \frac{\mathcal{Q}_b(b_{t+1}, z_{t+1}) u'(C(b_{t+1}, z_{t+1})) \}}{\beta \mathbb{E}_t u'(C(b_{t+1}, z_{t+1}))} \right\} + \mu_t + \xi_t u''(c_t) q_t \quad (38)$$

The constrained efficient equilibrium can be characterized by a collection of sequences $\{c_{t,t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}$ such that (2) (14)-(16) and $k_t = 1$

The regulated decentralized equilibrium is characterized by a sequence $\{c_{t,t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}$ such that (2), (3), (4), (5) and

$$u'(c_t) = \beta R (1 + \tau_t) \mathbb{E}_t [u'(c_{t+1})] + \mu_t \quad (39)$$

together with complementary slackness conditions. Using the expression for the tax (38) and (39), yields condition (15) and identical conditions characterizing the two equilibria.

A.2 Quantitative Model: Construction of the Planner's Problem and Proof of Decentralization

Households in the constrained efficient allocations solve

$$\max_{\{c_t, h_t, n_t, m_t, k_{t+1}\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \\ s.t. \quad c_t + q_t k_{t+1} = k_t q_t + z_t F(k_t, h_t, m_t) + T_t$$

$$\frac{b_{t+1}}{R} - \theta p_m m_t \geq -\kappa_t q_t k_t$$

This yields conditions

$$w_t = G'(n_t) \quad (40)$$

$$z_t F_h(t, h_t, m_t) = w_t \quad (41)$$

$$z_t F_m(t, h_t, m_t) = p_m (1 + \theta \mu / u') \quad (42)$$

$$q_t u'(t) = \beta \mathbb{E}_t \left\{ u'(t+1) (z_{t+1} F_k(k_{t+1}, h_{t+1}, m_{t+1}) + q_{t+1}) + q_{t+1} \kappa_{t+1} \mu_{t+1} \right\} \quad (43)$$

These equations together with market clearing in land ($k_{t+1} = 1$) and labor markets ($h = n$) are the implementability constraints in the planner's problem 34, which we repeat below. For

given future policies for labor \mathcal{H} , consumption C , inputs \mathcal{M} , shadow values μ and land prices Q the planner's problem can be written as:

$$\begin{aligned}
\mathcal{V}(b, \mathbf{s}) &= \max_{c, b', \mu, h, m} u(c - G(h)) + \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} \mathcal{V}(b', \mathbf{s}') & (44) \\
c + \frac{b'}{R} &\leq b + zF(k, h, m) - pm \\
zF_h(1, h, m) &= G'(h) \\
zF_m(1, h, m) &= p_m \left(1 + \frac{\theta \mu}{u'(c - G(h))} \right) \\
\mu \left(\frac{b'}{R} - p_m m + \kappa q \right) &= 0 \\
\frac{b'}{R} - p_m m &\geq -\kappa q \\
qu'(c - G(h)) &= \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} \{ u'(C(b', \mathbf{s}') - G'(\mathcal{H}(b, \mathbf{s}))) (Q(b', \mathbf{s}') + z'F_k(1, \mathcal{H}(b', \mathbf{s}'), \mathcal{M}(b', \mathbf{s}')) \\
&\quad + \kappa' \mu(b', \mathbf{s}') Q(b', \mathbf{s}') \}
\end{aligned}$$

Let v be the multiplier on the complementary slackness condition and μ^* the multiplier on the collateral constraint and η^m and η^h the multipliers on the first order conditions with respect to m and h .³⁰ First-order conditions with respect to c, b, h, q and μ and envelope condition are:

$$\begin{aligned}
c_t : \quad \lambda_t &= u'(t) + \frac{\eta_t^m p_m \theta \mu_t u''(t)}{u'(t)^2} - \xi_t u''(t) q_t \\
b_{t+1} : \quad \lambda_t &= \mu_t^* + \beta RE_t \left\{ V_b(b_{t+1}, s_{t+1}) + \xi_t \left(u''(t+1) C_b(b_{t+1}, \mathbf{s}_{t+1}) \right. \right. \\
&\quad \left. \left(Q_{t+1} + z_{t+1} F_k(t+1) \right) + u'(t+1) \left(Q_b(b_{t+1}, \mathbf{s}_{t+1}) + \right. \right. \\
&\quad \left. \left. z_{t+1} \left[F_h(t+1) H_b(b_{t+1}, s_{t+1}) + F_m(t+1) M_b(b_{t+1}, s_{t+1}) \right] \right) \right\} + \\
&\quad \mu_t \nu_t \\
q_t : \quad \xi_t u'(t) &= \kappa (\mu_t \nu_t + \mu_t^*) \\
h_t : \quad u'(t) G'(h) &= \lambda_t z_t F_h(t, h_t, m_t) + \eta_h [G''(h) - z_t F_{hh}(t, h_t, m_t)] + \eta_m \left[\frac{p_m \theta \mu u''(t)}{u'(t)^2} G'(h) \right. \\
&\quad \left. - z_t F_{mh}(t, h_t, m_t) \right] + q_t u''(t) G'(h) \xi_t \\
m_t : \quad \lambda_t z_t F_m(t, h_t, m_t) &= p_m [\lambda_t + \mu_t^* + \mu_t \nu_t] + \eta^h z_t F_{hm}(t, h_t, m_t) + \eta^m z_t F_{mm}(t, h_t, m_t)
\end{aligned}$$

³⁰Of course, μ_t is different from μ_t^* as the private and social values from relaxing the collateral constraint are different. Note that the relevant one for the asset price is μ_t .

$$\mu_t : \quad v_t \left(\frac{b'}{R} - p_m m + \kappa q \right) = p_m \frac{\theta \mu}{u'(c - G(h))} \eta^m$$

$$V'(b) = \lambda$$

The Euler condition for bonds is the following:

$$\begin{aligned} u'(t) = & \mu_t^* + \beta R \mathbb{E}_t \left\{ u'(t+1) + \frac{\eta_{t+1}^m p_m \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) q_{t+1} + \xi_t \left(u''(t+1) C_b(b_{t+1}, s_{t+1}) \right. \right. \\ & \left. \left. (Q_{t+1} + z_{t+1} F_k(k_{t+1}, h_{t+1}, m_{t+1})) + u'(t+1) (Q_b(b_{t+1}, s_{t+1}) + z_{t+1} [F_h(k_{t+1}, h_{t+1}, m_{t+1}) H_b(b_{t+1}, s_{t+1}) + \right. \right. \right. \\ & \left. \left. \left. F_m(k_{t+1}, h_{t+1}, m_{t+1}) M_b(t+1) \right] \right) \right) \right\} + \mu_t \nu_t - \frac{\eta_t^m p_m \theta \mu_t u''(t)}{u'(t)^2} + \xi_t u''(t) q_t \end{aligned}$$

Following the proof in section (), we can show that the tax that decentralizes the constrained-efficient allocations is given by:

$$\begin{aligned} \tau_t = & \frac{1}{\beta \mathbb{E}_t u'(t+1)} \left[\beta R \mathbb{E}_t \left\{ \frac{\eta_{t+1}^m p_m \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) q_{t+1} + \xi_t \left(u''(t+1) \right. \right. \right. \\ & \left. \left. \left(1 - \frac{B_b(b_{t+1}, s_{t+1})}{R} + z_{t+1} [F_h(k_{t+1}, h_{t+1}, m_{t+1}) H_b(b_{t+1}, s_{t+1}) + F_m(k_{t+1}, h_{t+1}, m_{t+1}) \right. \right. \right. \right. \\ & \left. \left. \left. M_b(b_{t+1}, s_{t+1}) \right] - G'(h_{t+1}) H_b(b_{t+1}, s_{t+1}) - p_m M_b(b_{t+1}, s_{t+1}) \right) (Q_{t+1} + z_{t+1} F_k(t+1)) + u'(t+1) (Q_b(b_{t+1}, \right. \right. \\ & \left. \left. (k_{t+1}, h_{t+1}, m_{t+1}) M_b(t+1) \right) \right) \right) \right\} + \mu_t \nu_t - \frac{\eta_t^m p_m \theta \mu_t u''(t)}{u'(t)^2} + \xi_t u''(t) q_t \left. \right] \end{aligned}$$

When the collateral constraint is not binding, the tax is the following.

$$\tau_t = \frac{\beta R \mathbb{E}_t \left(\frac{\eta_{t+1}^m p_m \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) q_{t+1} \right)}{\beta \mathbb{E}_t u'(t+1)}$$

A.3 Derivation of Collateral Constraint

We provide a derivation of the collateral constraint (26) as an incentive compatibility constraint resulting from a limited enforcement problem. Debt contracts are signed with creditors in a competitive environment. Financial contracts are not exclusive, i.e., agents can always switch to another creditor at any point in time. Households borrow at the beginning of the period, before the asset market open. Within period, households can divert future revenues and avoid any costs from defaulting next period when debt becomes due. At the end of the period, there are no more opportunities for households to divert revenues and repayment of previous bonds is enforced. Financial intermediaries can costlessly monitor diversion activities at time t . If creditors detect the diversion scheme, they can seize a fraction κ_t of the household assets. After defaulting, a household regains access to credit markets instantaneously and repurchases the assets that investors sell in open markets. Given this environment, a household that borrows \tilde{d}_{t+1} and engage in diversion activities gains \tilde{d}_{t+1} and loses a $\kappa_t q_t k_t$.

Formally, let V^R and V^d be the value of repayment and default respectively, and V be the continuation value. If a household borrows \tilde{d} at the beginning of the period and defaults, it gets

$$\begin{aligned}
 V^d(\tilde{d}, b, k, B, \mathbf{s}) &= \max_{b', k', c, n, h} u(c - G(n)) + \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} V(b', k', B', \mathbf{s}') & (45) \\
 \text{s.t.} \quad q(B, \mathbf{s})k' + c + \frac{b'}{R} &= \tilde{d} + q(B, \mathbf{s})k(1 - \kappa) + b + w(B, \mathbf{s})n + [zF(k, h, m) - w(B, \mathbf{s})h - p_m m] \\
 -\frac{b'}{R} + \theta p_m m &\leq \kappa q(B, \mathbf{s})k
 \end{aligned}$$

where the budget constraint reflects the fact that the household regain access to credit markets and can borrow b' and the collateral constraint reflects that agents buy back the assets from investors. If the household does not default, it gets the utility from current consumption plus the continuation value of starting next period with debt b' as stated in ??.

$$\begin{aligned}
 V^r(b, k, B, \mathbf{s}) &= \max_{b', k', c, n, h} u(c - G(n)) + \beta \mathbb{E}_{\mathbf{s}' | \mathbf{s}} V(b', k', B', \mathbf{s}') & (46) \\
 \text{s.t.} \quad q(B, \mathbf{s})k' + c + \frac{b'}{R} &= q(B, \mathbf{s})k + b + w(B, \mathbf{s})n + [zF(k, h, m) - w(B, \mathbf{s})h - p_m m] \\
 -\frac{b'}{R} + \theta p_m m &\leq \kappa q(B, \mathbf{s})k
 \end{aligned}$$

A simple inspection at the budget constraints implies that households repay if and only if $\tilde{d}_{t+1} \leq \kappa_t q_t k_t$

Notice that for the constrained-efficient equilibrium, the derivation of the feasible credit positions is analogous to the case in the decentralized equilibrium. If the planner engages in

diversion, creditors can seize a fraction κ_t of assets in the economy. Moreover, households can buy back the assets at the market price q_t . This implies that the same collateral constraint applies in the constrained-efficient equilibrium in this environment.

B Computational Algorithm

B.1 Numerical Solution Method for Decentralized Equilibrium

The computation of the competitive equilibrium requires solving for functions $\{\mathcal{B}(b, \mathbf{s}), \mathcal{Q}(b, \mathbf{s}), \mathcal{C}(b, \mathbf{s}), \mathcal{M}(b, \mathbf{s}), \mathcal{H}(b, \mathbf{s}), \mu(b, \mathbf{s})\}$ such that:

$$\mathcal{C}(b, \mathbf{s}) + \frac{\mathcal{B}(b, \mathbf{s})}{R} = zF(1, \mathcal{H}(b, \mathbf{s})) + b - p\mathcal{M}(b, \mathbf{s}) \quad (47)$$

$$-\frac{\mathcal{B}(b, \mathbf{s})}{R} + \theta p_m \mathcal{M}(b, \mathbf{s}) \leq \kappa \mathcal{Q}(b, \mathbf{s}) \quad (48)$$

$$u'(\mathcal{C}(b, \mathbf{s}) - G'(\mathcal{H}(b, \mathbf{s}))) = \beta R \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left[u'(\mathcal{C}(\mathcal{B}(b, \mathbf{s}), \mathbf{s}')) - G'(\mathcal{H}(\mathcal{B}(b, \mathbf{s}), \mathbf{s}')) \right] + \mu(b, \mathbf{s}) \quad (49)$$

$$zF_n(1, \mathcal{H}(b, \mathbf{s}), \mathcal{M}(b, \mathbf{s})) = G'(\mathcal{H}(b, \mathbf{s})) \quad (50)$$

$$zF_m(1, \mathcal{H}(b, \mathbf{s}), \mathcal{M}(b, \mathbf{s})) = p_m(1 + \theta\mu(b, \mathbf{s})/u'(\mathcal{C}(b, \mathbf{s}))) \quad (51)$$

$$qu'(c - G(h)) = \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left\{ u'(\mathcal{C}(b', \mathbf{s}')) - G'(\mathcal{H}(b, \mathbf{s})) (\mathcal{Q}(b', \mathbf{s}') + z'F_k(1, \mathcal{H}(b', \mathbf{s}'), \mathcal{M}(b', \mathbf{s}')) + \kappa' \mu(b', \mathbf{s}') \mathcal{Q}(b', \mathbf{s}') \right\}$$

We solve the model using a time iteration algorithm developed by [Coleman \(1990\)](#) modified to address the occasionally binding endogenous constraint. The algorithm follow these steps:

1. Generate a discrete grid for the economy's bond position $G_b = \{b_1, b_2, \dots, b_M\}$ and the shock state space $G_s = \{s_1, s_2, \dots, s_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.
2. Conjecture $\mathcal{B}_k(b, \mathbf{s}), \mathcal{Q}_k(b, \mathbf{s}), \mathcal{C}_k(b, \mathbf{s}), \mathcal{H}_k(b, \mathbf{s}), \mathcal{M}_k(b, \mathbf{s}), \mu_k(b, \mathbf{s})$ at time $K, \forall b \in G_b$ and $\forall \mathbf{s} \in G_s$
3. Set $j = 1$
4. Solve for the values of $\mathcal{B}_{k-j}(b, \mathbf{s}), \mathcal{Q}_{k-j}(b, \mathbf{s}), \mathcal{C}_{k-j}(b, \mathbf{s}), \mathcal{H}_{k-j}(b, \mathbf{s}), \mu_{k-j}(b, \mathbf{s})$ at time $k - j$ using (47)-(52) and $\mathcal{B}_{k-j+1}(b, \mathbf{s}), \mathcal{Q}_{k-j+1}(b, \mathbf{s}), \mathcal{C}_{k-j+1}(b, \mathbf{s}), \mathcal{H}_{k-j+1}(b, \mathbf{s}), \mu_{k-j+1}(b, \mathbf{s}) \forall b \in G_b$ and $\forall \mathbf{s} \in G_s$:

- (a) Assume collateral constraint (48) is not binding. Set $\mu_{k-j}(b, \mathbf{s}) = 0$ and solve for $\mathcal{N}_{k-j}(b, \mathbf{s})$ using (50). Solve for $\mathcal{B}_{k-j}(b, \mathbf{s})$ and $\mathcal{C}_{k-j}(b, \mathbf{s})$ using (47) and(49) and a root finding algorithm.
 - (b) Check whether $-\frac{\mathcal{B}_{k-j}(b, \mathbf{s})}{R} + \theta p_m \mathcal{M}_{k-j}(b, \mathbf{s}) \leq \kappa \mathcal{Q}_{k-j+1}(b, \mathbf{s})$ holds.
 - (c) If constraint is satisfied, move to next grid point.
 - (d) Otherwise, solve for $\mu(b, \mathbf{s}), \mathcal{M}_{k-j}(b, \mathbf{s}), \mathcal{H}_{k-j}(b, \mathbf{s}), \mathcal{B}_{k-j}(b, \mathbf{s})$ using (48, (49) and(50) with equality.
 - (e) Solve for $\mathcal{Q}_{k-j}(b, \mathbf{s})$ using (52)
5. Evaluate convergence. If $\sup_{B, \mathbf{s}} \|x_{k-j}(b, \mathbf{s}) - x_{k-j+1}(b, \mathbf{s})\| < \epsilon$ for $x = \mathcal{B}, \mathcal{C}, \mathcal{Q}, \mu, \mathcal{H}$ we have found the competitive equilibrium. Otherwise, set $x_{k-j}(b, \mathbf{s}) = x_{k-j+1}(b, \mathbf{s})$ and $j \rightsquigarrow j + 1$ and go to step 4.

B.2 Numerical Solution Method for Constrained-Efficient Equilibrium